Effects of Government Regulations on Under-Reporting of Carbon Emission Transfers by Enterprises in Supply Chains

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Abstract: In low-carbon environments, asymmetric carbon information causes the enterprises in a supply chain to face the risk of misstatements about carbon emissions. Such misstatements could affect the decisions about carbon emission transfers in the supply chain. To optimize carbon emission transfers among supply chain enterprises, this study formulates a supplier-led Stackelberg game model incorporating the government’s initial carbon emission allowances and fines. The study also examines the mechanism of the behaviors of enterprises in low-carbon supply chains, the proportions of initial quotas, the impact of government fines on carbon transfers, and the influence of the supply chain and carbon emission transfers on related supply chain decisions and profits. The main findings are as follows. First, the proportion of the government’s initial quota has a positive effect on the carbon emission transfer quantity of the supplier, while government fines and misstatement factors have a negative effect. Second, the carbon emissions of the unit product of the supplier decrease as the under-reporting factor and carbon emission transfer quantity of the supplier increase. The under-reporting factor has a stronger effect on the carbon emissions of the unit product. Third, in a carbon-free market, carbon emission transfers negatively affect the disclosed profits of the supply chain, whereas in a perfect carbon market, the carbon trading price has a certain endogenous regulation mechanism for the suppliers’ operational decisions. Fourth, the supplier’s wholesale price order quantity is negatively correlated with the supplier’s carbon emission transfer quantity, but positively correlated with the initial carbon quota ratio.

Keywords: supply chains; government regulations; under-reporting of carbon emission transfers; supplier-led Stackelberg game model

1. Introduction

Carbon emission transfers occur when the government of a country or region enacts strict emission reduction policies, which encourage that country’s or region’s enterprises to transfer their responsibilities for emission reduction to enterprises in other countries or regions that do not have such strict policies [1]. Zhong et al. [2], Wang et al. [3], and Xie et al. [4] verified the existence of carbon emission transfers at the macro-level, such as at national and provincial levels; those studies also discussed the directions of the flow of the transfers. Recent studies have examined carbon emission transfers in supply chains at the micro-level [4–6]. While meeting reduction targets set by the government, an enterprise in a supply chain may still attempt to maximize its own interests by transferring difficult emission-reducing activities to other enterprises, either upstream or downstream, in the chain. For example, the logistics outsourcing conducted by HP, the outsourcing of all machine manufacturing by Apple, and the vendor managed inventory (VMI) of Dell all lead to carbon emission transfers in the supply chain, to a certain extent [6–10]. Sun et al. [6] points out that, when carbon emission transfers take place among supply chain enterprises, accurately defining the carbon emission reduction responsibility of supply chain enterprises...
becomes difficult. Although these carbon emission transfers can help the transferor to achieve its emission reduction goals to a certain extent, under the condition of pursuing the maximum benefit, it may also have a negative impact on the emission reduction and profit of the carbon emission transfer undertaker and the overall supply chain, which will disrupt the operation of supply chain enterprises [7].

On the other hand, the implementation of carbon regulation policies causes carbon emission rights to become a production resource. In effect, the carbon emissions of each enterprise become an important form of the enterprise’s private information. Due to the asymmetry of carbon information, the information disclosed by supply chain enterprises is often inaccurate and untimely. Enterprises may even engage in the deceptive behavior of deliberately misreporting carbon emissions. For example, Volkswagen lied about their vehicles’ carbon emissions in 2015; approximately 98,000 gasoline-powered vehicles were involved in carbon dioxide emission data fabrication [11]. Yang et al. [12] pointed out that the misreporting of carbon emissions will further affect the emission reduction decisions of supply chain enterprises. This situation is not conducive to improving the overall emission reduction efficiency of the supply chain. Most carbon regulation policies are designed in a complete information environment [13]. However, in reality, information asymmetry is a common phenomenon. Graafland et al. [14] believed that the government’s non-regulation approach will reduce the social responsibility of enterprises. Zhang et al. [15] maintained that appropriate government intervention can improve the performance of the supply chain and help the supply chain achieve its emission reduction target.

From the above analysis, it can be seen that the behavior of underreporting carbon emissions per unit product, that widely exists in the market, not only directly affects the carbon emission transfer behavior of supply chain enterprises, but also indirectly affects the carbon emission transfer of the supply chain by disrupting the operation behavior of supply chain subjects. However, the existing studies rarely consider the impact of underreporting of carbon emissions, nor the role of carbon emission transfers in reducing supply chain emissions, or the impact of government supervision and quota definition on underreporting and carbon emission transfer. This issue is worth further study. The contributions of this paper are as follows. First, this research explores the effects of the amount of a government’s initial carbon quotas, the punishment intensity of supervision, and carbon trading prices on supply chain carbon emission transfers and other emission reductions. Second, the impact of carbon emission underreporting on supply chain carbon emission transfers is analyzed, as well as related emission reduction decisions under different carbon market situations. Third, this study also explores the effects of suppliers’ carbon emission transfers on the operational and reduction decisions of the various supply chain entities.

The remainder of this paper is organized as follows: Section 2 reviews the relevant literature and discusses the main contributions of these studies. Section 3 presents the problem description and related assumptions. Section 4 introduces the model, and the analysis of the model’s results under two scenarios, namely, a carbon-free market and carbon market improvement. Section 5 presents a numerical analysis. Finally, Section 6 summarizes the main conclusions and offers potential directions for future research.

2. Literature Review

In this section, we review the related literature dealing with supply chain misstate-
ments, government interventions in low-carbon supply chains, and supply chain carbon emission transfers.

2.1. Analysis of Supply Chain Misstatements

In reality, different types of enterprises in a supply chain have their own particular advantages. For example, manufacturers have advantages in production costs, while retailers have advantages in their access to information related to market demand. To maintain their dominant positions and earn more profit, enterprises in a supply chain often lie about their private information, after production begins [16]. This not only
affects operational decisions but also the decision-making behaviors of other entities in the supply chain. Occurrences of misstatements are mainly due to the information asymmetry between various subjects. In the supply chain, a subject’s misreporting behavior is mainly reflected in cost information, demand information, and quality information [17–21]. In actual operations, enterprises often deliberately conceal their cost information to obtain higher profit distributions, which could potentially cause a series of effects on a supply chain. For example, Liu et al. [22] proposed that suppliers who have the advantage of cost information often choose to overstate their costs so as to maximize their profits. Zhang et al. [23] found that buyers who conceal information about demand often earn lower profits than sellers who conceal information about production costs, regardless of whether the sellers are leaders or followers. Zhang et al. [24] showed that, under the private circumstances of a manufacturer’s cost information, their profits may increase, but the profits of the retailers and the entire supply chain will decrease, as well as the recovery rates and trading opportunities of all the members. Yan et al. [25] showed that suppliers sometimes misrepresent product quality information to deceive retailers. Ma et al. [26] found that retailers’ concealment of market demand information can distort order quantities and sales prices, ultimately leading to decreases in the supply chain’s profits.

The above studies indicate that, in a traditional supply chain, the subject of the supply chain could potentially influence decisions about misreporting information on costs, demand, or quality. However, these studies failed to consider the under-reporting of carbon information in low-carbon supply chains. Most scholars believe that improving the quality of the disclosure of carbon information in a low-carbon emission reduction environment will help enterprises reduce emissions and improve business performance [27–29]. However, Qian and Schaltegger [28] proved that, in high-energy-intensive enterprises, the correlation between changes in carbon information disclosures and performances was relatively weak. To meet the government’s emission reduction standards and their own profit requirements, enterprises will use their low-carbon information advantages in the supply chain to misrepresent their carbon information and gain a competitive advantage. For example, Zhou and Wu [11] proposed that, under both structures of supply chain rights, manufacturers may overstate their carbon information. Misrepresentation is always beneficial to the manufacturer but harmful to the interests of the retailer. Lou et al. [30] found that suppliers’ misrepresentations of their emission reduction costs are not entirely unfavorable to manufacturers and could even sometimes increase their profits. Li et al. [31] found that manufacturers’ concealment of emission reduction costs and efforts seriously affects the supply chain’s efficiency in terms of overall emission reductions. A literature review showed that, under the limitations of carbon policies, the amount of carbon emissions, emission reduction potential, and emission reduction efforts have become the private information of enterprises; this has also attracted the attention of some scholars.

However, existing studies only consider the influence of supply chain misstatements on internal decision-making, while ignoring the influence of government decision-making on misstatement behavior. Moreover, the emission reduction behavior of supply chains described in previous studies is still relatively traditional and does not involve the carbon emission transfers of the supply chains. Under the limitation of carbon policy, information on carbon emissions, cost reduction, reduction potential, carbon price, and the efforts of the enterprise have become private information, and misreporting of this carbon information will inevitably have an impact on the transfer of carbon emissions in the supply chain. Therefore, in this paper, carbon information misreporting and supply chain carbon emission transfers are both considered. This approach is conducive to an in-depth analysis of the impact of carbon emission transfer behavior on supply chain-related decisions under the current carbon trading policy.
2.2. Analysis of Government Intervention in Low-Carbon Supply Chains

In recent years, many scholars have analyzed emission reduction in low-carbon supply chains through government participation. To better promote supply chain emission reduction, the government has taken a series of measures, such as opening a carbon emission trading market, implementing a carbon tax, and subsidizing the cost of emission reduction. These steps have achieved certain results. Zhou et al. [32] found through empirical analysis that China’s emissions trading pilot program led to a significant reduction in carbon intensity. The average annual carbon intensity of pilot provinces decreased by about 0.026 tons per RMB 10,000 during the study period. Zhou et al. [33] pointed out that a carbon tax is an effective low-carbon policy tool, which could effectively reduce the use of fossil fuels and improve energy efficiency. The trade-off between cap-and-trade and carbon tax policies has also attracted the attention of researchers. Zeng [34] believed that the country should adopt a single approach (namely, a cap-and-trade or carbon tax) at enterprise level, in order to avoid repeated carbon costs. Camila et al. [35] pointed out that, although a carbon tax becomes effective quite rapidly, carbon trading is more effective in the long run, from the perspective of economic benefits and social recognition.

This study investigates the emission reduction in supply chains in a carbon market trading system. Xuan et al. [36] showed that a carbon market trading mechanism is an important way for the Chinese government to control environmental pollution. Carbon technology and low-carbon energy investments have significant but weaker effects on the intensity of carbon emissions and costs. As such, government fines on excess emissions are very necessary. The carbon market trading system, however, particularly in terms of the proportion of the initial quota, forms of punishment, and many other details, is not clear [37].

Based on the above analysis, the existing literature mainly focuses on the balance between carbon tax and carbon trading policy on the emission reduction effect of a low carbon supply chain under the participation of the government. Under the carbon trading policy, the initial quota ratio of the government and the punishment of excess emissions will further promote the implementation of the carbon trading policy, further regulate the emission reduction behaviors of supply chain enterprises, and have an impact on the emission reduction decisions of supply chain enterprises. However, existing literature is insufficient for the proportion of initial government quota and the penalty for excess emissions. In the context of the carbon market trading mechanism, and considering the existence of information asymmetry, this paper further studies the impact of the government’s penalties for excess emissions, as well as the impact of the government’s initial quota ratio, on supply chain related decisions.

2.3. Supply Chain Carbon Emission Transfer Analysis

Previous studies on carbon emission transfers are not rare. Zhang et al. [38] found that in 2002, 2007, and 2012, Europe had transferred carbon emissions to China through commodity trading. The study, using the Logarithmic Mean Divisia Index (LMDI) decomposition method, concluded that the main cause of the carbon emission transfers was the scale effect. Wang et al. [39] analyzed China’s carbon emission transfers from regional and sectoral perspectives and found that China’s net emissions inflow was generated by the trade between China and developed regions (including North America and Western Europe). According to Sun et al. [40], the eastern region transferred carbon emissions to the central and western regions. As such, the northwestern and northeastern regions became the worst-hit areas for carbon emission transfers and carbon leakages. A spatial structure of “western, central, and eastern” carbon emission transfers had formed. Zhou et al. [41] used input–output models and found that the carbon emission transfers between regions mainly occurred from carbon-intensive manufacturing industries from the underdeveloped areas in the northwest to the developed areas in eastern coastal areas. Chen et al. [42] argued that the energy industry and heavy industry departments have
provided a number of intermediate products and have, therefore, played a significant role in carbon emission transfers.

To sum up, most studies on carbon emission transfers in supply chains have been conducted at the macro-level. These studies examine the causes, influencing factors, and flow directions of carbon emission transfers among countries, regions, and industries. However, they have rarely touched upon the micro-level. In practice, carbon emission transfers among countries, regions, and industries cannot be separated from the commodity flows among different emission reduction subjects. The essence of this flow is the commodity circulation among the interrelated and interacting micro-emission reduction subjects in the supply chain [1,41], which, in turn, is inevitably accompanied by carbon emission transfers. Therefore, Shi et al. [5] aimed to combine input–output model and structural path analysis to trace carbon emission flows from primary producer sectors to final consumer sectors in Chinese supply chains. Sun et al. [6] analyzed the carbon emission transfer and emission reduction problem among enterprises within the supply chain, integrating the influence of government emission reduction policies and the low carbon market, considering the lag time of emission reduction technologies and the low carbon preferences of consumers. Sun and Fang [7] identified and optimized irrational transfers of carbon emissions in supply chains, and analyzed the influences of the changes in various coefficients which affect irrational transfers of carbon emissions in supply chains.

Considering the existence of carbon emission transfers further affects the decision-making of the main body of the supply chain, blurs the boundaries of the emission reduction responsibilities between enterprises, and weakens the effects of the implementation of the government’s carbon policy. Under the background of asymmetric information, this study explores the influence of carbon emission misrepresentation on carbon emission transfers, as well as the effect of carbon emission transfers on supply chain decision-making. The aim is to optimize carbon emission transfers among supply chain enterprises, and provide a theoretical basis for promoting supply chain enterprises and the government to undertake correct operation or emission reduction decisions.

3. Problem Description and Assumptions

This study first assumes a two-tier supply chain system consisting of suppliers and manufacturers who produce and reduce emissions according to the government’s initial carbon quotas. Considering China’s current carbon trading market is at its beginning, some enterprises in industries with high carbon emissions have entered the carbon trading market, such as those in industries with high pollution, such as electricity and transportation, while those in other industries are not included in the carbon trading market. Therefore, this paper mainly considers two scenarios: carbon-free market and carbon market perfection (as shown in Figure 1).

As shown in Figure 1, the low-carbon preferences of consumers not only directly affect the low-carbon operations of manufacturers, but they also indirectly affect suppliers. Therefore, suppliers tend to under-report their emissions, in order to earn more profits or to introduce greener products. To meet the government’s reduction targets, suppliers will take various measures to reduce emissions, but they will transfer particularly difficult reduction activities to the manufacturers. To ensure their own economic benefits, the manufacturers will fully accept such transfers. In a carbon-free market, the government will monitor emissions and impose fines if the reduced emissions are still higher than the quotas. In a perfect carbon market, the government will not monitor emissions, which would be voluntarily adjusted by enterprises whose emissions are lower than their quotas. Such enterprises will sell their remaining carbon emissions at the unit price $P_c$. 
In a low-carbon supply chain, suppliers provide manufacturers with intermediate products. A higher value of the emission reduction coefficient indicates a stronger consumer awareness of low carbon, and a zero value indicates a higher degree of difficulty in the reduction in emissions. The emission reduction cost per unit product of suppliers and manufacturers are $C_s = \frac{1}{2} \lambda \Delta e_s^2$ and $C_m = \frac{1}{2} \lambda \Delta e_m^2$, respectively [11].

The assumptions to build a vendor-led Stackelberg game model are as follows:

1. In a low-carbon supply chain, suppliers provide manufacturers with intermediate products, the outputs of which are equal to the market demand for the end products;
2. The initial carbon emissions quota is relevant only to a single cycle. Therefore, any unused portion of the quota cannot be transferred to the next cycle;
3. Except for the carbon emissions per unit of product in the supply chain, the remainder of the information is complete;
4. The emission reductions of the suppliers and manufacturers are independent of each other;
5. The emission reduction coefficients of the suppliers and manufacturers are equal to each other;
6. Carbon emission reductions and transfers have no effect on manufacturers’ production costs, which have been set to zero here, for the convenience of calculation. The variables mentioned in this paper and their definitions are given in Table 1.
Table 1. Variables and related definitions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>Factor of false reporting of carbon emissions per supplier</td>
</tr>
<tr>
<td>F</td>
<td>Fine that the government imposes on one unit of any carbon emissions exceeding the initial quotas</td>
</tr>
<tr>
<td>( e_1 )</td>
<td>Suppliers’ initial carbon emissions per unit of product</td>
</tr>
<tr>
<td>( e_2 )</td>
<td>Manufacturers’ initial carbon emissions per unit of product</td>
</tr>
<tr>
<td>c</td>
<td>Production cost per unit of product</td>
</tr>
<tr>
<td>( P_c )</td>
<td>Carbon trading price</td>
</tr>
<tr>
<td>( w )</td>
<td>Intermediate product wholesale price</td>
</tr>
<tr>
<td>( p )</td>
<td>Product market sales price</td>
</tr>
<tr>
<td>Q</td>
<td>Product market order quantity</td>
</tr>
<tr>
<td>t</td>
<td>Carbon emission transferred by suppliers</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Initial carbon quota allocation ratio (0 &lt; ( \tau ) &lt; 1)</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Sensitivity coefficients of demand to prices</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Sensitivity coefficients of demand to emission reduction levels</td>
</tr>
<tr>
<td>( E_{m} )</td>
<td>Carbon quota allocated by the government to manufacturers</td>
</tr>
<tr>
<td>( E_{s} )</td>
<td>Initial carbon quota allocated by the government to suppliers</td>
</tr>
<tr>
<td>( \Delta e_s )</td>
<td>Carbon emission reduction per product of suppliers</td>
</tr>
<tr>
<td>( \Delta e_m )</td>
<td>Carbon emission reduction per product of manufacturers</td>
</tr>
</tbody>
</table>

4. Model Construction and Analysis

4.1. Carbon-Free Market Scenario

In a carbon-free market, the government imposes a fine of \( F \) per unit of any carbon emissions exceeding the initial quotas. To obtain more benefits, the supplier underreports, with an under-reporting factor of \( Z \) (0 < \( Z \) < 1), for which a smaller value indicates a higher reporting strength. If the actual carbon emissions per unit product are \( e_s \), then the supplier’s externally disclosed carbon emissions are \( Z e_s \). Therefore, the reduction in the supplier’s product is \( \Delta e_s = e_1 - Ze_s \). If the supplier is under-reporting, then the product demand function in the market is as described below [43]:

\[
Q_L = a - \theta p + \gamma (e_1 - Ze_s + \Delta e_m)
\]  

Next, \( t > \tau e_2 - e_m \) means that the manufacturer has exceeded the quota after accepting a transfer of carbon emissions from the supplier; the manufacturer must, therefore, pay a penalty of \( [E_m - (e_m + t)Q_L]F \). The supplier’s public and actual profit functions, as well as those of the manufacturer, respectively, are:

\[
\pi_{sL} = (w - c)Q_L - \frac{1}{2} \lambda (e_1 - Ze_s)^2
\]  

\[
\pi_s = (w - c)Q_L - \frac{1}{2} \lambda (e_1 - e_s)^2
\]  

\[
\pi_m = (p - w)Q_L + [E_m - (e_m + t)Q_L]F - \frac{1}{2} \lambda \Delta e_m^2
\]  

According to the principle of game inverse solutions, this study first assumes that the supplier’s wholesale price, \( w \), and the emissions per unit product, \( e_s \), are given. The
manufacturer maximizes profits by deciding the product price, \( p \). This study finds the first derivative \( \frac{\partial \pi_m}{\partial p} \) of Equation (4) and sets it to zero. The action rule for \( p \) on \( w \) and \( e_s \) is:

\[
\frac{\partial \pi_m}{\partial p} = a - 2\theta p + \theta w + \gamma(e_1 - Ze_s + \Delta e_m) + \theta F(e_m + t - \tau e_2) = 0
\]  

(5)

Here, \( \frac{\partial^2 \pi_m}{\partial p^2} = -2\theta < 0 \) means that the manufacturer’s profit function is a concave function about \( p \), so the optimal retail price is:

\[
p(w, e_s) = \frac{a + \theta w + \gamma(e_1 - Ze_s + \Delta e_m) + \theta F(e_m + t - \tau e_2)}{2\theta}
\]  

(6)

The decision-making behavior of the supplier should be consistent with its misrepresentation, and the supplier’s decision should maximize public profits. Therefore, \( p(w, e_s) \) is incorporated into \( \pi_{sL} \). The first-order conditions of \( \pi_{sL} \) with respect to \( w \) and \( e_s \) are:

\[
\frac{\partial \pi_{sL}}{\partial w} = \frac{a - 2\theta w + \theta c + \gamma(e_1 - Ze_s + \Delta e_m) - \theta F(e_m + t - \tau e_2)}{2} = 0
\]  

(7)

\[
\frac{\partial \pi_{sL}}{\partial e_s} = -\frac{\gamma Z(w - c)}{2} + \lambda Z(e_1 - Ze_s) = 0
\]  

(8)

The Hesse matrix \( H(w, e_s) \):

\[
H(w, e_s) = \begin{bmatrix}
\frac{\partial^2 \pi_{sL}}{\partial w^2} & \frac{\partial^2 \pi_{sL}}{\partial w \partial e_s} \\
\frac{\partial^2 \pi_{sL}}{\partial e_s \partial w} & \frac{\partial^2 \pi_{sL}}{\partial e_s^2}
\end{bmatrix} = \begin{bmatrix}
-\frac{\theta}{2} & -\frac{\gamma Z}{2} \\
-\frac{\gamma Z}{2} & -\lambda Z^2
\end{bmatrix}
\]  

(9)

Let \( 4\theta \lambda > \gamma^2 \); \( H(w, e_s) \) is a semi-negative definite matrix. From Equations (7) and (8), the optimal decision of the supplier in a carbon-free market (indicated by *) is known by:

\[
w^* = \frac{2\lambda[a - \theta c + \gamma \Delta e_m - \theta F(e_m + t - \tau e_2)]}{4\theta \lambda - \gamma^2} + c
\]  

(10)

\[
e_{sL}^* = \frac{4\theta \lambda e_1 - \gamma[a - \theta c + \gamma(e_1 + \Delta e_m) - \theta F(e_m + t - \tau e_2)]}{Z(4\theta \lambda - \gamma^2)}
\]  

(11)

According to Equations (10) and (11), the following is obtained:

\[
p^* = \frac{a + \theta c + \gamma(e_1 + \Delta e_m) + \theta F(e_m + t - \tau e_2)}{2\theta}
\]  

(12)

\[
Q_L^* = \frac{\theta \lambda[a - \theta c + \gamma \Delta e_m - \theta F(e_m + t - \tau e_2)]}{4\theta \lambda - \gamma^2}
\]  

(13)

\[
\pi_{sL}^* = \frac{\lambda[a - \theta c + \gamma \Delta e_m - \theta F(e_m + t - \tau e_2)]^2}{2(4\theta \lambda - \gamma^2)}
\]  

(14)

\[
\pi_m^* = \frac{\lambda^2[a - \theta c + \gamma \Delta e_m - \theta F(e_m + t - \tau e_2)]^2}{(4\theta \lambda - \gamma^2)^2} - \frac{1}{2} \frac{\lambda \Delta e_m}{4\theta \lambda - \gamma^2}
\]  

(15)

\[
\pi_s^* = \frac{2\lambda^2[a - \theta c + \gamma \Delta e_m - \theta F(e_m + t - \tau e_2)]^2}{(4\theta \lambda - \gamma^2)^2} - \frac{\lambda [e_1 \gamma^2 + 4\theta \lambda e_1(1 - Z)]^2}{2Z^2(4\theta \lambda - \gamma^2)^4} + \gamma^2 \lambda[a - \theta c + \gamma(e_1 + \Delta e_m) - \theta F(e_m + t - \tau e_2)]
\]  

(16)
Proposition 1. In a carbon-free market, the supplier’s wholesale price is negatively related to the supplier’s emission transfers and positively related to the initial quota ratio and government fines. The emissions of the supplier’s unit products are negatively related to the under-reporting factor but positively related to the supplier’s emission transfers.

Proof. To find the supplier’s wholesale price, the emissions per unit product to the supplier’s carbon emission transfer quantity, $t$, the carbon quota ratio, $\tau$, the government penalty $F$, and the first derivative of the under-reporting factor, $Z$, this study obtains: $\frac{\partial w^*}{\partial t} < 0$, $\frac{\partial w^*}{\partial \tau} > 0$, $\frac{\partial w^*}{\partial F} > 0$, $\frac{\partial e^*_s}{\partial Z} < 0$, and $\frac{\partial e^*_s}{\partial t} > 0$. □

Proposition 1 states that, when a supplier transfers more emissions to a manufacturer, the supplier’s reduction costs will also decrease. The supplier can lower the wholesale price to ensure a smoother transfer of products between the upstream and downstream of the supply chain. If the government’s initial quota ratio and fines increase, then suppliers, under less pressure to cut emissions, are more likely to not lie about their carbon emissions. Instead, the suppliers will raise their wholesale prices to compensate for the increased costs of cutting their own emissions.

If a supplier minimizes the emissions per unit product, then their actual reduction capacity is less than the declared capacity. Hence, their initiative to reduce emissions has become debilitated, which may lead to an increase in the actual emissions per unit product. However, to decrease the emission reduction costs, the supplier may not choose to reduce emissions, because doing so would be difficult. Instead, the supplier could transfer the reduction tasks to the manufacturer.

Proposition 2. In a carbon-free market, the supply chain’s order quantity is directly proportional to the initial quota ratio but inversely proportional to the supplier’s transfer quantity.

Proof. The respective first-order derivatives of the order quantity to the supplier’s transfer amount, $t$, and the initial quota ratio, $\tau$, can be obtained: $\frac{\partial Q^*_L}{\partial \tau} > 0$ and $\frac{\partial Q^*_L}{\partial t} < 0$. □

Proposition 2 states that, when the initial quota ratio increases, enterprises only need to invest lower emission reduction costs to complete the emission reduction tasks stipulated by the government. The greenness of products and consumer demand both increase. In addition, the enterprises will have more funds to expand production and supply. Consequently, on-chain product transactions and accompanying order quantities increase. However, if the increased number of transactions requires the manufacturer to receive more emission transfers, then the manufacturer will tend to reduce their orders, so as to reduce their overall costs. When the supplier conducts carbon emission transfers, a deviation exists between the product greenness perceived by the consumers and the carbon emissions disclosed by the enterprises. As a result, the consumers experience regret and will, therefore, reduce their buyback behaviors and also reduce their product order quantities.

Proposition 3. There is an optimal carbon emission transfer quantity ($t_1^*$) by which the supplier can actually obtain the maximum profit. Assuming that the supplier’s actual profit is maximized, their transfer quantity is positively related to the initial quota ratio but negatively related to both government fines and the under-reporting factor.
Proof. To find the supplier’s actual equilibrium profit, a first-order partial deviation of the transfer amount, $t$, is obtained:

$$\frac{\partial \pi_s}{\partial t} = -4\theta^2 F(\theta^2 + \gamma(c_1 + \Delta c_m) - \theta F(e_w + \tau e_2) - c_1)$$

$$\frac{\partial^2 \pi_s}{\partial t^2} = \frac{\lambda \theta F^2 (4\theta^2 \gamma^2 - \gamma^2)}{Z^2 (4\theta^2 - \gamma^2)^2}, \text{ when } \frac{\partial \pi_s}{\partial t} = 0, \text{ then } t_1^* = \frac{(a - \theta c)(\gamma^2 - 4\theta^2 Z^2) + \gamma^3(c_1 + 3) - 4\theta^2 \gamma^2 Z^2 \Delta c_m - \gamma(c_1 Z^2 + 4\theta^2 \lambda c_1 (1 - Z))}{\theta F (\gamma^2 - 4\theta^2 Z^2)} - e_m + \tau e_2$$

From Equations (17) and (18), we know that, if $0 < Z < \sqrt{\frac{\tau^2}{2\theta^2}}$, then $\frac{\partial^2 \pi_s}{\partial t^2} < 0. \quad \square$

Additionally, $\pi_s^*$ is a concave function about $t$, so if $t < t_1^*$, then the supplier’s transfer amount, $t$, and the actual profit, $\pi_s^*$, have a positive relationship, i.e., the latter increases as the former increases. If $t > t_1^*$, then the two variables have a negative relationship, i.e., the latter decreases as the former increases. The transfer quantity can maximize the supplier’s profit within the abovementioned low reporting range.

Assuming that the supplier’s actual equilibrium profit is maximized, their optimal transfer amount, $t_1^*$, is the first-order partial derivative of the under-reporting factor $Z$, the initial carbon quota ratio $\tau$, and the government fine $F$. $\frac{\partial t_1^*}{\partial Z} < 0, \frac{\partial t_1^*}{\partial \tau} > 0, \text{ and } \frac{\partial t_1^*}{\partial F} < 0$.

Proposition 3 states that, to ensure that their actual profit is maximized, the supplier will choose between two options. The first option is to transfer more emissions. Since the government penalizes for emissions that exceed the initial quota, the supplier often reduces their under-reporting factor (i.e., increases the strength of under-reporting), thereby decreasing their ability to reduce emissions. The supplier’s ability to meet reduction goals depends mainly on increasing their transfer quantity. The second option is to reduce the emissions. Then, the optimum strategy would be to increase the under-reporting factor (i.e., reduce the strength of under-reporting) or to not under-report. The latter indicates that they can already meet or are more inclined to improve their own emission reduction capabilities, in order to meet the reduction goals in a timely manner.

If the manufacturer’s profits are insufficient to pay the government’s fines, then they will reject the supplier’s transfers and eventually refuse to cooperate. Hence, for the supplier to obtain the maximum benefits, they must ensure the normal conduct of product transactions, i.e., the manufacturer is profitable. The effective range within which manufacturers can accept carbon transfers is $\left(0, \frac{a - \theta c + \gamma \Delta c_m - e_m + \tau e_2 - \Delta c_m}{\theta F (\gamma^2 - 4\theta^2 Z^2)} \right)$. For the supplier’s optimal transfer amount, $t_1^*$, to be within this range, the supplier’s under-reporting factor, $Z$, must meet $\Delta c_m (\gamma^2 - 4\theta^2 Z^2) - \gamma c_1 (1 - Z) (4\theta^2 - \gamma^2) / 2\theta^2 < 0$. Suppliers can obtain maximum actual profits by transferring emissions.

**Proposition 4.** The overall public profit of the supply chain decreases continuously in line with the increase in the emissions transferred by the supplier. With different under-reporting intervals, the effects of the transfers on the overall actual profit of the supply chain are different.

Proof. To find the supply chain’s overall public profit, $\pi_{scL}^*$, this study takes the partial derivative of the supplier’s transfer amount, $t$, to obtain $\pi_{scL}^* = \pi_{sl}^* + \pi_m^* \frac{\partial \pi_{scL}}{\partial t} < 0$. To
calculate the supply chain’s overall actual profit, $\pi_{sc}^*$, for the first-order partial derivative of $t$, we obtain:

$$\frac{\partial \pi_{sc}^*}{\partial t} = \frac{-4\theta^2F(2\lambda Z-Z^2)}{2Z(4\theta - Z^2)} \gamma^2 + 4\theta \lambda_1 Z(1-Z) - 4\theta \gamma \lambda\gamma_1  \left(1 - \frac{\partial \pi_{sc}^*}{\partial t} = 0 \right.$$

(20)

From Equations (20) and (21), one can know that $0 < Z < \sqrt{\frac{\gamma^2}{4\theta \lambda\gamma_1}}$, then the transferred amount $t$ is positively related to $\pi_{sc}^*$, i.e., the latter increases with the former. If $t < t_2^*$, then the two variables have a negative relationship, i.e., the latter increases as the former decreases. When $Z \geq \sqrt{\frac{\gamma^2}{4\theta \lambda\gamma_1}}$, the above relationship is the opposite. □

Proposition 4 states that, in a supplier-led supply chain, the supplier has sufficient rights. When the supplier underreports the carbon emissions per unit of product, they meet the government’s reduction targets, mainly by transferring emissions to the manufacturer. The manufacturer must then bear most of the reduction costs, thereby leading to a certain reduction in the overall public profit of the supply chain.

As far as the actual profits of the entire supply chain are concerned, when the supplier’s misstatement factor is low $(0 < Z < \sqrt{\frac{\gamma^2}{4\theta \lambda\gamma_1}})$, i.e., when the supplier’s misstatement is strong, the greenness of the product is improved. When the quantity of carbon emission transfers is small $(t < t_2^*)$, the demand for the product, which is driven by consumers’ low-carbon preference, increases, resulting in a continuous increase in the profits of the supply chain. According to Proposition 1, with the continuous increase in carbon emission transfers $(t > t_2^*)$, consumers can obviously perceive the increase in the carbon emissions of the products; the resulting decrease in demand leads to a continuous decrease in the profit of the supply chain. When the misreporting factor is higher than a certain value $(Z \geq \sqrt{\frac{\gamma^2}{4\theta \lambda\gamma_1}})$, the relationship between carbon emission transfers and the actual profits of the supply chain is contrary to the above situation. This finding indicates that the misstatement factor could have a strong effect on the profits of the supply chain.

The comparison shows that $t_2^* < t_1^*$. Therefore, if the supplier chooses their own optimal carbon emission transfer amount, the supply chain’s profits will not reach the optimal state, i.e., the supplier sacrifices the manufacturer’s profit to ensure that the former maximizes their own profit.
4.2. Perfect Carbon Market

In a perfect carbon market, enterprises can buy and sell emission rights to meet their emission needs. Even after accepting transfers from the supplier, the manufacturer must purchase additional rights for any emissions that exceed the initial carbon quota. However, the manufacturer may also sell any surplus rights, should their emissions stay below the quota.

Under the above conditions, a Stackelberg game model with supplier-led manufacturers is established. The demand function for low-carbon products is:

\[ Q_L = a - \theta p + \gamma(e_1 - Zs + \Delta e_m) \]  

(22)

The public and actual profits of the supplier, as well as the profits of the manufacturer, are:

\[ \pi_{sl} = (w - c)Q_L + (E_S - ZsQ_L)P_c - \frac{1}{2}\lambda(e_1 - Zs)^2 \]  

(23)

\[ \pi_s = (w - c)Q_L + (E_S - ZsQ_L)P_c - \frac{1}{2}\lambda(e_1 - e_s)^2 \]  

(24)

\[ \pi_m = (p - w)Q_L + [E_m - (e_m + t)Q_L]P_c - \frac{1}{2}\lambda\Delta e_m^2 \]  

(25)

According to the principle of game inverse solutions, the first assumption is that the supplier’s wholesale price, \( w \), and emissions per unit product, \( e_s \), are given. The manufacturer maximizes profits by determining the product price, \( p \). The first derivative is obtained from Equation (25), so that \( \frac{\partial\pi_m}{\partial p} = 0 \). The action rule of \( p \) on \( w \) and \( e_s \) is:

\[ p(w, e_s) = \frac{a + \theta w + \gamma(e_1 - Zs + \Delta e_m) - \theta P_c(\tau e_2 - e_m + t - \tau e_1)}{2\theta} \]  

(26)

Equation (26) is substituted into the supplier’s public profit function, Equation (23). The first derivatives of \( p \) to \( w \) and \( e_s \) are:

\[ \frac{\partial\pi_{sl}}{\partial w} = \frac{a - 2\theta w + \theta c + \gamma(e_1 - Zs + \Delta e_m) + \theta P_c(\tau e_2 - e_m + t - \tau e_1 + Zs)}{2} = 0 \]  

(27)

\[ \frac{\partial\pi_{sl}}{\partial e_s} = -\frac{ZP_c[a - \theta w + \gamma(e_1 - Zs + \Delta e_m) + \theta P_c(\tau e_2 - e_m + t - \tau e_1)]}{2} \]  

(28)

\[ \frac{\partial^2\pi_{sl}}{\partial w^2} = \frac{-\gamma Z + 2\theta P_c^2}{2} \]  

(29)

Therefore, the Hessian matrix \( H(w, e_s) \) is:

\[ \begin{bmatrix} \frac{\partial^2\pi_{sl}}{\partial w^2} & \frac{\partial^2\pi_{sl}}{\partial w\partial e_s} \\ \frac{\partial^2\pi_{sl}}{\partial e_s\partial w} & \frac{\partial^2\pi_{sl}}{\partial e_s^2} \end{bmatrix} = \begin{bmatrix} -\frac{\theta}{Z(\theta P_c - \gamma)} & -\frac{-\gamma Z + 2\theta P_c}{2} \\ \frac{Z(\theta P_c - \gamma)}{2} & (\gamma P_c - \lambda)Z^2 \end{bmatrix} \]

Let \( 4\theta\lambda > (\theta P_c + \gamma)^2 \) be a negative definite matrix. Equations (27) and (28) can be simultaneously obtained for a perfect carbon market (indicated by *). The optimal decision of the supplier in a perfect carbon market is:

\[ w^* = \frac{a + \theta c + \gamma(e_1 + \Delta e_m) + \theta P_c(\tau e_2 - e_m - t - \tau e_1)}{2\theta} \]

(30)

\[ e_s^* = \frac{(\theta P_c + \gamma)(a + \gamma(e_1 + \Delta e_m) + \theta P_c(\tau e_2 - e_m - t) + (P_c^2(\theta P_c + \gamma)(\theta P_c + \gamma)^2 - 4\theta\lambda)}{Z(\theta P_c + \gamma)^2 - 4\theta\lambda} \]  

(31)
Incorporating Equations (30) and (31) into Equation (26), the optimal decision of the manufacturer can be obtained:

\[
p^* = \frac{3c + 3\gamma (c_1 + \Delta e_m) - \theta P_c (\tau e_2 - e_m - t + \tau e_1)}{(\theta P_c^2 + 3\gamma^2 - 2\gamma \theta P_c) \left[ a + \gamma (c_1 + \Delta e_m) + \theta P_c (\tau e_2 - e_m - t) \right] + (\theta \gamma - 3\gamma) \left[ \theta \gamma \tau P_c + \theta^2 \tau e_2 + c^2 P_c + 40 \lambda c_1 - \theta \gamma c \right]}{2(\theta P_c + \gamma)^2 - 40\lambda}
\]

The product demand and the supplier’s and manufacturer’s profits are:

\[
Q_L^* = \frac{a + \gamma (c_1 + \Delta e_m) + \theta P_c (\tau e_2 - e_m - t) - c \theta P_c - 4 \lambda e_1 - \gamma c}{\theta P_c + \gamma}
\]

\[
\pi_s^* = \frac{\left[ M(K^2 - 40\lambda) - K^2 (M - \theta \tau e_1 P_c + 8\lambda) - \theta KN \right] - 40 \lambda M + K^2 (\theta \tau e_1 P_c - 8\lambda) - \theta KN}{2\theta (K^2 - 40\lambda)^2}
\]

\[
\pi_l^* = \frac{\left[ M(K^2 - 40\lambda) - K^2 (M - \theta \tau e_1 P_c + 8\lambda) - \theta KN \right] - 40 \lambda M + K^2 (\theta \tau e_1 P_c - 8\lambda) - \theta KN}{2\theta (K^2 - 40\lambda)^2}
\]

\[
\pi_m = \frac{Z(K^2 - 40\lambda) \left[ M - 40 \lambda P_c (\tau e_2 - e_m - t) - K^2 (M - \theta \tau e_1 P_c + 8\lambda) - \theta KN \right] + \frac{\theta^2 \gamma P_c^2 + \theta \gamma (e_2 - e_1) P_c - \gamma^2 c_1 - 2\theta \lambda (e_2 - e_1) < 0}{\theta P_c + \gamma}}{4(\theta P_c + \gamma)^2 - 40\lambda}
\]

**Proposition 5.** When a perfect carbon market satisfies certain conditions, the wholesale price, \(w\), is negatively related to the transferred amount, \(t\), but positively related to the initial quota ratio, \(\tau\).

**Proof.** The first derivatives of \(w\) to \(t\) and \(\tau\) can be obtained as follows:

\[
P_c > \sqrt{\frac{\gamma^2 + 40\lambda - \gamma}{2\theta}}, \quad \frac{\partial w}{\partial t} < 0, \text{ i.e., } t \text{ is inversely proportional to } w. \text{ If } \theta^2 \gamma P_c^2 + \theta \gamma (e_2 - e_1) P_c - \gamma^2 c_1 - 2\theta \lambda (e_2 - e_1) < 0, \text{ then } \frac{\partial w}{\partial \tau} > 0, \text{ i.e., } w \text{ increases in line with } \tau. \quad \square
\]

Proposition 5, in contrast to Proposition 1, states that in a perfect carbon market, the relationships among the wholesale prices of supply chain products, the carbon emission transfer quantities of the suppliers, and the initial carbon quota proportion of the government are all affected by the carbon market trading prices. In addition, if the government’s initial quota ratio decreases, then the supplier tries to increase their profits by transferring the difficult-to-reduce emissions to the manufacturer. If the carbon price is higher than a certain value (which is the stop-loss value), the supplier then increases the wholesale price to ensure the normal operation of the supply chain. On the one hand, an overly-high or low initial quota ratio would lead to abnormal wholesale prices. On the other hand, the supplier should optimize their wholesale prices by determining the transferred amount, according to the carbon trading price.
Proposition 6. If the carbon trading price is \( 0, \frac{\theta \lambda - \gamma^2}{\theta \gamma} \), then the sales price is inversely proportional to the initial quota ratio but directly proportional to the transfers. If the carbon trading price is greater than \( 0, \frac{\theta \lambda - \gamma^2}{\theta \gamma} \), then the sales price is directly proportional to the initial quota ratio but inversely proportional to the transfers.

Proof. The respective first derivatives of the sales price to the transferred amount, \( t \), and the initial quota ratio, \( \tau \), can be obtained as follows:

\[
0 < P_c < \frac{\theta \lambda - \gamma^2}{\theta \gamma}, \quad \frac{\partial p^*}{\partial \tau} < 0, \quad \frac{\partial p^*}{\partial t} > 0
\]  

\[37\]

Proposition 6 states that if the initial quota decreases, the manufacturer not only must then reduce their own emissions, the manufacturer must also consider if the supplier is able to meet the reduction targets. If the latter is unable to meet the targets, then they will transfer the emissions that they cannot reduce to the former (the manufacturer). In that case, the manufacturer must invest more funds into reductions, or the manufacturer must purchase emission rights to accept the transferred amount. The manufacturer’s reduction costs will increase accordingly. To compensate for the lost profits, they may choose to increase the sales prices of their products.

If the carbon trading price is too high, then the optimal decision of the manufacturer to ensure their own profits is to accept fewer transferred emissions from the supplier. In that case, the supplier must undertake reduction independently, in which case the supplier’s reduction costs will consequently rise. This will inevitably lead to an increase in the prices of the supplier’s intermediate products and, eventually, of the sales prices of the end products.

Proposition 7. In a perfect carbon market, the effects of the supplier’s emissions per unit product, the product ordering quantities, the initial carbon quota ratio, and the transfer quantity are all the same as in a carbon-free market. However, all the coefficients of the effects are different.

Proof. The respective first derivatives of the carbon emissions from the supplier’s unit product to the transferred amount, \( t \), and the quota ratio, \( \tau \), are:

\[
\frac{\partial e_s^*}{\partial \tau} = \frac{(e_1 + e_2)(\theta P_c + \gamma)\theta P_c}{Z[4\theta \lambda - (\theta P_c + \gamma)^2]} = -\frac{e_1 + e_2}{Z} K_1
\]  

\[38\]

\[
\frac{\partial e_s^*}{\partial t} = \frac{\theta P_c(\theta P_c + \gamma)}{Z[4\theta \lambda - (\theta P_c + \gamma)^2]} = \frac{1}{Z} K_1
\]  

\[39\]

The first derivative of \( K_1 \) on the carbon price \( P_c \) is:

\[
\frac{\partial K_1}{\partial P_c} = \frac{(2\theta^2 P_c + \theta \gamma)[4\theta \lambda - (\theta P_c + \gamma)^2] + 2\theta^2 P_c(\theta P_c + \gamma)^2}{[4\theta \lambda - (\theta P_c + \gamma)^2]^2} > 0
\]  

\[40\]

The respective first derivatives of the product order quantities to \( t \) and \( \tau \) are:

\[
\frac{\partial Q_t^*}{\partial \tau} = \frac{\theta^2 \lambda P_c(e_1 + e_2)}{4\theta \lambda - (\theta P_c + \gamma)^2} = \frac{\theta^2 \lambda(e_1 + e_2) K_2}{4\theta \lambda - (\theta P_c + \gamma)^2} P_c, \quad \frac{\partial Q_t^*}{\partial t} = -\theta^2 \lambda K_2
\]  

\[41\]
The first derivative of $K_2$ on $P_c$ is:

$$\frac{\partial K_2}{\partial P_c} = 4\theta \lambda - (\theta P_c + \gamma)^2 + 2\theta P_c (\theta P_c + \gamma) \frac{\partial}{\partial P_c} \left[ 4\theta \lambda - (\theta P_c + \gamma)^2 \right] > 0 \quad (42)$$

The influencing factors $K_1$ and $K_2$ increase $P_c$. □

Proposition 7 shows that, in a perfect carbon market (in contrast to a carbon-free market), if the initial carbon quota is reduced, then the supplier’s costs of emissions per unit of product increase significantly, in line with the carbon trading price. To meet the reduction targets, the supplier must either improve their ability to reduce emissions by increasing investment in emission reduction technologies, or they must shift the responsibility for reducing emissions by increasing transfers. With the first option, the supplier’s carbon emissions per unit product will be reduced, but the unit product cost will then increase, and product orders will almost certainly eventually decrease. With the second option, after receiving the supplier’s transfers, the manufacturer would have to reduce their scale of production to reduce emissions, eventually leading to a decrease in orders.

**Proposition 8.** As in the case of a carbon-free market, there is an optimal carbon emission transfer that maximizes the supplier’s actual profit. The government’s initial quota ratio positively affects the supplier’s carbon emission transfer amount. The relationship of the carbon emission transfers will be further affected by the transaction price in the carbon market.

**Proof.** The first-order partial deviation of the supplier’s actual equilibrium profit to the transfer amount, $t$, is:

$$\frac{\partial \pi_s^*}{\partial t} = \frac{4\theta \lambda \left( 2(\theta P_c + \gamma)^2 (\theta P_c + \gamma) + (\theta P_c + \gamma) (4\theta \lambda + \theta^2 P_c + 8a P_c + 8\theta P_c)}{2(4\theta \lambda + \gamma)^2} \right) \left( 2 \theta P_c + \gamma ight) \left( 4\theta \lambda - (\theta P_c + \gamma)^2 \right) \quad (43)$$

Letting $\frac{\partial \pi_s^*}{\partial t} = 0$, then

$$t_3^* = \frac{Z(\theta^2 - \theta^2 P_c + 8a P_c + 8\theta P_c \gamma_1) + Z(\theta P_c + \gamma)}{\theta P_c (\theta P_c + \gamma)} = \frac{Z(\theta P_c + \gamma)}{\theta P_c (\theta P_c + \gamma)}$$

and $\frac{\partial^2 \pi_s^*}{\partial t^2} < 0$ is obtained.

The supplier’s actual profit, $\pi_s^*$, is a concave function around the transferred amount, $t$. Therefore, if $t < t_3^*$, then $t$ has a positive relationship with $\pi_s^*$, i.e., the latter increases with the former. If $t > t_3^*$, then the two variables have a negative relationship, i.e., the latter decreases as the former increases. The supplier’s optimal transfer quantity, $t_3^*$, for the first-order partial derivative of $Z$ is:

$$\frac{\partial t_3^*}{\partial Z} = \frac{Z(\theta^2 - \theta^2 P_c + 8a P_c + 8\theta P_c \gamma_1) + Z(\theta P_c + \gamma)}{\theta P_c (\theta P_c + \gamma)}$$

Let $\frac{\partial t_3^*}{\partial Z} = 0$, then $Z^* = \frac{\theta P_c + \gamma}{\theta^2 - \theta^2 P_c + 8a P_c + 8\theta P_c \gamma_1}$ and $\frac{\partial^2 t_3^*}{\partial Z^2} = \frac{Z(\theta^2 - \theta^2 P_c + 8a P_c + 8\theta P_c \gamma_1)}{\theta P_c (\theta P_c + \gamma)}$. When $\theta^2 - \theta^2 P_c + 8a P_c + 8\theta P_c \gamma_1 < 0$ and $P_c > 1 + \frac{8\theta P_c}{\theta P_c}$, then $\frac{\partial^2 t_3^*}{\partial Z^2} < 0$, i.e., the supplier’s carbon emission transfers are a concave function of the misstatement coefficient, $Z$, or conversely,
a convex function of the false reporting coefficient, \( Z \). Taking the first partial derivative of the supplier’s optimal carbon emission transfer quantity, \( t^*_S \), to the government’s initial quota proportion, \( \tau \), then 
\[
\frac{\partial t^*_S}{\partial \tau} = \frac{2(\gamma e_1 + \theta e_2) + \theta (e_2 - e_1)}{\partial \tau + \gamma} > 0
\]
is obtained. Therefore, the supplier’s optimal carbon emission transfers increase in line with the government’s initial quota proportion. □

Proposition 8 shows that, in order to ensure that the supplier’s actual profits are maximized under the constraints of emission reduction, higher carbon trading prices result in the supplier’s under-reporting factor’s being higher, i.e., the more inclined the supplier would be to not under-report and the more inclined the supplier would be to make fewer transfers. However, from the perspective of specific carbon trading prices, the supplier can always find the optimal amounts for under-reporting and transfers at each fixed price. This ability further illustrates that the trading prices have certain endogenous regulatory mechanisms for the operational decisions of the supplier.

From the properties of the above wholesale prices, sales prices, and order quantities, one can see that the government’s initial quota ratio reversely affects the carbon emission transfer behaviors of the supply chain, indicating that the government’s regulatory behavior could play a role in promoting emission reduction. When the supplier’s profit is maximized, the optimal carbon emission transfer amount will increase in line with the initial quota ratio. The latter result is consistent with the conclusion of Zhou and Wu [11]. When the government quota is higher, suppliers are more inclined to transfer more carbon emissions to obtain more profits. This means the suppliers could reduce the effectiveness of government policies in the pursuit of maximizing their own interests.

5. Numerical Analysis

In actual operations, while the carbon trading price and under-reporting factor are fluctuating, the government will adjust the initial quota ratio according to the capacity of the enterprise. To verify the validity of the propositions mentioned in the previous section, numerical analysis were conducted on the basis of parameter assignment. The values of the parameters are [11]:

- \( a = 100 \)
- \( \lambda = 16 \)
- \( \gamma = 3 \)
- \( \theta = 2 \)
- \( c = 5 \)
- \( e_1 = 10 \)
- \( e_2 = 10 \)
- \( \Delta e_m = 5 \)
- \( e_m = 5 \)
- \( t = 1.5 \)
- \( F = 15 \).

(1) This section discusses the effects of supplier carbon emission transfer quantities, under-reporting factors, and the government’s fines on the wholesale prices of the products. The effects of the transfer quantity and under-reporting factors on the suppliers’ emissions per unit of product in a carbon-free market are also examined.

Figure 2 shows that, if \( t \) decreases, then \( Z, F, \) and \( w^* \) increase. However, if the government penalty \( F \) increases, then \( t, Z, \) and \( w^* \) also increase, because the supplier will increase their under-reporting and transfers accordingly. To ensure that the product transactions with the manufacturer proceed normally, the supplier will reduce the wholesale prices. Figure 3 shows that, if \( t \) increases while \( Z \) decreases, then the supplier’s emissions, \( e_s^* \), of the unit product will increase, thereby confirming Proposition 1. If the supplier’s under-reporting increases, then the emissions will increase sharply. This happens because when the supplier’s misstatement coefficient is small, the supplier’s R&D investment is far less than the R&D cost that they should invest. The supplier is more willing to adopt carbon emission transfers to reduce emissions; the product emission reduction is, therefore, not obvious. Due to the under-reporting of suppliers, the nominal greenness of the products increases, as does the demand for the products. This, in turn, leads to a significant increase in the carbon emissions from the products. However, as the emissions continue to increase, the corresponding increase in emissions per unit product will not be significant. This finding is different from that of Sun et al. [6]. In this study, a portion of the carbon emission transfers is actually the product of the misstatements of the supply chain, whose influence also represents the influence of the transfers, to a certain extent. The additional carbon emissions generated during the supplier’s transfer and the carbon emissions generated by the difference in the levels of the technology of the enterprises will
not change significantly with the increase in the carbon emission transfers. The difference between the results of this study and those of Sun et al. [6] is probably due to the existence of carbon information misstatements.

Figure 2. Wholesale prices of supply chain products.

Figure 3. Carbon emissions of supplier’s unit products.

(2) This section discusses the effects of suppliers’ carbon emission transfers and the government’s initial carbon quota ratio on product order quantities in a carbon-free market. Figure 4 shows that, if $t$ decreases, then $Q_L^*$ increases. However, this trend would not be significant, because if the transfers accepted by the manufacturer decrease, then the costs of the manufacturer’s emission reduction would also decrease, while the number of products ordered from the supplier would increase. In addition, the reduction in carbon emission transfers indicates that the supplier is making significant efforts to reduce emissions. Therefore, the product has a better emission reduction effect, its greenness improves, and consumer demand increases. With higher $\tau$ and $Q_L^*$, the trend would be more significant. If the initial quota allocated by the government increases, then the pressure on the suppliers to reduce emissions will decrease. The suppliers can more effectively promote product emissions, and the manufacturers will not need to undertake too many carbon emission transfers.
transfers. With the impetus from the government and the increased market demand, the supplier will invest more capital in R&D, in order to further increase the degree of product greenness, as well as the quantity of the product.

Figure 4. Supply chain product order quantity chart.

(3) This section discusses the effects of under-reporting factors, the government’s initial carbon quota ratio, and the government’s fines on the suppliers’ optimal transfers in a carbon-free market. Figure 5 shows that if $Z$ decreases, the optimal transfer amount, $t_1^*$, will then increase at an accelerated rate, and the supplier’s under-reporting factor, $Z$, will decrease. The supplier presents a strong misrepresentation, but the supplier is still not able to reduce the extra carbon emissions when excess emissions are discovered by the government. In this case, they will face a fine; the benefits of the high degree of the greenness of the products conveyed by the false report will disappear, and the supplier’s optimal carbon emission transfer amounts will increase in line with the decrease in the under-reporting factor. If $\tau$ increases, then $t_1^*$ increases sharply with the initial quota ratio, $\tau$. This signifies that the carbon emission transfer has the characteristic of irrationality under the condition of the supplier’s pursuit of profit maximization. If $F$ increases, then $t_1^*$ increases. If the government’s fine, $F$, is 10 or 15, then $t_1^*$ changes significantly. This finding indicates that, where high government fines are possible, the manufacturers are limited in their ability to transfer carbon emissions, and their suppliers will be more willing to reduce their carbon emissions. Lin and Jia [37] pointed out that a higher emission trading penalty could potentially increase the cost of excessive emission risk; enterprises would, therefore, have a stronger willingness to reduce emissions. The findings of this study are consistent with Lin and Jia’s, but, in the absence of carbon transfer strategies, government fines may be more effective.
(4) This section discusses the effects of the suppliers’ transfers and the government’s initial quota ratio on the overall public and actual profits of the supply chain in a carbon-free market. Figure 6 shows that, if \( t \) increases and \( \tau \) decreases, the overall public profit will always decrease. Figure 7 shows that, if the supplier’s under-reporting factor interval is \((0, 0.35)\) and \( t > t_2^* \), then the supplier’s transfer amount, \( t \), has a negative relationship with the supply chain’s overall actual profit, \( \pi_{sc}^* \), i.e., the latter decreases as the former increases. If the supplier’s under-reporting factor range is \((0.35, 1)\) and \( t < t_2^* \), then, as the transferred amount increases, the overall actual profit still declines. The overall actual profit will continue to increase as the supplier’s low reporting strength decreases, while the actual profit’s growth rate first increases, then decreases. As can be seen, the overall actual profits of the suppliers are more vulnerable to the influence of false reporting factors. When a false reporting factor is equal to 1, i.e., when an enterprise tends not to misrepresent their carbon emissions, and the actual profit of the supply chain is maximized, the profit decreases slightly. When the amount of carbon transfers remains unchanged, the profit is still higher than with other under-reporting factors. This conclusion is similar to the findings regarding the carbon emission reduction per unit product, indicating that only when the carbon emissions of each product are reduced can the profit of the supply chain be improved.
Figure 7. Overall actual profit of supply chain.

(5) This section discusses the effects of the suppliers’ transfer quantities and the government’s initial quota ratio on the wholesale and sales prices, in a perfect carbon market. Figure 8 shows that, if the carbon trading price, \( P_c \), is 10, \( \theta P_c^2 + \gamma P_c - 2\lambda < 0 \), and \( \theta^2\gamma e_2P_c^2 + \theta\gamma(e_2 - e_1)P_c - \gamma^2e_1 - 2\theta\lambda(e_2 - e_1) < 0 \), then \( \tau \) increases, while the transferred amount, \( t \), and the wholesale price, \( w^* \), decrease. The wholesale price becomes more significant as the government’s initial quota changes. If \( P_c \) is 10, then it falls within the range \( \left(0, \frac{\theta\lambda - \gamma^2}{\theta\gamma} \right) \). The sales price, \( p^* \), is inversely proportional to the initial carbon quota ratio, \( \tau \). This relationship is directly proportional to \( t \). Compared with \( t \), \( \tau \) has a greater effect on \( p^* \). If \( \tau \) is within the range \((0.73, 1)\), then \( w^* \) is greater than or equal to \( p^* \). The manufacturer will then choose to interrupt the product transactions with the supplier.

Figure 8. Wholesale and sales prices of supplier’s products.

(6) This section discusses the effects of the under-reporting factors and carbon trading prices on the suppliers’ transfer quantities in a perfect carbon market. Figure 9 shows that, if the carbon trading price is too high or too low, the suppliers will choose the corresponding under-reporting factor. This will result in higher carbon emission transfers. When the carbon trading price is low, suppliers tend not to lower their own carbon emissions; rather, they will conduct a large number of carbon emission transfers. The manufacturers will be more willing to undertake carbon emission transfers, due to the low carbon trading price. However, if the suppliers do not actively research and develop emission reduction technology, a blind transfer will eventually lead to the reduction in supply chain profits.
On the one hand, suppliers want to reduce their own carbon emission reduction costs when the price is high. On the other hand, they also want to have surplus carbon quotas available for resale, in order to increase revenue. Therefore, suppliers often have low under-reporting factors and high amounts of carbon emission transfers. In this case, carbon transfer behavior often increases the difficulty of emission reduction, as well as increasing the emission reduction costs of downstream manufacturers, who have no bargaining power. Zhou and Wu [11] believed that lower carbon trading prices and larger false reporting coefficients lead to the manufacturers being inclined to increase the level of under-reporting, in order to obtain carbon profits. The conclusion of this paper supports the views of Zhou and Wu [11], but this paper also puts forward different views.

6. Conclusions and Management Implications

6.1. Conclusions
In a supply chain, the incomplete disclosure and asymmetry of carbon information causes occasional misrepresentations among the chain’s members. Such misrepresentations not only directly affect the supply chain members’ operational decisions, but they also complicate the carbon emission transfers caused by the flow of goods. By constructing a supplier-led Stackelberg game model, this study analyzes the transfers of a supplier’s unit products under government regulations in two separate scenarios, namely, a carbon-free market and a perfect carbon market. The aim is to further optimize the carbon emission transfer structure of the supply chain and to restrict the carbon misrepresentations of suppliers. The conclusions of this paper are summarized as follows:

(1) The government’s carbon trading policy has certain limitations with regard to regulating the irrational transfers of a supply chain. In reality, if the profit of each member of the supply chain and the overall profit of the supply chain increases after the carbon emission transfer among supply chain enterprises, it indicates that the transfer is a rational transfer of carbon emissions in the supply chain. On the contrary, if there is a profit reduction for any member in the supply chain, the transfer is an irrational transfer of carbon emissions in the supply chain [7]. Compared with no carbon [44,45], in a carbon market situation), supply chain-related decisions, carbon migration and the initial quota proportional relationship are all affected by the price of carbon trading. The carbon trading price impacts the supplier’s operating decisions regarding an endogenous regulation mechanism. However, if the carbon trading prices are too high or too low, the supplier can correspondingly choose to understate the relevant under-reporting factors, leading to higher carbon emission transfers. Therefore, relying solely on carbon regulation policies that do not consider carbon information asymmetry cannot effectively solve the problem of irrational carbon emission transfers in a supply chain;
(2) The consequences of enterprises misreporting carbon information are reflected through carbon emission transfers, which greatly reduce the advantage of transferring carbon emissions as a cost-sharing strategy. In a certain sense, neither carbon information misreporting nor supply chain carbon emission transfers are conducive to supply chain emission reduction. In addition, the misreporting of carbon information has a more significant negative impact on supply chain emission reduction than do carbon emission transfers. Some of the carbon emissions that have been misreported by enterprises will be transferred to the upstream and downstream of the supply chain in the form of carbon emission transfers. Therefore, the government needs to strengthen its supervision of enterprises based on fully understanding the influence of misreporting carbon information on carbon emission transfers of supply chain enterprises;

(3) Enterprises act in their own self-interests when they transfer carbon emissions. When the supplier has the highest profit, the optimal carbon emission transfer amount is higher than the carbon emission transfer amount when the supply chain has the highest profit. That is, the supplier squeezes the profit space of the upstream and downstream through the supplier’s own dominant position, in order to improve its own profits.

6.2. Management Implications

Supply chain carbon emission transfers can be an important way to optimize the resource allocation of a supply chain and to promote cooperative emission reduction. However, because of speculative behaviors in the form of false reports pertaining to carbon information, supply chain carbon emission transfers become an important way for enterprises to reduce emission reduction costs and nominally increase product greenness. These factors greatly reduce the regulatory effectiveness of the government. From the above results, a number of beneficial management implications can be drawn, as follows.

First of all, the government should actively encourage supply chain enterprises to accurately disclose carbon information, in order to solve the problem of carbon information asymmetry. The principal–agent mechanism should be adopted for the design of incentive contracts based on emission reduction results, in order to promote cooperation between suppliers and manufacturers in their emission reduction efforts. Second, the government should take into account the emission reduction capacities of enterprises and national targets; then, the government should reasonably adjust the initial carbon quota proportions. This should be conducted to avoid both overly high quotas, which increase the inertia of enterprises when it comes to reducing emissions, and overly small quotas, which dampen enthusiasm for emission reduction. Meanwhile, market supervision should be strengthened; false reporting and other negative emission reduction behaviors should be punished appropriately. Macro-control should be imposed on the carbon market, according to market fluctuations, in order to ensure that the trading price in the carbon market is within a reasonable range. Third, the government needs to encourage innovation in emission reduction technologies and increase innovation subsidies. In addition, the government and manufacturers need to strengthen and encourage low-carbon consumption, cultivate consumers’ low-carbon consumption concepts, and force suppliers to reduce emissions from the perspective of the consumers. Finally, when receiving carbon emission transfers from suppliers, manufacturers should limit the number of transfers that can take place, in order to avoid irrational transfers. Then, carbon emission transfers would be beneficial to the development of supply chains.

The limitations of this study also point out the direction for future research. First, the model only considers the suppliers’ misstatements about carbon information, while, in fact, manufacturers also make misstatements. In the case of two-way misstatements, how would the decision-making of the supply chain change, and how can the misstatements be restricted? Second, after false reporting, a supplier may actively reduce emissions, transfer all of them, or maintain the current level. Is there an equilibrium state that causes under-reporting to simultaneously have a positive effect on both the emission reduction and the economic benefits of the supply chain? Finally, this study assumes that carbon emission...
permits were used only for this cycle, but the problem may be more complicated when multiple cycles can be stored; especially when government monitors that supply chain enterprises under-report their carbon emissions, supply chain enterprises may be fined.

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