Decision Making in Green Supply Chain with Manufacturers’ Misreporting Behavior

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Abstract: To reduce the cost of energy conservation and emission reduction, some manufacturers will choose to misreport the green degree of their products, which may have some serious negative effects on the profit of the supply chain. To investigate the effects of the manufacturers’ misreporting strategy on the decisions and associated profit of the supply chain, we establish a two-period Stackelberg game model for a dual-channel manufacturer–retailer setting. Our results show that the manufacturers’ misreporting strategies lead to higher profits for them, which means that manufacturers tend to misreport the green degree of the products. However, the manufacturer’s misreporting strategy will put the retailer and the whole supply chain in a disadvantageous position and, hence, reduce the profit of the retailer and the whole supply chain. In addition, we also show that the manufacturer’s misreporting strategy only affects the greenness of the product and does not affect the pricing decisions of the manufacturer, the retailer and the whole supply chain. Finally, we design a revenue-sharing contract to achieve the coordination of the supply chain, which provides managerial insights for the decision makers of the supply chain.

Keywords: green supply chain; dual channel; misreporting; coordinate

1. Introduction

In recent decades, environmental pollution and its impact on social sustainable development has emerged as an issue of public concern [1–3], and the governments of various countries, such as France, Norway and China, have passed a series of strict environmental protection policies. These government policies result in both pressure and motivation for enterprises in the supply chain to improve their environmental performance. Moreover, due to the significant improvement of consumers’ environmental awareness in recent years, that is, consumers are willing to pay more for green products [4], green supply chain management (GSCM) has increasingly become one of the important ways to improve the performance of enterprises.

To build a good social image, many enterprises adopt the GSCM, which improves the economic performance of enterprises, and the improved economic performance in turn encourages enterprises to continue to take measures of energy conservation and emission reduction [5–7]. The adoption of green supply chain management has brought positive effects to enterprises in the supply chain, consumers and environmental protection [8–11]. However, a new problem has arisen in the GSCM. Specifically, to reduce the cost of energy conservation and emission reduction, some enterprises in the supply chain, as reported by Southern Weekend, misreported their efforts to reduce energy conservation and emission reduction [12]. Such behavior has a serious negative impact on the overall image of the supply chain and the economic interests of other enterprises in the supply chain [13]. However, in reality, it is difficult to define the misreporting behavior of enterprises. Moreover, in China, there is no law specifically for greenwashing marketing of enterprises, which

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brings great difficulties to the management of misreporting behavior. Therefore, in the following parts of this paper, we will establish a game model to study the misreporting behavior in the supply chain to provide effective guidance for policy making and enterprise decision making.

Furthermore, with e-commerce’s continuous development, several companies have promoted a multi-channel distribution system [14], which means that the manufacturer in the supply chain can sell products through both the retail channel and the direct channel. However, although the opening of online direct marketing channels will bring positive effects to enterprises [15–18], the differences in the attributes and prices of the two channels will affect consumers’ choice and cause changes in channel demand. Therefore, in the case of the coexistence of direct and retail channels, an effective pricing strategy of each channel can benefit the whole supply chain. Driven by this trend, consumers’ preferences also affect the pricing decisions of products to a certain degree. In such a dual channel supply chain environment, how to make pricing decisions for green products is an important issue.

There are many studies on the decision making and coordination of dual-channel green supply chain [19–23]. However, most of the studies consider the settings under the information symmetry, and few literature works have studied the misreporting of enterprises. Motivated by the significant impact of misreporting on the profits of enterprises and the research gap in the literature, we model a green supply chain consisting of a manufacturer and a retailer, where the manufacturer, as the leader, can decide whether to implement the strategy of misreporting.

- What pricing and production strategies should the manufacturer implement in the cases with and without misreporting?
- How does the manufacturer’s misreporting decision affect the decisions and profits of the retailer and the whole supply chain?
- Can a revenue-sharing contract effectively coordinate the supply chain and prevent the manufacturer from misreporting the green degree of the product?

To answer the questions above, we consider a supply chain where the manufacturer sells green products through a traditional channel retailer and its own online channel. In this supply chain, the manufacturer is the leader, and the retailer is the follower. A two-period Stackelberg game model is developed to determine the retailer’s optimal pricing strategy and the manufacturer’s optimal pricing and environmental protection strategy.

The main findings of this paper are the following. First, the manufacturer’s misreporting will reduce the green degree of the product, but it will not affect the price decisions of the manufacturer and the retailer. Second, the manufacturer’s misreporting can make the manufacturer gain more profits, but it will damage the profits of the retailer and the whole supply chain. Third, a revenue-sharing contract can effectively coordinate the decision making of the manufacturer and the retailer, and it can encourage the manufacturer to give up the strategy of misreporting.

The contribution of this study to the literature is three-fold. Firstly, the optimal pricing and environmental protection strategies are explored for a SC in which green products are sold in a dual channel by considering not only the influence of decision-making mode, but also the influence of channel structure on the decision making of each decision maker in the supply chain. Secondly, we construct a two-period Stackelberg game model of manufacturer and retailer, which is dominated by the manufacturer, to explore the influence of misreporting strategy on the optimal decision making and profit of the manufacturer, the retailer and the whole supply chain. Thirdly, we propose a revenue-sharing contract that coordinates supply chain decisions within a certain range and prevents the manufacturer from misreporting the green degree of the product.

The remainder of this paper is organized as follows. Sections 2 and 3 present the literature review and the model of this paper. In Section 4, we solve the optimal decision and profit of each model. Additionally, in Section 5, a comparative analysis of the optimal solution and optimal profit of different models is discussed. Section 6 illustrates the model
with a numerical example. The conclusions are presented in Section 7. All evidence is outlined in Appendix A.

2. Literature Review

This study is closely related to three research streams in the existing literature: dual-channel supply chain, green supply chain management and misreporting strategies in supply chains.

The first stream relates to dual-channel supply chain being the problem of optimal pricing decision. Dan [14], Yang [24,25], Xie [26] and Li [27] studied the optimal price decision in a centralized and decentralized supply chain and analyzed the influence of supply chain structure on price decision. However, the conclusions they came to were not entirely consistent. For example, the research conclusion of Dan [14] indicated that the optimal price of a centralized decision was higher than that of a decentralized decision, while Xie [26] indicated that the online and offline price under a decentralized decision was higher than that under a centralized decision. To determine the impact of channel structure on the optimal pricing decision, we solve the optimal pricing of a centralized and a decentralized decision, respectively, against the background of dual-channel green supply chain and make a comparative analysis.

In addition, the impact of dual channels on the profit of enterprises in the supply chain is also closely related to this study. Although the dual-channel strategy expands the market for the enterprise [28], it also leads to the competition between the direct channel and the retail channel [29]. Therefore, the dual-channel strategy will have a very complex effect on the profit of the enterprise. The research of Chiang [15] and Cai [30] showed that the manufacturer’s dual-channel strategy was profitable to the manufacturer, which meant that the opening of new channels can bring more profits to the manufacturer. Furthermore, Yue [31] and Chen [28] pointed out that the manufacturer’s dual channel strategy can not only improve the manufacturer’s own profits, but also bring positive effects to the profits of the whole supply chain. However, the dual-channel strategy of the manufacturer may have a negative effect on the retailer’s profit [15,32,33]. Different from the above literature, the channel strategy of enterprises in the supply chain is not the focus of our research but is one of the backgrounds of our research.

As for the literature of green supply chain management, the most relevant to this study are the decision making on product’s green degree and the contract coordination of the green supply chain.

The first stream relates to green supply chain management being the decision on the product’s green degree. Ghosh [34] analyzed the impact of decision-making models and power structures in green supply chains on player’s green strategies, and their research concluded that cooperation among players does indeed lead to higher greening levels. Zhu [35] also studied the factors that influence the “greenness” of the product, and they analyzed the impact of supply chain structure (concentrated and decentralized product), green product type (development-intensive or marginal cost-intensive) and the types of competition (price competition and greenness competition) on product greenness. Similarly, some other scholars have also conducted detailed research on emission reduction decision making [36–38] and reached similar conclusions. The difference between this study and the literature mentioned above is that we consider the influence of manufacturer’s misreporting behavior on product greenness, which has not been studied.

Contract coordination is also one of the focuses of green supply chain management. Many scholars have studied the coordination of dual-channel supply chain, which is the second stream related to dual-channel supply. Different types of contracts have been adopted by scholars, such as two-part tariff contract [22,34,38–40], revenue-sharing contract [41–44], cost-sharing contract [45–48] and buyback contracts [49]. All these studies are based on symmetrical information in the supply chain. However, in our study, the manufacturer may misreport the green degree of products, which falls into the category of information asymmetry.
Our work also relates to the literature on misreporting strategies in supply chains. The exciting literature on supply chain information sharing assumes that information is shared truthfully; in practice, however, unless each party can verify the authenticity of the other party’s information, manufacturers and retailers may misreport their own information for their own benefit [12,14]. As pointed by Lei [50], inefficiency occurs when each party of a supply chain makes decisions on the basis of inaccurate information. Yan’s [13] research on manufacturers’ misreporting of cost information indicated that manufacturers would overstate costs for their own benefit, which could have negative effects on the retailer and the entire supply chain, and this effect was related to consumers’ cross-price sensitivity. Qin [44] studied the misreporting of investment cost information of battery suppliers in a two-stage supply chain consisting of a battery supplier and an electric vehicle manufacturer, and their research results showed that the misreporting of cost information had adverse effects on participants and reduced the driving mileage level of electric vehicles. In addition to the loss of benefits in the supply chain, the behavior of misreporting will also aggravate the unfair distribution of profit in the supply chain [51]. Similarly, Lau [52] also studied cost misreporting by a manufacturer. However, they obtained a different result, which indicated that, in the case of linear demand, the manufacturer would overstate their own production costs, whereas in the case of iso-elastic demand, the manufacturer will underestimate their production costs to benefit themselves and the whole supply chain, which was counter-intuitive.

In addition, some scholars use empirical research methods to study misreporting behavior in the supply chain. Yin [53] explored the impact of consumer misreporting on suppliers’ investment decisions. Their research results showed that the over-investment of suppliers caused by customers’ misreporting behavior would also lead to poor performance of suppliers in the future, thus leading to negative market reaction. Spiliotopoulou [54] analyzed the influence of regional managers’ misreporting of forecast information on the inventory centralization decision, and their research concluded that if the regional managers have the motivation to misreport, the centralization decision makers will not trust the regional managers’ forecast information, which will lead to the absence of truth-telling equilibrium unless inventory competition or demand uncertainty is eliminated. However, to our knowledge, to date, no research has been conducted on the problem of misreporting the greenness of products in the supply chain, which is the focus of our study and one of the most important differences between our research and the above studies.

### 3. Model Construction and Basic Assumptions

In this section, we describe a game model in which the manufacturer sells green products with environmental protection features over a two-period horizon, denoted by $t \in \{f, s\}$. In the first period, the manufacturer announces the product information to the market; in the second period, since consumers have purchased the product in the first period, consumers can identify the real product information, which may be inconsistent with the information published by the manufacturer in the first period.

To solve the problems mentioned in Section 1, next, this section will analyze the decision under centralized and decentralized decision making, respectively. Against the background of centralized decision making, the manufacturer and the retailer negotiate to maximize the profit of the whole supply chain, while in the context of decentralized decision making, the manufacturer and the retailer individually make decisions to maximize their own profits.

#### 3.1. Model Structure

A two-period model with one manufacturer and one retailer is constructed. We model the interaction between the supplier and the manufacturer as a Stackelberg game, where the manufacturer serves as the leader, and the supplier is the follower. The manufacturer distributes their products to consumers through two channels: a retail channel, denoted by channel 1, and a direct channel, denoted by channel 2, as illustrated in Figure 1.
Similar to the past studies, we assume that demand is a linear function of the retail price and the degree of environmental protection, a greenness measure [36,55–57]. The demand functions for the retail and direct channels are expressed, respectively, as follows:

\[ d_1 = \delta a - \beta_1 p_1 + \theta_1 e \]
\[ d_2 = (1 - \delta) a - \beta_2 p_2 + \theta_2 e \]

where \( a \) is the total market demand for the product, which refers to the potential maximum market capacity. Parameter \( \delta \) represents the degree of consumers’ preference for the offline channel (i.e., retailer channel). Specifically, the higher \( \delta \) means that more consumers are inclined to buy products offline. When \( \delta = 1 \), consumers will not consume through the online channel. Parameter \( \beta_i \) \( (i = 1, 2) \) reflects the sensitivity of the demand for each channel to the product price, that is, the demand change for one unit price change. In addition, \( \theta_i \) \( (i = 1, 2) \) captures the sensitivity of market demand to changes in the degree of environmental protection.

Since our main focus is on the emission reduction strategies in the context of misreporting, in order to highlight more on this core issue, this paper will not consider the impact of price competition between two channels. Thus, we can assume the demand for channel \( i \) \( (i = 1, 2) \) is not affected by the retail price of channel \( j \) \( (j = 2 - i) \). Product demand decreases with its own retail price and increases with the product greenness level. This assumption is made for the simplicity of solving and analyzing the mathematical model, and it can also help us focus on the emission reduction in the context of misreporting.

Table 1 summarizes the meaning of all symbols and subscripts used in this paper.

**Table 1.** The description of the symbols.

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
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<tbody>
<tr>
<td>( a )</td>
<td>Potential market demand.</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Market share of the retail channel.</td>
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<tr>
<td>( \beta )</td>
<td>Elasticity coefficient of the retail price.</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Elasticity coefficient of the green degree.</td>
</tr>
<tr>
<td>( c )</td>
<td>Green degree of the product.</td>
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<tr>
<td>( k )</td>
<td>Cost coefficient of the green product.</td>
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<tr>
<td>( w )</td>
<td>The wholesale price charged by the manufacturer.</td>
</tr>
<tr>
<td>( p )</td>
<td>The retail price charged by the manufacturer and the retailer to consumers.</td>
</tr>
<tr>
<td>( \psi )</td>
<td>The profit function announced by the manufacturer when the manufacturer misreports the green degree.</td>
</tr>
<tr>
<td>( \pi )</td>
<td>The profit function of the manufacturer and the retailer.</td>
</tr>
</tbody>
</table>
Table 1. Cont.

<table>
<thead>
<tr>
<th>Superscripts/Subscripts</th>
<th>Description</th>
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<tbody>
<tr>
<td>$f$</td>
<td>The superscript $f$ denotes the first period.</td>
</tr>
<tr>
<td>$s$</td>
<td>The superscript $s$ denotes the second period.</td>
</tr>
<tr>
<td>*</td>
<td>The superscript * denotes the optimal state.</td>
</tr>
<tr>
<td>$m$</td>
<td>The subscript $m$ denotes the manufacturer.</td>
</tr>
<tr>
<td>$R$</td>
<td>The subscript $R$ denotes the retailer.</td>
</tr>
<tr>
<td>$i$</td>
<td>The subscript $i$ denotes channel $i$, $i = 1, 2$.</td>
</tr>
<tr>
<td>$c$</td>
<td>The subscript $c$ indicates the centralized decision making of the supply chain.</td>
</tr>
<tr>
<td>$s$</td>
<td>The subscript $s$ denotes the supply chain.</td>
</tr>
<tr>
<td>$d$</td>
<td>The subscript $d$ indicates the decentralized decision making of the supply chain without misreporting.</td>
</tr>
<tr>
<td>$h$</td>
<td>The subscript $h$ indicates the manufacturer misreporting the green degree of the product.</td>
</tr>
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</table>

3.2. Assumptions

Our model is based on the following assumptions:

**Assumption 1.** The investment in emissions reduction is a one-time decision, which has no effect on the unit production cost.

This assumption is along the same line of Krishnan [58] and Zhu [35]. Therefore, in our model, after the manufacturer decides the optimal degree of greenness in the first period, it remains unchanged in the second period.

**Assumption 2.** The manufacturer’s cost incurred by improving the green degree $e$ is given by $c_s = ke^2/2$.

This assumption has been employed in the literature, such as Ghosh [59] and Mukhopadhyay [60].

**Assumption 3.** Market demand is more sensitive to price than to greenness level. That is $\beta_i > \theta_i (i = 1, 2)$. In addition, the cost coefficient of energy conservation and emission reduction is sufficiently large with $k > \left\{ \frac{50\beta_1\beta_2\theta_2(1-\epsilon)\theta_2}{4\beta_1\beta_1-2(1-\epsilon)\theta_1\theta_2}, 3\beta_1 \right\} (i = 1, 2)$.

A similar assumption was made by researchers such as Tsay [61] and Xu [62].

4. Model Solutions and Discussions

In this section, we derive the optimal pricing strategies and associated optimal profits for the members of the supply chain.

4.1. The Centralized Model

First, we present the centralized decision model of supply chain as a benchmark. In such a model, a centralized decision maker decides the direct selling price, the retail price and the greenness degree of the two periods, respectively. The decision sequence, as shown in Figure 2, is as follows: First, the centralized decision maker determines the optimal degree of greenness based on the expected profits to maximize the total profits of the two periods. Next, the manufacturer decides the direct and retail prices for the first and second periods, respectively.

![Figure 2. Decision sequence in the centralized model.](image-url)
Clearly, the profit functions of the supply chain in the two periods are, respectively, given by
\[ \pi_c = p_f^1(\delta a - \beta_1 p^f_1 + \theta_1 e) + p^f_2((1 - \delta) a - \beta_2 p^f_2 + \theta_2 e) - \frac{k e^2}{2} \]  
(1)
\[ \pi_c = p^f_1(\delta a - \beta_1 p^f_1 + \theta_1 e) + p^f_2((1 - \delta) a - \beta_2 p^f_2 + \theta_2 e). \]  
(2)

Next, we can obtain the optimal values of the decision variables for the two periods, sequentially based on the reverse induction method. It follows from Equation (1) that the Hessian matrix of \( \pi_c^f \) is given by
\[ H = \begin{bmatrix}
\frac{\partial^2 \pi_c^f}{\partial p^f_1^2} & \frac{\partial^2 \pi_c^f}{\partial p^f_1 \partial p^f_2} \\
\frac{\partial^2 \pi_c^f}{\partial p^f_1 \partial p^f_2} & \frac{\partial^2 \pi_c^f}{\partial p^f_2^2}
\end{bmatrix} = \begin{bmatrix} -2\beta_1 & 0 \\ 0 & -2\beta_2 \end{bmatrix}. \]

Since \( |H| = 4\beta_1\beta_2 > 0 \), the optimal solution for the centralized decision model exists. Then, by solving \( \frac{\partial \pi_c^f}{\partial p^f_1} = 0 \) and \( \frac{\partial \pi_c^f}{\partial p^f_2} = 0 \), the optimal direct selling price and the optimal retail price of the first period are as follows:
\[ p^f_{1c} = \frac{a\delta + e\theta_1}{2\beta_1}, \]  
(3)
\[ p^f_{2c} = \frac{a - a\delta + e\theta_2}{2\beta_2}. \]  
(4)

Substituting (3) and (4) into (1) yields the optimal profit of the supply chain in the first period:
\[ \pi_c^f = \frac{1}{4}[-2e^2k + \frac{(a\delta + e\theta_1)^2}{\beta_1} + \frac{(a - a\delta + e\theta_2)^2}{\beta_2}]. \]  
(5)

Similarly, we obtain the Hessian matrix of \( \pi_c^e \) as
\[ H = \begin{bmatrix}
\frac{\partial^2 \pi_c^e}{\partial p^e_1^2} & \frac{\partial^2 \pi_c^e}{\partial p^e_1 \partial p^e_2} \\
\frac{\partial^2 \pi_c^e}{\partial p^e_1 \partial p^e_2} & \frac{\partial^2 \pi_c^e}{\partial p^e_2^2}
\end{bmatrix} = \begin{bmatrix} -2\beta_1 & 0 \\ 0 & -2\beta_2 \end{bmatrix}. \]

Obviously, the above Hesse matrix is negative definite, thus the optimal \( p^e_1 \) and \( p^e_2 \) maximizing \( \pi_c^e \) are given by
\[ p^e_{1c} = \frac{a\delta + e\theta_1}{2\beta_1}, \]  
(6)
\[ p^e_{2c} = \frac{a - a\delta + e\theta_2}{2\beta_2}. \]  
(7)

Similarly, the optimal profit of the supply chain in the second period is given by
\[ \pi_c^e = \frac{\beta_2(a\delta + e\theta_1)^2 - \beta_1[a(1 - \delta) + e\theta_2]^2}{4\beta_1\beta_2}. \]  
(8)

Next, the total profit of the whole supply chain is \( \pi_c = \pi_c^f + \pi_c^e \), that is
\[ \pi_c = \frac{1}{2}[-e^2k + \frac{(a\delta + e\theta_1)^2}{\beta_1} + \frac{(a - a\delta + e\theta_2)^2}{\beta_2}]. \]  
(9)

Since \( \frac{\partial^2 \pi_c}{\partial e^2} = -k + \frac{e^2}{\beta_1} + \frac{e^2}{\beta_2} < 0 \), the optimal green degree can be obtained as follows:
\[ e^*_c = \frac{a\delta\beta_2\theta_1 + a\beta_1\theta_2 - a\delta\beta_1\theta_2}{k\beta_1\beta_2 - \beta_2\theta_1^2 - \beta_1\theta_2^2}. \]  
(10)
Finally, substituting (10) into (3), (4), (6), (7) and (9) yields the optimal decision and optimal profit of the supply chain, as shown below:

\[
p^*_f = p^*_i = \frac{a\{k_1\theta_2 + \theta_2[(1 - \delta)\theta_1 - \delta\theta_2]\}}{2\beta_1 (k_2 - \theta_2^*) - 2\beta_2 \theta_1^*}\]

\[
p^*_c = p^*_i = \frac{a\{(k - k_1)\theta_1 - \theta_1[(1 - \delta)\theta_1 - \delta\theta_2]\}}{2\beta_1 (k_2 - \theta_2^*) - 2\beta_2 \theta_1^*}\]

\[
p^*_2 = p^*_c = \frac{a\{(k - k_2)\theta_1 - \theta_1[(1 - \delta)\theta_1 - \delta\theta_2]\}}{2\beta_1 (k_2 - \theta_2^*) - 2\beta_2 \theta_1^*}\]

**Corollary 1.** In a centralized model, if \(\beta_1 > \frac{k_1\theta_2 + \theta_2[(1 - \delta)\theta_1 - \delta\theta_2]}{k(1 - \delta)}\), the optimal direct price is greater than the optimal retail price; if \(\beta_1 \leq \frac{k_2\theta_1 + \theta_2[(1 - \delta)\theta_1 - \delta\theta_2]}{k(1 - \delta)}\), that is \(\beta_2 \geq \frac{(k - k_1)\theta_1 - \theta_1[(1 - \delta)\theta_1 - \delta\theta_2]}{k\theta_2^*}\), the optimal retail price is greater than the optimal direct price.

Corollary 1 is quite intuitive: when price elasticity \(\beta_i (i = 1, 2)\) is higher than a certain threshold, the price of the product in channel I will be reduced. Note that a price change will bring two opposite effects to the manufacturer. On the one hand, the increase in price will increase the manufacturer’s marginal profit and bring positive effect to the enterprise’s profit. On the other hand, the increase in price will lead to the decrease in product sales, which will lead to the decrease in the manufacturer’s sales revenue. When price elasticity exceeds a certain threshold, the increase in product price will lead to a substantial reduction in product sales. That is, the negative effects dominate the positive effects. Therefore, the manufacturer prefers to reduce the price of products.

### 4.2. The Decentralized Decision Model without Misreporting

In the context of decentralized decision making, we assume that the manufacturer and the retailer are rational, that is, the manufacturer and the retailer make decisions, respectively, to maximize their profits. Specifically, firstly, the manufacturer decides the optimal degree of greenness to maximize the total profit of the two periods; next, in the first period, the manufacturer, as the leader, determines the wholesale price of the product. Then, the manufacturer and the retailer decide the direct price and the retail price simultaneously under the given wholesale price and greenness degree. In the second period, the manufacturer determines the wholesale price; finally, the manufacturer and the retailer decide the optimal direct price and optimal retail price, respectively. Figure 3 describes the decision sequence of the manufacturer and the retailer in the decentralized model without misreporting.

![Figure 3. Decision sequence in the decentralized decision model without misreporting.](image)

The profit function of the manufacturer and the retailer at each period can be expressed as follows:

\[
\pi_{md}^f = w^f (\delta a - \beta_1 p_i + \theta_1 e) + p_c^f [(1 - \delta) a - \beta_2 p_c^f + \theta_2 e] - \frac{ke^2}{2} \tag{11}
\]
the only difference is that the manufacturer first determines the level of misreporting compared with the decentralized decision making in the context of information symmetry. Next, we will build a two-period game model to study the model of the manufacturer’s own green degree, which will bring them more market demand, and hence, more profits. Benefit themselves?

In the green supply chain, the manufacturer has the motivation to misreport their green degree, so the manufacturer must declare their own green degree first. However, this may not be the true case; more likely, one member cannot accurately grasp the other’s information, which will bring positive effects to the member who masters the other’s information, but it will bring negative effects to the other member of the supply chain. For example, when retailers make decisions, they cannot accurately know the green degree of products, so the manufacturer must declare their own green degree first.

Similar to the centralized model, the optimal solution and the optimal profit of the manufacturer and the retailer are solved by the backward induction, which is summarized in Proposition 1.

**Proposition 1.** In the model, which is a decentralized decision-making model without misreporting, we can obtain the optimal solution and the optimal profit of the manufacturer and the retailer as follows:

\[
\pi^*_{kd} = (p^*_1 - w^*) (\delta a - \beta_1 p^*_1 + \theta_1 e) 
\]

\[
\pi^*_{md} = w^* (\delta a - \beta_1 p^*_1 + \theta_1 e) + \frac{a}{\beta_1 (2k \beta_2 - 2 \theta^2_2) - \beta_2 \theta^2_1} (1 - \delta) a - \beta_2 p^*_2 + \theta_2 e
\]

\[
\pi^*_{kd} = (p^*_1 - w^*) (\delta a - \beta_1 p^*_1 + \theta_1 e).
\]

4.3. The Decentralized Decision Model with Misreporting

All the analyses above, whether centralized or decentralized, are carried out under the assumption that the manufacturer does not misreport the green level of the product. However, this may not be the true case; more likely, one member cannot accurately grasp the other’s information, which will bring positive effects to the member who masters the information, but it will bring negative effects to the other member of the supply chain. For example, when retailers make decisions, they cannot accurately know the green degree of products, so the manufacturer must declare their own green degree first. However, the manufacturer may misreport their green degree to obtain more benefits. In the case of asymmetric information, how can the manufacturer misreport their green degree to benefit themselves?

In the green supply chain, the manufacturer has the motivation to misreport their own green degree, which will bring them more market demand, and hence, more profits. Next, we will build a two-period game model to study the model of the manufacturer’s misreporting of the green degree. The decision sequence is shown in Figure 4. Specifically, compared with the decentralized decision making in the context of information symmetry, the only difference is that the manufacturer first determines the level of misreporting of product greenness before all pricing decisions are made.
Figure 4. Decision sequence in the decentralized decision model with misreporting.

Similar to Yan [13], when the manufacturer misreports the green degree of the product, the manufacturer makes decisions according to the announced profit function to be consistent with their own misreporting behavior. Suppose that the green degree of the product announced by the manufacturer is $e(1 + \epsilon)$. Thus, the announced profit functions of the manufacturer and the retailer in the first period are given by, respectively:

$$\pi_{mh}^f = w^f[\delta a - \beta_1 p_1^f + \theta_1 e(1 + \epsilon)] + p_2^f[(1 - \delta)a - \beta_2 p_2^f + \theta_2 e(1 + \epsilon)] - \frac{ke^2(1 + \epsilon)^2}{2} \tag{21}$$

$$\pi_{Rh}^f = (p_1^f - w^f)[\delta a - \beta_1 p_1^f + \theta_1 e(1 + \epsilon)] \tag{22}$$

However, the actual cost paid by the manufacturer is $ke^2/2$. Therefore, the actual profit functions of the retailer and the manufacturer in the first period are as follows:

$$\pi_{mh}^f = w^f[\delta a - \beta_1 p_1^f + \theta_1 e(1 + \epsilon)] + p_2^f[(1 - \delta)a - \beta_2 p_2^f + \theta_2 e(1 + \epsilon)] - \frac{ke^2}{2} \tag{23}$$

$$\pi_{Rh}^f = (p_1^f - w^f)[\delta a - \beta_1 p_1^f + \theta_1 e(1 + \epsilon)] \tag{24}$$

When consumers purchase the products, whether through the manufacturer or the retailer, they can realize that the green degree of the product is $e$, not $e(1 + \epsilon)$, which is announced by the manufacturer in the first period. Thus, in the second period, the market demand will change, which will lead to changes in the actual profit function of the manufacturer and the retailer.

According to Assumption 1, the cost of carbon emissions reduction is a one-time investment. Therefore, the decision functions of the manufacturer and the retailer in the second period are as follows:

$$\pi_{mh}^s = w^s[\delta a - \beta_1 p_1^s + \theta_1 e(1 + \epsilon)] + p_2^s[(1 - \delta)a - \beta_2 p_2^s + \theta_2 e(1 + \epsilon)] \tag{25}$$

$$\pi_{Rh}^s = (p_1^s - w^s)[\delta a - \beta_1 p_1^s + \theta_1 e(1 + \epsilon)] \tag{26}$$

As consumers of each channel realize the true green degree of the product in the second period, the final sales volume of each channel is $\delta a - \beta_1 p_1^s + \theta_1 e$. However, since all the products ordered in the first period have already been sold, the retailer does not know the true green degree of the product. Therefore, the retailer will still order $\delta a - \beta_1 p_1^s + \theta_1 e(1 + \epsilon)$ in the second period. The actual profit functions of the manufacturer and the retailer in the second period are as follows:

$$\pi_{mh}^s = p_2^s[(1 - \delta)a - \beta_2 p_2^s + \theta_2 e] + w^s[\delta a - \beta_1 p_1^s + \theta_1 e(1 + \epsilon)] \tag{27}$$

$$\pi_{Rh}^s = p_1^s \min \{[\delta a - \beta_1 p_1^s + \theta_1 e(1 + \epsilon)], (\delta a - \beta_1 p_1^s + \theta_1 e)] - w^s[\delta a - \beta_1 p_1^s + \theta_1 e(1 + \epsilon)] \tag{28}$$

Next, we obtain the optimal solution and the optimal profit for the case with the manufacturer misreporting the green degree by backward induction, which is summarized in Proposition 2.
Proposition 2. In a decentralized decision-making model with manufacturer’s misreporting, we have the optimal solutions and the associated profits of the manufacturer and the retailer given by

\[ w_h^* = \frac{a\{k\delta_2 + \theta_2[(1 - \delta)\theta_1 - \delta\theta_2]\}}{2\beta_1(\beta_2 - \theta_2)} - \beta_2\theta_1^2 \]  

(29)

\[ c_h^* = \frac{a\theta_2(2k(1 - \delta)\beta_1 - \theta_1[(1 - \delta)\theta_1 - \delta\theta_2])}{4k\beta_1(\beta_2 - \theta_2)} - 2k\beta_2\theta_1^2 \]  

(30)

\[ p_{1h}^* = p_{1h}^* = \frac{3a\{k\delta_2 + \theta_2[(1 - \delta)\theta_1 - \delta\theta_2]\}}{4\beta_1(\beta_2 - \theta_2^2) - 2\beta_2\theta_1^2} \]  

(31)

\[ p_{2h}^* = p_{2h}^* = \frac{a(2k\beta_1(1 - \delta) - \theta_1[(1 - \delta)\theta_1 - \delta\theta_2])}{4\beta_1(\beta_2 - \theta_2^2) - 2\beta_2\theta_1^2} \]  

(32)

\[ \pi_{wh}^* = \frac{a^2\{4k^2(4k\beta_2 - 3\theta_2)(1 - \delta)^2\beta_1^2 - \theta_1^2(1 - \delta)\theta_2 - (1 - \delta)\theta_1\} \{4k\beta_2\theta_1(1 - \delta) - \theta_2^2[\delta\theta_2 - (1 - \delta)\theta_1] + 4k\beta_1 \{2k^2\delta^2\beta_2 - 2k\beta_2[2(1 - \delta)^2\theta_2^2 - 3(1 - \delta)\delta\theta_1\theta_2 + 2\delta^2\theta_2^2]\}\}}{8k[\beta_2\theta_1^2 - \beta_1(2k\beta_2 - 2\theta_2^2)]^2} \]  

(33)

\[ \pi_{Rh}^* = \frac{a^2\{k\delta_2 + \theta_2[(1 - \delta)\theta_1 - \delta\theta_2]\} \{2k\beta_1[k\delta\beta_2 - (1 - \delta)\theta_1 - \delta\theta_2]\} - 2k(1 - \delta)\theta_2\theta_1 - \delta\theta_2^2\{2k\delta_2 - \theta_2[\delta\theta_2 - (1 - \delta)\theta_1]\}\}}{4k[\beta_2\theta_1^2 - \beta_1(2k\beta_2 - 2\theta_2^2)]^2} \]  

(34)

\[ \epsilon^* = \frac{2k\delta_2\beta_1 + \theta_2[2k(1 - \delta)\beta_1 + \theta_1(\theta_1 - \delta\theta_1 - \delta\theta_2)\}}{\theta_2[2k\beta_1(1 - \delta)\theta_1 - \theta_1[(1 - \delta)\theta_1 - \delta\theta_2]]} \]  

(35)

From the optimal solution of Propositions 1 and 2, we have the following Corollary 2.

Corollary 2. If \( \beta_1 > \frac{3k\delta_2 + (\theta_1 + 3\theta_2)(1 - \delta)\theta_1 - \delta\theta_2}{2k(1 - \delta)} \) is satisfied, the optimal direct price is greater than the optimal retail price. If \( \beta_1 \leq \frac{3k\delta_2 + (\theta_1 + 3\theta_2)(1 - \delta)\theta_1 - \delta\theta_2}{2k(1 - \delta)} \), that is, \( \beta_2 \geq \frac{2k(1 - \delta)\theta_1 - (\theta_1 + 3\theta_2)(1 - \delta)\theta_1 - \delta\theta_2}{3k\delta_2} \), the optimal retail price is greater than the optimal direct price in the decentralized decision-making model, regardless of whether the manufacturer misreports the green degree.

Similar to Corollary 1, Corollary 2 shows that price elasticity has a significant impact on pricing strategy. A larger \( \beta_i (i = 1, 2) \) means that consumers in channel \( i \) are more sensitive to the price of the product and, thus, a lower willingness for the manufacturer to increase the price.

5. Analysis of Optimal Decision and Optimal Profit and Contract Coordination

In this section, we will compare and analyze the optimal solution and optimal profit of the above three models to gain some managerial insights and propose a revenue-share contract to coordinate supply chain decisions.

5.1. Sensitivity Analysis

In this part, we will analyze the sensitivity of the optimal solutions, which are obtained in the fourth part.

Proposition 3.

\[ \frac{\partial w_{ij}^*}{\partial \beta_j} < 0, \quad \frac{\partial w_{ih}^*}{\partial \beta_j} = \frac{\partial w_{h}^*}{\partial \beta_j} < 0 \quad (i, j = 1, 2; \ t = f, \ s; \ l = c, d, h). \]  

(36)
As shown in Proposition 3, the optimal price, whether centralized or decentralized, has a negative correlation with price elasticity. The higher the price elasticity is, the more sensitive the demand is to price changes. That is, reducing the price can bring about a significant increase in the demand; thus, the manufacturer may reduce the wholesale price of the products, which means a decrease in marginal revenue. To maintain their own profits, the manufacturer will reduce the green degree of their products.

Similarly, the elasticity coefficient of the green degree also has a significant impact on the optimal decisions, as shown in Proposition 4.

Proposition 4.

\[
\frac{\partial p^*_t}{\partial \theta_j} > 0, \quad \frac{\partial w^*_d}{\partial \theta_j} = \frac{\partial w^*_h}{\partial \theta_j} > 0 \ (i, j = 1, 2; t = f, s; J = c, d, h).
\]

\[
\frac{\partial e^*_j}{\partial \theta_j} > 0 \ (j = 1, 2; J = c, d, h).
\]

On the contrary, there is a positive correlation between the elasticity coefficient of the green degree and the optimal price. This is consistent with our intuition, since the higher demand elasticity of the green degree, that is, a larger \( \theta_i \), means that the change of the green degree will have a greater impact on demand. To obtain more sales volume and more sales revenue, the manufacturer will increase the green degree of the products actively, which will lead the manufacturer to increase the wholesale price and the direct price of the products to make up for the extra cost of improving the green degree.

For the retailer, the increase in wholesale price means the increase in cost, which will lead to the decrease in retailer’s profit. Thus, to maintain their own profits, the retailer will also increase the retail price of the product.

Proposition 5.

\[
\frac{\partial p^*_t}{\partial k} < 0, \quad \frac{\partial w^*_d}{\partial k} = \frac{\partial w^*_h}{\partial k} < 0 \ (i = 1, 2; t = f, s; J = c, d, h).
\]

\[
\frac{\partial e^*_j}{\partial k} < 0 \ (J = c, d, h).
\]

Similar to price elasticity, the green cost coefficient of the product will also have a negative effect on the optimal decisions.

In other words, the increase in \( k \) will lead to the decrease in the optimal green degree and the optimal price, which is inconsistent with our intuition. For the manufacturer, the increase in \( k \) means the increase in energy saving and emission reduction costs. To maintain their own marginal profit, the manufacturer will reduce the green degree of the product, which will lead to the decrease in consumer preference for the product. To attract consumers to buy the product, as shown in Proposition 5, the manufacturer will reduce the wholesale price and the direct price, and the retailer will reduce the retail price of the product.

Proposition 6.

\[
\frac{\partial p^*_1}{\partial \delta} > 0, \quad \frac{\partial p^*_2}{\partial \delta} < 0, \quad \frac{\partial w^*_d}{\partial \delta} = \frac{\partial w^*_h}{\partial \delta} > 0 \ (t = f, s; J = c, d, h).
\]
If $\beta_2 > \frac{\theta_1 \theta_2}{\theta_1^2}$, that is $\theta_1 > \frac{\theta_2 \theta_1}{\theta_2^2}$ satisfied, then $\frac{\partial \hat{e}^*}{\partial h} > 0$, otherwise $\frac{\partial \hat{e}^*}{\partial h} \leq 0$; If $\beta_2 > \frac{2\theta_2 \theta_1}{\theta_1^2}$, that is $\theta_1 > \frac{2\theta_2 \theta_1}{\theta_2^2}$ satisfied, then $\frac{\partial \hat{e}^*}{\partial \delta} > 0$, otherwise $\frac{\partial \hat{e}^*}{\partial \delta} \leq 0$. (44)

From Proposition 6, several interesting conclusions can be drawn. First, the optimal price decision of the retail channel is positively correlated with the market share of the retail channel, while the optimal price decision of the direct channel is negatively correlated with $\delta$. The higher the $\delta$, the more market demand for the retail channel, which means that consumers prefer the retail channel. Therefore, to obtain more profits, the retailer will increase the retail price, and the manufacturer will also increase the wholesale price. Correspondingly, the higher the $\delta$, the less market demand for direct channels, that is, the less consumers prefer direct marketing channels. Therefore, the manufacturer reduces the direct selling price of the product.

Second, the optimal green degree is negatively correlated with the retail channel market share when the manufacturer misreports the green degree of the product, which can be explained by the following reasons. When the manufacturer misreports the green degree of the product, the consumers of the direct marketing channel will realize the true green degree in the second period, which will lead to the decrease in the sales volume of the direct marketing channel. A higher $\delta$ means more sales through the retail channel, that is, the retail channel is more important to the manufacturer. Therefore, the manufacturer will misreport the green degree to a greater extent. However, when $\delta$ decreases, the importance of the direct channel will be significantly improved. As the misreporting will lead to the reduction in demand for the direct channel in the second period, the manufacturer will misreport the green degree to a lesser extent.

Third, when the manufacturer does not misreport the green degree of the product, the relationship between the green degree and the retail channel market share depends on the elasticity coefficient of the product. Taking the centralized decision model as an example, if the price elasticity of the direct channel is greater than a certain threshold, or the elasticity coefficient of the green degree of the retail channel is greater than a certain threshold, that is, the consumers of the direct channel are more sensitive to the price, while the consumers of the retail channel are more sensitive to the green degree of the product, there is a positive correlation between the green degree and the market share of retail channels. As $\delta$ increases, the sales volume of the retail channel will increase accordingly. To cater to the preferences of consumers in the retail channel, the manufacturer will improve the green degree of the product. However, when $\delta$ decreases, the sales volume of the direct channel will increase. To cater to the preferences of consumers in the direct channel, the manufacturer will reduce the price of the product, which will lead to the reduction in the green degree of the product.

Next, we will analyze the influencing factors of the manufacturer’s misreporting level.

Proposition 7.

\[
\frac{\partial \hat{e}^*}{\partial \theta_1} > 0, \quad \frac{\partial \hat{e}^*}{\partial \delta} > 0, \quad \frac{\partial \hat{e}^*}{\partial \theta_2} < 0. \quad (45)
\]

If $\frac{\delta}{(1 - \delta)} > \frac{\theta_1}{\theta_2}$ satisfies, then $\frac{\partial \hat{e}^*}{\partial k} > 0$, otherwise $\frac{\partial \hat{e}^*}{\partial k} \leq 0$. (46)

Proposition 7 indicates that the elasticity coefficient of the green degree, the market share of the retail channel and the cost coefficient of energy conservation and emission reduction will have a significant impact on the manufacturer’s misreporting decision. Specifically, first, the manufacturer will misreport the green level of products to a greater extent when consumers in the retail channel are more sensitive to the green level of products. This is because the increase in $\theta_1$ can bring more sales to the retail channel. In order to reduce costs, manufacturers will misreport the green degree of products to a greater extent.
On the contrary, when \( \theta_2 \) increases, the manufacturer will misreport the green degree of the products to a lower extent. Since the misreporting will reduce the sales volume of the direct channel in the second period, the manufacturer will more truthfully announce the green degree of the product.

Second, there is a positive correlation between \( \delta \) and \( \epsilon \). The increase in \( \delta \) will reduce the importance of the direct channel, which will obviously lead the manufacturer to misreport the green degree of products to a greater extent.

Finally, when \( \frac{\delta}{(1-\epsilon)} > \frac{\theta_2}{\theta_1} \), \( \epsilon \) will increase with \( k \). \( \frac{\delta}{(1-\epsilon)} > \frac{\theta_1}{\theta_2} \) means that the sales volume of the retail channel is relatively high, and the consumers of the retail channel are not sensitive to the green degree, so when \( k \) increases, to reduce the cost, the manufacturer will misreport the green degree of the products to a greater extent. However, when \( \frac{\delta}{(1-\epsilon)} \leq \frac{\theta_1}{\theta_2} \), there are more consumers in the direct channel. To prevent the reduction in sales in the second period, the manufacturer will misreport the green degree of products to a lesser extent.

5.2. Comparative Analysis of Optimal Decision

In Section 4, we solve the optimal solutions of the centralized model, the decentralized model with information symmetry and the decentralized model with the manufacturer misreporting the green degree, respectively. Next, we will compare the optimal solutions of different models to provide decision-making guidance for practitioners.

**Theorem 1.**

\[
p_{ij}^{f_s} = p_{ij}^s; \quad w_d^{s} = w_d^{s*}, \quad w_h^{s} = w_h^{s*}, \quad (i = 1, 2; J = c, d, h) \quad (47)
\]
\[
p_{i}^{f_s} = p_{ih}^{s*} = p_{ih}^{s*}; \quad w_d^{s*} = w_d^{s*}, \quad w_h^{s*} = w_h^{s*}, \quad (i = 1, 2). \quad (48)
\]

According to Theorem 1, we can draw the conclusion that the retail prices of different periods are the same and so are the direct price and the wholesale price, either in the centralized model or the decentralized model. To see the intuition, first consider the centralized decision-making model as an example.

As shown in Equations (47) and (48), the profit of the central planner in the first period consists of three parts. The first item and the second item are the sales revenue of the whole supply chain. The third item is the cost of energy conservation and emission reduction for the manufacturer. The only difference between the second period and the first period is that in the second period, the central planner does not need to pay for energy conservation and emission reduction, since we have assumed that energy conservation and emission reduction is a one-time investment. Therefore, when the green degree of the first period is the same as that of the second period, the decision functions of the central planner in the first period and the second period are essentially consistent, which leads to the consistency of the optimal decision in the first period and the optimal decision in the second period.

In addition, Theorem 1 shows that the manufacturer’s misreporting strategy will not affect the optimal retail price, the optimal direct selling price and the optimal wholesale price. This is because in decentralized decision making, the manufacturer’s misreporting will affect the manufacturer’s actual cost, which is reflected in the manufacturer’s actual profit function. However, in our model, to conform to their own misreporting behavior, the manufacturer makes decisions according to the announced profit function. Since the green degree announced by the manufacturer in the model with misreporting is \((1+\epsilon)e_h\), which is equal to the green degree \(e\) of the model without misreporting, the decision function of the manufacturer in the model with misreporting is consistent with that of the model without misreporting. Therefore, the manufacturer’s misreporting will not affect the optimal retail price, the optimal direct price and the optimal wholesale price.

As shown in Theorem 1, the misreporting strategy will not affect the pricing decision of the firm, either the manufacturer or the retailer. However, it is obvious that the manufacturer’s misreporting behavior will have a significant impact on the decision making on the green degree, as shown in Theorem 2.
Theorem 2.

\[ p_{tc}^1 < p_{td}^1 = p_{th}^1, \quad p_{tc}^2 = p_{td}^2 < p_{th}^2, \quad t \in \{f, s\}. \]  
\[ e_c > e > e_h. \]  

From Theorem 2, several interesting conclusions can be drawn. First, the retail price in the centralized decision-making model is lower than that in the decentralized model, no matter whether the manufacturer misreports the green degree or not. This is consistent with our intuition, since in the context of decentralized decision making, due to the existence of the double marginalization effect, to obtain more profits, the manufacturer will increase the wholesale price of the product, and the retailer will increase the retail price of the product. This behavior leads to the decision making on the retail channel deviating from the optimal level in the context of decentralized decision making. However, in the context of centralized decision making, the central planner will determine a lower retail price from the overall perspective to increase the sales of the product, which results in \( p_{tc}^2 = p_{td}^2 < p_{th}^2, \quad t \in \{f, s\} \).

Second, different from the retail price, the direct price under centralized decision making is higher than that under decentralized decision making, which is mainly due to the highest green degree of the product in the context of centralized decision making. As the cost of the product increases, the central planner increases the direct selling price of the product.

In addition, the green degree is different from the above retail price and the direct selling price. Specifically, the green degree is affected not only by the decision-making mode, but also by the manufacturer’s misreporting strategy. From Theorem 2, we can draw the conclusion that the green degree is the highest under the centralized decision-making setting, and the green degree is the lowest under the model of manufacturer’s misreporting of the green degree. Obviously, centralized decision making improves the utilization efficiency of resources, which is better for consumers and the environment. Therefore, the manufacturer and the retailer should actively cooperate to bring more benefits to consumers and the environment. Second, the manufacturer’s misreporting of the green degree will reduce the manufacturer’s motivation of energy conservation and emission reduction and then bring negative effects to consumers and the environment.

5.3. Comparative Analysis of Optimal Profit

To guide the decision making of each member in the supply chain, we will compare the optimal profits of the manufacturer, the retailer and the whole supply chain under different models. Next, we will make a comparative analysis of the profits of the manufacturer and the retailer under different models.

Theorem 3. The manufacturer’s misreporting can bring more profits to the manufacturer, that is, \( \pi_{md}^* < \pi_{mh}^* \). However, the misreporting of the manufacturer will bring negative effects to the retailer’s profit, that is, \( \pi_R^* > \pi_{Rh}^* \).

As indicated in Theorem 3, misreporting can make the manufacturer more profitable, since misreporting can have two effects on the manufacturer. First, misreporting can increase the sales of products in the direct channel and the retail channel. Second, misreporting can reduce the cost of energy conservation and emission reduction for the manufacturer. Therefore, for the manufacturer, it is the best choice to misreport the green degree of the product.

For the retailer, however, misreporting can mislead the retailer to order too many products, which may not be fully sold in the second period. Therefore, the wrong green degree information will make the retailer incur extra purchase costs, which will lead to the reduction in the retailer profits.

Therefore, we can conclude that the manufacturer is motivated to misreport the green degree, which is inconsistent with the interests of the retailer. That is, the retailer has the
motivation to provide the contract to coordinate with the manufacturer to discourage them from misreporting.

Next, we will conduct a comparative analysis of the profits of the whole supply chain under different models, which is shown in Theorem 4.

**Theorem 4.** For the whole supply chain, the optimal profit under different models has the following relationship: \( \pi_{sh}^* < \pi_s^* < \pi_c^*. \)

From Theorem 4, some interesting conclusions can be drawn. First, the manufacturer’s misreporting strategy will lead to a reduction in supply chain profits. The manufacturer’s misreporting strategy will lead to information asymmetry, which further reduces the resource utilization efficiency of the whole supply chain. Even though the manufacturer’s misreporting strategy increases the manufacturer’s profit, it results in a larger decrease in the retailer’s profit. As a result, profits are ultimately reduced across the supply chain.

Second, the optimal profit under centralized decision making is higher than that under decentralized decision making, regardless of whether the manufacturer misreports the green degree or not. This shows that no matter what strategy the manufacturer chooses, it cannot eliminate the double marginalization effect, that is, it cannot make the whole supply chain achieve the optimal profit.

Therefore, we can conclude that misreporting of the green degree is not a favorable strategy in general. However, the manufacturer has the motivation to misreport the green degree, since their profits achieve growth after misreporting. From the perspective of the retailer, it is necessary to provide an incentive, such as a revenue-sharing contract, to urge the manufacturer to abandon the strategy of misreporting.

### 5.4. Coordination of Supply Chain

In this section, to achieve the coordination of the supply chain, the retailer will provide a "revenue-sharing contract" to stimulate the manufacturer to give up the misreporting strategy.

To achieve coordination, the retailer, as the follower, no longer decides the retail price of the product, and the manufacturer, as the leader, makes all the decisions in the supply chain. Then, when all the transactions are completed, the manufacturer will share the \( \tau \) part of the profit with the retailer. That is, as a centralized decision maker, the manufacturer decides the direct price, retail price and the green degree of the product, which is consistent with the centralized decision model.

The manufacturer, as a centralized decision maker, can achieve the optimal profit of the whole supply chain \( \pi_c^* \), which is shown in the fourth part. Note that only when the retailer and the manufacturer obtain at least the reservation profit in the decentralized model without misreporting do they accept the manufacturer’s contract. Hence, we can obtain the following constraints:

\[
\left\{ \begin{array}{l}
\tau \pi_c^* > \pi_{Rh}^* \\
(1 - \tau) \pi_s^* > \pi_{mh}^*
\end{array} \right. \tag{51}
\]

From inequality (51), we can observe that the value range of \( \tau \) is \([ \frac{\pi_{Rh}^*}{\pi_c^*}, \frac{\pi_s^* - \pi_{mh}^*}{\pi_c^*} ]\), that is, when \( \frac{\pi_{Rh}^*}{\pi_c^*} < \tau < \frac{\pi_s^* - \pi_{mh}^*}{\pi_c^*} \), the revenue-sharing contract can realize the coordination of the whole supply chain.

### 6. Numerical Experiments

In this section, using numerical experiments, we provide further managerial implications to prove the Propositions given above. Given the parameters employed in these studies, we let \( a = 2.5, \delta = 0.5, \theta_1 = \theta_2 = 0.6 \) and \( \beta_1 = \beta_2 = 1 \) illustrate the conclusions in our study. The experimental results are shown in Figures 5–9.
on the green degree of the product. Therefore, the price elasticity of the retail channel. Consequently, the manufacturer should have a deep understanding of the market before making a decision and try their best to meet consumers' demand for environmental protection.

The influence of \( \delta \) on the green degree of the product when the manufacturer misreports the green degree of the product, the optimal green degree of the product is the highest under the centralized decision-making setting. Moreover, when the manufacturer should pay more attention to the price elasticity of products in direct sales channels when they misreport the green degree of the product. This is because the retail channel has almost no influence on the decision on the green degree of the product, which will have negative effects on consumers and the environment.

**Figure 5.** The influence of \( \delta \) on optimal decisions.

**Figure 6.** The influence of \( \beta \) on the green degree of the product.

**Figure 7.** The influence of \( \theta \) on the green degree of the product.
From Figure 5, the following conclusions can be drawn: first, the direct price under centralized decision making is higher than that under decentralized decision making, while the retail price is lower than that under decentralized decision making. As for the green degree of the product, it is obvious that the green degree of the product is the highest under the centralized decision-making setting and is not affected by $\delta$. In addition, when the manufacturer misreports the green degree of the product, the optimal green degree is the lowest and is significantly affected by the market share of the retail channel. Furthermore, there is a negative relationship between the price of the direct channel and $\delta$, and a positive relationship between the price of the retail channel and $\delta$.

The above numerical analysis can provide some management insights for the decision makers of the enterprise. Obviously, under the background of decentralized decision making, there are double marginal effects in the supply chain, which will lead to lower resource utilization efficiency. Therefore, under the background of centralized decision making, the green degree of the product is higher than that of the product under the background of decentralized decision making, which is the fundamental reason why the direct selling price of the product under centralized decision making is higher than the direct selling price of the product under decentralized decision making. However, it should be noted that due to the influence of the double marginal effect, the retail price of the product under the background of decentralized decision making is higher than that under the background of centralized decision making, even though the green degree of the product is lower, which is harmful to consumers.

Second, the retailer and the manufacturer must pay attention to the impact of the retail channel market share on the process of competition. Specifically, the higher the market share of the retail channel is, the more favorable the retailer will be in the market, which can enable the retailer to set higher retail prices and obtain higher marginal profits.
addition, the retailer must be cautious about the manufacturer’s misreporting behavior, since it will not only induce the retailer to make wrong decisions, but also reduce the green degree of the product, which will have negative effects on consumers and the environment.

In addition to the conclusions obtained in Figure 5, some other conclusions can be drawn from Figure 6. First, there is a negative correlation between the optimal green degree and price elasticity, which means that the manufacturer should carefully analyze the characteristics of the product when deciding the greenness degree of the product. Specifically, if the product is a luxury, then the manufacturer should determine a lower degree of greenness; conversely, if the product is a necessity, the manufacturer should determine a higher degree of greenness.

Second, when the manufacturer misreports the green degree of the product, $\beta_2$, the price elasticity of the direct channel, will have a greater impact on the green degree of the product compared with $\beta_1$, the price elasticity of the retail channel. Therefore, the manufacturer should pay more attention to the price elasticity of products in direct sales channels when they misreport the green degree of the product. This is because the retail channel adopts the mode of resale, which means that even if consumers in the retail channel perceive the real green degree of the product, it will not affect the number of orders in the second period of the retailer. However, the consumer in the direct channel will have a direct impact on the manufacturer’s sales.

Similarly, the elasticity coefficient of the product’s green degree has a significant impact on the optimal green degree of the product, as shown in Proposition 4 and Figure 7. Specifically, the more sensitive consumers are to the green degree of the product, whether in the direct or the retail channel, the higher the green degree of the product. Thus, the manufacturer should have a deep understanding of the market before making a decision and try their best to meet consumers’ demand for environmental protection.

Besides, similar to the conclusion in Figure 5, the green degree of the product is the highest under the centralized decision-making setting. Moreover, when the manufacturers misreport the green degree of the product, the elasticity coefficient of consumers in the retail channel has almost no influence on the decision on the green degree of the product, while the elasticity coefficient of consumers in the direct channel has a significant influence on the decision on the green degree of the product.

As shown in Figure 8, the cost coefficient of the green degree of the product will have a negative impact on the price and the green degree of the product. Specifically, the increase in $k$ will lead to the decrease in the price and the green degree of the product. Therefore, it is advisable for company managers to combine the increase in the price of the product with the decrease in the green degree of the product, since this will neither lead to a significant reduction in the greenness of the product nor the profit of the enterprise.

Moreover, the cost coefficient $k$ has a small impact on $e^*_{h_r}$, since in this context, the manufacturer can reduce the impact of the change in the cost coefficient $k$ through the false positive strategy. In other words, as the cost coefficient $k$ increases, the manufacturer will misreport the greenness of the product to a greater extent. Therefore, for the retailer, when the cost coefficient $k$ is large enough, they should pay more attention to the product information published by the manufacturer and carefully distinguish its authenticity.

Figure 9 shows the relationship between the optimal profit of the manufacturer, the retailer and the whole supply chain, and shows the feasible range of contract coordination. From Figure 9, several interesting conclusions can be drawn. First, under the decentralized decision-making setting, the manufacturer will make more profits by misreporting the green degree of the product, while the retailer’s profits will be reduced. Second, the increase in the manufacturer’s profit is lower than the decrease in the retailer’s profit, which leads to the increase in the whole supply chain profit when the manufacturer misreports the green degree of the product. This means that the retailer must take measures to coordinate the decision making of the supply chain and prevent the manufacturer from adopting false-reporting strategy.
Then, in decentralized decision making, whether the manufacturer misreports the green degree of the product or not, there is a double marginalization effect. That is, the supply chain obtains the highest profit against the background of centralized decision making. Then, for member companies in the supply chain, they should actively coordinate to obtain more profits. Finally, there is a certain interval that makes the revenue-sharing contract effectively coordinate the decisions of the manufacturer and the retailer, so that the manufacturer and the retailer can obtain higher profits, which is shown in Section 5.

7. Conclusions

In this paper, we consider the issue of misreporting in the supply chain with the manufacturer selling green products through direct and retail channels. We obtained some results that are significantly different from the existing literature. First, the manufacturer’s misreporting strategy has no effect on product pricing decisions, either in direct or retail channels. However, the green degree of the product will be affected by the manufacturer’s misreporting strategy. Specifically, the manufacturer’s misreporting strategy will lead to the reduction in the green degree of the product. Therefore, for the government and regulatory authorities, it is necessary to strengthen the supervision of the green supply chain, since enterprises in the green supply chain are motivated to misreport their energy saving and emission reduction levels, which will have adverse effects on other enterprises in the supply chain, on consumers and on the environment. A variety of measures can be adopted to encourage enterprises in the green supply chain to give up the misreporting strategy, such as subsidies for energy conservation and emission reduction and policies to punish misreporting behavior.

Second, misreporting the green degree of products can bring more profits for the manufacturer, and thus, the manufacturer has an incentive to misreport the green degree of the product. However, for the retailer and the whole supply chain, the manufacturer’s misreporting strategy will result in the loss of profits. Obviously, for the retailer or other enterprises in the green supply chain, proactive measures must be taken, such as providing coordinated contracts or adopting technical means to share information in the green supply chain, to prevent the manufacturer from adopting the strategy of misreporting.

Besides, when certain conditions are met, the revenue-sharing contract can effectively discourage the manufacturer from engaging in the misreporting strategy and bring positive effects to the profits of the retailer and the whole supply chain, which provides a useful reference for the prevention of misreporting behavior in the green supply chain.

There are several limitations to our study. Firstly, we assume that the market demand for direct channel and retail channel is linear to the price and the green degree of the product. However, in practice, the market demand is uncertain, and companies in the supply chain are faced with more complex random market demand, which can be studied as a future research topic. In future research, the case of random market demand can be considered, which may lead to some more interesting and unexpected conclusions. Secondly, in this study, the supply chain is composed of a manufacturer and a retailer, which is obviously a simplified version of the actual situation. In practice, the supply chain may be composed of multiple manufacturers and retailers, which is more complex than the model in this paper. Therefore, more market subjects can be included into the model in future studies, such as competing manufacturers or competing retailers, government departments and third-party regulatory departments, which will provide more effective guidance and reference for the prevention of misreporting.

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Appendix A

Proof of Corollary 1. Since the optimal retail price and direct price against the background of centralized decision making are as follows:

\[ p_{1c}^{s} = \frac{a\{k\beta \theta_{2} + \theta_{2}(1 - \delta)\theta_{1} - \delta \theta_{2}\}}{2\beta_{1}(k\beta \theta_{2} - \theta_{2}) - 2\beta_{2}\theta_{1}^{2}} \]  \hspace{1cm} (A1)

\[ p_{2c}^{s} = \frac{a\{(k - k\delta)\beta_{1} - \theta_{1}(1 - \delta)\theta_{1} - \delta \theta_{2}\}}{2\beta_{1}(k\beta \theta_{2} - \theta_{2}) - 2\beta_{2}\theta_{1}^{2}} \]  \hspace{1cm} (A2)

Therefore, from Assumption 3, we can conclude that when \( \beta_{1} > \frac{k\beta \theta_{2} + (\delta_{1} + \theta_{2})(1 - \delta)\theta_{1} - \delta \theta_{2}}{k(1 - \delta)} \), \( p_{1c}^{s} < p_{2c}^{s} \), vice versa. \( \square \)

Proof of Proposition 1. The manufacturer’s and the retailer’s optimal profit can be expressed as follows:

\[ \pi_{md}^{l} = w^{l}(\delta a - \beta_{1}p_{1}^{l} + \theta_{1}e) + p_{2}^{l}(1 - \delta)a - \beta_{2}p_{2}^{l} + \theta_{2}e] - \frac{ke^{2}}{2} \]  \hspace{1cm} (A3)

\[ \pi_{rd}^{l} = (p_{1}^{l} - w^{l})(\delta a - \beta_{1}p_{1}^{l} + \theta_{1}e) \]  \hspace{1cm} (A4)

\[ \pi_{md}^{d} = w^{d}(\delta a - \beta_{1}p_{1}^{d} + \theta_{1}e) + p_{2}^{d}(1 - \delta)a - \beta_{2}p_{2}^{d} + \theta_{2}e] \]  \hspace{1cm} (A5)

\[ \pi_{rd}^{d} = (p_{1}^{d} - w^{d})(\delta a - \beta_{1}p_{1}^{d} + \theta_{1}e) \]  \hspace{1cm} (A6)

Next, we will solve the optimal decision in the second period by backward induction. Since \( \frac{d^{2}\pi_{md}^{l}}{dp_{1}^{l}} = -2\beta_{1} < 0 \) and \( \frac{d^{2}\pi_{md}^{d}}{dp_{1}^{d}} = -2\beta_{1} < 0 \), the optimal retail price and the optimal direct price are shown as follows:

\[ p_{1d}^{s} = \frac{a\delta + e\theta_{1} + \beta_{1}w^{s}}{2\beta_{1}} \]  \hspace{1cm} (A7)

\[ p_{2d}^{s} = \frac{a - a\delta + e\theta_{2}}{2\beta_{2}} \]  \hspace{1cm} (A8)

Therefore, we can rewrite the manufacturer’s profit function in the second period as

\[ \pi_{md}^{s} = \frac{a^{2}(1 - \delta)^{2} + 2ae(1 - \delta)\theta_{2} + e^{2}\theta_{2}^{2} + 2\beta_{2}\theta_{1}^{2}(a\delta + e\theta_{1} - \beta_{1}w^{s})}{4\beta_{2}} \]

Since \( \frac{d^{2}\pi_{md}^{s}}{dw^{s}} = -\beta_{1} < 0 \), \( \pi_{md}^{s} \) is a convex function about \( w^{s} \), that is, the optimal wholesale price can be obtained through the first derivative.
Therefore, by solving $\frac{\partial \pi_{md}^s}{\partial w_s} = 0$, we can obtain the optimal wholesale price as follows:

$$w_s^* = \frac{a \delta + e \theta_1}{2 \beta_1} \tag{A9}$$

By substituting (A9) into (A7) and (A8), $\pi_{md}^s$ and $\pi_{Rd}^s$, the optimal retail price, the optimal direct price and the optimal profits of the manufacturer and the retailer in the second stage can be obtained as follows:

$$p_{1d}^s = \frac{3(a \delta + e \theta_1)}{4 \beta_1} \tag{A10}$$

$$p_{2d}^s = \frac{a - a \delta + e \theta_2}{2 \beta_2} \tag{A11}$$

$$\pi_{md}^s = \frac{\beta_2 (a \delta + e \theta_1)^2 - 2 \beta_1 [a(1 - \delta) + e \theta_2]^2}{8 \beta_1 \beta_2} \tag{A12}$$

$$\pi_{Rd}^s = \frac{(a \delta + e \theta_1)^2}{16 \beta_1} \tag{A13}$$

Similarly, the optimal retail price, the optimal direct price and the optimal profits of the manufacturer and the retailer in the second stage can be obtained as follows:

$$w_d^f = \frac{a \delta + e \theta_1}{2 \beta_1} \tag{A14}$$

$$p_{1d}^f = \frac{3(a \delta + e \theta_1)}{4 \beta_1} \tag{A15}$$

$$p_{2d}^f = \frac{a - a \delta + e \theta_2}{2 \beta_2} \tag{A16}$$

$$\pi_{md}^f = \frac{1}{4} \{ -e^2 k + \frac{(a \delta + e \theta_1)^2}{4 \beta_1} - \frac{[a(1 - \delta) + e \theta_2]^2}{\beta_2} \} \tag{A17}$$

$$\pi_{Rd}^f = \frac{(a \delta + e \theta_1)^2}{16 \beta_1} \tag{A18}$$

Therefore, the total profit function of the manufacturer in the two periods is

$$\pi_{md} = \pi_{md}^f + \pi_{md}^s$$

Since $\frac{d^2 \pi_{md}}{dw_s^2} = -k + \frac{\delta^2_1}{2 \beta_1} + \frac{\delta^2_2}{\beta_2} < 0$, the optimal green degree is

$$e_d^* = \frac{a(\delta \beta_1 \theta_1 + 2 \beta_1 \theta_2 - 2 \delta \beta_1 \theta_2)}{2k \beta_1 \beta_2 - \beta_2^2 \beta_1^2 - 2 \beta_1 \theta_2^2} \tag{A19}$$

Substituting (A19) into (A9)–(A18), the optimal retail price and direct price can be obtained. Thus, the conclusion of Proposition 1 is proved. □

**Proof of Proposition 2.** If the manufacturer misreports the green degree of the product, the manufacturer and the retailer make decisions according to the announced profit functions.
Since $\frac{d^2\Psi_{mh}}{dp_1^2} = -2\beta_1 < 0$ and $\frac{d^2\Psi_{mh}}{dp_2^2} = -2\beta_2 < 0$, the optimal retail price and the optimal direct price are shown as follows:

$$p_{1h}^* = \frac{a\delta + e(1+\epsilon)\theta_1 + \beta_1 w^g}{2\beta_1} \quad (A20)$$

$$p_{2h}^* = \frac{a - a\delta + e(1+\epsilon)\theta_2}{2\beta_2} \quad (A21)$$

Therefore, we can rewrite the manufacturer’s profit function in the second period as

$$\Psi_{mh} = \frac{a^2(1-\delta)^2 + 2ae(1-\delta)(1+\epsilon)\theta_2 + e^2(1+\epsilon)^2\theta_2^2 + 2\beta_2 w^g[a\delta + e(1+\epsilon)\theta_1 - \beta_1 w^g]}{4\beta_2}$$

Since $\frac{d^2\Psi_{mh}}{dw_{1h}^2} = -\beta_1 < 0$, $\Psi_{mh}^f$ is a convex function about $w^f$, that is, the optimal green degree and wholesale price can be obtained through the first derivative.

Therefore, by solving $\frac{d\Psi_{mh}^f}{dw_{1h}} = 0$, we can obtain the optimal wholesale price as follows:

$$w_{1h}^* = \frac{a\delta + e\theta_1 + e\epsilon\theta_1}{2\beta_1} \quad (A22)$$

By substituting (A22) into (A20) and (A21), $\pi_{mh}^f$ and $\pi_{rkh}^f$, the optimal retail price, the optimal direct price and the optimal profits of the manufacturer in the second stage can be obtained as follows:

$$p_{1h}^f = \frac{3[a\delta + e(1+\epsilon)\theta_1]}{4\beta_1} \quad (A23)$$

$$p_{2h}^f = \frac{a - a\delta + e(1+\epsilon)\theta_2}{2\beta_2} \quad (A24)$$

$$\pi_{mh}^f = \frac{1}{8} \left\{ \frac{[a\delta + e(1+\epsilon)\theta_1]^2}{\beta_1} + 2[a^2(1-\delta)^2 + 2ae(1-\delta)\theta_2 + e^2(1-\epsilon^2)\theta_2^2] \right\} \quad (A25)$$

Similarly, the optimal retail price, the optimal direct price and the optimal profits of the manufacturer and the retailer in the first period can be obtained as follows:

$$p_{1h}^f = \frac{3[a\delta + e(1+\epsilon)\theta_1]}{4\beta_1} \quad (A26)$$

$$p_{2h}^f = \frac{a - a\delta + e(1+\epsilon)\theta_2}{2\beta_2} \quad (A27)$$

$$w_{1h}^f = \frac{a\delta + e\theta_1 + e\epsilon\theta_1}{2\beta_1} \quad (A28)$$

$$\pi_{mh}^f = \frac{1}{8} \left\{ -4e^2k + \frac{[a\delta + e(1+\epsilon)\theta_1]^2}{\beta_1} + \frac{2[a - a\delta + e(1+\epsilon)\theta_2]^2}{\beta_2} \right\} \quad (A29)$$

$$\pi_{rkh}^f = \frac{[a\delta + e(1+\epsilon)\theta_1]^2}{16\beta_1} \quad (A30)$$

Similar to Proposition 1, the optimal green degree is as follows:

$$e = \frac{a[a\delta + 2(1-\delta)\theta_1]\beta_1}{(1+\epsilon)[2\beta_1(k\beta_2 - \theta_2^2) - \beta_2\theta_2]} \quad (A31)$$

Next, the manufacturer determines the optimal $\epsilon$ to maximize the total profit of the two periods.
\[ \frac{d\pi_{\text{mh}}}{d\epsilon} = a^2[\delta \beta_2 \theta_1 + 2(1-\delta)\beta_1 \theta_2][2k \delta \beta_2 + \theta_2(2k(1-\delta)(1+\epsilon)\beta_1 - (1+\epsilon)\theta_1[\delta \theta_2 - (1-\delta)\theta_1]]}{2(1+\epsilon)^3/[\beta_2 \theta_2^2 - \beta_1(2k \beta_2 - 2\theta_2^2)]^2} \]

Since the green degree of the product is greater than 0, we can conclude from Equations (A19) and (A31) that \((1+\epsilon)\) is greater than 0. Thus, we only need to determine the sign of 

\[ 2k \delta \beta_2 \theta_1 + \theta_2(2k(1-\delta)(1+\epsilon)\beta_1 - (1+\epsilon)\theta_1[\delta \theta_2 - (1-\delta)\theta_1]) \]

When \(\epsilon < \frac{2k \delta \beta_2 \theta_1 + \theta_2(2k(1-\delta)\beta_1 + \theta_1(\delta \theta_2 - (1-\delta)\theta_1))}{\theta_2(2k(1-\delta)\beta_1 + \theta_1(\delta \theta_2 - (1-\delta)\theta_1))} \) is satisfied, we can conclude that \(2k \delta \beta_2 \theta_1 + \theta_2(2k(1-\delta)(1+\epsilon)\beta_1 - (1+\epsilon)\theta_1[\delta \theta_2 - (1-\delta)\theta_1]) > 0\), and vice versa. Therefore, we can conclude that \(\pi_m\) is positively correlated with \(\epsilon\) when \(\epsilon\) is greater than \(\frac{2k \delta \beta_2 \theta_1 + \theta_2(2k(1-\delta)\beta_1 + \theta_1(\delta \theta_2 - (1-\delta)\theta_1))}{\theta_2(2k(1-\delta)\beta_1 + \theta_1(\delta \theta_2 - (1-\delta)\theta_1))}\), otherwise, \(\pi_m\) decreases with the increase in \(\epsilon\).

Therefore, the optimal level of misreporting determined by the manufacturer is

\[ \epsilon^* = \frac{2k \delta \beta_2 \theta_1 + \theta_2(2k(1-\delta)\beta_1 + \theta_1(\delta \theta_2 - (1-\delta)\theta_1))}{\theta_2(2k(1-\delta)\beta_1 + \theta_1(\delta \theta_2 - (1-\delta)\theta_1))} \] (A32)

Substituting (A32) into (A22)-(A31), the optimal retail price and direct price can be obtained. Thus, the conclusion of Proposition 2 is proved. \(\square\)

**Proof of Corollary 2.** Similar to Corollary 1, Corollary 2 can be proved. \(\square\)

**Proof of Proposition 3.** Take the first derivative of \(p_{i,c}^{f}, p_{i,j}^{f}, w_{i}^{f}, w_{h}^{f}, e_{c}^{f}, e_{d}^{f}\) and \(e_{h}^{f}\) with respect to \(\beta_1\), and the following equations can be obtained:

\[ \frac{dp_{i,c}^{f}}{d\beta_1} = -2a(k \beta_2 - \theta_2^2)\{k \delta \beta_2 + \theta_2[(1-\delta)\theta_1 - \delta \theta_2] \} < 0 \] (A33)

\[ \frac{dp_{i,j}^{f}}{d\beta_1} = -a \theta_1 \theta_2 \{k \delta \beta_2 + \theta_2[(1-\delta)\theta_1 - \delta \theta_2] \} < 0 \] (A34)

\[ \frac{dp_{i,d}^{f}}{d\beta_1} = -12a(k \beta_2 - \theta_2^2)[k \delta \beta_2 + \theta_2[(1-\delta)\theta_1 - \delta \theta_2] \} < 0 \] (A35)

\[ \frac{dw_{i}^{f}}{d\beta_1} = -2a(k \beta_2 - \theta_2^2)[k \delta \beta_2 + \theta_2[(1-\delta)\theta_1 - \delta \theta_2] \} < 0 \] (A37)

\[ \frac{de_{c}^{f}}{d\beta_1} = -\frac{a \beta_2 \theta_1 \{k \delta \beta_2 + \theta_2[(1-\delta)\theta_1 - \delta \theta_2] \}}{[\beta_2 \theta_2^2 - \beta_1(2k \beta_2 - 2\theta_2^2)]^2} < 0 \] (A38)

\[ \frac{de_{d}^{f}}{d\beta_1} = -\frac{a \theta_1 \theta_2 \{k \delta \beta_2 + \theta_2[(1-\delta)\theta_1 - \delta \theta_2] \}}{[\beta_2 \theta_2^2 - \beta_1(2k \beta_2 - 2\theta_2^2)]^2} < 0 \] (A39)

\[ \frac{de_{h}^{f}}{d\beta_1} = -\frac{a \beta_2 \theta_1 \{k \delta \beta_2 + \theta_2[(1-\delta)\theta_1 - \delta \theta_2] \}}{[\beta_2 \theta_2^2 - \beta_1(2k \beta_2 - 2\theta_2^2)]^2} < 0 \] (A40)

Similarly, the derivatives of \(p_{i,c}^{f}, p_{i,j}^{f}, w_{i}^{f}, w_{h}^{f}, e_{c}^{f}, e_{d}^{f}\) and \(e_{h}^{f}\) with respect to \(\beta_2\) are also less than 0. Thus, Proposition 3 is proved. \(\square\)

**Proof of Proposition 4.** Since the proof process of Proposition 3 is similar to that of Proposition 4, we omit it. \(\square\)
Proof of Proposition 5. The proof process of Proposition 5 is similar to that of Proposition 4, so we omit it. □

Proof of Proposition 6. Since the proof of (41) and (42) is similar to the proof of Proposition 6, we omit it and only show the proof process of (43).

Take the first derivative of \( e^*_c \) and \( e^*_d \) with respect to \( \delta \), and the following equations can be obtained:

\[
\frac{de^*_c}{d\delta} = \frac{a\beta_2\theta_1 - a\beta_1\theta_2}{\beta_1(k\beta_2 - \theta_2^2) - \beta_2\theta_1^2} \quad (A41)
\]

Since the denominator is greater than 0, we only need to analyze the sign of the numerator.

Obviously, when \( \beta_2 > \frac{\theta_2\beta_1}{\theta_1} \), \( \frac{de^*_c}{d\delta} > 0 \); Otherwise \( \frac{de^*_c}{d\delta} < 0 \). Similarly, when \( \beta_2 > \frac{2\theta_2\beta_1}{\theta_1} \), \( \frac{de^*_d}{d\delta} > 0 \); Otherwise \( \frac{de^*_d}{d\delta} < 0 \). Thus, Proposition 6 is proved. □

Proof of Proposition 7. The proof process of Proposition 7 is similar to that of Proposition 6, so we omit it. □

Proof of Theorem 1. By comparing and analyzing the optimal solutions of different models, the conclusion of Theorem 1 can be obtained. □

Proof of Theorem 2. Through the above solution, we can obtain the following equation:

\[
p_{IC}^* - p_{id}^* = \frac{a(k\beta_2 + \theta_2)[(1-\delta)\theta_1 - \delta\theta_2]}{2\beta_1(k\beta_2 - \theta_2^2) - 2\beta_2\theta_1^2} - \frac{3a(k\beta_2 + \theta_2)[(1-\delta)\theta_1 - \delta\theta_2]}{4\beta_1(k\beta_2 - \theta_2^2) - 2\beta_2\theta_1^2} \quad (A42)
\]

Apparently, \( p_{IC}^* - p_{id}^* < 0 \), that is, \( p_{IC}^* < p_{id}^* \).

Similarly,

\[
p_{IC}^* - p_{id}^* = \frac{a[k\beta_2(1-\delta) - \theta_1][(1-\delta)\theta_1 - \delta\theta_2]}{2\beta_1(k\beta_2 - \theta_2^2) - 2\beta_2\theta_1^2} - \frac{a[2k\beta_2(1-\delta) - \theta_1][(1-\delta)\theta_1 - \delta\theta_2]}{4\beta_1(k\beta_2 - \theta_2^2) - 2\beta_2\theta_1^2} \quad (A43)
\]

Apparently, \( p_{IC}^* - p_{id}^* > 0 \), that is, \( p_{IC}^* > p_{id}^* \).

Besides,

\[
e^*_c - e^*_d = \frac{a\beta_2\theta_1 + a(1-\delta)\beta_1\theta_2}{\beta_1(k\beta_2 - \theta_2^2) - \beta_2\theta_1^2} - \frac{a\beta_2\theta_1 + 2a(1-\delta)\beta_1\theta_2}{2\beta_1(k\beta_2 - \theta_2^2) - 2\beta_2\theta_1^2} \quad (A44)
\]

Thus, \( e^*_c > e^*_d > e^*_h \). The conclusion of Theorem 2 is proved. □

Proof of Theorem 3. By comparing and analyzing the optimal profits of the manufacturer and the retailer in each model, we can obtain the following equation:

\[
\pi_{mh}^* - \pi_{md}^* = \frac{a^2[2k\delta\beta_2\theta_1 + \theta_2[2k(1-\delta)\beta_1 + \theta_1(\theta_1 - \delta\theta_2)]]^2}{8k[\beta_2\theta_1^2 - 2\beta_1(k\beta_2 - \theta_2^2)]^2} > 0 \quad (A46)
\]

\[
\pi_{Rd}^* - \pi_{Rh}^* = \frac{3a^2\theta_1[k\delta\beta_2\theta_2 - \theta_2(\delta\theta_2 - (1-\delta)\theta_1)] + 2k\delta\beta_2\theta_2 - \theta_2(\delta\theta_2 - (1-\delta)\theta_1) - 2k(1-\delta)\beta_1}{4k[\beta_2\theta_1^2 - 2\beta_1(k\beta_2 - \theta_2^2)]^2} > 0 \quad (A47)
\]

Thus, the conclusion of Theorem 3 is proved. □
Proof of Theorem 4. By comparing and analyzing the optimal profits of the supply chain in each model, we can obtain the following equation:

$$\pi_{sd}^* - \pi_{sh}^* = \frac{a^2 \{8k^2 \delta^2 \beta^2 \theta^2 + 2k \delta \beta_1 \theta_1 \{2k(1 - \delta) \beta_1 + 7 \theta_1 [(1 - \delta) \theta_1 - \delta \theta_2]\} + \theta_1^2 \{8k(1 - \delta) \beta_1 \theta_1 [(1 - \delta) \theta_1 - \delta \theta_2] + 5 \theta_1^2 [(1 - \delta) \theta_1 - \delta \theta_2]^2 - 4k^2 (1 - \delta)^2 \beta_1^2\}}{8k(\beta_2^2 \theta_1^2 + \beta_1 (-2k \beta_2^2 + 2 \theta_2^2))^2}$$  \hspace{1cm} (A48)

Moreover, when the assumptions are satisfied in Section 3, it is easy to prove that Equation (A49) is greater than 0, so we can draw the conclusion $\pi_{sd}^* > \pi_{sh}^*$.

Similarly,

$$\pi_{c}^* - \pi_{sd}^* = \frac{a^2 \beta_1^2 (k \beta_2^2 - \theta_2^2) \{k \delta \beta_2 + \theta_2 [(1 - \delta) \theta_1 - \delta \theta_2]\}^2}{2[\beta_1 (k \beta_2^2 - \theta_2^2) - \beta_2^2 \theta_2^2] [\beta_2^2 \theta_2^2 - \beta_1 (2k \beta_2^2 - 2 \theta_2^2)]^2} > 0$$  \hspace{1cm} (A49)

Therefore, Theorem 4 is proved. $\Box$

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