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Abstract: The economic uncertainty caused by COVID-19 has led governments around the world to attach more importance to green innovation to accomplish their carbon reduction schemes. To improve the green innovation encouraging effect of an environmental policy system, this study introduces a unit progressive carbon tax on the basis of a green innovation subsidy to discuss the synergy green innovation effect between them. We set up a dynamic evolutionary game model to analyze the respective influences of green innovation subsidies and an environmental policy system containing a unit progressive carbon tax on Low Carbon Technology (LCT) heterogeneous enterprises’ endogenous green innovation strategies. The Evolutionary Stable Strategy analysis of dynamic game models demonstrate that there does exist a synergy green innovation effect between green innovation subsidies and unit carbon taxes. The numerical simulation shows that the synergy green innovation effect of green innovation subsidies and carbon taxes contains both an overlapping policy effect and a more significant green innovation stimulating effect on enterprises with high LCT. Additionally, the introduction of a carbon tax will increase enterprises’ affordability on the green innovation cost coefficient. Furthermore, introducing a unit progressive carbon tax would also create additional stimulation for enterprises to pursue a larger carbon reduction amount for the carbon emission cost-saving advantage. Based on the synergy green innovation effect mentioned above, we also investigate the policy implications of varying the tax rate and subsidy proportion in different situations.

Keywords: synergy green innovation effect; green innovation subsidy; carbon tax; dynamic evolutionary game model

1. Introduction

Green innovation is the core of a low-carbon economy development and the key to mitigating global warming [1]. Following the Paris Agreement, the main economic bodies have been facing increasing pressure to reduce carbon emissions since 2020. Thanks to the global economic uncertainties caused by the COVID-19 pandemic, governments around the world are increasingly worried that restrictive carbon emission reduction policies will slow down economic recovery [2]. Therefore, governments must rely more on green innovation than on restrictive carbon emission policies to accomplish the carbon emission reduction target set out in the Paris Agreement. Though incentive environmental policies, such as green innovation subsidies, and punitive environmental policies, such as carbon taxes, can both stimulate green innovation in theoretical studies, governments mainly carry out green innovation subsidies to encourage the green innovation of enterprises in practice [3]. The question is if the persistence of green innovation encouraging policies cannot be guaranteed because of the governments’ increasing fiscal expenditure pressure from green innovation subsidies.
Because more and more carbon tax designs with clear green innovation orientations have been proposed, the green innovation-inducing effect of carbon taxes has attracted increasing attention [4]. There have been suggestions that governments introduce a carbon tax on the basis of green innovation subsidies to create a policy system having a synergy green innovation effect [5]. Yu et al. [6] proposed a unit progressive carbon tax with a significant green innovation-inducing effect, which can stimulate all enterprises’ green innovation precisely by building a direct link between their Low Carbon Technology (LCT) level and carbon emission costs. The unit progressive carbon tax provides a better carbon tax design to study the synergy green innovation effect of green innovation subsidies and carbon taxes, because it can improve the entire green innovation level of LCT heterogeneous enterprises.

The policy mechanisms of green innovation subsidies and carbon taxes are totally opposite in a green innovation-encouraging policy system. Green innovation subsidies support enterprises’ green innovations by co-sharing R&D investment, and carbon taxes stimulate enterprises’ green innovations by adding a carbon emission cost to production [7]. Green innovation subsidies only have an influence on the enterprises carrying out green innovation, while carbon taxes add a carbon emission cost sanction to every enterprise with carbon emissions [8]. Green innovation subsidies would increase governments’ fiscal pressure, while carbon taxes could raise fiscal revenue for governments. Therefore, introducing a carbon tax on the basis of a green innovation subsidy can not only encourage a green innovation environmental policy system covering all enterprises with carbon emissions, but also raise fiscal revenue for green innovation subsidies [9]. The most important point is that introducing a unit progressive carbon tax on the basis of a green innovation subsidy will form a complex policy synergy effect, which is just discussed as a simple policy-overlapping effect in the existing research [10]. The gap between the policy synergy effect and policy overlapping effect is worthy of further investigation.

In this study, we introduce a unit progressive carbon tax on the basis of a green innovation subsidy to analyze the synergy green innovation effect of an environmental policy system consisting of green innovation subsidies and carbon taxes. In particular, we discuss the influence of an introduced unit progressive carbon tax on the existing green innovation subsidy’s policy effect. Simultaneously, there have been studies indicating that the green innovation cost and carbon reduction amount have a great influence on the green innovation effect of environmental policies [11,12]. Therefore, in this study, we also discuss whether the unit progressive carbon tax works when the green innovation cost varies or whether the carbon emission reduction amount makes green innovation subsidies ineffective.

This paper employs a dynamic evolutionary game model to analyze the endogenous green innovation choice of enterprises under green innovation subsidies and carbon taxes, which can include the LCT path dependence and market competition advantage of green innovation as an endogenous factor. At first, we set up two dynamic evolutionary game models to describe the influence of green innovation subsidies and policy systems containing new introduced carbon taxes on LCT heterogeneous enterprises’ green innovation strategies. We used MATLAB R2018b to numerically simulate the green innovation effect of these two kinds of environmental policies. Next, we discuss the effect of introducing a unit progressive carbon tax when the increase of the innovation cost coefficient makes green innovation subsidies ineffective. Last, we discuss the effect of introducing a unit progressive carbon tax when variation of the carbon reduction amount makes green innovation subsidies ineffective. Our works expand the research about the synergy green innovation effect of green innovation subsidies and carbon taxes in the above aspects and provide policy implications on environmental policy choices for governments.

The remainder of this paper is arranged as follows: The relevant literature is reviewed in Section 2 to elaborate on the research context of this study. The dynamic evolutionary game models are set up in Section 3 to analyze the synergy green innovation effect of policy systems consisting of green innovation subsidies and carbon taxes. The Evolutionary Stable
Strategy (ESS) of the dynamic evolutionary game models is also discussed in Section 3. The unfolding synergy green innovation effect of green innovation subsidies and carbon tax is presented in a numerical simulation with MATLAB R2018b in Section 4. The discussion and conclusion are presented in Section 5.

2. Literature Review

As core environmental policy instruments, green innovation subsidies and carbon taxes encourage the green innovation of enterprises. Early in the study of the effects of green innovation on environmental policies, the green innovation effects of green innovation subsidies and carbon taxes were usually discussed. Currently, more studies are transferring the focus to the synergy green innovation effect of green innovation subsidies and carbon taxes. Thus, the influence of green innovation subsidies on green innovation, the influence of carbon taxes on green innovation, and the synergy influence of both green innovation subsidies and carbon taxes on green innovation are the main relevant literature topics for this study.

2.1. The Influence of Green Innovation Subsidies on Green Innovation

Green innovation subsidies consist in the government investing a part of its fiscal revenue to support enterprises’ green innovation, having positive externalities that can most directly stimulate enterprises’ green innovation [13,14]. Green innovation subsidies are a kind of incentive environmental policy instrument, meaning that it is a mild environmental policy intended to have only a policy guidance effect rather than a policy restriction effect [15]. At the same time, green innovation subsidies can stimulate enterprises’ green innovation in a more stable way than other environmental policies, because there is no intertemporal delay in their policy mechanism [16]. Therefore, almost all countries facing a carbon emission reduction pressure have provided different kinds of green innovation subsidies to enterprises and have witnessed the expected policy effectiveness [17,18]. According to the policy implementation experience, green innovation subsidies can directly increase the enterprises’ green innovation investments and reduce the green innovation risks on the condition of a certain high proportion of green innovation subsidies [19,20]. Besides the policy strength, external factors, such as product price, market demand, the innovation cost coefficient, and the carbon reduction amount, also have a great influence on the effect of a green innovation subsidy policy [21,22]. When the external factors mentioned above make green innovation subsidies ineffective at encouraging green innovation, governments can always increase the policy strength [23].

In fact, governments must refuse green innovation subsidies to some key industries, because the green innovation subsidies come totally from the governments’ limited fiscal income [24]. Concurrently, experts assert that green innovation subsidies acquired by key industries are not high enough to stimulate their green innovation completely. Thus, the gap between the growing green innovation subsidy expenditures and the limited fiscal incomes is becoming increasingly larger [25]. Some studies have demonstrated that governments can decrease the subsidy proportion gradually without affecting the policy effectiveness if green innovation can create a new market demand or bring a huge cost-saving advantage to enterprises [26]. However, because not all the green innovation from different industries can create a new market demand or a cost-saving advantage in a short time, it is not feasible to decrease the subsidy proportion [27]. With the fact that the gap between subsidy expenditures and fiscal income will continue to grow as the carbon reduction pressure faced by governments grows [28]. Governments should introduce more environmental policies to maintain the green innovation-encouraging effect of green innovation subsidies [29]. The system of a green innovation subsidy and a carbon tax has received the most attention because they do not intersect in the policy mechanism [30]. Furthermore, a synergy green innovation effect of green innovation subsidies and carbon taxes is expected beyond the significant overlapping effect [31]. In keeping with this idea, in this study, we introduce
a carbon tax on the basis of a green innovation subsidy to analyze their synergy green innovation effect, rather than the simple policy-overlapping effect.

2.2. The Influence of Carbon Taxes on Green Innovation

Carbon taxes consist in the government strongly restricting an enterprise’s carbon emission through negative externalities that can indirectly encourage its green innovation for a carbon emission cost-saving advantage [4,32]. Thus, carbon taxes are a kind of punitive environmental policy instrument, having a green innovation-inducing effect on enterprises’ green innovation [33]. Generally, carbon taxes can cover all enterprises with carbon emissions. In fact, as a punitive environmental policy, the carbon reduction effect is more significant than the green innovation-inducing effect of carbon taxes in the short term. The strong carbon reduction effect incurs great worries about the negative influence on economic growth and carbon tax burden inequality, with the result that carbon taxes are only implemented in several countries [34,35]. However, much research has demonstrated that the short-term negative effect of carbon taxes can be overcome during the implementation process by improving overall social welfare and allocating carbon taxes rationally [36,37]. After the essential position of carbon taxes was realized, researchers transferred their focus from the carbon reduction effect to the green innovation-inducing effect, trying to amplify the green innovation effect of carbon taxes [38]. New carbon tax designs with stronger green innovation-inducing effects, such as the unit progressive carbon tax, provide more appropriate policy choices for other countries to impose carbon taxes when expecting a significant green innovation-inducing effect [6,39]. Imposing carbon taxes to stimulate green innovation is a considerable policy advantage, because it can both increase the environmental policy strength and alleviate the subsidy expenditure pressure on governments [40].

Evidence from countries having imposed a carbon tax has verified that carbon taxes can induce enterprises’ green innovation by changing their cost–benefit mechanism of green innovation [41]. Besides the carbon tax rate, external factors, such as innovation cost, the carbon reduction amount, the learning-by-doing effect, and the technology substituting rate, also have a great influence on the green innovation-inducing effect of carbon taxes [30,42–44]. Increasing the tax rate does not always work when the green innovation-inducing effect is negatively affected by the external factors mentioned above. In this situation, other environmental policies are necessary to create a synergy policy effect [41]. Carbon taxes not only have an influence on enterprises’ benefits from green innovation, but also can raise fiscal income for governments. Global warming can be mitigated comprehensively if carbon tax revenues are invested in green innovation subsidies and environment governance [45]. As carbon taxes are introduced into environmental policy systems by more and more countries, the synergy green innovation effect of carbon taxes and other environmental policies is worthy of more discussion [46]. Once the carbon tax is introduced to the environmental policy portfolio, it will consist of a policy system, with the green innovation subsidy as the most implemented environmental policy in practice [47]. Thus, the research about the green innovation-inducing effect of carbon taxes should change to the synergy green innovation effect of green innovation subsidies and carbon taxes [48]. Our study focuses on this important transfer and aims to provide several policy implications for governments to improve their policy system’s performance.

2.3. The Influence of a Policy System including Green Innovation Subsidies and Carbon Taxes on Green Innovation

Green innovation subsidies, carbon taxes, carbon trading, and carbon capture are the main environmental policy instruments considered by governments [49]. The policy system of a green innovation subsidy and carbon tax is the most linked environmental policy system when stimulating green innovation becomes governments’ most important target [46,50]. In a policy system comprising green innovation subsidies and carbon taxes, governments provide green innovation subsidies to support enterprises’ green
innovation and impose carbon taxes to restrict enterprises’ carbon emissions to induce green innovation [5,7]. Because there is a significant overlapping policy effect, the policy system of green innovation subsidies and carbon taxes can stimulate enterprises’ green innovation effectively and compel enterprises to achieve a better level of carbon emissions [11,30]. However, the policy synergy effect is not entirely the same as the policy overlapping effect; the policy synergy effect is the interaction between green innovation subsidies and carbon taxes in dealing with the negative influence of external factors [51]. In a policy system of green innovation subsidies and carbon taxes, some researchers believe that green innovation subsidies are much more important than carbon taxes because their policy effect is more persistent [52]. Other researchers think carbon taxes are much more important because their policy strength is difficult to achieve with green innovation subsidies [53]. Furthermore, the policy system of green innovation subsidies and carbon taxes can reduce the reliance on a single policy, making sure that green innovation subsidies can prevent the overuse of carbon taxes or that carbon taxes can raise funds for green innovation subsidies to alleviate fiscal pressure [7,9].

When referring to the design of a policy system, some studies have suggested that a portfolio of dynamic and static policies will have a better effect, where a dynamic carbon tax or dynamic green innovation subsidy can amplify the green innovation effect [54]. Some studies suggest varying policy systems over a timeline, which should rely on a high green innovation subsidy in early stages and on a carbon tax in later stages [7,50,51]. It is only when the tax rate and subsidy level exceed a certain threshold that the synergy green innovation effect of this policy system will be significant [55,56]. Additionally, external factors, such as innovation cost, initial carbon emission level, and LCT progress speed, will negatively affect the synergy green innovation effect of a policy system consisting of a green innovation subsidy and carbon tax [24,54,55]. Increasing the tax rate and subsidy proportion is always the first choice to deal with the negative influence from the external factors mentioned above. Studies have also found that the policy system of green innovation subsidies and carbon taxes exerts different influences on different enterprises; the influence on enterprises with less market demand is weaker than on the enterprises with more market demand [56]. Summarily, research on the synergy green innovation effect of green innovation subsidies and carbon taxes has undergone a process from viewing it as a simple overlapping policy effect to a profound synergy policy effect [57]. Furthermore, our research tries to expand the synergy green innovation effect of green innovation subsidies and carbon taxes in more aspects.

Because green innovation subsidies have been provided by most governments in practice, the most likely future situation is introducing a carbon tax on the basis of green innovation subsidies. Though the related research has begun to discuss the synergy green innovation effect of green innovation subsidies and carbon taxes, it has tended to highlight the role of a single policy in the new consisted policy system rather than analyze the synergy effect of those two policies. Compared with these studies, our research will expand the synergy green innovation effect of green innovation subsidies and carbon taxes from the following three aspects: Firstly, introducing a unit progressive carbon tax will make the high LCT enterprises’ green innovation strategy become more sensitive to the new consisted environmental policy system. Next, introducing a unit progressive carbon tax will improve enterprises’ affordability on the green innovation cost coefficient. Finally, introducing a unit progressive carbon tax will also create an additional stimulation for enterprises to pursue a larger carbon reduction amount for the carbon emission cost saving advantage. In addition, our numerical simulation on the varying subsidy proportion and tax rate in different situations will also be helpful for governments to improve their green innovation subsidy and carbon tax policy system.
3. The Model

3.1. Participants and Game Description

In our study, enterprises with a large amount of carbon emissions in their production process were participants in the dynamic evolutionary game and had a green innovation strategy as their endogenous choice. Because of the path independence of green innovation, we assumed that the game participants were Enterprise 1 and Enterprise 2, who are heterogeneous in LCT [42]. Enterprise 1 has a higher LCT stock than Enterprise 2.

The LCT heterogeneous Enterprise 1 and Enterprise 2 played a duopoly game in the market. The green innovation as an endogenous choice of enterprises can not only reduce the carbon emission amount but also bring a market competition advantage [58]. Because of the positive externality of the enterprises’ green innovation, the governments introduced a carbon tax on the basis of green innovation as a policy system to stimulate the enterprises’ green innovation. The target of this policy system was to improve the green innovation probability of both Enterprise 1 with high LCT and Enterprise 2 with low LCT. Enterprise 1 and Enterprise 2 played a perfect information dynamic evolutionary game under the policy system of green innovation subsidies and carbon taxes carried out by governments to maximize their profit.

3.2. Assumptions

In accordance with previous studies, this paper made the following assumptions:

Assumption 1. On the basis of the classical duopoly game model, \( p(Q) = a - bQ \) is the market inverse demand function, \( a \) is the basic market demand, and \( b \) is the price elasticity of \( Q \). \( Q = q_1 + q_2 \) is the total production, \( q_1 \) is the production of Enterprise 1, \( q_2 \) is the production of Enterprise 2, and \( p \) is the market price. Enterprise 1 and Enterprise 2 have unit production costs of \( c_1 \) and \( c_2 \).

Assumption 2. The strategies of Enterprise 1 and Enterprise 2 are set as: \{Green Innovation; Non-Innovation\}. In the dynamic evolutionary game model \( x, y \in [0, 1] \) are used to describe the probability of Enterprise 1 and Enterprise 2 choosing the Green Innovation strategy. Both \( x \) and \( y \) are a function of time \( t \). Enterprise 1 and Enterprise 2 adjust their probability of selecting a Green Innovation or Non-Innovation strategy during a certain time period in which governments carry out different environmental policies, until they attain an ESS in the dynamic evolutionary game model.

Assumption 3. Green innovation will reduce the carbon emission of enterprises. Before green innovation, the unit carbon emissions of Enterprise 1 and Enterprise 2 are \( \varepsilon_1 \) and \( \varepsilon_2 \). Green innovation will take unit carbon reduction \( w_1 \) for Enterprise 1 and \( w_2 \) for Enterprise 2. According to the A-J classical assumption, the green innovation cost is \( \frac{1}{2} \beta_1 w_1^2 \) for Enterprise 1 and \( \frac{1}{2} \beta_2 w_2^2 \) for Enterprise 2, where \( \beta_1 \) and \( \beta_2 \) are the innovation cost coefficients [59]. Because Enterprise 1 has a higher LCT stock than Enterprise 2, we have \( \varepsilon_1 < \varepsilon_2 \), \( w_1 > w_2 \), and \( 0 < \beta_1 < \beta_2 < 1 \).

Assumption 4. At first, governments only provide Enterprise 1 and Enterprise 2 with a green innovation subsidy for co-sharing the green innovation cost. The subsidy proportions acquired by Enterprise 1 and Enterprise 2 are \( s_1 \) and \( s_2 \), and \( 1 > s_1 > s_2 > 0 \) on the condition that \( 0 < \beta_1 < \beta_2 < 1 \). After receiving green innovation subsidies from governments, the green innovation cost of Enterprise 1 and Enterprise 2 will reduce to \( \frac{1}{2}(1 - s_1)\beta_1 w_1^2 \) and \( \frac{1}{2}(1 - s_2)\beta_2 w_2^2 \). Then, governments introduced a unit progressive carbon tax to comprise a policy system of a green innovation subsidy and carbon tax. The unit progressive carbon tax determined the tax rate according to the unit carbon emission levels of enterprises. Hence, the tax rate of Enterprise 1 is \( \tau_1 \), and the tax rate of Enterprise 2 is \( \tau_2 \), where \( \tau_1 < \tau_2 \) because \( \varepsilon_1 < \varepsilon_2 \). The related variables and parameters are listed in Table 1.
Table 1. Model variables and parameters.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x/y)</td>
<td>Enterprise 1 or 2’s probability of choosing the Green Innovation strategy</td>
</tr>
<tr>
<td>(x(t)/y(t))</td>
<td>Enterprise 1 or Enterprise 2’s probability of choosing the Green Innovation strategy, assumed for time (t)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon)</td>
<td>Unit carbon emission of enterprises</td>
</tr>
<tr>
<td>(w)</td>
<td>Unit carbon emission reduction of enterprises caused by green innovation</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Green innovation cost coefficient of enterprises</td>
</tr>
<tr>
<td>(s)</td>
<td>Green innovation subsidy proportion of enterprises provided by governments</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Unit progressive carbon tax rate of enterprises imposed by governments</td>
</tr>
</tbody>
</table>

3.3. Basic Game Model

3.3.1. Basic Game Model under the Green Innovation Subsidy

Green innovation can not only help enterprises reduce the unit carbon emission in their production, but also take a leading position in market competition, which is the main driving force of enterprises’ green innovation. Thus, it is assumed that a green innovation strategy will change the market position in the duopoly game model when analyzing the green innovation effect of a green innovation subsidy provided by governments. According to the previous assumptions and the change in market position situations caused by enterprises’ green innovation strategies, we describe the static duopoly game process as follows:

**Strategy Profile 1.** Enterprise 1 chooses the green Innovation strategy, and Enterprise 2 chooses the Non-Innovation strategy. Because of green innovation, Enterprise 1 will gain a leading position in the market competition. Thus, Enterprise 1 and Enterprise 2 play a Stackelberg game, where Enterprise 1 is the market leader. Based on the equilibrium solution of the Stackelberg game model lead by Enterprise 1 (the solution process is listed in Appendix A), the payoff functions of Enterprise 1 and Enterprise 2 are:

\[
\pi_{S1} = \left[ a - 2c_1 + c_2 \right] \times \left[ a + 2c_1 + c_2 - c_1 - \frac{1}{2}(1 - s_1)\beta_1 w_1^2 \right],
\]

(1)

\[
\pi_{S2} = \left[ a + 2c_1 - 3c_2 \right] \times \left[ a + 2c_1 + c_2 - c_2 \right].
\]

(2)

In a situation where governments provide only a green innovation subsidy, carbon emissions will not increase the carbon emission cost to enterprises. Enterprise 1 and Enterprise 2 only need to afford the production cost and the green innovation cost if they invest. Enterprise 1’s investment in green innovation is reduced from \(\frac{1}{2}\beta_1 w_1^2\) to \(\frac{1}{2}(1 - s_1)\beta_1 w_1^2\) because the government’s funding proportion is \(s_1\). Enterprise 2 has neither a green innovation investment nor a green innovation subsidy.

**Strategy Profile 2.** Enterprise 1 chooses the Non-Innovation strategy, and Enterprise 2 chooses the Green Innovation strategy. In this strategy profile, Enterprise 2 becomes the market leader in the Stackelberg game model because of the green innovation. Based on the equilibrium solution of the Stackelberg game model lead by Enterprise 2 (the solution process is listed in Appendix A), Enterprise 1 and Enterprise 2’s payoff functions are:

\[
\pi_{S1} = \left[ a + 2c_1 - 3c_2 \right] \times \left[ a + 2c_1 + c_2 - c_1 \right],
\]

(3)

\[
\pi_{S2} = \left[ a - 2c_1 + c_2 \right] \times \left[ a + 2c_1 + c_2 - c_2 - \frac{1}{2}(1 - s_2)\beta_2 w_2^2 \right].
\]

(4)

In a situation where governments provide only green innovation subsidies and do not impose a unit progressive carbon tax, Enterprise 2’s investment in green innovation
is reduced from $\frac{1}{2} \beta_2 w_2^2$ to $\frac{1}{2} (1 - s_2) \beta_2 w_2^2$, acquiring an $s_2$ proportion subsidy from governments. Enterprise 1 has neither a green innovation investment nor a green innovation subsidy in this strategy profile.

**Strategy Profile 3.** Enterprise 1 chooses the Green Innovation strategy, and Enterprise 2 chooses the Green Innovation strategy as well. In this strategy profile, Enterprise 1 and Enterprise 2 have the same chance to be the market leader thanks to the green innovation. Thus, they play an equal probability Stackelberg game model. Based on the equilibrium solution of the equal probability Stackelberg game model for Enterprise 1 and Enterprise 2 (the solution process is listed in Appendix A), the payoff functions of Enterprise 1 and Enterprise 2 are:

\[
\pi_{11}^{S1} = \left[ \frac{3a - 2c_1 - c_2}{8b} \right] * \left[ \frac{a + 2c_1 + c_2}{4} - c_1 - \frac{1}{2} (1 - s_1) \beta_1 w_1^2 \right], \tag{5}
\]

\[
\pi_{11}^{S2} = \left[ \frac{3a - 2c_1 - c_2}{8b} \right] * \left[ \frac{a + 2c_1 + c_2}{4} - c_2 - \frac{1}{2} (1 - s_2) \beta_2 w_2^2 \right]. \tag{6}
\]

Both Enterprise 1 and Enterprise 2 acquire green innovation subsidies from the government and do not need to pay carbon taxes to the government in this policy situation. Enterprise 1’s investment in green innovation is reduced from $\frac{1}{2} \beta_1 w_1^2$ to $\frac{1}{2} (1 - s_1) \beta_1 w_1^2$ for an $s_1$ proportion funding from the government, while Enterprise 2’s investment in green innovation is reduced from $\frac{1}{2} \beta_2 w_2^2$ to $\frac{1}{2} (1 - s_2) \beta_2 w_2^2$ for an $s_2$ proportion funding from the government.

**Strategy Profile 4.** Enterprise 1 chooses the Non-Innovation strategy, and Enterprise 2 chooses the Non-Innovation strategy as well. In this strategy profile, the market positions of Enterprise 1 and Enterprise 2 do not change because neither of them carries out green innovation. Thus, they play a Cournot game model in the market competition. Based on the equilibrium solution of the Cournot game model played by Enterprise 1 and Enterprise 2 (the solution process is listed in Appendix A), Enterprise 1 and Enterprise 2’s payoff functions are:

\[
\pi_{22}^{S1} = \left[ \frac{a + 2c_2 - 2c_1}{3b} \right] * \left[ \frac{a - 2c_2 + 4c_1}{3} - c_1 \right], \tag{7}
\]

\[
\pi_{22}^{S2} = \left[ \frac{a + 2c_2 - 2c_1}{3b} \right] * \left[ \frac{a - 2c_2 + 4c_1}{3} - c_2 \right]. \tag{8}
\]

Because neither Enterprise 1 nor Enterprise 2 carry out green innovation, they neither need to invest in the green innovation nor receive green innovation subsidies from the government. They still need to pay no carbon taxes.

Based on Enterprise 1 and Enterprise 2’s payoff functions in different strategy profiles, we formulated the payoff matrix of the basic game model under green innovation subsidies, which is listed in Table 2.

**Table 2. Payoff matrix of the game model under green innovation subsidies.**

<table>
<thead>
<tr>
<th>Enterprise 1</th>
<th>Green Innovation: $x$</th>
<th>Non-Innovation: $1 - x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enterprise 2</td>
<td>Green Innovation: $y$</td>
<td>$\pi_{11}^{S1}$, $\pi_{11}^{S2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\pi_{12}^{S1}$, $\pi_{12}^{S2}$</td>
</tr>
</tbody>
</table>

3.3.2. Basic Game Model under the Green Innovation Subsidy and Carbon Tax

When the government imposes a unit progressive carbon tax on the enterprises’ carbon emissions on the basis of a green innovation subsidy, Enterprise 1 and Enterprise 2’s green
innovation strategy choices are affected by the carbon emission cost and green innovation investment at the same time. The concrete situations are:

**Strategy Profile 1.** Enterprise 1 chooses the Green Innovation strategy, and Enterprise 2 chooses the Non-Innovation strategy. Like the Stackelberg game model played in Section 3.3.1, where Enterprise 1 is the market leader, we can calculate the payoff functions of Enterprise 1 and Enterprise 2 as follows:

\[
\pi_{12}^{G1} = \left[ \frac{a - 2c_1 + c_2}{2b} \right] \times \left[ \frac{a + 2c_1 + c_2}{4} - c_1 - \tau_1 (\epsilon_1 - w_1) - \frac{1}{2} (1 - s_1) \beta_1 w_1^2 \right],
\]

\[
\pi_{12}^{G2} = \left[ \frac{a + 2c_1 - 3c_2}{4b} \right] \times \left[ \frac{a + 2c_1 + c_2}{4} - c_2 - \tau_2 \epsilon_2 \right].
\]

In a situation where the government imposes a unit progressive carbon tax on carbon emissions, Enterprise 1 and Enterprise 2 must pay carbon taxes as a carbon emission cost at first. Thanks to the green innovation, Enterprise 1’s carbon tax expenditure is reduced from \( \tau_1 \epsilon_1 \) to \( \tau_1 (\epsilon_1 - w_1) \). The green innovation cost of Enterprise 1 is reduced from \( \frac{1}{2} \beta_1 w_1^2 \) to \( \frac{1}{2} (1 - s_1) \beta_1 w_1^2 \) with green innovation subsidies from the government. Enterprise 2’s carbon tax expenditure is \( \tau_2 \epsilon_2 \) and has no green innovation investment.

**Strategy Profile 2.** Enterprise 1 chooses the Non-Innovation strategy and Enterprise 2 chooses the Green Innovation strategy. As in the Stackelberg game model played in Section 3.3.1, where Enterprise 2 is the market leader, we have Enterprise 1’s and Enterprise 2’s payoff functions as follows:

\[
\pi_{21}^{G1} = \left[ \frac{a + 2c_1 - 3c_2}{4b} \right] \times \left[ \frac{a + 2c_1 + c_2}{4} - c_1 - \tau_1 \epsilon_1 \right],
\]

\[
\pi_{21}^{G2} = \left[ \frac{a - 2c_1 + c_2}{2b} \right] \times \left[ \frac{a + 2c_1 + c_2}{4} - c_2 - \tau_2 (\epsilon_2 - w_2) - \frac{1}{2} (1 - s_2) \beta_2 w_2^2 \right].
\]

This time, Enterprise 1 needs to pay a carbon tax \( \tau_1 \epsilon_1 \) without a carbon emission cost saving from green innovation. Enterprise 2’s carbon tax is reduced from \( \tau_2 \epsilon_2 \) to \( \tau_2 (\epsilon_2 - w_2) \) because of the green innovation. Enterprise 2’s investment in green innovation is reduced from \( \frac{1}{2} \beta_2 w_2^2 \) to \( \frac{1}{2} (1 - s_2) \beta_2 w_2^2 \) with green innovation subsidies from the government.

**Strategy Profile 3.** Enterprise 1 chooses the Green Innovation strategy, and Enterprise 2 chooses the Green Innovation strategy as well. Like the equal probability Stackelberg game model played in Section 3.3.1, we calculate the payoff functions of Enterprise 1 and Enterprise 2 in this strategy profile as follows:

\[
\pi_{11}^{G1} = \left[ \frac{3a - 2c_1 - c_2}{8b} \right] \times \left[ \frac{a + 2c_1 + c_2}{4} - c_1 - \tau_1 (\epsilon_1 - w_1) - \frac{1}{2} (1 - s_1) \beta_1 w_1^2 \right],
\]

\[
\pi_{11}^{G2} = \left[ \frac{3a - 2c_1 - c_2}{8b} \right] \times \left[ \frac{a + 2c_1 + c_2}{4} - c_2 - \tau_2 (\epsilon_2 - w_2) - \frac{1}{2} (1 - s_2) \beta_2 w_2^2 \right].
\]

In this strategy profile, both Enterprise 1 and Enterprise 2 can gain a carbon emission cost saving from green innovation and green innovation subsidies from the government. Thus, Enterprise 1’s carbon tax expenditure is reduced from \( \tau_1 \epsilon_1 \) to \( \tau_1 (\epsilon_1 - w_1) \), and Enterprise 2’s carbon tax expenditure is reduced from \( \tau_2 \epsilon_2 \) to \( \tau_2 (\epsilon_2 - w_2) \). Enterprise 1’s investment in green innovation is reduced from \( \frac{1}{2} \beta_1 w_1^2 \) to \( \frac{1}{2} (1 - s_1) \beta_1 w_1^2 \), and Enterprise 2’s investment in green innovation is reduced from \( \frac{1}{2} \beta_2 w_2^2 \) to \( \frac{1}{2} (1 - s_2) \beta_2 w_2^2 \).

**Strategy Profile 4.** Enterprise 1 chooses the Non-Innovation strategy, and Enterprise 2 chooses the Non-Innovation strategy as well. As in the Cournot game model played by
Enterprise 1 and Enterprise 2 in Section 3.3.1, we have Enterprise 1’s and Enterprise 2’s payoff functions as follows:

\[ \pi_{G1}^{c2} = \left[ \frac{a + 2c_2 - 2c_1}{3b} \right] \cdot \left[ \frac{a - 2c_2 + 4c_1}{3} - c_1 - \tau_1 \varepsilon_1 \right], \quad (15) \]

\[ \pi_{G2}^{c2} = \left[ \frac{a + 2c_2 - 2c_1}{3b} \right] \cdot \left[ \frac{a - 2c_2 + 4c_1}{3} - c_2 - \tau_2 \varepsilon_2 \right]. \quad (16) \]

Because neither of the enterprises chooses the Green Innovation strategy, they need to pay carbon taxes without a carbon emission cost saving from green innovation. Enterprise 1’s carbon tax expenditure is \( \tau_1 \varepsilon_1 \), and Enterprise 2’s carbon tax expenditure is \( \tau_2 \varepsilon_2 \).

Based on Enterprise 1 and Enterprise 2’s payoff functions in different strategy profiles, we formulated the payoff matrix of the basic game model under a policy system consisting of green innovation subsidies and a carbon tax, which is listed in Table 3.

Table 3. Payoff matrix of the game model under green innovation subsidies and a carbon tax.

<table>
<thead>
<tr>
<th></th>
<th>Green Innovation:</th>
<th>Non-Innovation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enterprise 1</td>
<td>( x )</td>
<td>( 1 - x )</td>
</tr>
<tr>
<td>Enterprise 2</td>
<td>( y )</td>
<td>( 1 - y )</td>
</tr>
<tr>
<td>Green Innovation: ( y )</td>
<td>( \pi_{G1}^{c1} ), ( \pi_{G2}^{c1} )</td>
<td>( \pi_{G1}^{c2} ), ( \pi_{G2}^{c2} )</td>
</tr>
<tr>
<td>Non-Innovation: ( 1 - y )</td>
<td>( \pi_{G1}^{c2} ), ( \pi_{G2}^{c2} )</td>
<td>( \pi_{G1}^{c2} ), ( \pi_{G2}^{c2} )</td>
</tr>
</tbody>
</table>

3.4. Dynamic Evolutionary Game Model

3.4.1. Dynamic Evolutionary Game Model under Green Innovation Subsidies

In the dynamic evolutionary game model under green innovation subsidies, Enterprise 1 and Enterprise 2 adjust their probability of green innovation with time \( t \) changes to maximize their profits. According to previous assumptions, the probability of Enterprise 1 choosing the Green Innovation and Non-Innovation strategies are \( x (x \in [0, 1]) \) and \( 1 - x \) at time \( t \), respectively. Thus, the expected profit functions of Enterprise 1 choosing the Green Innovation and Non-Innovation strategies at time \( t + 1 \) are:

\[ u^{S1}_{G1} = y \pi^{S1}_{11} + (1 - y) \pi^{S1}_{12}, \quad (17) \]

\[ u^{S1}_{N1} = x \pi^{S1}_{21} + (1 - x) \pi^{S1}_{22}. \quad (18) \]

Therefore, the overall expected profit function of Enterprise 1 at time \( t + 1 \) is:

\[ u^{G1} = xu^{G1}_{G1} + (1 - x)u^{G1}_{N1}. \quad (19) \]

Similarly, the probability of Enterprise 2 choosing the green Innovation and Non-Innovation strategy are \( y (y \in [0, 1]) \) and \( 1 - y \) at time \( t \), respectively. The expected functions of Enterprise 2 choosing the Green Innovation and Non-Innovation strategies are, respectively:

\[ u^{S2}_{G1} = x \pi^{S2}_{11} + (1 - x) \pi^{S2}_{12}, \quad (20) \]

\[ u^{S2}_{N1} = x \pi^{S2}_{21} + (1 - x) \pi^{S2}_{22}. \quad (21) \]

Therefore, the overall expected profit function of Enterprise 2 at time \( t + 1 \) is:

\[ u^{S2} = yu^{S2}_{G1} + (1 - y)u^{S2}_{N1}. \quad (22) \]
To simplify the calculation, we assumed that \(c_1 = c_2 = c\) without any influence on the key parameters and variable values. Then, we could obtain the replicated dynamic equations of Enterprise 1 and Enterprise 2 through the equations of \(D(x) = dx/dt\) and \(D(y) = dy/dt\) (the derived Equations are listed in Appendix B). Let \(D(x) = 0\), and the equilibrium points of Enterprise 1’s replicated dynamic equation are \(x_1 = 0\), \(x_2 = 1\), and \(y_1^* = \frac{-(a-c)^2/72b+(1-s_1)b_1w_1(a-c)/4b}{10(a-c)^2/576b+(1-s_1)b_1w_1(a-c)/16b}\). Let \(D(y) = 0\), and the equilibrium points of Enterprise 2’s replicated dynamic equation are \(y_1 = 0\), \(y_2 = 1\), and \(x_1^* = \frac{-(a-c)^2/72b+(1-s_2)b_2w_2(a-c)/4b}{10(a-c)^2/576b+(1-s_2)b_2w_2(a-c)/16b}\).

Based on the replicated dynamic equations of Enterprise 1 and Enterprise 2, we can calculate the Jacobian matrix of the dynamic evolutionary game model for LCT heterogeneous enterprises, which is \(f(x, y)\) (the derived equation is listed in Appendix B). According to the equilibrium analysis process of the dynamic evolutionary game model, we can arrive at an ESS with the condition that \(det(J) > 0\) and \(tr(J) < 0\) [60]. The \(det(J)\) and \(tr(J)\) at the replicated dynamic equations’ equilibrium points can be calculated from Equation (A15), as shown in Table 4.

<table>
<thead>
<tr>
<th>Equilibrium Point</th>
<th>(det(J))</th>
<th>(tr(J))</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>+</td>
<td>+</td>
<td>Unstable</td>
</tr>
<tr>
<td>(0,1)</td>
<td>+</td>
<td>+</td>
<td>Unstable</td>
</tr>
<tr>
<td>(1,0)</td>
<td>+</td>
<td>+</td>
<td>Unstable</td>
</tr>
<tr>
<td>(1,1)</td>
<td>+</td>
<td>-</td>
<td>ESS</td>
</tr>
<tr>
<td>(x*, y*)</td>
<td>+</td>
<td>0</td>
<td>Center</td>
</tr>
</tbody>
</table>

Table 4 illustrates that [Green Innovation; Green Innovation] strategy profile is the only ESS in this dynamic evolutionary game model. Green innovation subsidies provided by the government can stimulate LCT heterogeneous enterprises to carry out green innovation effectively in a certain situation.

### 3.4.2. Dynamic Evolutionary Game Model under Green Innovation Subsidies and Carbon Tax

In the dynamic evolutionary game model under a policy system of green innovation subsidies and carbon tax, Enterprise 1 and Enterprise 2 also adjust their probability of green innovation with time \(t\) changes to maximize profits. Similar to the previous calculation processes for Enterprise 1 and Enterprise 2’s expected profit functions, Enterprise 1’s expected profit functions of Green Innovation and Non-Innovation at time \(t + 1\) are:

\[
\begin{align*}
    u_{G1}^{G1} &= y\pi_{G1}^{G1} + (1 - y)\pi_{G1}^{G1}, \quad (23) \\
    u_{G1}^{NI} &= y\pi_{G1}^{G1} + (1 - y)\pi_{G1}^{G1}. \quad (24)
\end{align*}
\]

The overall expected profit function of Enterprise 1 at time \(t + 1\) is:

\[
\begin{align*}
    u_{G1} &= xu_{G1}^{G1} + (1 - x)u_{G1}^{G1}. \quad (25)
\end{align*}
\]

Similarly, Enterprise 2’s expected profit functions of Green Innovation and Non-Innovation at time \(t + 1\) are:

\[
\begin{align*}
    u_{G2}^{G1} &= x\pi_{G2}^{G1} + (1 - x)\pi_{G2}^{G1}, \quad (26) \\
    u_{G2}^{NI} &= x\pi_{G2}^{G1} + (1 - x)\pi_{G2}^{G1}. \quad (27)
\end{align*}
\]

The overall expected profit function of Enterprise 2 at time \(t + 1\) is:
Let \( D(x) = dx/dt = 0 \), and the equilibrium points of Enterprise 1’s replicated dynamic equation are \( x_1 = 0, x_2 = 1, \) and \( y_1^* = -(-a-c)^2/(2b+\beta_1)(a-c) + t_1 w_1(a-c)/2b + (1-s_1)\beta_2 w_1^2(a-c)/4b \).

Let \( D(y) = dy/dt = 0 \), and the equilibrium points of Enterprise 2 are \( y_1 = 0, y_2 = 1, \) and \( x_1^* = -(-a-c)^2/72b+\beta_1\beta_2(a-c)/6b - t_2 w_2(a-c)/2b + (1-s_2)\beta_2 w_2^2(a-c)/4b \).

Based on the replicated dynamic equations of Enterprise 1 and Enterprise 2 under green innovation subsidies and a carbon tax, we can calculate the Jacobian matrix \( J(x,y) \) (the derived equation is listed in Appendix B). The \( det(J) \) and \( tr(J) \) at the replicated dynamic equations’ equilibrium points in this policy situation can be calculated from Equations (A16) and (A17), to arrive at Table 5.

### Table 5. Stability analysis under green innovation subsidies and carbon tax.

<table>
<thead>
<tr>
<th>Equilibrium Point</th>
<th>( det(J) )</th>
<th>( tr(J) )</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>+</td>
<td>+</td>
<td>Unstable</td>
</tr>
<tr>
<td>(0,1)</td>
<td>+</td>
<td>+</td>
<td>Unstable</td>
</tr>
<tr>
<td>(1,0)</td>
<td>+</td>
<td>-</td>
<td>Unstable</td>
</tr>
<tr>
<td>(1,1)</td>
<td>+</td>
<td>0</td>
<td>ESS</td>
</tr>
<tr>
<td>((x^<em>,y^</em>))</td>
<td>+</td>
<td>-</td>
<td>Center</td>
</tr>
</tbody>
</table>

Table 5 shows that \( det(J) > 0 \) and \( tr(J) < 0 \) at point (1,1), which means that the [Green Innovation; Green Innovation] strategy profile is the only ESS in the dynamic evolutionary game model under a policy system of a green innovation subsidy and carbon tax. Introducing a unit progressive carbon tax to the environmental policy system will not change the policy effectiveness of green innovation subsidies. The concrete synergy policy effect will be illustrated in the next section by numerical simulation.

### 4. Numerical Simulations

In this section, we used MATLAB R2018b to perform a numerical simulation of the ESS of the dynamic evolutionary game model. First, we analyzed the influence of introducing a unit carbon tax on the existing green innovation effect of green innovation subsidies, especially the different influence of a new policy system consisting of green innovation subsidies and a carbon tax on LCT heterogeneous enterprises. Second, we analyzed the influence of the green innovation cost coefficient’s change on the green innovation effect of existing green innovation subsidies. We considered the effect of introducing a carbon tax when the negative influence of the green innovation cost coefficient makes existing green subsidies ineffective. Finally, we analyzed the influence of the carbon reduction amount’s change on the green innovation effect of existing green innovation subsidies. We also calculated the effect of introducing a carbon tax when the negative influence of the carbon reduction amount makes existing green subsidies ineffective.

Because of the advantage of LCT, the initial carbon emission and green innovation cost coefficient of Enterprise 1 were lower than for Enterprise 2, and the carbon reduction amount of Enterprise 1 was higher than for Enterprise 2, according to previous assumptions. Because Enterprise 1 had an advantage in green innovation, the subsidy proportion received by Enterprise 1 was higher than Enterprise 2. Following the policy setting of the unit carbon tax, the tax rate imposed on Enterprise 1 was lower than Enterprise 2. The specific parameter values are listed in Table 6. The value of the green innovation cost coefficient, initial carbon emission amount, and carbon reduction amount were set to satisfy the equilibrium conditions of ESS. The value of green innovation subsidies’ proportion and carbon tax rate was set according to the Chinese government’s practice of subsidizing NEV industries and imposing an environmental protection tax. The parameter value of the market demand...
function was consistent with previous studies, where \( a = 50, b = 3, \) and \( c = 35 \) \[7,8,35\]. Then, we analyzed the evolutionary paths of the LCT heterogeneous Enterprise 1 and Enterprise 2 in different situations.

### Table 6. Simulation parameter assignment.

<table>
<thead>
<tr>
<th>Parameters of Enterprise 1 Assignment</th>
<th>Parameters of Enterprise 2 Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_1 ) 20</td>
<td>( \varepsilon_2 ) 22</td>
</tr>
<tr>
<td>( w_1 ) 10</td>
<td>( w_2 ) 8</td>
</tr>
<tr>
<td>( \beta_1 ) 0.5%</td>
<td>( \beta_2 ) 0.6%</td>
</tr>
<tr>
<td>( s_1 ) 0.4</td>
<td>( s_2 ) 0.35</td>
</tr>
<tr>
<td>( \tau_1 ) 0.1</td>
<td>( \tau_2 ) 0.2</td>
</tr>
</tbody>
</table>

### 4.1. Synergy Green Innovation Effect of Green Innovation Subsidies and Carbon Taxes

Figure 1 shows the green innovation evolutionary path of enterprises when only green innovation subsidies were provided for Enterprise 1 and Enterprise 2 by the government. The left part in Figure 1 demonstrates that Enterprise 1’s green innovation probability will converge to 1, which means Enterprise 1 will choose the Green Innovation strategy in a 100% probability, whether the initial green innovation probability of Enterprise 1 is low \( (x < 0.5) \) or high \( (x > 0.5) \). Similarly, the right part in Figure 1 illustrates that Enterprise 2’s green innovation probability under green innovation subsidies will converge to 1 like Enterprise 1. The convergence time to 1 of both Enterprise 1 and Enterprise 2 will shorten with the increase in the enterprises’ initial green innovation probability. Enterprise 2 is a little more sensitive to green innovation subsidies than Enterprise 1, because the convergence speed of Enterprise 2 is a little faster than Enterprise 1 when they have the same initial green innovation probability. Overall, green innovation subsidies are an effective environmental policy for stimulating enterprises’ green innovation, because they cause the green innovation probability of all of the LCT heterogeneous enterprises to converge to 1 at the same time.

![Figure 1. Policy effect of green innovation subsidies.](image-url)
As illustrated in Figure 2, when the unit carbon tax was introduced into a policy system, the policy effectiveness of existing green innovation subsidies did not change. Moreover, the green innovation effect of the policy system was more significant than the single green innovation subsidy, which, as demonstrated by both the convergence time and convergence speed to 1 in Figure 2, was much shorter or faster than that in Figure 1. The numerical simulation showed that there was a significant overlapping policy effect of green innovation subsidies and carbon taxes. In other words, green innovation subsidies were a milder environmental policy than the policy system. In addition, as opposed to the situation in Figure 1, Enterprise 1 was much more sensitive to the policy system of green innovation subsidies and carbon tax than Enterprise 2. In our understanding, when there were only green innovation subsidies in the policy system, Enterprise 2 was more willing to carry out green innovation with support from the government, because its disadvantage in LCT meant a poor affordability due to the failure of green innovation. When the government introduced a unit carbon tax on the basis of green innovation subsidies, Enterprise 1 wanted to acquire a bigger carbon emission cost-saving than Enterprise 2 because its advantage in LCT meant a higher carbon reduction amount than Enterprise 2. Thus, introducing the unit progressive carbon tax made Enterprise 1 more sensitive to the new policy system.

![Figure 2. Policy system of green innovation subsidies and a carbon tax.](image)

4.2. The Influence of the Green Innovation Cost Coefficient

The cost of green innovation was the most important factor when enterprises made green innovation decisions. In this section, we simulated the influence of the green innovation cost coefficient on the green innovation effect of green subsidies and the effect of introducing a carbon tax. Because Enterprise 2 was more sensitive to green innovation subsidies, we analyzed it as an example on the premise that it was risk-rational ($y = 0.5$). As illustrated in Figure 3, the evolutionary speed of Enterprise 2 converging to 1 was slower than before its green innovation cost coefficient increased from $\beta_2 = 0.6\%$ to $\beta_2 = 1.8\%$. If the green innovation cost coefficient of Enterprise 2 kept increasing, the evolutionary path of Enterprise 2 would not converge to 1 as before and might even converge in the opposite
direction until it reached 0. This implied that the green innovation effect of green subsidies was negatively affected by the increase in the green innovation cost coefficient. The green innovation effect of green innovation subsidies was ineffective when the innovation cost coefficient reached a certain level, which meant the policy did not work to encourage Enterprise 2’s green innovation.

\[ \beta_2 = 1.8\% \]

\[ \beta_2 = 3\% \]

\[ \beta_2 = 5\% \]

Figure 3. Influence of the green innovation cost coefficient under green innovation subsidies.

Figure 4 shows the policy effect of increasing the subsidy proportion when the increase in the green innovation cost coefficient led to a flip of Enterprise 2’s green innovation evolutionary path. In fact, it had a limited effect on increasing the subsidy proportion in a normal policy strength (\( s_2 \) increasing from 0.35 to 0.45) when \( \beta_2 = 3\% \). The policy effect became significant only if the subsidy proportion achieved 0.6, which is too high to be adopted by governments. In an extremely bad situation where \( \beta_2 = 5\% \), even if the government granted green innovation subsidies from 0.35 to 0.6%, the green innovation probability of Enterprise 2 improved only a little. Thus, when the green innovation cost coefficient was extremely high, a single green innovation subsidy was ineffective at stimulating the enterprise’s green innovation. This can help us understand why green innovation subsidies cannot achieve the expected policy effect in industries with an extremely high green innovation cost coefficient. Governments can introduce more environmental policies such as carbon taxes to pursue a synergy green innovation effect from the new consisted policy system.
Considering the limited or poor policy effect of increasing the subsidy proportion, we analyzed the synergy policy effect of introducing a unit progressive carbon tax in Figure 5. When introducing a unit progressive carbon tax under the same conditions in Figure 3, Enterprise 2’s ability to afford the green innovation cost coefficient became much higher. Compared with Figure 3, when the green innovation cost coefficient \( \beta_2 \) increased from 0.6 to 1.2% and 1.8%, the evolutionary path of Enterprise 2 converged to 1 in a shorter time. When \( \beta_2 \) increased to 2.4%, Enterprise 2’s green innovation evolutionary path converged to 1 rather than 0.76 in Figure 3 under the policy system consisting of green innovation subsidies and a unit progressive carbon tax. The green innovation evolutionary path converged to a lower level than 0.5 under the single green innovation subsidy when \( \beta_2 = 3.0\% \). Introducing a unit progressive caused the final green innovation probability to change to a higher level than 0.5 in Figure 5. Therefore, the introduction of a unit progressive carbon tax improved the enterprises’ ability to afford the green innovation cost coefficient. As analyzed in the dynamic evolutionary game model, enterprises can obtain a carbon emission cost saving advantage from green innovation when governments implement a policy system that contains carbon taxes. Introducing a unit progressive carbon tax provides more incentives to afford a higher green innovation cost coefficient. To sum up, improving the enterprises’ ability to afford the green innovation cost coefficient is one of the synergy green innovation effects of green innovation subsidies and a carbon tax. However, the current policy system is invalid if the green innovation cost coefficient increases to an extremely high extent (\( \beta_2 = 5.0\% \)), as seen in Figure 5. In the next section, we will discuss the different policy effects of increasing the carbon tax rate and subsidy proportion in an extremely bad situation (\( \beta_2 = 5.0\% \)).
As illustrated in Figure 6, the green innovation evolutionary path of Enterprise 2 gradually converged to 1 with the carbon tax rate increasing from 0.2 to 0.35. The improvement of Enterprise 2’s green innovation probability brought about by increasing the innovation subsidy was poorer than when increasing the carbon tax rate. Even if the green innovation subsidy proportion increased to 0.6, the policy effect was just the same as the carbon tax rate increasing to 0.25. According to the simulation in Figure 6, it is clear that increasing the carbon tax rate is more effective than increasing the funding proportion when the green innovation cost coefficient is extremely high in some industries. Compared with raising a higher carbon tax, increasing the green innovation subsidy proportion means a higher fiscal expenditure. Therefore, introducing carbon taxes is worth considering by governments in industries with extremely high green innovation coefficients to expand the policy adjustment space.
4.3. The Interaction between the Carbon Reduction Amount and the Policy Synergy Effect

In this section, we analyze the influence of the carbon reduction amount, which affects enterprises’ green innovation decisions by innovation cost and carbon reduction revenue. As shown in Figure 7, Enterprise 2’s green innovation evolutionary path slightly improved with the carbon reduction amount decreasing from 8 to 2 and the government only providing green innovation subsidies. On the contrary, Enterprise 2’s green innovation evolutionary path deteriorated to 0 with the carbon reduction amount increasing from 8 to 20. The reason for this discrepancy is that the increase in the carbon reduction amount would increase the innovation cost at first. When there is no punitive environmental policy, such as a carbon tax, the green innovation will not bring a carbon emission cost-saving advantage for enterprises in market competition. Thus, enterprises are more likely to choose the green innovation plan with lower carbon reduction amounts to maintain the market-leading advantage at a lower investment. However, it is not a good choice for a whole society to develop a green economy.

Figure 8 illustrates the policy effect of increasing the green innovation proportion when the increase in carbon emission reduction reversed the green innovation evolutionary path of Enterprise 2. When the carbon emission reduction was \( w_2 = 17 \), Enterprise 2’s green innovation evolutionary path gradually converged to 1, with the subsidy proportion increasing to 0.6. It is difficult, however, for governments to provide a green innovation subsidy to share the 60% green innovation cost with enterprises. When the carbon emission reduction was \( w_2 = 20 \), Enterprise 2’s green innovation evolutionary path improved a little, with the subsidy proportion increasing to 0.6. As seen in Figures 7 and 8, the green innovation effect of green innovation subsidies was limited when the carbon reduction amount kept increasing. Increasing the green innovation subsidy proportion did not offer an additional incentive affecting the enterprises’ green innovation decisions.
Figure 7. Influence of the carbon reduction amount.

Figure 8. Effect of increasing the subsidy proportion under green innovation subsidies.
In Figure 9, when the unit progressive carbon tax was introduced, the green innovation evolutionary path did not deteriorate as the carbon reduction amount increased. Moreover, Enterprise 2’s green innovation evolutionary path converged to 1 at a faster speed, with the carbon reduction amount increasing from 8 to 20. Under the policy system consisting of green innovation subsidies and a carbon tax, Enterprise 2’s green innovation evolutionary path deteriorated in the opposite direction, with the carbon reduction amount decreasing to 3. This was totally contrary to the situation in Figure 8. Introducing a unit progressive carbon tax caused the increase in the carbon reduction amount to amplify enterprises’ advantages in carbon emission cost saving. Thus, enterprises are motivated to afford the green innovation cost from the increase in the carbon reduction amount.

Figure 9. Influence of the carbon reduction amount under the policy system.

In Figure 10, we see the different policy effect of increasing the carbon tax rate or green innovation subsidy proportion when the decrease in the carbon reduction amount led to a flip of Enterprise 2’s green innovation evolutionary path under the policy system. According to the simulation result in Figure 10, increasing the carbon tax rate or green innovation subsidy proportion effectively improved Enterprise 2’s green innovation evolutionary path. The effective policy varied the direction and decreased the carbon tax rate. As shown in Figure 10, Enterprise 2’s green innovation evolutionary path converged to 1, with the carbon tax rate decreasing to 0.05 when $w_2 = 3$. Thus, increasing the carbon tax rate is not the only choice to optimize the carbon tax. Sometimes, governments should consider decreasing the carbon tax rate. To sum up, offering an additional incentive to enterprises to pursue a higher carbon reduction amount is also one of the synergy green innovation effects of green innovation subsidies and carbon taxes. In addition, a larger policy effect could be achieved by decreasing the carbon tax rate.
According to the numerical simulation results detailed in Section 4, the synergy green innovation effect of green innovation subsidies and carbon taxes was more than a simple overlapping policy effect. In fact, the introduction of carbon taxes made Enterprise 1 with a high LCT become more sensitive to the policy system consisting of green innovation subsidies and a carbon tax than Enterprise 2. The introduction of a carbon tax could increase enterprises’ ability to afford the green innovation cost coefficient. Additionally, introducing carbon taxes offers an additional incentive to enterprises to pursue a higher carbon reduction amount.

5. Conclusions and Discussions

This study tries to expand the synergy green innovation effect of green innovation subsidies and unit progressive carbon taxes. At first, we set up dynamic evolutionary game models with LCT heterogeneous enterprises as game participants to, respectively, analyze the policy effect of green innovation subsidies and a policy system containing unit progressive carbon taxes on enterprises’ endogenous green innovation decisions. The equilibrium analysis showed that \{Green Innovation; Green Innovation\} was the only ESS in the dynamic game model containing the unit progressive carbon tax. The dynamic evolutionary game model confirmed the synergy green innovation effect between green innovation subsidies and carbon taxes. Our basic conclusion obtained from the micro game model was same as the conclusion getting obtained from the macroeconomic model equilibrium by Acemoglu et al. [30]. Thus, more empirical research could be done on the basis of the consensus that green innovation subsidies and carbon taxes have the synergy green innovation effect.

The numerical simulation result presented by MATLAB R2018b verified the green innovation effect of a green innovation subsidy and carbon tax. When the government introduced a carbon tax into a policy system, both Enterprise 1 with high LCT and Enterprise 2 with low LCT converged to the Green Innovation strategy at a much faster speed.
than in the situation where the government provided only green innovation subsidies. Furthermore, the synergy green innovation effect was also reflected in that introducing a unit progressive carbon tax made the high LCT Enterprise 1 more sensitive to the new consisted policy system. The enterprises with high LCT are more sensitive to the environmental policy system containing carbon taxes, because introducing a unit progressive carbon tax stimulates enterprises to pursue a larger carbon emission cost-saving advantage in market competition [6]. In fact, this finding has always been ignored by policy makers.

The numerical simulation showed that enterprises’ green innovation evolutionary paths can flip to 0 with the increase in the green innovation cost coefficient at first. Furthermore, increasing the subsidy proportion has a limited effect in improving enterprises’ green innovation probability. When we introduced the unit progressive carbon taxes to the environmental policy system, the enterprises’ ability to afford the green innovation cost coefficient became much higher than before. Thus, increasing enterprises’ affordability of the green innovation cost coefficient is also one of the synergy green innovation effects of green innovation subsidies and carbon taxes. The increase in the green innovation cost coefficient means a high green innovation R&D investment for enterprises. The market advantage acquired from green innovation is not large enough to allow enterprises to afford the high green innovation cost coefficient, although governments co-share the innovation cost [8]. Introducing a carbon tax has put an additional carbon emission cost on enterprises, which has created the motivation for enterprises to reduce carbon emissions through green innovation [61]. Additionally, increasing the tax rate was more effective than increasing the subsidy proportion in the new environmental policy system when enterprises gave up green innovation because of the extremely high green innovation cost coefficient.

The numerical simulation on the different influences of carbon reduction amounts confirmed the probability of enterprises adopting green innovation declines with the increase in the carbon reduction amount in the beginning. The introduction of a unit progressive carbon tax could stimulate enterprises’ green innovation decisions, because green innovation can bring a carbon emission cost-saving advantage for them in the policy system containing unit progressive carbon taxes. Therefore, the synergy green innovation effect of green innovation subsidies and a carbon tax also includes that it creates additional encouragement for enterprises to pursue a higher carbon reduction amount. The increase in the carbon reduction amount means a higher investment in carbon reduction. Once the increase in investment is no longer compensated by the market leading position acquired from the green innovation, enterprises will give up the green innovation [62]. However, the increase in the carbon reduction amount means a smaller carbon emission cost, which will transfer to the market competition advantage for enterprises [63]. In addition, it is more effective to decrease the carbon tax rate than to increase the tax rate or subsidy proportion when enterprises give up green innovation because their carbon reduction amount is extremely low.

In summary, our research confirmed the synergy green innovation effect of green innovation subsidies and carbon taxes in the micro-dynamic evolutionary model, which has provided new theoretical evidence for it beyond the macroeconomic model. Furthermore, the numerical simulation results expand the synergy green innovation effect in three aspects, which have not been put forward in previous research.

Based on the profound contributions of our research, governments can try to adjust their environmental policies as follows: Firstly, countries like China that have set a carbon reduction scheme should consider introducing a unit progressive carbon tax. The unit progressive carbon tax will amplify the green innovation effect of green innovation subsidies and alleviate the subsidy expenditure pressure. Secondly, governments can encourage enterprises with a LCT advantage to carry out green innovation more actively by introducing a unit progressive carbon tax, because enterprises with high LCT are more sensitive to a policy system containing a unit progressive carbon tax. Thirdly, it is necessary to introduce a unit progressive carbon tax from the perspective of increasing the affordability of the green innovation cost coefficient or encouraging enterprises to continue increasing their
carbon reduction amount in specific industries. Finally, Governments should vary the carbon tax rate rather than subsidy proportion to amplify the synergy green innovation effect for increasing enterprises’ affordability relating to the innovation cost coefficient. Simultaneously, a decrease in the carbon tax rate offers an effective policy choice to deal with the problem of some industries with a low carbon reduction amount at the beginning of the green innovation transition.

This research discusses the synergy green innovation effect of green innovation subsidies and carbon taxes totally based on the theoretical deriving and numerical simulation of dynamic evolutionary game models. Though the set of simulation parameters has considered the subsidy proportion and tax rate in practice, there is still a lack of the empirical data to support the theoretical conclusion. In the future, researchers can study the green innovation effect of green innovation subsidies and carbon taxes in a specific industry, like steel industry, for instance, by the theoretical model and numerical simulation method of this research to evaluate the actual policy effect. In addition, we mentioned that tax revenue can alleviate the governments’ fiscal pressure of a green innovation subsidy, but we did not reflect the tax revenue transfer in the dynamic game model. For further study, researchers can set up an internal circulation between the carbon tax revenue and green innovation subsidy in the theoretical model to expand the related study. At last, this research studied the synergy green innovation effect of green innovation subsidies and carbon taxes, which did not pay attention to the influence of the synergy green innovation effect on macro social welfare. Thus, researchers can discuss the influence of green innovation subsidies and carbon taxes on social welfare through stimulating enterprises’ green innovation in the Computable General Equilibrium model.

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Appendix A

The solution process for the equilibrium market price and production of enterprises.

Strategy Profile 1. Enterprise 1 chooses the Green Innovation strategy, and Enterprise 2 chooses the Non-Innovation strategy. When Enterprise 1 takes the market leading position through green innovation, Enterprise 1 and Enterprise 2 play a Stackelberg game, led by Enterprise 1. In this Stackelberg game model, Enterprise 1’s revenue function is \( \pi_1(q_1, q_2^*, p) \). Enterprise 1 decides its profit maximization production according to the production of Enterprise 2. Based on the assumptions in Section 3, the revenue function of Enterprise 1 is:

\[
\pi_2(q_2, p) = [a - b(q_1 + q_2)] * q_2 - q_2 * c_2.
\]  

(A1)

According to the FOC,

\[
\frac{d\pi_2}{dq_2} = a - 2bq_2 - bq_1 - c_2 = 0.
\]  

(A2)
Thus,
\[ q_2^* = (a - bq_1 - c_2)/2b. \] \hfill (A3)

Then, the revenue function of Enterprise 1 is:
\[ \pi_1(q_1, q_2^*, p) = [a - b(q_1 + q_2^*)] * q_1 - q_1 * c_1. \] \hfill (A4)

Substituting Equation (A3) into Equation (A4), we have:
\[ \pi_1(q_1, q_2^*, p) = a - bq_1 + c_2 \frac{q_1}{2} * q_1 - q_1 * c_1. \] \hfill (A5)

According to the FOC,
\[ \frac{d\pi_2}{dq_2} = \frac{a}{2} - bq_1 + \frac{c_2}{2} - c_1 = 0. \] \hfill (A6)

Thus, the profit maximization production of Enterprise 1 is \( q_1^* = (a - 2c_1 + c_2)/2b \). Then, the best production of Enterprise 2 and market price are \( q_2^* = (a + 2c_1 - 3c_2)/4b \) and \( p = (a + 2c_1 + c_2)/4 \).

**Strategy Profile 2.** Enterprise 1 chooses the Non-Innovation strategy, and Enterprise 2 chooses the Green Innovation strategy. When Enterprise 2 takes the market leading position through green innovation, Enterprise 1 and Enterprise 2 play a Stackelberg game, led by Enterprise 2. Similar to the solution process of the Stackelberg game model led by Enterprise 1, we can calculate the profit maximization production of Enterprise 2 as \( q_2^* = (a - 2c_1 + c_2)/2b \). The best production of Enterprise 1 and market price are \( q_1^* = (a + 2c_1 - 3c_2)/4b \) and \( p = (a + 2c_1 + c_2)/4 \).

**Strategy Profile 3.** Enterprise 1 chooses the Green Innovation strategy, and Enterprise 2 chooses the Green Innovation strategy as well. When Enterprise 1 and Enterprise 2 both choose the Green Innovation strategy, Enterprise 1 and Enterprise 2 have the same probability of becoming the market leader. Thus, the profit maximization production of Enterprise 1 and Enterprise 2 is \( q_1^* = q_2^* = \left( \frac{a - 2c_1 + c_2}{2b} + \frac{a + 2c_1 - 3c_2}{4b} \right) * \frac{1}{2} = (3a - 2c_1 - c_2)/8b \), and the equilibrium market price is \( p = (a + 2c_1 + c_2)/4 \).

**Strategy Profile 4.** Enterprise 1 chooses the Non-Innovation strategy, and Enterprise 2 chooses the Non-Innovation strategy as well. When Enterprise 1 and Enterprise 2 both choose the Non-Innovation strategy, they play a Cournot game. The revenue functions of Enterprise 1 and Enterprise 2 are:
\[ \pi_1(q_1, p) = [a - b(q_1 + q_2)] * q_1 - q_1 * c_1. \] \hfill (A7)
\[ \pi_2(q_2, p) = [a - b(q_1 + q_2)] * q_2 - q_2 * c_2. \] \hfill (A8)

According to the FOC,
\[ \frac{d\pi_1}{dq_1} = a - 2bq_1 - bq_2 - c_1 = 0, \] \hfill (A9)
\[ \frac{d\pi_2}{dq_2} = a - 2bq_2 - bq_1 - c_2 = 0. \] \hfill (A10)

Then we have:
\[ q_1 = \frac{a - bq_2 - c_1}{2b}, \] \hfill (A11)
\[ q_2 = \frac{a - bq_1 - c_2}{2b}. \] \hfill (A12)

Combining Equations (A11) and (A12), we have \( q_1^* = q_2^* = (a + c_2 - 2c_1)/3b \), and the equilibrium market price is \( p = (a - 2c_2 + 4c_1)/3 \).
Appendix B

Equations in Section 3.4.

The replicated dynamic equation of Enterprise 1 under green innovation subsidies is:

\[
D(x) = \frac{dx}{dt} = x(1-x) \left\{ y \left[ 10(a-c)^2/576b + (1-s_1)\beta_1 w_1(a-c)/16b \right] + [(a-c)^2/72b - (1-s_1)\beta_1 w_1(a-c)/4b] \right\}. \tag{A13}
\]

The replicated dynamic equation of Enterprise 2 under green innovation subsidies is:

\[
D(y) = \frac{dy}{dt} = y(1-y) \left\{ x \left[ 10(a-c)^2/576b + (1-s_2)\beta_2 w_2(a-c)/16b \right] + [(a-c)^2/72b - (1-s_2)\beta_2 w_2(a-c)/4b] \right\}. \tag{A14}
\]

The Jacobian matrix of the dynamic evolutionary game model for LCT heterogeneous enterprises under green innovation subsidies and a carbon tax is:

\[
J = \begin{pmatrix}
(1-2x) \left[ y \left( \frac{10bL^2}{576} + \frac{L(1-s_1)\beta_1 w_1^2}{16} \right) + \left( \frac{bL}{72} - \frac{L(1-s_1)\beta_1 w_1^2}{4} \right) \right] & x(1-x) \left( \frac{10bL^2}{576} + \frac{L(1-s_1)\beta_1 w_1^2}{16} \right) \\
y(1-y) \left( \frac{10bL^2}{576} + \frac{L(1-s_1)\beta_1 w_1^2}{16} \right) & (1-2y) \left[ y \left( \frac{10bL^2}{576} + \frac{L(1-s_1)\beta_1 w_1^2}{16} \right) + \left( \frac{bL}{72} - \frac{L(1-s_1)\beta_1 w_1^2}{4} \right) \right]
\end{pmatrix}. \tag{A15}
\]

The replicated dynamic equation of Enterprise 1 under green innovation subsidies and a carbon tax is:

\[
D(x) = \frac{dx}{dt} = x(1-x) \left\{ y \left[ 10(a-c)^2/576b + \tau_1 \varepsilon_1 (a-c)/24b - \tau_1 \varepsilon_1 (a-c)/8b + (1-s_1)\beta_1 w_1(a-c)/16b \right] + [(a-c)^2/72b - \tau_1 \varepsilon_1 (a-c)/6b + \tau_1 \varepsilon_1 (a-c)/2b - (1-s_1)\beta_1 w_1(a-c)/4b] \right\}. \tag{A16}
\]

The replicated dynamic equation of Enterprise 2 under green innovation subsidies and a carbon tax is:

\[
D(y) = \frac{dy}{dt} = y(1-y) \left\{ x \left[ 10(a-c)^2/576b + \tau_2 \varepsilon_2 (a-c)/24b - \tau_2 \varepsilon_2 (a-c)/8b + (1-s_2)\beta_2 w_2(a-c)/16b \right] + [(a-c)^2/72b - \tau_2 \varepsilon_2 (a-c)/6b + \tau_2 \varepsilon_2 (a-c)/2b - (1-s_2)\beta_2 w_2(a-c)/4b] \right\}. \tag{A17}
\]

The Jacobian matrix of the dynamic evolutionary game model for LCT heterogeneous enterprises under green innovation subsidies and a carbon tax is:

\[
J = \begin{pmatrix}
(1-2x) \left[ y \left( \frac{10bL^2}{576} + \frac{L(1-s_1)\beta_1 w_1^2}{16} \right) + \left( \frac{bL}{72} - \frac{L(1-s_1)\beta_1 w_1^2}{4} \right) \right] & x(1-x) \left( \frac{10bL^2}{576} + \frac{L(1-s_1)\beta_1 w_1^2}{16} \right) \\
y(1-y) \left( \frac{10bL^2}{576} + \frac{L(1-s_1)\beta_1 w_1^2}{16} \right) & (1-2y) \left[ y \left( \frac{10bL^2}{576} + \frac{L(1-s_1)\beta_1 w_1^2}{16} \right) + \left( \frac{bL}{72} - \frac{L(1-s_1)\beta_1 w_1^2}{4} \right) \right]
\end{pmatrix}. \tag{A18}
\]

References


11. Hattori, K. Optimal combination of innovation and environmental policies under technology licensing. *Econ. Model.* 2017, 64, 601–609. [CrossRef]


27. Zhao, X.M.; Bai, X.L. How to motivate the producers’ green innovation in WEEE recycling in China?—An analysis based on evolutionary game theory. *Waste Manag.* 2021, 122, 26–35. [CrossRef]


44. Fried, S. Climate Policy and Innovation: A Quantitative Macroeconomic Analysis. *Am. Econ. J. Macroecon.* 2018, 10, 90–118. [CrossRef]