To Share or Not to Share? The Optimal Technology Investment in a Virtual Product Supply Chain

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Abstract: With the rapid rise of the virtual economy, the “brand + platform” virtual product distribution model led by virtual technology platforms has emerged, bringing a series of new virtual experiences to consumers. Considering that consumers have heterogeneous preferences for virtual technologies, we investigate a virtual product supply chain consisting of a brand owner, a virtual technology platform, and consumers (where the virtual technology platform has superior information about consumer preferences). We develop a signaling game model with preference information signaled by the virtual technology investment to study the optimal information-sharing strategy for the virtual product supply chain. We find that the virtual technology platform always prefers information sharing without paying any signaling cost. Moreover, we also observe that the brand owner and the platform can achieve alignment, that is, when consumers are more likely to prefer virtual technology, both the brand owner and the virtual technology platform are better off if the virtual technology platform chooses to share information with the brand owner. Finally, we analyze the consumer welfare and find that when consumers are moderately likely to prefer virtual technology, consumers can gain more benefits in the information-sharing scenario.

Keywords: virtual product; information asymmetry; information sharing; signaling; virtual technology investment

1. Introduction

Many brands and companies have placed forward-thinking technology concepts such as NFTs, virtual assets, and Metaverse in a more strategic position than ever before. According to the study published by Global Industry Analysts Inc., (GIA), the global market for Metaverse is expected to reach USD 758.6 billion by 2026 (https://www.strategyr.com/market-report-metaverse-forecasts-global-industry-analysts-inc.asp (accessed on 1 August 2020)). The Metaverse is an experiential virtual self-sustaining ecosystem powered by the virtual economy. According to Adroit Market Research, the global virtual goods market value is estimated to reach USD 189.76 billion by 2025 (https://www.adroitmarketresearch.com/press-release/virtual-goods-market (accessed on 1 March 2019)). Virtual goods are often used to improve the consumer experience in online games and social media [1]. Social life and the virtual Internet are converging and creating a large, fast-growing economy of virtual goods consumption.

The “Brand + Platform” model is an initial exploration for brands to enter the Metaverse market by selling virtual products on a virtual technology platform. Brands cooperate with virtual technology platforms to launch virtual products, while the platform provides technological support for virtual products. Technology platforms and companies have a lot to gain as consumer demand for virtual products increases. Driven by consumer demand to enhance their virtual image and wear in their personalized virtual spaces, many brands are trying digital fashion and virtual space through cooperation with games and social platforms. Collaborations between luxury brands and social games can generate more revenue, causing a broader halo effect (https://wwd.com/business-news/technology/...
In June 2020, Gucci launched an innovative “brand + social platform” cooperation model. Gucci collaborated with Snapchat, a photo–video social media platform, to launch the first digital virtual sneaker, which allows platform users to virtually try on Gucci sneakers using augmented reality (AR) technology and purchase them instantly online. Table 1 shows the recent examples of virtual collections.

### Table 1. Recent practices on virtual collections distribution.

<table>
<thead>
<tr>
<th>Brand</th>
<th>Virtual Technology Platform</th>
<th>Virtual Products</th>
<th>Practices</th>
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<tbody>
<tr>
<td>Gucci</td>
<td>Roblox</td>
<td>Bags, Clothes</td>
<td>Gucci and Roblox collaborate to launch the Gucci Garden Archetypes virtual exhibition. Balenciaga cooperates with Fortnite to launch a variety of designs, with virtual clothing and physical products available for purchase.</td>
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<tr>
<td>Balenciaga</td>
<td>Fortnite</td>
<td>Clothes</td>
<td>Forever 21 built a virtual store on Roblox. Vans launched a skateboard-themed virtual amusement park on Roblox.</td>
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<tr>
<td>Forever 21</td>
<td>Roblox</td>
<td>Clothes</td>
<td>Nars launched a virtual store on Zepeto, selling new makeup products, virtual clothing, and various accessories.</td>
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<tr>
<td>Vans</td>
<td>Roblox</td>
<td>Sneakers</td>
<td></td>
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<tr>
<td>Nars</td>
<td>Zepeto</td>
<td>Cosmetics</td>
<td></td>
</tr>
<tr>
<td>Stella Artois</td>
<td>ZED RUN</td>
<td>Game Props</td>
<td>Stella Artois issued a limited number of NFTs and designed a series of breed horses, themed skins, and 3D tracks.</td>
</tr>
</tbody>
</table>

**Remark 1.** *The information is obtained from public sources on the Internet.*

The virtual business environment is more competitive than ever before. Facing great uncertainty in business operations management, companies in different industries have invested in sophisticated systems to collect information to resolve uncertainty, including preferences, demand forecasting, etc. The problem of information asymmetry in the supply chain is an important factor that affects the efficiency of enterprise operation and cooperation and is also an important factor that hinders the future development of enterprises. Information sharing strategy becomes an important way to solve the information asymmetry problems in supply chain management and promote win–win cooperation. Different supply chain members are typically exposed to different sources of information. Retailers have point-of-sale data and marketing investment knowledge, while manufacturers study consumer demographics and motivations, so they have extensive knowledge of the market and future trends.

In reality, platforms (i.e., virtual technology providers) often have more information about virtual products than brands [2]. Virtual experiences have the advantage of being able to granularly track users’ activity, from tastes, to movement around the space, to demographic appeal and checkout process. The Metaverse economy strictly limits all aspects of the creation, exchange, and consumption of digital products to be carried out in the digital world. Therefore, virtual platforms, where transactions take place, can better capture information about consumer preferences through digital records.

We focus on the virtual product supply chain. Virtual products do not require inventory and can be produced at infinite capacity [2]. In the literature, traditional virtual products mainly refer to mobile applications, gift cards and coupons, e-books, e-music, services and extended warranties, memberships and subscriptions, etc. [3–5]. They assume that manufacturers have more information about these products than retailers. Different from traditional virtual products, virtual products in the Metaverse are based on a virtual technology platform with which the brand is contracted. The virtual technology platform is both a manufacturer of virtual products and a place where virtual product transactions take place, that is, the platform is both a manufacturer and a retailer. The virtual technology platform has more information about the consumer than the brand owner.
We establish a revenue-sharing commission contract that is typically applied to developers of virtual products and distribution platforms [6,7]. These contracts are widely used in online commerce, with several retail sites and platform distributors of virtual goods using them. Given its popularity in practice, revenue-sharing contracts have been applied to relevant research on virtual product operations management [5,8]. Our paper contributes to the aforementioned literature by considering consumer virtual technology preference information sharing in a virtual technology platform-based supply chain.

In this paper, we study two different information-sharing formats. The first format is “no information sharing”, which occurs when the virtual technology platform ex ante commits to not sharing the consumer virtual technology preference. The second format is “information sharing”, in which the virtual technology platform promises to reveal the true information after obtaining it, and the brand owner adjusts its pricing plan accordingly. We aim to investigate whether platforms should share their private information (consumer virtual technology preference) with upstream brands and examine the impacts of different information-sharing formats on supply chain participants.

Based on observed real-world practices, we characterize the virtual product release model of “brand + platform”. In our model, we consider a brand owner (e.g., Gucci) to cooperate with a virtual technology platform (e.g., Roblox) and release new virtual collections on the platform. Our analysis assumes that the platform chooses the information-sharing format at the beginning of the game and then the platform and the brand owner sign a revenue-sharing contract. After the virtual product enters the Metaverse market, the virtual technology platform has information superiority, and the brands update the posterior virtual technology preference type by the platform’s virtual technology investment as a signal. With this analytical framework, we investigate the research questions (RQs), which are as follows:

**RQ1:** If the virtual technology platform does not share its private information, then what is the platform’s optimal signaling strategy?

**RQ2:** Does a virtual technology platform have an incentive to share its superior preference information with the upstream brand owner?

**RQ3:** When can information sharing be a win–win strategy for both the brand owner and the virtual technology platform? How is consumer surplus affected?

In this paper, we characterize an analytical model to investigate the equilibrium outcomes under the two sharing formats and the firms’ preferences for these formats. The following are our significant contributions to this paper. First, we find that when consumers are less likely to like virtual technology, virtual technology platforms will increase the virtual technology investment to convince the brand owners that the market has a high preference for virtual products. As consumers are more likely to prefer virtual technology, the virtual technology platforms will no longer be eager to reveal the true type of consumer preference to the brand owners. Second, the virtual technology platforms are always willing to share their superior information, i.e., consumer virtual technology preference, with the brand owners, because in the no-information-sharing scenario, it will make the high type virtual technology platform pay an information cost to reveal their true information. Third, when consumers are more likely to like virtual technology, information sharing can benefit both the brand owners and the virtual technology platforms simultaneously. We also identify the consumer surplus in equilibrium. When consumers are moderately likely to prefer virtual technology, consumers can gain more benefits in the information-sharing scenario.

The conclusions of this paper not only reveal the signal motivation of enterprises’ virtual technology investment behavior but also provide decision-making references and managerial insights for enterprises to choose information-sharing strategies in virtual product supply chains. In reality, the phenomenon of information asymmetry in the supply chain is very common, and the decision-making and information-sharing strategies of
enterprises need to be considered from various aspects. Furthermore, this study provides novel and interesting research insights and future research directions for supply chain management research.

The remainder of this work is structured as follows. In Section 2, we review the relevant literature. The model setup is introduced in Section 3. In Sections 4 and 5, we describe the equilibrium analysis and investigate the optimal strategy in the no-information-sharing and information-sharing scenarios. In Section 6, we examine the main findings and derive conclusions. In Section 7, we discuss management implications and potential future research directions. All technological proofs are relegated to the Appendix A for the sake of presentation.

2. Literature Review

Our study is closely related to four streams of the literature: (i) operations management of the virtual product supply chain, (ii) information asymmetry in the supply chain, (iii) information sharing with signaling in marketing and operations, and (iv) agency pricing model.

2.1. Virtual Products

Our research is related to the literature about the virtual items and information products in supply chain management. Considering the consumer’s psychological behavior, Guo et al. [9] investigate how virtual currency affects player behavior, game provider strategy, and social welfare. Royo-Vela et al. [10] construct a structural equation model to study Chinese game players’ intention to purchase digital fashion. The pricing of virtual items is widely studied in the literature. Balasubramanian et al. [11] investigate two pricing strategies for information goods: selling and pay-per-use. They show that pay-per-use yields higher profits in a monopoly when the psychological cost is low. Some scholars combine virtual products with information asymmetry. Sundararajan [12] study the optimal pricing of information goods in an incomplete information scenario. He characterizes the optimal conditions for the usage-based and fixed-fee pricing model. Qu et al. [13] investigate the price and quality effort joint decisions in a virtual product supply chain with information sharing. They show that information sharing benefits both supply chain participants under certain conditions.

Some studies have also described the characteristics of virtual products. Huang and Sundararajan [14] analyze a pricing model of information products and indicate that the cost structure includes zero marginal costs and positive periodic fixed costs. Chernonog and Avinadav [15] compare traditional models designed for tangible products versus virtual product innovation models. These products allow the retailer to reduce holding costs while also ensuring timely demand fulfillment with no risk of shortage. Chernonog et al. [16] study how to determine pricing and quality in virtual product supply chains. Virtual items have sufficient production capacity, no holding costs, and zero lead time. Tan [1] develops an analytical model to study the economic implications of selling pre-owned virtual goods with blockchain technology. The author assumes that the value of virtual products will increase after resale.

Our study is fully motivated by the current real-life practice of the virtual business. In this paper, the virtual product supply chain involves information asymmetry. We focus on the virtual products in the context of the Metaverse economy, which is characterized by decentralization. These are different important features that differentiate this from the existing literature.

2.2. Information Asymmetry

Information asymmetry in supply chains has been widely studied in the literature. When signing contracts, supply chain members make plans and decisions depending on the information available to them. Lariviere and Padmanabhan [17] study the behavior of manufacturers to launch new products under slot-subsidy agreements with retailers. In this case, the manufacturer is either more informed than the retailer or equally as informed. Guo
and Iyer [18] argue that manufacturers can continuously access consumer information and affect downstream members’ decisions by sharing the information. Jiang et al. [19] analyze the strategic interaction between platform owners and independent sellers, arguing that the latter may be more informed than the platform. Zhang and Chen [20] study different information-sharing behaviors in a two-echelon supply chain wherein the manufacturer has complete demand information and the seller has partial information. They show that the supply-chain participants must sign a coordinative contract to ensure that they share their information. Shi et al. [21] argue that the latest advances in technology will intensify information asymmetry by allowing retailers to access more market information.

The literature has extensively studied the asymmetry of different information types. Guan and Chen [22] indicate that upstream members of the supply chain may gain more consumer preference information by conducting market research, product testing, focus groups, or by gaining access to joint research. Avinadav et al. [7] investigate the impact on virtual product supply chains of producers with superior demand information. The authors prove that known superiority is better for the retailer, whereas hidden superiority is better for the manufacturer. Vosooghidizaji et al. [23] study the coordination of corporate social responsibility in a supply chain in which a supplier and a manufacturer both possess private CSR cost information. They show that the supply chain profit decreases under different information asymmetry scenarios. Xu et al. [24] explore the influence of information asymmetry on optimal decisions and performance when the green marketing cost coefficient is the retailer’s private information. They indicate that information asymmetry is beneficial to the retailer.

Similar to the above studies, we also emphasize the problem of information asymmetry in the supply chain, and the virtual technology platforms have superior information. Many scholars have extensively studied the information asymmetry problem in supply chains, and most of them have focused on demand information and cost information asymmetry. However, we focus on the scenario that upstream members have more specific information about customer preferences. In this paper, we study the problem of consumer preference information asymmetry between upstream brand owners and downstream virtual technology platforms, and the type of information differs from related studies.

2.3. Information Sharing with Signaling

The supply-chain information-sharing strategy generates a lot of discussion in the operations-management literature. When only the virtual technology platform has consumer preference information, the brand owner will draw sensible deductions on the information based on the platform’s virtual technology investment decision. Thus, this study is also relevant to the literature that studies information sharing and signaling [25].

Jiang et al. [19] study how information is shared in a distribution channel where the supplier has more accurate demand forecasts than the downstream seller. They examine three different information-sharing formats and find that the manufacturer prefers the mandatory sharing format whereas the retailer prefers the no-sharing format. According to Stock and Balachander [26], a high-quality retailer may optimally choose to make the goods scarce to credibly signal the quality of its offering to uninformed customers. Jiang et al. [27] argue that the service provider may have more information about the customer’s problem than the customer and different costs on the service provider can signal customer preferences. Jiang and Yang [28] develop a dynamic game-theoretic model to investigate the quality and price decisions of a company’s new experience offerings. Early consumers can rationally infer cost and quality from a company’s price. They show that when a firm’s high efficiency is publicly known, they may reduce the product quality rather than increase it. Li et al. [29] study a two-stage model in which manufacturers have private information about the effectiveness of advertisements and they choose different information-sharing formats. Under the no-information-sharing format, the manufacturer and the retailer play a signaling game and can achieve alignment. Niu et al. [30] develop models to study whether the supplier will invest in decarbonizing innovation in a supply
chain where the retailer privately knows the true market size. They find that the supply chain performance under information asymmetry scenario may not necessarily be lower than under information symmetry scenario. Xu et al. [31] study the use of blockchain technology to solve the information-silo effect. The authors show that the information sharing cost-subsidy mechanism can reduce the signaling cost of information sharing and optimize the supply-chain structure.

This paper differs from the aforementioned papers in three ways. First, we consider two types of information sharing: signaling and voluntary disclosure. We study the two information-sharing strategies of virtual technology platforms and compare the preferences of supply chain members for these two information-sharing strategies. Second, we consider a supply chain where only virtual technology platforms have private consumer preference information, not brand owners. This assumption is very practical and important for new and virtual products. Third, most of the existing literature signals information through price and quality, etc. In this paper, we consider a game model in which platforms use virtual technology investment as a signal to transmit consumer preference information. This paper portrays the information transmission cost of virtual technology investment and specifies the profitability conditions of this transmission.

2.4. Agency Pricing Model

With the rapid growth of the Internet economy, the literature on agency pricing is growing. Avinadav et al. [5] provide a comprehensive study of the revenue-sharing contract formats commonly used in the mobile app business. They find that revenue-sharing contracts avoid the double marginalization effect related to vertical competition. Tan and Carrillo [32] compare an agency pricing model and a wholesale model for digital products. Their results show that the agency pricing model can mitigate the double marginalization effect of the supply chain. Hao et al. [33] identify a between-agent subsidization strategy for the platform and find that the agency pricing model leads to a higher price than the traditional model. Wang et al. [34] study a reselling agreement or an agency selling agreement between an upstream supplier and an online platform. They find that the agency fee affects the pricing strategy in two different ways. Bart et al. [35] review the literature on revenue-sharing contracts and indicate that few studies combine revenue-sharing contracts with information asymmetry. Tan [1] investigates an agent pricing model for trading second-hand virtual goods. He shows that developers do not always prefer a high revenue-sharing proportion.

In this article, following the above literature, we also employ the agency model. However, our supply chain involves virtual products and preference information asymmetry. We focus on consumer preference signaling with a typically used revenue-sharing contract. We further focus on the effect of agency pricing models on virtual product markets from a signaling perspective.

Table 2 summarizes the position of this study in the literature. We develop and evaluate a model to capture the formulation of a revenue-sharing consignment contract between a brand owner and a virtual technology platform, where the latter is the supply chain’s leader and has superior information about consumer virtual technology preferences. We provide a new perspective on virtual product introductions and operation systems. Our research provides implications for virtual product management as well as practical guidance for brands and virtual product developers (virtual technology platforms).

Considering the consumer preference information asymmetry problem in the virtual product supply chain, this paper studies the information sharing strategies and provides the following theoretical and practical implications: first, we study how consumer preference information from downstream virtual technology platforms is transmitted to upstream brand owners. The type of information differs from related studies, so this paper further enriches the theoretical study of information-sharing strategies in supply chains. Second, this paper characterizes the information signaling cost of virtual technology investment, specifies the profitability conditions of this signaling method, and provides practical guid-
ance to firms in choosing information-sharing strategies. Third, this paper introduces the signaling method into the enterprise decision-making in virtual product supply chain management, thus broadening the application area of signaling.

Table 2. Positioning of this study in the literature.

<table>
<thead>
<tr>
<th>Virtual Product</th>
<th>Agency Pricing</th>
<th>Information Asymmetry</th>
<th>Information Sharing</th>
<th>Signaling</th>
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3. Model

Consider a two-echelon supply chain consisting of a brand owner (she, denoted as B) and a virtual technology platform (he, VTP, denoted as P) where the brand owner wants to enter the Metaverse market by selling virtual products on virtual technology platforms. The brand owner cooperates with the platform to launch virtual products, and the platform provides technological support for virtual products. For example, Gucci has opened the Gucci Garden virtual space on the Roblox platform and sold digital bags in the virtual community. The platform will invest virtual technology investment (VTI) into virtual products. Virtual technology investment can be seen as a quality investment effort in virtual products. The virtual technology content of the virtual product reflects its quality. Product quality is regarded as a demand accelerator, and its positive effect on demand and the required investment to acquire it are examined in Huang et al. [37] and Chernonog et al. [15]. Thus, our fundamental assumption is that increased virtual technology investment implies higher virtual product quality, which leads to higher demand. We let \( \zeta \) be the virtual technology investment. We use a two-point distribution to characterize the information asymmetry of consumer virtual technology preference, which is consistent with previous research (Jiang et al. [19]; Jiang et al. [27]; Zhang et al. [38]; Li et al. [29]). \( \theta \) represents the consumer virtual technology preference, which can be either a high value \( \theta = \theta_h \) with an ex ante probability of \( \alpha \) or a low value \( \theta = \theta_l \) with an ex ante probability of \( 1 - \alpha \), where \( 0 < \theta_l < \theta_h \) and \( 0 \leq \alpha \leq 1 \). We assume that the VTP can know the virtual technology preference accurately and privately, whereas the brand owner only knows the prior distribution of the preference. Since all steps of digital products, including creation, exchange and consumption, have been limited by the metaverse economy, to be completed in a digital world, the virtual platform, as the place of transaction generation, can better understand the consumer preference information through digital records. Thus, only the VTP has private information about the consumer virtual technology preference. For simplicity, we denote the ratio of high preference to low preference \( (\sigma = \frac{\theta_h}{\theta_l}) \) as “preference variability”, which characterizes the uncertainty of virtual technology preference. Higher preference variability indicates a higher uncertainty level of virtual technology preference.

We establish a revenue-sharing contract which is commonly applied in a virtual product supply chain (Avinadav et al. [7]). In the agency model, the VTP serves as an agent and receives a proportion \( \beta \) of the revenue based on industry standards. Thus, the revenue-sharing proportion \( \beta \) is assumed to be exogenous. The game proceeds as depicted in Figure 1.
(1) In the first stage, the VTP decides whether to share his private information with the brand owner, before observing the preference information. We explore two information-sharing formats: no information sharing and information sharing. The first format is “no information sharing”, which occurs when the virtual technology platform ex ante commits to not sharing the consumer virtual technology preference. The second format is “information sharing”, in which the virtual technology platform promises to reveal the true information after obtaining it, and the brand owner adjusts their pricing plan accordingly. Then, the brand owner and the VTP sign a revenue-sharing contract. We assume that the revenue-sharing contract is determined in the first stage. Note that the VTP often takes time to build virtual spaces and test products before entering new markets, and the VTP expands into Metaverse markets with well-developed products (Li et al. [29]).

(2) In the second stage, the VTP launches a virtual product and enters the Metaverse market. The nature chooses the consumer virtual technology preference, $\theta_i$, where $i = \{h, l\}$. Then, the VTP can know the true consumer preference and determine the virtual technology investment $\zeta$. The preference information is shared according to the former arrangement. The brand owner observes the virtual technology investment $\zeta$, formulates their belief on the consumer preference type, $\theta_j$, where $j = \{h, l\}$, and decides the retail price $p$. We use the subscript "i" to refer to the true consumer virtual technology preference (i.e., $\theta_i$) and the second subscript "j" to refer to the brand owner’s belief (i.e., $\theta_j$). The brand owner’s belief may not always be consistent with the true information. We assume that the VTP’s virtual technology investment is observable to the brand owner since the brand owner can infer virtual technology investment from characteristics such as the complexity of the design and access/ability to use the software. Take the fashion industry as an example: depending on the complexity and fit of the garment, Dress-X has different levels of investment in virtual technology, ranging from realistic simulated virtual garments to dynamic versions of virtual garments.

The market size is normalized to one. We assume that consumers are heterogeneous in their valuation $v$ towards purchasing virtual products, following a uniform distribution $v \in [0, 1]$. Thus, consumer utility is

$$U = v - p + \theta_j \zeta$$

Then, the market demand is realized:

$$D = 1 - p + \theta_j \zeta$$

We assume that the VTP places an order following demand realization, so there is no demand uncertainty. Because virtual items can be produced with ample capacity and without the need for inventory, the sales volume of such products matches the demand (Chernonog et al. [16]). In the Metaverse economy, virtual products are decentralized and creatable. Thus, we assume that the VTP produces the virtual product at a unit cost $\gamma \zeta$ ($\gamma > 0$), $\gamma$ is the scale coefficient of the unit cost, respectively. Virtual technology ensures that platforms such as DressX can inspect, audit, update and adjust information.
and investments for each virtual product, each unit of virtual product will incur costs such as
transaction data volume and storage costs. We omit the trivial case ($2\theta_i - \theta_j \beta \theta_j - 2\gamma \theta < 0$
and $\beta \theta_i \theta_j - \theta_i \theta_j - \theta_j < 0$) where neither VTPs have an incentive to mimic the other’s type and
are inclined to reveal their true preference information. Regardless of realized demand, we assume $1 < \sigma < 2$ to ensure that both the brand owner and the VTP have nonnegative
demand in the Metaverse market in equilibrium.

We use the backward induction to solve the model. After observing the VTP’s virtual
technology investment, the brand owner’s belief on the preference is $\theta_j$, so they believe the
market demand is

$$D_B = 1 - p + \theta_j \zeta$$

The profit function of the brand owner is

$$\pi_{ij}^B(p) = p(1 - \beta)(1 - p + \theta_j \zeta) \tag{1}$$

The brand owner determines the retail price by maximizing $\pi_{ij}^B(p)$ given by (1). We
can easily derive the optimal retail price for the brand owner as $p_j(\zeta) = \frac{1 + \theta_j \zeta}{2}$. Considering
the optimal retail price $p_j(\zeta)$ and the true virtual technology preference $\theta_j$, the VTP’s profit,
denoted as $\pi_{ij}^V(\zeta)$, can be written as

$$\pi_{ij}^V(\zeta) = \beta p_j(\zeta) - \gamma \zeta \right) (1 - p_j(\zeta) + \theta_j \zeta) = \frac{1}{4} \left[ 1 + \zeta (2\theta_i - \theta_j) \right] \left[ \beta + \zeta (\beta \theta_j - 2\gamma) \right] \tag{2}$$

The VTP determines the virtual technology investment $\zeta$ by maximizing their profit
$\pi_{ij}^V(\zeta)$ given by (2). Based on the best response from the brand owner, the VTP’s optimal
virtual technology investment is $\zeta_{ij} = \frac{\gamma \beta \theta_j \theta_j - \beta \theta_j - 2\gamma}{(2\theta_i - \theta_j)(\beta \theta_j - 2\gamma)}$, respectively. Thus, the brand
owner’s corresponding retail price is $p_j = \frac{1 + \theta_j \zeta}{2} = \frac{(3\gamma - \beta \theta_j + \theta_j (\beta \theta_j - 4\gamma))}{(2\theta_i - \theta_j)(\beta \theta_j - 2\gamma)}$. By plugging in $p_j$ and $\zeta_{ij}$, we will obtain the brand owner and the VTP’s expected profits, respectively.

The following proposition summarizes the supply chain members’ optimal decisions
and outcomes for any given $\beta$, $\theta_i$ and $\theta_j$.

**Proposition 1.** Given a revenue-sharing proportion $\beta$, the VTP’s optimal virtual technology investment is $\zeta_{ij} = \frac{\gamma \beta \theta_j \theta_j - \beta \theta_j - 2\gamma}{(2\theta_i - \theta_j)(\beta \theta_j - 2\gamma)}$, the brand owner’s optimal retail price is $p_j = \frac{(\gamma \beta \theta_j - \beta \theta_j - 4\gamma)}{(2\theta_i - \theta_j)(\beta \theta_j - 2\gamma)}$, and the VTP and the brand owner’s profits are $\pi_{ij}^V = \frac{1}{4(2\theta_i - \theta_j)(\beta \theta_j - 2\gamma)}$, respectively.

Proposition 1 shows the optimal virtual technology investment and retail price for
any given revenue-sharing proportion $\beta$. Recall that the VTP first chooses between the two
information formats whether to share their private information. We apply backward analysis for each format to determine the optimal retail price of the brand owner and the VTP’s equilibrium virtual technology investment. In Section 4, we consider the no-information-sharing format, where the brand owner can infer consumer preference information from the VTP’s investment decisions even though they do not own it. That is, there is a signaling game in the no-information-sharing format. In Section 5, we investigate the outcomes of the information-sharing format and discuss the brand owner, the VTP and consumers’ preferences of the two information formats. Figure 2 shows the main structure of this paper.
Proposition 1 shows the optimal virtual technology investment and retail price for different consumer preferences. Thus, the brand owner cannot infer the true consumer preference perfectly. These two forms of information are the private information of the VTP, which they do not share with the brand owner. The VTP determines the virtual technology investment \( \zeta \) after observing the true consumer preference. Thus, the virtual technology investment \( \zeta \) reflects the information observed by the VTP. Then, the brand owner will update their belief about the consumer preference and set the retail price after observing the VTP’s virtual technology investment \( \zeta \). Thus, the VTP has the potential to distort the virtual technology investment \( \zeta \) and induce the brand owner to form a belief that can benefit them. Furthermore, the brand owner is also aware of the potential for information distortion. Therefore, there is a signaling game in this supply chain.

There exist two potential equilibria in a signaling game: separating equilibrium and pooling equilibrium. In the separating equilibrium, the VTPs set different investments for different consumer preferences. Thus, the brand owner can infer consumer preference accurately from the VTPs' investment decisions. In the pooling equilibrium, with various consumer preferences, the VTPs will make the same virtual technology investment, so the brand owner cannot infer the true consumer preference perfectly. These two forms of equilibria are discussed in the following subsections.

4. No Information Sharing

Under the no-information-sharing format, the consumer virtual technology preference is the private information of the VTP, which they do not share with the brand owner. The VTP determines the virtual technology investment \( \zeta \) after observing the true consumer preference. Thus, the virtual technology investment \( \zeta \) reflects the information observed by the VTP. Then, the brand owner will update their belief about the consumer preference and set the retail price after observing the VTP’s virtual technology investment \( \zeta \). Thus, the VTP has the potential to distort the virtual technology investment \( \zeta \) and induce the brand owner to form a belief that can benefit them. Furthermore, the brand owner is also aware of the potential for information distortion. Therefore, there is a signaling game in this supply chain.

There exist two potential equilibria in a signaling game: separating equilibrium and pooling equilibrium. In the separating equilibrium, the VTPs set different investments for different consumer preferences. Thus, the brand owner can infer consumer preference accurately from the VTPs’ investment decisions. In the pooling equilibrium, with various consumer preferences, the VTPs will make the same virtual technology investment, so the brand owner cannot infer the true consumer preference perfectly. These two forms of equilibria are discussed in the following subsections.

4.1. Separating Equilibrium

In this paper, if the VTP learns that the virtual technology preference is high (low), they will set a correspondingly high (low) virtual technology investment. Considering that the l-type VTP wants to mimic the h-type VTP, the latter will want to expose their information type to allow the brand owner to charge a correspondingly high retail price (to gain more marginal profit and prevent the high cost of the virtual technology investment). Hence, our analysis reveals that the VTP prefers the brand owner to perceive preference as high preference, allowing the brand owner to set a high retail price to maximize profit. Thus, the VTP that receives low preference information (i.e., l-type VTP) may have an incentive to pretend to receive high preference information (i.e., mimic h-type VTP). That is, the h-type VTP wishes to expose their type. Thus, for any given virtual-technology investment \( \zeta \), the VTP prefers the brand owner to choose a high retail price to gain more profit.

There are two types of separating strategies, costly separating and costless separating. Costless separating means that the separating strategy of the h-type VTP does not require any cost, which means that the difference between the virtual technology investment of the two VTPs is sufficiently high that the l-type VTP cannot mimic h-type VTP. The h-type and l-type VTPs are naturally separated, with no profit loss of the h-type VTP. In a costly separating, the h-type VTP’s investment requires a distortion to the optimal investment. The h-type VTP will distort their investment upward to increase the l-type VTP’s cost of mimicking. To successfully separate himself from the l-type VTP and reveal their preference.
information type accurately, the h-type VTP should pay a signaling cost. Thus, the h-type VTP will weigh the tradeoff between the signaling cost and the benefit of separating.

From Proposition 1, the brand owner’s optimal retail price is $p_j(\xi) = \frac{1 + \theta_j}{2}$, where $\theta_j$ is their belief about the consumer preference type. Recall the assumption that $\theta_h > \theta_l > 0$. Thus, if the VTP observes a low virtual technology preference $\theta_l$ (“l-type VTP”), they have the incentive to mimic that they observe high (“h-type VTP”) to induce the brand owner to set a high retail price. In the meanwhile, the h-type VTP also has the incentive to separate from the l-type VTP. We can verify the relation that $\pi^p_{lh} > \pi^p_{hl}$ and $\xi_{lh} > \xi_{hl}$; $\pi^p_{lh} > \pi^p_{lj}$ and $\xi_{lh} > \xi_{lj}$. This is also illustrated in Figure 3.

![Figure 3. The VTP’s optimal profit functions in separating equilibrium.](image)

We use $\theta_j(\xi)$ to denote the brand owner’s updated belief of virtual technology preference after observing the VTP’s virtual technology investment $\xi$. Then, we use the belief structure below, with a virtual technology investment threshold $\xi_{sep}$,

$$j(\xi) = \begin{cases} h, & \xi \geq \xi_{sep} \\ l, & \xi < \xi_{sep} \end{cases}$$

This situation allows the brand owner to identify the type of virtual technology investment of the VTP accurately when they observe the VTP’s virtual technology investment. The principle of belief updating for the brand owner is that in the separating strategy, there is a threshold of investment $\xi_{sep}$. When the brand owner observes that the VTP’s investment $\xi \geq \xi_{sep}$, the VTP is considered to be of h-type; when $\xi < \xi_{sep}$, the brand owner considers the VTP to be of l-type.

A perfect Bayesian separating equilibrium exists under the belief structure if and only if $\xi_{sep} \geq 0$ and the following conditions are satisfied

$$\max_{\xi \geq \xi_{sep}} \pi^p_{lh}(\xi) \leq \max_{\xi < \xi_{sep}} \pi^p_{lj}(\xi) \quad (3)$$

$$\max_{\xi < \xi_{sep}} \pi^p_{hl}(\xi) \leq \max_{\xi \geq \xi_{sep}} \pi^p_{lh}(\xi) \quad (4)$$

In separating equilibrium, even if the l-type VTP mimics the optimal investment of the h-type VTP and succeeds in misleading the brand owner into believing himself to be h-type, the l-type VTP still does not receive a higher profit from the mimicking. Thus, the condition for achieving separation is concluded in the following proposition.

**Proposition 2.** Under the no-information-sharing scenario, the separating strategy for the VTP is: there exists a unique perfect Bayesian separating equilibrium, where
The optimal virtual technology investment is \( \zeta_{sep}^l = \frac{\gamma - \beta \theta}{\beta \theta (\theta - 2 \gamma)} h < \zeta_{sep}. \)

(ii) The optimal price is \( p_h = \begin{cases} \left( \frac{1 - \alpha}{2 \gamma - \theta h} \right)^{\beta \theta} & \gamma \leq \gamma_1, \\ \frac{1}{2} \left( 1 + \frac{(1 - \alpha) \beta \theta}{(2 \gamma - \theta h - 2 \gamma)} \right) & \gamma > \gamma_1 \end{cases} \)

(iii) The VTP's and the brand owner's ex ante expected profit are

\[
\begin{align*}
\pi_p &= \begin{cases} \frac{1 - \alpha}{2 \gamma - \theta h} \left( \frac{1 - \alpha}{2 \gamma - \theta h} \right)^{\beta \theta} \gamma & \gamma \leq \gamma_1, \\ \frac{1}{2} \left( 1 + \frac{(1 - \alpha) \beta \theta}{(2 \gamma - \theta h - 2 \gamma)} \right) & \gamma > \gamma_1 \end{cases}, \\
\pi_\beta &= \begin{cases} \frac{1 - \alpha}{2 \gamma - \theta h} \left( \frac{1 - \alpha}{2 \gamma - \theta h} \right)^{\beta \theta} \gamma & \gamma \leq \gamma_1, \\ \frac{1}{2} \left( 1 + \frac{(1 - \alpha) \beta \theta}{(2 \gamma - \theta h - 2 \gamma)} \right) & \gamma > \gamma_1 \end{cases}.
\end{align*}
\]

where \( \Theta = \frac{(\theta_h - \gamma - \beta \theta - \Theta)}{\theta_j (\beta \theta - 2 \gamma)}. \)

Proposition 2 shows the virtual technology investment strategy for the VTP in a separating strategy. Note that \( \frac{\gamma - \beta \theta}{(2 \gamma - \theta h - 2 \gamma)} \) is the optimal virtual technology investment when there is no information asymmetry in the supply chain. In the separating strategy, l-type VTP does not mimic h-type VTP and does not require investment distortions to occur, so the separating investment and the profit for l-type VTP are the same as when information is symmetric.

From Proposition 2, when the virtual technology investment is very expensive, i.e., \( \gamma_1 < \gamma, \) the h-type VTP can signal the true consumer preference type to the brand owner without distorting their virtual technology investment. It is referred to as costless separating. If the virtual technology investment is expensive, it is difficult for l-type VTP to mimic h-type VTP. The mimicking behavior of l-type VTP requires higher investment in their virtual technology, resulting in higher costs of disguise. It is unprofitable for l-type to mimic h-type VTP. So, when \( \gamma_1 < \gamma, \) the h-type VTP does not need to distort the virtual technology investment and the two types of VTPs are naturally separated.

If the virtual technology investment is cheap, i.e., \( \gamma < \gamma_1, \) the h-type VTP will distort their investment to convince the brand owner that the true consumer preference is high. Specifically, it is easy to demonstrate that

\[
\frac{\gamma - \beta \theta}{(2 \gamma - \theta h) (\theta_j - 2 \gamma)} - \frac{\gamma - \beta \theta}{(2 \gamma - \theta h) (\theta_j - 2 \gamma)} > 0
\]

The last inequality holds if and only if \( \gamma < \gamma_1. \) When \( \gamma < \gamma_1, \) the h-type VTP has to distort up the virtual technology investment. Thus, they fail to achieve the maximum profit that they can gain from the information symmetry scenario. The h-type VTP's profit loss owing to distorting up the virtual technology investment is a signaling cost, and it is referred to as costly separating.

To achieve separating, the h-type VTP needs to distort their investment upwards to the point where the l-type VTP cannot mimic it, i.e., make their investment sufficiently high that the l-type VTP cannot mimic them with a higher profit than the maximum profit in the
The distorted result is that when the brand observes an investment of \( h \) in the information symmetry scenario. The shaded area in Figure 4 shows the distortion of the virtual technology investment caused by information asymmetry of the h-type VTP.

\[
\beta = 0.5, \; \theta_h = 1.5, \; \theta_l = 1
\]

![Costly separating vs. Costless separating](image)

**Figure 4.** The effect of \( \gamma \) on virtual technology investment in a separating equilibrium.

The distortion of investment in the separating equilibrium reduces the profit of the h-type VTP, implying that the h-type VTP has to pay a cost to achieve separating. The cost of separating for h-type VTP is \( \pi_{h,sep} - \pi_{h,ess} \). The distorted result is that when the brand observes an investment of \( h \) in the information symmetry scenario. The shaded area in Figure 4 shows the distortion of the virtual technology investment caused by information asymmetry of the h-type VTP.

\[
\Delta = \zeta_{sep} - \zeta_{h,sep} = \frac{\theta_h \gamma (\gamma - \theta_h)}{\theta_h (\theta_h - 2 \gamma)} \quad \text{is decreasing in} \; \gamma.
\]

**Lemma 1.** In separating equilibrium, when \( \gamma \) is small,

(i) The virtual technology investment \( \zeta \) is decreasing in \( \gamma \).

(ii) The retail price of virtual technology product \( p \) is decreasing in \( \gamma \).

(iii) The degree of upward distortion of virtual investments in h-type platform

\[
\Delta = \zeta_{sep} - \zeta_{h,sep} = \frac{\theta_h \gamma (\gamma - \theta_h)}{\theta_h (\theta_h - 2 \gamma)} \quad \text{is decreasing in} \; \gamma.
\]

Lemma 1 indicates that \( \gamma \) has an impact on the separating equilibrium of the h-type VTP. Figure 5 shows that as \( \gamma \) increases, the VTP invests less in the virtual technology and the brand owner sets a lower retail price. When \( \gamma \) is low, the cost of l-type VTP mimicking h-type VTP is low and the profit is high. Only when the investment of h-type VTP is distorted upward can l-type VTP give up mimicking. Thus, as \( \gamma \) increases, the cost of mimicking the h-type VTP is too high for the l-type VTP to mimic the h-type VTP. Then, the h-type VTP does not need to distort the investment to achieve separating. The magnitude of distortion required to separate the investment of the h-type VTP is also decreased.
4.2. Pooling Equilibrium

In this subsection, we discuss the pooling equilibrium of h-type and l-type VTP. In this case, the h-type VTP and the l-type VTP will formulate an identical VTP investment. When the brand owner observes the VTP’s investment, they cannot infer the true information type of VTP. Specifically, when the signaling cost is sufficiently high for the h-type VTP, then they may give up separating themselves and instead set a low virtual technology investment to pool with the l-type VTP. For an l-type VTP, they will gain the benefit from mimicking to be an h-type but have to pay the cost of mimicking due to the high virtual technology investment. The two types of VTP will choose to pool only if pooling yields a higher profit than separating. We can also verify the relation that $\pi_{il}^p > \pi_{ip}^p > \pi_{ih}^p$ and $\zeta_{ih} > \zeta_{ip} > \zeta_{il}$. This is also illustrated in Figure 6.

If the virtual technology investment is sufficiently low, the h-type VTP will separate himself. Thus, we adopt the pooling belief structure proposed by (Li et al. [29], Guo and Jiang [39]): in a pooling equilibrium, the brand owner’s belief updating principle is: there exists an investment threshold $\zeta_{pool}$ such that when the brand owner observes that the VTP gives an investment $\zeta \geq \zeta_{pool}$, then they cannot update their belief; when $\zeta < \zeta_{pool}$, the brand owner considers the VTP to be of low type. That is,

$$f(\zeta) = \begin{cases} \text{prior distribution}, & \zeta \geq \zeta_{pool} \\ \text{low type}, & \zeta < \zeta_{pool} \end{cases}$$
To indicate the brand owner’s belief in the pooling equilibrium, we replace $\pi_B^p$ with $\pi_{ip}^p$. If the VTP chooses to pool at virtual technology investment $\zeta$, the brand owner cannot update their belief on the consumer preference. Thus, the optimal retail price is determined by maximizing

$$\pi_{ip}^p(p, \zeta) = \alpha \pi_{ip}^p(p, \zeta) + (1 - \alpha) \pi_{ip}^p(p, \zeta).$$

If a VTP chooses a pooling strategy, then the brand owner’s optimal retail price is $p(\zeta) = \frac{1 + \theta}{2}$, where $\delta = \alpha \theta_0 + (1 - \alpha) \theta_1$ is the expectation on the VTP’s consumer virtual technology preference information. From (2), the i-type VTP’s profit is

$$\pi_{ip}^p(p; \zeta) = (\beta p_p(\zeta) - \gamma \zeta)(1 - p_p(\zeta) + \theta_1 \zeta) = \frac{[1 + \zeta(2\theta_1 - \delta)] \beta \zeta + \zeta(\beta \delta - 2\gamma)}{4}.$$

Under the belief structure, there exists a perfect Bayesian pooling equilibrium if and only if $\hat{\zeta}_{pool} \geq 0$ and the following constraints are satisfied.

A perfect Bayesian pooling equilibrium exists under the belief structure if and only if $\hat{\zeta}_{pool} \geq 0$ and the following conditions are satisfied

$$\max_{\zeta \geq \hat{\zeta}_{pool}} \pi_{ip}^p(\zeta) = \pi_{ip}^p(\hat{\zeta}_{pool})$$

$$\max_{\zeta \geq \hat{\zeta}_{pool}} \pi_{ip}^p(\zeta) = \pi_{lp}^p(\hat{\zeta}_{pool})$$

$$\max_{\zeta \geq \hat{\zeta}_{pool}} \pi_{ip}^p(\zeta) \geq \max_{\zeta < \hat{\zeta}_{pool}} \pi_{ip}^p(\zeta)$$

$$\max_{\zeta \geq \hat{\zeta}_{pool}} \pi_{ip}^p(\zeta) \geq \max_{\zeta < \hat{\zeta}_{pool}} \pi_{lp}^p(\zeta)$$

The conditions that guarantee that $\hat{\zeta}_{pool}$ is the optimal pooling investment is that neither the h-type VTP nor the l-type VTP is willing to deviate from the pooling investment $\hat{\zeta}_{pool}$, i.e., the gain from the VTP deviating from the pooling investment is lower than the gain from the pooling investment. Additionally, if the VTP deviates from the pooling, the brand owner will consider the VTP as a l-type VTP and set the corresponding retail price with a low preference belief.

We maximize the profit of i-type VTP under the constraint that both h-type and l-type VTP are more profitable when pooling rather than separating. Then, we can present the pooling equilibrium result in the following proposition.

**Proposition 3.** If $\gamma < \gamma_1$ and $\alpha_1 < \alpha$, a perfect Bayesian pooling equilibrium exists, where

(i) The optimal virtual technology investment is $\hat{\zeta}_{pool} = \frac{\gamma - \beta h_1}{(2h_1 - \delta)(\beta \delta - 2\gamma)}$.

(ii) The optimal price is $p_{pool} = \frac{(a - 1)\alpha \beta \theta_0^2 - (1 - a)\theta_1[3\gamma - (1 - a)\beta \theta_0] - \theta_1[3\alpha - 4\gamma + (1 - 3\alpha + 2\alpha^2)\beta \theta_0]}{2(\delta - 2h_1)(\beta \delta - 2\gamma)}$.

(iii) The VTP’s and the brand owner’s ex ante expected profit are

$$\pi_B = \frac{(\gamma + (\beta - a)\beta h_0^2 - (1 - a)\beta h_1)\left\{ (1 - a)\alpha \beta \theta_0^2 + (1 - a)\theta_1[3\gamma - (1 - a)\beta \theta_0] + \theta_1[3\alpha - 4\gamma + (1 - 3\alpha + 2\alpha^2)\beta \theta_0]\right\}}{4(\delta - 2h_0 + (\beta \delta - 2\gamma)^2)}$$

$$\pi_{ip} = \frac{(1 - \beta)\left\{ (1 - a)\alpha \beta \theta_0^2 + (1 - a)\theta_1[3\gamma - (1 - a)\beta \theta_0] + \theta_1[3\alpha - 4\gamma + (1 - 3\alpha + 2\alpha^2)\beta \theta_0]\right\}^2}{4(\delta - 2h_0 + (\beta \delta - 2\gamma)^2)}.$$
lead to a higher profit. Hence, the more likely consumers are to prefer virtual technology, the more VTP tends to pool.

\[(\beta = 0.5, \; \theta_h = 2.5, \; \theta_l = 1.5, \; \gamma = 0.9)\]

**Lemma 2.** In pooling equilibrium,

(i) The virtual technology investment \(\zeta\) is increasing in \(\alpha\) and \(\delta\).

(ii) The retail price of virtual technology product \(p\) is increasing in \(\alpha\) and \(\delta\).

Lemma 2 shows the impact of \(\alpha\) on supply chain members’ decisions. Figure 7 shows that as \(\alpha\) increases, both the virtual technology investment and the retail price are increasing. If \(\alpha\) is sufficiently low, the brand owner has a lower expectation of consumer preference and sets a low price based on the pooling investment. When the VTP anticipates this situation, they set a low pooling investment to maximize their profit. Then, the VTP has an incentive to deviate from the pooling with a higher investment to gain a higher profit. Thus, the pooling equilibrium exists only if consumers are more likely to prefer virtual technology.

![Figure 7](image_url)

**Figure 7.** The effect of \(\alpha\) on virtual technology investment and retail price in a pooling equilibrium.

### 4.3. The LMSE Equilibrium

In the no-information-sharing scenario, there is always a separating equilibrium, which includes both costly and costless separating. A pooling equilibrium exists only when \(\gamma < \gamma_1\) and \(\alpha_1 < \alpha\). When there are multiple equilibria, we need to refine the equilibrium. Thus, we use the lexicographically maximum sequential equilibrium (LMSE) (Mailath et al. [40]) concept to refine the multiple equilibria. This refining method has been widely used in many papers (e.g., Jiang et al. [19]; Guo and Jiang [39]; Li et al. [36]). In this model, the LMSE concept is used to choose the most profitable outcome for a VTP who wants to disclose their identity.

In this model, the h-type VTP has an incentive to expose their identity, while the l-type VTP wants to mimic. Hence, we use the LMSE refinements to choose the more profitable equilibrium for the h-type VTP, i.e., we compare \(\pi_{hp}(\zeta_{sep})\) with \(\pi_{hp}(\zeta_{pool})\) and choose the larger one. We compare the magnitude of the gains of the h-type VTP at the two equilibria to determine whether the h-type VTP would choose the separating equilibrium or the pooling equilibrium for a given parameter condition. We explain the basic logic for this refinement: the h-type VTP wants their type to be revealed, then they will choose to pool if the pooling outcome is more profitable than the separating outcome. Conversely, the h-type VTP chooses the separating equilibrium.
Proposition 4 shows the VTP’s optimal virtual technology investment and the brand owner’s retail price under the unique LMSE outcome. It is also illustrated in Figure 6.

**Proposition 4.** Under no information sharing scenario, there exists a unique LMSE outcome. Specifically, the VTP will pool if \( \gamma \leq \gamma_1 \) and \( \alpha \leq \alpha_2 \), and will separate if \( \gamma \leq \gamma_1 \) and \( \alpha \leq \alpha_2 \). Moreover, there is a threshold \( \alpha_2 \in [0, 1] \) in the equilibrium:

(i) if \( \alpha_2 \leq \alpha \), then the VTP will pool investment at \( \xi_{\text{pool}} = \frac{\gamma - \theta h}{(\theta h - 2)(\theta h - 2\gamma)} \), and the retail price is \( p_{\text{pool}} = \frac{(1 - \alpha)\beta\gamma(1 - \alpha)\beta - \theta l}{2(\theta h - 2)(\theta h - 2\gamma)} \).

(ii) if \( \alpha \leq \alpha_2 \), then the VTP will separate his investment, and the investment and the retail price are

\[
\begin{align*}
\begin{cases}
\xi_{\text{sep}}^h, p_{\text{sep}}^h = \left( \frac{\gamma - \beta \theta h}{\theta l(\theta h - 2\gamma)}, \frac{\gamma - \beta \theta h}{\theta l(\theta h - 2\gamma)} \right), & \gamma \leq \gamma_1, \\
\xi_{\text{sep}}^l, p_{\text{sep}}^l = \left( \frac{\gamma - \beta \theta l}{\theta h(\theta l - 2\gamma)}, \frac{\gamma - \beta \theta l}{\theta h(\theta l - 2\gamma)} \right), & \gamma > \gamma_1,
\end{cases}
\end{align*}
\]

where \( \theta = \sqrt{\frac{(\theta h - \theta l)(\gamma - \beta \theta l)(\beta(\theta h - \theta l) - 2\gamma)}{\theta h(\theta l - 2\gamma)}} \).

Note that \( \gamma_1 \) is the threshold between costly and costless separating. If the virtual technology investment is very expensive, i.e., \( \gamma_1 < \gamma \), the l-type VTP cannot afford the cost to mimic the h-type VTP. The h-type VTP can naturally separate themselves from the l-type VTP, resulting in a costless separating equilibrium. Thus, the brand owner can identify the difference between the two kinds of VTPs. If the virtual technology investment is cheaper, i.e., \( \gamma \leq \gamma_1 \), the VTP attains a costly separating. h-type VTP has an incentive to mimic the investment of h-type VTP to achieve higher profits. This requires a decision by the h-type VTP to choose a separating strategy to reveal their true type to the brand owner or a pooling strategy to keep the brand owner from knowing their true type. For the h-type VTP, when \( 0 < \alpha < \alpha_1 \), pooling equilibrium no longer exists and separating is the unique equilibrium. When \( \alpha_1 < \alpha \), both pooling and separating equilibria exist. Specifically, if \( \alpha_1 < \alpha < \alpha_2 \), pooling equilibrium is less profitable than the separating equilibrium; if \( \alpha_2 \leq \alpha \), pooling equilibrium is more profitable to the h-type VTP. In summary, if \( \alpha \leq \alpha_2 \), the VTP will conduct a costly separating; if \( \alpha_2 \leq \alpha \), the VTP will pool.

In general, as the probability of high consumer preference increases, the h-type VTP is no longer eager to reveal their true type to the brand owner. For the h-type platform, when consumers are more likely to prefer virtual technology (i.e., a high \( \alpha \)), the platform chooses a pooling equilibrium, i.e., the platform is no longer eager to reveal to the brand owner the true type of consumer virtual technology preference. When consumers are less likely to prefer virtual technology (i.e., a low \( \alpha \)), the platform chooses a separating equilibrium, i.e., the platform increases their investment to convince the brand owner that consumers in the marketplace prefer virtual technology. The reason is that, for h-type VTP, the magnitude of downward investment distortion decreases in \( \alpha \) when choosing the pooling equilibrium, and the profit loss due to pooling also decreases in \( \alpha \). In the separating equilibrium, the magnitude of investment distortions for the h-type VTP is independent of \( \alpha \), and the profit loss is also independent of \( \alpha \). Therefore, when \( \alpha_2 < \alpha \), the profit loss of the h-type VTP choosing the pooling equilibrium is lower than the cost of the separating equilibrium, so the pooling equilibrium is more profitable. When \( \alpha < \alpha_2 \), the profit loss of pooling is high enough that the h-type VTP can obtain higher profit by choosing the separating equilibrium, which is the shaded part between the blue and red lines in Figure 8. Although there is a pooling equilibrium at this point, the profit is higher for the h-type VTP by choosing the separating equilibrium. The conclusions can guide virtual technology platforms to make investment decisions based on the probability of consumer virtual technology preference.

It is consistent with real practices in the virtual industry. Roblox has collaborated with Gucci to launch the Gucci Garden Archetypes virtual exhibition, providing users
with a highly dynamic set of environments. Consumers can view the exhibit, chat in real-time and purchase Gucci’s virtual products on Roblox. With the widespread popularity of virtual technology, Roblox has launched a range of virtual products that continue to generate a buying frenzy.

We can summarize the optimal decisions of the supply chain members in Table 3.

Figure 8. The unique lexicographically maximum sequential equilibrium outcome.

Table 3. Optimal technology investment and retail price under no information sharing.

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>$\theta_i$</th>
<th>$\zeta$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costless separating $\gamma_1 &lt; \gamma$</td>
<td>$\theta_h$</td>
<td>$\frac{\gamma - \beta_0}{\beta_0(\beta_0 - 2\gamma)}$</td>
<td>$\frac{\gamma - \beta_0}{\beta_0(\beta_0 - 2\gamma)}$</td>
</tr>
<tr>
<td></td>
<td>$\theta_l$</td>
<td>$\frac{\gamma - \beta_0}{\beta_0(\beta_0 - 2\gamma)}$</td>
<td>$\frac{\gamma - \beta_0}{\beta_0(\beta_0 - 2\gamma)}$</td>
</tr>
<tr>
<td>Costly separating $\gamma &lt; \gamma_1$ and $\alpha &lt; \alpha_2$</td>
<td>$\theta_h$</td>
<td>$\frac{\gamma - \beta_0}{(2\gamma - \beta_0)\beta_0 - 2\gamma)}$</td>
<td>$\frac{1}{2} \left[ 1 - \frac{\beta_0(\gamma - \beta_0 + \Theta)}{(2\gamma - \beta_0)\beta_0 - 2\gamma)} \right]$</td>
</tr>
<tr>
<td></td>
<td>$\theta_l$</td>
<td>$\frac{\gamma - \beta_0}{(2\gamma - \beta_0)\beta_0 - 2\gamma)}$</td>
<td>$\frac{1}{2} \left[ 1 + \frac{(\gamma - \beta_0)\beta_0}{(2\gamma - \beta_0)\beta_0 - 2\gamma)} \right]$</td>
</tr>
<tr>
<td>Pooling $\gamma &lt; \gamma_1$ and $\alpha_2 &lt; \alpha$</td>
<td>$\frac{\gamma - \beta_0}{(2\beta_0 - 3)(\beta_0 - 2\gamma)}$</td>
<td>$\frac{(1 - \alpha)\beta_0^2 - (1 - \alpha)\beta_0(3\gamma - (1 - \alpha)\beta_0) - \beta_0[(3\alpha - 4)\gamma + (1 - 3\alpha + 2\alpha^2)\beta_0]}{2(3 - 2\beta_0)(\beta_0 - 2\gamma)}$</td>
<td></td>
</tr>
</tbody>
</table>

5. Information Sharing

We have determined the equilibrium in the signaling game between the VTP and the brand owner when the VTP does not share the consumer virtual technology preference information with the brand owner. In this section, we consider the full information sharing format in that the VTP shares their private consumer preference information with the brand owner. This format is consistent with the reality that VTPs and brand owners may develop more collaborative relationships. The VTP will share information with the brand owner, and the brand owner makes retail pricing decisions based on the shared information. In this section, we examine the optimal decisions of supply-chain participants under the information-sharing format.

Under the information-sharing format, the VTP will share their private consumer preference information with the brand owner. The brand owner will set the retail price according to shared information. Thus, the brand owner’s belief is always the same as the true information, i.e., $\theta_i = \theta_j$. To indicate this case of information sharing, we put a tilde above the variables ($\tilde{\theta}$).

Using backward induction, given the realized preference information $\theta_i (i = \{h, l\})$, the brand owner’s optimal retail price is $\tilde{p} = \frac{1}{2} (1 + \tilde{q} \theta_i)$, which is derived by solving $\max \tilde{\pi}_B$. Then, we substitute $\tilde{p}$ into the profit function $\tilde{\pi}_P$. By solving $\max \tilde{\pi}_P$, we can
calculate the VTP’s optimal investment: $\tilde{\tilde{\zeta}} = \frac{\gamma - \beta \theta_i}{\beta_i (\beta \theta_i - 2 \gamma)}$, respectively. Then, the brand owner’s corresponding retail price is $\tilde{p} = \frac{\gamma}{\beta_i (\beta \theta_i - 2 \gamma)}$, where $i \in \{h, l\}$.

Under the information sharing scenario, the optimal decision can be obtained by letting $\theta_i = \theta_i$. As a result, we conclude to the following proposition.

**Proposition 5.** When considering the information sharing format, given that the true virtual product preference is $i(i = h, l)$ and the revenue-sharing proportion is $\beta$, the VTP’s virtual technology investment is $\tilde{\zeta} = \frac{\gamma - \beta \theta_i}{\beta_i (\beta \theta_i - 2 \gamma)}$ and the brand owner’s optimal retail price is $\tilde{p} = \frac{\gamma}{\beta_i (\beta \theta_i - 2 \gamma)}$.

By plugging in $\tilde{\zeta}$ and $\tilde{p}$, the corresponding profits of the VTP and the brand owner are $\tilde{\pi}_p = \frac{\gamma^2}{8 \gamma \theta_i - 4 \beta \theta_i^2}$ and $\tilde{\pi}_B = \frac{(1 - \beta) \gamma^2}{(4 \gamma - 2 \beta \theta_i)^2}$. Then, we derive the expected profits of the VTP $E_i[\tilde{\pi}_i]$ and the brand owner $E_i[\tilde{\pi}_B]$, respectively, as follows:

$$\tilde{\pi}_p = \gamma^2 \left( \frac{\alpha}{8 \gamma \theta_i - 4 \beta \theta_i^2} + \frac{1 - \alpha}{8 \gamma \theta_l - 4 \beta \theta_l^2} \right) \tilde{\pi}_B = (1 - \beta) \gamma^2 \left[ \frac{\alpha}{(4 \gamma - 2 \beta \theta_i)^2} + \frac{1 - \alpha}{(4 \gamma - 2 \beta \theta_l)^2} \right]$$

### 6. Results and Insights

Under both the no-information-sharing and information-sharing formats, we achieved the optimal decisions of supply chain participants. Then, we can compare their ex ante profits to identify the VTP’s and brand owner’s preference between the two information formats.

The VTP’s optimal profit is unique under the information sharing format. However, under no information sharing format, the VTP’s signaling equilibrium depends on $\gamma$ and $\alpha$. Hence, the VTP’s profit has three potential values. We conclude proposition 6 by comparing the profits under the two information-sharing formats.

**Proposition 6.** The VTP always prefers information sharing format, i.e., $\tilde{\pi}_p \geq \pi_p$.

Proposition 6 shows that the virtual technology platform is always willing to share its superior information with the brand owner. The reason is that in the no information sharing scenario, the h-type VTP distorts his investment to signal information, thus incurring information signaling costs. Thus, it leads to a decrease in the expected profit of the VTP. This finding provides useful insights for platforms with private consumer preference information and guidelines for deciding whether to share that information with brands.

The results confirm the responses to the technology platforms’ strategies in the virtual industry. Digital fashion company The Fabricant has released a report analyzing how digital trends will influence consumer attitudes towards virtual fashion and their associated behaviors. It has collaborated with brands such as Adidas, Marques Almeida, and Buffalo London to produce virtual fashion, demonstrating the vast creative possibilities of digital fashion.

**Observation 1.** When $\alpha$ is sufficiently high, the brand owner prefers information sharing format, i.e., $\pi_B \leq \tilde{\pi}_B$; When $\alpha$ is sufficiently low, the brand owner prefers no information sharing format, i.e., $\pi_B \geq \tilde{\pi}_B$.

Figure 9 shows that when the probability of a high consumer preference is high, the brand owner prefers information sharing. The following reasons explain the conclusion. In the costless separating equilibrium, the investment decision of the VTP and the pricing decision of the brand owner are the same as those in the information sharing scenario, and thus $\pi_B = \tilde{\pi}_B$. In the costly separating equilibrium ($\alpha < \alpha_2$), h-type VTP pays information rents to the brand owner to signal his information correctly, and it allows the brand owners to earn more than in the natural separating equilibrium. Thus, we have $\pi_B > \tilde{\pi}_B$. In the pooling scenario ($\alpha > \alpha_2$), when $\alpha$ is sufficiently high, we have $\pi_B \leq \tilde{\pi}_B$. The relationship of the brand owner’s retail price in pooling equilibrium and information sharing scenarios is, $\tilde{\theta}_l < p_{pool} < \tilde{\theta}_h$. As $\alpha$ increases, the brand owner’s expected profits will mainly come from the high price ($\tilde{p}_h$) due to high consumer preference.
As can be seen from the arrows in Figure 9, the conclusion remains the same as the difference between high and low preferences becomes smaller, and the profit of the brand owner decreases. When the difference between high and low preferences is smaller, the platform is more likely to reach a pooling equilibrium.

Our analysis reveals that when the probability of high consumer virtual technology preference is sufficiently high (i.e., consumers are more likely to prefer the virtual technology), the brand owner can gain more profits in the information sharing scenario. That is, when consumers are more likely to prefer virtual technology, the brand owner prefers information-sharing formats.

\[ \pi = \int_0^1 (v - p + \theta_i \zeta) \, d\nu \]

\[ v_{CS} = \int_0^1 (v - p + \theta_i \zeta) \, d\nu \]

\[ v_{CS} = \alpha \pi_B + (1 - \alpha) \pi_i \]

Figures 9a, 9b, 9c, and 9d have the same format as Figure 9. The following corollary explains the potential reasons for the above practices.

**Corollary 1. (Win–Win Outcome).** When the probability of high consumer virtual technology preference (\( \alpha \)) is sufficiently high, both the brand owner and the VTP are better off if the VTP chooses to share information with the brand owner.

Based on Propositions 6 and Observation 1, we can easily derive Corollary 1. Corollary 1 explains that there exists a win–win outcome condition: both brands and virtual technology platforms prefer information sharing when consumers are more likely to prefer virtual technology. That is, under that condition, information sharing benefits both brands and platforms. This implies that the platforms should share the information with the brands when consumers have a high degree of virtual technology preference.

In reality, virtual image technology company Genies uses blockchain technology to sell virtual products, where it learns about consumer preferences by offering customized services and access to detailed data. With the popularity of virtual technology among consumers, Gucci has partnered with Genies to launch hundreds of virtual clothing looks for users to choose from. Corollary 1 suggests the potential reasons for the above practices. Thus, when deciding the appropriate virtual-product information-sharing strategy in terms of the profit of brands and platforms, it is important to identify the consumer information, such as the consumer virtual technology preference.

The surplus of a consumer is defined as the difference between the consumer’s willingness to pay for a product and the price that they pay. We calculated and compared the consumer surplus in two information-sharing scenarios. Recall that consumer utility is \( U = v - p + \theta_i \zeta \). We first investigate consumer welfare. We define the consumer surplus as follows.

\[ CS_i = \int_0^1 (v - p + \theta_i \zeta) \, d\nu = \alpha CS_i + (1 - \alpha) CS_i \]

We can obtain the corresponding consumer surplus in equilibrium by substituting the optimal retail prices as shown in Table 3. Furthermore, we compare consumer surplus in two scenarios by numerical study and obtain the following observation.
Observation 2 (Welfare Analysis). When the probability of high consumer virtual technology preference (α) is moderate, consumer surplus is higher when the technology platform shares the consumer preference information than when they do not share it, i.e., CS ≤ ĈS; When the probability of high consumer virtual technology preference (α) is sufficiently low or high, consumer surplus is lower when the technology platform shares the consumer preference information than when they do not share it, i.e., CS ≥ ĈS.

Observation 2 and Figure 10 demonstrates that the consumer virtual technology preference can affect the consumer surplus—i.e., having a sufficiently high or low probability of high virtual technology preference may make the consumers worse off.

As shown by the arrows in Figure 10, the conclusion remains the same as the difference between high and low preferences becomes smaller, and consumer surplus increases. When the difference between high and low preferences is smaller, the more likely the platform is to reach a separating equilibrium.

Expected consumer surplus is influenced by both the probability of high consumer preference, virtual technology investment and retail price. When α is sufficiently high, the negative effect of high retail price dominates the positive effect of technology investment; when α is sufficiently low, the negative effect of low technology investment dominates the positive effect of the retail price. Information sharing can benefit consumers more only when they have a moderate probability of high virtual technology preference. Hence, consumers virtual technology preference will affect their surplus and thus the information sharing strategy of the platform.

7. Conclusions

With the development of the virtual economy and the rapid expansion of the virtual commodity market, information sharing among enterprises becomes an important strategy to solve the information asymmetry of supply chain enterprises and promote them to achieve win–win cooperation. In this paper, we examine the choice of information strategies for virtual technology platforms in a virtual product supply chain. Specifically, we consider a virtual supply chain consisting of a brand owner and a virtual technology platform, where the brand owner sells virtual products through the virtual technology platform and enters the Metaverse virtual market. Consider a market with asymmetric information about consumers’ virtual technology preferences, where the platform provides virtual technology support and has an informational superiority. The brand owner signs a revenue-sharing contract with the virtual technology platform. The platform transmits market information signals through virtual technology investments. We investigate whether the virtual technology platform shares their superior information. We study the separating
and pooling equilibrium in the signaling game under the no-information-sharing format, and we derive the optimal solution using a stylized model under the information sharing format. Based on research questions, our main contributions to this study are as follows:

1. **What will be the virtual technology platform’s optimal signaling strategy if it chooses not to share information?**

   When consumers are less likely to prefer virtual technology, virtual technology platforms will increase the virtual technology investment to convince the brand owners that the market has a high preference for virtual products. As consumers are more likely to prefer virtual technology, the virtual technology platforms will no longer be eager to reveal the true type of consumer preference information to the brand owners.

2. **Does a virtual technology platform have an incentive to share its superior preference information with the upstream brand owner?**

   Information sharing is always beneficial to the virtual technology platform. This is because virtual technology platforms are always willing to share their superior information (i.e., consumer virtual technology preference) with brands to avoid generating “information rents”. However, information sharing is not always beneficial to brands, and brands are more likely to accept platforms to share information only when consumers are more likely to prefer virtual technology.

3. **When can information sharing be a win–win strategy for both the brand owner and the virtual technology platform? How is consumer surplus affected?**

   Information sharing may not be beneficial to brands under some conditions that have been numerically determined. Recall that the virtual technology platforms always prefer information sharing. Then we can achieve a “win–win” outcome. When consumers are more likely to like virtual technology, information sharing can benefit both the brand owners and the virtual technology platforms simultaneously. This conclusion suggests that when the virtual technology platform considers both the brand owner and its profit, the information sharing format should be determined based on the probability of consumer virtual technology preference.

   We also investigate how consumer surplus is affected by a numerical study. When consumers are moderately likely to prefer virtual technology, consumers can gain more benefits in the information-sharing scenario. Hence, consumers’ preference of virtual technology will affect their surplus and thus the information-sharing strategy of the platform.

This paper has several limitations. First, we assume there is no risk when entering a new market. As a result, another possible extension is to address business risks, such as the dot-com bubble economy or the risk attitudes of supply-chain members (Avinadav et al. [5]; Chernonog et al. [16]; Jiang et al. [19]; Li et al. [41]). Second, in our paper, we assume that the virtual technology platform accurately knows consumer preference information. Future research could consider analyzing situations where platforms do not know exact information about consumer preferences but have more information about their distribution than brand owners do. Finally, future studies could expand the analysis to supply chains with horizontal competition, which occurs when various developers and/or platforms compete in the market. (Li [42]; Gal-Or et al. [43]).

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Appendix A

Table A1. Notation table.

<table>
<thead>
<tr>
<th>Abbreviations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>The brand owner</td>
</tr>
<tr>
<td>P</td>
<td>The virtual technology platform</td>
</tr>
<tr>
<td>sep</td>
<td>The separating equilibrium case</td>
</tr>
<tr>
<td>pool</td>
<td>The pooling equilibrium case</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>The retail price determined by the brand owner</td>
</tr>
<tr>
<td>ζ</td>
<td>The virtual technology investment decided by the virtual technology platform</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>Consumer valuation, where ( v \in [0, 1] )</td>
</tr>
<tr>
<td>θ_i</td>
<td>the true consumer virtual technology preference, where ( i = {h,l} )</td>
</tr>
<tr>
<td>θ_j</td>
<td>the brand owner’s belief of the virtual technology preference, where ( j = {h,l} )</td>
</tr>
<tr>
<td>γ</td>
<td>The scale coefficient the unit cost</td>
</tr>
<tr>
<td>β</td>
<td>Revenue-sharing proportion</td>
</tr>
<tr>
<td>κ</td>
<td>The probability of high value virtual technology preference</td>
</tr>
<tr>
<td>σ</td>
<td>Preference variability</td>
</tr>
<tr>
<td>δ</td>
<td>The expectation on the VTP’s consumer virtual technology preference information, where ( δ = αθ_h + (1 − α)θ_l )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Functions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>consumer utility</td>
</tr>
<tr>
<td>D</td>
<td>Market demand</td>
</tr>
<tr>
<td>D_B</td>
<td>The brand owner’s market demand</td>
</tr>
<tr>
<td>D_P</td>
<td>The virtual technology platform’s market demand</td>
</tr>
<tr>
<td>π^B_{ij}</td>
<td>The brand owner’s profit in the no information sharing scenario, where ( i,j = {h,l} )</td>
</tr>
<tr>
<td>π^P_{ij}</td>
<td>The virtual technology platform’s profit in the no information sharing scenario, where ( i,j = {h,l} )</td>
</tr>
<tr>
<td>π_B</td>
<td>The brand owner’s profit in the information sharing scenario</td>
</tr>
<tr>
<td>π_P</td>
<td>The virtual technology platform’s profit in the information sharing scenario</td>
</tr>
<tr>
<td>CS</td>
<td>The consumer surplus</td>
</tr>
</tbody>
</table>

Proof of Proposition 2. It can easily be verified by the relation that \( ζ_{bh} > ζ_{hl} \) and \( ζ_{lh} > ζ_{ll} \) if and only if \( γ > \frac{1}{2} β (θ_h − θ_l) \).

The brand owner has the following belief structure: If \( ζ ≤ ζ_{sep} \), then the brand owner believes that the consumer virtual technology preference is \( β_l \); and if \( ζ > ζ_{sep} \), then the brand owner believes that the consumer virtual technology preference is \( β_h \). A perfect Bayesian separating equilibrium exists under this belief structure if and only if \( ζ_{sep} > 0 \) and the following constraints are satisfied.

\[
\max_{ζ \geq ζ_{sep}} π^P_{hl}(ζ) \leq \max_{ζ < ζ_{sep}} π^P_{hl}(ζ) \quad (A1)
\]

\[
\max_{ζ < ζ_{sep}} π^P_{lh}(ζ) \leq \max_{ζ \geq ζ_{sep}} π^P_{lh}(ζ) \quad (A2)
\]

According to condition (A1) and (A2), the l-type VTP cannot gain a higher profit if it mimics the h-type VTP, and the h-type VTP cannot make a higher profit if it mimics the l-type VTP. These two conditions ensure that the VTPs will be able to separate.
We first consider condition (A1). Then, for any belief structure with $\xi_{sep} \leq \zeta_{ll}$, we have

$$\max_{\zeta \geq \xi_{sep}} \pi^P_{lh}(\zeta) - \pi^P_{ll}(\zeta_{ll}) > \pi^P_{lh}(\zeta_{ll}) > \max_{\zeta < \xi_{sep}} \pi^P_{lh}(\zeta)$$

where the first inequality holds because $\xi_{sep} < \zeta_{ll}$ and $\zeta_{ll} > \zeta_{lh}$, the second inequality holds because $\pi^P_{lh}(\zeta) > \pi^P_{lh}(\zeta_{ll})$ for any $\zeta$, and the last inequality holds because $\xi_{sep} < \zeta_{ll}$; see also the following figure for illustration. Thus, condition (A1) will never be satisfied in this scenario.

![Figure A1](image-url)

**Figure A1.** Case I ($\xi_{sep} \leq \zeta_{ll}$) of the separating equilibrium satisfying condition (A1).

The blue line indicates that when $\zeta < \xi_{sep}$, the profit of the platform is $\pi^P_{lh}(\zeta)$. The red line represents that when $\zeta \geq \xi_{sep}$, the profit of the platform is $\pi^P_{lh}(\zeta)$.

Therefore, we focus on the scenario $\xi_{sep} > \zeta_{ll}$, in which $\max_{\zeta < \xi_{sep}} \pi^P_{lh}(\zeta) = \pi^P_{ll}(\zeta_{ll})$. From (1) and (2), condition (A1) can be rewritten as follows:

$$\max_{\zeta \geq \xi_{sep}} \left\{ \frac{(1 + 2\zeta\theta_l - \zeta\theta_h)(\beta + \zeta\beta\theta_h - 2\zeta\gamma)}{4} \right\} \leq \frac{(\gamma + \beta\theta_l - \beta\theta_l)^2}{4(2\theta_l - \theta_h)(2\gamma - \beta\theta_l)}$$

which implies that

$$\xi_{sep} \geq \xi_{sep} := \frac{\gamma - \beta\theta_l - \sqrt{(\theta_h - \theta_l)(\gamma - \beta\theta_l)^2(\beta\theta_h - \beta\theta_l - 2\gamma)}}{(2\gamma - \beta\theta_h)(\theta_h - 2\theta_l)}$$

(A3)

The illustration is shown in the figure. The x-axis value of the junction point where the horizontal line $y = \pi^P_{lh}(\zeta_{ll})$ crosses the curve $\pi^P_{lh}(\zeta)$ corresponds to this threshold. Regarding the l-type VTP’s profit, we can easily check that

$$\argmax_\zeta \pi^P_{lh}(\zeta) = \frac{\gamma - \beta\theta_l}{(2\theta_l - \theta_h)(2\gamma - \beta\theta_l)} < \xi_{sep} \leq 2\xi_{sep}$$

The optimal investment decision of the l-type VTP is $\xi_{sep} = \frac{\gamma - \beta\theta_l}{(2\theta_l - \theta_h)(2\gamma - \beta\theta_l)}$.

The blue line indicates that when $\zeta < \xi_{sep}$, the profit of the platform is $\pi^P_{lh}(\zeta)$. The red line represents that when $\zeta \geq \xi_{sep}$, the profit of the platform is $\pi^P_{lh}(\zeta)$.

Regarding the h-type VTP’s profit $\pi^P_{lh}(\zeta)$, proposition 1 demonstrates that $\pi^P_{lh}(\zeta)$ is concave in $\zeta$ and $\argmax_\zeta \pi^P_{lh}(\zeta) = \frac{\gamma - \beta\theta_l}{(2\theta_h - \theta_h)(2\gamma - \beta\theta_l)}$. Therefore, the decision of the h-type VTP is based on the magnitude of $\frac{\gamma - \beta\theta_h}{(2\theta_h - \theta_h)(2\gamma - \beta\theta_l)}$ and the threshold $\xi_{sep}$.
(i) If $\zeta^{\text{sep}} \leq \frac{\gamma - \beta h}{(2h - \theta_0) - (\beta h - 2)\beta}$, i.e., $\gamma_1 < \gamma$, then $\pi_{hh}^P(\zeta)$ reaches its global maximum value for $\zeta \geq \zeta^{\text{sep}}$, i.e., $\max_{\zeta > \zeta^{\text{sep}}} \pi_{hh}^P(\zeta) = \pi_{hh}^P\left(\frac{\gamma - \beta h}{(2h - \theta_0) - (\beta h - 2)\beta}\right)$. Condition (A2) is obviously satisfied in this scenario. The illustration is shown in the following figure. Therefore, it is most beneficial for the VTP to set $\zeta^{\text{sep}} = \frac{\gamma - \beta h}{(2h - \theta_0) - (\beta h - 2)\beta}$, and then allow $\zeta_{ll} = \zeta^{\text{sep}}$.

Figure A2. Case II ($\zeta^{\text{sep}} > \zeta_{ll}$) of the separating equilibrium satisfying condition (A1).

Figure A3. Case I ($\zeta^{\text{sep}} \leq \zeta_{ll}$) of the separating equilibrium satisfying condition (A2).

(i) The blue line indicates that when $\zeta < \zeta^{\text{sep}}$, the profit of the platform is $\pi_{hl}^P(\zeta)$. The red line represents that when $\zeta \geq \zeta^{\text{sep}}$, the profit of the platform is $\pi_{hh}^P(\zeta)$. If $\zeta^{\text{sep}} > \frac{\gamma - \beta h}{(2h - \theta_0) - (\beta h - 2)\beta}$, i.e., $\gamma < \gamma_1$, then $\pi_{hl}^P(\zeta)$ decreases for $\zeta \geq \zeta^{\text{sep}}$, and thus $\max_{\zeta > \zeta^{\text{sep}}} \pi_{hh}^P(\zeta) = \pi_{hh}^P(\zeta^{\text{sep}})$. Condition (A2) is more likely to be satisfied with a smaller $\zeta^{\text{sep}}$ in this scenario. Then, we can set $\zeta^{\text{sep}} = \zeta^{\text{sep}}$, and let $\zeta_{ll} = \zeta^{\text{sep}}$ to obtain the most profitable equilibrium result for the VTP.

The blue line indicates that when $\zeta < \zeta^{\text{sep}}$, the profit of the platform is $\pi_{hl}^P(\zeta)$. The red line represents that when $\zeta \geq \zeta^{\text{sep}}$, the profit of the platform is $\pi_{hh}^P(\zeta)$. In summary, Proposition 2 is proved. □
Figure A4. Case II ($\zeta_{sep} > \zeta_{ll}$) of the separating equilibrium satisfying condition (A2).

**Proof of Proposition 3.** The brand owner’s pooling belief structure is as follows: If $\zeta \geq \zeta_{pool}$, the brand owner is unable to update her prior belief; if $\zeta < \zeta_{pool}$, the brand owner believes that the virtual technology preference is $\theta_i$. There is a perfect Bayesian pooling equilibrium under this belief structure if and only if $\zeta_{pool} \geq 0$ and the following conditions are satisfied.

$$
\max_{\zeta \geq \zeta_{pool}^*} \pi_{ip}^P(\zeta) = \pi_{ip}^P(\zeta_{pool}^*) \quad (A4)
$$

$$
\max_{\zeta \geq \zeta_{pool}^*} \pi_{hp}^P(\zeta) = \pi_{hp}^P(\zeta_{pool}^*) \quad (A5)
$$

$$
\max_{\zeta \geq \zeta_{pool}^*} \pi_{ip}^P(\zeta) \geq \max_{\zeta < \zeta_{pool}^*} \pi_{il}^P(\zeta) \quad (A6)
$$

$$
\max_{\zeta \geq \zeta_{pool}^*} \pi_{hp}^P(\zeta) \geq \max_{\zeta < \zeta_{pool}^*} \pi_{hl}^P(\zeta) \quad (A7)
$$

Both the h-type VTP and the l-type VTP obtain the most profitable outcome at pool under conditions (A4) and (A5). Condition (A6) ensures that the l-type VTP prefers to pool because pooling is more profitable than separating, while condition (A7) ensures that the h-type VTP prefers to pool as well since pooling is more profitable than being considered the l-type one.

It is easy to show that

$$
\pi_{il}^M(\zeta) > \pi_{ip}^M(\zeta) > \pi_{ll}^M(\zeta)
$$

and $\zeta_{ih} > \zeta_{ip} > \zeta_{ll}$ if and only if $\gamma > \frac{1}{2} \beta(\theta_h - 2\theta_l + \delta)$ Then, consider the following conditions (A4) and (A5). It is clear from condition (5) that when the i-type VTP decides to pool, the profit $\pi_{ip}^M(\zeta)$ is concave in $\zeta$, and the first best VTI is $\zeta_{ip} = \frac{\gamma - \beta \theta_i}{(\gamma - \beta \delta)(2\gamma - \beta \delta)}$, where $i = h, l$. Thus, in order to satisfy constraints (A4) and (A5), we should have

$$
\zeta_{pool}^* \geq \max\{\zeta_{hp}, \zeta_{ip}\} = \zeta_{hp} = \frac{\gamma - \beta \theta_i}{(\gamma - \beta \delta)(2\gamma - \beta \delta)} \quad (A9)
$$

Then, we will discuss the conditions (A6) and (A7).

$\pi_{il}^P(\zeta)$ reaches its maximum value at $\frac{\gamma - \beta \theta_l}{\theta_l(-2\gamma + \beta \delta)}$. From (A9) and $\zeta_{ip} - \zeta_{ll} = \frac{\alpha(\theta_h - \theta_i)(\gamma - \beta \delta)(2\gamma - \delta + \theta_l)}{\theta_l(2\theta_h - \delta)(2\gamma - \beta \delta)(2\gamma - \alpha \beta \delta)} > 0$, we have $\zeta_{ll} < \zeta_{ip} \leq \zeta_{pool}^*$. Then, on the right side of (A6), we have

$$
\max_{\zeta < \zeta_{pool}^*} \pi_{il}^P(\zeta) = \pi_{il}^P\left(\frac{\gamma - \beta \theta_i}{\theta_l(-2\gamma + \beta \delta)}\right) = \frac{(\gamma + \beta \theta_i - \beta \theta_l)^2}{4(2\theta_l - \theta_i)(2\gamma - \beta \theta_l)}
$$
Note also that $\pi_{lp}^p(p)$ is concave in $\zeta$. Thus, (A9) implies that $\pi_{lp}^p(p)$ decreases for $\zeta \geq \zeta_{pool}$. Thus,

$$\max_{\zeta \geq \zeta_{pool}} \pi_{lp}^p(\zeta) = \pi_{lp}^p(\zeta_{pool}) = \left(1 + \zeta_{pool}^p(2\theta_1 - \delta)\right)\left(\beta + \zeta_{pool}^p(\beta\delta - 2\gamma)\right) \quad (\gamma + \beta\theta_h - \beta\theta_l)^2 \quad \frac{4(2\theta_h - \theta_l)(2\gamma - \beta\delta)}{(\delta - 2\theta_h)(\beta\delta - 2\gamma)}
$$

Therefore, from the above two equations,

$$\max_{\zeta \geq \zeta_{pool}} \pi_{lp}^p(\zeta) - \max_{\zeta < \zeta_{pool}} \pi_{ll}^p(\zeta) = \frac{1 + \zeta_{pool}^p(2\theta_1 - \delta)}{4} \left(\beta + \zeta_{pool}^p(\beta\delta - 2\gamma)\right) - \frac{4(2\theta_h - \theta_l)(2\zeta_{pool}^p - 2\gamma)}{(\delta - 2\theta_h)(\beta\delta - 2\gamma)}
$$

To satisfy condition (A6), we let $\max_{\zeta \geq \zeta_{pool}} \pi_{lp}^p(\zeta) - \max_{\zeta < \zeta_{pool}} \pi_{ll}^p(\zeta) \geq 0$ and obtain

$$\frac{\beta\theta_l - \gamma - \sqrt{(\gamma - \beta\theta_h)^2(2\gamma - \beta(\delta - \theta_h))}}{(\delta - 2\theta_l)(\beta\delta - 2\gamma)} \leq \zeta_{pool}^p \leq \frac{\beta\theta_l - \gamma + \sqrt{(\gamma - \beta\theta_h)^2(2\gamma - \beta(\delta - \theta_h))}}{(\delta - 2\theta_l)(\beta\delta - 2\gamma)} \quad (A10)
$$

Figure A5. The pooling equilibrium satisfies condition (A6).

The blue line indicates that when $\zeta < \zeta_{pool}$, the profit of the platform is $\pi_{ll}^p(\zeta)$. The red line represents that when $\zeta \geq \zeta_{pool}$, the profit of the platform is $\pi_{lp}^p(\zeta)$.

Regarding condition (A7), recall from (A9) that $\max_{\zeta \geq \zeta_{pool}} \pi_{hp}^p(\zeta) = \pi_{hp}^p(\zeta_{pool})$. From Proposition 1, $\pi_{hp}^p(\zeta)$ reaches its maximum value at $\zeta_{hl} \geq \zeta_{pool}^p(\zeta_{hl})$, i.e., $\pi_{hl}^p(\zeta_{hl})$. Then, $\max_{\zeta < \zeta_{pool}} \pi_{hp}^p(\zeta) = \pi_{hl}^p(\zeta_{hl})$. In this case, condition (A7) turns to be $\pi_{hp}^p(\zeta_{pool}) \geq \pi_{hl}^p(\zeta_{hl})$. Then obtain

$$\pi_{hp}^p(\zeta_{pool}) - \pi_{hl}^p(\zeta_{hl}) = \frac{1 + \zeta_{pool}^p(2\theta_h - \delta)}{4} \left(\beta + \zeta_{pool}^p(\beta\delta - 2\gamma)\right) - \frac{(\gamma + \beta\theta_h - \beta\theta_l)^2}{4(2\theta_h - \theta_l)(2\gamma - \beta\delta)} \geq 0
$$

Then obtain

$$\frac{\beta\theta_h - \gamma - \sqrt{(\gamma - \beta\theta_h)^2(2\theta_h - \theta_l)(\beta\delta - 2\gamma)}}{(\delta - 2\theta_h)(\beta\delta - 2\gamma)} \leq \zeta_{pool} \leq \frac{\beta\theta_h - \gamma + \sqrt{(\gamma - \beta\theta_h)^2(2\theta_h - \theta_l)(\beta\delta - 2\gamma)}}{(\delta - 2\theta_h)(\beta\delta - 2\gamma)}
$$

Recall that $\zeta_{pool} \geq \zeta_{hl} = \frac{\gamma - \beta\theta_h}{(2\theta_h - \theta_l)(\beta\delta - 2\gamma)}$. It is easy to show that
\[
\beta \theta_h - \gamma - \sqrt{\frac{a(\gamma - \beta \theta_h)^2(\theta_h - \theta_l)(\beta \theta_l - 2\gamma)}{(\delta - 2\theta_h)(\beta \delta - 2\gamma)}} < \gamma - \beta \theta_h < \beta \theta_h - \gamma + \sqrt{\frac{a(\gamma - \beta \theta_h)^2(\theta_h - \theta_l)(\beta \theta_l - 2\gamma)}{(\delta - 2\theta_h)(\beta \delta - 2\gamma)}}
\]

Therefore, condition (A7) requires that

\[
\frac{\gamma - \beta \theta_h}{(2\theta_h - \delta)(\beta \delta - 2\gamma)} \leq \hat{\xi} \leq \frac{\beta \theta_l - \gamma}{(2\theta_h - \delta)(\beta \delta - 2\gamma)}
\]

\[\text{Figure A6. The pooling equilibrium satisfies condition (A7).}\]

The blue line indicates that when \(\hat{\xi} < \hat{\xi}^{\text{pool}}\), the profit of the platform is \(\pi_{hl}^{M}(\zeta)\). The red line represents that when \(\hat{\xi} \geq \hat{\xi}^{\text{pool}}\), the profit of the platform is \(\pi_{hp}^{M}(\zeta)\).

To conclude, if a pooling equilibrium exists, it must satisfy (A9), (A10) and (A11). Then we can show that

\[
\beta \theta_h - \gamma + \sqrt{\frac{a(\gamma - \beta \theta_h)^2(\theta_h - \theta_l)(\beta \theta_l - 2\gamma)}{(\delta - 2\theta_h)(\beta \delta - 2\gamma)}} > \frac{\beta \theta_l - \gamma + \sqrt{a(\theta_l - \theta_h)(\gamma - \beta \theta_h)^2(2\gamma - \beta(\delta - \theta_l))}}{(\delta - 2\theta_l)(\beta \delta - 2\gamma)}.
\]

Therefore, the solution set that satisfy (A9), (A10), and (A11) is

\[
\frac{\gamma - \beta \theta_h}{(2\theta_h - \delta)(-2\gamma + \beta \delta)} \leq \hat{\xi} \leq \frac{\beta \theta_l - \gamma + \sqrt{a(\theta_l - \theta_h)(\gamma - \beta \theta_h)^2(2\gamma - \beta(\delta - \theta_l))}}{(\delta - 2\theta_l)(\beta \delta - 2\gamma)}.
\]

Therefore, a pooling equilibrium exists if

\[
\frac{\gamma - \beta \theta_h}{(2\theta_h - \delta)(-2\gamma + \beta \delta)} < \frac{\beta \theta_l - \gamma + \sqrt{a(\theta_l - \theta_h)(\gamma - \beta \theta_h)^2(2\gamma - \beta(\delta - \theta_l))}}{(\delta - 2\theta_l)(\beta \delta - 2\gamma)}.
\]

from which we obtain \(\gamma < \gamma_1\) and \(\alpha_1 < \alpha\).

\(\Delta\) is a biquadratic function of \(a\). By solving \(\Delta = 0\), we obtain four outcomes. Two of them are imaginary roots. And we can rule out the outcome which is not in the domain of the definition of the \(a\). Then, we obtain \(\alpha_1\).

Given \(\frac{\theta_h}{2} < \theta_l < \theta_h, 0 < a < 1, 0 < \beta < 1\) and \(\frac{\beta \theta_h}{2} < \gamma < \beta \theta_h\), we can verify that \(C > 0\) and \(\hat{\beta}_l(-2 + a)\theta_h + (-1 + a)\theta_l^3(-2\gamma + \beta \theta_l) < 0\). Thus, \(\frac{\partial \hat{\theta}_l}{\partial \gamma} < 0\) and \(\Delta\) is increase in \(a\).

Therefore, from \(\Delta > 0\), we obtain \(\alpha_1 < \alpha\).
In this case, the VTP’s most profitable pooling equilibrium is to set 
\[ \hat{\zeta}_{\text{pool}} = \frac{(\gamma - \beta \delta)}{(2\delta - 3\gamma + \beta \delta)}, \] and let \( \zeta_{\text{pool}} = \hat{\zeta}_{\text{pool}}. \) Then, Proposition 3 is proved. □

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