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An EOQ Model with Carbon Emissions and Inflation for Deteriorating Imperfect Quality Items under Learning Effect

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Abstract: We developed an economic order quantity (EOQ) model with a learning effect and carbon emissions under inflationary conditions and inspection for retailers where the items deteriorate naturally. Finally, the total profit of the retailer is maximized with respect to cycle length. A sensitivity analysis was also performed to understand the robustness of the model. In the sensitivity analysis, we discuss the impact of learning rate, inflation rate, and deterioration rate on lot size and length of the cycle, as well as the retailer's entire profit function. Observations and managerial insights are discussed. The effect of inventory parameters on the total profit is shown in the sensitivity section.

Keywords: carbon emissions; deterioration; inflation; inspection; learning effects

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1. Introduction

Many items are damaged due to deterioration, which is a natural process that cannot be ignored. Further, the freshness and quality of many products, such as food, flowers, vegetables, medicines, etc., does not decrease instantaneously but decreases after some time. However, some products suffer damage at a high rate of deterioration, and these are known as waste products. In ref. [1], Whitin presents an inventory model that considers the deterioration of fashion commodities and the expiry date of decaying items. In this way, in ref. [2], an EOQ model was developed for decaying items where the deterioration rate was considered to be exponential. Preservation technologies are managed with the help of electricity and generators; however, these systems produce very harmful gases such as carbon dioxide, methane, etc., and are damaging to the environment due to carbon emissions. The aim of the Paris Agreement of 12 December 2015 on environment change was to decrease global warming by 1.5 °C compared with pre-industrial levels. In ref. [3], the author suggests that carbon emissions are one of the most effective mechanisms to curb climate change and have the least negative impact on the economy of a country. Many governments have imposed a tax on industries for carbon emission due to various operational activities which, in turn, are associated with inventory. Hammami et al. [4] found that the attitudes of customers can force companies to produce green products and less carbon and suggested that the demand of consumers depends on the amount of carbon emissions. Research on inventory models has expanded rapidly and in the literature, different researchers have included different types of concepts in their models and results; thus, the total number of inventory models has also increased.

In the past, researchers have analyzed inventory systems under various realistic situations of deterioration, inflation, and learning. The basic EOQ model was developed by Buzacott [5] for deteriorating items with inflation conditions under different policies. Ref. [6] proposed an inventory model for the EOQ with inflation conditions under a different strategy. In this paper, it is considered that lots have some defective items. Salameh and Jaber [7] presented a model for EOQ that assumes that a proportion of the products in a lot under inspection will be defective. An EOQ model was proposed by the authors of [8] for defective quality items under the inspection process with a credit financing policy where the demand rate is less than the screening rate and shortages are allowed. Learning concepts provide good choices to the seller and buyer during the transaction of business. Retailers generally want less product and more profit and analyze how to minimize total cost. Ref. [9] was the first to formulate the behavior of learning in the form of a quantitative shape, which is known as the "learning curve". The aim of our work is to optimize the retailer's total current profit with respect to cycle length and to determine the effect of learning on the retailer's total profit under inflationary conditions and carbon emissions where defective items follow an S-shaped learning curve. Observations and managerial insights are discussed in the analysis part of the paper. The results and future extension of this study are explained in the conclusion with the help of sensitivity analysis.

2. Literature Review

2.1. Literature Review with Regard to Carbon Emissions and Inspection

Different authors have different strategies to control carbon emissions and many companies have willingly adopted mechanisms that help reduce carbon emissions. A number of researchers have studied which mechanisms are more effective in reducing carbon emissions. Ref. [10] reported that due to consumer awareness of the climate, supply chain management profits are declining while consumer utility is increasing. Ref. [11] applied a different carbon emission constraint on a lot size issue in order to limit carbon emissions per unit of item. Hu and Zhou [12] improved a model under carbon emissions with a trade credit strategy. Datta and Pal [13] presented two inventory models to maximize profits to reduce carbon emissions by adopting preservation technology and carbon tax policy. Daryanto et al. [14] provided a three-echelon inventory model to reduce carbon emissions and minimize the overall supply chain inventory costs. Ref. [15] presented an EOQ model for carbon emissions reduction. Ref. [16] derived a mathematical model for EPQ under shortages. Hovelaque and Bironneau [17] proposed an EOQ model that considered carbon emissions-dependent demand. The authors of ref. [18] derived a sustainable EOQ model and its theoretical formulation and applications, while Chen and Benjaafar [19] proposed an inventory model for the carbon constrained EOQ.

2.2. Literature Review with Regard to Defective Items, Inflation and Inspection

Inflation suggests one should procure more, meaning more investment inventory, which is highly correlated with the return on investment. As inflation and deterioration have a push and pull effect on the optimal cycle length, order quantity, and profit, their impact on the formulation of an the inventory model cannot be ignored. Due to the need to include inflation and deterioration in this paper, in this section, we discuss various studies related to inflationary conditions under different considerations. A model with inflationary condition under shortages where the demand rate is a linear function of time is discussed in Datta and Pal [13]. Sarker and Pan [20] proposed an inventory model with shortages under inflationary condition when inflation rate effects on order quantity. Moreover, Hariga [21] derived a mathematical model for decaying items under shortages where order rate is linear function of time. Hariga and Ben-Daya [22] generalized an EOQ model for lot sizing problem under inflationary situation. Jaggi et al. [23] improved a mathematical model with inflationary situation for decaying items influence of credit financing scheme where lots have defective items. Manna and Chaudhuri [24] proposed an inventory model with inflationary situation for decaying items under credit period strategy.

2.3. Literature Review with Regard to Defective Items, Inspection, and Learning Effect

We can say that learning includes the progress of the knowledge with the practice. The secreted information obtained through learning effects becomes essential to support the decision-making. During inflation how much order quantity should be ordered? In this situation, learning tools can rectify this situation when order quantity is not fixed, and it is changing per shipment. The number of shipments is the most important parameter during the transaction of order quantity in the business. Jaber and Goyal [25] explained an inventory model with a learning effect where a delivered lot has some defective items which follow the S-shaped learning curve. Jaber and Bonney [26] investigated the impact of the learning concept on the lot size. In a similar way, Khan et al. [27] developed a mathematical model with a screening rate where production cost followed the effect of learning. Konstantaras and Jaber [28] proposed an inventory model with shortages for defective items under learning effect. Jaggi et al. [8] derived a model with shortages under credit financing scheme. Tiwari et al. [29] introduced an inventory model with a fixed credit period policy for defective items when the demand rate depended on time.

Further, Agarwal et al. [30] proposed a model with learning and shortages for perishable items. Nobil et al. [31] derived a production model with shortages and rework under inspection. Jayaswal et al. [32] proposed an economic order quantity model with a learning and trade credit scheme. Jayaswal et al. [33] presented an economic production model with learning when demand was a function of the credit period. Kahin et al. [27] developed a model with the effect of learning under the credit period policy for perishable items for imperfect quality items. Jayaswal et al. [34] discussed fuzzy-based economic order quantity (EOQ) model with credit financing and backorders. Yadav et al. [35] proposed an inventory model which used the game theory approach for finding the optimal ordering policy for imperfect quality items. Kumar et al. [36] assumed new product launching with pricing, free replacement, rework, and warranty policies via a genetic algorithmic approach. Further, Jaggi et al. [23] developed a model with credit financing for deteriorating imperfect-quality items under inflationary conditions.

2.4. Research Gap

We studied the literature mentioned in the literature review section regarding carbon emissions, inflation with learning concepts for imperfect quality items. The contribution table of the authors is presented in Table 1. It is found from the literature survey that a lot of research papers were published with carbon emissions concepts under different situations, but there is no research work available regarding carbon emissions and inflation under learning concept for deteriorating imperfect quality items. The impact of waste management on the economic order quantity model is also studied due to the deteriorating nature of the product. The cost of waste management is also considered in this model. The objective of the present study is to develop an inventory model with carbon emissions and inflation under the learning effect for deteriorating imperfect quality items. Our contribution is shown at the bottom row of Table 1 with specified keywords. The present work has tried to fill this research gap.

2.5. Contribution Concern with Proposed Model

In Table 1, our contribution is shown at the bottom row of the contribution table with specified keywords. After formulation, the present model provided a positive effect on the order quantity, cycle length, and buyer’s total profit under carbon emissions. The importance of learning concepts in the presented model is demonstrated in the numerical example. From the numerical example, retailers obtain more profit as the learning parameter increases. Further, the best ordering policy is presented with all these concepts. The comparative results are presented in Table 2.

Table 1. Contribution table.

Author(s)	Impact of Learning	Inspection	Carbon Emissions	Deterioration	Defective Items	Inflation
Wright [9]	✔					
Salameh and Jaber [7]		✔			✔	
Jaber et al. [25]	✔	✔			✔	
Khan et al. [27]	✔	✔			✔	
Jaggi and Khanna [37]		✔		✔	✔	✔
Jaggi et al. [23]		✔		✔	✔	✔
Jaggi et al. [8]		✔			✔	
Jaggi et al. [38]		✔		✔	✔	✔
Patro et al. [39]	✔	✔		✔	✔	
Daryanto et al. [14]		✔	✔	✔	✔	
Liao et al. [40]				✔		✔
Daryanto and Christata [41]		✔	✔		✔	
Barman et al. [42]				✔		✔
Jayaswal et al. [32]	✔				✔	
Jayaswal et al. [43]	✔	✔		✔	✔	
Mashud et al. [44]		✔		✔	✔	
This paper	✔		✔	✔	✔	✔

Table 2. Comparison of cycle length, lot size, and buyer's whole profit with and without learning rate.

Model with No Learning Effect			Model with Learning Effect		
Cycle Length T_n^* (Year)	**Lot Size Q^* (Units)**	**Buyer's Total Profit $\Psi(T_n)$ (Dollars)**	**Cycle Length T_n^* (Year)**	**Lot Size Q^* (Units)**	**Buyer's Total Profit $\Psi(T_n)$ (Dollars)**
0.9032	46,694	1,472,210	1.0049	48,225	1,662,440

3. Assumptions

(i) CO_2 is directly emitted from electricity and fuels consumption in product storage Daryanto et al. [14].
(ii) Waste due to the deterioration process which is dangerous for the climate is properly arranged by investing in the waste management process.
(iii) The continuity of replacement is allowed.
(iv) Shortages and lead time are not involved in this model.
(v) The screening rate is greater than the demand rate [8].
(vi) The time horizon plane has been taken as finite.
(vii) Lots have some defective items as per consideration by [7].
(viii) Defective quality items follow the S-shaped learning curve suggested by Jaber and Goyal [25].
(ix) Imperfect items are sold at rebate prices.
(x) Lots have a constant deterioration rate in the whole cycle length.
(xi) The inflation rate is constant.
(xii) A carbon tax is allowed.

4. Mathematical Model

According to assumptions, the inventory level Q at time $t = 0$, may have defective and nondefective items. The entire delivered lot has been inspected at a constant rate of λ units/year and Q items are divided into perfect and imperfect quality items. Further, it is also considered that the inspection time is $t_n = \frac{Q}{\lambda}$. After the inspection process, the defective items are sold at the salvage price, c_s (see Figure A1). To remove the shortages, it is assumed $(1-P(n))Q \geq Dt_n$, which infers that, $P(n) \leq 1-\frac{D}{\lambda}$, where $t_n=\frac{Q}{\lambda}$. Further, it is also assumed that $I_1(t)$ is the inventory in the time period $[0,t_n]$ and equal to $I_1(t)=Q\,e^{-\theta\,t}+\frac{D}{\theta}\left[e^{-\theta\,t}-1\right]$ and $I_2(t)$ is the inventory in the time period $[t_n,T_n]$ which is equal to $I_2(t)=\frac{D}{\theta}\left[e^{\theta\,(t_n-t)}-1\right]+[(1-P(n))Q-D\,t_n]\,e^{\theta\,(t_n-t)}$ as well as order quantity Q at $t=0$, $\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)}$ and the total deteriorating quantity Dq is $(Q-D\,T-Q\,P(n))$ units because inventory level reduces due to both demand and deterioration. The retailer received few units which were completely damaged due to deterioration, namely as waste units. The total holding inventory in the warehouse under the impact of inflation is $\left[\frac{Q}{(\theta+R)}\left[1-e^{-T_n(R+\theta)}\right]+\frac{D}{\theta}\left[\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)}+\frac{e^{-RT_n}-1}{R}\right]+\frac{P(n)\,Q}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\right]$ units. Retailer's total income with inflation is the sum of demand met from the interval $[0,\ T_n]$, say SR_1, and sale of defective quality items, say SR_2, and these are equal to $\frac{pD}{R}\left[1-e^{-R\,T_n}\right]+c_s\,P(n)Qe^{-R\,t_n}$.

Now, the retailer's total cost is the sum of the holding cost, ordering cost, waste management cost, inspection cost, deterioration cost, and carbon emission cost. When items are damaged due to a high deterioration rate in the stock, then preservation controls the deterioration rate. Preservation is managed with the help of an electric generator which will generate carbon emissions. Thus, the new cost is added, which is termed as carbon emission cost. The emission cost due to electric energy,

$$(e_cE_eT_x)\left[\frac{Q}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right]+\frac{D}{\theta}\left[\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)}+\frac{e^{-RT_n}-1}{R}\right]+\frac{P(n)\,Q}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\right],$$

and due to electricity generation, is the

$$(e_cE_e)\left[\frac{Q}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right]+\frac{D}{\theta}\left[\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)}+\frac{e^{-RT_n}-1}{R}\right]+\frac{P(n)\,Q}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\right]$$

The buyer's total cost is addition of ordering cost C_k, screening cost C_sQ, waste management cost WMC is $C_wP(n)\quad Q+C_w(Q-D\,T_n-Q\,P(n))$, deterioration cost DC is $C_d(Q-D\,T_n-Q\,P(n))$ holding cost,

$$IHC \text{ is } C_h\left[\frac{Q}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right]+D\left[\frac{1-e^{-T_n(R+\theta)}}{(R+\theta)\theta}+\frac{e^{-RT_n}-1}{R\theta}\right]+\frac{P(n)\,Q}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\right]$$

total emission cost,

$$(e_cE_eT_x)\left[\frac{Q}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right]+\frac{D}{\theta}\left[\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)}+\frac{e^{-RT_n}-1}{R}\right]+\frac{P(n)\,Q}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\right]+$$
$$(e_cF_e)\left[\frac{Q}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right]+D\left[\frac{1-e^{-(\theta+R)T_n}}{(R+\theta)\theta}+\frac{e^{-RT_n}-1}{R\theta}\right]+P(n)\left(e^{-RT_n}-e^{-Rt_n}\right)\frac{Q}{R}\right]$$

and purchasing cost, C_pQ. By inserting all the costs into Equation (1) (see Appendix A), the retailer's total inventory cost is given below.

$$TC=C_k+C_sQ+C_pQ+WMC+IHC+EC+DC \tag{1}$$

$$
\begin{aligned}
TC = {} & C_k + C_s\left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}\right) + C_p\left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}\right) + C_w P(n)\left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}\right) \\
& + C_w\left(\left(\begin{array}{c} \frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)} - D\ T_n \\ -\left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}\right) P(n) \end{array}\right)\right) + C_h\left[\begin{array}{l} \frac{\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right] \\ +\frac{D}{\theta}\left[\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)}+\frac{e^{-RT_n}-1}{R}\right] \\ +\frac{P(n)\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}}{R}\left(e^{-RT_n}-e^{-Rt_n}\right) \end{array}\right] \\
& + (e_c E_e T_x)\left[\begin{array}{l} \frac{\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}}{(R+\theta)}\left[1-e^{-T_n(R+\theta)}\right]+ \\ \frac{D}{\theta}\left[\frac{1-e^{-(\theta+R)T_n}}{(R+\theta)}+\frac{e^{-RT_n}-1}{R}\right] \\ +\frac{P(n)\ Q}{R}\left(e^{-RT_n}-e^{-Rt_n}\right) \end{array}\right] + (e_c E_e)\left[\begin{array}{l} \frac{\frac{D\left(-1+e^{\theta\ T_n}\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}}{(R+\theta)}\left[1-e^{-T_n(\theta+R)}\right] \\ +\frac{D}{\theta}\left[\begin{array}{l} \frac{1-e^{-(\theta+R)T_n}}{(\theta+R)} \\ +\frac{e^{-RT_n}-1}{R} \end{array}\right] \\ +\frac{P(n)\ \frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}}{R}\left(e^{-RT_n}-e^{-Rt_n}\right) \end{array}\right] \\
& + C_d\left(\left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)} - D\ T_n - \left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}\right) P(n)\right)\right)
\end{aligned} \tag{2}
$$

The retailer's total revenue,

$$SR = \frac{pD}{R}\left[1-e^{-R\ T_n}\right] + c_s\ P(n)\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}e^{-R\ t_n} \tag{3}$$

The retailer's total profit,

$$SR - TC \tag{4}$$

Now, from Equations (2) and (3), the buyer's total profit is

$$
\begin{aligned}
\Psi(T_n) = {} & \frac{pD}{R}\left[1-e^{-R\,T_n}\right] + c_s\,P(n)\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)}e^{-R\,t_n} - C_k - C_s\left(\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)}\right) \\
& -C_p\left(\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)}\right) - C_w P(n)\left(\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)}\right) - C_w\left(\left(\begin{array}{l}\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)} - D\,T_n \\ -\left(\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)}\right)P(n)\end{array}\right)\right) \\
& -C_h\left[\begin{array}{l}\frac{\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)}}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right] \\ +\frac{D}{\theta}\left[\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)} + \frac{e^{-RT_n}-1}{R}\right] \\ +\frac{P(n)\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)}}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\end{array}\right] - (e_cE_eT_x)\left[\begin{array}{l}\frac{\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)}}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right]+ \\ \frac{D}{\theta}\left[\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)} + \frac{e^{-RT_n}-1}{R}\right] \\ +\frac{P(n)\ Q}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\end{array}\right] \\
& -(e_cE_e)\left[\begin{array}{l}\frac{\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)}}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right] \\ +\frac{D}{\theta}\left[\begin{array}{l}\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)} \\ +\frac{e^{-RT_n}-1}{R}\end{array}\right] \\ +\frac{P(n)\ \frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)}}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\end{array}\right] - C_d\left(\left(\begin{array}{l}\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)} - D\,T_n \\ -\left(\frac{D\left(e^{\theta\,T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\,T_n}\right)}\right)P(n)\end{array}\right)\right)
\end{aligned}
\tag{5}
$$

5. Solution Method

In this section, we optimize the buyer's total profit. The Mathematica 10.0 software tool is used to solve the complicated Equation (5). For maximum profit, the necessary condition will be $\frac{d\Psi(T_n)}{dT_n} = 0$, cycle length is obtained, $T_n = T_1$, (suppose) and after that, the second derivative is evaluated, $\frac{d^2\Psi(T_n)}{d{T_n}^2}$, and after substituting the value of $T_n = T_1$ in the second derivative, we obtain $\frac{d^2\Psi(T_n)}{d{T_n}^2} \leq 0$, then $T_n = T_1$ is the maximum value of T_n, which is represented by optimal cycle length ${T_n}^*$. As the concavity of total profit function $\Psi(T_n)$ has been shown graphically in Figure A2, solving the $\Psi(T_n)$ using the Mathematica tool function, the retailer's cycle time and total profit is obtained. Further, all the calculations and figures are presented in Appendix A.

Numerical Example

All the input parameters values have been taken from [14,23,32,45,46] for numerical discussion, and are given below in Table 3.

Table 3. Parameter values from [14,23,32,45,46].

D	$50,000$ units/year	F_e	0.0026 ton CO_2/L	b	1.40
λ	$175,000$ unit/year	E_e	0.0005 ton CO_2/kWh,	θ	0.1/year
p	USD50/unit	R	0.08	n^*	17
C_s	USD0.5/unit	C_k	USD2000/order	$p(17)$	2.36887×10^{-8}
C_p	USD25/unit	C_d	USD600/unit	T_n^*	1.00941 year
h	USD60/unit	a	40	Q^*	$48,225$ units,
e_c	1.44kWh/unit/year	g	999	t_n^*	0.2752 year
T_x	USD75/tonCO_2	C_h	USD2/unit	$\Psi(T_n^*)$	USD$1,662,440$

First of all, we calculate retailer's cycle time (decision variable) with the help of solution method $\frac{d\Psi(T_n)}{dT_n} = 0$, and then $T_n^* = 1.00941$ year, and after inserting the value of cycle length, the second derivative value at cycle length value is given as $\frac{d^2\Psi(T_n)}{dT_n{}^2} \leq 0 = -21$, then buyer's optimal cycle length is $T_n^* = 1.00941$ year, Then, we insert the value in the optimal order formula and obtain the value of optimal lot size $Q^* = \frac{D\left(e^{\theta\ T_n^*} - 1\right)}{\theta\left(1 - P(n)e^{\theta\ T_n^*}\right)} = 48,225$ units, inspection time $t_n = \frac{Q^*}{\lambda} = 0.2752$ year, and optimal retailer's total profit $\Psi(T_n{}^*) =$ USD$1,662,440$.

The order quantity is affected due to carbon emission and deterioration. The cycle time of 1.00941 years, order quantity 48,225 units, and total profit are 1,662,440 dollars with learning effect.

When the learning effect is relaxed in the present model, it means that the value of the learning rate becomes zero. The cycle length is 0.9032 years, lot size 46,694 units, and retailer's total profit USD1,472,210.

It is observed from the above results that cycle length, order size, and buyer's total profit are affected by the learning concepts. The results revealed that with the learning effect, total profit increases significantly.

6. Sensitivity Analysis

In this section, we are studying the impact of shipments, learning rate, inflation rate, and rate of deterioration on the cycle length, lot size, and buyer's total profit under carbon emission impact. First, the variation in the number of shipments is considered, and change in the retailer's total profit due to this variation is shown in Table 4. The effect of the inflation rate on the buyer's cycle length, order size, and buyer's total profit is presented in Table 5. The deterioration rate cannot be ignored, and its effect is shown in Table 6. Finally, the carbon emission impact is shown in Table 5 on the order quantity and buyer's profit.

Table 4. Effect of learning and shipments on the cycle length, order size, and buyer's total profit with carbon emissions.

Number of Shipment (n)	Rate of Learning								
	b=1.00			b=1.2 0			b=1.40		
	Cycle Time T_n	Lot Size Q	Retailer's Profit $\Psi(T_n)$(USD)	Cycle Time T_n	Lot Size Q	Retailer's Profit $\Psi(T_n)$(USD)	Cycle Length T_n	Lot Size Q	Retailer's Profit $\Psi(T_n)$(USD)
1	0.9034	46,669	1,472,500	0.9034	46,697	1,472,600	0.9035	46,698	1,472,720
2	0.9038	46,706	1,473,230	0.9040	46,709	1,473,750	0.9044	46,713	1,474,510
3	0.9047	46,720	1,475,010	0.9059	46,742	1,477,140	0.9078	46,755	1,480,720
4	0.9087	46,850	1,482,380	0.9109	46,824	1,486,610	0.9181	46,924	1,500,070
5	0.9124	46,852	1,489,280	0.9234	47,031	1,510,020	0.9422	47,331	1,545,150
6	0.9234	47,031	1,510,020	0.9467	47,400	1,553,570	0.9736	47,799	1,603,860
7	0.9422	47,331	1,545,150	0.9583	47,575	1,575,280	0.9937	48,075	1,641,570
8	0.9652	47,680	1,588,050	0.9918	48,052	1,637,910	1.0015	48,182	1,656,160
9	0.9841	47,954	1,623,590	1.0014	48,163	1,653,510	1.0036	48,195	1,660,660
10	0.9954	48,098	1,644,740	1.0032	48,204	1,659,370	1.0046	48,222	1,661,940
11	1.0091	48,586	1,654,940	1.0043	48,218	1,661,410	1.0048	48,222	1,662,300
12	1.0032	48,204	1,659,370	1.0047	48,223	1,662,100	1.0048	48,225	1,662,400
13	1.0042	48,217	1,661,200	1.0048	48,225	1,662,330	1.0049	48,225	1,662,430
14	1.0046	48,222	1,661,940	1.0048	48,223	1,662,400	1.0049	48,225	1,662,440
15	1.0048	48,224	1,662,240	1.0049	48,225	1,662,430	1.0049	48,225	1,662,440
16	1.0048	48,224	1,662,360	1.0049	48,225	1,662,420	1.0049	48,225	1,662,440
17	1.00491	48,225	1,662,440	1.00491	48,225	1,662,440	1.00491	48,225	1,662,440
18	1.00491	48,225	1,662,440	1.00491	48,225	1,662,440	1.00491	48,225	1,662,440

Table 5. Effect of deterioration rate on lot size, cycle length, and buyer's total profit with carbon emissions and fixed learning rate ($b = 1.4$) and no. of shipments ($n = 17$).

Deterioration Rate θ	Cycle Length T_n (Year)	Lot Size Q (Units)	Buyer's Total Profit $\Psi(T_n)$ (USD)
0.10	1.00491	48,225	1,662,440
0.15	0.6998	33,653	1,148,080
0.20	0.5365	25,828	876,186
0.25	0.4349	20,951	708,152

Table 6. Impact of inflation rate on cycle length, order size, and buyer's total profit with carbon emissions and fixed learning rate ($b = 1.4$) and no. of shipments ($n^* = 17$).

Inflation Rate R	Cycle Length T_n (Year)	Lot Size Q (Units)	Buyer's Total Profit $\Psi(T_n)$ (USD)
0.02	1.0349	49,735	1,710,570
0.04	1.0195	48,959	1,685,940
0.06	1.0049	48,225	1,662,440
0.08	0.9910	47,531	1,639,990

6.1. Observations and Managerial Insights

- From Table 4, it is seen that if the number of shipments and learning rate increase from top to bottom, then cycle length, lot size, and retailer's total profit rapidly increase up to the 10th shipment level with different learning rates. After the 10th shipment, cycle length, lot size, and retailer's total profit increase very slowly and approach the maturity phase up to the 16th, and this phase is called the learning phase. Finally, order size, cycle length, and buyer's total profit remain constant on 17th shipments and reach maturity phase. It means that retailers obtain the optimal length of cycle, maximum lot size, and maximum profit when the shipment is the 17th one and the learning rate is 1.4. Hence, the retailer obtains more profit due to decreased carbon emissions. It suggests that retailers should be aware of new strategies in the form of learning to obtain more profit.
- From Table 5, we analyzed that if the deterioration rate increases, then lot size, length of cycle, and buyer's profit reduce due to deterioration. Deterioration affects the cycle time and order quantity, as well as buyer's total profit. It reflects that the retailer should be aware during the transaction of business when products are deteriorating items. When this order quantity decreases, then carbon emissions increase due to deterioration. Hence, the retailer obtains less profit due to increased carbon emissions.
- From Table 6, we studied that if the rate of inflation increases, then the length of cycle, lot size, and retailer's profit decrease. Inflationary situations affect the lot size, cycle length, and buyer's total profit. It reflects that retailers should be aware during the transaction of business when products are deteriorating items. When this order quantity decreases, then the carbon emissions increase. Hence, retailers obtain less profit due to decreases in cycle time and order quantity and increase in carbon emission.

6.2. Discussion with Observations

We obtained more affirmative results from observations if the number of shipments increases from 1 to 16, then the order quantity and retailer's profit do not change when the learning rate is 1.40, whereas cycle length varies little. When the number of shipments increases after the 16th shipment level, then order quantity, cycle time, and retailer's profit do not change. It means that the order quantity, cycle time, and retailer's total profit become stable when the number of shipments is 17 and the learning rate is 1.40. Due to deterioration, the utility of the goods decreases, hence, it is optimal for the retailer to order for a shorter period, as under inflationary conditions the price of goods increases, therefore the retailer would like to order a large quantity for a longer period, which helps him to increase his profit. According to model assumptions, the combination of learning, inflationary condition, deterioration, carbon emissions, and inspection are favorable for the retailer. The effect of these parameters is separately shown in Tables 4–6. Currently, pharma companies are generating more profit due to the inflationary situation in COVID-19 for vaccination.

7. Concluding Remarks and Future Extension

We developed an EOQ model with carbon emissions and inflationary situation under learning effect where lots had imperfect deteriorating items and the retailer optimized his profit with respect to cycle time. The present study suggests that carbon emission is affected by cycle time (T_n^*) and order quantity, and carbon emission cost directly effects buyer's whole gain. A retailer wants to have definite cycle time and order quantity during the transaction of business, and this raises many questions in his mind regarding the shipments, cycle length, and order size. Due to variation of order quantity, retailers cannot make good decisions, and sometimes obtain profit or loss. Learning concepts suggest good decisions, and our work suggests that if a retailer wants to maximize his total profit with respect to the cycle time under a learning situation, then the retailer will have to manage shipments and learning rate up to the maturity phase. Further, Table 4 suggests that the cycle length (T_n^*), lot size (Q^*), and buyer's total profit $(\Psi(T_n^*))$ follow the S-shaped learning curve and achieve the maturity phase with variable shipment and learning rate. Furthermore, the retailer's cycle length (T_n^*), lot size (Q^*), and retailer's total profit $(\Psi(T_n^*))$ are affected by the inflation rate and deterioration rate under carbon emissions, which are discussed in the sensitivity analysis section. Our work is important for those who want to obtain an optimal lot size, optimal cycle length, and retailer's total profit with the various carbon emissions regulations imposed by the government or regulatory authorities. The observations revealed that (a) when the lot has more defective items, then the buyer should be more vigilant while ordering, (b) the present model provided good results when the learning rate is 1.40 and number of shipments is 17, (c) for high deterioration rate, the buyer should order less quantity more frequently, (d) in the highly inflationary market, the buyer should order a large quantity to increase his profit, and (e) the issue of environmental sustainability is addressed due to storage and is important for long-time existence. The present model is beneficial for business managers who want to obtain an optimal ordering quantity and strictly comply with carbon emission regulations imposed by the regulatory authorities or government. This research work can be extended by considering investments in green technology and exploring other mechanisms to decrease carbon releases.

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Notations

Q	Order quantity has been taken as decision variable (units).
D	Rate of demand (units/year).
C_h	Holding cost (USD/unit).
t_x	The imposed unit tax for emissions (USD/ton).
F_c	The carbon emission factor for fuels (tones/gallon).
E_c	The carbon emission factor for electricity (tones/kWh).
C_p	Unit purchasing cost (USD/unit).
p	Unit selling cost for perfect items (USD/units).
P	Percentage defective items are presents in Q.
$P(n)$	Imperfect quality items are following the S-shaped learning curve.
c_s	Unit selling price for imperfect items, $c_s < p$ (USD/unit).
C_s	Screening cost (USD/units).
θ	Deterioration rate (per year).
e_c	The variable amount of electricity utilized to store one unit of goods per time unit (KWh/year).
$C_h = h + t_x E_c e_c$	Holding cost due carbon emission from variable electricity (USD/unit/year).
$\widetilde{C}_h = e_c F_c$	Holding cost due carbon emission from generator fuels (USD/unit/year).
C_d	Deterioration cost (USD/unit).
C_w	Cost of waste management due to deterioration (USD/unit).
λ	Screening rate, $\lambda > D$ (USD/unit/year).
t_n	Inspection time (year).
T_n	Cycle length (year).
$I_1(t)$	Inventory at $t \in [0, t_n]$.
$I_2(t)$	Inventory at $t \in [t_n, T_n]$.
SR	Total sales revenue (USD).
TC	Total buyer's cost (USD).
$\Psi(T_n)$	Total buyer's whole profit (USD).
r	Discount rate at i inflation rate.
R	$r - i$, Inflation due to discount rate.

Appendix A

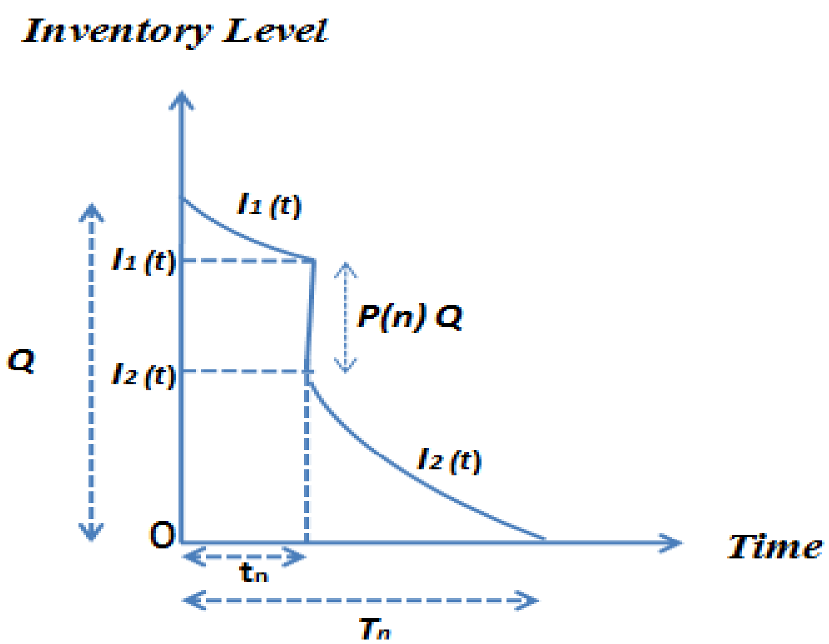

Figure A1. Inventory under inspection process.

$$\begin{aligned} &\tfrac{dI_1(t)}{dt} + \theta I_1(t) = -D, \quad t \in [0,\ t_n] \\ &\text{with boundary condition } I_1(0) = Q. \\ &I_1(t) = Q\, e^{-\theta\, t} + \left[e^{-\theta\, t}\tfrac{1}{\theta} - \tfrac{1}{\theta}\right] D \end{aligned} \tag{A1}$$

When the stock at $t = t_n$ is known as present inventory and it represented by *(EIL)* which is given in Equation (A2) [40]:

$$\begin{aligned} &EIL = I_1(t_n) - p(n)Q = Q\, e^{-\theta\, t} + \tfrac{D}{\theta}\left[e^{-\theta\, t} - 1\right] - p(n)Q \\ &= (1 - P(n))Q - D\, t_n \end{aligned} \tag{A2}$$

Again, find out the $I_2(T_n)$ in an interval $t \in [t_n, T_n]$ which follows as ODE with boundary condition which is given below:

$$\begin{aligned} &\tfrac{dI_2(t)}{dt} + \theta\, I_2(t) = -D,\ t \in [t_n,\ T_n] \\ &I_2(t) = IEL = (1 - P(n))z - D\, t_s,\ I_2(T_n) = 0. \\ &I_2(t) = \tfrac{D}{\theta}\left[e^{\theta(t_n - 1)} - 1\right] + [(1 - P(n))z - D\, t_n]\, e^{\theta(t_n - 1)} \end{aligned} \tag{A3}$$

where

$$t_n = \frac{Q}{\lambda} \tag{A4}$$

For the calculating of order quantity, from Equation (A3), as we know that $I_2(T_n) = 0$, then we obtain

$$Q = \frac{D\left(e^{\theta\, T_n} - 1\right)}{\theta\left(1 - P(n)e^{\theta\, T_n}\right)} \tag{A5}$$

Holding cost:

$$\begin{aligned} &IHC = C_h\left[\int_0^{t_n} I_1(t)\, e^{-Rt}dt + \int_{t_n}^{T_n} I_2(t)\, e^{-Rt}dt\right] \\ &C_h\left[\tfrac{Q}{(\theta+R)}\left[1 - e^{-(\theta+R)T_n}\right] + \tfrac{D}{\theta}\left[\tfrac{1-e^{-(\theta+R)T_n}}{(\theta+R)} + \tfrac{e^{-RT_n}-1}{R}\right] + \tfrac{P(n)\ Q}{R}\left(e^{-RT_n} - e^{-Rt_n}\right)\right] \end{aligned} \tag{A6}$$

Deterioration cost:

$$\begin{aligned} DC\ &= C_d\left(Q - \int_0^{T_n} Dtdt - P(n)Q\right) \\ &= C_d\left(\tfrac{D\left(e^{\theta\, T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\, T_n}\right)} - DT_n - P(n)\tfrac{D\left(e^{\theta\, T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\, T_n}\right)}\right) \end{aligned}$$

Waste management cost:

$$WMC = C_w\left(\frac{D\left(e^{\theta\, T_n} - 1\right)}{\theta\left(1 - P(n)e^{\theta\, T_n}\right)} - DT_n - P(n)\frac{D\left(e^{\theta\, T_n} - 1\right)}{\theta\left(1 - P(n)e^{\theta\, T_n}\right)}\right) + C_w P(n)Q$$

Inspection cost:

$$\text{Inspection cost} = C_s\left(\frac{D\left(e^{\theta\, T_n} - 1\right)}{\theta\left(1 - P(n)e^{\theta\, T_n}\right)}\right)$$

Remaining costs have been given in the mathematical part.
Retailer's total profit:

$$
\begin{aligned}
\Psi(T_n) = {} & \frac{pD}{R}\left[1-e^{-R\ T_n}\right] + c_s\ P(n)\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}e^{-R\ t_n} - C_k - C_s\left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}\right) \\
& -C_p\left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}\right) - C_w P(n)\left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}\right) - C_w\left(\left(\begin{array}{l} \frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)} - D\ T_n \\ -\left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}\right)P(n)\end{array}\right)\right) \\
& -C_h\left[\begin{array}{l} \frac{\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right] \\ +\frac{D}{\theta}\left[\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)}+\frac{e^{-RT_n}-1}{R}\right] \\ +\frac{P(n)\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\end{array}\right] - (e_c E_e T_x)\left[\begin{array}{l} \frac{\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right] + \\ \frac{D}{\theta}\left[\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)}+\frac{e^{-RT_n}-1}{R}\right] \\ +\frac{P(n)\ Q}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\end{array}\right] \\
& -(e_c F_e)\left[\begin{array}{l} \frac{\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}}{(\theta+R)}\left[1-e^{-(\theta+R)T_n}\right] \\ +\frac{D}{\theta}\left[\begin{array}{l}\frac{1-e^{-(\theta+R)T_n}}{(\theta+R)} \\ +\frac{e^{-RT_n}-1}{R}\end{array}\right] \\ +\frac{P(n)\ \frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\end{array}\right] - C_d\left(\left(\begin{array}{l} \frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)} - D\ T_n \\ -\left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}\right)P(n)\end{array}\right)\right)
\end{aligned}
\tag{A7}
$$

Now, we are following the numerical method and it differentiates Equation (A7). With respect to T_n, we obtain

$$
\frac{d\Psi(T_n)}{dT_n} = \left(\begin{array}{l}
pDe^{-RT_n} + c_s\ P(n)Q'' e^{-R\ t_n} - Rc_s\ P(n)Q\ t_n' e^{-R\ t_n} - C_s Q' - C_p Q' - C_w P(n) Q' - C_w\left(\left(\begin{array}{l} Q'-D \\ -Q'\ P(n)\end{array}\right)\right) \\
-C_h\left[\begin{array}{l} \frac{1}{(\theta+R)}\left[Q'-Q'e^{-(\theta+R)T_n}\right] + Qe^{-(\theta+R)T_n} + \frac{D}{\theta}\left[e^{-(\theta+R)T_n}-e^{-RT_n}\right] + P(n)Q\left(e^{-Rt_n}-e^{-RT_n}\right) \\ +\frac{P(n)Q'}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\end{array}\right] \\
-(e_c E_e T_x)\left[\begin{array}{l} \frac{1}{(R+\theta)}\left[Q'-Q'.e^{-(\theta+R)T_n}\right] + Qe^{-(\theta+R)T_n} + \frac{D}{\theta}\left[e^{-(\theta+R)T_n}-e^{-RT_n}\right] + P(n)Q\left(e^{-Rt_n}-e^{-RT_n}\right) \\ +\frac{P(n)Q'}{R}\left(e^{-RT_n}-e^{-Rt_n}\right)\end{array}\right] \\
-(e_c E_e)\left[\begin{array}{l} \frac{1}{(R+\theta)}\left[Q'-Q'.e^{-(\theta+R)T_n}\right] + Qe^{-(\theta+R)T_n} + \frac{D}{\theta}\left[e^{-(\theta+R)T_n}-e^{-RT_n}\right] \\ +P(n)Q\left(e^{-Rt_n}-e^{-RT_n}\right) \\ +\frac{P(n)}{R}\left(Q'e^{-RT_n}-Q'e^{-Rt_n}\right)\end{array}\right] - C_d\left(\left(\begin{array}{l} Q'-D \\ -Q'\ P(n)\end{array}\right)\right)
\end{array}\right) = 0
$$

Now, we find the value of Q' and t_n' from Equations (A4) and (A5).

$$\frac{dQ}{dT_n} = \frac{d}{dT_n}\left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}\right)$$

$$= \frac{\left(\theta\left(1-P(n)e^{\theta\ T_n}\right)\right)\frac{d}{dT_n}\left(D\left(e^{\theta\ T_n}-1\right)\right)-D\left(e^{\theta\ T_n}-1\right)\frac{d}{dT_n}\left(\theta\left(1-P(n)e^{\theta\ T_n}\right)\right)}{\left(\theta\left(1-P(n)e^{\theta\ T_n}\right)\right)^2}$$

$$\frac{dQ}{dT_n} = \frac{2De^{\theta\ T_n}}{\left(1-P(n)e^{\theta\ T_n}\right)}$$

$$and\ t_n' = \frac{Q'}{\lambda}$$

$$= \frac{\frac{2De^{\theta\ T_n}}{\left(1-P(n)e^{\theta\ T_n}\right)}}{\lambda}$$

Now, we are taking second derivative of Equation (A7); with respect to T_n, we obtain

$$\frac{d\Psi^2(T_n)}{dT_n{}^2} = \left(\begin{array}{l} -RpDe^{-RT_n} + c_sP(n)Q'''e^{-R\ t_n} - Rt_n'c_sP(n)Q''e^{-R\ t_n} \\ -Rc_sP(n)\frac{d}{dT_n}\left(Q\ t_n'e^{-R\ t_n}\right) - C_sQ'' \\ -C_pQ'' - C_wP(n)Q'' - C_w\left(\left(\begin{array}{l} Q'' \\ -Q''\ P(n) \end{array}\right)\right) \\ -C_h\left[\begin{array}{l} \frac{1}{(\theta+R)}\left[Q'' - Q''e^{-(\theta+R)T_n}\right] + 2Q'.e^{-(R+\theta)T_n} - (\theta + RQe^{-(\theta+R)T_n} \\ +\frac{D}{\theta}\left[-(\theta+R)e^{-(\theta+R)T_n} + \mathrm{Re}^{-RT_n}\right] \\ +Q'\left(e^{-Rt_n} - e^{-RT_n}\right)P(n) + RP(n)Q\left(-e^{-Rt_n} + e^{-RT_n}\right) \\ +\frac{1}{R}\left(Q''.e^{-RT_n} - Q''.e^{-Rt_n}\right)P(n) + P(n)Q'\left(-e^{-RT_n} + e^{-Rt_n}\right) \end{array}\right] \\ -(e_cE_eT_x)\left[\begin{array}{l} \frac{1}{(\theta+R)}\left[Q'' - Q''.e^{-(\theta+R)T_n}\right] + 2Q'e^{-(\theta+R)T_n} - (\theta + RQe^{-(\theta+R)T_n} \\ +\frac{D}{\theta}\left[-(\theta+R)e^{-(\theta+R)T_n} + \mathrm{Re}^{-RT_n}\right] \\ +P(n)Q'\left(e^{-Rt_n} - e^{-RT_n}\right) + RP(n)Q\left(-e^{-Rt_n} + e^{-RT_n}\right) \\ +\frac{P(n)Q''}{R}\left(e^{-RT_n} - e^{-Rt_n}\right) + P(n)Q'\left(-e^{-RT_n} + e^{-Rt_n}\right) \end{array}\right] \\ -(e_cE_e)\left[\begin{array}{l} \frac{1}{(\theta+R)}\left[Q'' - Q''e^{-(\theta+R)T_n}\right] + 2Q'.e^{-(R+\theta)T_n} - (\theta + RQe^{-(\theta+R)T_n} \\ +\frac{D}{\theta}\left[-(\theta+R)e^{-(\theta+R)T_n} + \mathrm{Re}^{-RT_n}\right] \\ +P(n)Q'\left(e^{-Rt_n} - e^{-RT_n}\right) + RP(n)Q\left(-e^{-Rt_n} + e^{-RT_n}\right) \\ +\frac{1}{R}\left(Q''e^{-RT_n} - Q''e^{-Rt_n}\right)P(n) + \left(-Q'e^{-RT_n} + Q'e^{-Rt_n}\right)P(n) \end{array}\right] - C_d\left(\left(\begin{array}{l} Q'' \\ -Q''\ P(n) \end{array}\right)\right) \end{array}\right) \leq 0$$

where Q'' and t_n'' can be obtained from Equations (A7) and (A4).

Now, Equations (A7) and (A4) differentiate with respect to with respect to T_n, which are given below:

As we know that $Q = \frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}$ and $t_n = \frac{Q}{\lambda}$,

$$\frac{dQ}{dT_n} = \frac{d}{dT_n}\left(\frac{D\left(e^{\theta\ T_n}-1\right)}{\theta\left(1-P(n)e^{\theta\ T_n}\right)}\right)$$

$$\frac{dQ}{dT_n} = \frac{2De^{\theta\ T_n}}{\left(1-P(n)e^{\theta\ T_n}\right)}$$

and its sec ond derivative with respect to T_n

$$\frac{d^2Q}{dT_n{}^2} = \frac{d}{dT_n}\left(\frac{2De^{\theta\ T_n}}{\left(1-P(n)e^{\theta\ T_n}\right)}\right)$$

$$= \frac{\left(\left(1-P(n)e^{\theta\ T_n}\right)\right)\frac{d}{dT_n}\left(2De^{\theta\ T_n}\right)-2De^{\theta\ T_n}\frac{d}{dT_n}\left(\left(1-P(n)e^{\theta\ T_n}\right)\right)}{\left(\left(1-P(n)e^{\theta\ T_n}\right)\right)^2}$$

$$\frac{d^2Q}{dT_n{}^2} = \frac{2D\theta\left(\left(1-P(n)e^{\theta\ T_n}\right)\right)e^{\theta\ T_n}+2D\ \theta\ e^{2\theta\ T_n}P(n)}{\left(\left(1-P(n)e^{\theta\ T_n}\right)\right)^2}$$

$$Q'' = \frac{2D\theta\left(\left(1-P(n)e^{\theta\ T_n}\right)\right)e^{\theta\ T_n}+2D\ \theta\ e^{2\theta\ T_n}P(n)}{\left(\left(1-P(n)e^{\theta\ T_n}\right)\right)^2}$$

and

$$\frac{d}{dT_n}(t'_n) = \frac{d}{dT_n}\left(\frac{Q'}{\lambda}\right)$$
$$= \left(\frac{Q''}{\lambda}\right)$$

Using mathematical software and obtaining optimal value of cycle time $T_n^* = 1.00491$ with input parameters, which implies that $\frac{d^2\Psi(1.00491)}{dT_n} = -21 \leq 0$, $T_n^* = 1.00491$ optimal cycle time.

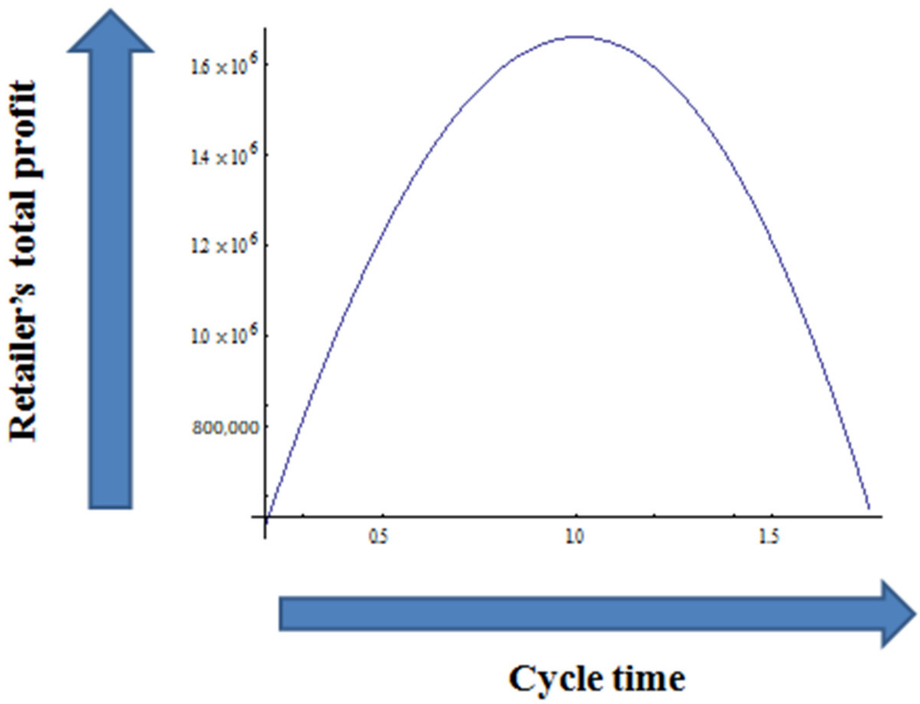

Figure A2. Concavity of buyer's whole profit.

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