The Effect of the Subsidiary on the Ultimate Controller’s Private Benefits: Enlightenment to the Risk Management Challenges for Sustainability of the Corporate

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Abstract: In a period of uncertainty about economic development, it is particularly important to maintain corporate sustainable growth in order to deal with the risk management challenge of sustainability. Private benefits of control in corporate governance play a crucial role in ensuring corporate financial sustainability to face the risk. The existing literature about private benefits of control mainly focuses on the assumption of absolute control by the ultimate controller, ignoring the influence of subsidiaries. This paper constructs a model of private benefits, based on a framework of the interaction of ultimate controllers and subsidiaries, and investigates how subsidiaries influence the ultimate controller’s expropriation. The model has proposed that: Subsidiary’s self-interest demand can prevent the ultimate controller’s private benefits; the autonomy owned by the subsidiary can be used to allocate resources, inhibiting the private benefits of control to some extent. Further research has found that when the proportion of funds that can be arranged by the subsidiary’s autonomy can meet the proportion of funds required for the subsidiary’s self-interest demand, as the subsidiary’s self-interest demand increases, the ultimate controller’s expropriation is reduced. This paper reveals the internal mechanism that private benefits of control are jointly determined by the ultimate controller and the subsidiary, expands the research on the decision mechanism of private benefits and provides new ideas for understanding the expropriation of the ultimate controller. Additionally, the solution to this problem can provide help and inspiration for risk management challenges for the sustainability of the corporate, as well as provide reference significance for economic sustainability.

Keywords: risk management; ultimate controller; private benefits of control; subsidiary autonomy; subsidiary self-interest demand

1. Introduction

Dramatic changes in the economic environment have led to challenges in the risk management sustainability of the corporate. One of the company’s measures to meet this challenge is to maintain financial sustainability. Among them, is how to improve the efficiency of corporate governance, which plays a crucial part in financial sustainability. Correspondingly, the expropriation of the ultimate controller is an essential element of corporate governance. It needs to be solved urgently nowadays how the listed company reduces the expropriation from the ultimate controller.

With the continuous in-depth reform and improvement of the financial market, the interests of small and medium investors obtain further protection. However, the problem of the ultimate controller’s expropriation still exists generally, especially in emerging market countries. Moreover, even one of the motivations of many controllers to hold shares is to obtain private benefits of control currently [1]. The pyramid ownership structure realizes the separation of control and ownership of the ultimate controller of a public company [2–5], which leads to agency conflicts between large and small shareholders [6,7], resulting in the ultimate controller’s behavior of expropriation [8–10], in turn, damaging the value of the
company and the interests of minority shareholders. Despite the strong interest in ultimate controllers of public firms; we know little about the internal mechanism of the ultimate controller’s behavior of expropriation. How public companies can reduce expropriation from the ultimate controller to promote sustainable development is a hot topic and an important one at present.

The existing literature has made a large number of studies on private benefits of control, but these studies only emphasize the dominant role of the ultimate controller, arguing that private benefits of control are absolutely controlled by the ultimate controller [11–13]. They ignore the role of the subsidiary. Jiang et al. (2010) not only quantify private benefits of control but also show that external regulation reduces private benefits of control while the degree of separation of powers increases [14]. Albuquerue and Wang found that under the low level of investor protection, the ultimate controller needs to weigh the benefits and losses when expropriating the private benefits [15]. The existing research implicitly assumes that expropriation is determined by the ultimate controller and the ultimate controller only considers the cost of legal regulation and loss of shared gains when expropriating private benefits of control, without considering the impact of subsidiaries, such as autonomy and self-interest demand on expropriation. In addition, the authors have found that the autonomy of a subsidiary has a positive impact on company performance, but there is no research to explore the mechanism of the subsidiary’s autonomy and self-interest demand on private benefits of control from the perspective of the internal market [16].

In fact, on the one hand, during the operation of the subsidiary in the pyramid structure, the ultimate controller’s control over the subsidiary is often not one-way, and its behavior is sometimes subject to the checks and balances as well as constraints of the subsidiary. In the early stage, it was assumed that the parent company had one-way control of the subsidiaries between the parent-subsidiary company [17]. Subsequently, this hypothesis was gradually expanded. Research based on the network relationship between parent and subsidiary companies believes that there is interdependence between parent and subsidiary companies which can make a mutual influence [18–20]. In the parent-subsidiary network, autonomy has attracted attention as an important factor affecting the overall behavior, strategy, and performance of parent-subsidiary companies [21]. The power of subsidiary management can affect the resource allocation tendency of the parent company [22].

At the same time, with the continuous increase in the number of subsidiaries and service markets, the group needs more and more professional agents to supervise, resulting in a substantial increase in the cost of supervision. Therefore, granting autonomy to subsidiaries is an inevitable trend in the development of subsidiaries under the pyramid ownership structure [23]. Subsidiaries with autonomy are no longer passive roles in terms of the ultimate controller’s private benefits, but can independently allocate the required resources, influencing private benefits of control furthermore [24]. Among them, the proportion of the business owned by the subsidiary and the cross-regional distribution of the business are important factors that affect the operating needs and autonomy of the subsidiary [25]. Furthermore, the increase in the business scale of subsidiaries will tilt the allocation of power between parent and subsidiary companies to subsidiaries, and the power of independent decision-making by subsidiaries will increase accordingly.

On the other hand, subsidiaries need to retain part of their earnings for future operational and investment needs to ensure financial flexibility and maintain the survival of the company so that the self-interest demand of the subsidiary can respond to the expropriation from the ultimate controller. Hart (2009) proposed that under incomplete contract conditions, both parties can bargain based on their own interests. Subsidiaries have control over resources due to their own demands [26]. With the expansion of the subsidiary’s business scale and the proportion of its business in the group increasing, the subsidiary enhances its bargaining power with the ultimate controller [24,27–29], in turn, obtaining a higher degree of autonomy. In this way, the subsidiary can determine the scope of business and make reasonable arrangements for assets, and it can also arrange funds in advance for
subsequent operations and investments within the permissions. That is to say, the ultimate controller’s expropriation will be affected as a result.

Last, the ultimate controller will also consider the autonomy and the demand of its own interests of the subsidiary to a certain extent, in order to be able to continuously expropriate the personal benefits of control.

As mentioned above, the subsidiaries can influence the expropriation. Although the subsidiary of the pyramid cannot directly oppose the ultimate controller, they can restrict private benefits of control. In this way, the ultimate controller’s expropriation of the public company will be affected by both the ultimate controller and the subsidiary. Therefore, in addition to the legal protections and the associated cost of expropriation, are private benefits of control also constrained by the self-interest demand and the autonomy of the subsidiary? This question reflects the necessity of the paper.

There has been research on the impact of ultimate controllers’ private benefits, which are mainly divided into three levels. First of all, at the institutional level, a transnational study by Dyck and Zingales (2004) shows that the degree of the legal protection of minority shareholder property rights, efficiency of law enforcement, and news media supervision has led to national differences in the ultimate controller’s private benefits [30]. They have also pointed out that appropriate tax policies can reduce the level of private benefits of control. Secondly, at the industry level, scholars’ research on expropriation shows that the ultimate controller’s private benefits present obvious industry differences. Demsetz and Lehn (1985) have found that media and entertainment have a relatively high scale of expropriation [31]. Finally, at the company level, studies have shown that corporate governance conditions, such as company size, company performance, company ownership structure, and control rights allocation, as well as corporate characteristics, such as share liquidity, have a significant impact on the ultimate controller’s expropriation. For example, the authors have found that the proportion of controlling shareholder ownership can affect expropriation, which in turn will affect block transactions [1].

To sum up, the above research on private benefits of control is very rich, but they all ignore the influence of the subsidiary, an important factor, on private benefits of control. Based on the above analysis, different from the existing literature on the theory that the ultimate controller’s expropriation is only determined by the ultimate controller itself, this paper believes that expropriation is the result of the joint action of the ultimate controller and the underlying subsidiary, rather than only by the ultimate controller. Although the subsidiary cannot directly compete with the ultimate controller, due to the subsidiary’s self-interest demand and autonomy, the subsidiary has bargaining power when the ultimate controller expropriates private benefits. Therefore, this paper conducts modeling research based on this hypothesis.

Therefore, this paper will use the model method to make a new definition of the background and scenario of the model discussion. Different from previous assumptions that believe that the ultimate controller’s expropriation is determined by the ultimate controller, the model of this paper intends to add the two important factors of the subsidiary’s self-interest demand and autonomy to study the influence mechanism of private benefits of control.

The innovation and contribution of this paper are embodied in the following four points. First, it reveals the inherent influence of the subsidiary’s self-interest demand and the autonomy on the ultimate controller’s expropriation. It breaks the hypothesis that the subsidiary can only passively accept private benefits of control, expands the research on the decision-making mechanism of pyramidal controlled groups’ internal private benefits of control, and enriches research on the economic consequences of the subsidiary’s autonomy and self-interest demand.

Second, it provides conditions for further in-depth exploration of the determinants of private benefits, at the same time, provides new ideas for the understanding of the ultimate controller’s expropriation.
Third, it makes up for the lack of qualitative conclusions about empirical research. This paper reveals the motives and internal mechanisms of the ultimate controller’s behavior, as well as provides a quantitative relationship between specific variables, which is something that empirical research cannot provide. It also provides theoretical support and research directions for subsequent relevant empirical studies, which have very important practical significance.

Fourth, It provides a new perspective on how to deal with risk management challenges of sustainability for enterprises.

2. The Design of The Ultimate Controller’s Private Benefits Model

2.1. Model Assumptions

This section will make a new model and intend to consider the subsidiary’s self-interest demand and autonomy. In the model construction, it is assumed that the ultimate controller controls the listed parent and subsidiary companies through an intermediate level, and simplifies the pyramid structure in the following way:

The shareholding ratio of the ultimate controller to the listed parent company is the product of the shareholding ratios of all levels, and the voting power of the ultimate controller to the listed parent company is equal to the minimum shareholding ratio of each level. The chain of “ultimate owner-parent-subsidiary” is shown in the following Figure 1a.

![Figure 1a. The chain of “ultimate owner-parent-subsidiary”](image)

Figure 1. (a). the chain of “ultimate owner-parent-subsidiary”, (b). Diagram of the two-stage model.

As shown in the diagram of the two-stage model in Figure 1b, this model considers the two-stage investment and private benefits model of listed parent and subsidiary companies. In period 0, the listed company uses all of its assets to invest and generates income in the first period, and the ultimate controller expropriates private benefits in the first period.

This paper believes that subsidiaries will have the need to arrange funds in advance, and can make arrangements within their autonomy to reduce the basis of private benefits of control. The ultimate controller’s private benefits are based on the residual income. On this basis minus what the subsidiary needs to retain, the ultimate controller makes a decision based on the maximization of its income. In the analysis of the model, we first consider the influence of the subsidiary’s self-interest demand on the expropriation of the ultimate controller, then consider the influence of the subsidiary’s autonomy on the expropriation of the ultimate controller, and finally consider the combined effect of the subsidiary’s self-interest demand and the autonomy of the subsidiary on expropriation. We describe this in detail next.

There are two prerequisites for the ultimate controller to expropriate private benefits from the member companies on the pyramid control chain. One is that there is a certain degree of separation of the two rights, which provides incentives for expropriation; the other is that the legal protection is weak and provides the possibility for expropriation. The essence of private benefits of control largely lies in public benefits.

From the perspective of the cost of expropriation, the cost of expropriation comes from: (1) the loss of public benefits; (2) related costs to evade legal supervision; (3) transaction costs paid for related exchanges for the implementation of the expropriation, such as organizational costs, information acquisition costs, rent-seeking costs, etc. The optimal
expropriation ratio makes the marginal revenue of expropriation equal to the marginal cost of expropriation.

More specifically, first, the loss of public benefits depends on the size of the final ownership. The greater the ownership, the greater the loss of public benefits; second, the cost of legal supervision increases with the improvement of the level of legal protection, law enforcement, and corporate governance. Third, the transaction cost in private benefits of control is determined by the complexity of expropriation behavior. For example, with the extension of the control chain between the ultimate controller and the subsidiary of a listed company, the greater the degree of information asymmetry, and the greater costs of related transactions may be.

Analyzed from the perspective of the benefits of expropriation, the more pyramid-type control levels, the greater the degree of separation of the two powers, the more wealth available for expropriation, and the more expropriation.

From the perspective of the subsidiary’s influence on the expropriation, the subsidiary’s self-interest demand and autonomy will affect the size of the private benefit of control. On the one hand, when the subsidiary’s self-interest demand and business scale increase, it needs to retain part of the funds, thereby reducing the basis for expropriation. On the other hand, the autonomy of the subsidiary allows it to allocate resources within the scope of its own reasonable demands, making the basis for the ultimate controller’s private benefits smaller, thereby inhibiting the expropriation.

2.2. Main Variables

According to the two-stage investment and private benefits of the control model of listed companies constructed, we select period 0 as the investment period and period 1 as the income and expropriation period.

The ultimate controller controls the listed parent company through the equity chain, holding its equity, and the listed parent company owns assets $A$ (without considering liabilities).

Assuming that the parent company’s business scale is certain and the equity investment ratio of the listed parent company to the subsidiary company is $m$ (i.e., the subsidiary business scale coefficient), that is, there is $m$ in the assets sinks to the subsidiary. Then the listed parent company holds the shares $\beta$ of the subsidiary. In this way, the parent company’s self-operated assets are $(1 - m)A$, and the subsidiary’s self-operated assets are $mA/\beta$.

The parent and subsidiary companies invest all their assets on date 0. To date 1, the parent company generates revenue $R(1 - m)A$, and the subsidiary company generates revenue $R(a)mA/\beta$. The details are as follows:

(i) $a$ represents the autonomy of the subsidiary, which is granted by the parent company; the existence of autonomy will change the profitability of the subsidiary $R(a)$.

(ii) For the subsidiary profit function $R(a)$, its relationship with subsidiary autonomy may have the following two situations. First, $R'(a) > 0$ means that the increase of autonomy will enhance the profitability of subsidiaries; second, $R'(a) < 0$, which means the increase of autonomy will weaken the profitability of subsidiaries.

(iii) Under the condition that other factors, such as the scale of business and the main business remain unchanged: assuming that the autonomy of the subsidiary is 0, that is, $R(0) = R$, it indicates that when the subsidiary does not have the autonomy, the parent and subsidiary have the same profitability.

In the period of date 1, the ultimate controller will expropriate private benefits. Regarding the expropriation, we describe them as follows.

The ultimate controller obtains private benefits from the parent and subsidiary company. Among them, the private benefit from the parent company is $R(1 - m)As_1$, and the private benefit from the subsidiary is $f(a, n)R(a)mAs_2$. In this function, $a$ represents the autonomy of the subsidiary, $n$ represents the self-interest demand of the subsidiary, and $s_1$ and $s_2$ represent the proportion of private benefits that the ultimate controller derives from the parent and subsidiary company, respectively. Moreover, $f(a, n)$ represents the
basic coefficient of expropriation (the percentage of total income minus the funds that the subsidiary needs to retain).

Regarding \( f(a,n) \), we define it like this:

(i) The value of the function \( f(a,n) \) is between \((0, 1)\), which means that the basic coefficient of private benefits expropriation affects the expropriation.

(ii) The independent variables \( a \) and \( n \), respectively, represent the autonomy and self-interest demand of the subsidiary, which has an impact on the coefficient \( f(a,n) \) for the expropriation.

In this model, subsidiaries’ self-interest demand and subsidiaries’ autonomy change the probability distribution of funds arranged in advance. The advance arrangement of funds by the subsidiary is in the range of 0 to \( R \). The specific amount depends on the result of the bargaining power of the subsidiary and the ultimate controller.

Regardless of the degree of autonomy of the subsidiary, the probability of arranging small funds in advance is relatively high. In contrast, the probability of arranging large funds in advance is relatively low.

Specifically, for the independent variable \( a \), the subsidiary with low autonomy has a higher probability to arrange small funds in advance. For the subsidiary with high autonomy, the probability of arranging large amounts of funds is higher.

In terms of the expected value of arranging funds in advance: when the subsidiary is at low autonomy, the expected value of arranging funds in advance is small; when the subsidiary is at high autonomy, the expected value of arranging funds in advance is higher. Therefore, the derivative function is \( f_a < 0 \), that is, the higher the autonomy, the larger the funds arranged in advance, and the smaller the basis of the ultimate controller’s expropriation.

In the same way, the independent variable \( n \) is the self-interest demand of the subsidiary. When the self-interest demand of the subsidiary is small, the expected value of arranging funds in advance is small; when the self-interest demand of the subsidiary is large, the expected value of arranging funds in advance is large. There is derivative function \( f_n < 0 \), that is, the larger the demand of the subsidiary’s self-interest, the smaller the expropriation basis of the ultimate controller. When ignoring the self-interest, demand and autonomy of the subsidiary, there is \( f(0,0) = 1 \), at this time the expropriation’s basis is the entire income. As shown in Figure 2:

![Figure 2](image-url)

**Figure 2.** Probability Distribution of Autonomy Difference Funds. The solid curve represents high autonomy. The dotted curve represents low autonomy.

Furthermore, when the ultimate controller expropriates private benefits ratios \( s_1 \) and \( s_2 \) from the parent and subsidiary companies, it has to pay the cost of \( c \). \( c \) is a function of the private benefit ratio, that is, \( c(s) \). Here, \( c_s > 0 \), that is, the greater the proportion of private benefits, the greater the cost; \( c_{ss} > 0 \), that is, the greater the proportion of private benefits, the greater the marginal cost. Among them, referring to the related research of La Porta et al (2002), the cost function \( c(s) \) is specified as the function form \( c(s) = \frac{1}{2}ks^2 \), where \( k \) is the
degree of the legal protection of shareholders, and \( s \) is the proportion of private benefits expropriation \([11]\).

Key notations in this paper are presented in Table 1.

**Table 1. Key notations.**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>( n )</td>
<td>The self-interest demand of the subsidiary</td>
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<tr>
<td>( a )</td>
<td>The autonomy of the subsidiary</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>The shareholding ratio that the ultimate controller holds</td>
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<td>( A )</td>
<td>Parent company’s assets</td>
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<tr>
<td>( m )</td>
<td>The subsidiary business scale coefficient</td>
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<tr>
<td>( \beta )</td>
<td>The proportion of the parent company’s equity investment in subsidiaries</td>
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<tr>
<td>( R(a) )</td>
<td>The profitability of subsidiaries</td>
</tr>
<tr>
<td>( R'(a) )</td>
<td>The derivative function of profitability</td>
</tr>
<tr>
<td>( f(a,n) )</td>
<td>the basic coefficient of the expropriation</td>
</tr>
<tr>
<td>( s )</td>
<td>the proportion of the expropriation</td>
</tr>
<tr>
<td>( s_1 )</td>
<td>the proportion of private benefits that the ultimate controller derives from the parent</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>the proportion of private benefits that the ultimate controller derives from the subsidiary</td>
</tr>
<tr>
<td>( c(s) )</td>
<td>the function of the private benefit ratio</td>
</tr>
<tr>
<td>( c_s )</td>
<td>the first derivative of ( s )</td>
</tr>
<tr>
<td>( c_{ss} )</td>
<td>the second derivative of ( s )</td>
</tr>
<tr>
<td>( k )</td>
<td>the degree of the legal protection of shareholders</td>
</tr>
<tr>
<td>( P )</td>
<td>private benefits</td>
</tr>
<tr>
<td>( U )</td>
<td>The ultimate controller’s total revenue</td>
</tr>
<tr>
<td>( M )</td>
<td>Abbreviated function of ( R(1 - a)\beta/f(a) - f(a)a\beta )</td>
</tr>
<tr>
<td>( \phi(a) )</td>
<td>Subsidiary’s function of arranging funds in advance</td>
</tr>
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3. The Model of Private Benefits under the Influence of Subsidiaries

According to the basic assumptions of the model, private benefits \( P \) can be expressed as:

\[
P = R(1 - m)A_\text{S1} + f(a,n)R(a)\frac{mA}{\beta}s_2
\]  

(1)

Among them, \( R(1 - m)A_\text{S1} \) represents the private benefits obtained from the parent company level, and \( f(a,n)R(a)\frac{mA}{\beta}s_2 \) represents the private benefits obtained from the subsidiary level.

From the expression of private benefits of control, it can be seen that private benefits of control are related to the level of parent-subsidiary companies and is also dynamic. The factors that affect private benefits of control are: Subsidiary’s self-interest demand; Subsidiary’s autonomy.

Among them the subsidiary’s self-interest demand affects private benefits of control by affecting the basic coefficient of expropriation \( f(a,n) \); the autonomy of the subsidiary affects private benefits of control by affecting the basic coefficient of expropriation \( f(a,n) \) and the profitability of the subsidiary \( R(a) \).

In the expression of the private benefits, the ratios of \( s_1 \) and \( s_2 \) for expropriation are determined by the ultimate controller. The basis for making the decision is to maximize its total revenue. The ultimate controller’s total revenue \( U \) is expressed as:

\[
U = aR(1 - m)A(1 - s_1) + R(1 - m)A[s_1 - c(s_1)] + a\beta R(a)\frac{mA}{\beta} [1 - f(a,n)s_2] + f(a,n)R(a)\frac{mA}{\beta} [s_2 - c(s_2)]
\]  

(2)

In the income expression \( U \) of the ultimate controller, \( aR(1 - m)A(1 - s_1) \) represents the public benefits obtained from the parent company’s self-operated business, \( R(1 - m)A[s_1 - c(s_1)] \) represents the private benefits obtained from the parent company, \( a\beta R(a)\frac{mA}{\beta} [1 - f(a,n)s_2] \) represents the public benefits obtained from the subsidiary’s business, and \( f(a,n)R(a)\frac{mA}{\beta} [s_2 - c(s_2)] \) represents the private benefits obtained from the subsidiary.
Next, about this function \( c(s) = \frac{1}{2}ks^2 \), the independent variables are brought into \( s_1 \) and \( s_2 \), respectively, and the private benefits ratios \( s_1 \) and \( s_2 \) are determined by the ultimate controller according to the maximization of its income.

Here we take the first-order derivative of \( s_1 \) and \( s_2 \), respectively, and calculate the maximum value in function \( U \) and the proportion of the private benefits can be obtained:

\[
\frac{dc(s_1)}{ds_1} = 1 - a, \quad \frac{dc(s_2)}{ds_2} = 1 - a\beta
\]

Solve the Equation (3), we can know:

\[
s_1 = c_s^{-1}(1 - a), \quad s_2 = c_s^{-1}(1 - a\beta)
\]

Substitute the function \( c(s) = \frac{1}{2}ks^2 \) into the Equation (4):

\[
s_1 = k^{-1}(1 - a), \quad s_2 = k^{-1}(1 - a\beta)
\]

The above formula indicates that when the ultimate controller’s total income \( U \) is maximum, the following relationship exists:

(i) The higher the ultimate controller’s equity ratio \( \alpha \) in the parent company, the lower the proportion of expropriation from the parent company;

(ii) The higher the product \( a\beta \) of the ultimate controller’s equity ratio to the parent company and the parent company’s equity ratio to the subsidiary, the lower the proportion of expropriation from the subsidiary.

4. Model Analysis

The subsidiary’s self-interest demand and the autonomy affect the ability of the subsidiary’s funds to be arranged in advance, and the autonomy of the subsidiary can affect the profitability of the subsidiary. The change in profitability and the advanced arrangement of profit together affect the ultimate controller’s private benefits. The model first analyzes the influence of the subsidiary’s self-interest demand and autonomy on private benefits separately and further analyzes the joint influence of the two of them on private benefits.

4.1. The Influence of the Subsidiary’s Self-Interest Demand on Private Benefits

First of all, the model only considers the influence of the subsidiary’s self-interest demand on private benefits \( P \), which can be expressed as:

\[
P = R(1 - m)As + f(n)R(a)\frac{mA}{\beta}s
\]

\[s.t. \quad s_1 = c_s^{-1}(1 - a), \quad s_2 = c_s^{-1}(1 - a\beta)\]

In order to research the influence of the self-interest demand of subsidiaries \( n \) on private benefits of control, it is necessary to seek guidance on the self-interest demand of subsidiaries, and we can obtain:

\[
\frac{dP}{dn} = f'(n)R(a)\frac{mA}{\beta}c_s^{-1}(1 - a\beta) < 0
\]

It can be inferred from Inequality (7) as the following Result 1.

**Result 1.** The available derivatives above are all less than 0, indicating that if other conditions remain unchanged: when the subsidiary’s self-interest demand increases, the ultimate controller’s private benefits become less.
4.2. The Influence of the Subsidiary’s Autonomy on Private Benefits

In the previous section, we considered the influence of subsidiaries’ self-interest demand on private benefits of control. In this section, we considered the influence of the subsidiary’s autonomy on private benefits. There are two possibilities for the influence of a subsidiary’s autonomy on private benefits: restraining or promoting. Therefore, there will be many situations.

First, the autonomy of subsidiaries can influence the profitability, then the profitability influences the basis of expropriation, thereby changing the amount of expropriation; second, the autonomy improves the ability to arrange funds in advance, which can reduce expropriation. When the increase in the basis for private benefits caused by autonomy is greater than the decrease in the basis for private benefits caused by the advance arrangement of funds, private benefits of control increase. In other cases, private benefits of control decrease. Private benefits of control can be expressed as:

\[ P = R(1 - m)A\xi_1 + f(a)R(a)\frac{mA}{\beta}s_2 \]  \hspace{1cm} (8)

s.t. \( s_1 = c_s^{-1}(1 - a) \), \( s_2 = c_s^{-1}(1 - a\beta) \)

In order to research the influence of subsidiary autonomy on private benefits of control, we seek guidance on subsidiary autonomy and obtain:

\[ \frac{dP}{da} = \left[ f'(a)R(a) + f(a)R'(a)\right]\frac{mA}{\beta}s_1 + (1 - a\beta) \]  \hspace{1cm} (9)

It can be inferred from function (9) as the following Result 2.

Result 2.
(i) When \( R(a) \) is a monotonously decreasing function, there is \( \frac{R'(a)}{R(a)} < |\frac{f'(a)}{f(a)}| \), this indicates that the greater the subsidiary’s autonomy, the smaller the ultimate controller’s private benefits.

(ii) When \( R(a) \) is a monotonic increasing function, there are two situations. When \( \frac{R'(a)}{R(a)} > |\frac{f'(a)}{f(a)}| \), the value of the function (9) is greater than zero. This indicates that the bigger autonomy of the subsidiary, the larger private benefits of control expropriated. When \( \frac{R'(a)}{R(a)} < |\frac{f'(a)}{f(a)}| \), the value of the function (9) is less than zero. This indicates that the bigger autonomy of the subsidiary, the smaller the private benefits of control expropriated.

In addition, in order to research the influence of a subsidiary’s business scale on private benefits, it is necessary to the derivative of the subsidiary’s business scale. The following equation can be drawn:

\[ \frac{dP}{dm} = \left[ -Rs_1 + f(a)R(a)\frac{mA}{\beta} \right] A \]  \hspace{1cm} (10)

s.t. \( s_1 = c_s^{-1}(1 - a) \), \( s_2 = c_s^{-1}(1 - a\beta) \)

It can be inferred from Equation (10) as the following Result 3.

Result 3.
(i) When \( f(a)R(a)\frac{mA}{\beta} < Rs_1 \), that is to say, \( R(a) < R(1 - a)\beta / |f(a) - f(a)\alpha\beta| \), let \( R(1 - a)\beta / |f(a) - f(a)\alpha\beta| = M \). It shows that when the profitability of a subsidiary is \( R(a) < M \), the greater the proportion of the subsidiary’s business scale, the smaller the private benefits expropriated by the ultimate controller.

(ii) when \( f(a)R(a)\frac{mA}{\beta} > Rs_1 \), that is to say, \( R(a) > R(1 - a)\beta / |f(a) - f(a)\alpha\beta| \), let \( R(1 - a)\beta / |f(a) - f(a)\alpha\beta| = M \). It shows that when the profitability of the subsidiary is \( R(a) > M \), the greater the proportion of the subsidiary’s business scale, the greater the private benefits expropriated by the ultimate controller.
4.3. The Influence of Subsidiary’s Self-interest Demand and Subsidiary’s Autonomy on Private Benefits

The previous sections separately discussed the influence of the subsidiary’s self-interest demand and autonomy on private benefits. They can influence private benefits together. In this section, we discuss the influencing factors of private benefits from the perspective of the two-factor combination.

The amount of funds that the subsidiary can allocate in advance for future investment and operation depends on the ability brought by autonomy and the demand for funds for subsequent investments. Assuming that the subsidiary’s future investment needs to arrange the profit of proportion \( n \) in advance, and autonomy \( a \) enables the subsidiary to give the subsidiary profit of proportion \( \varphi(a) \) (\( \varphi(a) > 0 \)), then it can be known that the subsidiary can arrange funds of \( \min\{\varphi(a) R(a) \frac{m_A}{P}, nR\frac{m_A}{P}\} \), and the basis of the ultimate controller’s private benefits is \( R - \min\{\varphi(a) R, nR\} \). In this way, the basic coefficient of expropriation caused by the autonomy and the self-interest demand of the subsidiary can be combined into:

\[
f(a, n) = 1 - \min[\varphi(a), n]
\]  

(11)

Derivative to \( a \) and \( n \),

\[
f_a(a, n) = \begin{cases} -\varphi'(a) & a \leq \varphi^{-1}(n) \\ 0 & a > \varphi^{-1}(n) \end{cases}
\]  

(12)

\[
f_n(a, n) = \begin{cases} -1 & n \leq \varphi(a) \\ 0 & n > \varphi(a) \end{cases}
\]  

(13)

Private benefits of control: \( P \) can be expressed as:

\[
P = R(1 - m)As_1 + f(a, n)R(a)\frac{mA}{\beta} s_2
\]  

(14)

s.t. \( s_1 = c_s^{-1}(1 - a) \), \( s_2 = c_s^{-1}(1 - a\beta) \)

Derivative to \( a \),

\[
\frac{dP}{da} = \left[f_a(a, n)R(a) + f(a, n)R'(a)\right] \frac{mA}{\beta} s_2 = \begin{cases} [-\varphi'(a) R(a) + f(a, n) R'(a)] \frac{mA}{\beta} s_2 & a \leq \varphi^{-1}(n) \\ [f(a, n) R'(a)] \frac{mA}{\beta} s_2 & a > \varphi^{-1}(n) \end{cases}
\]  

(15)

From the analysis of the above equation, we can know the relationship between the ultimate controller’s private benefits \( P \) and the subsidiary autonomy \( a \) and combined with the influence of the subsidiary’s autonomy on private benefits, the following Figure 3 can be obtained.

Among them, the influence of the subsidiary’s autonomy on private benefits is mainly divided into four situations:

(i) When the disposable funds owned by the subsidiary’s autonomy do not meet the funds that the subsidiary needs to retain for its own self-interest demand, two situations arise: Scenario 1: If the autonomy of the subsidiary is positively correlated with profitability, and \( \frac{R'(a)}{R(a)} > \frac{f'(a)}{f(a)} \) is established: private benefits of control increase with the increase in autonomy; Scenario 2: If the autonomy of the subsidiary is negatively related to profitability, that is, \( \frac{R'(a)}{R(a)} < \frac{f'(a)}{f(a)} \) is established: private benefits of control decrease with the increase of the autonomy.
(ii) When the disposable funds of the subsidiary’s autonomy meet or even exceed the funds that the subsidiary needs to retain for its own interests, that is, the autonomy \(a>\phi^{-1}(n)\), the increase in autonomy no longer affects the number of funds in advance, so that the retained earnings remain unchanged. However, the autonomy of the subsidiary still has an impact on the profitability of the subsidiary, resulting in another two situations: Scenario 3: When \(R'(a) > 0\): as the autonomy increases, the private benefits increase; Scenario 4: When \(R'(a) < 0\): as the autonomy increases, the private benefits decrease. Result 4 can be inferred as the following.

Result 4. When the disposable funds of the subsidiary from autonomy are greater than the funds that the subsidiary needs to retain, the influence of autonomy on private benefits of control is uncertain. It depends on the effectiveness of the autonomy’s impact on profitability. When the disposable funds of the subsidiary from autonomy are less than the funds that the subsidiary needs to retain, the result is the same as Result 2.

In order to research the influence of the self-interest demand of subsidiaries on private benefits of control, this paper seeks the guidance of the private benefits on the self-interest demand of subsidiaries:

\[
\frac{dP}{dn} = f_n(a,n)R(a)\frac{mA}{\beta}s_2 = \begin{cases} 
-R(a)\frac{mA}{\beta}s_2 & n \leq \phi(a) \\
0 & n > \phi(a)
\end{cases}
\tag{16}
\]

Based on the analysis of the above results, the relationship between the ultimate controller’s private benefits \(P\) and the subsidiary’s self-interest demand \(n\) is shown in Figure 4.

When the funds that the subsidiary needs to retain are less than the subsidiary’s disposable funds within its autonomy, that is, \(n<\phi(a)\), the increase in demand will increase the number of funds arranged in advance. In this way, the basis for expropriation decreases, thereby resulting in a decrease in private benefits. When the funds that the subsidiary needs to retain are greater than the disposable funds from autonomy, that is, \(n>\phi(a)\), the increase in demand no longer affects the funds arranged in advance. Therefore, the basis of expropriation remains the same, and the private benefits remain unchanged.
n, the increase in demand will increase

Figure 4. The relationship between the ultimate controller’s private benefits and the subsidiary’s self-interest demand.

It can be inferred from Equation (16) as the following Result 5.

Result 5.
(i) When the funds that the subsidiary needs to retain are less than the subsidiary’s disposable funds within its autonomy: as the self-interest demand increases, the private benefits decrease;
(ii) When the funds that the subsidiary needs to retain are greater than the subsidiary’s disposable funds within its autonomy: as the self-interest demand increases, the private benefits remain unchanged.

5. The Private Benefits Expropriation Proportion to the Public Parent—Subsidiary Company

The previous sections have analyzed the influence of the self-interest demand of the subsidiary and the autonomy of the subsidiary on the private benefits, considering the absolute number of private benefits of control. Then, what influence do the subsidiary’s self-interest demand and autonomy have on the proportion of private benefits of control expropriated by the public parent company and subsidiary? The public is also very interested in this issue.

First of all, this section considers the proportion of private benefits of control from the public parent company. The private benefits expropriated by the public parent company during date 1 is \( R(1 - m)A_s \), when the income generated by the listed parent company during date 1 is \( R(1 - m)A \). In this way, the proportion of the private benefits expropriated by the public parent company is \( s_1 \) that is, \( s_1 = c_s^{-1}(1 - \alpha) \). It can be seen that the proportion of the private benefits expropriated by the public parent company has nothing to do with the autonomy of the subsidiary \( a \) and the self-interest demand of the subsidiary \( n \).

Secondly, we consider the proportion of the private benefits expropriated by subsidiaries. The private benefits from the subsidiary during date 1 is \( f(a, n)R(a)\frac{mA}{P}s_2 \), when the subsidiary’s income generated during date1 is \( R(a)\frac{mA}{P} \). It can be expressed as \( S = f(a, n)s_2 \), that is, \( S = f(a, n)c_s^{-1}(1 - \alpha\beta) \).

Takes the derivative of \( a \) and \( n \):

\[
\frac{dS}{da} = \begin{cases} -\varphi'(a)c_s^{-1}(1 - \alpha\beta) < 0 & a \leq \varphi^{-1}(n) \\ 0 & a > \varphi^{-1}(n) \end{cases}
\]

(17)

\[
\frac{dS}{dn} = \begin{cases} -c_s^{-1}(1 - \alpha\beta) < 0 & n \leq \varphi(a) \\ 0 & n > \varphi(a) \end{cases}
\]

(18)

It can be inferred from Equations (17) and (18) as the following Result 6. We conclude as follows.
Result 6. When the proportion of funds that the subsidiary can arrange in advance is less than the proportion of funds that the subsidiary needs, the bigger the autonomy is, the lower the proportion of the private benefits from the subsidiary. On the contrary, the increase in autonomy does not affect the proportion of the private benefits from the subsidiary.

When the proportion of funds that the subsidiary needs is less than the proportion of funds that the subsidiary can arrange in advance, as the subsidiary’s self-interest demand increases, the proportion of the private benefits obtained by the subsidiary is reduced. On the contrary, as the subsidiary’s self-interest demand increases, the proportion of private benefits of control expropriated by subsidiaries remains unchanged.

6. Discussions

Based on the above model analysis, we can find the results that the subsidiary autonomy and self-interest demands can influence private benefits of control. However, the previous studies related to expropriation did not consider these two important factors, only the effect of the ultimate controller. This paper is innovative compared to previous studies and enriches the research on the influencing factors of expropriation.

At the same time, it plays an important role for companies to meet the risk management challenges for the sustainability of the corporate. First, we can reduce the expropriation of the ultimate controller by granting autonomy to the subsidiary and considering the demands of the subsidiary’s self-interest. Second, as the expropriation decreases, the company will have more cash flow and funds retained to ensure its financial and economic sustainability. In this way, economic sustainability is strengthened, so enterprises can cope with risk management challenges for sustainability better, which is vital to the current and future environment. Additionally, the ability of the corporate to resist economic crises and risks is enhanced.

In addition, many companies have branches all over the world nowadays. On the one hand, the influence of the control function of the main headquarter of the company for their subsidiaries headquartered in other countries is an important issue. On the other hand, subsidiaries that play main roles in local economies (such as in CE Europe), have to perform decisions made by the main headquarters located elsewhere [32]. How does the headquarters grant autonomy to subsidiaries, and how can subsidiaries develop the economy in combination with relevant local policies, so that they can better cope with economic crises and challenges? This is a very meaningful topic for further research in the future.

Additionally, there are some difficulties with the empirical work due to the lack of data. In the future, we will obtain relevant data through different means, such as field visits, and questionnaires for the empirical work to improve our research.

7. Conclusions

According to the research on the ultimate controller’s expropriation, this paper believes that the self-interest demand and the autonomy of the subsidiary can bring funds and resources retention for the subsidiary, and change the basis of expropriation and profitability, affecting private benefits of control. Through the model analysis and demonstration, this paper has the following findings:

First, the subsidiary’s self-interest demand needs to retain profits, which reduces the basis for the ultimate controller’s expropriation, thereby inhibiting private benefits of control.

Second, the autonomy of the subsidiary affects the basis of expropriation through two paths: the autonomy of the subsidiary affects the ability of the subsidiary to arrange funds in advance; additionally, the subsidiary’s autonomy can affect the profitability of subsidiaries. The basis of private benefits of control increases with the subsidiary’s profitability. Therefore, the influence of autonomy on private benefits is uncertain. It depends on the combined interaction of factors, such as profitability and the ability to arrange funds in advance.
Third, when the proportion of funds that the subsidiary needs is less than the proportion of funds that the subsidiary can arrange in advance, as the subsidiary’s self-interest demand increases, the proportion of the expropriation obtained by the subsidiary is reduced. When the proportion of funds that the subsidiary can arrange in advance is less than the proportion of funds that the subsidiary needs, the autonomy is negatively related to the proportion of the expropriation from the subsidiary.

The findings of this paper are very meaningful and inspire how companies can develop sustainably.

First of all, this research reveals the influence of subsidiary self-interest demand and subsidiary autonomy on the ultimate controller’s private benefits. It shows that the subsidiary does not completely passively accept expropriation from the ultimate controller but can influence the ultimate controller’s behavior. Secondly, this paper proves that the ultimate controller’s expropriation depends on the autonomy of the subsidiary, the subsidiary’s self-interest demand, profitability, and the basis of the private benefits, which expands the research paradigm of private benefits. The impact of the autonomy and self-interest demand of the subsidiary on private benefits of control is not just a single inhibition or promotion. The final impact depends on the game relationship between the four of them. Even to a certain extent, the increase in autonomy can promote the ultimate controller’s private benefits.

Therefore, it provides very interesting and instructive directions for future research: according to this paper as the theoretical basis, future research can carry out further and in-depth research. On one hand, how to grant the autonomy and self-interest demand of a subsidiary, and how much autonomy is granted to reduce expropriation of the ultimate controller is a topic and direction that worth research. On the other hand, in this way, as the private benefits of the ultimate controller are reduced, the corporate governance effect and financial sustainability will be improved. Moreover, the ultimate controller can shift their focus from private benefits to sustainable development. The ability to meet risk management challenges is enhanced accordingly.

Author Contributions: Conceptualization, M.C. and Z.M.; methodology, M.C.; validation, M.C.; formal analysis, M.C.; investigation, M.C., Z.M. and Y.L.; resources, M.C. and Z.M.; writing—original draft preparation, M.C. and Z.M.; writing—review and editing, M.C., Z.M. and Y.L.; supervision, Z.M.; funding acquisition Z.M. and Y.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Social Science Foundation of China, grant number 14BGL039 and the Beijing Municipal Office of Philosophy and Social Science Planning, grant number 19GLB021.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References
3. La Porta, R.; Lopez-De-Silanes, F.; Shleifer, A. Corporate ownership around the world. J. Financ. 1999, 54, 471–517. [CrossRef]


