The Modeling and Simplification of a Thermal Model of a Planar Transformer Based on Internal Power Loss

Zhan Shen 1,2, Bingxin Xu 1, Chenglei Liu 1, Cungang Hu 2, Bi Liu 2, Zhike Xu 1, Long Jin 1,*, and Wu Chen 1

1 Jiangsu Provincial Key Laboratory of Smart Grid Technology and Equipment, Southeast University, Nanjing 210096, China
2 Power Quality Engineering Research Center, The Ministry of Education, Anhui University, Hefei 230601, China
* Correspondence: jinlong@seu.edu.cn; Tel.: +86-136-0517-5556

Abstract: With the development of high-performance wide-band-gap devices and increasing converter frequency, planar transformers are widely used in high-frequency and high-power-density power conversions. Due to the skin effect and proximity effect, accurate thermal analysis and a simplified thermal model of planar transformers are needed for quick thermal verification as well as system design. This paper proposes two thermal simplification models based on the planar transformer's thermal impedance network. The internal power loss and thermal coupling between each component are first analyzed. Then, based on thermal radiation theory, the simplified thermal model of the planar transformer is presented. It only requires the input of the total power loss of the planar transformer to calculate the temperature rise, and it does not need the power loss of each component. Finally, the simulation and experimental verification are carried out on a MHz prototype.

Keywords: planar transformer; internal power loss; thermal impedance; thermal coupling; thermal simulation; thermal model simplification

1. Introduction

The development of wide-band-gap (WBG) semiconductor technology, e.g., gallium nitride (GaN) HEMTs and silicon carbide (SiC) devices, enables the tolerance of higher temperatures and higher efficiency and switching frequency than conventional Si material devices [1]. Using those WBG devices, power electronic converters are capable of operating at a higher frequency, e.g., hundreds of kHz and MHz [2,3]. Generally, in magnetic design, the area product, which is the product of the core cross-section area and the winding area, is inversely proportional to the operation frequency [4]. Therefore, with the increase in operating frequency, the cross-section area as well as the winding area, is inversely proportional to the operation frequency [4]. Therefore, with the increase in operating frequency, the cross-section area as well as the winding area of magnets decrease significantly; in turn, the volume decreases, and the power density increases accordingly [5]. Planar transformers are becoming increasingly popular in high-frequency and high-power-density converters because of their advantages of small size, ease of manufacturing, and large heat balance surface [6,7]. Its structure is shown in Figure 1. With the demand for miniaturization, accurate thermal stress information of the transformer is needed to improve its thermal reliability. The high-frequency skin effect and proximity effect will increase the internal temperature of the transformer, leading to the overheating of the winding and insulation part of the printed circuit board and affecting its reliability [8]. The FR-4 of the printed circuit board (PCB) degrades under thermal stress and collides at the edge of the board. The key parameters such as inductance and parasitic capacitance will also drift. This condition can affect the normal operation of the circuit [9]. Therefore, it is essential to establish the thermal model of a planar transformer accurately and quickly for a model-based design, especially in harsh applications requiring high temperatures, such as aerospace and military systems.
Thermodynamic models of different components of planar transformers have been established in the past. In [10], based on the principle of fluid mechanics, the thermal characteristics of PCB are modeled by the finite volume method. The maximum connection temperature, the thermal distribution and the influence of different ambient temperatures on the thermal characteristics of PCB are obtained. As described in [11], a new SPICE ferroelectric–thermal model of ferromagnetic core is given, and the calculation methods for the relevant magnetic parameters, geometric parameters, and thermal parameters are introduced. On this basis, an inverse thermal model based on the temperature rise of the transformer core is proposed to determine core loss [12]. For this purpose, PCB with thermal sensors is initially used to measure temperature rises, whereas these models in [10–12] lack comprehensive comparison, and different winding structures will lead to different power loss distributions inside. In order to predict the temperature distribution of each component and provide the basis for high reliability, the thermal network modeling of planar transformers is essential [13]. The analysis of temperature modeling requires knowledge of transformer structures, power loss models and temperature points of interest [14,15]. In [16], a thermal model of the transformer considering multi-physics coupling is proposed. It can improve the accuracy of the hotspot temperature calculation but with a significant increase in model complexity. A steady-state thermal network model is proposed on an underwater ring transformer [17]. This model is more straightforward than the finite element thermal simulation model and shows acceptable error, but the application range of the model is limited in underwater scenarios.

Currently, there is much research on the thermal model analysis of planar transformers based on component modeling [18]. In [19], in order to analyze the internal space of the transformer in detail and improve the heat dissipation capability, the finite element method is used to conduct the thermal analysis of magnetic cores with different shapes. The inhomogeneous temperature distribution in a planar transformer is obtained by computational fluid dynamics (CFD) simulation [20]. In [21], a thermal simulation method for planar magnetic elements based on a lumped element thermal network model is proposed. The method is suitable for planar magnetic elements composed of double planar E-shaped cores and the combination of planar E core and plate. In [22], a three-dimensional frequency-thermal model of planar transformers based on lumped parameters network was established according to different thermal resistances, including convection, conduction and radiation heat transfer. It is worth noting that this model not only considers the three-dimensional geometry effect of the reactor core but also considers the surface thermal resistance to simulate the high-frequency operation effect. The transient thermal change of the transformer is analyzed with a lumped capacitance thermal model. To perform the thermal design of the planar transformer, a new thermal network model is proposed in [23]. The thermal resistance is measured based on the linear principle, and temperature increase formulas for the magnetic core and coil are proposed. However, its thermal resistance needs to be obtained in multiple experiments. The whole magnetic component is regarded as a compact block, assuming homogenous internal temperature, and the equivalent thermal resistance is then obtained by fitting the third-order analytical model with CFD simulation results as in [24], or by analytical calculation as in [25]. As described in [26], linear and nonlinear thermal models are fitted experimentally to obtain temperature dynamics. These studies show that the inhomogeneity of internal temperature is caused by power loss and uneven structure. However, apart from the detailed analysis model, there is no efficient modeling method for the temperature difference of each component. In [27], a three-dimensional finite element thermal simulation model of high frequency DC-DC converter plane transformers is presented. It is established based on the detailed information of transformer structure. The influence of frequency and output current on temperature distribution can be evaluated to improve transformer reliability during the design phase. However, the heat transfer coefficient of the model needs to be fitted, which requires highly accurate model input parameters. In [28], two kinds of transformers in a high-frequency LLC resonant converter are analyzed by finite element analysis (FEA), and the loss of transformer is
calculated. Using thermal transient software for long-time thermal simulation, the error between the simulation and the experiments is small. However, there are many thermal transient simulations to be performed in the whole procedure, which are complicated and time-consuming. In [29], based on the electro-thermal-magnetic coupling effect of planar transformers, a thermal model capable of detecting the winding temperature is proposed. Using the concepts of a multi-source alternative thermal model in the calculation process can effectively shorten the calculation time and analyze the thermal characteristics under various boundary conditions; however, the modeling process is also complicated.

**Figure 1.** Structure of a planar transformer and its components: the primary windings, the secondary windings, and the core.

The comparison of the thermal modeling methods is summarized in Table 1. The computer-aid modeling methods such as FEA and CFD are accurate but require both significant time and computing resources. On the other side, the thermal circuit and network models can achieve quick computation time without sacrificing too much accuracy, but the procedure to determine the related parameters is complex and requires highly accurate parameter input. Therefore, in this paper, two simplified methods of the thermal modeling of a planar transformer are proposed on the basis of the thermal network model. Their parameter calculation is simple and needs no precise information on the detailed structure of the planar transformer. Moreover, the analytical model can be obtained based on simplification. Finally, all the simplifications are based on reasonable assumptions in order to maintain good accuracy, especially for a compact transformer design. For the scenarios with high percentages of air space in the transformer core window, the accuracy of the proposed models decreases accordingly. Therefore, the proposed model is especially suitable for high-density planar transformers. It can be used in model-based iteration design implemented in a design code flow or for quick thermal verification.

**Table 1.** Comparisons of existing and proposed models and solutions.

<table>
<thead>
<tr>
<th>Model or Solution</th>
<th>Description</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA [19,27–29]</td>
<td>finite element analysis</td>
<td>small error</td>
<td>high accurate parameter requirements, time consuming</td>
</tr>
<tr>
<td>CFD [20,24,25]</td>
<td>computational fluid dynamics</td>
<td>accurate fluid analysis</td>
<td>complex modeling process, time consuming</td>
</tr>
<tr>
<td>Multi-physics Model [16,20,29]</td>
<td>multi-physics coupling</td>
<td>accurate, including multi-physics impacts</td>
<td>complex simulation procedure, time consuming</td>
</tr>
<tr>
<td>Steady-state Thermal Circuit Model [17,23]</td>
<td>to establish the thermal circuits</td>
<td>simpler than simulation</td>
<td>higher error than simulation</td>
</tr>
<tr>
<td>Thermal Network Model [21,22]</td>
<td>to establish thermal network model of planar transformers</td>
<td>analytical model</td>
<td>complex model parameter calculation procedure</td>
</tr>
<tr>
<td>Simplified Models in This Paper</td>
<td>two kinds of simplification methods</td>
<td>simple modeling procedure, good accuracy level for compact design</td>
<td>accuracy decreases with the increase of air space in the transformer</td>
</tr>
</tbody>
</table>
In this paper, first, the thermal impedance network of a planar transformer is introduced by further considering the environment and thermal coupling between iron core and winding. Then, two simplification methods are presented based on thermal analysis and thermal equivalence. Finally, the accuracy of the simplified models is compared and verified on the MHz prototype. The proposed simplification model greatly reduces the difficulty and complexity of thermal analytical modeling and simulation of the planar transformer without sacrificing the accuracy of the thermal modeling.

2. Electromagnetic and Thermal Modeling of Planar Transformer

2.1. Power Loss Model for Windings and Core

2.1.1. Winding Losses

The losses of the magnetics are the sources in the thermal modeling. To determine and predict the temperature distribution at different positions in the winding and core of a planar transformer, the high-frequency losses of each part should be analyzed first, and the non-uniform thermal models under different winding structures can be established later.

Typically, there are three kinds of losses in high-frequency transformers, i.e., the dielectric loss in the insulation material, the resistive loss in the winding, and the core loss in the magnetic core. The dielectric loss is generated in the high electric field. It can take as high as 17% of the total loss in medium-frequency, high-voltage, and high-power wiring transformers [30]; however, planar magnetics normally works in the high-frequency range with lower voltage applied in the insulation material [31,32], and the related loss is generally neglectable [33,34]. Hence, only the winding and core loss-induced thermal response is discussed in this paper.

The ac resistance is calculated by the classical Dowell’s equation [35] considering both the impact of the skin and proximity effect in the high-frequency range:

\[ R_{ac} = R_{dc} F_r = R_{dc} \Delta \left[ v_3 + \frac{2}{3} \left( p^2 - 1 \right) v_2 \right], \]

where \( F_r \) is the ac resistance coefficient between winding ac resistance \( R_{ac} \) and winding dc resistance \( R_{dc} \) and \( p \) is the number of layers. \( v_2 \) and \( v_3 \) are the proximity and skin effect coefficients, which are calculated by:

\[ v_2 = \frac{\sinh \Delta - \sin \Delta}{\cosh \Delta + \cos \Delta}, v_3 = \frac{\sinh (2\Delta) + \sin (2\Delta)}{\cosh (2\Delta) - \cos (2\Delta)} \]

where \( \Delta \) is the penetration ratio, which is defined by:

\[ \Delta = \Delta' \sqrt{\frac{h}{d_c}}, \Delta' = \sqrt{\frac{h}{d_c}}, \frac{h}{d_c} = \frac{d_w}{\delta} \sqrt{\frac{h}{d_c}} \]

where \( h \) is the width of the winding, \( d_c \) is the height of the core window, \( d_w \) is the thickness of the PCB copper layer, and \( \delta \) is the skin depth at the frequency under consideration. With Dowell’s equation, the power loss of each layer is then calculated.

In Dowell’s equation, the magnetic field is assumed to be parallel to the winding layer, which means the one-dimensional flux distribution, and is very suitable for magnetic field distribution in planar magnetics [36]. Further, the coefficient \( \sqrt{\frac{h}{d_c}} \) is used to compensate the magnetic field distortion in the high-frequency range.

From Equation (1) and Figure 2, the magnetic field as well as the loss of the planar transformer in each layer are different due to the proximity effect. Therefore, the loss distribution in different layers of winding is not uniform, and different winding layers are considered different sources in the subsequent thermal modeling.
In Dowell’s equation, the magnetic field is assumed to be parallel to the winding and is the phase angle, $B$ is the core flux density, $K$, $a$, and $b$ are inherent characters determined by the material property, and these are obtained from the datasheet provided by the core manufacturer.

2.2. Thermal Network Modeling and Analysis

Once the loss of the planar transformer is obtained, the thermal model can be established by building the thermal network and modeling the thermal impedances. In hundreds of kHz and MHz range, the planar transformer is normally designed with a compact, high-power-density structure. The internal side of the transformer is composed of PCB windings and insulation materials, and the core surrounds those components with a huge coverage area. Therefore, either there is little space for the air in the transformer, or the air inside is difficult to exchange heat with the air outside the core. The core acts as a cabinet of the whole system, and the heat exchange between the external environment and the internal winding, insulation and air can be neglected.

The system thermal network modeling is illustrated in Figure 3, where $P_{w1}$, $P_{w2}$, ..., $P_{wn}$ and $T_{w1}$, $T_{w2}$, ..., $T_{wn}$ are the losses and temperature of the winding at each layer, $R_{12}$, $R_{13}$, ..., $R_{1n}$ are the thermal resistances between each layer, and $P_{core}$ is the loss of the core.

![Figure 2. The cross-section and magnetic field intensity distribution of transformer winding.](image)
Once the loss of the planar transformer is obtained, the thermal modeling includes the core and the winding. The thermal modeling of the planar transformer is:

\[ R_{\text{core}} \text{ and } R_{\text{rad}} \]

The thermal exchange inside and outside of the core is neglected for the external thermal resistance in the previous analysis. Therefore, the thermal resistance between them is infinite, and only the resistance between the core and ambient \( R_{\text{core}} \) is needed outside the core. Generally, \( R_{\text{core}} \) is the parallel connection of the convection resistance \( R_{\text{conv}} \) and radiation resistance \( R_{\text{rad}} \). The convection resistance model \( R_{\text{conv}} \) is [39]:

\[
R_{\text{conv}} = \frac{1}{h_{\text{conv}} A_{\text{conv}}} = \frac{1}{\frac{\lambda}{\rho C} \cdot A_{\text{conv}}} \tag{5}
\]

where \( h_{\text{conv}} \) is the convective coefficient and is determined by \( \text{Nu} \), \( \lambda \), and \( L \). \( \text{Nu} \) is the Nusselt number, \( \lambda \) is the thermal conductivity of the air, and \( A_{\text{conv}} \) is the convective area, which is the surface area of the core; \( L \) is the defined surface length of different surfaces. The model of thermal radiation resistance is:

\[
R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_{\text{rad}}} = \frac{1}{\epsilon \sigma \left( \frac{T_a^4}{T_s^4} - 1 \right) \cdot A_{\text{rad}}} \tag{6}
\]

where \( h_{\text{rad}} \) is the radiation transfer coefficient determined by \( \epsilon \), \( \sigma \), and the temperature difference of ambient temperature and \( T_a \) and \( T_s \). \( T_a \) refers to the core and winding temperature \( T_{\text{core}}, T_{w1}, T_{w2}, \ldots, T_{wn} \) when calculating corresponding \( R_{\text{rad}} \). Both \( T_a \) and \( T_s \) are in absolute temperatures (Kelvin). \( \epsilon \) is the emissivity of the surface material, i.e., ferrite core, \( \sigma \) is Stefan-Boltzmann constant, \( A_{\text{rad}} \) is the radiation area, i.e., the surface area of the core.

Further, all the internal heat transfers to the core and then to the external environment, so the internal thermal resistance considering the thermal conduction is:

\[
R_{\text{cond}} = \frac{1}{h_{\text{cond}} A_{\text{cond}}} = \frac{1}{\frac{\lambda}{l} \cdot A_{\text{cond}}} \tag{7}
\]

where \( h_{\text{cond}} \) is the conductivity coefficient determined by \( \lambda \), \( l \) is the length in the direction of heat flow, and \( A_{\text{cond}} \) is the conduction area between the winding and insulation, or the insulation and the core.
Finally, the thermal network in Figure 3 is expressed as below with the loss and thermal resistance derived from the previous analysis:

\[
\begin{bmatrix}
T_{w1} \\
T_{w2} \\
\vdots \\
T_{wn}
\end{bmatrix} =
\begin{bmatrix}
R_{11} & R_{12} & \cdots & R_{1n} \\
R_{21} & R_{22} & \cdots & R_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
R_{n1} & R_{n2} & \cdots & R_{nn}
\end{bmatrix}
\begin{bmatrix}
P_{w1} \\
P_{w2} \\
\vdots \\
P_{wn}
\end{bmatrix} + T_{core}
\]

(8)

This equation is the general expression of the thermal impedance network, which suits arbitrary winding configurations whether of parallel, series, or interleaved structure. Moreover, the radiation resistance \( R_{rad} \) by (6) is a nonlinear equation that depends not only on the material property and transformer surface but also on the ambient and transformer temperatures. Therefore, an iterative process is needed to solve the thermal network analytically. Finally, due to the big volume of the winding and core, there is the temperature distribution and difference in the same layer and core, and the temperature calculated in this formula is the hotspot temperature, which is normally located in the middle of the layer or the core.

3. Thermal Model Simplification

The thermal modeling presented in the previous section considers the temperature difference between each layer and helps to locate the hotspot location in the windings during the thermal analysis. However, most design procedures normally only limit the highest temperature of the structure, and the heat location information is not necessary. Therefore, this paper presents an equivalent simplified model for the thermal calculation of a planar transformer that is simpler than the detailed layer-by-layer thermal modeling.

3.1. Planar Transformer System to the Simplification Models

The structure of the planar transformer itself is very compact, and the core acts as a cabinet of the winding, insulation and air inside. Therefore, these internal components are thermally isolated from the external environment, and heat conduction is the main internal heat transfer method. The total loss of the planar transformer is the main factor affecting the temperature rise. The structure of the core-enclosed winding largely cuts off the conduction between the winding and the air. Therefore, as the first kind of simplification, the whole planar transformer structure is simplified as a discrete magnetic core model in the thermal analysis, as shown in Figure 4, the total loss of the transformer is added to the core:

\[
P_{\text{core}}' = P_{\text{total}} = \sum_{i=1}^{n} P_{\text{ei}} + P_{\text{core}}
\]

(9)

Figure 4. The simplified core thermal impedance model.
For the second kind of simplification, the whole planar transformer is further simplified as a cuboid model, as shown in Figure 5. The equivalent cuboid has the same external dimension and heat transfer capability as the planar transformer but with a much simpler structure. Similarly, the total loss in Equation (9) is applied to the cuboid model.

Figure 5. The simplified cuboid thermal impedance model.

3.2. The Equivalent Calculation

The radiation thermal resistance of each part is obtained in Equation (6). Assuming the $T_a$, $T_s$ and $\varepsilon$ are consistent in all directions due to the small volume of the transformer, all thermal radiation resistances are equivalent to parallel connection. Similarly, the thermal convection resistance of each part can be obtained in Equation (7) where $Nu$ is also a constant. The total thermal radiation resistance and convective thermal resistance are:

$$
R_{rad} = \frac{1}{\varepsilon} \frac{\sum_{i=1}^{n} A_{rad,i}}{Nu \sum_{i=1}^{n} A_{conv,i}}
$$

$$
R_{conv} = \frac{1}{Nu \sum_{i=1}^{n} A_{conv,i}}
$$

Normally, the planar transformer is with the cuboid dimension, so $n = 6$ in the formula represents six radiation thermal resistances of the transformer on six surfaces of the cuboid. In Equation (10), $A_{rad,i}$ and $A_{conv,i}$ are simplified as the projection area of the transformer in these six directions.

In the analytical calculation procedure, the transformer in both two simplified models can be further considered as one node in thermal network modeling. Therefore, the whole system in Figures 4 and 5 is further simplified as a thermal network of two nodes (the ambient and transformer), an equivalent thermal resistance between them (combing convection and radiation resistances), a power input in the transformer node, and an assumed temperature for the ambient $T_a$. Equation (10) is used to calculate the thermal resistance, and a simple equation is used to calculate the temperature of the transformer $T_{trans}$:

$$
T_{trans} = R_{total} P_{total} + T_a
$$

where $R_{total}$ and $P_{total}$ are the total thermal resistance and loss of the transformer, respectively.

When using the simplified magnetic core and cuboid to represent the planar transformer in thermal modeling, the thermal resistance $R_{rad}$ before and after simplification are equivalent. Therefore, the emissivity of the surface $\varepsilon$ of the magnetic core or cuboid in the simplified model is modified according to their total projection area in six directions:

$$
\frac{1}{\varepsilon \sum_{i=1}^{6} A_{rad,i}} = \frac{1}{\varepsilon^m \sum_{i=1}^{6} A_{rad,i}^m} = \frac{1}{\varepsilon^c \sum_{i=1}^{6} A_{rad,i}^c}
$$
In Equation (11), $\varepsilon_m$ and $\varepsilon_c$ are the emissivity of the surface of magnetic core and the surface of cuboid, respectively. The equivalent thermal emissivity $\varepsilon_m$ and $\varepsilon_c$ are:

$$
\varepsilon_m = \frac{\sum_{i=1}^{6} A_{rad,i} \varepsilon_i}{\sum_{i=1}^{6} A_{rad,i}}, \quad \varepsilon_c = \frac{\sum_{i=1}^{6} A_{rad,i} \varepsilon_i}{\sum_{i=1}^{6} A_{rad,i}}
$$

(13)

To achieve the same thermal transfer capability, the density and specific heat capacity of the simplified magnetic core and cuboid are modified. Assuming that the temperature rise $\Delta T$ of different materials and parts of the planar transformer is consistent, the heat absorbed $Q$ of the original planar transformer is:

$$
Q = \Delta T \sum_{i=1}^{n} V_i \rho_i c_i
$$

(14)

where $V_i$, $\rho_i$, and $c_i$ represent the volume, density, and specific heat capacity of different materials of the transformer, respectively. In order to ensure that the heat absorption process before and after the two kinds of simplification is consistent, the following equations is followed:

$$
\rho_m c_m = \frac{\sum_{i=1}^{n} V_i \rho_i c_i}{V_m}, \quad \rho_c c_c = \frac{\sum_{i=1}^{n} V_i \rho_i c_i}{V_c}
$$

(15)

where $\rho_m c_m$ and $\rho_c c_c$ are the product of the density and specific heat capacity of the core and the cuboid. $V_m$ is the volume of the core. $V_c$ is the volume of the cuboid.

To use the model in simulation, the simplified structure in Figures 4 and 5 are used, Equations (12) and (14) shall be followed, and the parameters in (13) and (15) can be set in the simulation accordingly.

4. Thermal Simulation and Experimental Verification

4.1. Planar Transformer Model Parameters

In order to verify the presented two kinds of simplification methods, a case study of the planar transformer is presented in this section. Specifications of the planar transformer are shown in Table 2:

<table>
<thead>
<tr>
<th>Core Type</th>
<th>Core Material</th>
<th>Windings Structure</th>
<th>PCB Copper Thickness</th>
<th>FR4 Thickness</th>
<th>Insulation Layer Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>ER32</td>
<td>Hitachi ML91S</td>
<td>Interleaved</td>
<td>0.1 mm</td>
<td>0.26 mm</td>
<td>29 mm × 27.1 mm × 0.375 mm</td>
</tr>
<tr>
<td>Pri-Copper Thickness</td>
<td>Sec-Copper Thickness</td>
<td>Pri-Terminal Size</td>
<td>Sec-Terminal Size</td>
<td>Primary Turns</td>
<td>Secondary Turns</td>
</tr>
<tr>
<td>70 um</td>
<td>0.07 mm</td>
<td>Φ4 mm × 10.01 mm</td>
<td>Φ1.8 mm × 5.8 mm</td>
<td>Pri1 + Pri2 + Pri3 = 1 × 3 = 3 turns</td>
<td></td>
</tr>
<tr>
<td>Core Size</td>
<td>Core Window Size</td>
<td>Core Mid-Terminal Size</td>
<td>Distance between Insulation to Core</td>
<td>Distance between Pri-Copper to Core</td>
<td>Distance between Sec-Copper to Core</td>
</tr>
<tr>
<td>32.1 mm × 25.4 mm × 12 mm</td>
<td>5.8 mm × 27.2 mm</td>
<td>Φ0.8 mm × 1.18 mm</td>
<td>0.05 mm</td>
<td>0.05 mm</td>
<td>0.5 mm</td>
</tr>
</tbody>
</table>

The plane transformer model is shown in Figure 1, and the power loss of each component is calculated by the winding and core loss equations, as shown in Table 3. The total loss of the plane transformer is 1.6004 W.
When calculating the core loss, $\theta$ is the phase angle which is an integral parameter ranging from 0 to $2\pi$, $K = 2.69 \times 10^{-11}$, $a = 2.73$, and $b = 2.927$ are inherent characters determined by the material property. They are obtained from the datasheet provided by the core manufacturer. With the above parameters, the coefficient $k_i$ is calculated as $3.52 \times 10^{-13}$.

Then by applying Faraday’s law, the relationship between the flux density $B$ and input voltage, magnetic core cross-section area $A_c$, operating frequency $f_s$, and primary winding turns $N_p$ can be obtained:

$$B = \frac{V_{in}}{4N_p A_c f_s} \quad \text{(16)}$$

Then Equation (4) can be simplified below [40]

$$P_s = k_i (2f_s)^{a-\beta} \left( \frac{V_{in}}{N_p A_c} \right)^\beta \quad \text{(17)}$$

The transformer normally operates at $f_s = 1 \text{ MHz}$, the core cross-section area $A_c$ is $141 \text{ mm}^2$, the primary winding turns $N_p = 3$, and the testing input voltage of the transformer is $123 \text{ V}$, so the total core loss is calculated as $1.08 \text{ W}$.

4.2. Thermal Simulation of the Original Planar Transformer

The program ANSYS Icepak is used in this paper for the thermal calculation, it utilizes the computational fluid dynamics (CFD) method for thermal and fluid flow analyses. Other calculation methods such as thermal lumped parameters (TLP) method can also be used if needed [41]. The simulation setting in Ansys ICEPAK are shown in Table 4.

Table 4. Icepak Settings.

<table>
<thead>
<tr>
<th>Gravity Vector</th>
<th>Solution Type</th>
<th>Problem Types</th>
<th>Air Region</th>
<th>Boundary Conditions</th>
<th>Radiation Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global::Y (Negative)</td>
<td>Steady state</td>
<td>Temperature and flow</td>
<td>$120 \text{ mm} \times 96.3 \text{ mm} \times 36 \text{ mm}$</td>
<td>Free opening</td>
<td>Discrete ordinates</td>
</tr>
<tr>
<td>Maximum Number of Iterations</td>
<td>Flow Regime</td>
<td>Flow Iterations per Radiation Iteration</td>
<td>Auto Mesh Setting</td>
<td>Facet Level</td>
<td>Angular Discretization</td>
</tr>
<tr>
<td>100</td>
<td>Laminar</td>
<td>5</td>
<td>Resolution</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

In the simulation, when the ambient temperature is set at 23.4 °C and 50 °C, the thermal simulation of the original complex planar transformer is obtained by setting the losses of each component, as shown in Table 3. All the material and structure details of the windings, insulation, and air are included in the simulation, and the simulation results are shown in Table 5 and Figures 6 and 7.
Table 5. Material parameters settings.

<table>
<thead>
<tr>
<th>Components</th>
<th>Material</th>
<th>Mass Density (kg/m$^3$)</th>
<th>Specific Heat (J/(kg · °C))</th>
<th>Thermal Conductivity (W/m·C)</th>
<th>Thermal Expansion Coefficient (1/C)</th>
<th>Thermal Diffusivity (m$^2$/s)</th>
<th>Dynamic Viscosity (kg/m·s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>Zn-ferrite</td>
<td>5330</td>
<td>585</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Insulation layer</td>
<td>mica</td>
<td>2500</td>
<td>500</td>
<td>0.34</td>
<td>1.05 × 10$^{-5}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PCB</td>
<td>FR-4</td>
<td>1250</td>
<td>1300</td>
<td>0.35</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Windings</td>
<td>Cu-pure</td>
<td>8933</td>
<td>397</td>
<td>387.6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Region</td>
<td>Air</td>
<td>1.1614</td>
<td>1005</td>
<td>0.0261</td>
<td>0.00333</td>
<td>2.885 × 10$^{-5}$</td>
<td>1.84 × 10$^{-5}$</td>
</tr>
</tbody>
</table>

Figure 6. The structure of the planar transformer under test.

Figure 7. Cont.
Figure 7. The thermal simulations of the original planar transformer model: (a) the result of setting the ambient temperature to 23.4 °C with gravity; (b) the result of considering the radiation of windings with gravity; (c) the result of setting the ambient temperature to 50 °C with gravity, (d) the result of setting the ambient temperature to 50 °C without gravity.

If gravity is considered in the simulation, then the movement of air flow around the planar transformer is affected by the gravity. The air with higher temperature has lower density, and therefore it flows upward. The airflow through the transformer can carry the heat away from the transformer and leads to a lower hotspot temperature than that without considering the gravity. It can be seen that under the influence of gravity and thermodynamics, the temperature distribution of the plane transformer is not uniform, and the hotspot is usually on the winding. When the ambient temperature is 23.4 °C, the
maximum temperature reaches 58.1 °C if the heat radiation is set only on the magnetic core without considering the heat radiation of the winding. In (b), the thermal radiation of the winding is taken into account. However, the difference is only 0.4 °C from that of only considering the thermal radiation of the magnetic core. This is due to the small radiation coefficient of copper. Therefore, the radiation of the windings can be ignored.

### 4.3. The First Kind of Thermal Simplification Simulation Results

Using the first kind of thermal simplification, the whole planar transformer is simplified as a single core model, and the total loss of the planar transformer is added to the discrete core model. Again, set the ambient temperature to 23.4 °C and 50 °C. The thermal simulation results obtained are shown in Figure 8.

![Figure 8](image_url)

**Figure 8.** The thermal simulations of the discrete core model: (a) the result of setting the ambient temperature to 23.4 °C; (b) the result of setting the ambient temperature to 50 °C.

The Nussle number $Nu$ is 1.42, the thermal conductivity of the air $λ$ is 0.023 W/m-K, the defined surface length $L$ which is the height of the magnetic core is 12 mm, and the Stefan-Boltzmann constant $σ$ is $5.67 \times 10^{-8}$. In the first kind of thermal simplification, the convective area $A_{\text{comp}}$ and the radiation area $A_{\text{rad}}$ which are the surface area of the core can be considered as their total projection area in six directions, which are both 2838 mm². The surface thermal radiation coefficient of ferrite $ε$ is 0.8. From Equation (13), the first kind of thermal simplification’s equivalent surface thermal radiation coefficient is 0.99. According to Equations (5) and (6), the thermal radiation resistance and the thermal convection resistance can be calculated, and their parallel connection is the equivalent thermal resistance of the plane transformer. From Equation (11), the temperature of the
transformer $T_{\text{trans}}$ can be calculated. The parameters and results used in the analytical calculation are shown in Table 6.

**Table 6. Calculation Parameters and Results of the First Kind Simplification.**

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\sigma$</th>
<th>$T_a$</th>
<th>$A_{\text{rad}}$</th>
<th>$A_{\text{conv}}$</th>
<th>$h_{\text{rad}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>$5.67 \times 10^{-8}$</td>
<td>23.4 °C</td>
<td>2838 mm$^2$</td>
<td>2838 mm$^2$</td>
<td>6.9549</td>
</tr>
</tbody>
</table>

By comparing the simplification result with Figure 7, it can be seen that the temperature rise difference is quite large when the ambient temperature is 23.4 °C. When the ambient temperature is 50 °C, the temperature with adding the loss on the core alone is about 0.3 °C higher than that of the original model. According to the formula derived in Section 3.2, the radiation coefficient of the surface material and the model density and specific heat capacity of the magnetic core model are modified. The simulation results obtained after modification are shown in Figure 9. It can be seen that this modification can reduce the temperature difference in two cases, which verifies the presented thermal simplification method.

**Figure 9.** The thermal simulations of the core model with new surface emissivity: (a) the result of setting the ambient temperature to 23.4 °C; (b) the result of setting the ambient temperature to 50 °C.
4.4. The Second Kind of Thermal Simplification Simulation Results

The second kind of thermal simplification, a cuboid model with the same length, width, and height as the core, is used, and the total loss is added to the cuboid. The difference with the first kind of thermal simplification is that the convective area and the radiation area are 3010 mm$^2$. The second kind of thermal simplification’s equivalent surface thermal radiation coefficient is 0.93. The parameters and results used in the analytical calculation are shown in Table 7.

Table 7. Calculation Parameters and Results of the Second Kind Simplification.

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon$</th>
<th>$\sigma$</th>
<th>$T_a$</th>
<th>$A_{rad}$</th>
<th>$A_{conv}$</th>
<th>$h_{rad}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>0.93</td>
<td>$5.67 \times 10^{-8}$</td>
<td>23.4 °C</td>
<td>3010 mm$^2$</td>
<td>3010 mm$^2$</td>
<td>6.5331</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$h_{conv}$</th>
<th>$R_{rad}$</th>
<th>$R_{conv}$</th>
<th>$R_{total}$</th>
<th>$P_{total}$</th>
<th>$T_{trans}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.3881</td>
<td>50.8416 $\Omega$</td>
<td>31.9740 $\Omega$</td>
<td>19.6293 $\Omega$</td>
<td>1.6004 W</td>
<td>54.81 °C</td>
</tr>
</tbody>
</table>

Modify the model parameters according to Section 3.2, and the parameters used in the simulations are presented in Table 8. The thermal simulation results obtained are shown in Figure 10.

Table 8. Comparison of the material parameters.

<table>
<thead>
<tr>
<th>Model</th>
<th>Density (kg/m$^3$)</th>
<th>Specific Heat (J/(kg·°C))</th>
<th>Surface Material Emissivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>5330</td>
<td>580</td>
<td>0.8</td>
</tr>
<tr>
<td>Cuboid</td>
<td>5330</td>
<td>814</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Figure 10. The thermal simulations of a cuboid model: (a) the result of setting the ambient temperature to 23.4 °C; (b) the result of setting the ambient temperature to 50 °C.
The temperature rise error is approximately 0.5 °C. The errors of both temperatures are very small, and the temperature rise of the original model can be well predicted. It validates the effectiveness of the simplification model.

4.5. Experimental Results Analyses

Experimental measurements are carried out on the MHz prototype to compare with thermal simulation and theoretical calculation and verify the simplified model [15, 16]. The experimental configuration is shown in Figure 11, and the testing bench is shown in Figure 12. The planar transformer is excited by a DC-DC converter with two full bridges to emulate the 1 MHz rectangle waveform voltage excitation in operation scenarios. \( Q_1, Q_2, Q_3 \) are the semiconductor switches, \( C_H, C_L \), and \( R_L \) are input and output DC link capacitors and output load, respectively. The input current and voltage \( i_H, v_H \), and output current and voltage \( i_L, v_L \) of the transformer are measured with the power analyzer Newtons4th PPA5500. The core loss is tested when the output side full bridge is open-circuited, and the core loss \( P_{\text{core}} \) is calculated by:

\[
P_{\text{core}} = \frac{1}{T} \int_0^T v_L(t)i_H(t)dt
\]

(18)

where \( T \) is the period counted. When output side full bridge and load are connected, the full load loss of the planar transformer \( P_{\text{full}} \) is calculated by:

\[
P_{\text{full}} = \frac{1}{T} \int_0^T v_L(t)i_L(t) - v_H(t)i_H(t)dt
\]

(19)

The winding loss is obtained by subtracting the full load loss from the core loss. The thermal camera FLIR X8400sc is used to obtain the thermal image and the temperature rise of the planar transformer.

The total loss of the plane transformer is 0.414 W, where the highest temperature of the plane transformer is 34.7 °C. Using the simplified model, the temperature rise is obtained analytically by multiplying the total thermal resistance and the total loss, then adding to the ambient temperature. The above results are compared and analyzed, and the error is shown in Figure 13.
The winding loss is obtained by subtracting the full load loss from the total loss. The winding loss of the planar transformer is 0.41 W. When the total loss is 0.414 W, the maximum temperature of the planar transformer is measured by the thermal camera is 34.7 °C. In this case, the error label in the bar in Figure 13 is the error of temperature rise obtained by different methods compared with the thermal camera. The thermal simulation error with the simplified model is very small, about 0.28%. However, due to the low thermal radiation coefficient of the windings in the real object, the actual thermal resistance will be higher than the calculated thermal resistance, so the measured temperature rise of the real planar transformer is 34.7 °C. Overall, the analytical calculation, thermal simulation, and experimental measurements agree well with the results.
each other, and the error is neglectable, which verifies the proposed simplification model in this paper.

5. Conclusions

This paper analyzes the thermal network of a planar transformer based on the power loss and thermal coupling of transformer components and proposes two simplified thermal modeling methods based on reasonable assumptions. Firstly, the loss calculation models and thermal network modeling methods are introduced in detail. Then two thermal simplification models, i.e., magnetic core simplification and cuboid simplification, are proposed. Based on the theory of thermal radiation, conduction, and convection, the parameters of the thermal simplification models are derived analytically. Moreover, the thermal simulation results of various models and planar transformers with different losses are compared and analyzed in ANSYS Icepak. The temperature rises of the thermal simplification models are basically similar to that of the original model. Finally, the simplified models are compared with the measurement results on the MHz prototype, which verifies the effectivity of the simplified thermal model of the planar transformer. The proposed models are helpful for the establishment of analytical thermal models and the design of high-density planar magnetics.

Several vital conclusions are obtained based on the analysis and case study, which are as follows.

1. The proposed simplification models are applicable for both CFD simulation and analytical methods. When used as the simplified model in simulation, the models do not need the detailed loss distribution as well as the transformer internal structure information, and the obtained hotspot temperature and temperature distribution are close to the original detailed model. When used as the simplified model in analytical modeling, the model can predict the hotspot temperature of the planar transformer precisely.

2. Generally, the simplification is based on the assumptions that the windings and insulation materials occupy most of the planar transformer core window space; therefore the accuracy of the simplified model can increase with the decrease of the free air space in the core window. Moreover, the proposed simplification modeling method is also applicable for other high-density components in power electronics applications with proper modification.

3. By using the simplified simulation model, the first kind of simplified method can provide more detailed and precise temperature distribution than the second kind of simplified model, but it also requires more core structure information.

4. The first simplified model is convenient for quick CFD simulation, while the second model is more suitable for analytical calculation. Therefore the second one has great application value in the model-based analytical iteration design procedure, while the first one is helpful for quick simulation verification during planar transformer design.

Author Contributions: Conceptualization, Z.S. and L.J.; Data curation, B.X.; Formal analysis, C.L.; Investigation, B.X., C.L., C.H. and B.L.; Methodology, Z.X.; Supervision, L.J. and W.C.; Writing—original draft, Z.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Fundamental Research Funds for the Central Universities grant number 2242022R10162, Power Quality Engineering Research Center, the Ministry of Education, Anhui University grant number KFKT202210, and Jiangsu Provincial Key Laboratory of Smart Grid Technology and Equipment of Southeast University grant number 4216002101.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.


