



# Article Robust Design of Relief Distribution Networks Considering Uncertainty

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**Abstract**: In the post-disaster response phase, an efficient relief distribution strategy plays a vital role in alleviating suffering in disaster-stricken areas, which sometimes becomes challenging in humanitarian logistics. Most governments pre-located the relief goods at the pre-determined warehouses against possible disasters. Those goods must be shipped to the relief distribution centers (RDCs) to be further distributed to the victims in impacted areas upon the disasters. Secondary disasters can occur due to the first disaster and can occur relatively close in time and location, resulting in more suffering and making the relief distribution activities more challenging. The needs of additional RDCs must be determined as well in response to the secondary disasters. A robust optimization model is proposed to hedge against uncertainties in RDCs' capacity and relief demand. Its objective is to minimize the sum of transportation cost, additional RDC cost, and shortage of commodities. The computational results are given to demonstrate the effectiveness of the proposed model. The sensitivity analysis gives an insight to the decision-makers.



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**Copyright:** © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). **Keywords:** humanitarian logistics; relief distribution; multi-commodity; robust optimization; disaster logistics

# 1. Introduction

Humanitarian logistics has recently gained significant attention due to the increasing frequency of natural and human-made disasters, and relief distribution has played a central role. As the number of disasters grows steadily, studies on relief distribution models have been growing extensively. Because of the deadly nature of disasters, a well-designed disaster management system or humanitarian logistics operation is necessary to aid affected people and reconstruct the affected areas. Various disasters such as earthquakes, tsunamis, hurricanes, floods, volcanic eruptions, explosions, nuclear disasters, and contagious diseases constantly threaten human lives. Among these disasters, earthquakes are one of the most catastrophic. For instance, the Great Sichuan earthquake struck on 12 May 2008, in China's Sichuan province, brought massive suffering to the stricken areas. Approximately 374,000 people were injured, 88,670 people died, and there was enormous property damage [1]. Another devastating earthquake that struck Kathmandu, the city of Nepal, in 2015 had shaken northern India, northwestern Bangladesh, southern parts of Tibet, and western Bhutan. The Nepal earthquake of 2015 killed more than 9000 people and toppled more than 600,000 structures in Katmandu and nearby cities [2]. In the last few years, there were many earthquakes worldwide, including but not limited to the 2018 Peru earthquake, 2019 Indonesia earthquake and 2020 Turkey earthquake [3]. Because of the devastation of a disaster, it is challenging to transport essential supplies to the affected areas to support basic living needs (water, medicine, food, and so on) for those who are trapped in disaster-affected areas. Hence, humanitarian logistics is an important field of study.

In post-disaster relief logistics, decision-makers need to make two critical decisions, i.e., tactical, and operational. Tactical decisions include selecting the locations of RDCs, the number of multiple relief supplies to be mobilized, and transportation modes. Operational decisions are about transportation plans between RDCs and affected areas. According to [4], tactical decisions are made in the deployment phase of humanitarian logistics. The period of the deployment phase is within 6 h after a disaster strike. In this phase, the decision about RDCs is made. The existing buildings or vacant places are usually pre-determined for the initial set of RDCs. This initial set of RDCs may have capacity ranges according to the tactical decisions. However, the unpredictable nature of a disaster, including prolonged disaster periods or secondary disasters, can make the initial set of RDCs in the deployment phase unable to meet the demand of affected areas. Therefore, it is necessary to reform the humanitarian logistics network.

The formation or reformation of logistics operations following large-scale disasters is complex [5]. Reformation can come in front of public or private agencies because of the uncertain capacity of RDCs and the unavailability of roads between RDCs and demand points. Moreover, it is impossible to repair blocked, unavailable, or damaged roads quickly. In such cases, it is necessary to build additional RDCs. A secondary disaster is a similar kind of disaster, or different kind, that occurs at a relatively close time and location. For example, the second earthquake or aftershock following a large earthquake after a few hours or days causes mountain collapses, debris flows, landslides, flooding, and other problems called secondary disasters, or a disaster chain. For instance, after the Great Sichuan earthquake in 2008, some aftershocks continued to occur in and around the primary focal area for up to several months causing further casualties and damages [6].

In addition, Omi [7] discussed a vast number of aftershocks that occur after a largescale earthquake which can cause secondary disasters. After the first disaster, impromptu organizations and government agencies attempt to work together, quickly provide relief commodities and services that help save lives, provide comfort, and even restore entire communities [5]. Nevertheless, the complexity of distributing multiple commodities becomes more severe for aid providers when more than one disaster occurs in the same area or closely spaced areas.

In general, the first 72 h after any disaster are called the "golden relief period." During this period, it is difficult for aid providers to measure the exact demand for relief goods because of the damaged communication systems and the scarcity of relief goods (e.g., water, medicine, food, etc.). Therefore, the demand for relief goods remains uncertain.

A robust optimization model is proposed for post-disaster relief distribution considering the demand and the initial RDC's capacity uncertainties. The consideration of uncertainty in humanitarian logistics led many studies to adopt a stochastic programming approach.

However, there are two significant disadvantages of stochastic programming [8]. Firstly, a stochastic programming approach requires the probability distributions of uncertain parameters. It is difficult to estimate the probabilities in real-life cases because of the insufficient historical data of disasters. Secondly, the literature shows that most stochastic programming approaches in humanitarian logistics are scenario-based, and the different values of uncertain parameters represent different scenarios. An extensive set of scenarios can lead to significant computational complexity.

Conversely, a robust optimization approach does not require the probability distribution of uncertain parameters and scenarios [9]. In robust optimization, each uncertain parameter is denoted by a specific interval, and the feasibility of the solution is guaranteed by using a min-max approach. This study is motivated by the complex nature of disasters, especially when it involves secondary disasters. Therefore, increasing the resilience of the affected areas is the main priority of this study. In summary, the objectives of this study are as follows.

- To propose a deterministic optimization model for the relief distribution problem.
- To propose a robust optimization model for the relief distribution problem.

- To understand the significance of conservatism degrees and data variabilities of robust models.
- To minimize transportation cost, additional RDC cost, and shortage of commodities.
- To understand the sensitivities of the proposed model against variations of shortage cost.

The main contributions of this study are summarized as follows: (1) A robust optimization model for the relief distribution is proposed with uncertain demand at the demand points and the uncertain capacity of initial RDCs. (2) The proposed model can manage relief distribution activities and select the candidate locations for additional RDCs, given road unavailability. (3) The influence of decisions in humanitarian logistics is analyzed and discussed. (4) The proposed model can assist decision-makers in accurately identifying necessary data.

The rest of the paper is organized as follows. The relevant literature is reviewed in Section 2. The problem description and the deterministic model of this study are presented in Section 3. Section 4 proposes the robust optimization model. A case study is given and the computational results are analyzed through the proposed model in Section 5. Finally, the concluding remarks and future study directions are drawn in Section 6.

### 2. Literature Review

In disaster management, uncertainties often arise because of the nature of the emergencies. To consider those uncertainties, stochastic programming has been widely used. Stochastic models for emergency relief distribution, emergency medical services, and facility location are widely studied. Mete and Zabinsky [10] proposed a stochastic optimization approach for storing and distributing medical supplies. This model is used for multiple types of medical supply distributions in the context of the uncertainties of disaster events. Another model and solution approach for facility location of medical supplies was proposed [11]. They addressed the uncertain demand and proposed three heuristics to solve the problem: a genetic algorithm, a locate/allocate heuristic, and a Lagrangian relaxation heuristic. Mohamadi and Saeed Yaghoubi [12] proposed a bi-objective stochastic optimization model to determine the transfer points of evacuees and medical supplies distribution centers.

Relief distribution involves multiple stages of relief operations and multiple kinds of relief goods, so many have proposed multi-stage stochastic programming. Barbarosoğlu and Arda [13] presented one of the pioneering works in relief distribution by using the two-stage stochastic programming approach for planning the transportation of first-aid commodities to disaster-affected areas. They formulated a multi-commodity, multi-modal network flow formulation to describe the flow of material over an urban transportation network. Döyen, Aras, and Barbarosoğlu [14] proposed a two-echelon facility location problem for humanitarian logistics network design where they only focused on relief distribution and facility location decisions. Li, Jin, and Zhang [15] proposed a two-stage stochastic programming approach for disaster preparedness and responses. Their model can help decide the locations, capacities, and resources to shelters in the preparedness phase and the distribution of evacuees and resources to shelters in the response phase. Cavdur, Kose-Kucuk, and Sebatli [16] proposed a two-stage stochastic programming model for facility allocation and relief distribution where facility allocation and service decisions are performed in the first and second stages, respectively.

Nilay Noyan [17] developed a two-stage stochastic programming model for lastmile distribution in disaster response while considering demand and network-related uncertainties. [18] proposed a dynamic truck and trail routing problem where they did not consider any uncertainty. Another last-mile distribution problem was proposed where the study used a drone as a fleet, and the objective was to minimize the travel distance of the drone [19]. Their model incorporated a hybrid allocation policy and achieved high levels of accessibility and equity simultaneously. Rawls and Turnquist [20] and Chang, Tseng, and Chen [21] independently proposed a two-stage stochastic model to decide facility location and resource distribution. The model of Rawls and Turnquist [20] provided a pre-positioning strategy for hurricanes and considered uncertain demands.

An integrated stock pre-positioning and relief distribution model was proposed with a two-stage stochastic scenario-based probabilistic-stochastic programming approach by Tofighi, Torabi, and Mansouri [22]. The uncertainties of both supply and demand were considered with the availability level of the transportation route after the earthquake. Chen [23] compared two models, namely the stochastic and  $\Psi$ -expander models, for relief pre-positioning. Geng, Hanping Hou, and Shaoguang Zhang [24] proposed a model considering both the relief pre-positioning and location of emergency shelters. Few studies have considered the redistribution strategy in disaster response. A two-stage stochastic programming approach for redistribution strategies was proposed to minimize the dissatisfaction cost in the first stage and minimized total transportation time in the second stage [25]. Noham and Michal Tzur [26] defined the humanitarian supply chain network by addressing the facility location and inventory allocation decisions, combined with an incentive system that aims to enhance population cooperation.

Due to the devastation of a large-scale disaster, many existing RDC or warehouses can be destroyed, making the relief distribution activities challenging. Few studies consider this disruption scenario when modeling a relief distribution network. Yahyaei and Bozorgi-Amiri [27] proposed a robust and reliable humanitarian relief network that considers the risk of facility disruption and designed their relief distribution network by integrating the shelter and supply facility locations. Fereiduni and Shahanaghi [28] proposed a robust optimization model for relief distribution and evacuation in the disaster response phase by considering uncertainties and the disruption of bridges. Apart from the disruption scenario, Akbarpour, S. Ali Torabi and Ali Ghavamifar [29] proposed a bi-level model for the pharmaceutical relief chain, where they developed a min-max robust model to tackle the demand uncertainty. For earthquake preparedness, location-allocation network design and perishable product supply chain designs are modeled [30]. Sarma, Das, and Bera [31] proposed a multi-objective model for emergency relief operations. They used Facebook postings to estimate the uncertain demand. Another study by Sarma [32] considered the redistribution plan after the disaster considering disruption scenarios to minimize the total cost of relief operation.

Secondary disasters are often neglected in prior studies. For instance, Zhang [33] proposed an emergency resource allocation problem for secondary disasters. They tried to minimize the total time of dispatching emergency resources. Alem, Clark, and Moreno [34] proposed a two-stage stochastic network flow model to help decide how to rapidly supply humanitarian aids to victims of disasters under demand, supply, network, and budget uncertainties. They considered the uncertain budget and conducted a case study for floods and landslides in Rio de Janeiro State, Brazil, by a two-phase heuristic. Jianfang Shao [35] proposed a model for calculating the demand for relief in multiple disasters, and their proposed model is divided into two parts: supply classification and demand calculation.

However, most models overlook the road disruption, the minimum demand satisfaction rate of multiple commodities at demand points, the required number of distributions, and sensitivity against shortage cost, which are considered in this paper. Tzeng, Cheng, and Huang [36] proposed a mathematical model with a few similarities but did not consider uncertainty. Among non-deterministic models, Liu [37] proposed a robust optimization model to deal with uncertainty for evacuating people and transferring relief personnel to affected areas. This study denotes the uncertainties in demand and capacity of the initial RDCs using interval data to remove the requirement for the probability distribution of uncertain parameters found in models based on stochastic programming.

## 3. Problem Description

The relief distribution network considered in this study is depicted in Figure 1, which involves a two-echelon disaster relief chain: (1) relief warehouses (RWs), (2) initial set of RDCs, (3) additional RDCs at candidate locations, and (4) demand points (DPs). Com-

modities are sent from RWs and to RDCs in the first echelon. RDCs include an initial set of RDCs selected in the deployment phase of humanitarian logistics operations and additional RDCs at candidate locations.



Figure 1. Relief distribution network.

It is assumed that commodities from public or private organizations are already stored in RWs before the disasters. The following hypothesis is postulated for relief operations based on the relief distribution network shown in Figure 1. Given the occurrence of a disaster (e.g., an earthquake), secondary disasters (e.g., debris flows, landslides, and flooding) may result in the roads' unavailability and demand fluctuations. The functionality of the proposed model is triggered immediately after the disaster's strike. Uncertain roads' availability and demand may require reformation of relief distribution network by establishing additional RDCs at candidate locations of specific capacity and re-routing the transportation.

This study assumes that the initial set of RDCs selected in the deployment phase may not have enough capacity to cover the demand of affected areas after the primary disaster. Some affected areas may not be accessible from the RDCs due to road disruptions caused by secondary disasters. Secondary disasters like landslides and debris flow block the existing roads. This kind of secondary disaster makes the initial set of RDCs unable to meet the demand at DPs. Candidate locations for additional RDCs are identified around a disaster region, and additional RDCs are established at such locations that the relief distribution can be facilitated at a low cost. Commodities initially transported between RWs to RDCs in the first echelon are delivered from RDCs to DPs in the second echelon.

A robust optimization model is proposed to take the demand and RDCs' capacity uncertainties into account. The uncertain parameters are represented as the intervals for those parameter values. Moreover, the roads' unavailability between RDCs and DPs is considered using the indefinite travel distance between the departure and the destination. In addition to the road unavailability, the minimum demand satisfaction rate for each commodity at each DP is considered to ensure fairness and criticality among DPs. The minimum demand satisfaction rate for each commodity at each DP must be satisfied.

The assumptions in this study are summarized as follows.

- 1. Multiple relief goods need to be distributed to the DPs of which demand is uncertain.
- 2. The locations of the RWs and the initial set of RDCs are known.
- 3. The candidate locations for additional RDCs and their potential capacities are known.
- 4. The available quantities of relief goods at RWs are known.
- 5. Heterogenous vehicles of different capacities are allowed to deliver mixed commodities.

The following sets, parame	eters, and decision variables are given in this model as
follows:	
Ι	Set of RWs, where $i \in I$ . For simplicity, $i = 1, 2, 3,, I$ .
J	Initial set of RDCs $j \in J$ . For simplicity, $j = 1, 2, 3,, J$ .
A	Set of candidate locations for RDCs $a \in A$ . For simplicity,
	$a=1,2,3,\ldots,A.$
R	Set of DPs $r \in R$ . For simplicity, $r = 1, 2, 3,, R$ .
G	Set of commodities $g \in G$ . For simplicity, $g = 1, 2, 3,, G$ .
	Set of vehicle type $l \in L$ . For simplicity, $l = 1, 2, 3,, L$ .
Parameters	
$k_i^s$	Total quantity of available commodity $g$ in RW $i$
Na	Potential capacity of RDCs at candidate location <i>a</i>
Н	Maximum number of additional RDCs at candidate locations
$C_{a}$	Establishment cost of an RDC at candidate location <i>a</i>
$C^{i}$	Operating cost for vehicle type <i>l</i> per unit distance
W <sup>g</sup> , V <sup>g</sup>	Weight and volume of commodity $g$ , respectively
$W^l, V^l$	Weight and volume capacity of vehicle type <i>l</i> , respectively
$S_{ij}$	Distance between RW $i$ and RDC $j$
E <sub>ia</sub>	Distance between RW <i>i</i> and candidate location <i>a</i>
M <sub>jr</sub>	Distance between RDC $j$ and DP $r$
F <sub>ar</sub>	Distance between candidate location $a$ and DP $r$
$T_{max}$	Maximum allowable travel time of vehicles
$T_{ij}$	Round-trip time between RW $i$ and RDC $j$
T <sub>jr</sub>	Round-trip time between RDC $j$ and DP $r$
T <sub>ar</sub>	Round-trip time between candidate location $a$ and DP $r$
$D_r^{g}$	Demand for commodity $g$ at DP $r$
$P_j$	Capacity of RDC <i>j</i>
$ au_{rg}$	Shortage cost for unsatisfied demand of commodity $g$ at DP $r$
N	Minimum demand satisfaction rate that must be satisfied for
urg	commodity $g$ at DP $r$
$\eta_{i}^{l}$	Number of vehicle type <i>l</i> at RW <i>i</i>
$\mu_{j}^{l}$	Number of vehicle type <i>l</i> at RDC <i>j</i>
	Number of vehicle type <i>l</i> at a potential RDC at candidate
$\omega_a$	location <i>a</i>
Decision variables	
. gl	Quantity of commodity <i>g</i> transported from RW <i>i</i> to RDC <i>j</i> by
$x_{ij}^{\circ}$	vehicle type <i>l</i>
gl	Quantity of commodity $g$ transported from RW $i$ to potential
Wia	RDC at candidate location <i>a</i> by vehicle type <i>l</i>
~ <sup>gl</sup>	Quantity of commodity $g$ transported from RDC $j$ to DP $r$ by
<sup>2</sup> jr	vehicle type <i>l</i>
-sgl	Quantity of commodity <i>g</i> transported from potential RDC at
Var	candidate location $a$ to DP $r$ by vehicle type $l$
$TR_{ii}^l$	Number of trips made by vehicle type <i>l</i> from RW <i>i</i> to RDC <i>j</i>
	Number of trips made by vehicle type $l$ from RW $i$ to
$IR_{ia}^{*}$	potential RDC at candidate location <i>a</i>
$TR_{ir}^{l}$	Number of trips made by vehicle type <i>l</i> from RDC <i>j</i> to DP $r$
ر ۱	Number of trips made by vehicle type from potential RDC a
$TR_{ar}^{\iota}$	to DP r
	1, if potential RDC is established at candidate location $a_{i}$ 0.
<i>Y</i> <sub>a</sub>	otherwise.
SHra	Shortage or unsatisfied demand of commodity $\sigma$ at DP $r$
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The multi-commodity relief distribution model is formulated as follows:

$$\begin{aligned} Minimize & \sum_{a \in A} C_a y_a + \sum_{l \in L} C^l \left( \sum_{i \in I} \sum_{j \in J} S_{ij} T R_{ij}^l + \sum_{i \in I} \sum_{a \in A} E_{ia} T R_{ia}^l + \sum_{j \in J} \sum_{r \in R} M_{jr} T R_{jr}^l \right) \\ & + \sum_{a \in A} \sum_{r \in R} F_{ar} T R_{ar}^l \right) + \sum_{r \in R} \sum_{g \in G} \tau_{rg} S H_{rg} \end{aligned}$$

Subject to,

$$\sum_{l \in L} \sum_{j \in J} z_{jr}^{gl} + \sum_{l \in L} \sum_{a \in A} v_{ar}^{gl} + SH_{rg} = \widetilde{D}_r^g \quad \forall g \in G, r \in R$$
(1)

$$\sum_{l \in L} \sum_{j \in J} z_{jr}^{gl} + \sum_{l \in L} \sum_{a \in A} v_{ar}^{gl} \ge \alpha_{rg} \widetilde{D}_r^g \quad \forall g \in G, r \in R$$
<sup>(2)</sup>

$$\sum_{l \in L} \sum_{j \in J} x_{ij}^{gl} + \sum_{l \in L} \sum_{a \in A} w_{ia}^{gl} \le k_i^g \quad \forall g \in G, i \in I$$
(3)

$$\sum_{l \in L} \sum_{i \in I} x_{ij}^{gl} \le \widetilde{P}_j \quad \forall g \in G, j \in J$$
(4)

$$\sum_{l \in L} \sum_{i \in I} w_{ia}^{gl} \le N_a y_a \quad \forall g \in G, a \in A$$
(5)

$$\sum_{l \in L} \sum_{i \in I} x_{ij}^{gl} \ge \sum_{l \in L} \sum_{r \in R} z_{jr}^{gl} \quad \forall g \in G, j \in J$$
(6)

$$\sum_{l \in L} \sum_{i \in I} w_{ia}^{gl} \ge \sum_{l \in L} \sum_{r \in R} v_{ar}^{gl} \quad \forall g \in G, a \in A$$
(7)

$$\sum_{a \in A} y_a \le H \tag{8}$$

$$\sum_{g \in G} x_{ij}^{gl} W^g \le T R_{ij}^l W^l \quad \forall j \in J, i \in I, l \in L$$
(9)

$$\sum_{g \in G} x_{ij}^{gl} V^g \le TR_{ij}^l V^l \quad \forall j \in J, i \in I, l \in L$$
(10)

$$\sum_{g \in G} w_{ia}^{gl} W^g \le T R_{ia}^l W^l \quad \forall a \in A, i \in I, l \in L$$
(11)

$$\sum_{g \in G} w_{ia}^{gl} V^g \le TR_{ia}^l V^l \quad \forall a \in A, i \in I, l \in L$$
(12)

$$\sum_{g \in G} z_{jr}^{gl} W^g \le T R_{jr}^l W^l \quad \forall j \in J, r \in R, l \in L$$
(13)

$$\sum_{g \in G} z_{jr}^{gl} V^g \le T R_{jr}^l V^l \quad \forall j \in J, r \in R, l \in L$$
(14)

$$\sum_{g \in G} v_{ar}^{gl} W^g \le T R_{ar}^l W^l \quad \forall a \in A, r \in R, l \in L$$
(15)

$$\sum_{g \in G} v_{ar}^{gl} V^g \le T R_{ar}^l V^l \quad \forall a \in A, r \in R, l \in L$$
(16)

$$\sum_{j \in J} TR_{ij}^l T_{ij} + \sum_{a \in A} TR_{ia}^l T_{ia} \le \eta_i^l T_{max}^i \quad \forall l \in L, i \in I$$
(17)

$$\sum_{r \in R} TR_{jr}^l T_{jr} \le \mu_j^l T_{max}^j \quad \forall l \in L, j \in J$$
(18)

$$\sum_{r \in R} TR_{ar}^{l} T_{ar} \le \omega_{a}^{l} T_{max}^{a} \quad \forall l \in L, a \in A$$
(19)

$$x_{ij}^{gl}, w_{ia}^{gl}, z_{jr}^{gl}, v_{ar}^{gl}, SH_{rg}, TR_{ij}^{l}, TR_{ia}^{l}, TR_{jr}^{l}, TR_{ar}^{l}$$
 are positive numbers (20)

The objective function minimizes the total cost, including the establishment cost of additional RDCs, the transportation cost, and the shortage cost for unsatisfied demand. Constraint (1) shows the relationship between demand and shortage. Constraint (2) implies that minimum demand satisfaction rates for commodities should be satisfied at DPs. Constraint (3) ensures that the total outgoing quantity of commodities from RWs to RDCs is less than or equal to the total available quantity of a commodity at RWs. Constraint (4) indicates that the quantities of incoming commodities to RDCs from RWs are not greater than the capacities of RDCs.

In addition, Constraint (5) guarantees that once any additional RDC is established, the commodity must be sent there, and the total incoming commodities to it are less than or equal to its capacity. Constraints (6) and (7) ensure that the total amount of commodities transported from RWs to RDCs are more than or equal to those from RDCs to DPs. Constraint (8) limits the maximum number of RDCs that can be established. Constraints (9)–(16) are related to vehicle capacities in weight and volume for the transportations among RWs, RDCs, and DPs. These constraints guarantee that the total weight or volume of the commodities loaded in a vehicle exceeds neither the weight nor the volume capacities of the corresponding vehicle. Constraints (17)–(19) limit the number of trips for each vehicle, considering the maximum allowable travel time and the number of vehicles available at RWs and RDCs. Constraint (20) defines all the variables.

## 4. Robust Model Formulation

The deterministic model in Section 3 assumes that the input parameters are precisely known nominal values and does not consider data uncertainties. Data uncertainties affect the quality and feasibility of the solutions. If any input parameter is different from its nominal value, some constraints may be violated, and the solution obtained by the deterministic model may not be optimal or feasible. Therefore, the approach to compromise the quality of solution against the data uncertainty is called "robust." Therefore, to protect the decision-makers against the worst realization of outcomes, it is necessary to deal with the conservatism of the robust model.

For the first time, Soyster in 1973 [38] proposed a linear mathematical programming model to produce a feasible solution for all constraints that belong to a convex set. However, the solution obtained by his model was too conservative, compromising the quality of the solution with the robustness excessively [39].

The notable works were done independently by Ben-tal and Nemirovski in 1999 [39], H U Y En in 1997 [40], El Ghaoui in 1998 [41], and Ben-Tal, Aharon, Laurent El Ghaoui, and Arkadi Nemirovski in 2009 [42]. They proposed less conservative models by considering ellipsoidal uncertainties, which involve solving the robust counterparts of the nominal problem in the form of conic quadratic problems [39]. However, their models were nonlinear and added computational complexity.

A vital step to deviate from this undesirable complexity was made by [9]. Their model was linear and offered control on the degree of conservatism for every constraint. They defined a prespecified number  $\Gamma_p$  of uncertain coefficients in the *p*-th constraint of the deterministic model. If less than  $\Gamma_p$  uncertain coefficients take different values than nominal values, their model guarantees the feasibility of the solution. Since their model is linear and computationally tractable, we have adopted their model to propose our robust model in this paper.

Mulvey in 1995 [43] proposed a robust stochastic programming approach using a soft constraint concept by considering a penalty for constraint violation. In humanitarian logistics, where our lives are at stake, there are some hard constraints to be satisfied in addition to soft constraints, which are flexible due to the uncertainty. Sometimes, uncertain

parameters can be described only by an interval without probability distribution estimation. The robust optimization approach proposes and handles hard constraints and interval uncertainty [9].

To obtain the robust counterpart of our proposed model, we first briefly describe the robust optimization approach introduced by Bertsimas and Sim in 2004 [9] and the extended research by Hatefi and Jolai [44], in which they considered the following deterministic linear model.

Consider the following nominal mixed-integer programming problem, where *c* is a *n*-dimensional vector, *A* is a  $m \times n$  matrix, *b* is an *m*-dimensional vector, and *x* is a polyhedron.

$$\begin{array}{l}
\text{Max } cx \\
\text{s.t. } Ax \leq b \\
x \in X
\end{array}$$
(21)

The uncertainty can exist on *A*, *b*, and *c*. The objective coefficient vector *c* can be assumed to be certain without loss of generality. If *c* has any uncertainty, we can set z = cx and maximize *z* by adding constraint  $z - cx \le 0$  to  $Ax \le b$ . That is, we can move the uncertainty from *c* to *A*.

Bertsimas and Sim [9] considered the uncertainty of the coefficient matrix *A* and Hatefi and Jolai [44] further considered the uncertainty of the right-hand-side (RHS) constants *b*.

Assume that some parameters of the coefficient matrix A are uncertain. For constraint p, let  $\Omega_p$  denote the set of uncertain coefficients in constraint p. The uncertain coefficient,  $a_{pq}(q \in \Omega_p)$ , can be modeled as a random variable  $\tilde{a}_{pq}$  (see Ben-tal and Nemirovski [39]) that takes values according to a symmetric distribution with a mean equal to nominal value  $a_{pq}$  in interval  $[a_{pq} - \hat{a}_{pq}, a_{pq} - \hat{a}_{pq}]$ , where  $\hat{a}_{pq}$  denotes the maximum deviation from the nominal value. For random variable  $\tilde{a}_{pq}$ , another random variable  $\eta_{pq}$ , called a scaled deviation, is defined as follows.

$$\eta_{pq} = \frac{\widetilde{a}_{pq} - a_{pq}}{\widehat{a}_{pq}},$$

It is also symmetric and takes values in [-1, 1], i.e.,  $|\eta_{pq}| \le 1$ .  $\tilde{a}_{pq}$  is a random variable for uncertain coefficients and is a constant for a certain coefficient.

For every possible realization of random variable  $\tilde{a}_{pq}$ , Equation (21) can be formulated as:

$$\operatorname{Max} cx$$
s.t.  $\sum_{j=1} \widetilde{a}_{pq} x_q \leq b_p \quad \forall \widetilde{a}_{pq} \in \Omega_p$ 

$$x \in X$$

$$(22)$$

The left-hand side can be transformed as follows.

$$\sum_{q=1} \widetilde{a}_{pq} x_q = \sum_q (a_{pq} + \hat{a}_{pq} \eta_{pq}) x_q = \sum_q a_{pq} x_q + \sum_{q \in \Omega p} \hat{a}_{pq} \eta_{pq} x_q \le \sum_q a_{pq} x_q + \sum_{q \in \Omega p} \hat{a}_{pq} |x_q| \le b_p \quad \forall p$$
(23)

Here, a new formulation can be formulated.

#### Max cx

s.t. 
$$\sum_{q} a_{pq} x_q + \max_{\eta_{pq} \in \mathbb{Z}_p} \left( \sum_{q \in \Omega_p} \hat{a}_{pq} \eta_{pq} x_q \right) \le b_p, \quad \forall p$$
 (24)  
 $x \in X$ 

where  $Z_p = \left\{ \eta_{pq} \middle| \sum_{q=1}^n \eta_{pq} \leq \Gamma_p \right\}$ ,  $\forall p$ . For every *p*-th constraint, the term,  $\max_{\eta_{pq} \in Z_p} \left( \sum_{q \in \Omega_p} \hat{a}_{pq} \eta_{pq} x_q \right)$  gives the necessary "protection" of the constraint by maintaining a gap between  $\sum_{q} a_{pq} x_q$  and  $b_p$  to avoid violation. Bertsimas and Sim [9] introduced a parameter,  $\Gamma_p$ , not necessarily integer, that takes values in the interval  $[0, |\Omega_p|]$ . It is called the *budget of uncertainty* or *conservatism degree*. This parameter adjusts the robustness or protection against uncertainty.  $\Gamma_p = 0$  indicates the nominal (deterministic) case for constraint p.  $\Gamma_p = |\Omega_p|$  represents the worst uncertainty for constraint p [38]. For  $\Gamma_p \in [0, |\Omega_p|]$ , the decision-maker can make a tradeoff between the protection level of constraint p and the degree of conservatism of the solution. Decision-makers can choose the value of  $\Gamma_p$  According to their tolerance against uncertainty. Note  $\sum_{q \in J_p} \eta_{pq} \leq \Gamma_p$ ,  $\forall p$ .

The model by [9] protects itself against all cases that up to  $\lfloor \Gamma_p \rfloor$  of coefficients  $a_{pq}$  are allowed to change, and one coefficient  $a_{pt}$  changes by  $(\Gamma_p - \lfloor \Gamma_p \rfloor) \hat{a}_{pt}$ .

Now, Equations (22) and (23) produce a new formulation as follows.

Max cx

s.t. 
$$\sum_{j} \overline{a}_{pq} x_{q} + \beta_{p} (x, \Gamma_{p}) \leq b_{p}, \quad \forall p \qquad (25)$$
$$x \in X$$

where  $\beta_p(x, \Gamma_p) = \max_{\{S_p \cup \{t_p\} | S_p \subseteq \Omega_p, |S_p| = \Gamma_p, t_p \in \Omega_p \setminus S_p\}} \left\{ \sum_{q \in S_p} \hat{a}_{pq} |x_q| + (\Gamma_p - \lfloor \Gamma_p \rfloor) \hat{a}_{pt_p} |x_{t_p}| \right\}$ is called the protection function of constraint *p*.

Considering both Formulations (24) and (25),  $\beta_p(x, \Gamma_p)$  equals the objective function of the following linear optimization problem (See proposition (1) of [9]),

$$\beta_{p}(x,\Gamma_{p}) = \max_{\eta_{pq} \in z_{p}} \left( \sum_{q \in J_{p}} \hat{a}_{pq} \eta_{pq} |x_{q}| \right)$$
s.t. 
$$\sum_{q=1}^{n} \eta_{pq} \leq \Gamma_{p} \quad \forall p$$

$$0 \leq \eta_{pq} \leq 1 \quad \forall \Omega_{p}$$
(26)

Now the dual problem of Formulation (26) is written as follows by introducing dual variables  $z_p$  and  $v_{pq}$ .

$$Max \Gamma_{p} z_{p} + \sum_{q \in \Omega_{p}} v_{pq}$$
  
s.t.  $z_{p} + v_{pq} \ge \hat{a}_{pq} |x_{q}|, \forall p, q \in \Omega_{p}$   
 $v_{pq} \ge 0, \forall p, q \in \Omega_{p}$   
 $z_{p} \ge 0, \forall p$  (27)

By setting  $y_q = |x_q|$ , Formulation (27) can be reformulated.

Maximize *cx* 

Subject to

$$\sum_{q \in \Omega_{p}} \overline{a}_{pq} x_{q} + z_{p} \Gamma_{p} + \sum_{q \in \Omega_{p}} v_{pq} \leq b_{p} \forall p$$

$$z_{p} + v_{pq} \geq \hat{a}_{pq} y_{q} \forall p, q \in \Omega_{p},$$

$$-y_{q} \leq x_{q} \leq y_{q} \forall q,$$

$$v_{pq} \geq 0 \forall p, q \in \Omega_{p},$$

$$y_{q} \geq 0 \forall q$$

$$z_{p} \geq 0 \forall p$$

$$(28)$$

 $x \in X$ 

By duality, because Equation (26) is feasible and bounded, then the dual problem (Equation (28)) is also feasible and bounded. Their objective values coincide. It proves that Equations (26) and (28) are equivalent (see Theorem 1 of Bertsimas and Sim [9]).

In case of uncertainty on the RHS constant  $b_p$  for constraint p, we define the conservatism degree  $\Gamma$  taking values between [0, m] to denote the number of uncertain constants  $b_p(1 \le p \le m)$ . Similarly,  $\tilde{b}_p \in [b_p - \hat{b}_p, b_p + \hat{b}_p]$ , where  $b_p$  and  $\hat{b}_p$  are the nominal value and maximum deviation from the nominal value, respectively. An adjusted upper bound of this interval by the common budget of uncertainty is  $b_p + \frac{\Gamma}{m}\hat{b}_p$ . This upper bound is used to convert our constraints involving uncertain constants  $b_p(1 \le p \le m)$ .

The model proposed in Section 3 has uncertain parameters in Constraints (1), (2), and (4). There is no uncertainty in the coefficient matrix. The uncertainty exists only in the right-hand side constant.

$$\sum_{l \in L} \sum_{j \in J} z_{jr}^{gl} + \sum_{l \in L} \sum_{a \in A} v_{ar}^{gl} + SH_{rg} = \widetilde{D}_r^g \ \forall g \in G, \ r \in R$$
$$\sum_{l \in L} \sum_{j \in J} z_{jr}^{gl} + \sum_{l \in L} \sum_{a \in A} v_{ar}^{gl} + SH_{rg} = \widetilde{D}_r^g \ \forall g \in G, \ r \in R$$
$$\sum_{l \in L} \sum_{i \in I} x_{ij}^{gl} \leq \widetilde{P}_j \ \forall g \in G, \ j \in J$$

The robust optimization approach works only with stochastic models, where the constraints have "less than or equal to" inequality [37]. However, constraint (1) in this paper is equality constraint, and this constraint is rewritten as an equality constraint.

If considering the relationship between the shortage and the demand, the shortage occurs only when the demand for a commodity exceeds the quantity of commodity transported from RDC. Therefore, constraint (1) can be rewritten as follows:

$$SH_{rg} \ge \widetilde{D}_r^g - \left(\sum_{l \in L} \sum_{j \in J} z_{jr}^{gl} + \sum_{l \in L} \sum_{a \in A} v_{ar}^{gl}\right), \ \forall g \in G, r \in R,$$
(29)

where  $\tilde{D}_r^g$  is an uncertain parameter. Assume that  $\tilde{D}_r^g$  distributes symmetrically in the ranges of  $\left[D_r^g - \hat{D}_r^g, D_r^g + \hat{D}_r^g\right]$  where  $D_r^g$  and  $\hat{D}_r^g$  denote the nominal demand (mean of the interval) and the constant maximum deviation from the nominal demand, respectively. Taking the adjusted upper bound of the interval for the worst-case realization, which should be minimized, constraint (29) can is rewritten in the following robust form.

$$SH_{rg} \ge D_r^g + \frac{\Gamma}{||G| * |R||} \hat{D}_r^g - \left(\sum_{l \in L} \sum_{j \in J} z_{jr}^{gl} + \sum_{l \in L} \sum_{a \in A} v_{ar}^{gl}\right), \ \forall g \in G, r \in R,$$
(30)

where  $\Gamma \in [0, |G| * |R|]$ .

Constraint (2) ensures that the minimum demand satisfaction rate at each DP, at which the quantity of the commodities from RDCs must be greater than the demand of the DP. Similarly, constraint (2) is rewritten in the following robust form.

$$\sum_{l\in L}\sum_{j\in J}z_{jr}^{gl} + \sum_{l\in L}\sum_{a\in A}v_{ar}^{gl} \ge \alpha_{rg}\left(D_r^g + \frac{\Gamma}{||G|*|R||}\hat{D}_r^g\right), \ \forall g\in G, r\in R,\tag{31}$$

where  $\Gamma \in [0, |G| * |R|]$ .

For constraints (4), the uncertainty exists on the capacity of initial RDCs,  $P_j$  is also distributes symmetrically in the ranges of  $[P_j - \hat{P}_j, P_j + \hat{P}_j]$  where  $P_j$  and  $\hat{P}_j$  denote the nom-

inal capacity and the constant maximum deviation from the nominal capacity. Similarly, the robust form of constraint (4) is as follows.

$$\sum_{l \in L} \sum_{i \in I} x_{ij}^{gl} \le P_j - \frac{\Gamma}{\parallel J \parallel} \hat{P}_j \ \forall g \in G, j \in J$$
(32)

where  $\Gamma \in [0, |J|]$ .

The robust model is obtained by replacing the constraints (1), (2), (4) with the new constraints (30), (31), and (32), which are represented as follows.

$$Minimize \sum_{a \in A} C_a y_a + \sum_{l \in L} C^l \left( \sum_{i \in I} \sum_{j \in J} S_{ij} T R^l_{ij} + \sum_{i \in I} \sum_{a \in A} E_{ia} T R^l_{ia} + \sum_{j \in J} \sum_{r \in R} M_{jr} T R^l_{jr} + \sum_{a \in A} \sum_{r \in R} F_{ar} T R^l_{ar} \right) + \sum_{r \in R} \sum_{g \in G} \tau_{rg} S H_{rg}$$

s.t. (3), (5)–(20), (30)–(32).

This robust optimization model can produce the solutions by setting the conservatism degree according to the decision-maker's preference. In the next section, we solve the proposed robust model with a test problem for various conservatism degrees and data variabilities. The computational results are also provided.

## 5. Numerical Analysis

5.1. Test Problem

For numerical experiments and analysis, a test problem is generated. In the preparation stage of the disaster management cycle, it is assumed that there are two RWs ( $i_1$  and  $i_2$ ), three initial RDCs ( $j_1$ ,  $j_2$ , and  $j_3$ ), and three candidate locations for additional RDCs ( $a_1$ ,  $a_2$ , and  $a_3$ ). The nine most severely stricken disaster areas in a province or state are considered DPs ( $r_1$ ,  $r_2$ ,  $r_3$ ,  $r_4$ ,  $r_5$ ,  $r_6$ ,  $r_7$ ,  $r_8$ , and  $r_9$ ). Their pairwise distances in km are given in Tables 1 and 2.

Table 1. Distance between RWs to initial RDCs (in km).

	$j_1$	$j_2$	$j_3$	<i>a</i> <sub>1</sub>	<i>a</i> <sub>2</sub>	<i>a</i> <sub>3</sub>
$i_1$	13.8	7.3	22.1	18.3	10.6	18.4
<i>i</i> <sub>2</sub>	28.8	28.8	11	27	23.6	18.3

	$r_1$	<i>r</i> <sub>2</sub>	<i>r</i> <sub>3</sub>	$r_4$	$r_5$	<i>r</i> <sub>6</sub>	$r_7$	<i>r</i> <sub>8</sub>	<b>r</b> 9
<i>j</i> 1	UA *	1.19	1.54	3.4	1.83	4.71	UA *	4.52	3.76
j <sub>2</sub>	5.1	4.57	UA *	2.1	5.4	1.42	UA *	1.5	3.94
j3	3.8	2.26	3.95	4.96	3.06	3.11	5.79	2.04	2.63
$a_1$	3.5	3.1	2.9	2.83	1.93	0.82	1.76	2.26	1.35
<i>a</i> <sub>2</sub>	2.85	1.28	0.97	2.08	3.24	2.12	0.6	1.71	0.64
<i>a</i> <sub>3</sub>	3.45	1.84	1.56	0.86	3.83	2.72	1.4	1.94	2.23

Table 2. Distance between RDCs and DPs (in km).

\* UA means unavailable road (debris on the road or shutdown).

According to [6], China's Sichuan province experienced severe landslides because of the main earthquake and aftershocks. Most of the landslides occurred within the earthquake-stricken region. Therefore, this study also considers the landslide as a secondary disaster. As a consequence of the landslide, the unavailability of roads is also considered as given by "UA" in Table 2.

Water bottles and emergency medical kits are considered relief goods. We assumed that twelve 1-L water bottles are in a box, and various medicines and medical equipment are contained in a kit. The volumes are measured in cubic centimeters. The two types of relief goods are represented as RG  $g_1$  and RG  $g_2$ . The parameters of relief goods are shown in Table 3. Table 3 shows the available supply in stock at RWs  $i_1$  and  $i_2$ .

Relief Goods	Supply in Stock (Box)		Unit	Unit Weight,	Unit Volume,	
	RW <i>i</i> <sub>1</sub>	RW i <sub>2</sub>	enit	W <sup>g</sup> (kg)	$V^{g}$ (cm <sup>3</sup> )	
RG $g_1$	16,000	17,000	Water Box (1000 mL, 12 bottles)	12	$36 \times 26 \times 30$	
RG <i>g</i> <sub>2</sub>	12,000	10,000	Emergency Medical Kit	2	$43 \times 24 \times 15$	

Table 3. Parameters for relief commodities.

As explained earlier, the initial set of RDCs have uncertain capacities with nominal means. It may be related to the uncertainties caused due to the degradation of the facilities or supplies at existing RDCs. It is assumed that the nominal capacities of all initial RDCs are assumed to be 5000 water boxes and 5000 emergency medical kits. The additional RDCs can be established at candidate locations to satisfy the demand caused by unexpected second disasters. It is assumed that these additional RDCs have specific capacities in this study because they are new facilities. The capacities of those RDCs at candidate locations and their establishment costs are given in Table 4. Each candidate location has its establishment costs.

 Table 4. Capacities of additional RDCs at candidate locations.

Candidate	Capacity	y (in Box)	Establishment Cost ( (¢)
Location	RG $g_1$	RG $g_2$	Establishment Cost, $C_a$ (\$)
$a_1$	4500	4500	14,000
<i>a</i> <sub>2</sub>	4500	4500	24,000
<i>a</i> <sub>3</sub>	4000	4500	16,000

Big and small trucks with heterogenous capacities are used to transport relief goods. The weight and volume capacities of the trucks are summarized in Table 5. The transportation costs per unit distance are 5 USD and 3 USD for big and small vehicles, respectively.

Table 5. Parameters of the vehicles.

Type of Vehicle	Weight Capacity, $W^l$ (kg)	Volume Capacity, V <sup>1</sup> (cm <sup>3</sup> )
Big truck	3500	465  imes 175  imes 180
Small truck	1500	$231 \times 150 \times 160$

Finally, the nominal demand of each DP is given for each RG in Table 6. As explained earlier, these demands are uncertain because it is almost impossible to assess the size of needs or impacts on DPs quickly.

Table 6. Demands of RGs at DPs.

Demand Points	Demand, $\tilde{D}_r^q$				
	<b>RG</b> $g_1$	RG $g_2$			
$r_1$	2500	2000			
<i>r</i> <sub>2</sub>	2000	1820			
<i>r</i> <sub>3</sub>	3000	2000			
$r_4$	2500	2200			
$r_5$	4000	2500			
$r_6$	3080	1900			
<i>r</i> <sub>7</sub>	2900	2040			
$r_8$	3900	2800			
<i>r</i> 9	5000	3000			

#### 5.2. Computational Results

Numerical experiments were conducted on a computer with an Intel (R) Core (TM) i-5-2500 @ 3.30 GHz CPU with 4 GB of RAM. The deterministic and robust models proposed in this paper were coded using LINGO 10.0.

Without loss of generosity, the minimum demand satisfaction rate for commodities and DPs was set to 0.4 in both models. Then, a sensitivity analysis of the minimum demand satisfaction rate is also carried out. The data variability of uncertain parameters was varied by 10%, 15%, 25%, and 35% relative to their nominal values.

Since there are nine DPs and two RGs, the corresponding degree of conservatism  $\Gamma$  associated with the coefficients of uncertain demand has values within the interval [0, 18]. For the uncertain capacity of the initial set of RDCs, the conservatism degree is within the interval [0, 3] since there are three initial RDCs in the relief distribution network. Figure 2 shows the changes in the objective function value for different conservatism degrees ( $\Gamma$ ) and the different data variabilities for uncertain demands and capacities of initial RDCs. This figure shows that the objective function value is sensitive to the small changes of  $\Gamma$  at the low range of conservatism degrees. The objective function value for the deterministic model is 52,288. The deterministic model is solved without considering the uncertainty of demand and capacity.

According to the sensitivity analysis, low conservatism degree ranges for demand and capacity parameters cause a higher deterioration in the robust objective function value. In the experiments of this study, the most significant degradation of the robust objective function value or worst-case objective function value is achieved when setting the conservatism degree to 9 and data variability to 35% for demand uncertainty. For the capacity uncertainty, the worst-case objective function value is achieved by setting the conservatism degree to 3 and perturbating uncertain capacities to 35%. It is trivial that higher conservatism degree and higher data variability lead to higher uncertainty, and they make the robust model have more conservative solutions.



(a) For Demand Uncertainty

(**b**) For Capacity Uncertainty

Figure 2. Sensitivity of the proposed robust model for various conservatism degrees on demand and capacity uncertainty at different data variabilities.

The relative extra cost (*REC*) measures the quality deterioration in robust solutions obtained from the proposed robust model [45]. The REC is defined as follows,

$$REC = \frac{Z(\Gamma > 0) - Z(\Gamma = 0)}{Z(\Gamma = 0)},$$

where Z ( $\Gamma$ ) is the optimal objective function value of the robust model when conservatism degree  $\Gamma$  is given.

The REC represents the extra cost that should be counted for the different levels of protection according to the conservatism degrees ( $\Gamma > 0$ ) in comparison with the determin-

istic status ( $\Gamma$  = 0). Tables 7 and 8 summarize the results of robust models, including robust optimal solutions (ROS) and RECs, for different data variabilities of demand and capacity at various levels of protection, i.e., conservatism degrees ( $\Gamma$ ). Both tables show that the more significant data variabilities from the nominal demand and capacity result in larger RECs. When the conservatism degree is 0, the optimal objective function value of 52,288 was obtained for the deterministic and robust models with nominal value and no data variability. Columns 1 and 2 in Tables 7 and 8 show the robust model settings.

**Table 7.** Experimental results of the robust optimal solutions for various conservatism degrees of demand parameters and data variabilities compared to the deterministic optimal solution.

Conservatism Degree	Data Variability (%)	ROS	<b>REC (%)</b>	
0	-	52,288	0	
	10	59,075	12.98	
2	15	62,095	18.75	
3	25	65,124	24.54	
	35	67,223	28.56	
5	10	60,566	15.83	
	15	64,304	22.98	
	25	67,093	28.31	
	35	69,700	33.30	
	10	63,346	21.14	
-	15	66,403	26.99	
7	25	69,351	32.63	
	35	71,978	37.65	
	10	68,103	30.24	
	15	70,804	35.41	
9	25	73,650	40.85	
	35	75,835	45.03	

**Table 8.** Experimental results of the robust optimal solutions for various conservatism degrees of capacity parameters and data variabilities compared to the deterministic optimal solution.

Conservatism Degree	Data Variability (%)	ROS	<b>REC (%)</b>
0	-	52,288	0
	10	54,450	4.13
1	15	56,179	7.44
1	25	57,854	10.64
	35	59,769	14.30
	10	55,909	6.92
2	15	57,990	10.90
2	25	60,854	16.38
	35	63,590	21.61
	10	58,980	12.79
2	15	61,930	18.44
3	25	65,478	25.22
	35	68,478	30.96

Tables 7 and 8 show that the objective function values of the robust model are higher (more deteriorate) than that of the deterministic model. To understand the significance of the robust model and the protection by the conservatism degrees, we have conducted different experiments on the data variabilities to the coefficients and RHS constants of the deterministic models. Even with slight variations on the coefficients and RHS constants, the optimal solutions of the deterministic model without data variabilities violate some constraints, becoming infeasible. It is conjectured that the optimality is still maintained for a data variation on some loose constraints, but on most tight constraints, the optimality is easily broken by a slight variation on the data. It is essential to note that the robust

model provides feasible solutions even under higher data variabilities. The ROS under various data variability guarantees the optimality of the robust model even if it offers more conservative (or higher) objective function values than the deterministic model. Tables 7 and 8 show the apparent trend of increasing ROS as the conservatism degree or the data variability increases. Therefore, it is concluded that our proposed robust model can produce robust solutions immune to data variabilities on uncertain parameters.

We discuss optimal solutions of the robust models with conservatism degrees of 3, 5, 7, and 9 and data variabilities of 10%, 15%, 25%, and 35% compared to that of the deterministic model. Table 9 shows how many RGs are transported from RWs to RDCs at the first echelon relief distribution activities. The deterministic model has an optimal solution that does not require additional RDCs at candidate locations. In the first row with the conservatism degree of 0, RG  $g_1$  of 4460 water boxes and RG  $g_2$  of 3570 boxes of emergency medical kits are transported from RW  $i_1$  to RDC  $j_1$ . Similarly, RG  $g_1$  of 4570 and RG  $g_2$  of 3920 to RDC  $j_2$ . RDC  $j_3$  is not used for the relief distribution, and no additional RDC is necessary for the optimal solution of the deterministic model. In the deterministic model, constraint (4) concerns the initial RDC's capacity that the initial RDCs have enough capacity to receive all incoming commodities from RWs.

However, in the robust model, with the increase of the conservatism degree and data variability for the initial RDC's capacities, the robust optimal solution may require additional RDCs at candidate locations. According to constraint (32), the capacities of initial RDCs may decrease with the increase of the conservatism degree and data variability. In the second row of Table 9, with the conservatism degree of 1 for uncertain capacity of the initial RDCs and the data variability of 10%, we still do not need additional RDCs. However, in the third row of Table 9, with the conservatism degree of 1 and data variability of 15%, the optimal solution of the robust model determines to establish an additional RDC at candidate location  $a_1$ . The experimental results show that the test problem only requires an additional RDC at candidate location  $a_1$  in our robust model with various robust settings.

Table 9 shows that only RW  $i_1$  sends commodities to RDCs  $j_1$  and  $j_2$ , and the RDC at the candidate location  $a_1$  and RW  $i_2$  sends commodities only to RDC  $j_3$ . It is observed that this transportation is reasonable considering the distance matrix among all locations. Another important observation from Table 9 is the quantity of RGs transported to RDCs. With the increase of conservatism degree and data variability, the transported RGs did not increase linearly. Decision-makers could select a suitable degree of conservatism to make a wise tradeoff between the quantity of transported RGs and the total cost because a higher degree of conservatism leads to a higher cost.

Figures 3–5 show the optimal amounts of RGs transported from RDCs to all DPs at the second echelon relief distribution activities. Figure 4 shows the optimal solution for the deterministic case, in which the nominal demand and the nominal capacity of the initial set of RDCs are used without considering any uncertainties. Figures 5 and 6 present the amounts of RGs transported from RDCs to all DPs, under the given robust settings of conservatism degrees of 3, 5, 7, and 9 and a data variability of 25%. As explained in Table 9, note that the deterministic model produces an optimal solution that does not build an additional RDC, and the robust model generates the optimal solutions to establish an additional RDC.

RWs	Conservatism	Data	RD	C <i>j</i> <sub>1</sub>	RD	C <i>j</i> <sub>2</sub>	RD	C <i>j</i> <sub>3</sub>	RDC at <i>a</i> <sub>1</sub>	
KVV5	Degree	Variability (%)	<i>8</i> <sub>1</sub>	<i>8</i> <sub>2</sub>	$g_1$	<i>8</i> <sub>2</sub>	$g_1$	<i>8</i> <sub>2</sub>	$g_1$	<i>8</i> <sub>2</sub>
	0	-	4460	3570	4570	3920				
	1	10	4351	3707	4302	3870				
		15	3100	3130	4280	3813			4511	3530
		25	3324	2737	4133	3670			4430	3640
		35	3400	2590	3980	3530			4504	3501
	2	10	3170	3010	4200	3751			4500	3500
RW $i_1$		15	3320	2720	4050	3600			4500	3512
		25	3750	1940	3310	3170			4510	3500
		35	3450	2490	3130	3070			4500	3302
-	3	10	3320	2720	4050	3604			4500	3500
		15	3830	2640	3170	3011			4500	3405
		25	2180	1880	2250	1750			4503	3500
		35	2930	2600	2630	2607			4500	3500
	0	10–35					4030	3405		
	1	10					4351	3780		
		15					1130	1870		
		25					1135	1870		
		35					1130	1875		
	2	10					1135	1870		
RW i <sub>2</sub>		15					1130	1857		
		25					1130	1870		
-		35					2350	1575		
	3	10					1130	1870		
		15					1233	1878		
		25					2420	1440		
		35					2650	1030		

**Table 9.** The first echelon relief distribution activities in optimal solutions of deterministic and robust models for uncertain capacity of the initial RDCs.



Figure 3. The number of commodities transported from RDCs to DPs in the deterministic model.



**Figure 4.** The amount of RG  $g_1$  transported to DPs with various conservatism degrees for uncertain demand when data variability is 25%.



**Figure 5.** The amount of RG  $g_2$  transported to DPs with various conservatism degrees for uncertain demand when data variability is 25%.



Figure 6. Sensitivity analysis of shortage amounts against unit shortage cost.

In Figure 3, most DPs receive the commodities from the closer RDCs. It is possible because the first echelon distribution supports the optimal decision at the second echelon transportation. However, DP 8 receives the split delivery of RG  $g_1$  from RDCs  $j_1$  and  $j_2$ . We have assumed that some roads are unavailable or blocked due to landslides or other secondary disasters. Considering the distance and accessibility, the deterministic model decides the number of commodities transported from each RDCs to DPs.

Recall that the uncertainty increases as the conservatism degree or data variability increases. It indicates that the demands of DPs increase so that the optimal solution stays feasible and robust in the robust models. Figures 4 and 5 show the experimental results of the robust models, where the amounts of RGs transported to each DP with various conservatism degrees and a data variability of 25% show the increasing trend as the conservatism degree increases. For these experiments, we use the conservatism degree of 1 and the data variability of 10% as the default robust setting for the uncertain capacity of initial RDCs.

The influence of unit shortage cost on the shortage amounts in our robust model is studied through the sensitivity analysis. Since we have multiple commodity types, the multi-dimensional sensitivity analysis may be appropriate, but a sensitivity analysis for the shortage cost of an RG was conducted by setting the shortage cost of the other RG to be a fixed value in this study. In addition, it was conjectured that the relative importance of the shortage costs of multiple commodity types might exhibit interesting behaviors. In this experiment, the sensitivity analysis was conducted with conservatism degrees of 5 for the demand and of 1 for the capacity of initial RDCs while considering 10% data variability from the nominal values.

The results of sensitivity analysis are given in Figure 6. In Figure 6a, the shortage amount of RG  $g_1$  decreases as unit shortage cost of RG  $g_1$  increases when unit shortage cost of RG  $g_2$  is set to 5 USD. Figure 6b shows a similar trend for the shortage amount of RG  $g_2$  when the unit shortage cost of RG  $g_1$  is set to 5 USD. In other words, the greater unit shortage penalties of RGs result in lower shortage amounts.

Another experiment to understand the effects of the minimum demand satisfaction rate ( $\alpha_{rq}$ ) was conducted. According to dynamic situations at DPs, the minimum demand satisfaction rate can be chosen differently per the DPs and RG types. However, for simplicity, in our experiments, the minimum demand satisfaction rate is identical throughout all DPs and all RG types. Its effects on the optimal objective function values of the robust model and the number of newly established RDCs are analyzed. The robust model was studied with the conservatism degrees of 5 for the demand uncertainty and 2 for capacity uncertainty, and 10%, 15%, 25%, and 35% data variabilities from nominal values of both uncertain parameters.

Figure 7 indicates that optimal objective function values increase with the minimum demand satisfaction rate, regardless of the data variability. Figure 8 shows that the number of additional RDCs at candidate locations under different robust settings. When the minimum demand satisfaction rate is 0.4 and the data variability is 10%, no additional RDC is required to satisfy all demands at DPs. However, if we increase the data variability to 15% in the same experimental setting, an additional RDC must be established to satisfy the uncertain demand. Even with higher data variabilities of 25% and 35%, an additional RDC can satisfy all demands. As the minimum demand satisfaction rate increases, more RDCs are required to be established.



Figure 7. Relationship between objective function values and the minimum demand satisfaction rate.



Figure 8. Relationship between the number of additional RDCs and the minimum demand satisfaction rate.

Beyond our experiments, we set the minimum demand satisfaction rate to 0.75, and then the proposed robust model cannot find feasible solutions with three candidate locations for RDCs. It indicates additional candidate location must be considered at a higher minimum demand satisfaction rate than 0.7. Otherwise, it is concluded that the supply or capacity of initial RDCs should be increased with the increase of data variability and demand satisfaction rates. The minimum demand satisfaction rate analysis could help decision-makers decide about the required number of additional RDCs.

# 6. Conclusions

A multi-commodity relief distribution model under uncertainty is discussed in this paper. The emergency disaster situations cause unpredictable and unexpected uncertainty in the relief distribution model. Our relief distribution model considers the uncertainty on the demand and the capacities of initial RDCs. Motivated by secondary disasters, the disruption of roads and the establishment of additional RDCs were considered to satisfy the dispersed demands while highlighting a real-life disaster scenario and makING the model more realistic.

In a disaster management system, decision-makers want to satisfy the demand of victims or affected areas and dispatch relief goods and teams as soon as possible despite

having complexation (e.g., road unavailability, facility disruption, uncertainties, etc.). The objective of this robust model is to minimize the logistics cost while minimizing the penalized cost of shortage amount to determine the required number of additional RDCs at candidate locations to be established, the allocation of RGs to RDCs, the shipments to DPs, and the numbers of vehicle trips. Since it is difficult to estimate the probability distribution of those uncertain parameters, the robust optimization approach was adopted over the stochastic programming approach.

A deterministic mixed-integer programming model was formulated to design the relief distribution network, and a robust counterpart was obtained through the robust modeling by Bertsimas and Sim [9]. To illustrate the effectiveness of this proposed model, a numerical analysis is carried out. The required number of additional RDCs among candidate locations of candidate RDCs was determined to cover the DPs that are isolated from the initial set of RDCs due to road unavailability.

The results revealed critical managerial insights as follows. (1) Among two uncertain parameters, the demand uncertainty leads to a more significant increase in the total cost than capacity uncertainty. (2) The demand uncertainty influences the total cost and the unmet demand more significantly than the capacity uncertainty. (3) The relationship between the shortage amount of commodities and shortage cost is reciprocal, meaning an increase in the shortage cost leads to a decrease in the shortage amount of commodities. In other words, an increase in shortage cost increases demand satisfaction. (4) The minimum demand satisfaction rate could assist decision-makers in predicting whether the capacity of existing RDCs is enough, and how many new centers are needed. (5) Finally, the effects of conservatism degree and the data variability on the objective function and decision variables were revealed by a sensitivity analysis, and decision-makers should be cautious when assessing uncertainties. In uncertain environments, the optimal solution of the deterministic model leads to infeasibility or violates the optimality easily. Therefore, the conservative solution can be obtained to offer protection against those uncertainties using the robust optimization approach.

The sensitivity of the results also reflects the effectiveness of the proposed model. Authorities need to have a concrete model that can help them make a decision quickly. Although a case study was analyzed, some limitations will be considered in future study directions. (1) a multi-period setting will be considered to show the effects of secondary disasters; (2) vehicle routing decisions are aggregated with the current model; and (3) the model is extended by considering a multi-modal transportation system, traffic congestion during relief delivery, and minimum response time. The limitation of this study can be addressed as well. Our model only considers the total disruption of the roads, which makes the road blocked. Since this study focused on connectivity and distance in our relief distribution network model, road capacity reduction or traffic congestion can be considered uncertain in future research.

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