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A Dynamic Failure Rate Forecasting Model for Service Parts Inventory

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Abstract: This study investigates one of the reverse logistics issues, after-sale repair service for in-warranty products. After-sale repair service is critical to customer service and customer satisfaction. Nonetheless, the uncertainty in the number of defective products returned makes forecasting and inventory planning of service parts difficult, which leads to a backlog of returned defectives or an increase in inventory costs. Based on Bathtub Curve (BTC) theory and Markov Decision Process (MDP), this study develops a dynamic product failure rate forecasting (PFRF) model to enable third-party repair service providers to effectively predict the demand for service parts and, thus, mitigate risk impacts of over- or under-stocking of service parts. A simulation experiment, based on the data collected from a 3C (computer, communication, and consumer electronics) firm, and a sensitivity analysis are conducted to validate the proposed model. The proposed model outperforms other approaches from previous studies. Considering the number of new products launched every year, the model could yield significant inventory cost savings. Managerial and research implications of our findings are presented, with suggestions for future research.

Keywords: green supply chains; reverse logistics; third-party repair service providers; failure rate forecast; service parts; bathtub curve theory; Markov Decision Process

1. Introduction

Growing concern about environmental protection makes reverse logistics more important than ever. This study investigates one of the reverse logistics issues, after-sale repair service, especially for in-warranty products. After-sale service is critical to customer service, since it could turn customer complaints into customer satisfaction, or even prevent customer complaints. Nonetheless, the uncertainty in the number of defective products returned makes forecasting and inventory planning of service parts difficult, causing a backlog of returned defectives (shortage of service parts) or an increase in inventory costs (over stock of service parts) [1]. This problem leads to poor after-sale repair service and customer dissatisfaction. In practice, the forecast and inventory planning of service parts depend on accurate predictions of product failure rates. However, the literature and current business practices erroneously assume that product failure rates are constant [2] or follow a particular statistical distribution [3,4]. This unrealistic assumption could be very costly, causing poor after-sale service and inventory cost increase [2]. Based on Bathtub Curve (BTC) theory and Markov Decision Process (MDP), this study develops a dynamic product failure rate forecasting (PFRF) model to enable third-party repair service providers (3PSPs) to predict demand quantities of service parts and, thus, mitigate risks of over- or under-stocking of service parts.

BTC theory and MDP are adopted to develop the proposed model, since they can be used to describe and predict the pattern of product failures. In particular, BTC describes how product failure in one period may be affected by product failure in the previous period. On the other hand, MDP can revise product failure rates obtained from BTC in each warranty period. The proposed PFRF model is a two-phase approach for managing the service parts inventory of in-warranty products. In practice, there are three patterns of the product failure rate in warranty periods: the early/infant failure rate (the rate is high), normal failure rate (the rate is low), and wear-out/end of life failure rate (the rate is high). In the first period, we apply BTC theory as a foundation to estimate the early failure rate, normal failure rate, and wear-out failure rate of a product, and determine a set of initial product failure rates in each warranty period. In the second period, based on the actual product failure rate (the actual returned defectives rate) of the previous period, we apply MDP to revise/update the product failure rate for the next period. Additional updated supply chain information, including a rolling production plan (sometimes called rolling plan) for the next period, and actual shipments of previous periods from the original equipment manufacturers (OEMs) and original design manufacturers (ODMs), is incorporated to predict the demand for service parts in the next period. Considering the number of new products launched onto the market in some of the high-tech industries (e.g., mobile phone, laptops), this model could improve current service parts inventory management practices with potentially tremendous cost savings.

The remainder of this article is organized as follows. The next section reviews the literature related to product failure, service parts, BTC, and MDP applications, followed by the development of a product failure rate forecasting (PFRF) model. A simulation, based on the data collected from a 3C (computer, communication, and consumer electronics) firm in China, is then conducted to validate the proposed model. We conclude with a discussion of the managerial and research implications of our findings, as well as research limitations.

2. Literature Review

Closed-loop supply chains integrate forward and reverse logistics. The concept emphasizes product returns after sales and the activities of capturing return value through reuse, remanufacturing, and recycling [1]. Over the last decade, many studies have verified the benefits related to reverse logistics and developed numerous approaches to manage various after-sales activities [2–10]. This study examines a specific reverse logistics program, after-sales product returns, and services. This section reviews the nature of the problem and relevant literature.

2.1. Current Business Practices and Issues

After-sale activities can generate significant revenues for industrial suppliers [11]. However, most brands have more than one manufacturer and supplier, making it challenging to manage after-sale activities. SONY, for example, sources from Compel, Quanta, and Inventec, among others. Since individual manufacturers have their own product designs involving confidentiality, manufacturers have to build their own repair lines or find local partners to handle return products, which increases their overhead costs. Specifically, in a repair line, defectives are received, diagnosed, disassembled, repaired, re-assembled, and tested. Operations required in repair lines can be very different from those in regular assembly lines. Manufacturers usually cannot leverage their existing facilities to perform the repair operation. Figure 1 displays a closed-loop supply chain with product returns. Returned products from customers are consolidated in the warehouse before they are sent to the repair center for inspection and disassembly. Dismantled parts are then sent for remanufacturing or to the secondary market as spare parts [12].

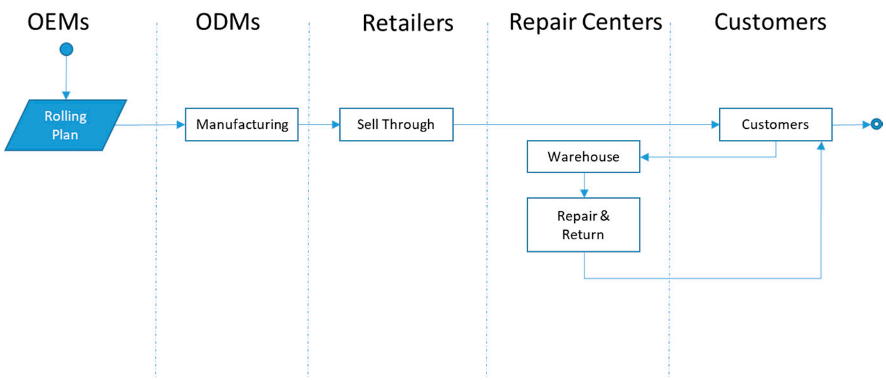

Figure 1. A Closed-Loop Supply Chain with Product Return.

In the current practice, OEMs send the rolling plan to ODMs for product assembly without product failure information. ODMs develop the product (parts) failure rate for production planning, to be utilized for after-sale services. The product failure rate, however, is not transparent to OEMs and it could distort OEMs' service part orders since the return rate is not constant and the number of returned defectives for each month is unknown. Most of the time, this product return and repair process is not considered a core competence of manufacturers. Therefore, manufacturers frequently outsource product repair to local third-party service providers (3PSPs). Unfortunately, third-party service partners have their own reporting systems, and to consolidate reports from different manufacturers becomes extremely challenging. From the 3PSPs' perspective, forecasting product returns is extremely difficult, if not impossible, due to the uncertainty related to the timing and quantity of returned products [13]. As third-party service partners fail to plan their stock and order service parts properly and in a timely manner, after-sale service deteriorates. There are very few studies examining the issue of forecasting product returns [14,15]. Hsueh [3] suggests fixed/constant demand rates and fixed return rates, both with a normal distribution, during different product lifecycle stages. The proposed PFRF model focuses on the forecasting and planning problems that 3PSPs currently face, and a decision model is developed to improve their repair operations.

In addition to forecasting product failures and returns, information sharing among various supply chain parties is also critical to after-sale service. Supply chain information sharing can shorten the lead time and reduce inventory [16,17]. In the case of a product return process, the common industry practice is to use a fixed failure rate for service parts for returned products [18]. Lin and Chen [2] propose a "3PSP" solution, where a 3PSP, as an authorized service provider (ASP), not only processes the returned defectives, but also orders new service parts to fulfill the turnaround time [19]. Ma et al. [20] find that it is beneficial for an OEM to license ASPs to remanufacture, while Yan et al. [21] suggest that OEMs conducting take-back operations can improve the overall remanufacture operations. Usually, a 3PSP would order new service parts from ODMs according to the rolling plan from OEMs. However, the revised return rates are used without any adjustment for return rate fluctuations. The return rate is called epidemic if it goes higher than expected. A low return rate could indicate that few products were sold in the market and the products might be terminated soon [18]. The proposed model includes necessary adjustments to revise failure rates, for more accurate and effective service parts inventory management.

2.2. BTC

A BTC depicts the pattern of failures during a product's life time. There are three stages of product failures: early failure, normal failure, and wear-out failure (Figure 2). Normal failure falls between early and wear-out failure, and the failure rate during this time is usually constant, with a lower rate [22]. The product warranty period falls within the 'normal life' stage of the BTC [23].

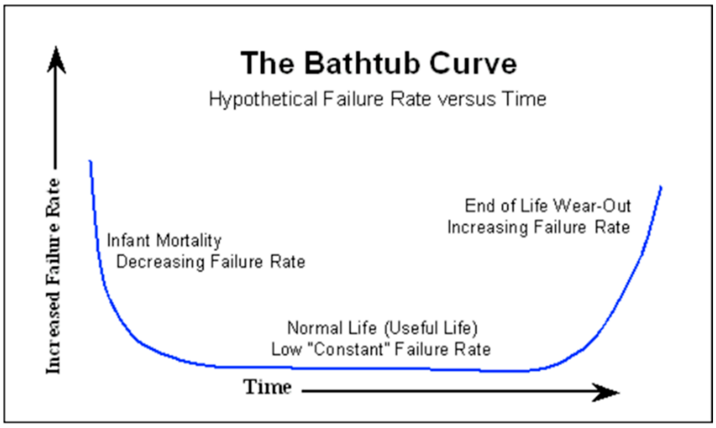

Figure 2. A Typical Life Cycle Bathtub Curve (Sourced from "Reliability and MTBF Overview", Victor Reliability Engineering).

The term "regression" was originally introduced by Francis Galton in the nineteenth century to describe the biological phenomenon of the heights of descendants of tall ancestors tending to regress toward a more normal average (a phenomenon now known as regression toward the mean) [24,25]. In this research, a historical product annual failure rate (AFR) is used as the basis of the failure/return rate. Since AFR is a static concept, we thus combine the concepts of regression and MDP with the 'constant failure rate' of the BTC, in order to generate a dynamic failure rate for the product warranty period to reflect the real situation. The initial failure rate (IFR) is derived from AFR, and is then used for service parts preparation when a product is launched. After that, failure rates will be revised through the application of MDP. IFR is likely to be higher than revised failure rates because it falls at the end of the early failure rate.

2.3. MDP

The adjustment of the failure rate can be guided by a discrete-time Markov Decision Process (MDP). Introduced in the 1950s [17], MDP is the model for sequential decision-making when outcomes are uncertain. The key components in the sequential decision model include a set of states of the systems, a set of decisions and their corresponding actions, rewards/costs, and transition probabilities, which are dependent on the state and the action.

The product warranty period falls within the normal life of the product BTC. During the product warranty period, product rolling plans and actual shipment become the bases of service parts planning, and the returned defectives are another factor added to predict the failure rate for the next service planning stage. The components required in MDP applications are presented in the following.

(a) Decision epochs and periods

Decisions are made at points of time referred to as decision epochs. Let T denote the set of decision epochs. In discrete time problems, time is divided into periods or stages.

(b) State and action sets

At each decision epoch, the system occupies a state S. If, at some decision epoch, the decision-maker observes the system in state $s \in S$, they may choose action a from the set of allowable actions in state s, A_s. Let $A = \cup_{s \in S} A_s$.

(c) Reward and transition Probabilities

As a result of choosing action $a \in A_s$, in state s at epoch t,

(1) The decision-maker receives a reward, $r_t(s, a)$ and

(2) The system state at the next decision epoch is determined by the probability distribution $p_t(\cdot \mid s, a)$.

When the reward depends on the state of the system at the next decision epoch, we let $r_t(s, a, j)$ denote the value at time t of the reward received when the state of the system at decision epoch t is s, action $a \in A$, is selected, and the system occupies state j at decision epoch t + 1. Its expected value at decision epoch t may be evaluated by computing:

$$r_t(s,\ a) = \sum\nolimits_{j \in S} r_t(s,\ a,\ j) p_t(j|s,\ a) \tag{1}$$

The function $p_t(j \mid s, a)$, denotes the probability that the system is in state $j \in S$ at time t + 1. When the decision-maker chooses action $a \in A$, in state s at time t, it is called a transition probability function.

We usually assume that:

$$\sum_{j \in S} p_t(j|s,\ a) = 1 \tag{2}$$

We refer to the collection of objects as a *Markov decision process*.

$$\{T,\ S,\ A_s\ p_t(\cdot|s,\ a),\ r_t(s,\ a)\} \tag{3}$$

(d) Decision rules

A *decision rule* prescribes a procedure for action selection in each state at a specified decision epoch. Decision rules are functions d_t: $S \to A_s$, which specify the action choice when the system occupies state s at decision epoch t. For each $s \in S$, $d_t(s) \in A_s$. This decision rule is said to be Markovian (memoryless) because it depends on previous system states and actions, only through the current state of the system, and *deterministic* because it chooses an action with certainty.

We classify decision rules as history-dependent and randomized (HR), history-dependent and deterministic (HD), Markovian and randomized (MR), or Markovian and deterministic (MD), depending on their degree of dependence on past information and on their method of action selection. We denote the set of decision rules at time t by ${D_t}^K$, where K designates a class of decision rules K = (HR, HD, MR, MD); ${D_t}^K$ is called a decision rule set.

(e) Policies/Plan

A *policy*, *contingency plan*, or *strategy* specifies the decision rule to be used in all decision epochs. A policy π is a sequence of decision rules, i.e., $\pi = (d_1, d_2, \ldots\ d_{N-1})$ when $d_t \in {D_t}^K$ for $t = 1, 2, \ldots$, $N - 1$ for $N \leqq \infty$, in which K represents any of the above classes.

In summary, this research constructs a service part inventory control model with information-sharing in warranty services. Information includes a rolling plan, shipping quantities, and historical product or parts failure rates. Specifically, we utilize both MDP and BTC to estimate the failure rates and to plan the service parts supply during the product warranty period with information sharing among OEMs, ODMs, and third party service providers.

3. Research Framework

This section develops the dynamic product failure rate forecasting (PFRF) model, taking advantage of information sharing among OEMs, ODMs, and 3PSPs. Based on the shared information, the model is able to employ BTC and MDP to update and improve the forecasting of the failure rate.

3.1. Closed-Loop Supply Chain with Information-Shared Service Parts Planning

Information shared among ODMs, OEMs, and 3PSPs can reduce the inventory cost [2,16]. Shared information includes rolling plans (or new product introduction (NPI) plan), product (parts) failure rates, product warranty periods, and shipment. Most of the time, the actual production that ODMs make would be different from the NPI plan. Information on actual shipment deliveries from ODMs to

3PSPs would enable 3PSPs to plan for service parts production. Since 3PSPs will also have the latest product (parts) failure rates, they can prepare service part orders more accurately. Figure 3 shows the flows of the relevant information, with this research focusing on service part supply during the warranty period.

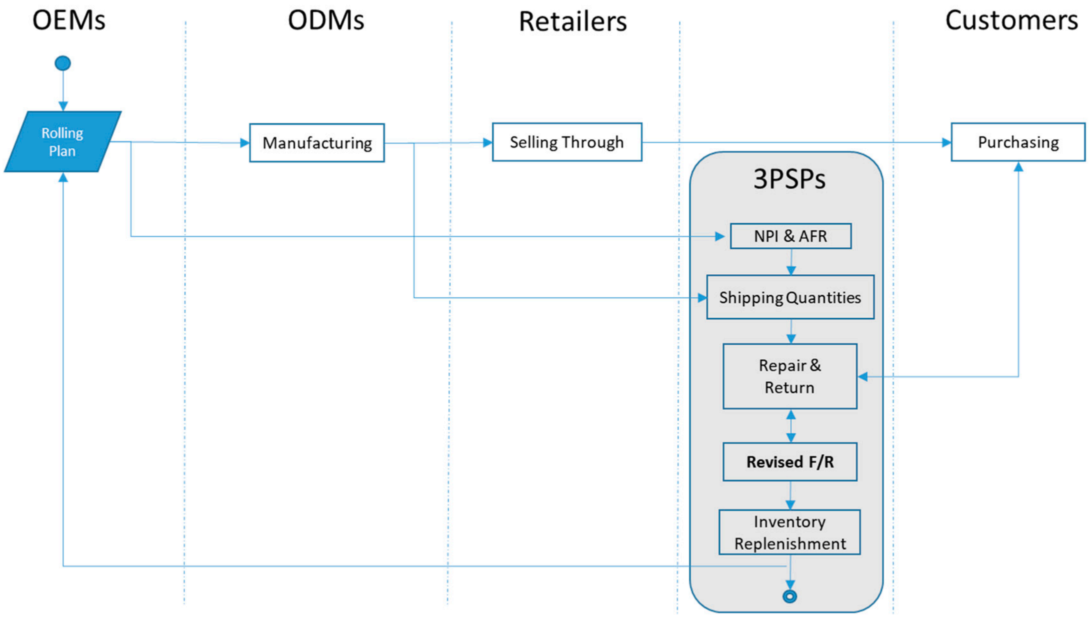

Figure 3. Proposed Information Sharing.

First, OEMs develop the product rolling/NPI plan, passing it on to ODMs for manufacturing. OEMs then share the NPI and annual failure rate (AFR) information with 3PSPs for service parts preparation. After products are delivered to retailers, ODM reveals shipment information to 3PSPs for revising service parts preparation. Retailers sell products to customers, and customers return to 3PSPs when defectives are found during the warranty period.

Our proposed model suggests that 3PSPs prepare initial service parts production at the same time that the product is launched onto the market. Meanwhile, 3PSPs collect product return and shipment information to revise failure rates; moreover, they will use an NPI plan to forecast the service parts demand. This process will continue until the end of the product warranty period. After that, 3PSPs send product failure information to OEMs and ODMs to improve new product planning for the next period. Furthermore, AFR is divided into monthly-based IFR and the failure rate base line (or normal failure rate). IFR is used for the first three-month service parts planning, together with the rolling plan. The initial service parts are thus sent to the service inventory of repair service centers. After that, the real shipment and returned quantities are collected, enabling calculation of the temporary failure rate for the next period of inventory planning. Some of the defectives can be repaired and become serviceable parts later.

Overall, we believe that an accurate product failure rate during the warranty period is key to the success of service parts inventory planning. Most studies are based on static failure/return rates. Irrespective of the methodologies used, the assumption of a constant rate may result in large demand distortion after several planning periods. The proposed PRFR model develops a more accurate failure/return rate to improve the service parts inventory replenishment decision.

3.2. Model Development

Based on the information sharing among ODMs, OEMs, and 3PSPs, the proposed PFRF model derives a dynamic failure rate by the MDP and an initial failure rate by the BTC. Table 1 lists the notations used in the mathematical model, and Figure 4 displays the research framework and algorithm of the proposed model (for detail explanation of the algorithm, refer to Appendixs A and B).

Table 1. Mathematical Notation.

Parameters	
Notation	**Descriptions**
AFR	Annual failure rate
NPI_i	NPI quantities in planning period i
rp	NPI period
IFR_i	Initial failure rate for every month i, derived from initial annual failure rate by Bathtub Curve Theory of previous NPI
IRP	Quantities of initial repair service parts
FR_i	Actual failure rate of Bathtub Curve in month i
$MDPfr_i$	Revised failure rate of Bathtub Curve in month i
qs_i	Shipping quantities in period i
$qr_{j,i}$	Returned defective quantities of NPI jth period in the end of planning period i
Q_i	Purchased quantities service parts in planning period i
IL_i	Inventory level in the beginning of planning period i
Input Variables	
qr_{ij}	Returned defective quantities of NPI jth period in the end of planning period i
Function	
Convert_AFR_to_IFR_by_BathtubCurve: A function that converts annual failure rate into product life time failure rate first, then distributes failure rates to every month by its MTC	

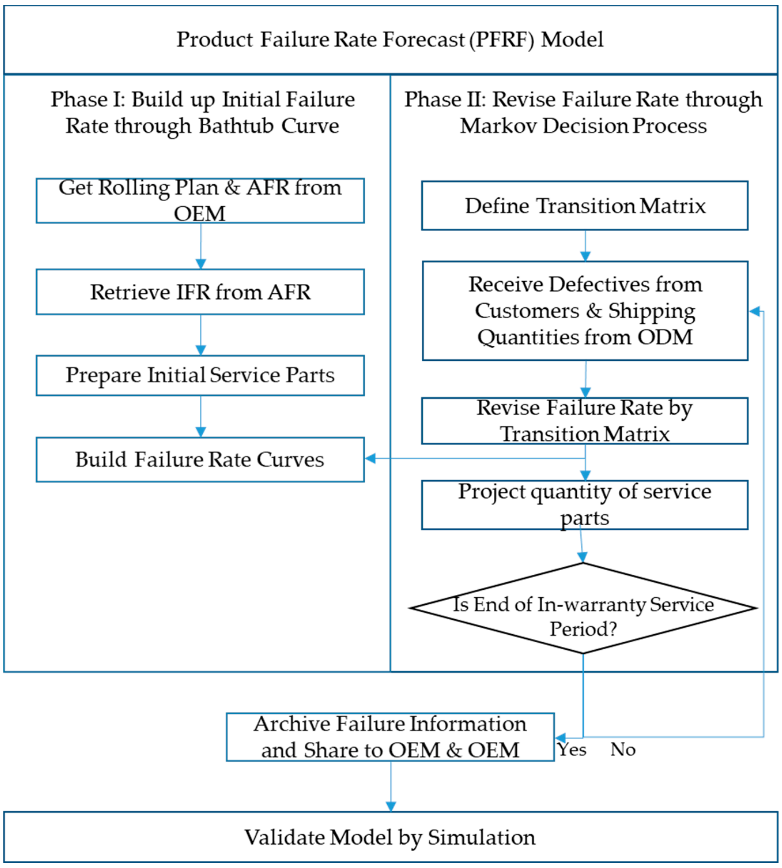

Figure 4. The Product Failure Rate Forecasting (PFRF) Model.

A rolling plan is an array of planned shipments to ODMs from OEMs. This plan is given to ODM and 3PSP for manufacturing and service parts preparation, respectively. AFR from OEM can be divided into monthly-based IFR. IFR is used together with NPI, which means the service parts must be ready when new products are launched in the market.

Phase 1: Develop Initial Failure Rate Using BTC

(1) Retrieve rolling plan and AFR from OEM

The original BTC is selected by any artificial intelligence system or expert system with a case-based reasoning mechanism [18]. The selected BTC with historical failure information has the attributes most similar to the NPI product.

(2) Retrieve IFR from AFR

$$IFR_i = Convert\_AFR\_to\_IFR\_by\_BathtubCurve(AFR) \tag{4}$$

Convert_AFR_to_IFR_by_BathtubCurve is a function that converts the *AFR* into *IFR*. *IFR* is not a constant since it contains the early failure rates and normal failure rates. It may not be equal every month. The beginning of *IFR* will be higher because of product immaturity.

(3) Prepare Initial Service Parts & build BTC

Each NPI iteration forms its own BTC. Therefore, the failure rate of the ith month can be re-calculated after the NPI is launched.

Consistent with the industry practice, the first batch of three-month service parts is pushed to 3PSP.

$$IRP = IL_1 = \sum_{i=1}^{3} IFR_i * NPI_i \tag{5}$$

IRP is used when a new product is launched in the market, which means that service parts should and must be ready when a new product is launched.

At the beginning of the NPI, the inventory level of service parts equals *IRP*. The inventory level at the beginning of planning period 2 is the inventory level of planning period 1 minus the total returned defectives in this period.

$$IL_2 = IL_1 - q_{11} \tag{6}$$

The inventory level at the beginning of period 3 equals:

$$IL_3 = IL_2 - qr_{12} - qr_{21} \tag{7}$$

qr_{12} denotes the returned defectives, which was the second return of the first batch delivered to the market, while qr_{21} denotes the first return of the second NPI launch. Two situations must be considered at this moment:

1. Since the actual shipped quantity is not always equal to the planned quantity, inaccuracy occurs in service parts planning.
2. The new purchase quantity should arrive at service providers at the beginning of each planning period after period 4. The delivery lead time, for simple modeling, is set to be equal to one planning period. Therefore, the purchasing activity will begin at the end of each planning period after Phase two.

Phase 2: Revise failure rate using the MDP

At this point, two-months of data are available for revising the failure rate for every BTC period. A failure rate is provided by the transition probability matrix, and the decision relating to the order of new service parts is subsequently made.

(1) Define Transition Matrix

The actual failure rate in each planning period i is calculated as the sum of every ith returned defective divided by all of its shipped quantities.

$$FR_j = \frac{\sum_{b=1}^{i} qr_{b,\,j}}{\sum_{a=1}^{j} qs_a} \tag{8}$$

However, the actual failure rate from this period may not be not used for the next planning period. Instead, the proposed model adopts the concept of "regression" for adjustment: if f_t is greater than or equal to the upper bound of the initial failure rate f_{UP}, the failure rate for the next planning period is set as the upper bound. Conversely, if f_t is less than or equal to the lower bound of the initial failure rate f_{LB}, then f_{t+1} equals the lower bound. Otherwise, f_t is planned for the next period.

$$f_{t+1} = \begin{cases} f_{LB},\ if\ f_t \leq f_{LB} \\ f_{UB},\ if\ f_t \geq f_{UB} \\ f_t; otherwise \end{cases} \tag{9}$$

(2) Revise Failure Rate

The failure rate for the next planning period will be adjusted by the transition matrix (Equation (10)). The base of market demand is the shipment plus the NPI planned quantities for the next period, which is considered in the MDP probability transition matrix.

$$MDPfr_{tmp\_j} = TP(FR_{tmp\_j}) \tag{10}$$

(3) Project Quantity of Service Parts

The total service-parts demand estimated for the next planning period will be the total quantities in each BTC period multiplied by the revised MDP failure rate.

$$pQty_{k+1} = pQty_{k+1} + \sum_{x=tmp_{j+1}}^{rp} qs_x * MFPfr_{tmp\_j} \tag{11}$$

The decision variable, Q_i, will be the number described above minus all previously shipped service-parts.

$$Q_{J+1} = pQty_{j+1} - Q_J \tag{12}$$

The MDP decision space relates to the decision of how many service parts should be purchased at the end of each planning period, which is the MDP state. The MDP reward function is defined as the expected purchasing cost per period.

The process will be iterated to the end of the last warranty period. The complete algorithms are shown in Appendixs A and B.

4. Simulation Experiment

In this section, an experiment is conducted with real data from a 3PSP. Three models are tested and compared. Model 1 represents the current industry practice [3], using fixed return rates and fixed demand rates for service parts preparation. It calculates the differences of the two rates to determine the inventory level. Model 2 is a more advanced approach offered by the extant literature (Lin and Chen, 2014, [2], where the demand and rates are updated every period for future planning. The proposed PFRF model, Model 3, extends Model 2, with the establishment and implementation of upper bound and lower bound return rates. We expect the proposed model to outperform the other two models regarding the total inventory cost, including holding, purchasing, and stock-out.

4.1. Initial Conditions

The NPI plan used in the simulation (Table 2) is from a 3PSP, a 3C (computer, communication, and consumer electronics) company in China. Table 2 presents an eight-month NPI plan with a nine-month warranty, which means there are 17 planning periods for service parts. The NPI plan is generated by the computer brand OEM, and the actual shipment is provided by a manufacturer ODM. Those two numbers are different, suggesting that the actual shipment is different from the plan. Products are sold in China, and in-warranty defectives are collected and shipped to a 3PSP in Shanghai. Figure 5 displays the NPI plan, together with the numbers during in-warranty period.

Table 2. Example of an NPI Plan.

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8
NPI (Rolling plan)	22,985	45,743	47,987	25,976	73,860	39,753	38,294	6005
Shipment	22,838	45,200	46,907	27,600	74,000	41,000	37,025	5000

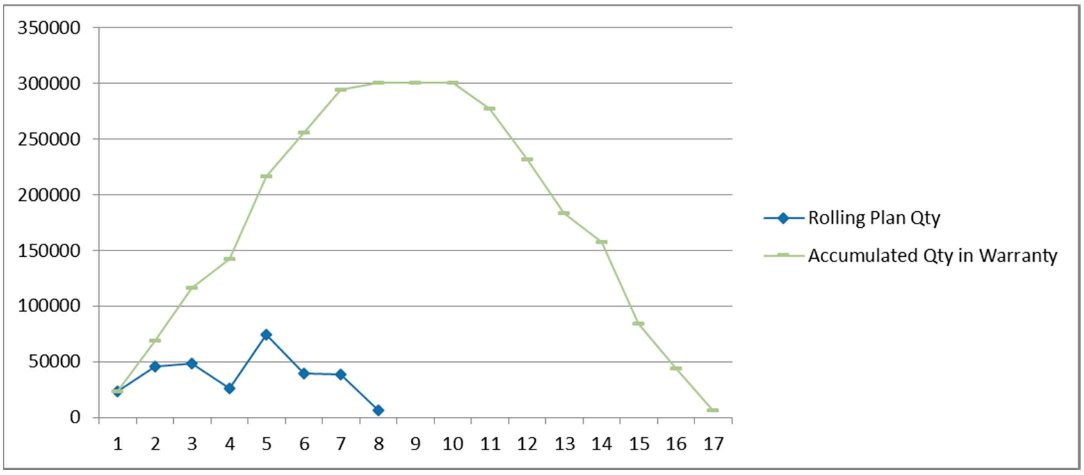

Figure 5. Examples of Quantities of NPI and Products under Warranty.

Table 3 presents the returned defectives during the warranty period. The data are used to evaluate the cost performance of three models in the simulation: (1) Model 1, a baseline model with constant failure rates; (2) Model 2, failure rates are updated every period; (3) Model 3, the proposed approach with the regression of failure rate.

For the sake of fair comparison, safety stock policy is not realized and minimum order quantity (MOQ) is eliminated in the simulation. Model 1 represents the current industry practice, assuming a fixed failure rate for service parts replenishment policy, as presented by Hsueh [3,18]. Model 2 implements Lin and Chen's approach [2], and the failure rates are re-calculated at every period and used for the next planning period. Model 3 is proposed by this research, and failure rates are adjusted and calculated by MDP. In other words, Model 1 implements the fixed rates of demand and return. hBoth Model 2 and Model 3 implement the dynamic failure rate; however, Model 3 applies the regression of failure rate. The inventory will be replenished at the beginning of each stage, for example, 1/S, and be deducted at the end of each stage, as in 1/E.

Table 3. Returned Quantities.

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10	Period 11	Period 12	Period 13	Period 14	Period 15	Period 16	Period 17
Actual Returned Qty	1195	4018	2309	3141	5550	5552	6407	5992	6227	7025	6419	5038	4348	3569	1922	701	73

The initial stocks for the three models are the same. Following industry practice, all models use failure rates of 0.06 and 0.05 for the first two planning periods. Model 1 assumes a constant rate of 0.02 for the remaining planning periods, which is used as the base line in our analysis. Model 2 and Model 3 use failure rates of 0.06 and 0.05 for the initial two-period service parts planning, and re-calculate the failure rate at the end of every planning period. Model 3 uses 0.02 for the base failure rate, with plus/minus 25% of base failure rate as its upper bound and lower bound, respectively. Purchasing occurs at the end of the planning period, and the inventory will be fulfilled at the beginning of the next planning period. The first re-calculation of the failure rate is made at the end of the second planning period, using two-months of collected data. Model 2 revises the failure rate at the end of every period, and Model 3 (the proposed PFRF model) adjusts the revised failure rate by means of MDP.

4.2. Results

The inventory level and purchasing quantities of the three models are shown in Table 4 and Figure 6. Both Table 4 and Figure 6 display the inventory level at every planning period. Every period is divided into two separate segments: start of the period (for example, 1/S) and end of the period (for example, 2/E). As shown, stock-out happens in all three models.

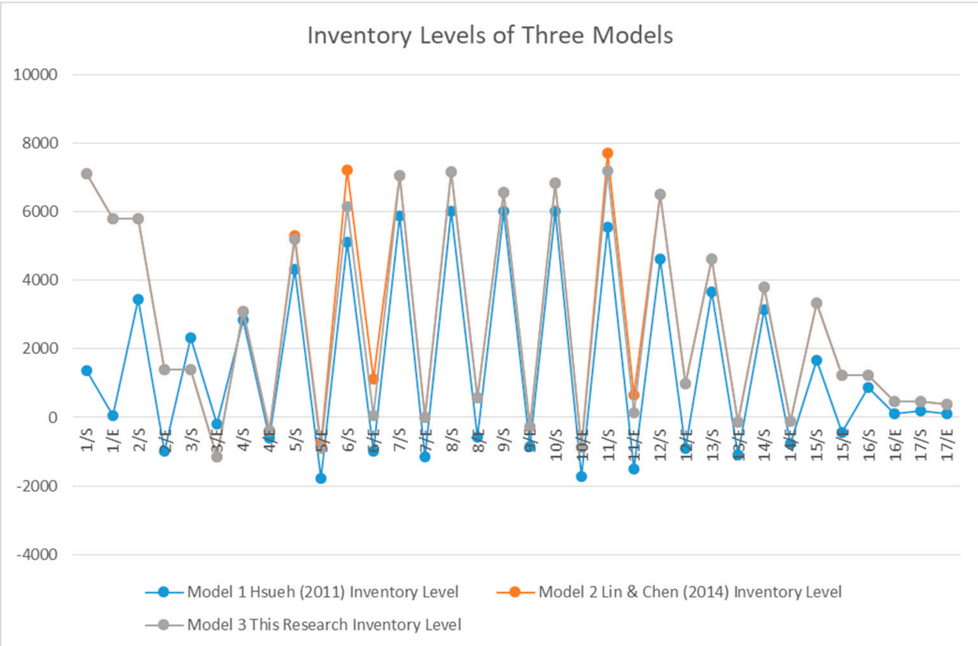

Figure 6. Inventory Levels of Three Models.

Table 4. Inventory Level and Purchase Quantities of Three Models.

(a)

		Situation	1/S	1/E	2/S	2/E	3/S	3/E	4/S	4/E	5/S	5/E	6/S	6/E	7/S	7/E	8/S	8/E	9/S	9/E
Model 1	**Hsueh (2011)**	**Inventory Level**	1379	65	3436	−983	2334	−205	2853	−602	4331	−1774	5126	−982	5891	−1157	6012	−579	6012	−837
		Purchasing Quantities	1379	3371		3317		3058		4933		6900		6873		7169		6591		6849
Model 2	**Lin & Chen (2014)**	**Inventory Level**	7128	5814	5814	1395	1395	−1144	3112	−343	5245	−860	7225	1117	7016	−32	7191	600	6591	−258
		Purchasing Quantities	7128					4256		5588		8085		5899		7223		5991		7107
Model 3	**PFRF Model**	**Inventory Level**	7128	5814	5814	1395	1395	−1144	3112	−343	5245	−860	6407	299	7016	−32	7191	600	6591	−258
		Purchasing Quantities	7128					4256		5588		7267		6717		7223		5991		7107

(b)

		Situation	9/S	9/E	10/S	10/E	11/S	11/E	12/S	12/E	13/S	13/E	14/S	14/E	15/S	15/E	16/S	16/E	17/S	17/E
Model 1	**Hsueh (2011)**	**Inventory Level**	6012	−837	6012	−1715	5552	−1508	4637	−904	3677	−1105	3158	−767	−1681	−433	885	114	184	104
		Purchasing Quantities		6849		7267		6145		4581		4263		2448		1318				
Model 2	**Lin & Chen (2014)**	**Inventory Level**	6591	−258	6849	−878	7727	667	6521	980	4635	−147	3813	−112	3338	1224	1224	453	453	373
		Purchasing Quantities		7107		8605		5854		3655		3960		3450						
Model 3	**PFRF Model**	**Inventory Level**	6591	−258	6849	−878	7489	429	6521	980	4635	−147	3813	−112	3338	1224	1224	453	453	373
		Purchasing Quantities		7107		8367		6092		3655		3960		3450						

4.3. Comparison of the Three Models

The ratio of holding cost, purchasing cost, and stock-out cost is defined as 1:2:3 for a unit. The ratio could vary due to different costs of service parts. Table 5 shows the holding cost, purchasing cost, stock-out cost, and total cost from one service part of a particular product, in one batch of production. Depending on the type of product, there could be as many as thousands of batches over the entire warranty period. Therefore, in practice, the actual cost differences among the three models could be much more significant than those displayed in Table 5.

Table 5. Cost Comparison.

	Holding Cost	**Purchasing Cost**	**Stock-Out Cost**	**Total Cost**
Model 1 Hsueh (2011)	238	152,924	40,653	193,815
Model 2 Lin & Chen (2014)	12,623	153,602	11,322	177,547
Model 3 PFRF Model	11,567	153,602	11,322	176,491

Model 1 has the lowest holding cost and purchasing cost and the highest stock-out and total cost, suggesting under-predication/purchase of the service parts. Models 2 and 3 have the same purchasing and stock-out costs, while Model 3 provides a lower holding cost.

Table 6 displays the failure rates. In Model 2 and Model 3, the inventory level changes dramatically when the failure rates go up and down from period 2 to period 4. The failure rates in model 2 are updated in every period, based on the actual rate in the most recent period. The "normal failure" rate used in the simulation is 0.02, with plus 25% of the upper bound and minus 25% of the lower bound. The revised failure rates in Model 3 appear in period 4, period 5, period 10, period 15, and period 17. The former three are revised downward because they exceed the upper bound, which leads to lower inventory levels in period 5, period 6, and period 11 in Model 3.

Table 6. Failure Rates Comparison.

(a)

		Failure Rates Used in Calculation								
		Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9
Model 1	**Hsueh (2011)**	0.06	0.05	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Model 2	**Lin & Chen (2014)**	0.06	0.05	0.022089	0.024238	0.028193	0.023716	0.023926	0.022002	0.022863
Model 3	**PFRF Model**	0.06	0.05	0.022089	0.024238	0.025	0.023716	0.023926	0.022002	0.022863

(b)

		Failure Rates Used in Calculation								
		Period 9	Period 10	Period 11	Period 12	Period 13	Period 14	Period 15	Period 16	Period 17
Model 1	**Hsueh (2011)**	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Model 2	**Lin & Chen (2014)**	0.022863	0.025794	0.023567	0.020023	0.020654	0.021259	0.013463	0.018346	0.016
Model 3	**PFRF Model**	0.022863	0.025	0.023567	0.020023	0.020654	0.021259	0.015	0.018346	0.015

At the end of period 5, the calculated failure rate is 0.028211, exceeding the upper bound of Model 3. Model 2 uses this rate in period 7, while Model 3 would 'regress' the rate into its upper bound, causing a lower purchasing cost than Model 2, with a lower inventory level in early period 7. The beginning inventory level is equal to the failure rate multiplied by the shipping quantities, and the purchasing quantities are the difference between the expected inventory and the current inventory. If the current inventory in the early period is positive, it means the purchasing will be less than the expected inventory level. In contrast, a negative inventory means lost sales and the demand should be fulfilled in the next period, when the purchasing will be higher than the expected inventory level. In Table 4(a), Model 2 and Model 3 have the same inventory levels and purchasing before 5/S. However, the calculated failure rate 5/E is higher than the upper bound of the failure rate, which means lost sales in 5/E. Therefore, Model 3 will have less purchasing at 5/E and a lower inventory level at 7/S than Model 2, because the calculated failure rates are between the failure rate upper bound and lower bound. A high failure rate occurs in period 5 (0.028193 = 6015/(22,838 + 45,200 + 46,907 + 27,600 + 74,000)), which leads to a higher inventory level of Model 2 (7225 = (22,838 + 45,200 + 46,907 + 27,600 + 74,000 + 39,753) × 0.028193) than that of Model 3 (6407 = (22,838 + 45,200 + 46,907 + 27,600 + 74,000 + 39,753) × 0.025) at 6/S, and more purchasing for Model 2 (8085 = 7225 − (−860)) than for Model 3 (7267 = 6407 − (−860)). At the end of period 6 (6/E), after deducting the quantities of return defectives, the inventory level at Model 2 becomes (1117 = 7225 − 6108) and Model 3 comes to (299 = 6407 − 6108). The accumulated failure rate in period 6 comes to (0.023716 = 6108/(22,838 + 45,200 + 46,907 + 27,600 + 74,000 + 37,025)). Inventory levels for both Models 2 and 3 at 7/S are 7016 = (22,838 + 45,200 + 46,907 + 27,600 + 74,000 + 41,000 + 38,294) × 0.023716). Purchasing for both Models 2 and 3 at 7/E is 7223 = (22,838 + 45,200 + 46,907 + 27600 + 74,000 + 41,000 + 37,025 + 6005) × 0.023926 − (−32)). After deducting the returns, the inventory levels for both Model 2 and Model 3 are the same again at 8/E.

Accordingly, in period 5 and period 6, Models 2 and 3 have the same purchasing cost and stock-out cost. Model 3 has a lower holding cost than Model 2.

Figure 7 displays the failure rates of the three models. For a fair comparison, Model 1 also uses the same values of 0.06 and 0.05 as Model 2 and Model 3. If not, the simulation performance for Model 1 would be worse because it underestimates the failure rates in the warranty period. Model 2 uses the updated failure rates, and the failure rates of Model 3 are regressed between the upper bound and lower bound. Therefore, both the highest failure rate and lowest failure rate occur in Model 2. Failure rates below the upper bound are found in period 15. The existing inventory level exceeds the projected inventory level; therefore, the inventory level does not change in period 16. There is no action in period 17, since it is the last period in the warranty period.

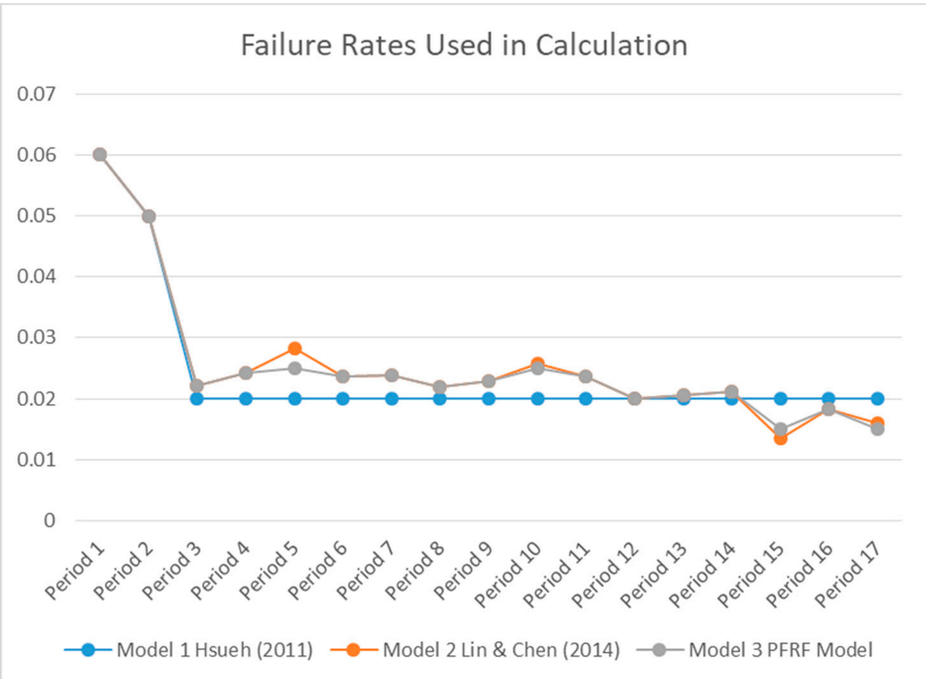

Figure 7. Failure Rates Chart of the Three Models.

Additionally, this case reveals that most of the failure rates are higher than the rate during the stage of normal failure. If the range between the upper bound failure rate and lower bound rate is large enough, for example, plus and minus 30%, respectively, Model 3 will perform the same as Model 2 does. This particular issue is reviewed in the next section.

5. Sensitivity Analysis

To validate the proposed model, we examine the impact of changes in return defectives, failure rate base line, and the upper/lower bounds of failure rates.

5.1. Increase Return Defectives

Figure 8 shows the increase in inventory levels with an increase of 3% of return defectives

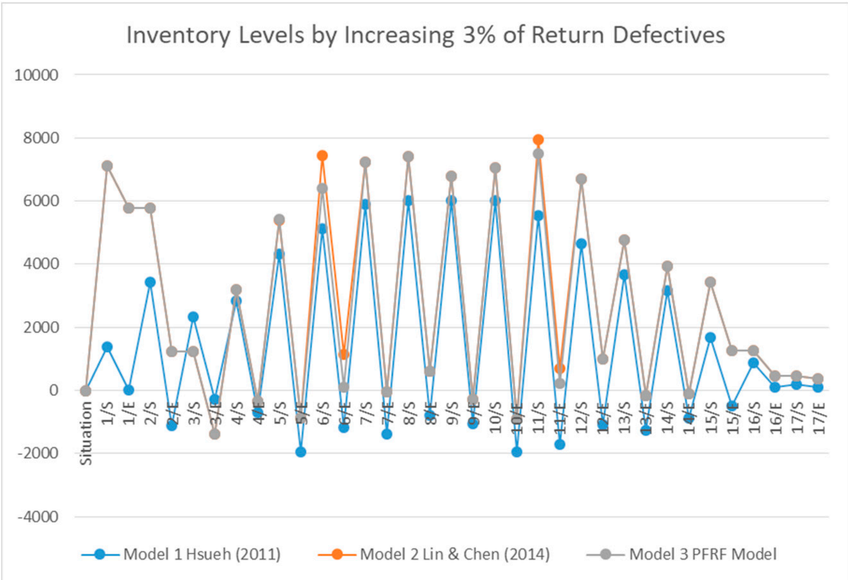

Figure 8. Sensitivity Analysis on Inventory Levels (+3% of Return Defectives).

In this case, the increase of return defectives increases the stock-out in Model 3, since the regression reduces the inventory level when return defectives exceed the upper bound (Table 7). However, Model 3 still has the lowest total cost among all models (Table 8).

In this case, Model 1 has the best holding cost with the worst stock-out performance, leading to the highest total cost. Model 3 has the best holding cost and stock-out cost and produces the lowest total cost. Specifically, when return defectives increase by 3%, the total cost increases by 5.77% in Model 1, 3.11% in Model 2, and 0.47% in Model 3.

Table 9 shows the results of failure rates used in the three models. Apparently, the regression of return rates creates the best performance with the lowest cost.

Table 7. Sensitivity Analysis: Return Defectives (+3%)—Inventory and Purchasing.

(a)

		Situation	1/S	1/E	2/S	2/E	3/S	3/E	4/S	4/E	5/S	5/E	6/S	6/E	7/S	7/E	8/S	8/E	9/S	9/E
Model 1	**Hsueh (2011)**	**Inventory Level**	1379	26	3436	−1115	2334	−281	2853	−705	4331	−1957	5126	−1165	5891	−1368	6012	−776	6012	−1042
		Purchasing Quantities	1379	3410		3449		3134		5036		7083		7056		7380		6788		7054
Model 2	**Lin & Chen (2014)**	**Inventory Level**	7128	5775	5775	1224	1224	−1391	3205	−353	5401	−887	7442	1151	7226	−33	7406	618	6788	−266
		Purchasing Quantities	7128					4596		5754		8329		6075		7439		6170		7320
Model 3	**PFRF Model**	**Inventory Level**	7128	5775	5775	1224	1224	−1391	3205	−353	5410	−878	6407	116	7226	−33	7406	618	6788	−266
		Purchasing Quantities	7128					4596		5763		7285		7110		7439		6170		7320

(b)

		Situation	9/S	9/E	10/S	10/E	11/S	11/E	12/S	12/E	13/S	13/E	14/S	14/E	15/S	15/E	16/S	16/E	17/S	17/E
Model 1	**Hsueh (2011)**	**Inventory Level**	6012	−1042	6012	−1946	5552	−1719	4637	−1070	3677	−1248	3158	−884	1681	−496	885	91	184	102
		Purchasing Quantities		7054		7498		6356		4747		4406		2565		1381		0		0
Model 2	**Lin & Chen (2014)**	**Inventory Level**	6788	−266	7054	−904	7958	687	6716	1009	4774	−151	3927	−115	3437	1260	1260	466	466	384
		Purchasing Quantities		7320		8862		6029		3765		4078		3552		0		0		0
Model 3	**PFRF Model**	**Inventory Level**	6788	−266	7054	−904	7489	218	6716	1009	4774	−151	3927	−115	3437	1260	1260	466	466	384
		Purchasing Quantities		7320		8393		6498		3765		4078		3552		0		0		0

Table 8. Sensitivity Analysis: Return Defectives (+3%)—Cost.

		Return Defectives +3%			
		Holding Cost	Purchasing Cost	Lost Sales Cost	Total Cost
Model 1	Hsueh (2011)	219	157,444	47,316	204,979
Model 2	Lin & Chen (2014)	12,574	158,194	12,300	183,068
Model 3	PFRF Model	11,070	158,194	8,061	177,325

Table 9. Return Rates used in Calculation (+3%).

(a)

		Failure Rates Used in Calculation								
		Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9
Model 1	**Hsueh (2011)**	0.06	0.05	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Model 2	**Lin & Chen (2014)**	0.06	0.05	0.02275	0.024961	0.029038	0.024427	0.024643	0.022659	0.023547
Model 3	**PFRF Model**	0.06	0.05	0.02275	0.025	0.025	0.024427	0.024643	0.022659	0.023547

(b)

		Failure Rates Used in Calculation								
		Period 9	Period 10	Period 11	Period 12	Period 13	Period 14	Period 15	Period 16	Period 17
Model 1	**Hsueh (2011)**	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Model 2	**Lin & Chen (2014)**	0.023547	0.026565	0.024271	0.020623	0.021271	0.021893	0.013864	0.018894	0.0164
Model 3	**PFRF Model**	0.023547	0.025	0.024271	0.020623	0.021271	0.021893	0.015	0.018894	0.0164

5.2. Decrease Return Defectives

We also review the scenarios when return defectives are decreased by 3%, 5%, 10%, and 15%. Tables 10–12 present the results of reducing 3% of return defectives.

As expected, the decrease of return defectives improves the performance for all three Models. Total cost is decreased by 6.25% in Model 1, 3.33% in Model 2, and 3.03% in Model 3, respectively. Model 3 remains the best performer.

The reduction of 3% yields similar results, as Model 3 continues to dominate both Models 1 and 2. In the case of the 10% and 15% decrease, Models 2 and 3 actually yield the same cost performance. Note that an MDP transition for the new failure rate is determined by the upper bound and lower bound of failure rates. If the range of the upper bound and lower bound failure rates is high (e.g., 10%), the results of the proposed model (Model 3) will be the same as those of Model 2 (Lin & Chen, 2014) because the MDP transition would simply revise the failure rate, as it never reaches the upper bound or lower bound. Overall, the sensitivity analysis validates the effectiveness of the proposed model.

Table 10. Sensitivity Analysis: Return Defectives (−3%)—Inventory and Purchasing.

(a)

			1/S	1/E	2/S	2/E	3/S	3/E	4/S	4/E	5/S	5/E	6/S	6/E	7/S	7/E	8/S	8/E	9/S	9/E
		Situation	1/S	1/E	2/S	2/E	3/S	3/E	4/S	4/E	5/S	5/E	6/S	6/E	7/S	7/E	8/S	8/E	9/S	9/E
Model 1	**Hsueh (2011)**	**Inventory Level**	1379	105	3436	−850	2334	−128	2853	−498	4331	−1493	5126	−701	5891	−945	6012	−381	6012	−631
		Purchasing Quantities	1379	3331		3184		2981		4829		6619		6592		6957		6393		6643
Model 2	**Lin & Chen (2014)**	**Inventory Level**	7128	5854	5854	1568	1568	−894	3018	−333	5087	−737	6893	1066	6693	−143	6975	582	6393	−250
		Purchasing Quantities	7128					3912		5420		7630		5627		7118		5811		6893
Model 3	**PFRF Model**	**Inventory Level**	7128	5854	5854	1568	1568	−894	3018	−333	5087	−737	6407	580	6693	−143	6975	582	6393	−250
		Purchasing Quantities	7128					3912		5420		7144		6113		7118		5811		6893

(b)

			9/S	9/E	10/S	10/E	11/S	11/E	12/S	12/E	13/S	13/E	14/S	14/E	15/S	15/E	16/S	16/E	17/S	17/E
		Situation	9/S	9/E	10/S	10/E	11/S	11/E	12/S	12/E	13/S	13/E	14/S	14/E	15/S	15/E	16/S	16/E	17/S	17/E
Model 1	**Hsueh (2011)**	**Inventory Level**	6012	−631	6012	−1483	5552	−1296	4637	−737	3677	−961	3158	−649	1681	−369	885	138	184	107
		Purchasing Quantities		6643		7035		5933		4414		4119		2330		1254				
Model 2	**Lin & Chen (2014)**	**Inventory Level**	6393	−250	6643	-852	7495	647	6325	951	4496	−142	3698	−109	3237	1187	1187	440	440	363
		Purchasing Quantities		6893		8347		5678		3545		3840		3346						
Model 3	**PFRF Model**	**Inventory Level**	6393	−250	6643	−852	7489	641	6325	951	4496	−142	3698	−109	3237	1187	1187	440	440	363
		Purchasing Quantities		6893		8341		5684		3545		3840		3346						

Table 11. Sensitivity Analysis: Return Defectives (−3%)—Cost.

		Return Defectives −3%			
		Holding Cost	Purchasing Cost	Lost Sales Cost	Total Cost
Model 1	Hsueh (2011)	350	147,986	33,366	181,702
Model 2	Lin & Chen (2014)	12,658	148,590	10,380	171,628
Model 3	PFRF Model	12,166	148,590	10,380	171,136

Table 12. Failure Rates used in Calculation (−3%).

(a)

		Failure Rates Used in Calculation (−3% of Return Defectives)								
		Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9
Model 1	**Hsueh (2011)**	0.06	0.05	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Model 2	**Lin & Chen (2014)**	0.06	0.05	0.021419	0.023508	0.026895	0.022625	0.023207	0.021341	0.022175
Model 3	**PFRF Model**	0.06	0.05	0.021419	0.023508	0.025	0.022625	0.023207	0.021341	0.022175

(b)

		Failure Rates Used in Calculation (−3% of Return Defectives)								
		Period 9	Period 10	Period 11	Period 12	Period 13	Period 14	Period 15	Period 16	Period 17
Model 1	**Hsueh (2011)**	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Model 2	**Lin & Chen (2014)**	0.022175	0.025019	0.022859	0.019420	0.020032	0.020620	0.013055	0.017775	0.0154
Model 3	**PFRF Model**	0.022175	0.025	0.022859	0.019420	0.020032	0.020620	0.015	0.017775	0.0154

6. Conclusions and Further Research

This study investigates an important reverse logistics issue, the after-sale service for in-warranty products. A PFRF model is developed, utilizing supply chain information, to more accurately predict the failure rates and, thereby, improve service parts inventory decisions and reduce the total cost. Specifically, the model applies BTC theory and MDP to effectively manage the demand for service parts, and mitigates risk impacts of over stock or shortage of service parts. We assume that the failure rates of in-warranty products fluctuate within a reasonable range, and such a range can be estimated based on historical data. The results of the numerical examples and sensitivity analysis suggest that the proposed PFRF model outperforms the baseline model (Model 1, [3]) and Lin and Chen's approach (Model 2) [2]. Evidently, taking into consideration possible fluctuations of failure rates and adjusting the demand forecasting from period to period significantly improves the total inventory cost and after-sales service.

Note that the total cost presented in the simulation and the sensitivity analysis are based on the service parts involved in a single delivery batch. In practice, the cost saving from the proposed model would be more significant than what is demonstrated in Table 5, considering the amount of new desktop PCs, laptops, tablets, and cell phones launched every year [26]. HP can be used as an example as they shipped 58.8 million PCs in 2017 [27]. Assuming a 2% return rate, five service parts required for every defective product, and each part valued at $50, the potential inventory cost saving of service parts from the application of the PRFR model could be more than US$3 million. The proposed model is particularly valuable to electronic industries with short product life cycles and an unpredictable market demand, especially when product shipment is very different from the production plan. Statista [26] suggests there will be 561 million desktop PCs, laptops, and tablets launched in 2019. Accordingly, the potential saving for the entire electronic industry could be significant. We would not claim that the proposed model would revolutionize the forecasting and inventory management of service parts, but that the model should offer practitioners an opportunity to improve their service parts planning.

Overall, the PFRF model is particularly valuable to those industries (e.g., mobile phone) with short product life cycles and an unpredictable market demand, especially when shipment is very different from production plan. This research contributes to the issue of in-warranty product return management by proposing a model that can reduce the total cost of managing service parts and improve after-sale service.

It is important to point out that the PFRF model is not without limitation. The first limitation concerns the service parts supply in the end-of-life service period. Due to changes in technology, companies may terminate service parts supply at some point of time. The so-called last time buy (LTB) [28] order is not considered in this model. Furthermore, the BTC and AFR may not be accurate in early usage because they are provided by OEMs, which do not reflect the customer demand.

For future research, the model can be extended to derive optimal or near-optimal normal failure rates and the upper/lower bound of failure rates to minimize the total inventory cost. Moreover, the PFRF model considers a single item in one product; however, it can be used for all service items of the product. There could and will be many BTCs and MDPs in one product, as long as the service histories of its service conditions are recorded, and can be referred to for the next product launch as long as the same service parts will be used. Additionally, 3PSP can apply the simulation with the new failure rate base line and the associated upper/lower bounds of the failure rate to identify a better inventory plan, and keep the parameters in the system for reference, if the same or a similar product will be used in the future. Finally, some countries demand products be serviced beyond the warranty period. In that case, the PFRF model can be extended to cover service for the entire product life cycle.

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Appendix A. Failure Rate of Coordinate Cell (*i*, *j*) of Planning Period *k*

Coordinate cell (i, j) denotes the defective return quantities

		Planning Period																
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Iteration	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6	1, 7	1, 8	1, 9	1, 10							
	2		2, 1	2, 2	2, 3	2, 4	2, 5	2, 6	2, 7	2, 8	2, 9	2, 10						
	3			3, 1	3, 2	3, 3	3, 4	3, 5	3, 6	3, 7	3, 8	3, 9	3, 10					
	4				4, 1	4, 2	4, 3	4, 4	4, 5	4, 6	4, 7	4, 8	4, 9	4, 10				
	5					5, 1	5, 2	5, 3	5, 4	5, 5	5, 6	5, 7	5, 8	5, 9	5, 10			
	6						6, 1	6, 2	6, 3	6, 4	6, 5	6, 6	6, 7	6, 8	6, 9	6, 10		
	7							7, 1	7, 2	7, 3	7, 4	7, 5	7, 6	7, 7	7, 8	7, 9	7, 10	
	8								8, 1	8, 2	8, 3	8, 4	8, 5	8, 6	8, 7	8, 8	8, 9	8, 10

Coordinate cell (i, j) denotes the ith iteration of this NPI in the jth planning period.

Failure rate at coordinate cell (i, j) in k planning period can be calculated by the following pseudo code.

$$i \leq rp; j \leq wp; rp \leq wp;\ MDPfr_x = NPI_x \ for\ all\ x \leq j$$

$$FR_j = \frac{\sum_{b=1}^{j} qr_{b,\,j}}{\sum_{a=1}^{j} qs_a}$$

$$MDPfr_j = TP(FR_j)$$

The $(i + 1)$th delivery iteration will calculate its expected return defectives by this failure rate.

$$pQty_{j+1} = (\sum_{a=1}^{i} qs_a) * MDPfr_j + NPI_{i+1} * IFR_{i+1}$$

Appendix B. Generic Calculation of Failure Rate for any Coordinate Cell (*i*, *j*) at *k*th Planning Period

The following is a generic algorithm for the calculation of the failure rate for any coordinate cell (i, j) at the kth planning period.

initial $MDPfr_x = NPI_x$
initial $pQty_x = 0$
$\mathrm{j} = \mathrm{k} - \mathrm{i} + 1$
i_quotation = k mod wp
i_iteration = i_quotation + 1
if k < wp
  for tmp_i = 1 to k
    tmp_j = k - tmp_i + 1//tmp_j = j
  $FR_{tmp\_i} = \frac{\sum_{y=1}^{tmp\_j} qr_{y,\ tmp\_i}}{\sum_{x=1}^{tmp\_j} qs_x}$
    $MDPfr_{tmp\_i} = TP(FR_{tmp\_i})$
    If tmp_i = 1 then
    $pQty_{k+1} = pQty_{k+1} + (\sum_{x=1}^{tmp\_j} qs_x + NPI_{tmp\_j+1}) * MDPfr_{tmp\_i}$
  else
    $pQty_{k+1} = pQty_{k+1} + \sum_{x=1}^{tmp\_j+1} qs_x * MDPfr_{tmp\_i}$
    End if
  next tmp_i
else
  for tmp_i = k – wp × i_quotation + 1 to rp
    tmp_j=k-tmp_i+1
    $FR_{tmp\_j} = \frac{\sum_{y=1}^{tmp\_i} qr_{y,\ tmp\_j}}{\sum_{x=1}^{tmp\_i} qs_x}$
    $MDPfr_{tmp\_j} = TP(FR_{tmp\_j})$
    $pQty_{k+1} = pQty_{k+1} + \sum_{x=tmp_{j+1}}^{rp} qs_x * MFPfr_{tmp\_j}$
  next tmp_i
end if
$Q_{J+1} = pQty_{j+1} - Q_J$

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