Optimization and Performance Analysis of Francis Turbine Runner Based on Super-Transfer Approximate Method under Multi-Energy Complementary Conditions

Xiaobo Zheng 1, Yaping Zhao 1,*, Huan Zhang 2, Yongjian Pu 1, Zhihua Li 3 and Pengcheng Guo 1

1 Institute of Water Resources and Hydropower, Xi’an University of Technology, Xi’an 710048, China
2 Zhejiang Fuchunjiang Hydropower Equipment Co., Ltd., Hangzhou 311121, China
3 Xi’an Thermal Power Research Institute Co., Ltd., Xi’an 710054, China

* Correspondence: zyp0168@xaut.edu.cn

Abstract: Hydropower unit is the compensation power generation of the energy regulating unit in wind–solar–water multi-energy complementary systems that often require the turbine to operate in a partial working condition area, thereby resulting in problems of low hydraulic efficiency and severe vibration during operation. A multi-objective and multi-condition optimization design method for Francis turbine runner based on the super-transfer approximation approach was proposed in this study. The proposed method aims to improve the hydraulic performance of the turbine, enhance and suppress the vibration of the turbine, and expand the operation range of the turbine on the basis of the actual situation given that Francis turbine frequently operates in low- and ultralow-load areas under the condition of multi-energy complementarity and continuous adjustment of operating conditions. Different operating conditions from low load to full load were selected as performance evaluation conditions. The super-transfer approximation method was used to select the weight co-efficient of water turbine operating conditions, and a multi-objective optimization function with the efficiency and cavitation performance of the water turbine as optimization objectives was constructed to ensure that the optimized water turbine can achieve the optimal performance in the full working condition range. Results showed that the pressure distribution on the blade surface of the optimized runner was uniform and the working ability was enhanced under the condition of ensuring the performance stability of optimal and rated conditions when the original runner was optimized. The hydraulic efficiency of the turbine under the low-load conditions OP1 and OP2 increased by 4.61 and 3.17%, respectively. Hence, the optimized runner is suitable for hydraulic turbines under multi-energy complementary conditions. The results of this study can provide a reference for the optimal design and operation of the turbine runner.

Keywords: multi-energy complementation; super transfer approximation; multiple working conditions; optimal design

1. Introduction

The integration of wind turbine, photovoltaic, and hydropower units into a multi-energy complementary system has become a new research direction for improving energy utilization given that carbon neutrality has become an important objective worldwide [1,2]. Each energy source in traditional energy systems is independent of one another. Each single energy source needs to consider the coordination between different energy sources in various aspects, such as energy production, conversion, and storage, to adapt to the energy utilization mode under the condition of multi-energy complementarity, improve energy utilization rate, and achieve efficient utilization [3]. The strong randomness and uncertainty of wind and solar power generation in multi-energy complementary power generation systems [4] require the real-time adjustment of the hydraulic turbine output of hydropower as compensation power generation to suppress the fluctuation of wind and
Solar power generation effectively. Meanwhile, the operation of hydropower unit in the off-design condition area for a long time poses a great risk to the operation stability of the hydropower unit. Therefore, optimizing the design of the turbine runner and realizing the efficient and stable turbine operation under the condition of multi-energy complementarity are important to achieving the “double carbon” goal.

Many local and foreign scholars have recently carried out a series of investigations on the structural and performance optimization of rotating machinery using various optimization methods [5,6]. Ding and others [7] used the fourth-order Bezier curve and multiple control points to parameterize the middle arc of the blade and optimized the blade profile of the cross-flow fan with weighted average flow under multiple working conditions as the optimization goal by combining Latin hypercube sampling, entropy method, and multi-island genetic algorithm (MIGA). The volume flow of the optimized impeller improved compared with that of the original impeller at different speeds. Wang and others [8] constructed a comprehensive optimization system using the inverse problem design method, numerical simulation, Latin hypercube sampling method, Kriging model, and nondominated genetic algorithm, with blade load as the design parameter and pump section efficiency at 0.7Qdes and 1.1Qdes as the optimization objective. The efficiency of the pump section is significantly improved in a large flow range, and the efficiency at the design point increases by about 4.2%. Shao and others [9] optimized centrifugal compressor blades on the basis of free-form deformation technology (FFD) and computational fluid dynamics (CFD); increased the compression ratio and isentropic efficiency by 0.46% and 0.84%, respectively, through the optimized centrifugal compressor model; and reduced the low-speed area in flow channel and loss. Leonel Alveyro Teran [10] optimized the blade profile of the Francis turbine through an artificial neural network (ANN) and genetic algorithm (GA). The highest efficiency was 14.77%, which was higher than the original scheme, and the occurrence of cavititation in the blade was reduced. The shape class function (CST) method proposed by Kulafe [11] has been increasingly used because of its excellent geometric description ability. Its main idea is to use shape functions constructed by various rational spline polynomials in describing the geometric characteristics of blades.

The multi-objective and multi-condition optimization design method of hydraulic turbine runners has been widely used in analyzing the research status of the optimization design of hydraulic turbines. However, the selection of optimization conditions is mainly performed in the high-load area, and the optimization objective function is typically the average value of different conditions without considering the operation weight factors of different conditions. Therefore, it is unsuitable for the optimal design of turbines under the condition of multi-energy complementation. A multi-objective and multi-condition optimization design method for Francis turbine runner based on the super-transfer approximation method is proposed in this work to improve the hydraulic performance of the turbine in the full working condition range and broaden the working range of the turbine given that the Francis turbine frequently operates in low- and ultralow-load areas under the condition of multi-energy complementarity and continuous adjustment of operating conditions.

The hydraulic turbine is required to operate as an energy regulating unit under the condition of multi-energy complementarity of wind, solar, and water to expand the efficient and stable operation range of the hydraulic turbine as well as improve the operational performance of the low-load area and even the ultralow-load area under the premise of ensuring the operation performance of the rated load area. Therefore, blade placement angle and water head are used as the respective design variable and constraint condition in this study on the basis of impeller optimal design theory and basic requirements of the energy regulation unit to improve the hydraulic performance of the turbine, enhance and suppress the vibration of the turbine, and expand the operating range of the turbine. Weighted values of hydraulic efficiency and cavititation performance based on the super-transfer approximation method are regarded as optimization objectives. A multi-condition and multi-objective optimization method of the turbine based on the super-transfer approxi-
imation method is then constructed. Finally, a turbine runner suitable for the multi-energy complementary condition is designed.

The remainder of this study is structured as follows. The parameterization of the runner is introduced through the Bezier curve and the super-transfer approximation method is used to determine the optimal operating condition weight coefficient in the first section. The efficiency and the weighted value of the minimum pressure value of the draft tube are utilized as optimization objectives. Meanwhile, an approximate response model is constructed. A multi-condition and multi-objective optimization method of the turbine runner combined with the multi-objective genetic algorithm (MOGA) under the condition of multi-energy complementation is established. The numerical calculation method and the selection of operating points are then discussed in the second section. Differences in speed, turbulent kinetic energy, and load of the runner before and after blade optimization are compared and analyzed and the influence of the optimized blade on the hydraulic efficiency of the Francis turbine and flow field characteristics in the runner is explored in the third section. Finally, conclusions of the study on the methods and advantages of the multi-condition and multi-objective optimal design are presented in Section 4.

2. Establishment of Optimization Method

2.1. Blade Parameterization

The parameterization of runner blades is an important part of the optimization design because it aims to use few design parameters, describe the geometry of runner blades accurately [12], adjust the shape of the Bezier curve, and express the blade profile accurately by changing the coordinates of a small number of control points [13,14].

The interpolation of the $n$-th Bezier curve for a point is expressed as follows:

$$ P(t) = \sum_{i=0}^{n} P_i B_{i,n}(t), \quad 0 \leq t \leq 1 $$ (1)

where $t$ is the independent variable implicitly expressed by the Bezier curve. $P_i$ is the $i$-th control point in the Bezier curve. For $n$-th Bernstein basis function $B_{i,n}(t)$ is defined as:

$$ B_{i,n}(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}, \quad i = 0, 1, 2, 3, \cdots n $$ (2)

Runner blades are divided into five three-dimensional sections along the blade height direction in this study. Each three-dimensional section is constructed using cubic Bezier curves. The geometric shape of the runner is described by combining the blade angle and thickness distribution law of the runner. New control blades are generated by stacking profile lines of each section to realize the parametric modeling process of the runner. Twenty design parameters are determined by checking the accuracy of parameterized blades to reflect the original blade shape accurately with minimal design variables. Figure 1 illustrates the runner blade parameterization and Bezier curve control. Figure 1a shows the parameterization results, where gray indicates the original blade and blue indicates the five three-dimensional sections obtained after parameterizing the blade placement angle distribution according to the Bessel curve. Among them, 1–5 are 5 sections of the blade equally divided along the blade height direction. Figure 1b presents the diagram of the parameterization of blade placement angle via the Bezier curve for a single section airfoil. The X-axis is the relative chord length of the blade section airfoil from head to tail, and the Y-axis is the airfoil placement angle. P0–P3 are the four Bezier control points that constrain the placement angle of the blade.
The existing multi-condition optimal design mainly focuses on improving the operational performance of the high-load area, and is not suitable for the hydraulic unit under the condition of multi-energy complementary. Therefore, based on the super-transfer approximation method proposed by Narasimhan [15–17], this paper comprehensively considers the frequency of the turbine in different operating conditions, determines the operating weight factor of the different operating conditions in the optimization design process, and predicts the hydraulic performance, which is more suitable for the hydraulic unit under the condition of multi-energy complementary. Therefore, based on the super-transfer approximation method, the specific structural process is presented as follows:

1. Compare the importance of any two optimization objectives to generate a binary comparison matrix \( A \), where

\[
A = (a_{ij})_{N \times N}, \quad a_{ij} = \frac{1}{a_{ji}}, \quad \text{where } N \text{ is the number of objects.}
\]

Figure 1. Blade parameterization. (a) Schematic diagram of blade parameterization; (b) Bezier curve control diagram.

Figure 2. Comparison of the weight coefficients of each working condition between the super-transfer approximation method and the original design method.

2.2. Super-Transitive Approximate Method

Affected by the wide application of new energy power generation such as wind power generation and solar power generation, hydropower units under the condition of multi-energy complementation need to undertake the task of adjusting the load, and operate in low-load areas with low hydraulic efficiency and frequent hydraulic instability for a long time, namely the operation weight factor of each working condition is different from that of conventional units. Therefore, in the design process of converting a conventional unit into an energy regulating unit, it is necessary to redistribute the weight coefficients of each working condition to ensure the stable operation of the unit in the low-load area. The existing multi-condition optimal design mainly focuses on improving the operational performance of the high-load area, and is not suitable for the hydraulic unit under the condition of multi-energy complementary. Therefore, based on the super-transfer approximation method proposed by Narasimhan [15–17], this paper comprehensively considers the frequency of the turbine in different operating conditions, determines the operating weight factor of the different operating conditions in the optimization design process, and predicts the hydraulic performance, which is more suitable for the hydroelectric unit in the new power system. The weight factor comparison of each working condition after optimization is shown in Figure 2.
(2) Use the complementary matrix $B_i$ to find the super-transitive approximation matrix $A^*$. Complementary Matrix $B_i = (b_{i1}, b_{i2}, \ldots, b_{iN})^T$, where $i = 1, 2, \ldots, N$; $b_{ij} = a_{ij}$. That is, the $i$-th row of the complementary matrix $B_i$ is equal to the $i$-th row of matrix $A$; $b_{iN} = (a_{iN})^{-1}b_{ij}$. $A^* = (a^*_{ij})_{N \times N}$, where $a^*_{ij} = \sqrt[\sqrt{b_{i1}^1 \times b_{i2}^2 \times \cdots \times b_{iN}^N}]; a_1^1, a_2^2, \ldots, a_N^N$ are the elements in the $i$-th row and the $j$-th column of the complementary matrices $B_1, B_2, \ldots, B_N$ respectively.

(3) Determining Weight Coefficients Using Eigenvector Method.

$(A^* - \lambda_{\text{max}} I)W = 0$, $\sum_{i=1}^{N} W_i = 1$, where $a^*_{ij} = \sqrt[\sqrt{b_{i1}^1 \times b_{i2}^2 \times \cdots \times b_{iN}^N}$ is the maximum eigenvector value of $A^*$, $W$ is the characteristic vector corresponding to $\lambda_{\text{max}}$. Then $W_i$ is the weight coefficient of the desired optimization objective.

According to these methods, the final weight coefficients of each working condition are 0.42, 0.28, 0.16, and 0.14.

2.3. Establishment of Optimization System

A multi-condition and multi-objective optimization system is established on the basis of the parameterization of blades and weight coefficients of different operating conditions. This method regards the head as the constraint condition and considers the hydraulic efficiency and the weighted value of the minimum pressure value of the draft tube under various working conditions as optimization objectives, as shown in Figure 3. Low-load conditions A and B are 40 and 60% outputs, respectively; and rated and optimal working conditions are 70 and 100% outputs, respectively. Multiple sample points are generated on the basis of the Latin hypercube sampling experimental design method [18] to ensure the diversity of the sample space and form the sample space. The number of sample points for the optimal design of the Francis turbine runner in this paper is set to 200 given the calculation time and accuracy. Sample space parameters are listed in Table 1. The generated sample space is subjected to CFD calculation [19] to determine the performance parameters through the super-transfer approximation method. The weighted average value of each performance parameter is obtained from the weight coefficient of each working condition, and a reasonable approximate response model is selected. The Kriging model [20] is selected as the approximate framework of the optimal design in this work, and the constructed approximate model is verified using the method of inserting verification points.

Table 1. Sample space.

<table>
<thead>
<tr>
<th>Serial Number</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>...</th>
<th>$x_{17}$</th>
<th>$x_{18}$</th>
<th>$x_{19}$</th>
<th>$x_{20}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original blade</td>
<td>40.35</td>
<td>18.08</td>
<td>25.86</td>
<td>49.71</td>
<td>...</td>
<td>49.02</td>
<td>40.36</td>
<td>68.60</td>
<td>77.55</td>
</tr>
<tr>
<td>1</td>
<td>37.10</td>
<td>17.63</td>
<td>24.24</td>
<td>54.21</td>
<td>...</td>
<td>49.02</td>
<td>40.36</td>
<td>68.60</td>
<td>77.55</td>
</tr>
<tr>
<td>2</td>
<td>42.38</td>
<td>16.37</td>
<td>23.31</td>
<td>45.56</td>
<td>...</td>
<td>51.20</td>
<td>43.05</td>
<td>67.60</td>
<td>75.42</td>
</tr>
<tr>
<td>3</td>
<td>42.75</td>
<td>16.64</td>
<td>27.06</td>
<td>51.18</td>
<td>...</td>
<td>44.83</td>
<td>38.73</td>
<td>73.09</td>
<td>78.36</td>
</tr>
<tr>
<td>4</td>
<td>42.79</td>
<td>19.08</td>
<td>27.40</td>
<td>46.65</td>
<td>...</td>
<td>49.83</td>
<td>42.00</td>
<td>63.07</td>
<td>85.26</td>
</tr>
<tr>
<td>5</td>
<td>42.83</td>
<td>19.75</td>
<td>26.67</td>
<td>51.92</td>
<td>...</td>
<td>53.90</td>
<td>41.55</td>
<td>65.20</td>
<td>80.30</td>
</tr>
<tr>
<td>6</td>
<td>42.87</td>
<td>17.16</td>
<td>27.89</td>
<td>47.40</td>
<td>...</td>
<td>53.36</td>
<td>38.81</td>
<td>74.87</td>
<td>73.71</td>
</tr>
<tr>
<td>7</td>
<td>42.91</td>
<td>16.68</td>
<td>25.85</td>
<td>47.40</td>
<td>...</td>
<td>53.36</td>
<td>38.81</td>
<td>74.87</td>
<td>73.71</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>194</td>
<td>44.36</td>
<td>16.63</td>
<td>23.81</td>
<td>54.53</td>
<td>...</td>
<td>49.27</td>
<td>36.61</td>
<td>75.11</td>
<td>77.21</td>
</tr>
<tr>
<td>195</td>
<td>44.31</td>
<td>16.75</td>
<td>28.07</td>
<td>54.19</td>
<td>...</td>
<td>49.61</td>
<td>44.10</td>
<td>74.77</td>
<td>75.57</td>
</tr>
<tr>
<td>196</td>
<td>44.36</td>
<td>16.63</td>
<td>23.81</td>
<td>54.61</td>
<td>...</td>
<td>49.27</td>
<td>37.55</td>
<td>75.23</td>
<td>78.10</td>
</tr>
<tr>
<td>197</td>
<td>44.33</td>
<td>19.87</td>
<td>28.04</td>
<td>54.55</td>
<td>...</td>
<td>48.57</td>
<td>44.35</td>
<td>74.99</td>
<td>77.26</td>
</tr>
<tr>
<td>198</td>
<td>44.15</td>
<td>18.43</td>
<td>28.30</td>
<td>54.33</td>
<td>...</td>
<td>48.96</td>
<td>42.07</td>
<td>75.41</td>
<td>79.10</td>
</tr>
<tr>
<td>199</td>
<td>44.32</td>
<td>18.58</td>
<td>28.26</td>
<td>54.43</td>
<td>...</td>
<td>50.46</td>
<td>43.30</td>
<td>73.47</td>
<td>76.29</td>
</tr>
<tr>
<td>200</td>
<td>44.15</td>
<td>18.43</td>
<td>27.99</td>
<td>54.40</td>
<td>...</td>
<td>47.88</td>
<td>42.18</td>
<td>75.44</td>
<td>79.19</td>
</tr>
</tbody>
</table>
Figure 3. Multi-condition and multi-objective optimization design method.

Swarm intelligence algorithm and multi-objective genetic algorithm are global optimization algorithms, but the traditional swarm intelligence algorithm is prone to local optimization, long calculation time, and low convergence accuracy; the NSGA-II algorithm can process a group of solutions at the same time, improve the operation speed and robustness of the algorithm, and help to select solutions of multiple Pareto sets for decision-makers in one algorithm run. The multi-objective optimization algorithm can simultaneously optimize tasks with multiple objective functions. Multiple Pareto optima can simultaneously obtain a uniformly distributed solution from the existing Pareto optimal set, thus reducing production time and cost [21,22].

Global optimization based on a multi-objective genetic algorithm (MOGA) is performed in this study given the conflict of multiple optimized objective functions [22–24]. A mathematical model is established by taking the maximum weighted average value of hydraulic efficiency and the minimum pressure of the draft tube under each working condition as the optimization objective, the head under each working condition as the constraint condition, and the $Y$ coordinate of the beta angle as the design variable. The $Y$ coordinate of a single beta angle is its corresponding genetic gene, and the weighted average of hydraulic efficiency and a minimum pressure of draft tube under various working conditions is its corresponding objective function. The initial population number in this work is 10,000, and the fitness evaluation is carried out according to the objective function. Cross mutation iteration is then conducted to obtain the candidate solution before Pareto. Finally, the optimal solution is manually selected. The specific process of genetic algorithm implementation is shown in Figure 4. The maximum allowable Pareto percentage and the convergence stability ratio are used as the convergence criteria of the optimization algorithm to ensure the diversity of the population and obtain the global optimal solution within a reasonable time. The convergence of the algorithm and end of the optimization process is verified when the maximum allowable Pareto percentage reaches 95% and the convergence stability ratio is 0.5. The setting of the multi-objective genetic algorithm is presented in Table 2.
hydraulic efficiency and the minimum pressure of the draft tube under each working condition as the optimization objective, the head under each working condition as the constraint condition, and the Y-coordinate of the beta angle as the design variable. The Y-coordinate of a single beta angle is its corresponding genetic gene, and the weighted average of hydraulic efficiency and a minimum pressure of draft tube under various working conditions is its corresponding objective function. The initial population number in this work is 10,000, and the fitness evaluation is carried out according to the objective function. Cross-mutation iteration is then conducted to obtain the candidate solution before Pareto. Finally, the optimal solution is manually selected. The specific process of genetic algorithm implementation is shown in Figure 4. The maximum allowable Pareto percentage and the convergence stability ratio are used as the convergence criteria of the optimization algorithm to ensure the diversity of the population and obtain the global optimal solution within a reasonable time. The convergence of the algorithm and end of the optimization process is verified when the maximum allowable Pareto percentage reaches 95% and the convergence stability ratio is 0.5. The setting of the multi-objective genetic algorithm is presented in Table 2.

Table 2. Related parameters of multi-objective genetic algorithm.

<table>
<thead>
<tr>
<th>Algorithm Type</th>
<th>Parameter</th>
<th>Numerical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOGA</td>
<td>Initial sample number</td>
<td>10,000</td>
</tr>
<tr>
<td></td>
<td>Number of iteration samples</td>
<td>4000</td>
</tr>
<tr>
<td></td>
<td>Maximum Allowed Pareto Percentage</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>Convergence Stable Ratio</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Maximum number of iterations</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>number of candidate points</td>
<td>5</td>
</tr>
</tbody>
</table>

3. Numeric Calculation Method and Selection of Operating Point

**Numerical Computation Model**

The Francis turbine investigated in this study is shown in Figure 5, and its geometric parameters are listed in Table 3.

![Figure 5. Full channel calculation model.](image-url)
As shown in Figure 6, the head basically remains stable when the total number of grids in the whole fluid field is more than 4.75 million. Therefore, the number of grids in the fluid field is determined at 4.75 million, in which the number of runner grids is 1.967 million. The grid distribution is presented in Figure 7.

Hexahedron grid is used in this work to discrete the computational domain. The calculation efficiency is improved as extensively as possible and the optimization time is shortened to ensure that the calculation accuracy meets the requirements during the optimization design process of the runner. The head of the small-flow condition (OP2) is selected as the basic performance parameter in this study to verify the grid independence. As shown in Figure 6, the head basically remains stable when the total number of grids in the whole fluid field is more than 4.75 million. Therefore, the number of grids in the fluid field is determined at 4.75 million, in which the number of runner grids is 1.967 million. The grid distribution is presented in Figure 7.

**Table 3.** Design parameters of Francis turbine.

<table>
<thead>
<tr>
<th>Geometry Parameter Name</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel diameter D1 (m)</td>
<td>2.46</td>
</tr>
<tr>
<td>Relative guide vane height</td>
<td>0.29</td>
</tr>
<tr>
<td>Rated speed n_r (rpm)</td>
<td>300</td>
</tr>
<tr>
<td>Rated water head (m)</td>
<td>79.5</td>
</tr>
<tr>
<td>Number of runner blades</td>
<td>14</td>
</tr>
<tr>
<td>Number of active guide vanes</td>
<td>24</td>
</tr>
<tr>
<td>Number of fixed guide vanes</td>
<td>24</td>
</tr>
</tbody>
</table>

In this study, CFX is used for numerical simulation, and the SST k-ω turbulence model is used. The following constraints are assumed: boundary conditions of the volute inlet are uniform and the distribution of mass flow is continuous; the outlet boundary of the draft tube is static pressure; the wall is nonslip; and the interface between the outlet of the movable guide vane and the inlet of the runner, outlet of the runner, and the inlet of the draft tube is the “frozen rotor”.

In order to verify the reliability of the numerical simulation results, the turbine efficiency is used as the evaluation value, and the numerical simulation results are compared.
Runner optimization aims to improve the operating efficiency and cavitation performance in the low-load area while ensuring the operating performance under optimal and rated operating conditions as extensively as possible to adapt to hydropower as compensation under the condition of multi-energy complementation of wind, solar, and water. The actual operation of power generation is adjusted. Therefore, the study selects the four operating points shown in Figure 9 for performance evaluation in the process of runner optimization. Each operating point from OP1 to OP4 corresponds to 40% output, 60% output, 70% output and 100% output of the unit respectively. The blue line is the specific speed corresponding to each water head, and the red line is the output limit line.

![Efficiency comparison between the simulated value and experimental value.](image)

**Figure 8.** Efficiency comparison between the simulated value and experimental value.

Four operating points from low load to full load are selected in this work to ensure that the optimized turbine can guarantee the rated operating performance and improve the operating performance of the turbine in the low-load area. The position of the final selected working point is marked on the model characteristic curve, and working points OP1–OP4 correspond to 40, 60, 70, and 100% outputs of the unit.

![Comprehensive characteristic curve of hydraulic turbine model.](image)

**Figure 9.** Comprehensive characteristic curve of hydraulic turbine model.
4. Results and Comparative Analysis

Optimization Results

The runner of the Francis turbine is optimized through the constructed approximate response model combined with the multi-objective genetic algorithm (MOGA). The Pareto solution set under each working condition is obtained after the convergence of the optimization system (Figure 10). The abscissa is the minimum pressure value of the draft tube, the ordinate is the hydraulic efficiency, and the blue point is the Pareto-recommended solution set. The hydraulic efficiency of each working condition affects and restricts one another. Finally, a blade is selected as the final optimized blade in the Pareto solution set on the basis of actual needs of the power station, and the red point in the figure represents the final selected optimized blade. The shape comparison of runners before and after optimization is shown in Figure 11.

Figure 10. Pareto solution set of the optimized blade. (a) OP1; (b) OP2; (c) OP3; (d) OP4.

Figure 11. Comparison of blade geometry before and after optimization.

Figure 12 presents the comparison of the hydraulic efficiency of the turbine before and after optimization. The hydraulic efficiency of the turbine in the low-load area is higher than that of the original blade, except for a slight decrease in the rated point.
after the optimization of the runner. The hydraulic efficiency of working condition OP1 demonstrated the most significant increase from 89.1% before optimization to 92.2%, while that of operating points OP2 and OP3 increased by 1.5 and 0.1%, respectively. The optimized runner exhibits satisfactory energy conversion capacity when running in the low-load area.

![Hydraulic efficiency comparison of hydraulic turbine.](image)

**Figure 12.** Hydraulic efficiency comparison of hydraulic turbine.

Figure 13 illustrates the circulation distribution at the inlet and outlet of the runner before and after the optimization of the turbine. The circumferential velocity of the water flow at the inlet of the runner is reduced, the circulation volume at the inlet of the runner at the lower ring is reduced, and the stable distribution of the water flow inside the runner channel can help improve the internal flow of the runner after optimization under partial load conditions. Unstable flow phenomena, such as off-flow and vortex, play an important role. Reduction of the outlet circulation volume of the runner under partial and optimal working conditions is very important to improve the flow state of the draft tube. The red dashed box is the area with obvious optimization effect.

![Circulation distribution at the inlet and outlet of the runner.](image)

**Figure 13.** The circulation distribution of inlet and outlet of the runner before and after optimization.

(a) inlet of the runner; (b) outlet of the runner.

Three different blade height positions were considered from the upper crown to the lower ring to explore the load distribution on the blade surface before and after runner optimization under different working conditions (Figure 14) as well as obtain the pressure distribution on the blade surface under different working conditions (Figure 15).
Figure 14. The position diagram of each span.

Figure 15. Blade load distribution (100% output). (a) OP1 Blade load distribution; (b) OP4 Blade load distribution.

Figure 15 shows the pressure distribution at different positions of the blade under the two operating conditions of OP1 and OP4. It can be seen from the figure that the low pressure caused by the head impact at the inlet edge of the blade in the low load area and the low pressure within 60% of the blade back side from the head are well improved after optimization. The pressure distribution on both sides of the blade is uniform, and the overall pressure of the blade increases by about 26.8%. After the optimization of the rated load area, the pressure at the inlet of the blade increases significantly, the pressure gradient from the head to the tail of the blade changes greatly, and the difference in the pressure at the outlet of the blade is small. The optimized runner blades can well improve the impact cavitation of the blade head and the airfoil cavitation on the back of the blade in the low load area, thereby reducing the hydraulic loss and improving the hydraulic efficiency of the turbine.

Figure 16 presents the blade turbulent kinetic energy and velocity vector before and after optimization at different relative blade height positions. The position of large turbulent kinetic energy in the runner is mainly distributed at the blade outlet under the OP1 optimization condition when the relative blade height position is 0.05. Evident flow separation occurs in the place where the turbulent kinetic energy is large and forms a small part of the vortex before optimization. The turbulent kinetic energy in the flow...
channel gradually reduces after optimization due to the change of the flow direction of the water flow and the corresponding reduction of backflow at the tail of the blade effectively improves the phenomenon of water flow stability. The turbulent kinetic energy of the original blade is mainly distributed on one side of the blade when the relative blade height is 0.50. The turbulent kinetic energy of the blade inlet part is reduced and a clear vortex is absent after optimization. The turbulent kinetic energy of the blade is mainly distributed in the tail and the turbulent kinetic energy in the flow channel is significantly reduced after optimization when the relative blade height is 0.95. Phenomena, such as flow separation and flow separation inside the runner before and after optimization, are absent under the OP4 working condition. The water flow distribution at the upper crown of the blade before and after optimization is uniform, and the span is 0.50. A small part of the pressure surface is observed at the blade inlet before optimization. The turbulent kinetic energy distribution area and the maximum value in the flow turbulence area are reduced after optimization, thereby indicating that the energy loss in this area is reduced. The turbulent kinetic energy before and after the optimization of the blade is mainly distributed on the pressure surface and the trailing edge at the lower ring of the blade. A and B in the Figure 16a are the local enlarged views of Span = 0.05 before and after blade optimization, respectively, and C is the local enlarged view of Span = 0.50 before blade optimization.

Figure 16. Cont.
1.5% under OP1 and OP2 working conditions, respectively, to ensure the stable performance of the turbine.

Author Contributions: X.Z. and H.Z. carried out the numerical simulations, analyzed data, and wrote the first draft of the manuscript. X.Z., Y.Z. and Z.L. conceived and supervised the study and edited the manuscript. Y.P. and P.G. edited the manuscript. All authors have read and agreed to the published version of the manuscript.

5. Discussion and Conclusions

Runner blades of the Francis turbine operating under the condition of multi-energy complementation of wind and water are optimized with multiple working conditions and multiple objectives to improve the operation performance of the turbine in the partial working condition area. The results of this study can provide guidance for the hydraulic design and operation of the energy-sparing turbine. The following conclusions can be drawn:

1. A multi-condition and multi-objective optimal design system for the Francis turbine under the condition of multi-energy complementation is established. The Bezier curve of integrated blades, calculation of the weight coefficient of the turbine operating condition based on the super-transfer approximation method, the Latin hypercube sampling test design method, the CFD numerical simulation, and multi-condition multi-objective optimization design system of Francis turbine runner based on multi-objective genetic algorithm. As well, using this optimal design system, with the blade placement angle as the design variable, the water head as the constraint condition, and the weighted hydraulic efficiency and cavitation performance of the turbine as the target parameters, the multi-energy-regulated Francis turbine in a multi-energy complementary system is carried out.

2. The hydraulic efficiency of the optimized hydraulic turbine increases by 3.1 and 1.5% under OP1 and OP2 working conditions, respectively, to ensure the stable performance of the hydraulic turbine in the rated working condition. The significant increase of pressure on the head and back of the blade remarkably improves the head impact and back airfoil cavitations of the turbine.

3. The significant reduction of the circulation volume at the outlet of the runner helps reduce the kinetic energy loss at the outlet of the runner and improves the flow state of water flow in the draft tube. The turbulent kinetic energy inside the runner is reduced, and the backflow and eddy current are significantly improved. It can be seen that the optimized runner is more suitable for operation under multi-energy complementary conditions.

Author Contributions: X.Z. and H.Z. carried out the numerical simulations, analyzed data, and wrote the first draft of the manuscript. X.Z., Y.Z. and Z.L. conceived and supervised the study and edited the manuscript. Y.P. and P.G. edited the manuscript. All authors have read and agreed to the published version of the manuscript.

Figure 16. Cloud diagram of turbulent kinetic energy and velocity vector distribution before and after runner optimization (under OP4 condition, turbulent kinetic energy $\geq 1.5 \, \text{m}^2/\text{s}^2$). (a) Comparison of blade turbulent kinetic energy and velocity vector before and after OP1 condition optimization; (A) (B) (C) (b) Comparison of blade turbulent kinetic energy and velocity vector before and after OP4 condition optimization.
**Funding:** This work was supported by National Natural Science Foundation of China (51839010, 52009105), Shaanxi Natural Science Basic Research Program-Yin Han Ji Wei Joint Fund (2019JLP-25), Scientific Research Program of Engineering Research Center of Clean Energy and Eco-hydraulics in Shaanxi Province (QNZX-2019-06) and The Youth Innovation Team of Shaanxi Universities (2020-29).

**References**


