Pricing and Contract Coordination of BOPS Supply Chain Considering Product Return Risk

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Abstract: This study proposes a BOPS (buy online and pick up in store) model for a two-echelon supply chain, considering the product return risk to reach an equilibrium result of two omni-channel integration modes, managed by the manufacturer or the retailer, respectively. Furthermore, this study analyzes the impacts of the service cost coefficient and the consumer loyalty of the offline channel on the optimal price, service decision, and profit of different integration modes. This study also compares the profits of enterprises under centralized and decentralized decisions. The results show that centralized decision making can improve the service level of the SA (Store Assistance) effort and the sustainable performance of the supply chain. Whether the BOPS channel is managed by the manufacturer or retailer, the service cost, the product return rate, and the offline channel consumer loyalty have an impact on the decision of product sale price, wholesale price, and SA effort service level. Offline customer loyalty is less sensitive in the BOPS channel managed by the retailer than that managed by the manufacturer. It is suitable for the manufacturer to manage the BOPS channel when offline customer loyalty is low. Otherwise, it is appropriate for the retailer to manage the BOPS channel. The profit of the supply chain under the decentralized decision is less than that under the centralized decision. When executing a wholesale price contract, different omni-channel integration modes can alleviate the double marginalization effect. It can fully coordinate the supply chain members, motivate retailers to improve the service level of the SA effort, and realize Pareto improvement to support supply chain sustainability.

Keywords: omni-channel supply chain; buy online and pick up in store; return risk; pricing decision; wholesale price contract

1. Introduction

Recently, the Internet economy has been growing rapidly, with enterprises generally using the coexisting marketing model of both online and offline sales. The enterprises open online channel sales in order to expand the scope of propaganda, promote the brand influence, and attract more consumers as well. It is difficult, however, for consumers to accurately evaluate the qualities of goods, relying only on the text, picture, and video information released on online sale channels [1]. In order to relieve consumers’ doubts and encourage purchases, online sale channels adopt a liability-free return policy. If the uncertainty of goods fails to match the consumer’s preferences, then the return of goods happens. However, the product return rate of online sales is high, especially for some goods that need to be tried on personally to know whether the consumers like them [2]. To solve the mismatch issue, some well-known retailers have arranged omni-channel sales, such as Zara, Uniqlo, Suning, GAP and Taiping Bird, etc., which have integrated multiple sales channels [3]. People can buy a complete set of suits online and pick them up offline. Physical stores offer free fitting, trouser leg modification and ironing services. The buy online and pick up in store (BOPS) model brings consumers a higher-quality...
shopping experience [4,5]. Therefore, more and more firms have adopted omni-channel operations for business sustainability [6]. The omni-channel strategy is one that can reduce risks and increase opportunities for firms because it employs various channels, such as offline, online, and mobile [7]. Nearly 70% of US shoppers used BOPS more than once in 2019 [8]. However, offline brick-and-mortar stores provide high-quality service, which often involves high labour costs, management costs, and venue rent. For having a long-term sustainable development of the BOPS models, we need to find answers to the following questions: How can traditional contractual relations motivate offline channels to serve consumers? How can the unified sales price of online and offline channels achieve a win–win situation for all the members of the supply chain? All these factors pose a severe challenge to balance the costs and benefits among all the supply chain members for the sustainable development of BOPS.

In modelling research, Gao and Su investigated the applicable conditions of BOPS from the perspective of inventory management and examined the effect of the omni-channel model on expanding consumer groups [9]. They established an analytical model to explore the impacts of the BOPS strategy on the pricing and expected a profit of the dual-channel retailer, as well as the environment, and found that the retailer can benefit from the BOPS strategy when the operating cost is lower, and the fraction of consumers without channel preference is higher [10]. Liu and Xu determined that in the retailers’ optimal decision in reducing the price and increasing the order quantity before and after opening a BOPS channel, the proportion of online channel buyers increases [11]. Previous studies in BOPS were mainly in retailers [10,11] or manufacturers [9]. Enhancing active coordination among all the supply members is a prerequisite for sustainable supply chain management [12,13]. However, there are a lack of studies in determining the profits of manufacturers, retailers, and supply chains in the situations of centralized decision vs. decentralized decision, the BOPS channel managed by the manufacturer vs. by the retailer, and with vs. without wholesale price contract in omni-channel business. This study attempts to fill this research gap to provide a contribution to the sustainability of supply chain development of BOPS business.

This study proposes a BOPS model for two-echelon supply chains, accounting for the product return risk to achieve the equilibrium results of two omni-channel integration modes, managed by the manufacturer or the retailer, respectively. More specifically, this study attempts to analyze the impacts of the service cost coefficient and the consumer loyalty of the offline channel on the optimal price, service decision, and profit of different integration modes. It also compares the profits of each supply chain member under two omni-channel integration modes. This study contributes to supply chain research in omni-channel sustainable development by providing researchers with a research model that can be extended to conduct further studies. In addition, this study contributes to our knowledge of how to implement an integration mode of different BOPS strategies, as mentioned above. It helps to coordinate the supply chain members, motivate retailers to improve the SA effort service level, and realize Pareto improvement to support the supply chain sustainability in omni-channel business. The remainder of this work is organized as follows. Section 2 reviews the related existing literature. The research problems and the assumptions are described in Section 3. Section 4 analyzes the centralized and decentralized BOPS models by the manufacturer and the retailer, respectively. The omni-channel supply chain coordination mechanism under the wholesale price contract is implemented in Section 5. Section 6 tests the feasibility of the models and investigates the influence of contract coefficient, Sales Assistance (SA) effort service cost coefficient and offline channel consumer loyalty on supply chain members’ decision making and profit by numerical analysis. Section 7 summarizes the findings, provides some theoretical and managerial implications, and proposes some possible issues for future research.
2. Literature Review

2.1. Management of Channels

The management of channels includes dual-channel and omni-channel management. In terms of dual-channel supply chain management, many scholars conducted research from different perspectives, such as the influence of supply chain power structure [14], free riding [15,16], risk aversion [17,18], complementary products [19], logistics service [20] and information forecasting [21]. Ofek found that opening online channels would improve the service level of stores, thus, reducing the product return rate, but damaging the profits of retailers when retailers were highly competitive [22]. From the perspective of consumers, Chiang found that direct sales channels could improve the total profit of suppliers by reducing the double marginalization [23].

In terms of omni-channel supply chain management, some studies suggested that BOPS strategy would enhance the overall income of the retailers. Gallino and Moreno concluded, based on the data of a large omni-channel retail store, that BOPS could improve the accuracy of inventory information and promote the extra consumption behavior of offline channel buyers [24]. Cao et al. analyzed the influence of BOPS on demand for different channels, product price and profit, and indicated that online retailers would bring themselves higher profits by implementing BOPS alone, while dual-channel retailers would not directly benefit from providing BOPS [25]. Bell analyzed the impact of hall effect on its operation efficiency, relying on the data of American retailer WarbyParker [26]. Fan et al. developed game models dominated by manufacturers and retailers, respectively, from the perspective of merchants’ up-selling behavior, and discussed the optimal decision and profit of enterprises implementing BOPS [27].

However, some scholars suggested that implementing the BOPS strategy would not always improve the sustainable performance of retailers. For example, Gao and Su, based on the newsboy model, analyzed the implementation conditions of BOPS from the perspective of inventory management and proposed that only products with poor sales in physical stores should adopt the BOPS Model [9]. Shi et al. analyzed the interactive impact of implementing BOPS on product pre-sale and product return policies and found that implementing the BOPS strategy would reduce the retailer’s revenue [28]. Zhang et al. discussed the impact of consumers’ returning and cancelling orders on product pricing and inventory decisions when retailers implemented the BOPS strategy [1]. The study concluded that the implementation of the BOPS strategy was not good for the retailers when the operating cost of offline entity pickup was high. Jin et al. analyzed the impact of the BOPS strategy on the size of the offline store service area and found that the ratio of inventory cost per unit product to consumer arrival rate determined the size of the BOPS service area [29]. Through a survey of 436 potential customers, Kim et al. found that performance expectations, job expectations and convenience conditions would affect consumers’ willingness to use the BOPS mode of buying [30]. Saha and Bhattacharya discussed an inventory control strategy when stores have walk-in orders and BOPS orders [31]. When online channels and physical stores are under different ownership, the stores may choose an inventory policy that favours walk-in orders over BOPS orders.

2.2. Product Return Risks

Relevant studies mainly focused on the aspects of refund and compensation policy and the optimal strategy of supply chain members. Some scholars have studied the product return policy and its effect on supply chain performance [32–35]. Some analyzed the effect of product return behaviour on wholesale price, ordering and supply chain coordination [36–38]. Further, some other studies discussed strategies of multi-channel retailers [22], product return risks [39], product return competition [40], and consumers’ purchase delay decisions [41]. Most of the papers argued that product returns are due to the uncertainty in product valuations. Liu et al. discovered that when customers’ sensitivity to pricing is high and the product return rate is low, retailers and manufacturers should oppose
signing return processing contracts [42]. However, when consumers’ sensitivity to pricing and the product return rate is high, retailers and manufacturers should sign contracts to maximize profits. Zhang and Ma compared and analyzed the optimal equilibrium strategy of fresh food e-commerce under different product return modes (refund mode and exchange mode) [43]. No matter which product return mode, the product return amount and the purchase cost have a significant impact on the optimal equilibrium strategy, but the exchange purchase mode is the most favourable for suppliers. Ren et al. constructed four models to study pricing, return policies, and return insurance policies [44]. The study found that when the net residual value of a product is greater than or equal to zero, online retailers should offer a money-back guarantee, but they do not have to offer free product return insurance, as this will not increase their market share or profits. The key related literature are shown in Table 1.

In the existing research on the BOPS model, although some researchers studied pricing and service decision in the supply chain environment, few scholars considered the product return risk factor. Moreover, the research is very limited on how to realize omni-channel supply chain coordination through a wholesale price contract under BOPS mode.

### Table 1. Key Related Literature.

<table>
<thead>
<tr>
<th>Literature</th>
<th>Channel Type</th>
<th>Decision Variables</th>
<th>Demand Characteristics</th>
<th>Game Process</th>
<th>Coordination</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1] Omni-channel</td>
<td>Pricing &amp; Inventory</td>
<td>consumer utility</td>
<td>N/A</td>
<td>return shipping fee sharing</td>
<td></td>
</tr>
<tr>
<td>[9] Omni-channel</td>
<td>Inventory</td>
<td>consumer utility</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[18] Dual-channel</td>
<td>Pricing &amp; Ordering</td>
<td>price-dependent stochastic</td>
<td>Nash</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[19] Dual-channel</td>
<td>Pricing &amp; Service</td>
<td>linear involving service factor</td>
<td>Bertrand; Stackelberg</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[21] Dual-channel</td>
<td>Pricing &amp; Service</td>
<td>consumer utility</td>
<td>Bertrand</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[25] Omni-channel</td>
<td>Pricing</td>
<td>consumer utility</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[28] Omni-channel</td>
<td>Pricing</td>
<td>consumer utility</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[31] Omni-channel</td>
<td>Inventory</td>
<td>consumer utility</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[45] Dual-channel</td>
<td>Pricing</td>
<td>linear</td>
<td>Stackelberg;</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[46] Omni-channel</td>
<td>Pricing &amp; Service</td>
<td>linear involving service factor</td>
<td>Stackelberg</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>[42] Omni-channel</td>
<td>Pricing</td>
<td>linear involving service and the return risk</td>
<td>N/A</td>
<td>wholesale price contract</td>
<td></td>
</tr>
</tbody>
</table>

### 3. Problem Statement

This study considers a supply chain that consists of a manufacturer and a retailer, in which the manufacturer controls the channel on the line and the retailer owns and manages the offline physical stores. Each channel has a certain proportion of loyal consumers, among which the offline channel loyalty of consumers is \( \theta \), \( 0 < \theta < 1 \), that is, the online channel loyalty of consumers is \( 1 - \theta \). The manufacturer sells the product to the retailer at a wholesale price of \( \omega \).

Compared with the online channel, the offline channel can provide on-site explanations and guided experiences for consumers. The SA service helps consumers to meet their visual, tactile, auditory and other sensory experiences, so as to reduce consumers’ uncertainty in product value evaluation. While improved service level of SA can help the retail-
ers gain a comparative competitive advantage, the retailers also have to bear the corresponding cost, including the expenses of hiring professional shopping guides and training, renting places, purchasing experience facilities, and so on.

Consumers’ uncertainty in product value assessment will lead to the return of goods, which will reduce consumers’ expected utility and eventually lead to a decline in market consumer demand. For the uncertainty, this study measures the product return rate by \( r, \theta < r < 1 \) [22]. Assuming that the offline channel does not provide SA service, the product return rate of both the online and the offline channels is the same, taking the product return rate \( r \) of the online channel as the standard, namely \( r_1 = r \). When the offline channel begins to provide SA service, let the product return rate of the offline channel be \( r_2 \), and \( r_2 \) is bound to be less than \( r_1 \). Let \( r_2 = (1 - s)r \) where \( s (\theta < s < 1) \) represents the level of SA service, namely the degree to reduce the product return rate. The cost of improving the service level of SA is expressed as \( 1/2\theta s^2 (\theta \geq 0) \) [47], where \( \theta \) is the cost coefficient. Consumer unit return cost is \( g \), including return time, opportunity, logistics cost, etc. Therefore, in the traditional dual-channel model, the expected return cost of offline channels is \( rg \) while that of the offline channel is \( (1 - s)rg \).

According to the assumption, channel demand can be expressed in the following forms:

The demand function of the online channel market is

\[
D_E = (1 - \varepsilon)(1 - \theta)a - p - b_1rg
\]

The offline channel market demand function is

\[
D_S = \theta a - p - b_1(1 - s)rg
\]

The demand function of the BOPS channel market is

\[
D_B = \varepsilon (1 - \theta)a - p - b_1(1 - s)rg
\]

where, \( a \) represents the total potential market demand. After the implementation of the BOPS model, the demand of consumers who originally preferred offline channel shopping remains unchanged, and the proportion of \( \varepsilon \) consumers who originally preferred online channel shopping become BOPS channel consumers. \( b_1 (b_1 > 0) \) represents the elasticity coefficient of market demand to expected return cost. Then, the above variables are marked with \( C, DE, DS, HE \) and \( HS \), respectively, representing the equilibrium game situation of the centralized decision, the BOPS channel managed by the manufacturer, the BOPS channel managed by the retailer, and the BOPS channel with contract coordination.

4. Model Formulation and Analysis

This section analyzes the solution under the centralized decision and the decentralized decision. The decentralization decision investigates the optimal solutions of the supply chain with two different omni-channel integration modes, the BOPS channel managed by the manufacturer and the BOPS channel managed by the retailer, respectively.

4.1. Centralized Decision

The centralized decision is an ideal situation for channel integration; that is, manufacturers and retailers belong to a unified system. The profit of the supply chain system can be maximized by determining the optimal SA service level \( s \) and the sales price \( p \). The profit of the supply chain can be expressed as follows.

\[
Max \pi^C = p(D_S + D_B + D_E) - 1/2hs^2 = p[a - 3p - 3b_1rg + 2sb_1rg] - 1/2hs^2 \quad (1)
\]

The supply chain’s profit includes the revenue of product sales through the online channel, the offline channels and the BOPS channel, while the supply chain’s expenditure includes the cost of improving the service level of SA.

To derive the optimal solution, obtain the Hessian matrix as follows.
The first-order principal of the Hessian matrix is $D^c_\ell = -\ell < 0$, when the parameters meet the condition $3\ell - 2(b_1 r g)^2 > 0$, and the second-order principal $D^c_{\ell \ell} > 0$. In real business, the cost coefficient $\ell$ is much greater than the expected return cost $g$, and $0 < r < 1$. Therefore, the matrix can be negative definite, and there exists an optimal solution that can maximize the profit.

Take the first-order derivative of the profit function of the supply chain with respect to $p$ and $s$ and set the result to 0. Proposition 1 can be obtained by a simultaneous solution.

$\frac{\partial \pi_{\ell \ell}}{\partial s} = 2p b_1 r g - \ell s$

$\frac{\partial \pi_{\ell \ell}}{\partial p} = a - 6p - 3b_1 r g + 2s b_1 r g$

Proposition 1. When the parameters meet the condition $3\ell - 2(b_1 r g)^2 > 0$, the optimal strategy for implementing BOPS mode under the centralized decision is

$p^{c^*} = \frac{h(a - 3b_1 r g)}{6\ell - 4(b_1 r g)^2}$ (2)

$s^{c^*} = \frac{b_1 r g(a - 3b_1 r g)}{3\ell - 2(b_1 r g)^2}$ (3)

By bringing the optimal unified pricing and service level $(p^{c^*}, s^{c^*})$ in Proposition 1 to the original supply chain profit function, the optimal profit of the supply chain implementing BOPS mode under the centralized decision making can be obtained:

$\pi^{c^*} = p^{c^*}[a - 3p^{c^*} - 3b_1 r g + 2s^{c^*} b_1 r g] - 1/2\ell(s^{c^*})^2 = \frac{h(a - 3b_1 r g)^2}{4[3\ell - 2(b_1 r g)^2]}$

According to Proposition 1, the derivative of the optimal strategy on service cost $\ell$ can be obtained as follows.

Corollary 1. Optimal sales price $p^{c^*}$ and SA service level $s^{c^*}$ are negatively correlated with service cost coefficient $\ell$, respectively.

Corollary 1 shows that, under the centralized decision, the higher the service cost coefficient $\ell$ is, the lower the optimal service level and the optimal product pricing will be. That is, for enterprises that master multiple channels of online stores and offline stores at the same time, the enterprise will reduce the service level and commodity price at the same time to ensure the maximum profit if the service cost increases in the market. The possible reason is that increasing the service cost may lead to a lower service level. On the other hand, enterprises reduce sales prices to stimulate consumer demand, hoping to offset SA service costs through sales revenue.

4.2. Decentralized Decision

In a decentralized omni-channel supply chain, the manufacturer and the retailer are risk-neutral and completely rational individuals, making decisions based on the principle of maximizing their profits. In the Stackelberg Game between the manufacturer and the retailer, the manufacturer first decides the wholesale price $\omega$, and then the retailer decides
the product sales price $p$ and the service level $s$ of SA. This section considers two different omni-channel integration modes, BOPS channels managed by the manufacturer or the retailer, respectively, and analyzes the optimal decision of supply chain members.

### 4.2.1. BOPS Channel Managed by the Manufacturer

The BOPS channel managed by the manufacturer means that the profit from the BOPS orders belongs to the manufacturer. For example, the cooperation between T-mall supermarkets and physical convenience stores adopts this strategy, and the retailer does not need to purchase the BOPS order at the wholesale price. In this case, the manufacturer’s profit can be expressed as follows.

$$
\max \pi_M = \omega D_S + p(D_B + D_G) \\
= \omega[\theta a - p - b_1(1 - s)r_g] + p[(1 - \theta)a - 2p - b_1(2 - s)r_g]
$$

(4)

The manufacturer’s revenue includes the profit of product sales through the online channel and the BOPS channel, and the revenue from the offline channel at the wholesale price.

The retailer’s profit can be expressed as follows.

$$
\max \pi_R = (p - \omega)D_S - 1/2hs^2 \\
= (p - \omega)[\theta a - p - (1 - s)b_1r_g] - 1/2hs^2
$$

(5)

The retailer’s revenue includes the profit of product sales through the offline channel while the expenditure includes the cost of improving the service level of SA and the purchasing cost from the manufacturers with the wholesale price.

The inverse method is adopted to find the equilibrium solution. The second partial derivative of the retailer’s profit on $p$ and $s$ can be obtained below.

$$
H_R = \begin{pmatrix}
\frac{\partial^2 \pi_R}{\partial s^2} & \frac{\partial^2 \pi_R}{\partial s \partial p} \\
\frac{\partial^2 \pi_R}{\partial p \partial s} & \frac{\partial^2 \pi_R}{\partial p^2}
\end{pmatrix} = \begin{pmatrix}
h & b_1r_g \\
b_1r_g & -2
\end{pmatrix}
$$

The first-order principal of the Hessian matrix is $D_R = -h < 0$. When the parameter meets the condition $2h - (b_1r_g)^2 > 0$, the second-order principal $D_R > 0$. In real business, the cost coefficient $h$ is much greater than the expected return cost $g$, and $0 < r < 1$. Therefore, the matrix is judged to be negative definite, and there exists a solution that can maximize profit.

Take the first-order derivative of the retailer profit function with respect to $p$ and $s$ and set as 0. The optimal decision of the retailer can be obtained under a given wholesale price $\omega$.

$$
\frac{\partial \pi_R}{\partial s} = (p - \omega)b_1r_g - hs \\
\frac{\partial \pi_R}{\partial p} = \theta a - 2p - b_1(1 - s)r_g + \omega
$$

Thus, the solution is:

$$
s_{DE}^* = \frac{b_1r_g[\theta a - \omega - b_1r_g]}{2h - (b_1r_g)^2}
$$

(6)

$$
p_{DE}^* = \frac{k[\theta a + \omega - b_1r_g] - \omega(b_1r_g)^2}{2h - (b_1r_g)^2}
$$

(7)

Substitute Formulas (6) and (7) into the manufacturer’s profit formula and find the first and second derivatives of $\omega$.

$$
\frac{\partial \pi_M}{\partial \omega} = \frac{h[(1 + \theta)a - 6p - 4b_1r_g + 3b_1r_g\omega - (b_1r_g)^2(a - 4p - 3b_1r_g + 2b_1r_g\omega)]}{2h - (b_1r_g)^2}
$$
\[
\frac{\partial^2 \pi_M^{DE}}{\partial \omega^{DE^2}} = -2
\]

Since \( \frac{\partial^2 \pi_M^{DE}}{\partial \omega^{DE^2}} < 0 \), the manufacturer profit function is known to be a convex function with respect to \( \omega \), and there is a unique optimal solution.

**Proposition 2.** When \( 2h - (b_2 r g)^2 > 0 \), the optimal strategy for the manufacturer to manage the decentralized decision in the BOPS channel is

\[
\omega_{DE^*} = \frac{[h - (b_1 r g)^2](a - 2\theta a - b_1 r g)}{2[2h - (b_1 r g)^2]}
\]

\[
p_{DE^*} = \frac{[h - (b_1 r g)^2]^2(a - 2\theta a - b_1 r g) + 2h[2h - (b_1 r g)^2](a \theta - b_1 r g)}{2[2h - (b_1 r g)^2]^2}
\]

\[
s_{DE^*} = \frac{(6\theta a - 3b_1 r g - a)b_1 r g h - (4\theta a - b_1 r g - a)(b_1 r g)^3}{2[2h - (b_1 r g)^2]^2}
\]

Put the optimal decision \((\omega_{DE^*}, p_{DE^*}, s_{DE^*})\) in Proposition 2 into the original profit function, and obtain the optimal profit of the supply chain without BOPS mode under decentralized decision:

\[
\pi_M^{DE^*} = \omega_{DE^*}[(\theta a - p_{DE^*} - b_1(1-s_{DE^*})r g) + p_{DE^*}((1-\theta)a - 2p_{DE^*} - b_1(2-s_{DE^*})r g)]
\]

\[
\pi_k^{DE^*} = (p_{DE^*} - \omega_{DE^*})[(\theta a - p - (1-s_{DE^*})b_1 r g) - 1/2h(s_{DE^*})^2]
\]

According to Proposition 2, the derivative of the optimal strategy for the offline channel consumer loyalty \( \theta \) and service cost \( h \) can be obtained as follows.

**Corollary 2.** When the manufacturer manages the BOPS channel, the optimal selling price \( p_{DE^*} \) and SA service level \( s_{DE^*} \) are positively correlated with \( \theta \), and the optimal wholesale price \( \omega_{DE^*} \) is negatively correlated with \( \theta \).

Corollary 2 shows that the retailer would like to keep a higher selling price and provide more SA service level when the consumers more prefer the offline channel. However, the more consumers prefer to shop through offline channels, the fewer benefits manufacturers get from BOPS consumers, and their main profits come from supplying retailers with the wholesale price. At this point, the manufacturer will not raise the wholesale price but will lower the wholesale price to stimulate offline consumer demand. The possible reason is that the market share held by manufacturers is relatively small when consumers prefer to shop through offline channels. In this case, manufacturers are in a weak position, so they cannot make full use of channel advantages to set higher wholesale prices, and tend to adopt a low-price strategy.

**Corollary 3.** When the manufacturer manages the BOPS channel, the optimal sales price \( p_{DE^*} \) and SA service level \( s_{DE^*} \) are negatively correlated with the service cost coefficient \( h \), and optimal wholesale price \( \omega_{DE^*} \) is positively correlated with \( h \).

Corollary 3 shows that with the increase in service cost, the retailer tends to reduce service level and sales price. As the service level continues to decline, the uncertainty of consumer evaluation of product value increases, and consumer demand is greatly affected. The manufacturer, affected by a sharp drop in consumer demand, will cut the wholesale price. This proposition is consistent with Corollary 1, which indirectly confirms the robustness of the proposition.
4.2.2. BOPS Channel Managed by the Retailer

In omni-channel practice, some manufacturers will include all BOPS orders in the sales performance of the offline retailers, in order to improve the service enthusiasm of the offline retailers. For example, in the process of channel integration, Uniqlo (Ube, Japan), a Japanese fast-moving clothing brand owned by Fast Retailing Co., Ltd., will include all online orders of store pick up into the sales volume of stores. Therefore, this section analyzes the coordinated decision of the supply chain when a retailer manages the BOPS channel, and the profit of the manufacturer can be expressed as follows:

\[
\max \pi_M^{DS} = \omega(D_b + D_s) + pD_E
\]

(11)

The manufacturer’s revenue includes the profit of product sales through online channels, and revenue from supplying to offline channels and BOPS channels at the wholesale price.

Retailer profit optimization can be expressed as follows:

\[
\max \pi_R^{DS} = (p - \omega)(D_b + D_s) - \frac{1}{2}hs^2
\]

(12)

The retailer’s revenue includes the profit of product sales through the offline channel and the BOPS channel while the expenditure includes the cost of improving the service level of SA and the cost of buying from manufacturers at the wholesale price.

The inverse method is used to find the equilibrium solution. The second partial derivative of retailer profit on p and s is obtained:

\[
H^{DS} = \begin{pmatrix}
\frac{\partial^2 \pi_R^{DS}}{\partial s^2} & \frac{\partial^2 \pi_R^{DS}}{\partial s \cdot \partial p} \\
\frac{\partial^2 \pi_R^{DS}}{\partial p \cdot \partial s} & \frac{\partial^2 \pi_R^{DS}}{\partial p^2}
\end{pmatrix}
= \begin{pmatrix}
-h & 2b_1r_1g \\
2b_1r_1g & -4
\end{pmatrix}
\]

The first-order principal of the Hessian matrix is: \(D^{DS}_R = -h < 0\). When \(h - (b_1r_1g)^2 > 0\), the second-order \(D^{DS}_R > 0\). In real sales, the cost coefficient \(h\) is much greater than the expected return cost \(g\), and \(0 < r < 1\). Therefore, the matrix is judged to be negative definite and there exists a unique optimal decision value.

Taking the first-order derivative of the retailer profit with respect to \(p\) and \(s\), respectively, and setting it as 0, the optimal decision of the retailer can be obtained simultaneously under a given wholesale price \(\omega\).

\[
\begin{align*}
\frac{\partial \pi_R^{DS}}{\partial s} &= 2(p - \omega)b_1r_1g - hs \\
\frac{\partial \pi_R^{DS}}{\partial p} &= (\theta + \varepsilon - \varepsilon\theta)a - 4p - 2b_1r_1g + 2b_1r_1gs + 2\omega
\end{align*}
\]

Thus, the solution is:

\[
\begin{align*}
s_R^{DS^*} &= \frac{b_1r_1g[(\theta + \varepsilon - \varepsilon\theta)a - 2b_1r_1g - 2\omega]}{2h - 2(b_1r_1g)^2} \\
p_R^{DS^*} &= \frac{[(\theta + \varepsilon - \varepsilon\theta)a - 2b_1r_1g + 2\omega]h - 4\omega(b_1r_1g)^2}{4h - 4(b_1r_1g)^2}
\end{align*}
\]

(13)

(14)

Substitute Formulas (13) and (14) into the manufacturer’s profit formula, and find the first and second derivatives of \(\omega^{RD}\):

\[
\frac{\partial \pi_M^{DS}}{\partial \omega^{DS}} = \frac{h[4(\theta + \varepsilon - \varepsilon\theta)a + 2(1 - \varepsilon)(1 - \theta)a - 12p - 10b_1r_1g + 8b_1r_1gs - 4\omega]}{4h - 4(b_1r_1g)^2}
\]
\[ -4(b_r g)^2[(\theta + \epsilon - \epsilon \theta)a + (1 - \epsilon)(1 - \theta)a - 4p - 3b_r rg + 2b_r rgs] \]
\[
\frac{\partial^2 \pi_M^{DS}}{\partial \omega^{DS}^2} = \frac{-8h[5h - 4(b_r g)^2] + 32(b_r g)^2[6 - 4(b_r g)^2]}{[4h - 4(b_r g)^2]^2}
\]

Since \( \frac{\partial^2 \pi_M^{DS}}{\partial \omega^{DS}^2} < 0 \), the manufacturer profit function is convex with respect to \( \omega^{DS} \), and there is a unique optimal solution. Make \( \frac{\partial^2 \pi_M^{DS}}{\partial \omega^{DS}^2} = 0 \), then the optimal wholesale price for the manufacturer is \( \omega^{DS*} \).

**Proposition 3.** When \( 2h - (b_r g)^2 > 0 \), the optimal strategy for the retailer to manage the decentralized decision making in the BOPS channel is:

\[
\omega^{DS*} = \frac{E_1}{8(b_r g)^4 - 16(b_r g)^2h + 10h^2}
\]
\[
p^{DS*} = \frac{-[4(b_r g)^2 - 2h]E_1 + [8(b_r g)^4 - 16(b_r g)^2h + 10h^2]E_2}{[4h - 4(b_r g)^2][8(b_r g)^4 - 16(b_r g)^2h + 10h^2]}
\]
\[
s^{DS*} = \frac{2b_r rge_1 + [8(b_r g)^4 - 16(b_r g)^2h + 10h^2]e_3}{[2h - 2(b_r g)^2][8(b_r g)^4 - 16(b_r g)^2h + 10h^2]}
\]

Among them
\[ E_1 = \{h^2(\theta a \epsilon - a \epsilon - \theta a - 4b_r r g + 2a) + (b_r g)^2[6 - 4(b_r g)^2](\theta a + a \epsilon - \theta a \epsilon - a + b_r r g)\} \]
\[ E_2 = aeh + a \theta h - aeh \theta - 2b_r r gh \]
\[ E_3 = aeb_r r g + \theta ab_r r g - \theta aeb_r r g - 2(b_r g)^2 \]

By inserting the optimal decision in Proposition 3 into the profit function, the optimal profit of the supply chain without BOPS mode under decentralized decision making can be obtained as follows:

\[
\pi_M^{DS} = \omega^{DS*}[(\theta + \epsilon - \epsilon \theta)a - 2p^{DS*} + 2b_r r gs^{DS*} - 1] + p^{DS*}[(1 - \epsilon)(1 - \theta)a - p^{DS*} - b_r r g]
\]
\[
\pi_p^{DS} = (p^{DS*} - \omega^{DS*})[(\theta + \epsilon - \epsilon \theta)a - 2p^{DS*} - 2b_r r g + 2b_r r gs^{DS*}] - 1 / 2h(s^{DS*})^2
\]

**Corollary 4.** The optimal SA service level in decentralized decision making is lower than that in centralized decision making, whether the manufacturer or the retailer manages the BOPS channel, \( s^{DS*} < s^C, s^{DS*} < s^C \), the omni-channel supply chain cannot realize the coordination under the decentralized decision making. The main reason is that both the manufacturer and the retailer consider the problem from their own interest maximization in decentralized decision making, which leads to a double marginalization effect. This situation is also a common phenomenon in reality, so it is necessary to adopt a certain contract mechanism to coordinate the omni-channel supply chain, so as to maximize the benefits of the whole omni-channel supply chain and all other parties.

5. The BOPS Omni-Channel Supply Chain Coordination and Optimization Research

According to the above analysis, decentralized decision making will cause damage to the benefits of the supply chain system, and the omni-channel supply chain cannot be coordinated without a contract mechanism. Therefore, the manufacturer, as the dominant player in the supply chain, should provide cooperative contracts to competing retailers for centralized pricing, so as to maximize the benefits of the supply chain system. Combined with the characteristics of omni-channel supply chain, manufacturers can introduce a revenue distribution mechanism based on the wholesale price contract to motivate retailers to cooperate in implementing BOPS.

5.1. BOPS Channel Managed by the Manufacturer
When the manufacturer manages the BOPS channel, it is assumed that the revenue distribution coefficient is \(\tau (0 < \tau < 1)\), which depends on the bargaining power of the manufacturer and the retailer. That is, the optimal profit of the whole supply chain distributes the profit part \(\tau\) to the manufacturer and the remaining part \(1 - \tau\) to the retailer.

\[
\pi^M_{HE} = \omega[(\theta a - p - b_1 s)rg] + p[(1 - \theta)a - 2p - b_1 (2 - s)rg] = \tau \pi_S^c \\
\pi^R_{HE} = (p - \omega)[(\theta a - p - (1 - s) b_1 rg] - 1/2 hs^2 = (1 - \tau) \pi_S^c
\]

(18)

In order to coordinate the supply chain, the supply chain members should execute the optimal selling price and the optimal service level of SA under the centralized decision.

\[
\begin{align*}
p^M_{HE*} &= p^{c*} = \frac{h(a - 3b_1 rg)}{6h - 4(b_1 rg)^2} \\
s^M_{HE*} &= s^{c*} = \frac{b_1 rg(a - 3b_1 r g)}{3h - 2(b_1 rg)^2}
\end{align*}
\]

The manufacturer’s optimal wholesale price \(\omega^M_{HE*}\) is the coordinated price. In order to ensure that both parties actively accept the contract, it is necessary to ensure that the profits of both manufacturers and retailers under this contract are Pareto improved; that is, the profits of supply chain members are greater than those under the decentralized decision, so,

\[
\begin{align*}
\pi^M_{HE*}(\omega^M_{HE*}, p^{c*}, s^{c*}) &= \tau \pi_S^{c*} \\
\pi^R_{HE*}(\omega^M_{HE*}, p^{c*}, s^{c*}) &= (1 - \tau) \pi_S^{c*} \\
\pi^M_{HE*}(\omega^M_{HE*}, p^{c*}, s^{c*}) &\geq \pi^D_{M*} \\
\pi^R_{HE*}(\omega^M_{HE*}, p^{c*}, s^{c*}) &\geq \pi^D_{R*}
\end{align*}
\]

Proposition 4. When the coordinating wholesale price \(\omega^M_{HE*}\) and parameter \(\tau\) satisfy the following conditions, the manufacturer’s profit and retailer’s profit can achieve Pareto optimality and the supply chain system can be perfectly coordinated.

\[
\begin{align*}
\omega^M_{HE*} &= \frac{p^{c*}(6\tau - 4) + h s^{c*} + a p^{c*}(2 - 2\tau - 2\theta) + b_1 r g p^{c*}(6\tau + 2s - 4\tau s^{c*} - 4)}{2(b_1 r g + p^{c*} - \theta a - b_1 r g s^{c*})} \\
\pi^M_{HE*} &\leq \tau \leq 1 - \frac{\pi^D_{R*}}{\pi^S_{c*}}
\end{align*}
\]

5.2. BOPS Channel Managed by the Retailer

When the retailer manages the BOPS channel, it is assumed that the revenue distribution coefficient is \(\varphi (0 < \varphi < 1)\); that is, the overall optimal profit of the supply chain distributes the profit of part \(\varphi\) to the manufacturer and the remaining part \(1 - \varphi\) to the retailer.

\[
\begin{align*}
\pi^M_{HS} &= \omega[(\theta + \epsilon - \epsilon \theta)a + 2(b_1 r g s - p - b_1 r g)] + p[(1 - \theta)(1 - \theta)a - p - b_1 r g] = \varphi \pi_S^c \\
\pi^R_{HS} &= (p - \omega)[(\theta + \epsilon - \epsilon \theta)a - 2p - 2b_1 r g + 2b_1 r g s] - 1/2 hs^2 = (1 - \varphi) \pi_S^c
\end{align*}
\]

(20)

(21)

Similarly, in order to coordinate the supply chain, the supply chain members should execute the optimal selling price and the optimal service level of SA under the centralized decision.

\[
\begin{align*}
p^M_{HS*} &= p^{c*} = \frac{h(a - 3b_1 rg)}{6h - 4(b_1 rg)^2} \\
s^M_{HS*} &= s^{c*} = \frac{b_1 rg(a - 3b_1 r g)}{3h - 2(b_1 rg)^2}
\end{align*}
\]
The manufacturer’s optimal wholesale price $\omega^{HE}$ is the coordinated price. In order to ensure that both parties actively accept the contract, it is necessary to ensure that the profits of both manufacturers and retailers under this contract are Pareto improved [47]; that is, the profits of supply chain members are greater than the profits under the decentralized decision, so,

$$
\begin{align*}
\pi_M^{HE} &= \pi^C_S, \\
\pi_R^{HE} &= (1-\tau)\pi^C_S, \\
\pi_M^{HS} &= \pi^C_S, \\
\pi_R^{HS} &= \pi^C_S,
\end{align*}
$$

**Proposition 5.** When the coordinating wholesale price $\omega^{HE}$ and parameter $\varphi$ satisfy the following conditions, the manufacturer’s profit and retailer’s profit can achieve Pareto optimality and the supply chain system can be perfectly coordinated.

$$
\begin{align*}
\omega^{HE} &= \frac{p^C - 2h \varphi s^C + b_1 r g p^C - 2\varphi - 4\varphi s^C}{2(2h r g + 2p^C - a \varphi - 2b_1 r g s^C + \varepsilon \theta a)} \\
\frac{\pi^C_M}{\pi^C_S} &\leq \varphi \leq 1 - \frac{\pi^C_R}{\pi^C_S}
\end{align*}
$$

6. **Numerical Analysis**

Due to a large number of parameters and complex analytical formulas, it is difficult to directly compare the solution results. In order to further investigate the coordination effect of the wholesale price contract under the omni-channel integration mode of the BOPS, the design and calculation of model parameters are firstly used to conduct a comparative analysis of decisions and benefits. Secondly, the impacts of contract coefficient, SA service cost coefficient, and the offline channel consumer loyalty on supply chain members’ decision making and profit are analyzed. In order to get more abundant management enlightenment, the calculation and inference are carried out through numerical analysis.

6.1. **Optimal Decision and Profit Comparative Analysis**

Assuming that the parameters meet the above constraint conditions, the omni-channel supply chain owns the following market characteristics: $a = 150, \theta = 0.4, \varepsilon = 0.5, h = 50, r = 0.4, b_1 = 1, g = 1$. After implementing the BOPS sales model, the contract coefficient $\tau = 0.6$ when the manufacturer manages BOPS channels, and $\varphi = 0.6$ when the retailer manages BOPS channels. The optimal decision and profit of different BOPS integration modes are shown in Table 2.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\omega^*$</th>
<th>$s^*$</th>
<th>$p^*$</th>
<th>$\pi^C_M$</th>
<th>$\pi^C_R$</th>
<th>$\pi^C_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>--</td>
<td>0.4769</td>
<td>24.8363</td>
<td>--</td>
<td>--</td>
<td>1844.8401</td>
</tr>
<tr>
<td>DE</td>
<td>7.3630</td>
<td>0.2509</td>
<td>33.5071</td>
<td>681.6598</td>
<td>934.7571</td>
<td>1616.4169</td>
</tr>
<tr>
<td>HE</td>
<td>3.5368</td>
<td>0.4769</td>
<td>24.8363</td>
<td>737.9361</td>
<td>1106.9041</td>
<td>1844.8401</td>
</tr>
<tr>
<td>DS</td>
<td>19.3275</td>
<td>0.3153</td>
<td>35.7494</td>
<td>536.8751</td>
<td>948.3325</td>
<td>1485.2075</td>
</tr>
<tr>
<td>HS</td>
<td>11.2728</td>
<td>0.4769</td>
<td>24.8363</td>
<td>737.9361</td>
<td>1106.9041</td>
<td>1844.8401</td>
</tr>
</tbody>
</table>

Table 2 shows that the interesting game between the manufacturer and the retailer under decentralized decision making significantly reduces the total supply chain profit. In addition, the SA service level of retailers under centralized decision making is signifi-
cantly higher than that under decentralized decision making with different BOPS integration modes. Therefore, consumers can enjoy higher service quality in physical stores under centralized decision making. For the BOPS channel managed by the manufacturer or the retailer, a wholesale price contract can effectively improve supply chain performance, motivate retailers to improve SA service level, realize effective coordination of the omni-channel supply chain, and make supply chain members achieve Pareto improvement.

This is probably because when the manufacturer reduces the wholesale price, the increasing revenue motivates the retailer to provide high-quality services. The high level of SA service stimulates consumers’ purchase intention in the store, so as to stimulate the market demand. Increasing consumer demand generates more revenue shared by the manufacturer and retailer. Therefore, from the perspective of the profits of supply chain members, the executing of wholesale price contracts satisfies the individual rationality of the benefit distribution of the alliance. This implies that appropriate contract coordination is necessary to eliminate conflicts and achieve a win–win status under centralized decision making.

6.2. Sensitivity Analysis of Contract Coefficients $\tau$ and $\phi$

In the wholesale price contract, the proportion of revenue distribution has a direct impact on the service level of retailers’ physical stores, assuming that $a = 150, \theta = 0.2, \varepsilon = 0.2, h = 50, r = 0.4, b_1 = 1.2, g = 1$. The following will analyze the influence of contract coefficient on pricing decisions and profits of supply chain members under different BOPS channel integration modes. According to the assumption above, when the manufacturer manages the BOPS channel, the overall optimal profit of the supply chain distributes the profit of $\tau$ part to the manufacturer and the remaining $1 - \tau$ part to the retailer. When the retailer manages the BOPS channel, the overall optimal profit of the supply chain distributes the profit of part $\phi$ to the manufacturer and the remaining part $1 - \phi$ to the retailer. Under different coefficients of the wholesale price contract, the optimal profit of the model is shown in Figure 1.

Figure 1. Influence of contract coefficient on optimal profit. (a) Influence of contract coefficient $\tau$ on manufacturer profit; (b) Influence of contract coefficient $\tau$ on retailer profit; (c) Influence of contract coefficient $\tau$ on supply chain profit; (d) Influence of contract coefficient $\phi$ on manufacturer profit; (e) Influence of contract coefficient $\phi$ on retailer profit; (f) Influence of contract coefficient $\phi$ on supply chain profit.
As shown in Figure 1a,d and Tables 3 and 4, no matter whether the manufacturer or the retailer manages the BOPS channel, as the wholesale price contract coefficient keeps increasing, the manufacturer’s profit keeps increasing compared with that without the contract. It implies that the manufacturer gets the benefits from the improvement in the SA service level of physical stores under the contract. When the manufacturer manages the BOPS channel, and the wholesale price contract coefficient is greater than 0.54, the manufacturer’s profit is higher than that under the decentralized decision. When the retailer manages the BOPS channel, and the wholesale price contract coefficient is greater than 0.52, the manufacturer’s profit with the contract is higher than that under the decentralized decision.

Table 3. The influence of contract coefficient \( \tau \) on decision and profit.

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( \omega^{HE} )</th>
<th>( \pi_M^{HE} )</th>
<th>( \pi_M^{HE} - \pi_M^{DE} )</th>
<th>( \pi_R^{HE} )</th>
<th>( \pi_R^{HE} - \pi_R^{DE} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.54</td>
<td>0.3663</td>
<td>996.2137</td>
<td>61.4566</td>
<td>848.6265</td>
<td>166.9666</td>
</tr>
<tr>
<td>0.56</td>
<td>1.4231</td>
<td>1033.1105</td>
<td>98.3534</td>
<td>811.7297</td>
<td>130.0698</td>
</tr>
<tr>
<td>0.58</td>
<td>2.4800</td>
<td>1070.0073</td>
<td>135.2502</td>
<td>774.8329</td>
<td>93.1730</td>
</tr>
<tr>
<td>0.60</td>
<td>3.5368</td>
<td>1106.9041</td>
<td>172.1470</td>
<td>737.9361</td>
<td>56.2762</td>
</tr>
<tr>
<td>0.62</td>
<td>4.5936</td>
<td>1143.8009</td>
<td>209.0438</td>
<td>701.0393</td>
<td>19.3794</td>
</tr>
</tbody>
</table>

Table 4. The influence of contract coefficient \( \varphi \) on decision and profit.

<table>
<thead>
<tr>
<th>( \varphi )</th>
<th>( \omega^{HS} )</th>
<th>( \pi_M^{HS} )</th>
<th>( \pi_M^{HS} - \pi_M^{DS} )</th>
<th>( \pi_R^{HS} )</th>
<th>( \pi_R^{HS} - \pi_R^{DS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.52</td>
<td>8.5808</td>
<td>959.3169</td>
<td>10.9844</td>
<td>885.5233</td>
<td>348.6482</td>
</tr>
<tr>
<td>0.54</td>
<td>9.2538</td>
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<td>237.9578</td>
</tr>
<tr>
<td>0.60</td>
<td>11.2728</td>
<td>1106.9041</td>
<td>158.5716</td>
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<td>0.66</td>
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<td>0.68</td>
<td>13.9648</td>
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<td>590.3488</td>
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<td>0.70</td>
<td>14.6378</td>
<td>1291.3881</td>
<td>343.05613</td>
<td>553.4520</td>
<td>16.5770</td>
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</table>

As shown in Figure 1b,e and Tables 3 and 4, no matter whether the manufacturer or the retailer manages the BOPS channel, with the continuous increase in contract coefficient, the retailer’s profit shows a changing trend of decreasing after adopting the wholesale price contract. When the manufacturer manages the BOPS channel, and the contract coefficient is less than 0.62, the retailer’s profit after making the wholesale price contract is always higher than before. When the retailer manages the BOPS channel, and the contract coefficient is less than about 0.70, the retailer’s profit is higher than that under the decentralized decision. Therefore, within a certain range of wholesale price contract coefficients, the wholesale price contract can realize Pareto improvement of supply chain members. Therefore, a wholesale price contract is the best choice for supply chain members. Finding an appropriate wholesale price ratio under different integration modes can motivate retailers and manufacturers to participate in cooperation. This phenomenon supports Propositions 4 and 5.

Figure 1c,f further visually show the influence of the coefficient of wholesale price contract on the total profit of the omni-channel supply chain. Whether the manufacturer or the retailer manages the BOPS channel, the profit of the supply chain after executing the wholesale price contract increases significantly compared with that without the contract. It reaches the profit of the supply chain under a centralized decision.
In summary, from the above analysis, it can be seen that based on the wholesale price contract, the revenue distribution mechanism of the omni-channel supply chain coordination can effectively stimulate the manufacturer and the retailer to cooperate and execute the optimal decisions under centralized decision making. This mechanism expands the overall consumer demand. Therefore, it achieves a win–win situation for all the members of the supply chain by improving the profits of the manufacturers, retailers, and the whole supply chain.

6.3. Sensitivity Analysis of SA Effort Service Level Cost Coefficient

With other parameters unchanged, the influence of SA service level cost coefficient \( h \) on SA effort service level, optimal selling price and profit of supply chain members is analyzed when the contract coefficient \( \tau = 0.6 \) and \( \varphi = 0.6 \). The results are shown in Figures 2 and 3.

![Figure 2](image2.png)

**Figure 2.** Influence of contract coefficient on optimal profit.

![Figure 3](image3.png)

**Figure 3.** Influence of SA effort service cost coefficient on optimal profit.

As shown in Figure 2, with the increase in cost coefficient \( h \), the SA service level of the retailer is decreased whatever under the centralized decision, the decentralized decision, and having wholesale price contract situations. Compared to these three situations, the service level of the retailer under the centralized decision decreases faster. In addition, whether the manufacturer or the retailer manages the BOPS channel, the SA level under the contract is equal to the SA level under the centralized decision. It is significantly higher than the SA level under the decentralized decision without the contract. With the increase in cost coefficient \( h \), the optimal wholesale price and the optimal sales price of products do not show a significant change before and after having a contract. Comparing the two BOPS modes, the wholesale price, the selling price, and the SA service level in the retailer-
managed BOPS mode are significantly higher than those in the manufacturer-managed BOPS mode. It implies that consumers have a better shopping experience in the retailer-managed BOPS mode. The possible reason is that retailers have more market shares and are motivated to provide high-quality SA service to attract consumers and stimulate consumer demand when they manage BOPS channels. Retailers in a strong position in the consumer market can make use of channel advantages and raise sales prices. However, manufacturers with a smaller market share can only set lower wholesale prices.

As shown in Figure 3 and Table 5, before and after the wholesale price contract, the profits of the manufacturer, the retailer, and the supply chain system gradually decrease with the increase in the service cost coefficient. These results are consistent with Corollary 1 and Corollary 3, which indirectly confirm the robustness of these results. The profit of the supply chain system under the centralized decision is much higher than that under the decentralized decision. The profit of the decision making subject under the contract is always higher than that under the non-contract coordination mechanism. No matter whether the manufacturer or the retailer manages the BOPS channel, the profit of both parties can achieve Pareto improvement after the supply chain adopts the wholesale price contract. In addition, with the increase in effort cost coefficient, each different sales BOPS integration mode of the decision making under the contract and contract of gap shrinking profits. It suggests that for the retailer in the process of entity shop management, who pays SA hard service costs, the more the wholesale price contract coordination superiority will be reduced.

Table 5. The influence of service level cost coefficient on profit before and after contract.

<table>
<thead>
<tr>
<th>h</th>
<th>$\pi^M_{HE} - \pi^M_{DE}$</th>
<th>$\pi^R_{HE} - \pi^R_{DE}$</th>
<th>$\pi^M_{HS} - \pi^M_{DS}$</th>
<th>$\pi^R_{HS} - \pi^R_{DS}$</th>
</tr>
</thead>
<tbody>
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<td>5</td>
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<td>56.8740</td>
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</table>

6.4. Sensitivity Analysis of Offline Consumer Loyalty $\theta$

With other parameters unchanged, the influence of the offline channel consumer loyalty on the SA service level, the optimal sales price and the profit of supply chain members are analyzed when the contract coefficient $\tau = 0.6$ and $\phi = 0.6$. The results are shown in Figures 4 and 5.

As shown in Figures 4 and 5, when the manufacturer manages the BOPS channel, as $\theta$ increases, the selling price and SA service level increase, while the wholesale price decreases. However, after the implementation of the wholesale price contract, the wholesale price as a coordination price keeps increasing. This shows the increasing cooperation between the manufacturer and the retailer since the implementation of the contract. In terms of profits, the profit of the manufacturer is decreasing, while the retailer’s profit is increasing. The overall profit of the supply chain is reduced. The main reason for this phenomenon is that with the increase in consumers’ recognition of the offline channel, the market share of the retailer gradually increases, while the position of the manufacturer is constantly weakening. When consumers prefer offline shopping, they are more willing to accept products from offline channels, retailers have a greater advantage in the competition, and corporate profits increase significantly, while manufacturers’ profits fall down with the reduction in online demand.
As shown in Figures 4 and 5, when the retailer manages the BOPS channel, the sales price and service level keep rising with the increase in $\theta$, while the wholesale price keeps decreasing, but the fluctuation range is relatively small, and the change is relatively gentle. The profit of the manufacturer keeps decreasing, while the retailer profit keeps increasing, and the change range is more gentle than that of the BOPS channel managed by the manufacturer. The overall profit of the supply chain is reduced. The main reason for this phenomenon is that with the increase in consumers’ recognition of offline channels, the market share of the retailer gradually increases. As the retailer needs to provide SA service for all consumers, they need to spend a lot on service cost, so the profit growth of the retailer is small.

By comparing the two omni-channel integration modes managed by the manufacturer and the retailer in Figures 4 and 5 and Table 6, it can be seen that, when the retailer manages the BOPS channel, the change range of both price and SA service level slows down. From the perspective of profit change amplitude, compared with the significant change in the manufacturer’s profit in the BOPS channel management, the retailer’s profit change in the BOPS channel is more gentle and smaller, and its sensitivity to the offline channel customer loyalty $\theta$ is less than that under the manufacturer’s management. In terms of absolute profit, when $\theta < 0.38$, the supply chain members prefer the manufacturer to manage the BOPS channel, and when $\theta > 0.38$, the supply chain members prefer the retailer to manage the BOPS channel. For the whole supply chain, when $\theta < 0.5$, the whole supply chain tends to the BOPS channel managed by the manufacturer, as the profit of the whole supply chain is higher. The opposite is true when $\theta > 0.5$. From the perspective of consumers’ feelings, when customer loyalty in offline channels is low, consumers can obtain a higher quality SA service level and enjoy a more intimate experience and service when the retailer manages the BOPS channel. While in the offline channel with high customer loyalty, consumers can obtain a higher quality SA service level when the manufacturer manages the BOPS channel. The management revelation of this section is that cus-
Customer loyalty in the offline channel, as a realistic external condition of enterprise operation, will not only affect the price and demand of products in the market but also directly affect the profits of supply chain members. In order to optimize the overall profit of the omni-channel supply chain, supply chain members must choose appropriate and corresponding channel integration strategies, according to the actual specific market situation and environment. This also fully explains why each sales channel strives to increase consumer loyalty, improve the channel preference rate of consumers, and the ultimate goal is to own more market share, and increase the sales profit of the enterprise.

Table 6. The influence of offline consumer loyalty $\theta$ on profit margin.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\pi_{DS}^M - \pi_{DE}^M$</th>
<th>$\pi_{DS}^R - \pi_{DE}^R$</th>
<th>$\pi_{S}^M - \pi_{S}^DE$</th>
<th>$s_{DS}^* - s_{DE}^*$</th>
</tr>
</thead>
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<tr>
<td>0.2</td>
<td>-450.3341</td>
<td>269.7349</td>
<td>-180.5992</td>
<td>0.1942</td>
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<tr>
<td>0.22</td>
<td>-458.8537</td>
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<td>0.1812</td>
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<tr>
<td>0.24</td>
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<td>0.1682</td>
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<tr>
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<td>540.8048</td>
<td>-571.6794</td>
<td>-30.8746</td>
<td>0.0124</td>
</tr>
</tbody>
</table>

7. Discussion and Conclusions

7.1. Conclusions

Under the trend of omni-channel sales, this study takes the BOPS model of online purchase and offline pickup as the research object, constructs and studies two omni-channel integrated supply chain models, the BOPS channels managed by the manufacturer and the retailer. Through model derivation and numerical analysis, the conclusions are drawn as follows.

Centralized decision making helps to improve the SA service level and achieve the global optimal supply chain performance. To realize the profit distribution under centralized decision making, this study proposes the wholesale price contract to stimulate the supply chain members to cooperate, under two BOPS models, execute the centralized decisions and obtain the Pareto optimality.

The BOPS channel managed by the retailer is less sensitive to offline customer loyalty than that managed by the manufacturer. It is suitable for the manufacturer to manage the BOPS channel when offline customer loyalty is low. Otherwise, it is appropriate for the retailer to manage the BOPS channel.

Whether the manufacturer or the retailer manages the BOPS channel, the decisions of product sales price, the wholesale price, and SA service level depend on the service cost coefficient, the product return rate, and the offline channel consumer loyalty. By evaluating the above factors, effective coordination can be achieved to improve the performance of the BOPS market and fulfil the sustainability requirement of stakeholders.
7.2. Theoretical Implications

One of the most critical issues for any supply chain is the trade-off between different channels. An efficient supply chain is achieved when all the stakeholders in the supply chain are satisfied by optimizing their profits and costs. This study contributes to omni-channel supply chain research by exploring and enriching the theories in the research field of BOPS. Based on the new omni-channel BOPS sales model, this work constructs a supply chain model of BOPS that takes the product return risk into account. By comparing the equilibrium solutions implementing the integration mode of different BOPS, the study explores different optimal decisions of the supply chain, discusses the best market situation due to the implementation of BOPS, and analyzes the coordination mechanism of overall channels of the supply chain with wholesale price contract. This study provides researchers with a research model of BOPS, for which researchers can extend this model to conduct further studies.

In addition, this study contributes to our knowledge in showing how to improve the sustainable performance of a supply chain in the BOPS business. It shows that the profits of manufacturers, retailers, and supply chains can be improved under different situations, such as offline channel consumer loyalty, as a realistic external condition, and service quality of SA effort, as an internal condition. According to different situations, different omni-channel business strategies can be applied to form the coordination of the supply chain members, motivate retailers to improve the service level of the SA effort, and realize Pareto improvement to support the supply chain sustainability in omni-channel business. Optimizing the total system profit for a supply chain is the outcome of successful sustainable supply chain management. This study shows that economic benefits for each stakeholder can be increased, and a high service level of SA effort can be delivered through adopting BOPS strategies. This study shows how to coordinate a BOPS supply chain more sustainably compared to the traditional one.

7.3. Practical Implications

In real business, the manufacturer and retailer pricing decisions will be influenced by the SA service level, and this kind of influence is two sided. The manufacturer, when determining the coefficient of the wholesale price contract, needs to give full consideration to the SA service level of double-sided effects brought by the cost coefficient, in order to bring more effective decision-making advantages into full play. If the BOPS model is implemented, the enterprise should pay attention to the reduction in the service cost, such as the human cost of sales staff, the management cost of physical stores and the site rent, etc. SA service level decisions can be used as an effective measure to improve the performance of supply chain members.

As a realistic external condition of enterprise operation, consumer loyalty in offline channels will not only affect the price and demand of products in the market, but also directly affect the profits of supply chain members. In order to achieve the optimization of the overall profit of the omni-channel supply chain, enterprises must choose appropriate and corresponding channel integration strategies, according to the actual specific market situation and environment. The BOPS channel managed by the retailer is less sensitive to consumer loyalty of the offline channel than that under the BOPS channel managed by the manufacturer.

When the customer loyalty of offline channels is low, it is suitable for the manufacturer to manage the BOPS channel in sales; otherwise, it is suitable for the retailer to manage and operate the BOPS channel in sales. From the perspective of consumers' feelings, when customer loyalty in offline channels is low, consumers can obtain a higher quality SA service level and enjoy a more intimate experience and service when the retailer manages the BOPS channel.

In terms of the market sales, each channel tries to increase consumer loyalty, so as to improve the channel preference rate of consumers, and its ultimate purpose is to expand
market share, and increase the profit of the enterprise. For manufacturers, they should make full use of the convenience of the network. By opening live broadcasts, they can invite anchors to promote products, so as to introduce customer flow for online sales. As the most direct customer contact terminal, retailers should grasp the dynamic market information, customer preferences, and flexible response to market changes. Furthermore, retailers and manufacturers should achieve a deepened cooperation, strengthen information exchange and internal communication, and work together to enhance brand competitiveness.

7.4. Limitation and Future Research

There still exist some limitations in the method and research of this study. Firstly, different supply chain game models, such as the retailer’s dominant position, are not considered. Secondly, the retailer only considers the impact of providing services on the product return rate, without considering the reality that high-quality offline services can also promote consumers’ additional consumption behaviours in physical stores. In the future, we will further take the game mode of supply chains and other omni-channel sales factors into the study, so as to get richer enlightenment on omni-channel sales management.

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References


