Multistage Game Model Based Dynamic Pricing for Car Parking Slot to Control Congestion

Sowmya Karri † and Meera M. Dhabu *,†

Department of Computer Science and Engineering, Visvesvaraya National Institute of Technology, Nagpur 440010, India
* Correspondence: meeradhabu@cse.vnit.ac.in
† These authors contributed equally to this work.

Abstract: Car parking has become expensive both in terms of money and the time to search for a free parking slot, especially in metropolitan cities. The parking lots nearer to shopping complexes or corporate buildings often get occupied, leaving the farther parking lots empty and leading to traffic congestion near these already busy areas. With a dynamic pricing strategy, the prices of high-demand parking lots can be surged up during peak hours, deviating the traffic towards the unoccupied, cheaper parking lots. A dynamic pricing strategy for car parking slots helps in fairly distributing the traffic among all the parking lots and helps in increasing the revenue of the parking lot management as well. However, increasing the prices unconditionally will affect the drivers. In this paper, a game-theory-based dynamic pricing strategy is proposed that aims to optimize the benefits for both the car parking management and the drivers. The goal is to find the Nash equilibrium price for peak hours and off-peak hours where the car parking management’s revenue is maximized while the price paid by the drivers is minimized. Simulation results show that the proposed model manages to reduce the congestion at peak hours and the profits of parking lot owners are increased.

Keywords: car parking; dynamic pricing; game theory; Nash equilibrium

1. Introduction

Statistics show that there is a constant growth in the number of vehicles, especially in cities. However, the number of parking spaces remains almost constant due to limited resources such as land, lack of proper management and increasing population. Drivers cruise for some time in searching for a free parking lot. This results in wastage of time, energy, and natural resources such as fuel, in addition to the pollution [1] that is caused.

Many cities have adopted smart parking systems such as automated parking systems to park cars in multilevel parking lots by utilizing the space in an intelligent way. Some cities are using Internet of Things (IoT) to detect and send the information regarding empty parking slots to the drivers using sensors. Early researches were mainly focused on different prediction techniques [2–4] to predict the occupancy of parking lots at different time slots. These predictions may help drivers to find free parking spaces, avoiding the need to cruise to find one. Drivers generally prefer on-street parking lots [5] to off-street parking lots as the on-street lots are cheaper and nearer. They cruise to find an on-street parking lot even though the smart parking system informs about a free slot in an off-street parking lot. This leads to traffic congestion near on-street car parks. The multilevel parking systems or the prediction techniques alone cannot resolve parking problems until the demand is properly managed. Even some academics [6] argue that instead of increasing the parking lots, it is better to control the demand. A proper distribution of vehicles looking for parking spaces among all the available parking lots (on-street and off-street) resolves the problem of congestion near on-street parking lots during peak hours and loss of revenue faced by the off-street parking lots. Refs. [7,8] analyzed the behavior of the drivers while parking...
their vehicles and found that parking behavior is majorly influenced by the pricing factor. On the other hand, it is also important not to overburden the drivers with high tariffs.

One way to manage the demand is by using dynamic pricing [9]. Surging up the prices of on-street car parks at peak hours can make some drivers deviate towards the off-street car parks if the cost of the off-street parking slot and the traveling cost to reach the parking slot (total cost induced for parking) is cheaper than the cost of the on-street parking slot. By doing this, the traffic can be distributed between on-street and off-street car parks. Even though the occupancy of on-street car parks will be reduced at peak hours, its revenue can be increased due to the surged prices at these hours. Surged prices may not influence all the drivers. A few drivers may not prefer to drive to a farther parking lot and may still wish to park at the nearer parking lot despite the surged prices. However, most of the drivers prefer to drive to an off-street parking lot to reduce the total cost of the parking; this can help in boosting up the occupancy rate as well as revenue of the off-street car parks. Many researchers have proposed dynamic pricing strategies where the dynamic prices for different time slots are calculated using game theory [10,11].

Game theory is the study of the different ways in which economic agents make interactive choices to produce outcomes (which might not be planned by the individual agents) with respect to the preferences of those agents. In the work presented in this paper, we consider a multistage game between the drivers and the parking lot management. The goal is to influence the behavior of drivers so that there is a fair distribution of traffic among all the parking lots during the peak hours. In order to find a Nash equilibrium price, the parking lot owner acts as a first player by setting a price, while the drivers act as the second players and respond to the price that has been set. Both the players try to maximize their respective utility functions. For the parking lot owners, increasing their profits will be the main priority while the drivers try to minimize the total cost to be paid for the parking. The proposed model is tested on both real-time data and synthetic data for different days, locations of parking lots and time of the day. This is necessary since the number of vehicles searching for parking lot varies vastly based on these factors.

2. Literature Review

A lot of research has been conducted on how to detect, manage and assign parking slots among the vehicles. A parking system was proposed in [12] where the system assigns a parking lot based on the request received from the user. This request includes proximity to the user’s destination (distance the user is willing to walk from the parking lot to their destination) and the maximum cost the driver is willing to pay for the parking lot. The system optimally allocates a parking lot to the user and, at each decision point, the system sends a better choice until the user reaches and occupies the parking space. The allocation process is solved as a sequence of mixed integer linear programming problems at each decision point. The authors focused more on how to manage the reservation of parking lots to avoid resource reservation conflict. According to this model, a user might never obtain a parking lot of his/her choice if his/her request does not match any of the parking lots’ cost or proximity. It was specified that the parking lots have different prices but the pricing strategies followed by parking lots are not mentioned.

Different prediction strategies (Fourier series, k-means clustering, Fourier series and time series) were applied on the occupancy data of Birmingham city council to predict the car parking occupancy rate [4]. The results of predictions were displaced in the designed website that allows drivers to see the forecast a day in advance. These predictions can be rendered useless if the parking slot is taken by the time the driver arrives, since parking availability is a stochastic process. In order to overcome this problem, a multivariate spatiotemporal model [3] was suggested that predicts the occupancy of both on-street and off-street parking lots. This model predicts the parking occupancy at the estimated time of arrival. Both temporal and spatial correlations were considered and the results were tested on the real-time data of San Francisco parking lots. Predicting the occupancy can be
used as additional data to solve the parking management problem but cannot solve the problem alone.

Behavior as a response to cordon pricing and parking tariff in Central Business District (CBD) was analyzed in [7]. According to the study [7], the Parkers may choose to park inside the cordonized pricing zone or outside the CBD and use public transport or walk to the CBD, or even may cancel their trip. This model determines how the Parkers respond to price changes, which is also called Price Elasticity. Various experiments were performed by changing the prices, toll price, etc., and results show how these attributes influence the driver’s choice of parking lot. [8] analyzed the parking behavior of drivers based on different factors such as proximity, parking cost and charging stations. A survey was conducted in this work and the authors tried to find the interdependency of these factors that influence the driver’s behavior. From [7,8], it is clear that parking price has a direct impact on Parkers’ behavior.

A smart parking system was proposed in [13,14] that dynamically allocates parking resources using the dynamic pricing policy. The main aim of the authors of [13] is to allow drivers to make reservations for the parking lots in advance. Prices are determined based on the utilization of parking lots. Prices increase as the utilization of parking lots increases. The percentage of change in price with the percentage of change in utilization is decided by the parking management. In addition to the above model proposed [13], the authors of [14] developed a dynamic pricing model that calculates the price based on three factors: utilization of parking lot, operational cost of the parking lot and late arrival cost when the user arrives late in the booked parking slot. The authors of [15] analyzed parking time using the Discrete Batch Markovian Arrival Process (D-BMAP), where the system identifies the optimal arrival rate. The price proportionally changes once the arrival rate of the on-street parking lot exceeds the optimal value. The strategies proposed by the authors [13–15] update the prices using predefined functions, which are based on the utilization and may have an inverse effect on the revenue if the updated prices are too high for the drivers. It may also lead to another problem if the updated price at a high utilization period is not high enough to divert drivers to another parking lot. Unlike these strategies, in our proposed system, the change in the prices are determined based on the received responses of drivers. Further, finding an equilibrium price will ensure that the updated prices are optimal for both the drivers and the parking lot owners.

The authors of [16] proposed a dynamic pricing policy, which is formulated as a stochastic dynamic programming problem. With the dynamic programming approach, the authors tried to maximize the overall revenue for the entire service horizon in the long run as the reservations are made randomly. The authors [17] focused on competition of the drivers for a parking lot, based on which prices are decided. The models developed by the authors [17] attempted to minimize the system wide driving distances by setting the prices accordingly. A machine-learning-based model was used in [18] to predict occupancy-based prices. Random forest algorithm was applied to predict the occupancy of on-street parking lots and the results were used to determine the parking prices for an intelligent transport system. Unlike these pricing models, the pricing strategy proposed in this work implements Game theoretic strategy since game theory [19] is helpful in understanding the choices in a given situation when there are competing players. It is also well known for helping independent players in making optimal decisions.

Game theory has been employed in various studies of pricing in the transportation field where there exists competition among drivers or between parking lot owners [20,21]. A bilevel model to optimize parking based on Stackelberg’s leader–follower game was developed in [10,11]. The parking management acts as the leader (upper level) [10] and its objective is to have a minimized difference of occupancy among different parking lots and maximize the profits. The drivers act as the followers (lower level) and their objective is to find the free parking slots with minimum price and proximity. The authors [11] proposed a strategy to find optimal parking rate and aimed to balance the spatial and temporal distribution of the vehicles. An auction-based algorithm was used to determine prices
in [22] and to make both Nash equilibrium assignment and optimal assignment of drivers to the parking lots nearly identical. A similar pricing model was developed in [23,24], where prices are decided based on the number of vehicles competing for a particular parking space as in [22]. In [25], Principal-agent theory was applied to the parking pricing model and tested on real-time data for the city of Shanghai. A Macroscopic Fundamental Diagram (MFD)-based strategy is used [26] to find congestion dynamics at the network level and feedback-based dynamic pricing is implemented.

Unlike the strategies in [10,11,22,23], where the authors tried to establish Nash equilibrium among drivers or among the parking lot owners, the pricing strategy model presented in this paper tries to find the Nash equilibrium between drivers and the parking lot management on the bases of prices for a specific time slot. The prices as outcomes of the proposed strategy are updated at every time slot and vary for different locations. In this paper, game-theory-based dynamic pricing for distributed car parking is proposed.

3. Methodology

The general formulation of our parking model is as follows:

- Parking slot is an individual parking space in which one car can be parked. Parking lot refers to the parking area that is dedicated to vehicle parking and comprises multiple parking slots.
- Time slot \( k \) is defined as the time interval in which occupancy of the parking lot is calculated and during this slot parking price will remain constant. In the work proposed in this paper, the duration of time slot \( k \) is 1 h.
- The set of cost prices \( C = \{cp_1, cp_2, \ldots cp_k\} \) represents the cost borne by the parking lot management for a particular parking slot for a different time slot \( k \). We assume that this cost includes the rent of the parking space and maintenance cost.
- The set \( N = \{sp_1, sp_2, \ldots sp_k\} \) represents the fixed prices (nominal or static prices) that are being charged by the parking lot owners for each time slot \( k \) in the absence of dynamic pricing mechanism.
- The set of dynamic prices are represented as \( P = \{dp_1, dp_2, \ldots dp_k\} \) where \( k \) represents the time slot. These prices are the outcome of our proposed model. The dynamic price of a parking slot is always greater than or equal to the cost price of a single parking slot.
- The set \( O = \{so_1, so_2, \ldots so_k\} \) represents the total occupancy (nominal occupancy) of the parking lot for the time slot \( k \) when fixed pricing strategy is used.
- The set \( L = \{do_1, do_2, \ldots do_k\} \) represents the total occupancy of the parking lot when our proposed dynamic pricing strategy is used.
- The Price Elasticity of Demand (PED) is the percentage change in demand for every 1% change in the price. PED is denoted by \( e_k \). The value of \( e_k \) is always negative since the occupancy always decreases with the increase in price.

In this section, we present a game model where we tried to find Nash equilibrium between the two players—namely, drivers and the parking lot owners—regarding prices. A Nash equilibrium finds an optimal state of the game where both players make optimal moves by considering the moves of their opponent.

3.1. System Overview and Assumptions

Metropolitan cities have parking lots where drivers pay and park their vehicles on an hourly basis. On-street parking lots are generally cheaper or even free of cost in many cities. Drivers generally prefer to park in on-street parking slots [6,27] since they are cheaper as well as easily commutable. Off-street parking lots are farther from the city centers and generally require little more effort to commute to their destination. Information regarding vacancy of parking slots can be obtained by the drivers through mobile applications or display screens outside the parking lots. The dynamic prices are displayed at least an hour prior. With the advent of the smart parking system, drivers access information regarding vacancy and prices, based on which they make parking lot choices. These dynamic prices
influence the drivers’ behavior, by encouraging them to choose the lesser priced parking lot compared with the crowded on-street parking lot with higher prices. Drivers may sometimes change their trip mode too and choose public transportation because of the surged prices, thereby reducing pollution and traffic congestion. Parking lot owners’ revenue is calculated using (1). The profit gained by the parking lot owners is the difference between revenue and the cost prices. Profit can be calculated using (2).

\[ R = \sum_{k=1}^{N} d_p d_o_k \]  
(1)

\[ P = \sum_{k=1}^{N} d_p d_o_k - \sum_{k=1}^{N} c_p t_k \]  
(2)

The following assumptions are made for the proposed pricing strategy:

- Cost price of every parking slot of a particular parking lot is the same and includes rental of the space and maintenance cost.
- Sensors are installed at each parking slot and the information about occupancy is accurate.
- PED of the parking lots of different areas at different time slots are known prior.
- Peak hours and off-peak hours depend on the location of the parking lots. For instance, parking areas near shopping malls will have peak hours on weekends, whereas parking areas near corporate buildings will have peak hours on weekdays.
- It is assumed that if any car is parked for more than one time slot, the updated price for the next time slot should be paid by the driver.
- The pricing strategy is modeled exclusively for noncommuters since it is assumed that commuters’ occupancy is almost fixed and does not change frequently with the dynamic pricing. Monthly or yearly subscription parking is recommended for commuters.

The aim of the parking lot owners is to increase their profits. Increasing the prices unconditionally may increase the profits of the parking lot owners for the particular instance, but those prices can be harsh on the drivers. Some drivers may choose not to park in the overpriced parking lots. Eventually, the occupancy of these overpriced parking lots may decrease, thereby decreasing the revenue of the parking lot. So, parking lot owners should consider user satisfaction regarding the prices as well in order to avoid the future loss. The user satisfaction function \( s_k \) [28] is calculated as shown below in (3):

\[ s_k = s_0 \beta_k \left( \frac{d_o_k}{s_0} \right)^{\alpha_k} - 1 \]  
(3)

where \( \alpha_k = \frac{1}{\beta_k} + 1 \) (always \( < 1 \)) and \( \beta_k = -\frac{s_{p_k}}{s_0} \).

If the nominal occupancy \( (s_0) \) is greater than the dynamic pricing occupancy \( (d_o) \), the drivers are not satisfied, i.e., \( s_k \) value will be positive as shown in Table 1. Loss of satisfaction will be added as a cost to the driver. If the dynamic pricing occupancy is greater than the nominal occupancy, it implies that the drivers are satisfied. So, negative satisfaction function is preferable for both drivers and the parking lot owners. The parking lot owners also make sure to increase the satisfaction of the drivers.
3.2. Essential Components of the Game

The important components of the proposed game are the players, strategy sets of each player, utility functions of the players and Nash Equilibrium.

3.2.1. Players

In the proposed model, the two players are the parking lot management and the drivers. We tried to establish a game between a single parking lot owner and a set of drivers.

3.2.2. Strategy Sets of the Players

In the game theory model, every player can choose different moves called strategies from the entire set of possible strategies. The strategy sets of the parking lot owners and drivers are given as follows:

$$S_1 = \{ dp \mid dp \in \mathbb{R}^N, d_{o_{\min}} \leq d_{o(p)} \leq d_{o_{\max}}, dp \geq cp \}$$  \hspace{1cm} (4)

$$S_2 = \{ do \mid do \in \mathbb{R}^N, d_{o_{\min}} \leq do \leq t_k \}$$  \hspace{1cm} (5)

3.2.3. Utility Functions of the Players

The utility functions are used for calculating payoffs (real numbers) for the strategic interactions that occur among the players when the players choose a particular strategy. In the proposed game model, the parking lot owners wish to maximize their profits and also to satisfy the drivers, whereas the aim of the drivers is to minimize the cost to be paid for a parking slot.

The utility function of the parking lot owners for the game model proposed in this work is given by

$$U_1 = \sum_{k=1}^{N} dp_k d_{o_k} - \sum_{k=1}^{N} cp_k t_k - \sum_{k=1}^{N} s_k (d_{o_k}, s_{o_k})$$  \hspace{1cm} (6)

Refs. \cite{29,30} tried to predict the behavior of drivers in choosing parking lots. Factors such as cost of the parking slot, occupancy and walking distances have significant effects on the drivers. The parking lot choice behaviors of most of the drivers are similar. Therefore, the utility function of the drivers is collectively represented as (7)

$$U_2 = - \sum_{k=1}^{N} dp_k d_{o_k} - \sum_{k=1}^{N} s_k (d_{o_k}, s_{o_k})$$  \hspace{1cm} (7)

This is considered as the average behavior of the individual drivers, where $p_k d_{o_k}$ is the price paid by the drivers. Negative sign of the first term in (7) indicates that the drivers wish to minimize the cost to be paid. Drivers’ satisfaction should be added to both the utilization functions $U_1$ and $U_2$. If $s_k$ is negative (i.e., customers are satisfied), the second

---

<table>
<thead>
<tr>
<th>$s_k$</th>
<th>Relation between $d_{o_k}$ and $s_{o_k}$</th>
<th>Occupancy Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative</td>
<td>$d_{o_k} &gt; s_{o_k}$</td>
<td>Dynamic pricing occupancy is greater than the nominal occupancy</td>
</tr>
<tr>
<td>Zero</td>
<td>$d_{o_k} = s_{o_k}$</td>
<td>Dynamic pricing occupancy is equal to the nominal occupancy</td>
</tr>
<tr>
<td>Positive</td>
<td>$d_{o_k} &lt; s_{o_k}$</td>
<td>Dynamic pricing occupancy less than the nominal occupancy</td>
</tr>
</tbody>
</table>

---

Table 1. Satisfaction function.
term in (7) becomes positive. Both the drivers and parking lot owners try to maximize their utility functions under the following conditions.

$$\begin{align*}
(d_p^*, d_o^*) &= \arg \max_{d_p, d_o} U_1 = \sum_{k=1}^{N} (d_p_k d_o_k - c_{p_k} l_k) - s_k \\
d_o^* &= \arg \max_{d_o} U_2 = - \sum_{k=1}^{N} (d_p_k d_o_k + s_k)
\end{align*}$$

Subject to the following:

- The dynamic occupancy should be always less than or equal to the total number of slots in a parking lot.
- The dynamic occupancy should be greater than or equal to the minimum occupancy required. In order to guarantee the minimum occupancy, the parking lot managers should make sure not to increase the prices very high.

$$\begin{align*}
d_{o,k,min} &\leq d_{o,k} \leq d_{o,k,max} \\
\text{and the dynamic price should be always greater than or equal to the cost price of each slot.}
\end{align*}$$

3.2.4. Nash Equilibrium

In game theory, Nash Equilibrium is the point where both players have made their decisions and an outcome is reached. In our model, the goal is to find the optimal price $p^* \in S_1$ and optimal occupancy $l^* \in S_2$ such that a Nash equilibrium $(p^*, l^*) \in S_1 x S_2$ is obtained.

$$\begin{align*}
\forall p \in S_1, p \neq p^*; U_1(p^*, l^*) &\geq U_1(p, l^*) \\
\forall l \in S_2, l \neq l^*; U_2(p^*, l^*) &\geq U_2(p^*, l)
\end{align*}$$

These constraints will make sure that the payoffs of the drivers and parking lot owners are maximized only when the optimal occupancy $l^*$ and optimal price $p^*$ are considered.

3.2.5. Backward Induction to Solve the Game

Backward induction, similar to all game theory, uses the assumptions of rationality and maximization. In this proposed model, the parking lot owners act as the first player by setting a price. The drivers respond to the prices set by the parking lot owners. So, the drivers act as the second players and try to maximize their payoffs. To achieve this, we differentiate $U_2$ with respect to $d_{o,k}$ and equate it to zero. The optimal occupancy $d_{o,k}^*$ obtained will be plugged into the utility function $U_1$. Now, we try to maximize $U_1$ with respect to $d_{p,k}$.

$$\begin{align*}
\frac{\delta U_2}{\delta d_{o,k}} &= -d_{p,k} - \alpha_k \beta_k \left( \frac{d_{o,k}}{s_{o,k}} \right)^{\alpha_k - 1} = 0 \\
d_{o,k} &= \left( -\frac{d_{p,k}}{\alpha_k \beta_k} \right)^{1/\alpha_k - 1} s_{o,k}
\end{align*}$$

Now, we take the second-order derivative of $U_2$ to verify whether the function is a decreasing or increasing function.

Since $\alpha_k < 0$ and $\alpha_k \beta_k < 0$, the diagonal elements of the Hessian matrix are all negative and the off-diagonal elements are zero. Since the Hessian matrix is negative definite, it is proved that $d_{o,k}^*$ is the optimal occupancy for the price chosen by the parking lot management.
Let us consider \( \frac{1}{\alpha_k - 1} = \epsilon_k \) and \( \alpha_k \beta_k = \eta_k \); (14) can be rewritten as

\[
d_o^* = \left( \frac{d p_k}{\eta_k} \right)^{\epsilon_k}
\]  

(16)

Now, we write \( U_1 \) as a function of \( d o_k \) by plugging (15) into (6)

\[
U_1(p) = \sum_{k=1}^{N} d p_k \left( \frac{d p_k}{\eta_k} \right)^{\epsilon_k} - c p_k t_k - s_k \left[ \left( \frac{d p_k}{\eta_k} \right)^{\epsilon_k}, s o_k \right]
\]  

(17)

On taking the first derivative of (16) and equating it to zero, we obtain

\[
d p_k = \left( \frac{d o_k}{s o_k} \right)^{1/\epsilon_k} \eta_k
\]  

(18)

The constraints on occupancy can be written as

\[
d p_{k,\min} < d p_k < d p_{k,\max}
\]  

(19)

where

\[
p_{k,\min} = \max \left\{ c p_k, \left( \frac{d o_{k,\max}}{s o_k} \right)^{1/\epsilon_k} \right\}
\]  

(20)

\[
d p_{k,\max} = \left\{ \left( \frac{d o_{k,\min}}{s o_k} \right)^{1/\epsilon_k} \right\}
\]  

(21)

4. Results

The simulations are performed on an Intel core i5 1.60 GHz CPU running Windows 10. Python 3.7.4 is used for the implementation.

4.1. Real-Time Data Set

The data set of parking lots in Birmingham [31] is used to model our strategy. This data set consists of four attributes, namely, System code number (reference to the parking lot location), Capacity (total capacity of the parking lot), Occupancy (number of parking slots occupied) and Last updated (time stamp at which the data is updated). This data set contains data of 30 parking lots with their occupancy information for different time slots from October 2016 to December 2016. For the game theory model proposed in this paper, the data of a single parking lot with system code “Shopping” are selected. The parking capacity of this parking lot is 1920. A day is divided into 10 slots (k = 7 to 16) from 7 a.m. to 5 p.m. Multiple entries for a single time slot are resolved by taking the maximum occupancy value that is recorded throughout the time slot. The data that are missing for some days due to some technical problem or because of defective sensors are filled using the “Forward Filling” strategy.

The proposed model works for parking lots (capacity higher or lower than the data used in this work) of any city as long as the PED of the considered area is known since the relative distribution of vehicles for different time slots will remain the same for a particular parking lot. The time slots in which the occupancy of parking lot is more than 60% can be considered as peak hours and less than that can be considered as off-peak hours. Since the peak and off-peak hours are dependent on the percentage of occupancy, the capacity of parking lot does not change the results of the proposed model. The parking lots are also categorized based on the activities near parking lots. Two different categories are considered in this paper: parking lots near shopping areas and those near business areas. This categorization is necessary since the PED and peak hours vary for these areas. PED values used for all the game models simulated for the proposed strategy are shown in Tables 2 and 3.
Table 2. PED values for different time slots on weekdays.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PED</td>
<td>−0.001</td>
<td>−0.8</td>
<td>−0.78</td>
<td>−0.7</td>
<td>−0.59</td>
<td>−0.41</td>
<td>−0.37</td>
<td>−0.3</td>
<td>−0.60</td>
<td>−0.75</td>
</tr>
</tbody>
</table>

Table 3. PED values for different time slots on weekends.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PED</td>
<td>−0.001</td>
<td>−0.8</td>
<td>−0.78</td>
<td>−0.7</td>
<td>−0.65</td>
<td>−0.6</td>
<td>−0.45</td>
<td>−0.3</td>
<td>−0.4</td>
<td>−0.55</td>
</tr>
</tbody>
</table>

We considered the minimum and maximum occupancy values (in %) as a response to the dynamic prices, as shown in Table 4. For instance, \( d_{o_{k,max}} = 200 \) at time slot 7, meaning the dynamic occupancy is 200% of the nominal occupancy for that particular time slot. These values are experimentally obtained by changing different values.

Table 4. Minimum and maximum occupancy for different time slots.

<table>
<thead>
<tr>
<th>Slot</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_{o_{k,min}} ) (in %)</td>
<td>100</td>
<td>80</td>
<td>75</td>
<td>50</td>
<td>55</td>
<td>60</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>55</td>
</tr>
<tr>
<td>( d_{o_{k,max}} ) (in %)</td>
<td>200</td>
<td>150</td>
<td>120</td>
<td>70</td>
<td>75</td>
<td>80</td>
<td>75</td>
<td>70</td>
<td>65</td>
<td>65</td>
</tr>
</tbody>
</table>

In this example, we considered the cost price of each parking slot to be USD 50 and fixed price to be USD 100 with a profit of USD 50 for each time slot. The minimum price \( p_{k,min} \) is calculated using (19). It is observed that \( E_k \) has similar properties as the PED value. So, \( E_k \) is considered as the PED value for the time slot \( k \) and \( \eta_k \) is considered as \( sp_k \), i.e., nominal price. The maximum price \( p_{k,max} \) is calculated using (20).

Next, the parking lot manager, as a first player, chooses any price between \( p_{k,min} \) and \( p_{k,max} \) randomly and sets the price for the respective time slot. The responses of the drivers for the newly set prices are calculated as dynamic occupancy using (15).

Figure 1 shows the occupancy of a parking lot during weekdays and weekends. From the graph, it can be observed that the occupancy during weekdays are higher than the occupancy during weekends.

The graphs in Figure 2 show the dynamic prices vs. static prices for different time slots during weekdays and weekends. It is observed that the dynamic prices are lower than the static prices during off-peak hours, whereas they are higher than the static prices during peak hours.
One of the objectives of the model proposed in this paper is to maximize the occupancy during off-peak hours and minimize the occupancy during peak hours. Figure 3 demonstrates that using the proposed model, the occupancy during weekends and weekdays is increased during off-peak hours and are decreased during the peak hours. The proposed model also aims to increase the overall profit of the parking lot management, which helps in improving the revenue.

Figure 4 shows that, with the proposed model, the overall profit of the parking lot owners increased.
The proposed algorithm is also tested for the scenario where the dynamic prices can be lesser than the cost price during off-peak hours. Making the prices of the parking slots lesser than the cost price could cover the sunk costs of the parking lot owners. So, (20) becomes

\[ p_{k,\text{min}} = \max\{0.7 \times c_{p_k}, (d_{k,\text{max}} / s_{0_k})^{1/\epsilon_k} \} \]  

The graphs in Figure 5 show the dynamic prices vs. static prices for different time slots during weekdays and weekends when the dynamic price is allowed to be lower than the cost price and greater than 70 percent of the cost price.

![Figure 5. Dynamic prices vs. static prices on (a) weekdays and (b) weekends.](image)

4.2. Synthetic Data Set

To evaluate the proposed model further, we generated synthetic data using Poisson distribution. Two different parking areas are considered: one near the shopping mall and another near a corporate building. The Poisson distribution is used to generate nominal occupancy (includes both arrival rate and departure rate) for both parking lots from 7 a.m. to 5 p.m., i.e., 10 time slots of one hour each. We considered that each parking lot has a total of 100 parking slots. The cost price is taken as USD 50 for each parking slot and the fixed price is considered as USD 100. It is observed that the traffic near the shopping mall parking lots is greater during the evening on weekdays (peak hours) and throughout the day during weekends; whereas, the number of vehicles waiting to park near corporate buildings is greater in the morning hours and decreases gradually in the evenings as shown in Figure 6. The early hours are the off-peak hours for the parking lot near the shopping mall since the traffic is lesser during these hours.

![Figure 6. Occupancy of parking lots at different time slots during weekdays and weekends. (a) Near shopping mall. (b) Near corporate building.](image)
The dynamic prices obtained using the proposed model for synthetic data are shown in Figure 7. For the parking lot near a shopping mall, dynamic prices are lesser than the fixed price for the time slots 7, 8 and 9 (off-peak hours), whereas the dynamic prices are higher at peak hours, i.e., during time slots 10 to 16. In the case of the corporate building parking lot, dynamic prices are lesser than the fixed price during off-peak hours (7 to 10 time slots and 15 to 16 time slots) and higher than the fixed price during peak hours (time slots 10 to 15).

Figure 8 shows the dynamic occupancy at different time slots near the shopping mall parking lot and the corporate building parking lot. The graphs show that the occupancy is controlled during peak hours and increases during off-peak hours when the proposed dynamic pricing strategy is used.

![Figure 7](image1.png)  
**Figure 7.** Dynamic prices vs. static prices for different time slots. (a) Near shopping mall. (b) Near corporate building.

![Figure 8](image2.png)  
**Figure 8.** Dynamic occupancy vs. static occupancy for different time slots. (a) Near shopping mall. (b) Near corporate building.

The graphs in Figure 9 show the profit earned by the parking lot owners near the shopping mall and the corporate building. Even though there is loss during off-peak hours, there is relatively more profit during peak hours. It is observed that the overall profit is increased.
5. Discussion

Finding a free parking space has become a hectic job for the drivers in major cities. Even though there are enough parking spaces, most of the drivers compete for the on-street cheaper parking lots, leaving the off-street parking lots empty. Proper management of this traffic will resolve the problem of congestion near on-street parking lots, thereby saving lots of time and fuel that would be wasted cruising to find a free parking slot. Early studies tried to analyze the behavior of drivers and, of many factors that influence drivers’ behavior, price plays a major role in choosing a parking lot. Our research tries to find an optimal dynamic pricing strategy and also to improve the revenue of the parking lots (both on-street and off-street.)

Previous researches proposed different dynamic pricing strategies where the price of the parking lot depends on factors such as arrival rate, competition between drivers and proximity. Some studies even focused on implementing game theoretical models to find the Nash equilibrium among drivers or among the parking lot owners. The pricing strategy proposed in this paper uses a multistage game model where the two players—drivers and parking lot owners—try to maximize their benefits and the model finds a Nash equilibrium between these two players regarding the prices of the parking slots. Unlike previous studies, the proposed pricing strategy is developed for different parking locations, for different days and for different parking hours (peak and off-peak hours). The outcomes of previous studies regarding the occupancy predictions of parking lots and PEDs can be used to improve the accuracy of the pricing strategy proposed in this paper.

6. Conclusions

The proposed dynamic pricing model can be used as a pricing tool to control the traffic near on-street parking lots during peak hours and deviate this traffic towards off-street parking lots, which remain unoccupied otherwise. The proposed model ensures that the prices will be high enough during peak hours such that the revenue of parking lot owners is increased. In addition, our proposed model also ensures that the prices are not so high that there is negative impact on the drivers. The proposed strategy is developed and tested on both real-time data of parking lots of Birmingham city and a synthetic data set for different days, i.e., weekdays and weekends, since the number of vehicles searching for parking lots varies according to the time of day, type of day and location. The results show that the game-theory-based pricing strategy is a promising strategy to obtain the best prices and also helps in controlling traffic congestion near parking lots. Proper distribution of traffic among parking lots will not only save drivers’ cruising time, but also reduce fuel consumption and pollution.

In the future, we aim to evaluate our model on a more appropriate data set of parking lots in Seattle city. We will also aim to develop a competitive game model in which the
dynamic prices consider the pricing competition among the parking lots nearby as well. We will try to add other parameters such as proximity to the destination in the satisfaction function of the drivers.

The contribution of the proposed work is outlined as follows:

• In the proposed model, dynamic prices for parking slots at different time slots are obtained using the multistage gaming model.
• Unlike the existing game-theory-based dynamic price strategies, we tried to find the Nash equilibrium between the parking lot owners and the drivers on the basis of prices.
• In the proposed system, the changes in the prices are obtained on the basis of received responses of drivers.
• Finding an equilibrium price will ensure that the updated prices will be optimal for both the drivers and the parking lot owners.
• The proposed strategy is tested on the parking data set of Birmingham city.
• Data of two different parking lots such as the parking lot near a shopping mall and the parking lot near a corporate building are generated and the proposed strategy is tested on the synthetic data sets as well.

Author Contributions: The research was designed and performed by S.K. and M.M.D. The data was collected and analyzed by S.K. The methodology was proposed and implemented by S.K. under the supervision of M.M.D. The paper was written by S.K., reviewed and edited by M.M.D. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: On behalf of all authors, the corresponding author states that there is no conflict of interest.

Abbreviations

- \( k \) Time slot
- \( cp_k \) Cost price
- \( sp_k \) Static price charged by the parking lot owners
- \( c_k \) Price Elasticity of Demand (PED)
- \( dp_k \) Dynamic prices
- \( so_k \) Occupancy when static prices are used (Nominal Occupancy)
- \( do_k \) Occupancy when dynamic prices are used
- \( t_k \) Total available parking slots
- \( s_k \) Satisfaction function of the drivers
- \( do_{k,min} \) Minimum occupancy when dynamic pricing is used
- \( do_{k,max} \) Maximum occupancy when dynamic pricing is used
- \( dp_{k,min} \) Minimum dynamic price
- \( dp_{k,max} \) Maximum dynamic price
- \( dp^* \) Optimal price
- \( do^* \) Optimal occupancy

References


11. Zong, F.; He, Y.; Yuan, Y. Dependence of parking pricing on land use and time of day. *Sustainability* 2015, 7, 9587–9607. [CrossRef]


27. Hoang, P.H.; Zhao, S.; Houn, S.E. Motorcycle drivers’ parking lot choice behaviors in developing countries: Analysis to identify influence factors. *Sustainability* 2019, 11, 2463. [CrossRef]
