Multi-Depot Electric Bus Scheduling Considering Operational Constraint and Partial Charging: A Case Study in Shenzhen, China

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Abstract: Electric buses (e-buses) demonstrate great potential in improving urban air quality thanks to zero tailpipe emissions and thus being increasingly introduced to the public transportation systems. In the transit operation planning, a common requirement is that long-distance non-service travel of the buses among bus terminals should be avoided in the schedule as it is not cost-effective. In addition, e-buses should begin and end a day of operation at their base depots. Based on the unique route configurations in Shenzhen, the above two requirements add further constraint to the form of feasible schedules and make the e-bus scheduling problem more difficult. We call these two requirements the vehicle relocation constraint. This paper addresses a multi-depot e-bus scheduling problem considering the vehicle relocation constraint and partial charging. A mixed integer programming model is formulated with the aim to minimize the operational cost. A Large Neighborhood Search (LNS) heuristic is devised with novel destroy-and-repair operators to tackle the vehicle relocation constraint. Numerical experiments are conducted based on multi-route operation cases in Shenzhen to verify the model and effectiveness of the LNS heuristic. A few insights are derived on the decision of battery capacity, charging rate and deployment of the charging infrastructure.

Keywords: electric bus; scheduling; Large Neighborhood Search; partial charging; multi-depot; vehicle relocation

1. Introduction

Buses account for only a small number of all the vehicles on city roads, but their emissions take up a much higher portion of the total road emissions owing to their long operation time and distance [1]. With increasing eco-awareness around the globe, electric buses (e-buses) are regarded as an effective solution to reduce urban air pollution and greenhouse gas emissions. In recent years, e-buses are increasingly introduced into the transit systems with government support. Shenzhen, a metropolitan in southern China, had switched all the diesel buses to electric ones by the end of 2017 [2]. According to [3], the number of e-buses in operation worldwide has reached approximately 500,000, which is predicted to take up over 67% of the total share by 2040. A major concern for operating e-buses in the public transportation system is their limited battery capacity [4]. Most of the diesel buses have a maximum driving range of larger than 300 km under urban driving conditions [5]; while, the driving range of the e-buses currently available in the market varies from 100 to over 300 km, depending on their battery capacity and driving condition. For example, an e-bus with 350 kWh of battery capacity can cover a range between 190 and 210 km, depending on the local driving conditions [6]. As such, e-buses often need...
recharging during the day to prolong their service time. For the convenience of e-bus charging activities, charging facilities are required to be established at bus depots, bus stops on the routes, or both.

In the transit operation planning stage, e-bus scheduling aims to assign e-buses to carry out timetabled trips on bus routes considering their charging demand. E-bus scheduling should also take local operational requirements into consideration to make applicable schedules. A common requirement is that long-distance non-service travel of the e-buses among bus terminals should be avoided as it requires a large amount of manual labor and is not cost-effective. Furthermore, e-buses should begin and end operation of the day at their base depots [7,8]. In many e-bus operation cases in Shenzhen, these two requirements add further restriction to the form of feasible schedules and make the e-bus scheduling problem more difficult. Take bus route M133 in Shenzhen as an example. Figure 1a displays the layout of route M133 the length of which is around 37.5 km. Timetabled service trips are carried out on the route where terms “up” and “down” are used to distinguish their direction. Two depots with charging infrastructure are located close to the end terminals of the route. Thus, non-service travel incurred by e-bus charging is always short—once the e-bus has a low battery State of Charge (SoC) after completing a trip at one of the terminals, it would head to the closest depot for charging. Figure 1b shows the form of feasible and infeasible e-bus schedules. Two e-bus schedules are presented using the time-space network. The first graph is a feasible schedule because it does not include any non-service travel between the two depots; while the second graph is an infeasible schedule because it includes a non-service travel from Changlingdong Depot to Shekou Port Depot. In the feasible schedule, the e-bus starts one day’s operation from Shekou Port Depot, carrying out four service trips (two up trips and two down trips) after finishing which it can return to the base depot at the end of the day without non-service travel. While in the infeasible schedule, the e-bus starts one day’s operation from Shekou Port Depot, carrying out three service trips (two up trips and one down trip) after finishing, which needs a long-distance non-service travel to return to the base depot, Shekou Port Depot, at the end of the day. Such kind of long-distance non-service travel should be avoided as much as possible according to the transit agency. We name this requirement on the form of feasible e-bus schedule as the vehicle relocation constraint.

As introduced above, the e-bus operation scenario we considered in Shenzhen has the unique route configuration and operational requirements which we named as the vehicle relocation constraint. It causes difficulty in generating feasible high-quality e-bus schedules and has not been considered in the existing studies. Based on the aforementioned considerations, with the aim to generate applicable e-bus schedules in Shenzhen involving multi-depot, vehicle relocation constraint and partial charging, this paper developed an LNS heuristic which can effectively solve real-world e-bus scheduling instances including hundreds of trips. In the LNS heuristic, we devised a novel solution formulation procedure which is used in the initial solution generation and solution repair process to generate schedules satisfying the vehicle relocation constraint. A case study based on the e-bus operation scenarios and time-of-use tariff in Shenzhen is conducted to demonstrate the performance and application of the LNS heuristic. A few insights are derived on the decisions of battery capacity, charging rate, and deployment of the charging infrastructure.

The remainder of the paper is organized as follows. Section 2 reviews the related studies of e-bus scheduling. Section 3 presents the mixed integer programming (MIP) model for the problem. Section 4 introduces our LNS algorithm. In Section 5, numerical experiments based on the real-world e-bus operation cases are conducted. Section 6 concludes the paper.
Figure 1. (a) Layout of bus route M133 in Shenzhen; (b) Form of feasible and infeasible e-bus schedules.

2. Literature Review

2.1. E-bus Scheduling Problem

The e-bus scheduling problem is derived from the Vehicle Scheduling Problem (VSP) which considers the assignment of diesel buses to timetabled trips on bus routes to minimize the total number of vehicles used. The difficulty of the VSP is dependent on the number of depots involved. Single-depot VSP is polynomial solvable while multi-depot VSP has proven to be NP-hard by Bertossi et al. [9]. Electric Vehicle Scheduling Problem (EVSP) arises with the introduction of e-buses to the transit systems and has attracted wide research interests. The problem can be distinguished into different categories based on the charging technology considered. Currently, the commonly adopted charging technologies include opportunity charging, depot charging, and battery swapping [10]. Opportunity charging and depot charging are plug-in charging technologies which differ in the charging rates and charging locations. Fast opportunity charging utilizes high electricity power to restore the battery energy rapidly when e-buses are holding at the stops and end terminals of the bus routes without incurring dead mileage; while depot charging uses a lower electricity power to restore the battery energy when e-buses return to the depots, and it usually takes...
several hours to fully charge a battery. To avoid long non-service travel time, e-buses with high operation mileages of over 200 km usually adopt fast opportunity charging while those with lower operation mileages will go for depot charging during off-time.

Under plug-in charging mode, e-bus charging can be time-consuming if the charging power is low, thus has a great impact on the operation. To manage the e-bus charging activity, full charging and partial charging policies are employed with the latter being more flexible and complex from the planning side. Considering the full battery charging policy, Wang and Shen (2007) addressed a VSP with route and fueling time constraints assuming a limited driving range and fixed charging time [11]. Liu et al. (2019) developed a model and a Genetic Algorithm (GA) to optimize the e-bus schedule with the aim of minimizing the fleet size, charging facility, and empty driving mileage [12]. Liu and Ceder (2020) proposed a deficit function theory-based model and an integer programming model to minimize the number of vehicles and chargers [8].

Some researches considered the partial charging policy where e-buses can be recharged for a flexible amount instead of to a full battery. van Kooten Niekerk et al. (2017) considered single-depot scenarios and introduced two models with a different level of detail resembling the actual nonlinear charging processes. A Column Generation algorithm was proposed to solve the problem [22]. Yıldırım and Yıldız (2021) considered the fleet composition and scheduling problem. An IP-column-generation algorithm was developed to solve the large-scale instances [23]. Janoveca and Kohánia (2019) proposed a single depot EVSP model considering the capacity of the charging facilities which is solved by the standard solver [24]. Li et al. (2020) addressed the EVSP in joint with the charger deployment problem. An adaptive GA was designed to solve the problem [25].

2.2. Application of LNS in Solving EVSP and EVRP

The LNS heuristic was first introduced by Shaw (1998) [26] to solve the Vehicle Routing Problem (VRP). The LNS heuristic explores the neighborhood of a solution extensively by destroy-and-repair operators. These operators function to partially destroy and then repair the current solution to obtain a new one. Node removal and reinsertion are the most commonly adopted destroy-and-repair operators in solving the EVSP problems. Wen et al. (2016) proposed a connection-based network model for a multi-depot EVSP. An Adaptive LNS (ALNS) heuristic was developed to solve the problem with customized bus trips that are geographically dispersed [7]. Considering nonlinear charging process and multi-vehicle type, Zhang et al. (2021) developed an MIP model for the EVSP with linear approximation of the nonlinear charging function. An ALNS heuristic was devised to solve the problem [27]. Perumal et al. (2021) studied the e-bus scheduling problem integrated with the crew scheduling problem considering a single depot and full charging policy. They developed an ALNS heuristic utilizing branch-and-price heuristics to address the problem [28].

The LNS and its extension, the ALNS, have been proven to be successful in solving many Electric Vehicle Routing Problem (EVRP) problems in the logistics context. EVRP aims to route a set of electric vehicles to deliver or pick up customer packages in certain sequences to minimize the operational cost. The EVRP and EVSP have similarities in
that both of them need to optimize the schedule for electric vehicles to serve customers considering the charging demand. Due to the problem complexity, constructive and local search-based heuristics were proposed to solve the large-scale problem instances. Koç et al. (2019) devised an ALNS-based heuristic to solve the EVRP with shared charging stations [29]. References [30–32] developed LNS-based heuristics to address the EVRP considering partial recharging.

From the above discussion we can see that although many studies have focused on EVSP, a heuristic for solving the multi-depot EVSP considering multiple depots, vehicle relocation constraint, and partial charging has not been proposed in the literature. This paper aims to meet this gap and address the multi-depot e-bus scheduling problem considering vehicle relocation constraint and partial charging.

3. Mathematical Formulation

3.1. Problem Description

In the transit network, a bus route is characterized by two end terminals and a series of intermediate stops. On a bus route, a service trip starts from one end terminal at a planned time and ends at the other end terminal to carry passengers. The timetable of the route includes all the trips in one day with their planned start time from the start terminal and expected end time at the end terminal. On the electrified bus routes, these trips are carried out by a fleet of e-buses. Each e-bus undertakes a sequence of trips in a day which is called a trip chain. An operation schedule consists of the trip chains for all the e-buses. The end terminals function as bus depots to serve for vehicle parking, maintenance, and charging if the charging facilities are established, we therefore referred to them as depots.

The e-bus scheduling problem is formulated on an acyclic direct graph \( G = (V, A) \). Each timetabled trip is represented by a trip node in graph \( G \). Denote \( K \) as the set of depots. Each depot \( k \in K \) is created with two nodes in graph \( G \): an operation begin node \( \phi^k \) and an operation end node \( d^k \), representing the e-bus begins/ends the operation from/at depot \( k \), respectively. For each node \( \phi^k \) and \( d^k \), \( k \in K \), denotes \( z_{\phi^k} \) as the earliest operation start time and \( z_{d^k} \) as the latest operation end time. Denote \( T \subseteq V \) as the set of trip nodes. For a trip node \( i \in T \), \( z_i \) denotes the scheduled start time; \( s_i \) and \( e_i \) represent the scheduled trip start time and trip energy consumption, respectively.

The arc set \( A \) includes three kinds of arcs: (i) The pull-out arc connects an operation begin node \( \phi^k \), \( k \in K \) and a trip node \( i \in T \), representing that an e-bus begins its operation from depot \( k \) to carry out the first trip \( i \); (ii) the pull-in arc connects a trip node \( i \) and an operation end node \( d^k \), \( k \in K \), representing that an e-bus completes its last trip \( i \) and return to depot \( k \); (iii) the trip connection arc connects two trip nodes, representing that two trips are carried out consecutively by and e-bus. Each arc \((i, j) \in A \) is associated with a non-service travel time \( t_{ij} \) and energy consumption \( c_{ij} \). Denote \( t^b \) as the buffer time between two successive trips. To ensure the trips are carried out without delay, trip nodes \( i \) and \( j \) can be connected only if the time compatible condition \( z_i + s_i + t_{ij} + t^b \leq z_j \) is satisfied. Each arc is associated with a cost \( c_{ij} \) including the non-service travel cost and vehicle usage cost \( c' \) on the pull-out arcs. We define \( z_{ij} \), \((i, j) \in T \) as the average unit charging cost in the time interval between the end of trip node \( i \) and the start of trip node \( j \) which is calculated according to the time-of-use tariff.

An example of the graph for an instance with two depots and four trips is shown in Figure 2. Each depot is created with an operation start node \( \phi^k \) and an operation end node \( d^k \), \( k = 1, 2 \). Each trip is represented by a trip node \( i \) associated with a start time and an expected end time, \( i = 1, 2, 3, 4 \). The nodes are connected with each other by the three kinds of arcs. The feasible trip chain illustrated in Figure 1b corresponds to the graph in the following way: trip 1 to trip 4 in Figure 1b correspond to trip node \( i = 1, 2, 3, 4 \) in the graph; Shekou Port Depot corresponds to node \( \phi^1 \) and \( d^1 \) while Changlingdong Depot corresponds to node \( \phi^2 \) and \( d^2 \). The waiting and non-service travel arcs in Figure 1b are represented by the trip connection arcs in the graph.
We assume that the e-buses begin the operation with a fully charged battery and can get recharged at the depots anytime during the day. A safety range of the battery SoC is $[u^{\text{min}}, u^{\text{max}}]$. The energy consumption of a trip is proportional to the trip time with a rate $r_d$ (kWh/min). The charging rate of the e-bus battery is $r_s$ (kWh/min) with a charging preparation time $p^p$. Based on the above settings, the problem looks for the minimum cost schedule that satisfies the following constraints: (i) Each trip is carried out exactly once; (ii) the start time of each trip is respected; (iii) the vehicle relocation constraint is satisfied; (iv) the vehicle battery SoC is always kept within the safety range $[u^{\text{min}}, u^{\text{max}}]$. The list of notations for the MD-EVSP model is provided in Table 1.

**Table 1. Definitions of the sets, parameters and variables for the MD-EVSP model.**

<table>
<thead>
<tr>
<th>Sets</th>
<th>Parameters</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>$B$ Battery capacity of the e-bus</td>
<td>$X_{ijk}$ Binary variable that equals 1 if an e-bus housed at depot $k$ traverses arc $(i,j) \in A$, and 0 otherwise</td>
</tr>
<tr>
<td>$T$ Set of trip nodes</td>
<td>$a_0$ Earliest operation start time</td>
<td>$Z_i$ Lowest battery SoC of the vehicle at the trip start time of trip node $i$; At each operation start</td>
</tr>
<tr>
<td>$V$ Set of nodes</td>
<td>$b_0$ Latest operation end time</td>
<td>$Y_i$ (end) node $d^k$ ($d^k$), $k \in K$, $Y_i$ is the lowest battery SoC of the vehicle at the operation start (end) time.</td>
</tr>
<tr>
<td>$A$ Set of arcs</td>
<td>$s_i$ Trip time of trip node $i$</td>
<td>$W_{ij}$ Amount of energy to be charged after completing trip $i$ before starting trip $j$</td>
</tr>
</tbody>
</table>
3.2. MIP Model

Based on the above problem description, the MD-EVSP model is formulated as follows.

\[
\begin{align*}
\text{min} & \quad \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} X_{ijk} + \sum_{(i,j) \in A} g_{ij} W_{ij} \\
\text{s.t.} & \quad \sum_{k \in K} \sum_{j \in (i,j) \in A} X_{ijk} = 1, \quad i \in T \\
& \quad \sum_{j \in (i,j) \in A} X_{ijk} - \sum_{j \in (j,i) \in A} X_{jik} = 0, \quad i \in T, \quad k \in K \\
& \quad \sum_{\beta \in K \setminus \{k\}} \sum_{i \in V} X_{oi\beta k} = 0, \quad k \in K \\
& \quad \sum_{\beta \in K \setminus \{k\}} \sum_{i \in V} X_{id\beta k} = 0, \quad k \in K \\
& \quad Y_i - e_i + W_{ij} - e_{ij} + M \left(1 - \sum_{k \in K} X_{ijk}\right) \geq Y_j, \quad (i,j) \in A \\
& \quad W_{ij} - (z_j - z_i - s_i - t_{ij} - t^p) r_s \sum_{k \in K} X_{ijk} \leq 0, \quad (i,j) \in A \\
& \quad Y_i - e_i + W_{ij} - u_{\text{max}} \leq 0, \quad (i,j) \in A \\
& \quad u_{i,j}^{\text{min}} \leq Y_i \leq u_{i,j}^{\text{max}}, \quad i \in V \\
& \quad W_{ij} \geq 0, \quad (i,j) \in A \\
& \quad X_{ijk} \in \{0,1\}, \quad (i,j) \in A, \quad k \in K
\end{align*}
\]

Objective (1) minimizes the sum of vehicle usage, empty travel and charging cost. Constraints (2) ensure that each trip is performed only once. Constraints (3) are the flow conservation constraints. Constraints (4) and (5) require that each vehicle begin and end its operation at its base depot. Constraints (6) ensure the vehicle battery energy consistency of two successive trips. Constraints (7) restrict that charging on arc \((i,j)\) is performed only if it is selected and time is available. Constraints (8) require that the upper battery SoC level cannot be exceeded after recharging. Constraints (9) keep the vehicle battery SoC within the safety range. Constraints (10) and (11) define the domains of the variables.

4. Large Neighborhood Search Heuristic

In this section, we present the LNS heuristic for the multi-depot e-bus scheduling problem. Given an initial solution, the LNS heuristic explores better solutions regarding the objective function extensively by the solution destroy-and-repair mechanism. Node removal and reinsertion are the commonly adopted destroy-and-repair operators in solving the EVRP related problems. However, these operators are not effective in generating good-quality solutions for our problem. Figure 3a shows the expected form of a feasible trip chain which does not need vehicle relocation (long-distance non-service travel) to return to the base depot, and an infeasible trip chain which requires the e-bus to travel from depot \(b\) to depot \(a\) after completing the operation. Figure 3b illustrates that applying the trip node removal and reinsertion operators may result in neighborhood solutions with vehicle relocation between the end depots thus violating the vehicle relocation constraint. In addition, applying the node exchange operator cannot change the solution structure on a great extent and is not an effective way to search for the neighborhood of the current solution. Considering the above issue, we developed a novel solution formulation heuristic (Section 4.2) and applied it in the initial solution generation and neighborhood searching process of the LNS heuristic to generate high-quality solutions.
return to the base depot, and an infeasible trip chain which requires the e-bus to travel from depot \( b \) to depot \( a \) after completing the operation. Figure 3 (b) illustrates that applying the trip node removal and reinsertion operators may result in neighborhood solutions with vehicle relocation between the end depots thus violating the vehicle relocation constraint. In addition, applying the node exchange operator cannot change the solution structure on a great extent and is not an effective way to search for the neighborhood of the current solution. Considering the above issue, we developed a novel solution formulation heuristic (Section 4.2) and applied it in the initial solution generation and neighborhood searching process of the LNS heuristic to generate high-quality solutions.

Figure 3. (a) The form of feasible and infeasible trip chains; (b) Applying trip removal and reinsertion would result in infeasible trip chains.

4.1. Heuristic Framework

The LNS heuristic framework is presented in Algorithm 1. Given a set of unassigned trips as input, an initial solution \( x_0 \) is generated by a novel constructive heuristic Algorithm 2 (Section 4.2). \( x_0 \) is initialized as the current solution \( x_{cur} \) and best solution \( x^* \). In each iteration, a new temporary solution \( x' \) is generated by the Schedule Destroy and Repair procedure (Section 4.3). \( x' \) is subsequently improved by the local search operators (Section 4.4). The evaluation function \( f(x) \) consists of the cost of vehicle usage, empty travel and charging. If a neighborhood solution \( x' \) is interior to the current solution \( x_{cur} \) in terms of \( f(x) \), a simulated annealing method is used as the acceptance criteria: the probability of accepting \( x' \) as the current solution is determined by \( p = e^{(x_{cur} - x')/t} \); as \( t \) decreases with the searching process underway, \( p \) decreases. In the diversification process, a larger portion of the trip chains in \( x_{cur} \) are destroyed and then repaired by the solution destroy and repair operators. The searching procedure will end if there has been \( N_{iter} \) iterations without improvement in \( x^* \). Then \( x^* \) is returned as the best feasible solution.
The outline of Algorithm 2 is described as follows.

1. **Input**: A set of trips $T$
2. $x_0 \leftarrow$ Generate an initial solution by **Algorithm 2**
3. **Initialization**: $x^{cur} \leftarrow x_0$; $x^* \leftarrow x_0$; $t \leftarrow t_0$; $n = 0$;
4. Repeat
5. If diversification criterion satisfied
   6. $x^{cur} \leftarrow$ diversity ($x^{cur}$)
   7. $x' \leftarrow$ **Schedule Destroy and Repair** ($x^{cur}$)
   8. $x' \leftarrow$ **Local Search** ($x'$)
   9. If accept ($x'$, $x^{cur}$)
   10. $x^{cur} \leftarrow x'$
   11. If $f(x') < f(x^*)$
   12. $x^* \leftarrow x'$
   13. $t \leftarrow 0$
   14. $t \leftarrow at$
   15. $n \leftarrow n + 1$
6. Until $n > N_{iter}$
7. **Return** $x^*$

### 4.2. Solution Formulation Heuristic

In this section, we introduce a novel construction heuristic, Algorithm 2 to formulate an e-bus schedule given a set of trips. Algorithms 2 includes two steps: In the first step, a set of short trip chains named trip chain segments (TCSs) are generated, which starts from and ends at the same depot. In the second step, the TCSs are merged with each other to formulate complete trip chains. The reason for first formulating TCSs and then merging them is to satisfy the vehicle relocation constraint and avoid unnecessary vehicle relocation. The outline of Algorithm 2 is described as follows.

<table>
<thead>
<tr>
<th><strong>Algorithm 2</strong> Schedule Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input</strong>: A set of trips</td>
</tr>
<tr>
<td><strong>Output</strong>: A complete schedule</td>
</tr>
<tr>
<td><strong>Step 1</strong>: Formulate a set of TCSs by <strong>Algorithm 3</strong></td>
</tr>
<tr>
<td><strong>Step 2</strong>: Merge the trip chain segments by <strong>Algorithm 6</strong></td>
</tr>
</tbody>
</table>

An illustration of the solution formulation procedure of Algorithm 2 is presented in Figure 4.

**Figure 4.** An illustration of Algorithm 2 to generate a set of trip chains.
In the first step, a set of TCSs are generated by Algorithm 3 taking a set of trips \( S^l \) as input. The procedure of Algorithm 3 is as follows: Firstly, the trips are sorted in non-decreasing order of their start times. A new trip chain is created for each of the first \( n \) trips that cannot be appended to other trips where \( n \) is generated by a geometric distribution \( n \sim G(p_1) \). Then, for each unassigned trip \( t \), we append it to the end of the existing trip chains, generating a set of candidate TCSs; the TCS with the lowest cost will be retained. If the trip cannot be successfully appended, a new trip chain will be created for it. After all the trips are assigned, the trip chains that do not end at the base depot are regarded as infeasible and removed. The feasible set of TCSs is denoted as \( S^c \) and the unassigned set of trips is denoted as \( S^l_{\text{left}} \). \( S^u \) and \( S^l \) are input to Algorithm 4 to generate new TCSs.

**Algorithm 4 TCS Formulation with Unassigned Trips**

**Input:** Set of trips \( S^l_{\text{left}} \); set of trip chain segments \( S^c \)

**Output:** Set of trip chain segments \( S^c_{\text{new}} \)

**Repeat**

**Repeat**

- Select the trip chain \( u \in S^c \) with the latest operation start time
- Add the trips in \( u \) and \( S^l_{\text{left}} \) into a new empty set \( S^l_1 \)
- Generate a set of TCSs \( S^c_u \) with unassigned trips \( S^l_u \) by Algorithm 5 (\( S^l_1 \))
- If \( |S^c_u| < |S^l_{\text{left}}| \)
  - \( S^c \leftarrow S^c \setminus u \cup S^c_u \) and \( S^l_{\text{left}} \leftarrow S^l_u \)
- Until \( S^l_{\text{left}} = \emptyset \) or \( S^c \) is traversed
- Relax the vehicle relocation constraint

**Until** \( S^l_{\text{left}} \neq \emptyset \)

In Algorithm 4, in each iteration, we select and destroy an existing TCS \( u \in S^c \) and reformulate new TCSs using the trips in \( u \) and \( S^l_{\text{left}} \). This reformulation is achieved by a trip chain formulation subroutine (Algorithm 5) using the k-regret trip insertion principle introduced in Wen et al. [7]. If the number of unassigned trips is reduced after the reformulation procedure, the new TCSs will be retained. The iteration will terminate if \( S^l_{\text{left}} \) is empty or the TCSs in \( S^c \) have been traversed. If all the trips are not assigned after the iteration terminates, the procedure of Algorithm 4 will repeat again to generate new TCSs without feasibility checking. This setting will ensure that all the free trips in \( S^l_{\text{left}} \) can be assigned.

**Algorithm 5 K-regret Insertion TCS Formulation**

**Input:** Set of trips \( S^l_1 \)

**Output:** Set of TCSs \( S^c_u \) and set of unassigned trips \( S^l_u \)

Initialize set \( S^c_u = \emptyset \)

**Repeat**

- Select a trip \( t \in S^l_1 \)
- Search for feasible insertion positions for \( t \) into the TCSs in \( S^c_u \)
- If the insertion positions exist
  - Generate a set of candidate TCSs \( S^c_u \) for all feasible insertion positions of \( t \)
  - Calculate the \( k \) criteria \( \Sigma_{i=1}^{k} (f^i(u) - f^i(\text{previous})) \), where \( f^i(u) \) is the \( i \)-th best evaluation value of \( u \in S^c_u \)
  - Retain the best candidate TCS \( u^* \) in \( \cup_{t \in S^l_1} S^c_u \) and the corresponding trip \( t^* \)
  - \( S^c_u \leftarrow S^c_u \cup u^* \); \( S^l_1 \leftarrow S^l_1 \setminus t^* \)
- Else
  - Create a new TCS \( u \) for trip \( t \)
  - \( S^c_u \leftarrow S^c_u \cup u \); \( S^l_1 \leftarrow S^l_1 \setminus t \)
- Until \( S^l_1 = \emptyset \)

Remove all TCSs \( u \in S^c_u \) with vehicle relocation and add the trips of \( u \) into set \( S^l_1 \).
In the second step, after all the trips are assigned, the TCSs are merged together to formulate complete trip chains by Algorithm 6. In Algorithm 6, in each iteration, two TCSs are selected and merged into one TCS by Algorithm 7. Firstly, one TCS \( u \) is selected and merged with each of the remaining TCSs in \( S' \), creating a set of candidate TCSs \( S_{\text{cand},u}^c \). Then the candidate TCS \( u \in S_{\text{cand},u}^c \) with the \( i \)-th lowest evaluation value \( f(u) \) is retained as the merged TCS and \( S'' \) is renewed; \( i \) is generated according to a geometric distribution \( i \sim G(p_2) \). The iteration will terminate when no feasible merging exists. The set of complete trip chains will be returned.

**Algorithm 6: Merge Trip Chain Segments**

**Input:** A set of TCSs \( S^c \)

**Output:** Set of merged trip chain segments \( S_{\text{merge}}^c \)

\[
S_{\text{merge}}^c \leftarrow S^c
\]

**Repeat**

1. Sort the trip chains in \( S^c \) in increasing order of the operation start time;

2. **Repeat**
   - Select the first trip chain \( u \in S^c \),
   - Attempt to merge \( u \) with the remaining trip chains in \( S^c \) by **Algorithm 7**
   - If merge succeeds
     - Add the merged trip chain into \( S_{\text{merge}}^c \)
     - Remove the two original trip chains in \( S^c \) and \( S_{\text{merge}}^c \)

3. Until \( S^c = \emptyset \)

4. Add all the trips chains in \( S_{\text{merge}}^c \) into \( S^c \)

5. Until no feasible merge exists

**Algorithm 7: Merge One Trip Chain Segment**

**Input:** Trip chain \( u \) and set of trip chains \( S^c \)

**Output:** Merged trip chain \( u_m \)

\[
S^c \leftarrow S^c \setminus \{u\}; S_{\text{cand},u}^c = \emptyset
\]

**Repeat**

1. Select the next \( u' \in S^c \)
2. Generate a candidate TCS \( u'' \) by merging \( u \) with \( u' \);
   \[
   S_{\text{cand},u}^c = S_{\text{cand},u}^c \cup \{u''\}
   \]
3. Until the TCSs in \( S^c \) are traversed
4. If \( S_{\text{cand},u}^c \neq \emptyset \)
   - Select the TCS \( u_m \) with the \( i \)-th lowest evaluation value from \( S_{\text{cand},u}^c \) by evaluation function \( f^i(u) \) where \( i \sim G(p_2) \);
   - Else
     - \( u_m = \text{null} \)

4.3. Neighborhood Solution Generation

A neighborhood solution is generated by the **Schedule Destroy Repair** procedure.

**Schedule Destroy Repair Procedure**

**Step 1:** **Schedule destroy**. A number of \( N_d \) trip chains are destroyed. Firstly, a number of \( \beta N_d \) trip chains with the highest evaluation value \( f(x) \) are destroyed. The remaining \( (1 - \beta) N_d \) trip chains are destroyed randomly.

**Step 2:** **Schedule repair**. The unassigned trips from the destroyed trip chains are reformulated into a new set of trip chains by **Algorithm 2**, the infeasible trip chains among which is then fixed by **Algorithm 8**.
In this procedure, a number of $\beta N_d$ trip chains of the current solution are destroyed and then the unassigned trips are reformulated into new trip chains by Algorithm 2. The reformulated trip chains together with the remaining ones become a new neighborhood solution. In the reformulation process, the battery SoC constraint (iv) introduced in Section 3.1 is relaxed to allow for a larger searching range. This may cause some of the reformulated trip chains to be infeasible. Algorithm 8 is then used to fix the infeasible trip chains.

Algorithm 8 is a local search heuristic based on two operators: Trip exchange, trip remove and reinsertion. In each iteration, the trip chain $P$ violating the battery SoC constraint most is repaired. To repair $P$, we select a trip $t$ in $P$ and apply the trip exchange operator to exchange $t$ with a trip of other trip chains. If no feasible trip exchange exists and $t$ is a round trip, the trip remove-and-reinsert operator is applied to remove $t$ from $P$ and reinsert it into another trip chain. The overall process will terminate if all of the trips of $P$ are traversed. The overall process will terminate if all of the trip chains satisfy the battery SoC constraint or a maximum number of $N_{iter}$ iterations has been reached. After the above process, a new schedule is generated which will be accepted as a neighborhood solution only if it is feasible.

4.4. Local Search

In the local search step, we improve the schedule by the following three operators, which are applied in random order: (i) The trip subsequence remove and reinsertion: This operator removes a subsequence of trips from one trip chain and tries to insert it into another trip chain. (ii) Trip remove and reinsertion: This operator is applied if the timetable includes loop trips. It removes a loop trip from one trip chain and tries to reinsert it into another trip chain. (iii) Trip exchange: This operator selects two trips from different trip chains and tries to exchange their positions. The solution obtained by the first move, improving in the objective function $f(x)$, is accepted.

5. Numerical Experiments

5.1. Data Preparation

In this section, we conduct numerical experiments based on the bus routes in Shenzhen with the aim to demonstrate the effectiveness of our LNS algorithm in solving real-world problem instances. The layout of the bus routes is displayed in Figure 5. This case includes six routes and four bus depots located at the end stops of the routes. All the depots are equipped with charging facilities for e-bus charging. A total number of 778 trips are scheduled on these routes. Figures 6 and 7 show the number of scheduled trips, average travel time and trip frequency at each depot during the day.

We first conducted experiments on single-route scheduling and then on multi-route scheduling where the e-buses are allowed to carry out trips on different routes. We acquired the parameters related to the charging technology and type of the e-bus from the transit agency. The BYD e-buses with a battery capacity of 260 kWh are used with the safety range of the battery SoC set between 30 and 100 percent of the battery capacity. According to the typical charging technology specifications of the investigated fleets, the charging rate is estimated as 1.6 kWh/min. Based on the analysis of historical running data, we estimate the electricity consumption rate as 0.45 kWh/min. Then, one day’s timetables were generated for the bus routes based on the headway requirements and historical running data. The time-of-use tariff in Shenzhen as shown in Table 2, is considered.
Figure 5. A display of the routes layout of the case.

Figure 6. Number of scheduled trips and average travel time of the routes.

Figure 7. Trip start frequency at each depot during the day.
Table 2. Time-of-use tariff in Shenzhen.

<table>
<thead>
<tr>
<th>Start</th>
<th>End</th>
<th>Price (RMB/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6:00</td>
<td>6:59</td>
<td>0.26</td>
</tr>
<tr>
<td>7:00</td>
<td>8:59</td>
<td>0.70</td>
</tr>
<tr>
<td>9:00</td>
<td>11:29</td>
<td>1.05</td>
</tr>
<tr>
<td>11:30</td>
<td>13:59</td>
<td>0.70</td>
</tr>
<tr>
<td>14:00</td>
<td>16:29</td>
<td>1.05</td>
</tr>
<tr>
<td>16:30</td>
<td>18:59</td>
<td>0.70</td>
</tr>
<tr>
<td>19:00</td>
<td>20:59</td>
<td>1.05</td>
</tr>
<tr>
<td>21:00</td>
<td>22:59</td>
<td>0.70</td>
</tr>
<tr>
<td>23:00</td>
<td>23:59</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Before the experiments, a subset of instances including small and large-scale ones was selected for parameter tuning. Table 3 provides the parameter setting of the LNS heuristic which has proved to be robust in the preliminary tests.

Table 3. Parameter setting in the numerical experiments.

<table>
<thead>
<tr>
<th>Para.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_{\text{iter}})</td>
<td>3000</td>
</tr>
<tr>
<td>(N_{\text{iter}})</td>
<td>100</td>
</tr>
<tr>
<td>(l_0)</td>
<td>1</td>
</tr>
</tbody>
</table>

In the numerical experiments, the MIP model was solved by optimization solver Cplex 12.6 with a time limit of 3600 s and the LNS heuristic was coded in Java. All the experiments were run on a PC with Windows 10, Intel Core i5-8250U, 1.80 GHz and 8 GB RAM.

5.2. Cases with Single Route

We analyzed the performance of the MIP model and LNS heuristic based on the six bus routes. The computational results are presented in Table 4. It shows the number of timetabled trips (#Trips), e-buses used (#V), objective of the MIP model solved by Cplex and LNS heuristic (Obj), computational time, and the solution gap. The MIP gap is the gap between the objective and lower bound value obtained by the Cplex. The gap of the LNS heuristic is the gap between the solution and objective of the MIP model. If the MIP gap is higher than 95% within 3600 s of computational time, we report the lower bound obtained by Cplex in the bracket.

Table 4. Computational results of the single route instances.

<table>
<thead>
<tr>
<th>Route</th>
<th>#Trips</th>
<th>#V</th>
<th>Obj</th>
<th>Run time (s)</th>
<th>Gap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MIP</td>
<td>LNS</td>
<td>MIP</td>
<td>LNS</td>
</tr>
<tr>
<td>M299</td>
<td>78</td>
<td>12</td>
<td>12</td>
<td>12,132.1</td>
<td>12,130.5</td>
</tr>
<tr>
<td>M409</td>
<td>92</td>
<td>8</td>
<td>8</td>
<td>8127.1</td>
<td>8128.4</td>
</tr>
<tr>
<td>90</td>
<td>114</td>
<td>6</td>
<td>6</td>
<td>6000.0</td>
<td>6000.0</td>
</tr>
<tr>
<td>42</td>
<td>136</td>
<td>14</td>
<td>14</td>
<td>14,327.2</td>
<td>14,315.8</td>
</tr>
<tr>
<td>43</td>
<td>142</td>
<td>16</td>
<td>16</td>
<td>16,581.1</td>
<td>16,562.0</td>
</tr>
<tr>
<td>81</td>
<td>216</td>
<td>38</td>
<td>38</td>
<td>(38,000)</td>
<td>38,659.9</td>
</tr>
</tbody>
</table>

The results show that the MIP model can be solved to near optimal by Cplex with small MIP gap for all the instances except the instance for route No. 81. The LNS heuristic can generate near-optimal solutions in a short time for all the instances. The computational time is positively related to the number of trips involved.
5.3. Cases with Multiple Routes

We obtained the optimal schedule by the LNS heuristic on multiple routes under two scenarios regarding the configuration of the charging facilities. In scenario I, all the depots are equipped with charging facilities; in scenario II, we assume that Moon Bay Depot is not equipped with charging facilities. As such, the e-buses housed at Moon Bay Depot need to be moved to out-of-depot charging stations for recharging during the off time. We assume that 10 percent of energy will be consumed traveling from the depot to the charging station. Therefore, the e-buses will start the operation from the depot with 90 percent of battery SoC and should maintain at least 40 percent of battery SoC at the end of the operation.

Table 5 presents the computational results of the multi-route case including the number of e-buses used (#V), objective obtained by the LNS heuristic (Obj), charging cost and computational time. The LNS heuristic can obtain high-quality solutions within a reasonable computation time and enable the operator to organize the fleet in an efficient way. Compared with scenario I, scenario II requires more e-buses to be utilized, as the e-buses housed at Moon Bay Depot have higher daily charging demand than those housed at other depots. With more time being spent on charging, the utility of the e-buses on service trips decreases. Therefore, equipping charging infrastructure at bus depots is critical and beneficial in maintaining high operation efficiency of the e-buses.

Table 5. Computational results of the multi-route instances.

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>#V</th>
<th>Obj</th>
<th>Charging Cost</th>
<th>Run time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario I (All depots charging available)</td>
<td>93</td>
<td>95,328</td>
<td>2327.7</td>
<td>3545.3</td>
</tr>
<tr>
<td>Scenario II (Moon Bay charging unavailable)</td>
<td>98</td>
<td>100,413</td>
<td>2412.8</td>
<td>4096.5</td>
</tr>
</tbody>
</table>

Compared with the single-route scheduling mode under which 94 e-buses are used in total, the multi-route scheduling mode saves one e-bus being utilized. Figure 8 shows the comparison of the total cost and charging cost of single-route and multi-route scheduling. We can see that the daytime charging cost under multi-route scheduling is higher than that under single route scheduling. This is because under multi-route scheduling, 93 e-buses are used, causing each e-bus to charge more during the day; however, the total operational cost is reduced greatly under multi-route scheduling. As such, there is a trade-off between the fleet size and charging demand which together determine the operational cost.

![Figure 8. Comparison of the total cost and charging cost of single route and multi-route scheduling.](image-url)
To determine the capacity required for the charging facilities at the depots, we calculated the number of e-buses charging at the depots during different time of the day as shown in Figure 9. We can observe that Shenzhen North Railway Station Depot has the highest charging demand among all the depots. 6 and 8 charging piles are required to satisfy the e-bus charging without waiting under single route and multi-route scheduling, respectively. The e-buses tend to charge during the off-peak time regarding the time-of-use tariff which can result in a lower charging cost. Compare Figure 9b for multi-route scheduling with Figure 9a for single route scheduling, we can see multiple route scheduling requires larger number of chargers. This is because a smaller number of e-buses are used and the daily charging amount is increased, which coincides with the results in Figure 8. Figure 10 gives the maximum number of chargers required at each depot during the daytime operation under single-route and multi-route scheduling mode. This value can provide a reference for the transit agency to determine the number of chargers that should be reserved for e-bus charging during the day to ensure the e-buses can be charged without waiting upon arrival at the depots. However, to determine the total number of chargers that should be established at the depots, the capacity of the depot, fleet size, and night-time charging activity should also be taken into consideration.

Figure 9. The number of e-buses charging at the depots during the day under (a) single route scheduling and (b) multi-route scheduling.
5.4. Sensitivity Analysis

We also analyzed the impact of the vehicle battery capacity and charging rate on the number of vehicles used and operational cost on the multi-route case with all depots charging available. The ranges of battery capacity and charging rate take the values in the intervals [180, 340] kWh and [0.8, 2.4] kWh/min, respectively. Within the ranges of the charging rate and battery capacity, the fleet size of the schedules stays the same, all being 93 vehicles. Figure 11 shows the operational cost of the e-bus fleet under different charging rate and battery capacity. In Figure 11a, under the battery capacity of 260 kWh, the operational cost decreases with the increasing of the charging rate. As the charging efficiency is improved, e-buses operate more efficiently and can choose to charge in the periods with lower tariff, resulting in a decrease in the charging cost. In Figure 11b, under the charging rate of 1.6 kWh/min, the operational cost decreases with the increasing of the battery capacity. The longer driving range per charge and less time spent in charging make the e-buses operate more efficiently. It is interesting to find that the speed of the decrease tends to lower down, meaning that the effect of cost-saving by increasing the battery capacity decrease gradually. As such, it is best to adopt e-buses with moderate battery capacity for operation from the economic viewpoint.

Figure 10. Number of chargers required each depot under single route and multi-route scheduling.

Figure 11. Sensitivity analysis of the charging rate and battery capacity on the operational cost of the e-buses. (a) The impact of charging rate on the operational cost; (b) The impact of battery capacity on the operational cost.
6. Conclusions

In this paper, we addressed a large-scale multi-depot e-bus scheduling problem considering the vehicle relocation constraint, which is required in the e-bus operational scenarios in Shenzhen. Partial charging is allowed at the bus depots. An MIP model is formulated to minimize the total operational cost. An LNS heuristic is developed to solve the large-scale problem instances with hundreds of trips. The vehicle relocation constraint renders the destroy-and-repair operators commonly used in the existing LNS heuristics for EVSP problems ineffective in our problem. As such, we devised a novel solution formulation procedure being used in the initial solution generation and solution repair process of our LNS heuristic.

We conducted numerical experiments based on the e-bus operation cases in Shenzhen. The results showed that the LNS heuristic can generate high-quality solutions for both single-route and multi-route scheduling cases. Sensitivity analysis was conducted to investigate the impact of the charging rate and battery capacity on the operational cost of the e-buses fleet. The number of charging piles reserved for daytime charging of the e-buses at the depots can be determined based on the optimal charging plan. The following managerial insights are derived based on numerical experiments:

- Compared with single-route operation mode, multi-route operation mode can save the number of vehicles utilized at the expense of a higher charging cost. As such, the optimal schedule with the lowest total operational cost requires a balance between the vehicle usage cost and charging cost.
- Equipping enough charging facilities at bus depots is critical in maintaining high operation efficiency of the e-buses.
- With a certain number of scheduled trips, increasing the battery capacity can reduce the operational cost; however, the effect of the reduction tends to decrease. Increasing the charging rate can reduce the operational cost.

In large bus depots connecting multiple bus routes with small headways, the charging activities of the e-buses can have a significant impact on the electricity grid. As such, it is interesting for future study to optimize the e-bus operation schedule with the charging plan that balances the charging demand over the day to minimize the impact of the charging activities on the electricity grid.

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