Article
Loss Aversion Order Strategy in Emergency Procurement during the COVID-19 Pandemic

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Abstract: The COVID-19 pandemic has had a serious impact on firms’ sourcing strategies. Since COVID-19 disrupted the supply chain, firms have had to make emergency purchases from other suppliers. In addition, emergency ordering is one of the most effective strategies to achieve sustainable operations because such a strategy can save inventory costs. We aim to address a retailer’s emergency procurement strategies during the COVID-19 pandemic. We use prospect theory and the newsvendor model to uncover the retailer’s inventory decisions. In our study, we find that retailers have the choice to order items before the selling period at the normal purchase price, and, if available, they can order them before the end of the selling period at the urgent purchase price. We perform a comparison of the optimal ordering policy and margins in this case with the conventional and loss aversion models. The influence of emergency procurement on the optimal order policy and margins is investigated as well. This paper contributes in theory that we innovatively capture the uncertainty of emergency sourcing, which is a feature that has never been considered in current research.

Keywords: COVID-19; loss aversion; newsvendor problem; emergency procurement; prospect theory; order strategy

1. Introduction

One of the impacts of COVID-19 was supply chain disruption, which led to many companies having difficulty in purchasing raw materials. Eventually, companies had to turn to other suppliers for emergency raw material purchases [1]. Meanwhile, emergency procurement is one of the most effective means to achieve sustainable operations. Firstly, it saves the cost of inventory, which helps firms to achieve sustainable inventory management. Secondly, the implementation of emergency ordering strategy helps firms to have flexible capital flow. The firm will not be tied up with excessive inventory. Finally, emergency ordering is an indispensable tool for sustainable supply chain management. Therefore, it is necessary to study the emergency ordering strategy for the sustainable operation of a firm.

The newsvendor model is one of the most important models in operation management [2]. It is also the fundamental term to research for other problems, such as inventory management, supply chain management, and the cooperation and competition between firms [3]. The classical newsvendor model research is used for deciding the optimal order quantity to maximize the expected revenue when there is only one random order chance before the selling period for the risk-moderate newsvendor. In this paper, we study a newsvendor model with a loss aversion strategy. It is important to emphasize that loss aversion is also one of the effective means to achieve sustainable supply chain operations. The loss aversion strategy implies that firms will try to avoid losses as much as possible. In this case, the firm’s operation will achieve maximum efficiency and minimum cost, which means that the firm’s development is sustainable. The newsvendor model with loss
aversion is significant for the sustainable inventory management of the enterprise. In addition, companies’ procurement and inventory strategies are also affected by COVID-19. In the event of a supply disruption, companies must make emergency purchases. The procurement strategy during COVID-19 is also different from regular emergency procurement. This is because emergency procurement during the COVID-19 pandemic is also subject to uncertainty.

Currently, there is a wealth of research on the issue of child reporting with risk appetite, particularly with regard to emergency orders. However, very few studies have bothered to consider the uncertainty of emergency ordering. Especially in the context of COVID-19, where retailers’ ordering is fraught with uncertainty, the research in this paper fills this gap. The objective of this paper is to theoretically provide retailers with some emergency ordering decisions that address the uncertainty posed by COVID-19.

In this paper, we innovatively capture the uncertainty of emergency sourcing, a feature that has never been considered in current research. We combine the loss aversion strategy and the emergency ordering strategy to study the problem of a news vendor with these two strategies. Additionally, we apply prospect theory to capture loss aversion as a feature. We first model the news vendor without the emergency-order strategy and calculate the optimal order quantity and profit. Subsequently, we model the news vendor with an emergency ordering strategy and calculate the optimal order quantity and profit. To highlight the value of the emergency ordering strategy, we compare the two models. The numerical case demonstrates the robustness of the analytical results. Since the COVID-19 pandemic, emergency procurement strategies have been uncertain. If the alternate supplier has an interruption in supply due to COVID-19, then emergency procurement will also be affected. To reflect the impact of COVID-19, we consider the uncertainty of emergency procurement in the extended model.

2. Literature Review

Many researchers have expanded the classical news vendor model and modified some basic assumptions. The classical news vendor model assumes news vendors are risk-moderate, but many of the news vendors are risk-averse in the real world. Therefore, some risk measuring tools are invited into the news vendor model, such as mean variance analysis, CVaR (conditional value at risk), the downside risk constraint method, and prospect theory. Based on the research domains, this paper reviews the relevant literature on the purchasing strategy and news vendor problems with risk aversion.

As the introduction and implementation of quick response, the ordering opportunity for a news vendor could occur more than once. If the demand is too high, during the selling period, the emergency order could be placed at a price higher than the one before the selling period started. In other words, the news vendor has two approaches to placing the order: the ordinary order and the emergency order. Chen and Federgrune [4] studied the problem of the risk-averse news vendor with emergency order choice. Wu et al. (2009) [5] studied the production model with feedback. The model enables one more production opportunity when the demand could be observed, which means the order could be placed twice. Choi et al. (2008) [6] studied the news vendor model with an additional order. Qiao (2008) [7] researched the second-order and second-sales problems for news vendors. Gotoh and Takano (2007) [8] studied the loss-averse news vendor problem considering backordering using the risk measuring tools of prospect theory and the CVaR. Similarly, Huang et al. (2022) [9] established a loss-averse news vendor’s preordering decision model based on the CVaR measure to provide the optimal solutions in new product launching in selling seasons. The data-driven decision making is also conducted in a risk-averse news vendor problem with an unknown demand distribution with a profit risk constraint [10]. Xu et al. (2006) [11] studied the loss-averse news vendor problem considering emergent replenishment with and without shortage cost. Chen et al. (2009) [12] and Xu (2010) [13] explored the optimal ordering decision for a loss-averse news vendor facing CVaR. The loss-averse news vendor problem has been studied [14–18], and prospect
theory based newsvendor problem has also been considered [19–21]. However, the joint impacts of loss-averse and prospect theory on newsvendor problem are not clear.

Some studies have incorporated the reference point and its effect on the optimal solutions in loss-averse newsvendor problems. For example, Liu et al. (2022) [22] explored a single-period inventory problem with a quantity-oriented reference point, where the newsvendor has loss-averse preferences and the conditional CVaR measure is used to hedge against the risk. Qiu et al. (2022) [18] investigated the joint pricing and stocking decisions for a loss-averse retailer with a reference point effect under stochastic demand based on prospect theory. They also compared the optimal pricing and stocking decisions with the classical newsvendor. Bai et al. (2019) [23] incorporated the effect of reference dependence in the loss-averse newsvendor model and examined the optimal pricing and ordering decisions. The two-time ordering newsvendor problem was also studied [18,24]. Other related research extended the loss-averse newsvendor by considering sustainability regulations [25–27], reference dependence [22,23], the supply contract [28–30], supply uncertainty [31,32], emergency replenishment [33], etc. Choi (2018) [34] incorporated cap-and-trade carbon emissions regulation in the loss-averse newsvendor problem. Market information updating [35] and backordering [36] were also considered. Lee et al. (2015) [37] explored the loss-averse newsvendor considering a contract with multiple supply options. As for the importance of sustainability in supply chains, the related studies in the fields of supply chain and inventory management are also included [38,39]. Intelligent algorithms [40,41] are also proposed to solve inventory decisions.

In contrast to the studies above, this study utilizes prospect theory to describe the loss-averse newsvendor. This paper studies the decision problem with emergency ordering opportunities and analyzes the impact of emergency ordering opportunities on the optimal ordering quantity and revenue of a loss-averse newsvendor, as well as the loss-averse index, retail price, wholesale price, and residual value. In addition, this paper compares the classic newsvendor model with the loss-averse newsvendor model.

3. Materials and Methods

Kahneman and Tversky established prospect theory in 1979. It shows people’s different attitudes to the various risks of loss and revenue. There is a parameter $W_0$ to measure the perceptional wealth of the decision-maker. The response is significantly higher when revenue is lower than $W_0$. This is the presence of the aversion to losing. There is a loss-averse coefficient $\lambda \geq 1$. The greater the coefficient, the higher the degree of loss aversion. When $\lambda = 1$, the risk is moderate.

The utility function for the decision-maker is usually periodic linear. Following Wu et al. (2021) [32] and Liu et al. (2021) [2], we set $W_0$, and let $W$ be the revenue for the decision-maker; then the utility function is

$$U(W) = \begin{cases} W - W_0, & W \geq W_0, \\ \lambda(W - W_0), & W < W_0. \end{cases}$$

(1)

Suppose the decision-maker (newsvendor) has a random demand $x$ with a density function $f(x)$, distribution function $F(x)$, and mean value $\mu$. Before the selling period, a newsvendor ordered the products with quantity $Q$ at price $w$, and then sold the products at price $p$ in the selling period. If the demand $x$ in the period is more significant than $Q$, the newsvendor may place an emergency order at price $e(e > w)$ to meet the demand. If the demand $s$ in the period is smaller than $Q$, then no more orders will be placed, and the unsold products will be sold at the residual value $s$. The rational assumption is $s < w < e < p$. This is a general setting regarding the newsvendor problem (See [27,29]).

When the loss-averse newsvendor does not have the choice for an emergency order, the revenue function is
\[ \pi_r(x, Q) = \begin{cases} \pi_r^-(x, Q) = (p - s)x - (w - s)Q, & x \leq 0, \\ \pi_r^+(x, Q) = (p - w)Q, & x > Q. \end{cases} \] (2)

4. Loss-Averse Newsvendor Model with Emergency Ordering Opportunity

When the loss-averse newsvendor has the choice for an emergency order, the revenue function is

\[ \pi_e(x, Q) = \begin{cases} \pi_e^-(x, Q) = (p - s)x - (w - s)Q, & x \leq 0, \\ \pi_e^+(x, Q) = (p - e)x + (e - w)Q, & x > Q. \end{cases} \] (3)

The expected revenue for the classical newsvendor model is \( E[\pi(x, Q)] \). The expected revenue for a loss-averse newsvendor without an emergency order choice is \( E[\pi_r(x, Q)] \). The expected revenue for a loss-averse newsvendor with an emergency order choice is \( E[\pi_e(x, Q)] \).

Let \( q_0(Q) = Q\left(\frac{w}{p - w}\right) \). Because \( w < p \), so \( q_0(Q) < Q \). If \( x < q_0(Q) \), then \( \pi(x, Q) = \pi^-(x, Q) < 0 \).

Thus, we have

\[
E_r[\pi_r(x, Q)] = \lambda \int_{0}^{q_0(Q)} \pi_r^-(x, Q)f(x)d(x) \\
+ \int_{q_0(Q)}^{Q} \pi_r^-(x, Q)f(x)d(x) \\
+ \int_{q_0(Q)}^{+\infty} \pi_r^-(x, Q)f(x)d(x),
\] (4)

and

\[
E_e[\pi_e(x, Q)] = \lambda \int_{0}^{q_0(Q)} \pi_e^-(x, Q)f(x)d(x) \\
+ \int_{q_0(Q)}^{Q} \pi_e^-(x, Q)f(x)d(x) \\
+ \int_{q_0(Q)}^{+\infty} \pi_e^+(x, Q)f(x)d(x).
\] (5)

The upcoming Theorem 1 shows the convexity of \( E_r[\pi_r(x, Q)] \) and \( E_e[\pi_e(x, Q)] \). The theorem also shows the optimal order strategies for the loss-averse newsvendors with or without an emergency order choice.

**Theorem 1.** (i) \( E_r[\pi_r(x, Q)] \) and \( E_e[\pi_e(x, Q)] \) are strict concave functions regarding \( Q_r^* \) and \( Q_e^* \), respectively.

(ii) Max\( Q_r \)\( E_r[\pi_r(x, Q)] \) has the only optimal solution \( Q_r^* \), and

\[
(p - s)F(Q_r^*) + (\lambda - 1)(w - s)F(Q_r^*\frac{w - s}{p - s}) = p - w.
\] (6)

(iii) Max\( Q_r \)\( E_e[\pi_e(x, Q)] \) has the only optimal solution \( Q_r^* \), and

\[
(e - s)F(Q_r^*) + (\lambda - 1)(w - s)F(Q_r^*\frac{w - s}{p - s}) = e - w.
\] (7)

Proof see Appendix A.

Equations (4) and (5) show that the expected utility of the loss-averse newsvendor is the sum of the respective expected profit and expected loss. Specifically, \( \lambda - 1 \) is the determinant of loss aversion. Especially when \( \lambda = 1 \), the second term is canceled out, and then the expected utility equals the expected profit. In this condition, the newsvendor is risk-moderate. When \( e = p \), Equations (6) and (7) are the same. \( e = p \) means that there is
no extra profit from the emergency order, so there is no need to place an emergency order at the moment. Equation (4) shows that the expected utility of a loss-averse newsvendor with the opportunity for an emergency order is the sum of expected utility of the loss-averse newsvendor and the expected revenue of the emergency order. Equation (5) shows that the expected utility is the sum of the revenue of the classic newsvendor model, expected loss, and the expected revenue from the emergency order. When $\lambda = 1$ and $e = p$, the equation appears to be our model. Therefore, the model in this study is an expansion of the risk-moderate model and the model with the opportunity for an emergency order.

5. Model Analysis

5.1. Analysis for the Impact on Revenue and Order Quantity

Suppose the optimum solution and optimum value of the classical newsvendor model are $Q^*$ and $E^*$, respectively. The optimum solution and value of the loss-averse newsvendor with an emergency order opportunity are $Q^*_r$ and $E^*_r$, respectively. The impact of the emergency order opportunity provided by the supplier on the revenue and order quantity of the newsvendor is analyzed as follows.

**Theorem 2.** $E^*_r < E^*$, $E^*_r < E^*_e$, and $Q^*_r < Q^*_e < Q^*$

Theorem 2 shows that the revenue of the loss-averse newsvendor is lower than that of the classic newsvendor. In this condition, because the newsvendor is loss-averse with the conservative decision, the order quantity is smaller than the optimum. As a result, the revenue decreases. The revenue of the loss-averse newsvendor with emergency order opportunities is higher than that without emergency order opportunities. The additional emergency order opportunity increases the room available and flexibility for decision-making and increases the potential solution. This, in turn, increases the revenue. As the emergency order opportunity is available, some orders from the newsvendor may not be placed until demand occurs, so the order quantity before the selling period starts to reduce. Therefore, the opportunity for an emergency order provided by the supplier enables the newsvendor to place the orders in two different periods: a reduced amount of normal orders, and an emergency order. As a result, the flexibility of decision choice improves the newsvendor’s profitability.

We do not compare the revenue $E^*_e$ of the loss-averse newsvendor with the emergency order opportunity to the revenue $E^*$ of the classic newsvendor because there is no necessary relationship between the size of the two. A previous analysis shows that the decision from the loss-averse newsvendor tends to be conservative, so that the revenue may reduce. On the other hand, the additional emergency order opportunity brings flexible decision-making, so the revenue could increase. In contrast to the classic newsvendor model, revenue for the loss-averse newsvendor depends on the dynamics of the degrees for conservativeness and decision flexibility. When the coefficient for loss-averse $\lambda$ is very low, such as $\lambda = 1$, by Equation (5), we have $E_e[U(\pi(x, Q))] \geq E[U(\pi(x, Q))]$. Thus, we have $E^*_e \geq E^*$

5.2. Static State Comparison Analysis

The upcoming content is to make a static state comparison analysis on the loss-averse coefficient, emergency order price, retail price, and residual value to maintain some inspiration for management. The study focuses on the impact of the changes in those terms on order quantity and revenue.

**Theorem 3.** The results of sensitivity analyses are as follows:

(i) As the loss-averse coefficient $\lambda$ increases, the order quantity and revenue reduce;
(ii) As the price $e$ of emergency orders increases, the order quantity increases, but revenue decreases;
(iii) As the retail price $p$ increases, the order quantity and revenue increase;
(iv) As the wholesale price $w$ increases, the order quantity and revenue reduce;
(v) As the residual value $s$ increases, the order quantity and revenue increase.

Proof see Appendix A.

The conclusions for Theorem 3 are intuitive. As the newsvendor is loss-averse, they tend to reduce the order quantity to prevent the potential loss brought by the possible oversupply. The reduced order quantity induces decreased revenue. Especially when $\lambda$ is high enough, there must be $E^*_e \geq E^*$. The higher the price of an emergency order, the lower the marginal profit from selling the products by emergency order. Because of the lower profitability, the newsvendor increases ordinary order quantity and reduces the emergency order quantity. This small room for decision-making induces decreased revenue. The rest of the three conclusions are also similar in logic. Theorem 3 also indicates that, in most situations, the more ordinary order placed, the more revenue occurs.

6. Numerical Examples

This section explains the conclusions above by numerical examples focusing on explaining Theorem 2, especially comparing the values of $E^*_e$ and $E^*$.

Example 1. Suppose $w = 10$, $p = 16$, $e = 12$, $s = 8$, $x \sim U[100, 200]$, and $=2$, then we have $Q^* = Q^*_e = 175$, $Q^*_E = 150$, $E = E^*_e = 825$, and $E^*_e = 825$. These results confirm $Q^*_e < Q^*_e = Q^*$ and $E^*_e = E^* < E^*_e$.

Example 2. Suppose $w = 10$, $p = 13$, $e = 12$, $s = 8$, $x \sim U[0, 200]$, and $=2$, then we have $Q^* = 120$, $Q^*_e = 103.45$ $Q^*_E = 83.33$, $E = 180$, $E^*_e = 155.17$ and $E^*_e = 183.33$. These results confirm $Q^*_e < Q^*_e < Q^*$ and $E^*_e < E^* < E^*_e$.

Example 3. If $\lambda =5$ in Example 2, then we have $Q^* = 120$, $Q^*_e = 73.17$ $Q^*_E = 55.55$, $E = 180$, $E^*_e = 109.76$ and $E^*_e = 155.56$. These results confirm $Q^*_e < Q^*_e < Q^*$ and $E^*_e < E^* < E^*_e$.

Examples 1–3 show the establishment of the conclusions for Theorems 2 and 3. Example 1 also shows that the equalization signs in Theorems 2 and 3 are rationalized. Examples 2 and 3 show that $E^*_e$ may be higher or lower than $E^*$, depending on the newsvendor’s attitude to risk. As the loss-averse coefficient $\lambda$ increases, the order quantity and profit of newsvendor reduce. When $\lambda$ is high enough, the profit for the loss-averse newsvendor must be lower than that for the classic newsvendor.

7. Extended Analysis: Uncertain Emergency Procurement Due to COVID-19

In this section, we consider the uncertainty of emergency procurement brought by COVID-19. During the COVID-19 pandemic, suppliers for emergency procurement may also face supply disruptions, which leads to the failure of emergency procurement. In order to obtain the best emergency procurement strategy during COVID-19, it is reasonable to consider emergency procurement uncertainty.

The emergency procurement is characterized by $zQ$. $z$ is random with mean $\mu_z$ and variance of $\sigma^2_z$. $g(z)$ is the probability density function of the emergency procurement quantity. When the loss-averse newsvendor has the choice for an emergency order, the revenue function is

$$
\pi_e(x, Q) = \begin{cases} 
\pi_-(x, Q) = (p - s)x - (w - s)Q, & x \leq 0, \\
\pi_+(x, Q) = (p - e)x + (e - w)zQ, & x > Q.
\end{cases}
$$
Thus, we have
\[
E_e[U(\pi(x, Q))] = \lambda \int_0^{+\infty} \int_0^{q_0(Q)} \pi_-(x, Q) f(x) d(x) d(z) \\
+ \int_1^{Q} \int_0^{q_0(Q)} \pi_-(x, Q) f(x) d(x) d(z) \\
+ \int_1^{+\infty} \int_0^{+\infty} \pi_+(x, Q) f(x) d(x) d(z).
\]

Then we have the following results.

**Theorem 4.** (i) \(E_e[U(\pi(x, Q))]\) are strict concave functions of \(Q^*_e\). (ii) \(\max_{Q} E_e[U(\pi(x, Q))]\) has the only optimal solution \(Q^*_e\), and

\[
(e - s)(\int_0^{1} F(zQ^*_e) d(z) + \int_1^{+\infty} F(zQ^*_e) d(z)) \\
+ (\lambda - 1)(w - s)(\int_0^{1} F(zQ^*_e) \frac{w - s}{p - s} d(z) + \int_1^{+\infty} F(zQ^*_e) \frac{w - s}{p - s} g(z) dz) = e - w. \tag{10}
\]

The proof of the above results are similar to the model that does not consider the emergency procurement uncertainty. Thus, we omit it here. These results show the robustness of the results we derive from the basic models.

8. Conclusions and Discussion

8.1. Conclusions

The newsvendor problem is one of the fundamental problems in management. Many extended loss-averse newsvendor problems and emergency order opportunities by the supplier are incorporated into the newsvendor model. This study discusses the impact of emergency order opportunities on the optimum order strategy and revenue of a loss-averse newsvendor and conducts a static state analysis of the parameters. Emergency order choice not only influences the order quantity from the loss-averse newsvendor but also influences the newsvendor’s revenue. Emergency order opportunities expand the scope and flexibility of the newsvendor’s decision-making. The newsvendor is able to properly arrange the order quantity in two separate periods to improve profitability.

8.2. Discussions

We have several interesting results against extant literature. For example, we find that the profit of the loss-averse newsvendor is lower than that of the classic newsvendor. The profit of the loss-averse newsvendor with emergency order opportunities is higher than that without emergency order opportunities. In addition, we find that as the newsvendor is loss-averse, this tends to reduce the order quantity to prevent the potential loss brought by possible oversupply. In most situations, the more ordinary order placed, the more profits occurs. Our findings provide essential managerial insights for real-world practice.

We provide managerial guidance for firms’ decision making concerning emergency purchasing during the COVID-19 pandemic. In particular, the retailers of healthcare systems need to make the most urgent purchasing decisions when medical supplies are in short supply. Therefore, the retailers are not only able to hedge their purchasing risks, but also to secure their profits. This study will attract the attention of the industry and provide guidance on emergency purchasing decisions.

This paper has several limitations. We hence propose research directions for exploration in the future. First, this paper has not taken the cost of shortage in supply into account. Future studies may add to the factor of shortages in supply. This may bring interesting results, as in Wang (2009) [15]. Second, some decisions in other scenarios can
be included in this study. Future studies may also consider the scenario of decision making for order placing and price setting at the same time. Even though the demand is random, people can still manipulate demand distribution through price setting. Therefore, it makes sense to decide on placing an order and setting a price.

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**Appendix A**

**Proof of Theorem 1.** (i) With (4), we have \( \frac{dE_r[U(\pi(x,Q))]}{dQ} = (p-w) - (p-s)F(Q) + (\lambda-1)\frac{(s-w)F\left(\frac{w-s}{p-s}Q\right)}{p-s} \). With (5), we have \( \frac{dE_r[U(\pi(x,Q))]}{dQ} = (e-w) + (s-e)F(Q) + (\lambda-1)(s-w)\frac{F\left(\frac{w-s}{p-s}Q\right)}{p-s} \). Considering the second-order conditions of (4) and (5), respectively, we have \( \frac{d^2E_r[U(\pi(x,Q))]}{dQ^2} = (s-p)f(Q) - (\lambda-1)\frac{(w-s)^2}{(p-s)}f\left(\frac{w-s}{p-s}Q\right) < 0 \) and \( \frac{d^2E_r[U(\pi(x,Q))]}{dQ^2} = (e-s)\frac{(w-s)^2}{(p-s)}f\left(\frac{w-s}{p-s}Q\right) < 0 \). Thus, \( E_r[U(\pi(x,Q))] \) and \( E_r[U(\pi(x,Q))] \) are strict concave functions of \( Q^*_r \) and \( Q^e_r \), respectively.

(ii) Theorem 1(i) explains that there is only one optimal solution for \( \max_{Q^*_r}E_r[U(\pi(x,Q))] \); that is, the root of \( E_r[U(\pi(x,Q))] = 0 \), which is denoted as \( Q^*_r \). Then, we have (6).

(iii) Theorem 1(i) explains that there is only one optimal solution for \( \max_{Q^*_r}E_r[U(\pi(x,Q))] \); that is, the root of \( E_r[U(\pi(x,Q))] = 0 \), which is denoted as \( Q^*_r \). Then, we have (7). \( \square \)

**Proof of Theorem 3.** (i) With (5), we have \( [(e-s)f(Q^*_r) + (\lambda-1)\frac{(w-s)^2}{p-s}f\left(\frac{w-s}{p-s}Q^*_r\right)] \frac{dQ^*_r}{dx} < 0 \). Assume that \( \lambda_1 < \lambda_2 \). When \( \lambda = \lambda_1 \), the optimal order quantity is \( Q^*_r \). When \( \lambda = \lambda_2 \), the optimal order quantity is \( Q^e_r \). We solve for \( Q^*_r \) and the decrement of expected utility related to \( \lambda \). Then we have \( E_r[U(\pi(x,\lambda_1,Q^*_r))] \geq E_r[U(\pi(x,\lambda_2,Q^e_r))] \geq E_r[U(\pi(x,\lambda_2,Q^e_r))] \). Thus, the revenue decreases.

(ii) With (5), we have \( [(e-s)f(Q^*_r) + (\lambda-1)\frac{(w-s)^2}{p-s}f\left(\frac{w-s}{p-s}Q^*_r\right)] \frac{dQ^*_r}{dx} = 1 - F(Q^*_r) > 0 \), so \( \frac{dQ^*_r}{dx} > 0 \). The, we can show that the revenue decreases as \( e \) increases.

(iii) With (5), we have \( [(e-s)f(Q^*_r) + (\lambda-1)\frac{(w-s)^2}{p-s}f\left(\frac{w-s}{p-s}Q^*_r\right)] \frac{dQ^*_r}{dp} = (\lambda-1)\frac{(w-s)^2}{p-s}f\left(\frac{w-s}{p-s}Q^*_r\right) \). Then, we have \( \frac{dE_r[U(\pi(x,Q))]}{dp} = \mu + (\lambda-1)\frac{p}{p-s}QF\left(\frac{w-s}{p-s}Q\right) - \int_0^{\frac{w-s}{p-s}Q} F(x) \). Thus, we can show that the revenue increases with \( p \) increases.

(iv) With (5), we have \( [(e-s)f(Q^*_r) + (\lambda-1)\frac{(w-s)^2}{p-s}f\left(\frac{w-s}{p-s}Q^*_r\right)] \frac{dQ^*_r}{dx} = -\frac{(w-s)^2}{p-s}f\left(\frac{w-s}{p-s}Q^*_r\right) < 0 \). Thus, we can show that the revenue decreases as \( w \) increases.

(v) With (5), we have \( [(e-s)f(Q^*_r) + (\lambda-1)\frac{(w-s)^2}{p-s}f\left(\frac{w-s}{p-s}Q^*_r\right)] \frac{dQ^*_r}{ds} = F(Q^*_r) + (\lambda-1)[F(Q^*_r) + \frac{w-s}{p-s}Q^*_r f\left(\frac{w-s}{p-s}Q^*_r\right) Q^*_r] > 0 \), so \( \frac{dQ^*_r}{ds} > 0 \). Thus, we can show that the revenue increases as \( s \) increases. \( \square \)
References


5. Qiu, R.; Yu, Y.; Sun, M. Joint pricing and stocking decisions for a newsvendor problem with loss aversion and reference point. Omega 2009, 37, 724–730. [CrossRef]


27. Xie, W.; Chen, B.; Huang, F.; He, J. Coordination of a supply chain with a loss-averse retailer under supply uncertainty and marketing effort. J. Ind. Manag. Optim. 2021, 17, 3393. [CrossRef]


31. Dong, C.; Li, Q.; Shen, B.; Tong, X. Sustainability in supply chains with behavioral concerns. Sustainability 2019, 11, 4051. [CrossRef]


34. Choi, S. A loss-averse newsvendor with cap-and-trade carbon emissions regulation. Sustainability 2018, 10, 2126. [CrossRef]

35. Ma, L.; Zhao, Y.; Xue, W.; Cheng, T.C.E.; Yan, H. Loss-averse newsvendor model with two ordering opportunities and market information updating. Int. J. Prod. Econ. 2012, 140, 912–921. [CrossRef]


41. de Souza, T.A.Z.; Pereira, J.L.J.; Francisco, M.B.; Sotomonte, C.A.R.; Jun Ma, B.; Gomes, G.F.; Coronado, C.J.R. Multi-objective optimization for methane, glycerol, and ethanol steam reforming using lichtenberg algorithm. *Int. J. Green Energy* 2022, 1–18. [CrossRef]