Abstract: Digital financial innovation is a new impetus for economic and social development. However, lack of regulation will also have a huge impact on economic and social development. In this paper, an evolutionary game model of digital finance innovation is constructed, the evolutionary strategies of both sides of the game are discussed, and a simulation analysis is carried out, based on the dynamic reward and punishment mechanism of the government. The results show that the system can achieve evolutionary stability under the dynamic reward and punishment mechanism, and that the evolutionarily stable strategy is unique. We also find that when the punishment of regulators increases, the probability of compliance innovation of digital financial enterprises will increase, and the probability of active supervision of regulatory agencies will decrease. When regulators increase incentives, the probability of the compliance innovation of digital financial enterprises will decrease. Similarly, the probability of active supervision by regulators will also decrease and the decrease will be more obvious. To achieve the win-win development of digital financial innovation and regulation, it is necessary to continuously improve the regulatory capacity and level, reduce regulatory costs, and build a dynamic reward and punishment mechanism. Our research contributes to enhancing compliance innovation in digital financial enterprises.

Keywords: digital finance; evolutionary game; financial supervision; reward and punishment mechanism
when enterprises carry out digital financial business, they often aim to maximize their own interests. In the case of insufficient regulation, they will tend to carry out innovative behaviors with more benefits and greater risks [3].

Therefore, it is necessary to deeply analyze the motivation, mechanism and strategy of financial institutions to carry out digital financial innovation, and discuss how regulators build a scientific regulatory system. While effectively protecting the enthusiasm of digital financial innovation, we should strengthen the supervision of digital financial innovation and give full play to the role of digital financial innovation in promoting the economy and industry, and promote the sustainable and healthy development of digital finance. This has both important theoretical value and practical significance.

The contribution of this paper is mainly reflected in the following aspects: First, to the best of our knowledge, this paper is one of the few to discuss innovation and regulation in digital finance. Secondly, by introducing a dynamic reward and punishment mechanism, the dynamic evolution game model of digital finance innovation and regulation is constructed, and the evolutionary game relationship between digital finance innovation and regulation is further discussed. Third, not only is the evolutionary game relationship between digital financial innovation and regulation studied in this paper, but the impact of different rewards and punishments on the digital financial innovation behavior is also deeply analyzed, which is complementary to research in related fields.

The arrangement of this paper is as follows: Section 2 reviews the relevant literature. Section 3 analyzes the evolutionary game model under the static reward and punishment mechanism. Section 4 analyzes the evolutionary game model under the dynamic reward and punishment mechanism. Section 5 simulates our proposed model and analyzes the key influencing factors. Section 6 summarizes the conclusions of this paper.

2. Literature Review

The trend of sustainable development has prompted people to pay more and more attention to the importance of compliance innovation for the healthy development of digital finance. In recent years, scholars have conducted in-depth research on digital financial innovation and regulation, mainly on the positive role of digital financial compliance innovation [4–13], the negative impact of digital financial violation innovation [14–21], and the impact of regulatory constraints and regulations on digital financial innovation [22–28]. The results can provide reference for the follow-up research.

In terms of the positive role of digital finance, the innovative development of digital finance has an obvious driving effect on the technological progress of enterprises, which can better correct the structural problems existing in traditional financial support for enterprise innovation activities [4–6]. Gorgeous Xie et al. [7] and Tian et al. [8] argue that digital finance can effectively promote total factor productivity by improving the technological level of enterprises and regional innovation capacity. Zhang et al. [11] found that digital finance can not only effectively meet residents’ demand for financial services, but also promote residents’ consumption in multiple dimensions. Some scholars believe that digital finance makes traditional financial business more convenient and brings new business opportunities and models for financial enterprises [12,13].

Digital finance not only has a positive impact on the development of economy and society, but also can have a negative impact on economy and society if it is not regulated and its development is not regulated [14–16]. The innovation of digital finance makes mixed operations and cross-border operations common, which increases the difficulty of supervision and is prone to transnational money laundering, illegal operations and other risks [17,18]. The accumulation of these risks can easily lead to systemic risks and ultimately affect the stability of the financial system [19,20]. In addition, Cumming et al. [21] also pointed out that the promotion of digital currency increases the risk exposure degree of consumers and increases the risk probability of consumers.

In terms of the impact of regulatory constraints and regulations on digital financial innovations, the existing regulatory system manages financial businesses in different fields,
respectively, so it cannot effectively supervise the phenomenon of mixed business caused by digital financial innovation. Therefore, some scholars believe that the imperfections of the existing regulatory system can be improved by establishing the basic principles of digital financial regulation, adopting more regulatory means and clarifying the scope of regulation [22,23]. For example, Arner et al. [24] and Ran et al. [25] argue that regulators should establish a standardized regulatory instrument and regulatory system to achieve a balance between digital financial innovation and financial stability. Zhou et al. [26] also found that a high intensity of punishment by regulatory agencies can effectively curb illegal behaviors in financial innovation.

Evolutionary game theory assumes bounded rationality and is closer to reality than traditional game theory. Therefore, it has been widely used in the study of dynamic relationship in the fields of industry–university–research cooperation and public administration [29–34]. The dynamic relationship between digital financial innovation and regulation shows an obvious feature of “bounded rationality”, so some scholars have studied it by using the evolutionary game method [35–40]. Among them, Song et al. [35] used the evolutionary game method and found that when moderate innovation and regulation are both greater than the set critical value, digital financial innovation and regulation can achieve a “win-win” situation. Based on the evolutionary game method, An et al. [36] made an in-depth analysis of regulatory costs, regulatory strategies and punishment measures in the process of financial innovation and regulation. Cui et al. [38] believe that government agencies and other parties need to cooperate in order to build a good financial system and promote financial innovation. Based on the dynamic game model, Lumpkin [40] analyzed and concluded that digital financial innovation and regulation are mutually opposite and interrelated dialectical relations. Only by maintaining the dynamic balance between the two can the positive interaction between compliance innovation and effective regulation be realized.

To sum up, scholars have studied digital financial innovation and regulation from different perspectives and with different methods, and have achieved rich results. However, there are still the following shortcomings: First, there are few literatures on the dynamic evolutionary game process of digital financial innovation and regulation, and the existing research is not clear about the setting of indicators related to digital financial innovation and regulation, and does not distinguish the cost and other indicators of different strategy choices of both sides of the game in detail. Second, although some scholars have studied the role of a reward and punishment mechanism in supervision, most of the researches have not deviated from the research framework of the general evolutionary game model, and often set the reward and punishment value as a static fixed value, without fully considering the impact of the dynamic reward and punishment mechanism on digital financial innovation. Therefore, this paper will consider the impact of strategic choices on costs and other indicators to establish an evolutionary game model between digital financial innovation and regulation. On the one hand, a dynamic reward and punishment mechanism will be introduced to deeply analyze the impact of reward and punishment intensity on the dynamic evolution of digital financial innovation behavior when regulators implement a dynamic reward and punishment mechanism. On the other hand, a MATLAB R2019b simulation is used to analyze the influence of each parameter on the evolutionary game equilibrium, so as to put forward targeted countermeasures and suggestions.

3. Evolutionary Game Analysis of Digital Financial Innovation under the Static Reward and Punishment Mechanism

3.1. Base Assumption and Payoff Matrix

In this section, an evolutionary game model of digital financial enterprise innovation behavior under the static reward and punishment mechanism will be established, and the cooperation strategies between different subjects will be analyzed. Based on this, the hypothesis is as follows:
(1) It is assumed that digital financial innovation and regulation mainly involve two game subjects: digital financial enterprises and regulators. Both of them are bounded rational and affected by randomness, information asymmetry and other factors. It is difficult for game subjects to get the optimal strategy in one game, and they need to learn and improve many times before gradually approaching the optimal strategy [41]. As the main body of innovation, whether the innovation activities of digital financial enterprises comply with relevant regulatory norms will directly affect social stability and economic development. As the management department of digital financial enterprises, the regulator is responsible for supervising the innovation activities such as product innovation, business innovation and service innovation carried out by digital financial enterprises [42].

(2) Digital financial enterprises can choose “compliance innovation” strategies or “illegal innovation” strategies in the process of innovation. Among them, compliance innovation refers to the innovation activities carried out by digital financial enterprises in accordance with relevant regulatory norms. For example, by relying on the relevant technologies, financial products can be rationally optimized, service forms can be enriched, and operation efficiency can be continuously improved. Illegal innovation refers to the use of new technology by digital finance enterprises to illegally absorb deposits, monopolize the market and evade regulation, which harms the social interest [43]. Regulators can also choose one of two strategies: “active regulation” or “negative regulation”. Among them, active regulation refers to regulators actively fulfilling their regulatory responsibilities and investing more financial and material resources to supervise and review the standardization of digital financial enterprises. Negative regulation refers to regulators adopting a laissez-faire attitude and ignoring their regulatory responsibilities, which makes it difficult to detect the illegal innovation behaviors of digital financial enterprises in a timely manner [44]. At the same time, assuming that the probability of digital financial enterprises choosing compliance innovation is \( x \), the probability of illegal innovation is \( 1 - x \), the probability of regulators choosing active regulation is \( y \), and the probability of negative regulation is \( 1 - y \), and the trend of \( x, y \in (0,1) \).

(3) When digital financial enterprises choose compliance innovation, the revenue is \( R_1 \), and when they choose violation innovation, the excess revenue is \( L \) (\( L > 0 \)). The cost of compliance innovation for digital financial enterprises is \( C_1 \), and the cost of non-compliance innovation is \( C_2 \).

(4) When digital financial enterprises choose compliance innovation, the regulator will gain \( R_2 \), such as the benefits brought by the promotion of government reputation and the healthy development of social economy. If digital financial enterprises choose to innovate in the violation of regulations, the digital financial security incidents will lead to the reduction of credibility of the government regulatory agencies [45]. At this time, the regulatory agencies will gain \( R_3 \) and cause a loss of \( W \) to the social public interests, \( W \). The cost of active regulation is \( C_3 \), such as investing more manpower and material resources, purchasing and using higher-end supervision technology and equipment. The cost of negative regulation is \( C_4 \), for example, if regulators do not carefully check in accordance with relevant regulations whether digital financial enterprises illegally innovate.

(5) When regulators choose to actively regulate, there will be incentives and penalties. For digital financial enterprises with compliance innovation, the regulatory authorities will provide direct subsidies or preferential policy support, and the additional income obtained by digital financial enterprises is denoted as \( V \). Digital financial enterprises that choose innovative strategies that violate regulations will also be punished by the regulatory authorities, such as fines and business suspension, marked as \( F \). In the case of negative regulation, the regulatory authorities will neither reward the compliance innovation of digital finance enterprises with subsidies nor punish the innovation behavior of digital finance enterprises in violation of regulations.
According to the above assumptions, the payoff matrix of the strategic choice game between digital financial enterprises and regulators can be obtained, as shown in Table 1.

**Table 1.** Game return matrix between regulators and digital financial enterprises.

<table>
<thead>
<tr>
<th>Digital finance enterprise</th>
<th>Compliance innovation (x)</th>
<th>Illegal innovation (1-x)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Regulators</strong></td>
<td>Active regulation (y)</td>
<td>Negative regulation (1-y)</td>
</tr>
<tr>
<td></td>
<td>(R_1 + V - C_1)</td>
<td>(R_1 - C_1)</td>
</tr>
<tr>
<td></td>
<td>(R_2 - C_3 - V)</td>
<td>(R_2 - C_4)</td>
</tr>
<tr>
<td></td>
<td>(R_1 + L - C_2 - F)</td>
<td>(R_1 + L - C_2)</td>
</tr>
<tr>
<td></td>
<td>(R_3 + F - C_3 - W)</td>
<td>(R_3 - W - C_4)</td>
</tr>
</tbody>
</table>

3.2. Copy the Dynamic Equation and Equilibrium Point

Assuming that \(E_{11}\) and \(E_{12}\) are the expected returns of digital financial enterprises adopting the strategies of “compliance innovation” and “illegal innovation”, respectively, and \(E_1\) is the average expected returns of digital financial enterprises, it can be obtained according to Table 1:

\[
\begin{align*}
E_{11} &= y(R_1 + V - C_1) + (1-y)(R_1 - C_1) \\
E_{12} &= y(R_1 + L - C_2 - F) + (1-y)(R_1 + L - C_2) \\
E_1 &= xE_{11} + (1-x)E_{12}
\end{align*}
\]  

(1)

According to the Malthusian dynamic equation, the replication dynamic equation for digital financial enterprises choosing compliance innovation strategy can be obtained as follows:

\[
F_1(x) = \frac{dx}{dt} = x(E_{11} - E_1)
\]  

(2)

Substitute Equation (1) into Equation (2) to obtain:

\[
F_1(x) = x(1-x)[y(V + F) + C_2 - C_1 - L]
\]  

(3)

Similarly, assuming that \(E_{21}\) and \(E_{22}\) are, respectively, expected earnings of “positive regulation” and “negative regulation” adopted by regulators, and \(E_2\) is the average expected earnings of regulators, then:

\[
\begin{align*}
E_{21} &= x(R_2 - C_3 - V) + (1-x)(R_3 + F - C_3 - W) \\
E_{22} &= x(R_2 - C_4) + (1-x)(R_3 - W - C_4) \\
E_2 &= yE_{21} + (1-y)E_{22}
\end{align*}
\]  

(4)

Thus, the replicative dynamic equation of positive regulatory strategy selected by regulators can be obtained as follows:

\[
F_1(y) = \frac{dy}{dt} = y(E_{21} - E_2)
\]  

(5)

Substitute Equation (4) into Equation (5) to obtain:

\[
F_1(y) = y(1-y)[x(-V - F) + F - C_3 + C_4]
\]  

(6)

A two-dimensional dynamic system \(D_1\) can be obtained from the duplicated dynamic equation of both sides of the above game:

\[
\begin{align*}
F_1(x) &= x(1-x)[y(V + F) + C_2 - C_1 - L] \\
F_1(y) &= y(1-y)[x(-V - F) + F - C_3 + C_4]
\end{align*}
\]  

(7)
Let $F_1(x) = 0$ and $F_1(y) = 0$ in Formula (7), thus five equilibrium points of the two-dimensional dynamic system $D_1$ can be obtained, which are $P_1(0,0), P_2(0,1), P_3(1,0), P_4(1,1), P_5(x_0, y_0)$, where $x_0 = \frac{C_4 + C_3 + F}{V + F}, y_0 = \frac{C_4 + C_3 + L}{V + F}$.

### 3.3. Stability Analysis of Evolutionary Game Model

The equilibrium points obtained from the above two-dimensional dynamic system $D_1$ are not the complete evolutionary stability strategy (ESS) of the system. According to Friedman’s method, the evolutionary stability strategy can be obtained by analyzing the stability of the Jacobian matrix of the two-dimensional dynamic system. The Jacobian matrix $J_1$ of the system can be obtained from the two-dimensional differential dynamic equations:

$$ J_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} $$

(8)

Among them:

$$ A_{11} = (1 - 2x)[y(V + F) + C_2 - C_1 - L] $$
$$ A_{12} = x(1 - x)(V + F) $$
$$ A_{21} = y(1 - y)(-V - F) $$
$$ A_{22} = (1 - 2y)[x(-V - F) + F - C_3 + C_4] $$

The stability of the equilibrium point of the two-dimensional dynamic system $D_1$ can be determined by the determinant value and trace value of matrix $J_1$. When $\text{Det}J_1 > 0$ and $\text{Tr}J_1 < 0$, the equilibrium point of the two-dimensional dynamic system $D_1$ can be said to be the evolutionarily stable strategy (ESS).

$$ \text{Det}J_1 = A_{11} A_{22} - A_{12} A_{21} > 0 $$

(9)

$$ \text{Tr}J_1 = A_{11} + A_{22} < 0 $$

(10)

Based on the above analysis, the values of the five equilibrium points can be obtained, as shown in Table 2:

**Table 2. Values at $A_{11}, A_{12}, A_{21}, A_{22}$ of equilibrium points.**

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>$A_{11}$</th>
<th>$A_{12}$</th>
<th>$A_{21}$</th>
<th>$A_{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1(0,0)$</td>
<td>$C_2 - C_1 - L$</td>
<td>0</td>
<td>0</td>
<td>$F - C_3 + C_4$</td>
</tr>
<tr>
<td>$P_2(0,1)$</td>
<td>$V + F + C_2 - C_1 - L$</td>
<td>0</td>
<td>0</td>
<td>$-F - C_3 + C_4$</td>
</tr>
<tr>
<td>$P_3(1,0)$</td>
<td>$-(C_2 - C_1 - L)$</td>
<td>0</td>
<td>0</td>
<td>$C_4 - C_3 - V$</td>
</tr>
<tr>
<td>$P_4(1,1)$</td>
<td>$-(V + F + C_2 - C_1 - L)$</td>
<td>0</td>
<td>0</td>
<td>$-(V - C_3 + C_4)$</td>
</tr>
<tr>
<td>$P_5(x_0, y_0)$</td>
<td>0</td>
<td>$A$</td>
<td>$B$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Among them:

$$ A = \frac{(C_4 - C_3 + F)(V - C_4 + C_3)}{V + F}, \quad B = \frac{(C_2 - C_1 - L)(V + F - C_1 + C_2 - L)}{V + F} $$

As can be seen from Table 2, equilibrium point $P_5$ is not an evolutionary stable strategy since $\text{Tr}J_1 = 0$ does not satisfy Equation (10). The characteristic root corresponding to the equilibrium point $P_5$ is $\lambda_1, \lambda_2 = \pm i\sqrt{K}$, where:

$$ K = -(C_4 - C_3 + F)(V - C_4 + C_3)(C_1 - C_2 + L)(V + F - C_1 + C_2 - L) $$

$$ (V + F)^2 $$

The characteristic roots $\lambda_1$ and $\lambda_2$ are a pair of pure imaginary roots, so the equilibrium point $P_5$ is the center point of the two-dimensional dynamic system $D_1$. At this time, the
evolution trajectory of the system is a closed rail loop with periodic movement around the center point.

The above equilibrium points are analyzed as follows:

(1) When $F < C_3 - C_4$ and $L > C_2 - C_1$, the stable evolution strategy of system $D_1$ is $P_1(0, 0)$;

(2) When $C_3 - C_4 < F < L$ and $V < C_1 - C_2$, the stable evolution strategy of system $D_1$ is $P_2(0, 1)$;

(3) When $V > C_4 - C_3$ and $L < C_2 - C_1$, the stable evolution strategy of system $D_1$ is $P_3(1, 0)$;

(4) When $F > L$ and $C_1 - C_2 < V < C_4 - C_3$, the stable evolution strategy of system $D_1$ is $P_4(1, 1)$.

According to the local stability analysis method of the Jacobian matrix, the stability of the equilibrium points is analyzed, and the corresponding $DetJ_1$ and $TrJ_1$ of the four equilibrium points can be obtained, respectively, as shown in Table 3.

Table 3. Local stability of each equilibrium points in cases (1) to (4).

<table>
<thead>
<tr>
<th>Situation</th>
<th>Equilibrium</th>
<th>$DetJ_1$</th>
<th>$TrJ_1$</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$P_1(0, 0)$</td>
<td>+</td>
<td>-</td>
<td>ESS</td>
</tr>
<tr>
<td></td>
<td>$P_2(0, 1)$</td>
<td></td>
<td></td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$P_3(1, 0)$</td>
<td></td>
<td></td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$P_4(1, 1)$</td>
<td></td>
<td></td>
<td>Saddle point</td>
</tr>
<tr>
<td>(2)</td>
<td>$P_1(0, 0)$</td>
<td>-</td>
<td></td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$P_2(0, 1)$</td>
<td>+</td>
<td>-</td>
<td>ESS</td>
</tr>
<tr>
<td></td>
<td>$P_3(1, 0)$</td>
<td></td>
<td></td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$P_4(1, 1)$</td>
<td>+</td>
<td>+</td>
<td>Instability point</td>
</tr>
<tr>
<td>(3)</td>
<td>$P_1(0, 0)$</td>
<td>+</td>
<td>+</td>
<td>Instability point</td>
</tr>
<tr>
<td></td>
<td>$P_2(0, 1)$</td>
<td></td>
<td></td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$P_3(1, 0)$</td>
<td>+</td>
<td>-</td>
<td>ESS</td>
</tr>
<tr>
<td></td>
<td>$P_4(1, 1)$</td>
<td></td>
<td></td>
<td>Saddle point</td>
</tr>
<tr>
<td>(4)</td>
<td>$P_1(0, 0)$</td>
<td>-</td>
<td></td>
<td>Saddle point</td>
</tr>
<tr>
<td></td>
<td>$P_2(0, 1)$</td>
<td>+</td>
<td>+</td>
<td>Instability point</td>
</tr>
<tr>
<td></td>
<td>$P_3(1, 0)$</td>
<td>+</td>
<td>+</td>
<td>Instability point</td>
</tr>
<tr>
<td></td>
<td>$P_4(1, 1)$</td>
<td>+</td>
<td>-</td>
<td>ESS</td>
</tr>
</tbody>
</table>

Based on the stability analysis of the equilibrium points above, the evolution process of digital financial enterprises and regulatory agencies can be obtained. The phase diagrams of evolution under various conditions can be drawn and corresponding analysis results can be obtained.

The evolutionary stability strategy of case (1) is $P_1(0, 0)$, and the corresponding evolutionary stability strategy is (illegal innovation, negative regulation). At this time, the evolution phase diagram of the system is shown in Figure 1a. In this case, $F < C_3 - C_4$ indicates that the benefits of active regulation are less than the costs. $L > C_2 - C_1$ indicates that digital financial enterprises can obtain greater benefits from illegal innovation. As a result, digital finance companies and regulators tend towards illegal innovation and negative regulation, respectively. At this point, the system will evolve into a state of frequent digital financial security incidents.
The evolutionary stability strategy of case (2) is $P_2(0, 1)$, and the corresponding evolutionary stability strategy is (illegal innovation, active regulation). At this time, the evolution phase diagram of the system is shown in Figure 1b. In this case, $C_3 - C_4 < F < L$ means that the cost paid by the regulator for active regulation is small, and the regulator can obtain a large penalty income. However, the excess profits obtained by digital financial enterprises through illegal innovation are greater than the fines they need to bear. When $V < C_1 - C_2$, the extra income of digital finance enterprises due to compliance innovation is less than the extra cost of compliance innovation. At this point, digital financial enterprises will tend to innovate against regulations, and regulators will tend to actively regulate.

The evolutionary stability strategy of situation (3) is $P_3(1, 0)$, and the corresponding evolutionary stability strategy is (compliance innovation, negative regulation). At this time, the evolution phase diagram of the system is shown in Figure 1c. In this case, when $V > C_4 - C_3$, the subsidy that regulators need to pay to digital financial enterprises in active regulation is greater than the extra cost they need to pay in negative regulation, so regulators will gradually tend to negative regulation. $L < C_2 - C_1$ indicates that the extra cost for illegal innovation of digital finance enterprises is greater than the excess income they can obtain. Therefore, digital finance enterprises will gradually tend towards compliance innovation.

The evolutionary stability strategy of situation (4) is $P_4(1, 1)$, and the corresponding evolutionary stability strategy is (compliance innovation, active regulation). At this time, the evolution phase diagram of the system is shown in Figure 1d. In this case, when $C_1 - C_2 < V < C_4 - C_3$, the regulatory cost of active regulation is less than the cost of negative regulation, so the regulatory agency will gradually tend to active regulation.

The evolutionary stability strategy of situation (5) is $P_5(0, 1)$, and the corresponding evolutionary stability strategy is (illegal innovation, negative regulation). At this time, the evolution phase diagram of the system is shown in Figure 1e. In this case, when $V < C_3 - C_4$, the extra cost for illegal innovation of digital finance enterprises is less than the excess income they can obtain. Therefore, digital finance enterprises will gradually tend to negative regulation.
regulators actively regulate, digital financial enterprises will tend to innovate in compliance because \( F > L \), that is, the excess income obtained by digital financial enterprises is less than the cost of the penalty they need to pay. It can be seen that the active regulation of regulatory bodies plays a role in promoting compliance innovation of digital financial enterprises.

According to the stability analysis, equilibrium point \( P_3 \) is the center point of the system, and the characteristic roots \( \lambda_1 \) and \( \lambda_2 \) are a pair of pure virtual roots, that is, equilibrium point \( P_3 \) is the non-asymptotic stable point of the system. At this point, the evolutionary game process of the system is a closed rail loop with periodic movement around the central point. At this point, the evolutionary phase diagram of the system is shown in Figure 1e. At this point, the two game groups of digital financial enterprises and regulatory agencies show cyclical behavior patterns.

### 4. Evolutionary Game Analysis of Digital Financial Innovation under the Dynamic Reward and Punishment Mechanism

In view of the situation that the system cannot achieve evolutionary stability under the above static reward and punishment mechanism, the dynamic reward and punishment mechanism is introduced as a decision variable to improve the system stability, and the influence of related parameter changes on the evolutionary stability strategy of both sides of the game is analyzed.

#### 4.1. Stability Analysis under the Dynamic Reward and Punishment Mechanism

Assuming that the regulator of rewards and punishments to digital financial companies is related to digital financial enterprise innovation behavior, a regulator of subsidies and incentives to digital financial companies to \( g(x) = xV \), where \( V \) is the biggest amount of subsidies and incentives, the punishment for \( m(x) = (1 - x)F \), including \( F \) as the biggest limit, punishing the available digital financial firms and regulators of \( D_2 \) two-dimensional dynamic system:

\[
\begin{align*}
F_2(x) &= x(1 - x)\{y[g(x) + m(x)] + C_2 - C_1 - L\} \\
F_2(y) &= y(1 - y)\{x[g(x) - m(x)] + m(x) - C_3 + C_4\}
\end{align*}
\]  

(11)

The five equilibrium points of the two-dimensional dynamic system \( D_2 \) can be obtained as follows: \( P'_1(0,0), P'_2(0,1), P'_3(1,0), P'_4(1,1) \) and \( P'_5(x_1,y_1) \). Let \( C_5 = C_3 - C_4 \) represent the difference between the costs of choosing different regulatory strategies; Ling \( C_E = C_1 - C_2 + L \) represents the cost difference between different innovation strategies for digital finance enterprises. Where \( x_1 = \frac{F - \sqrt{F^2 - (F - V)(F - C_5)}}{F - V}, \ y_1 = \frac{C_5 \sqrt{F^2 - (F - V)(F - C_5)}}{F^2 - (F - V)(F - C_5)} \).

The Jacobian matrix of two-dimensional dynamic system \( D_2 \) is:

\[
J_2 = \begin{bmatrix}
A'_{11} & A'_{12} \\
A'_{21} & A'_{22}
\end{bmatrix}
\]  

(12)

Among them:

\[
\begin{align*}
A'_{11} &= (1 - 2x)[y(g(x) + m(x)) - C_E] + xy(1 - x)[g'(x) + m'(x)] \\
A'_{12} &= x(1 - x)[g(x) + m(x)] \\
A'_{21} &= y(1 - y)(1 - x)m'(x) - xg'(x) - g(x) - m(x) \\
A'_{22} &= (1 - 2y)[x[g(x) - m(x)] + m(x) - C_3]
\end{align*}
\]

Then, according to the local stability analysis method of the Jacobian matrix, the stability of the equilibrium point of the two-dimensional dynamic system \( D_2 \) is analyzed, and the results are shown in Table 4.
Therefore, $P_{S}^{*}(x_{1}, y_{1})$ is the evolutionary stability strategy of system $D_{2}$ under the dynamic reward and punishment mechanism, and the system has asymptotic stability.

### 4.2. Parameter Analysis under the Dynamic Reward and Punishment Mechanism

The equilibrium point $P_{S}^{*}(x_{1}, y_{1})$ is the stable evolution strategy of the two-dimensional dynamic system $D_{2}$. It can be seen that the stable evolution strategy of digital financial enterprises is only related to the difference in the cost of regulatory strategy and the upper limit of reward and punishment intensity, while the stable evolution strategy of regulatory institutions is related to the difference in cost of innovation strategy, the difference in the cost of regulatory strategy and the upper limit of reward and punishment intensity. As the calculation process of partial derivatives of $x_{1}$ to $F$ and $V$ is too complicated, the influence of $F$ and $V$ on $x_{1}$ will be further analyzed in the following chapter of simulation analysis. Therefore, by taking partial derivatives of $x_{1}$ with respect to $C_{S}$ and $y_{1}$ with respect to $C_{E}$, $C_{S}$, $F$ and $V$, respectively, the influence of the changes of reward and punishment intensity on the system evolution stability was analyzed.

1. Cost difference between different regulatory strategies $C_{S}$. The partial derivative of $x_{1}$ with respect to $C_{S}$ yields $\frac{\partial y_{1}}{\partial C_{S}} = -\frac{1}{2\sqrt{F^{2} - (F - V)(F - C_{S})}} < 0$, so $x_{1}$ is the monotone minus function with respect to $C_{S}$. That is to say, when the difference between the costs of the different regulatory strategies of regulators $C_{S}$ decreases, the value of $x_{1}$ increases. At this time, because the difference between the costs of different regulatory strategies keeps decreasing, regulators tend to actively regulate, and digital financial enterprises also tend to innovate in compliance under regulatory pressure. The partial derivative of $y_{1}$ with respect to $C_{S}$ yields $\frac{\partial y_{1}}{\partial C_{S}} = -\frac{C_{E}(F - V)}{2[F^{2} - (F - V)(F - C_{S})]} < 0$, so $y_{1}$ is the monotone minus function with respect to $C_{S}$. In other words, when the difference between the costs of different regulatory strategies $C_{S}$ decreases, the value of $y_{1}$ increases, and the impact of the costs of different regulatory strategies on the regulatory agencies‘ strategic choices decreases continuously. In order to enhance the credibility of the government and ensure the healthy development of society and economy, the supervisory authorities tend to actively supervise [46]. Thus, effective control of regulatory costs can make digital financial innovation and regulatory strategies evolve toward a stable state (compliance innovation, active regulation).

2. The upper limit of reward and punishment intensity $F$ and $V$. Partial derivatives of $y_{1}$ with respect to $F$ and $V$ yield $\frac{\partial y_{1}}{\partial F} = -\frac{C_{E}(F - C_{S})}{2[F^{2} - (F - V)(F - C_{S})]^{\frac{3}{2}}} < 0$ and $\frac{\partial y_{1}}{\partial V} = -\frac{C_{E}(F - C_{S})}{2[F^{2} - (F - V)(F - C_{S})]^{\frac{3}{2}}} < 0$, so $y_{1}$ is a monotone minus function of $F$ and $V$. That is, the value of $y_{1}$ decreases when the rewards and punishments of the regulatory agencies $F$ and $V$ increase. In this case, the higher the fine $F$ is, the more severe the punishment will be for digital financial enterprises. Under the deterrent of high punishment intensity, the regulatory agencies do not need to carry out frequent regulatory activities, so the probability of active regulation is reduced. When the incentive subsidy $V$ for digital financial enterprises is larger, the regulator will have more incentive subsidy expenditure, and the regulator tends to be negative regulation. Therefore, the application of incentive and punishment mechanism should not blindly

### Table 4. Stability analysis of system $D_{2}$ equilibrium point.

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Det$J_{2}$</th>
<th>Tr$J_{2}$</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{1D}^{*}(0, 0)$</td>
<td>-</td>
<td>-</td>
<td>Saddle point</td>
</tr>
<tr>
<td>$P_{1D}^{*}(0, 1)$</td>
<td>-</td>
<td>-</td>
<td>Saddle point</td>
</tr>
<tr>
<td>$P_{1D}^{*}(1, 0)$</td>
<td>-</td>
<td>-</td>
<td>Saddle point</td>
</tr>
<tr>
<td>$P_{1D}^{*}(1, 1)$</td>
<td>-</td>
<td>-</td>
<td>Saddle point</td>
</tr>
<tr>
<td>$P_{1D}^{*}(x_{1}, y_{1})$</td>
<td>+</td>
<td>-</td>
<td>ESS</td>
</tr>
</tbody>
</table>
increase the incentive and complement efforts, but should be within a reasonable range of dynamic adjustment, in order to promote the system (compliance innovation, active regulation) development.

(3) The difference of innovation strategy cost of digital finance enterprises $C_E$. The partial derivative of $y_1$ with respect to $C_E$ yields $\frac{\partial y_1}{\partial C_E} = \frac{1}{\sqrt{F^2-(F-V)(F-C_0)}} > 0$, so $y_1$ is a monotonically increasing function of $C_E$. That is, when the difference $C_E$ between the costs of different innovation strategies of digital financial enterprises is larger, the value of $y_1$ is larger. The higher the $C_E$ value, the lower the cost of illegal innovation for digital finance enterprises, and the greater the excess income on illegal innovation. Digital finance enterprises often choose to innovate illegally. However, the negative impacts of illegal innovation, such as diminished government credibility and disruption of the digital financial system, will lead regulators to opt for active regulatory strategies [47]. Therefore, in order to move the system toward “compliance innovation and active regulation”, regulators need to strengthen supervision and constantly squeeze the profit margin of illegal innovation.

5. The Simulation Analysis

In order to more clearly and intuitively reflect the dynamic evolution behavior of digital financial enterprises and regulatory agencies, MATLAB software was used to carry out numerical simulation analysis of the game system. At the same time, the influence of the upper limits $F$ and $V$ on the dynamic evolution of digital financial enterprises is discussed.

5.1. Simulation Analysis of System Unable to Achieve Evolutionary Stability under the Static Reward and Punishment Mechanism

In this section, by referring to the data settings of relevant papers [35,36,38] and combining with the actual situation, the initial values of various parameters in the system are assumed as follows when the system cannot achieve evolutionary stability: $C_1 = 4$, $C_2 = 3$, $C_3 = 3$, $C_4 = 1$, $F = 6$, $V = 2$, $L = 1$. According to the initial values of each parameter, $(x_0, y_0) = (0.5, 0.25)$ can be easily obtained. When the two-dimensional dynamic system $D_1$ has the same initial value, let us say that the initial values of $x$ and $y$ are both 0.5, and the evolutionary game process of system $D_1$ is shown in Figure 2.

![Figure 2](image-url)

*Figure 2. Evolutionary game diagram of system $D_1$.*

From Figure 2, it can be seen that the evolutionary game process is a closed-track line loop with periodic motion around the central point $(x_0, y_0)$. The system has no stable equilibrium point. It indicates that the game process of two groups, digital financial enterprises and regulators, shows a cyclical behavior pattern at this time.

The above conditions remain unchanged. In order to discuss the influence of different values on the evolutionary game curve, when the initial value $x = 0.5$ of compliance innovation probability of digital financial enterprises, $y = 0.2$ and $y = 0.6$ are, respectively,
taken as the initial value of regulatory institutions’ evolutionary game strategy, as shown in Figure 3a. When the regulator actively supervises the initial value \( y = 0.25 \) of probability, \( x = 0.2 \) and \( x = 0.6 \) are, respectively, taken as the initial value of the evolutionary game strategy of digital financial enterprises, as shown in Figure 3b.

![](image1.png)

**Figure 3.** Influence of different values on evolutionary path under the static reward and punishment mechanism. (a) Evolutionary path of regulators; (b) evolutionary path of digital financial enterprises.

It can be seen from Figure 3a,b that for different initial values of \( x \) and \( y \), the evolution trajectory fluctuates with time, and the system will not be stable at the central point \((x_0, y_0)\), and there is no evolutionary stability strategy. In terms of fluctuation amplitude, the fluctuation amplitude of evolution trajectory is obviously different with the initial value. When \( x = 0.2 \) and \( y = 0.6 \), \( x \) and \( y \) are farther from the center point, and the fluctuation range is larger than that of \( x = 0.6 \) and \( y = 0.2 \). It can be seen that the initial value has a certain influence on the game strategy choice of the two sides, and the supervisory authorities should take the initial value fully into account when formulating the supervisory policy. Therefore, based on the probability of compliance innovation in digital financial enterprises, it is reasonable for regulators to implement incentive and punishment mechanisms.

5.2. Simulation Analysis When the System Reaches Evolutionary Stability under the Dynamic Reward and Punishment Mechanism

5.2.1. Simulation Analysis When the System Reaches the Stability Result

In system \( D_2 \) of the dynamic reward and punishment mechanism, the initial values of each parameter are consistent with Section 4.1. According to each parameter value, \((x_1, y_1) = (0.38, 0.45)\) can be obtained. When the initial values of \( x \) and \( y \) are both 0.5, the evolutionary game process of system \( D_2 \) is shown in Figure 4.

![](image2.png)

**Figure 4.** Evolutionary game diagram of system \( D_2 \).
As can be seen from Figure 4, when the initial values of \( x \) and \( y \) are both 0.5, the evolution trajectory of the game system between digital financial enterprises and regulators shows a trend of spiral convergence with the increase of the number of iterative steps of system \( D_2 \), and finally reaches stability.

The influence of different values of both sides of the game on the evolutionary game curve under the dynamic reward and punishment mechanism is further analyzed. When the initial value \( x = 0.38 \) of compliance innovation probability of digital financial enterprises, \( y = 0.2 \) and \( y = 0.6 \) are, respectively, taken as the initial value of regulatory institutions’ evolutionary game strategy, as shown in Figure 5a. When the regulator actively supervises the initial value \( y = 0.45 \) of probability, \( x = 0.2 \) and \( x = 0.6 \) are, respectively, taken as the initial value of the evolutionary game strategy of digital financial enterprises, as shown in Figure 5b.

\[ \begin{align*}
\text{(a)} & \quad \text{Evolutionary path of regulators;} \\
\text{(b)} & \quad \text{evolutionary path of digital financial enterprises.}
\end{align*} \]

It can be seen from Figure 5a,b that \( x \) and \( y \) have temporary shocks in their evolutionary trajectories at the initial stage due to the influence of strategies of both sides of the game and market environment. The initial values \( x = 0.6 \) and \( y = 0.2 \) are more volatile. However, no matter what the initial value of digital finance companies and regulators is, both strategies can gradually reach stability.

### 5.2.2. Simulation Analysis of the Influence of Upper Limit of Reward and Punishment Intensity on Evolutionary Path

Considering the effect of the upper limit of penalty \( F \) on the behavioral strategies of digital financial firms and the behavioral strategies of regulators, and holding all other parameters constant, both \( x \) and \( y \) stabilize after a brief wobble as the regulator’s penalty limit \( F \) increases. The probability \( x \) of compliance innovation of digital financial enterprises increases, while the probability \( y \) of active regulation by regulators decreases, as shown in Figure 6a,b. It can be seen that the greater the upper limit of punishment imposed by regulators, the more severe the punishment may be imposed on digital financial enterprises for innovation violations. Under the deterrent of high punishment, more digital financial enterprises tend to adopt compliance innovation strategies. As a result, there is no need for regulators to conduct frequent regulatory activities and the likelihood of regulation is greatly reduced [48]. At the same time, it is also possible that the increased penalties and large fines will make the regulatory behavior more closely watched by the public and financial enterprises. Regulators will therefore be more cautious and less likely to opt for active regulation [49].
Figure 6. Influence of punishment intensity on the evolutionary path. (a) The impact of penalties on digital finance firms; (b) the impact of penalty levels on regulators.

Considering the impact of the reward cap \( V \) on the behavioral strategies of digital finance firms and regulators, and holding other parameters constant, both \( x \) and \( y \) will stabilize after a brief wobble as the regulator’s reward cap \( V \) increases. Both the probability of compliance innovation \( x \) of digital financial enterprises and the probability of active regulation \( y \) of regulatory institutions decrease accordingly, but the decrease of \( x \) is smaller than that of \( y \), as shown in Figure 7a,b. Therefore, appropriate incentives, subsidies, and preferential policies can promote compliance innovation in digital finance enterprises. However, in the long run, excessive incentive policies make it less likely that digital finance enterprises will innovate in compliance. The reason may be that the amount of incentive subsidies is too high, which will put more pressure on the financial expenditure of the regulatory authorities and discourage them from adopting an active regulatory strategy. At the same time, when the probability of active regulation becomes smaller, the probability of digital financial enterprises getting rewards and subsidies will also become smaller, so the probability of digital financial enterprises choosing compliance innovation will also decrease. In addition, because of information asymmetry, excessive incentive policy tend to bring arbitrage space to enterprises. Digital finance companies can fake innovation results and use them for other purposes unrelated to innovation to obtain more money. A similar situation has been argued by scholars in other fields, such as Wang et al. [50], who found in their research on project management that the continuous increase of rewards and punishments would not change the direction of investment and high effort. When Zuo et al. [51] and Sun et al. [52] studied the new energy vehicle industry, they found that when the government subsidy intensity exceeded a reasonable range, the subsidy policy would fail.

Figure 7. Influence of reward intensity on evolutionary path. (a) The impact of incentives on digital finance firms; (b) the impact of incentive levels on regulatory bodies.
6. Conclusions and Recommendations

6.1. Conclusions

This paper constructs an evolutionary game model of government supervision and innovation behavior in digital financial enterprises, analyzes the influencing factors in the game process, and discusses the institutional evolution under a static reward and punishment mechanism and a dynamic reward and punishment mechanism. The following conclusions can be drawn:

First, when regulators adopt the static reward and punishment mechanism, the evolution process of digital financial enterprises and regulators cannot reach a stable state. The system is a closed rail loop with periodic movement around the central point, and the game process presents periodic behavior without a stable equilibrium point. However, when regulators adopt the dynamic reward and punishment mechanism, that is, when the intensity of reward and punishment is adjusted in time with the probability change of compliance innovation of digital financial enterprises, the game process of the two game subjects can achieve evolutionary stability. It can realize the effective supervision of digital financial enterprises’ innovation behavior by regulators, indicating that the dynamic reward and punishment mechanism is more effective.

Secondly, under the dynamic reward and punishment mechanism, when the regulators increase the punishment intensity, the probability of digital financial enterprises choosing compliance innovation will increase, and the probability of regulators choosing active regulation will decrease. However, when the upper limit of regulators’ rewards is larger, the probability of compliance innovation of digital financial enterprises will decrease, as will the probability of active regulation by regulators, and the decrease will be more obvious. Therefore, for the regulator, it is necessary to set a reasonable threshold of rewards and punishments, and cannot increase rewards or aggravate punishments without limit [53].

Thirdly, under the dynamic incentive and punishment mechanism, when the cost difference between different regulatory strategies is small and the cost difference between different innovation strategies is large, digital finance enterprises often choose compliance innovation. When the cost difference between different regulatory strategies of regulators is smaller, the upper limit of reward and punishment is smaller, and the cost difference between different innovation strategies of digital financial enterprises is smaller, regulators will tend to actively regulate. This requires regulators to make efforts to reduce the regulatory costs, while making illegal innovation pay more costs, and then compress the illegal innovation profits of digital financial enterprises [54].

6.2. Recommendations

Based on the above analysis conclusions, in order to achieve the win-win development of digital financial innovation and regulation, the following suggestions are put forward:

First, regulatory authorities should continuously improve their regulatory capacity and level in terms of sector synergy, infrastructure and expertise, and continuously reduce regulatory costs. The smaller the difference in costs between regulatory strategies, the fewer additional costs regulators will have to pay when choosing active regulatory strategies. This is a time when regulators can reap greater benefits, such as social prestige and socioeconomic development, at less additional cost. Therefore, on the one hand, there should be a clear division of labour at all levels of government supervision to reduce the inefficiencies caused by duplication of work. Regulators at all levels should fully communicate and coordinate to ensure the effective operation of digital financial regulation. On the other hand, we should vigorously develop regulatory equipment and technology, train digital finance professionals, reduce the cost of traditional regulatory tools, encourage regulators to choose active regulatory strategies, and improve the effectiveness of digital financial regulation [55,56].

Second, the management should constantly improve the supervision system and establish a dynamic reward and punishment mechanism. Fixed rewards and disincentives not only increase supervision cost, but also lead to regulatory failures due to opportunistic
behavior by digital finance enterprises. Therefore, the supervisory authorities should dynamically adjust the incentive and punishment measures according to the actual situation [57]; for example, increasing subsidies to encourage new digital financial innovations. However, as digital financial innovation gradually matures and forms a scale, regulators should also gradually reduce the amount of subsidies, and through stricter punishment measures to promote digital financial enterprises to choose compliance innovation.

Third, digital finance enterprises should strengthen their own construction and self-regulatory management and conduct digital finance business in compliance with the law. We will give full play to the positive role of the industry self-regulatory mechanism in regulating digital financial innovation behaviors, promoting timely disclosure of information by digital financial enterprises, enhancing transparency and actively cooperating with supervision by regulatory bodies and others [58,59]. At the same time, digital financial enterprises should take the responsibility of financial education and help users to raise awareness of risk prevention. While implementing compliance financial innovation, we should take full account of corporate social responsibility and promote the sustainable and healthy development of digital finance [60].

6.3. Shortcomings and Prospects

Although this paper has some theoretical contributions, there are still certain limitations. For example, since there are many factors that affect the innovation behavior of digital financial enterprises and the regulatory behavior of regulators, only some important factors are studied in this paper. Other factors, such as regional differences and the role of social media, need further research. In addition, there are also differences in the regulatory strategies of local regulators and central regulators. Future research can compare and analyze the regulatory strategies of different regulatory agencies. Further research considering more relevant factors will help government regulatory authorities to formulate sustainable development strategies and create a good innovation environment for digital financial enterprises.

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