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Abstract: This paper considers the protection of intellectual property rights and financial development in the green low-carbon endogenous economic growth model, and also considers the total financing scale of the firms in the financial development sector, the transformation ability of the R&D sector to the advanced technology of developed countries, and the intensity of intellectual property protection, which gives the household utility function to a household. After maximizing the utility function, this paper analyzes the economic growth rate and mainly finds that the economic growth rate increases with the increase of technological transformation capacity parameters, two kinds of production efficiency parameters, and the total financing scale of the firms, and in addition it decreases with the increase of the technical level of developing countries relative to developed countries. Then, considering the improvement degree of intermediate goods, R&D efficiency and financial frictional coefficient, the relationship between it and the economic growth rate is obtained. This paper finds that the economic growth rate increases with the increase of the degree of improvement; R&D efficiency parameter; the probability that any R&D project can bring positive returns; technical level; the investment in reducing carbon emissions; the amount of energy invested in the final goods production sector; and it decreases with the increase of the financial frictional coefficient.

Keywords: financing scale; technological transformation; financial friction; intellectual property right; improvement

1. Introduction

What impact will intellectual property protection and financial development have on green low-carbon endogenous economic growth? This paper is based on this problem.

Comprehensively promoting the green transformation of economic development cannot be done without the strong support of the financial sector. Predecessors have done a lot of research on financial development. They studied the relationship between finance and health [1], renewable energy [2], carbon dioxide emissions [3–5], green growth [6], natural resources, economic growth [7], export diversity, inflation [8], foreign direct investment [9], environmental sustainability [10], growth of the firm [11], renewable energy consumption [12–14], energy intensity [15], energy efficiency [16], energy security [17], total reserves and ecological footprint [18,19], environmental regulation [20], patience [21], government [22] and so on. Previous research found that: health promoted financial development; the interaction between financial development and renewable energy reduced carbon dioxide emissions; financial development increased carbon emissions; and the interaction between financial development and technological innovation increased the negative...
impact on green growth. Foreign direct investment had a significant negative impact on economic growth and financial development. Financial development promoted firm growth and renewable energy consumption, it promoted economic growth and showed a linear relationship. Financial development and technological innovation enhanced energy security. Specifically, financial development can improve energy security through technological innovation. There was also a two-way causal relationship between financial development and total reserves and ecological footprint. Financial development was positively related to patience and energy efficiency. Financial development had a positive impact on governance and the impact depended on a country’s development level and openness. Meanwhile, Yang and Ni [23] found that financial development affected green development through different paths.

As for intellectual property protection, many scholars have conducted relevant research and found that developed countries had the strongest intellectual property protection, the least developed countries had the weakest protection, and the intellectual property protection level in developed countries is higher than that in developing countries [24]. Moreover, the level of intellectual property protection is also used to regulate the relationship between the aggressiveness of firm technology and the degree of development, and the exploration of R&D activities abroad [25]. Intellectual property rights have a positive impact on innovation and growth in general, and the impact of intellectual property rights in developed countries on innovation is higher than that in developing countries [26]. At the same time, intellectual property protection affected the export quality of firms through innovation, which promote quality upgrading [27]. Gmeiner and Gmeiner [28] found that domestic innovation was positively related to respect for foreign and domestic intellectual property rights. Respect of domestic intellectual property rights encouraged innovation. Roh et al. [29] explored the impact of intellectual property rights on green innovation and found that firm intellectual property rights significantly affected open innovation, green process innovation and green goods innovation. Tanaka and Iwaisako [30] studied how intellectual property protection affected innovation and found that strengthening intellectual property protection promoted innovation. On this basis, Zheng et al. [31] analyzed the influence of intellectual property protection in the south and the north. The extent of intellectual property protection in the two countries is different in terms of patent breadth. However, there is a lack of research on green low-carbon endogenous economic growth by intellectual property protection and financial development; this paper is based on this research.

The innovations of this paper are as follows:

This paper combines financial development and intellectual property protection, and considers two sectors, namely, the financial development sector and the R&D sector, which act on green low-carbon endogenous economic growth. Then, this paper discusses the impact of the total financing scale and financial frictional coefficient of firms in the financial development sector, and the transformation ability of the R&D sector to advanced technologies in developed countries on green low-carbon endogenous economic growth.

The rest of this paper is described as follows. Section 2 constructs an endogenous economic growth model including intellectual property protection and financial development, which mainly considers: the relationship between economic growth rate and total human capital; technological transformation capacity parameters; production efficiency parameters; social capital; the human capital allocation of the R&D sector; the total financing scale of firms; the share of government subsidies; the share of subsidies for firms and residents actively, consciously and autonomously practicing green low-carbon behavior; the technical level of developing countries relative to developed countries; the degree of improvement; the R&D efficiency parameter; the probability that any R&D project can bring positive returns; technical level; the investment in reducing carbon emissions; the amount of energy invested in the final goods production sector; and the financial frictional coefficient. Section 3 is the competitive market equilibrium of the model. Section 4 discusses the probability of success or failure of intermediate goods improvement in the intermediate goods sector. Section 5 is the empirical analysis, which mainly analyzes the economic
growth rate and its influencing factors in Section 2, according to the China Statistical Yearbook 2021 [32]. Section 6 is the conclusion. Section 7 is policy recommendations.

2. Model and Method

This paper divides the economic system into two parts, which are production and household. In the part of production, the final goods sector, intermediate goods sector, financial development sector, research and development (R&D) sector and the energy sector are considered. The total financing scale of firms is considered in the financial development sector; the transformation ability of advanced technology in developed countries and the intensity of intellectual property protection is considered in the R&D sector. The household utility function is given in the household part, and the Hamilton function is constructed to solve the equilibrium economic growth rate. This is then used to analyze and discuss: the relationship between the economic growth rate and total human capital; technological transformation capacity parameters; production efficiency parameters; social capital; the proportion of human capital used by R&D sectors for advanced technological transformation in developed countries in total human capital; the technical level of developing countries relative to developed countries; the total financing scale of firms; the share of government subsidies and the share of subsidies for firms and residents actively, consciously and autonomously practicing green low-carbon behavior; the degree of improvement; R&D efficiency parameters; the probability of a positive return that any R&D project can bring; financial frictional coefficient; technical level; investment in reducing carbon emissions; and the amount of energy invested in the production sector for final goods.

2.1. Production Sector

In the production sector, this paper considers eight parts which are: final goods sector, intermediate goods sector, financial development sector, R&D sector, capital, investment in reducing carbon emissions, and the environmental quality and energy sector. In the financial development sector, this paper considers the total financing scale of firms. In the R&D sector, this paper considers the transformation ability of advanced technology in developed countries and the intensity of intellectual property protection, and so on.

2.1.1. Final Goods Sector

In the final goods section, this paper considers five factors which are: environmental quality, human capital invested in the green low-carbon final goods sector, investment in reducing carbon emissions, R&D, and the amount of energy in the total production function. Therefore, according to the extended D-S (Dixit-Stiglitz) function, the total production function is obtained:

$$Y = A(N)L^\beta e^{1-s-\beta}\left[\int_0^D x_i^\phi di\right]^\phi$$

where $N$ is the environmental quality, $A(N)$ is the function related to environmental quality, $D$ is the number of R&D sectors, $x_i$ is the $i$th intermediate goods, $H$ is the total human capital, $nH$ is the human capital invested in the final goods production sector, $n$ is the proportion of human capital of final goods sector in total human capital, $E$ is the amount of energy invested in the final goods production sector, $e$ is the investment in reducing carbon emissions, $\phi \gamma_1 = \alpha$ is the output share of intermediate goods produced by the R&D sector, $\gamma_2$ is the output share of intermediate goods of the $i$th sector, $\phi$ is the output share of R&D sector, $\beta$ is the output share of the human capital used for final goods production, $\gamma$ is the output share of energy used for final goods production, $1 - \alpha - \beta - \gamma$ is the output share of investment in reducing carbon emissions. Here, according to Wan et al. [33], $A(N) = N^{\gamma_1}, K = \int_0^D x_i di$. 

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Therefore, Equation (1) can be rewritten as:

\[ Y = N^n K^\alpha D^{\beta-a} (nH)^\beta E^\gamma e^{1-a-\beta-\gamma} \]  

(2)

To maximize the profit of the final goods sector, this paper can get:

\[
\text{Max } \pi_Y = p_Y Y - w_Y(nH) - \int_0^D p_{x_i} x_i \, d\bar{i} 
\]  

(3)

where \( p_Y \) is the human capital return rate for final goods sector, might as well set \( p_Y = 1 \). \( w_Y \) is the wage rate of labor used for the production of final goods (return rate of human capital). \( p_{x_i} \) is the price of intermediate goods.

The first-order condition for the profit maximization of the final goods sector is:

\[
w_Y = N^n \beta (nH)^{\beta-1} D^\beta x_i^\alpha E^\gamma e^{1-a-\beta-\gamma} 
\]  

(4)

\[
p_{x_i} = N^n (nH)^\beta D^{\beta-1} x_i^\alpha E^\gamma e^{1-a-\beta-\gamma} 
\]  

(5)

According to Equation (5), this paper can obtain:

\[
x_i = \left( \frac{\alpha N^n (nH)^\beta D^{\beta-1} E^\gamma e^{1-a-\beta-\gamma}}{p_{x_i}} \right)^{\frac{1}{\alpha}} 
\]  

(6)

### 2.1.2. Intermediate Goods Sector

The profit maximization of intermediate goods sector is:

\[
\text{Max } \pi_{x_i} = p_{x_i} x_i - r x_i - p_D 
\]  

(7)

where \( p_D \) is the patent price of intermediate goods, \( r \) is the rental rate of capital.

According to Yu [34] and Yang et al. [35], in the perfect competition of the R&D market, the patent price of intermediate goods is equal to the discounted value of monopoly income of intermediate goods producers, so:

\[
p_D = \int_t^\infty \pi_{x_i} e^{-r(s-t)} \, ds = \frac{\pi_{x_i}}{r} 
\]  

(8)

According to Equations (5), (7) and (8) and solve the first derivative of \( x_i \), this paper obtains:

\[
p_{x_i} = \frac{r}{\alpha} 
\]  

(9)

Bring Equations (8) and (9) into Equation (7) to obtain:

\[
\pi_{x_i} = \frac{r^2 (1-a)}{\alpha(r+1)} x_i 
\]  

(10)

\[
p_D = \frac{r (1-a)}{\alpha(r+1)} x_i 
\]  

(11)

### 2.1.3. Financial Development Sector

Based on Liu [36], firm funds mainly come from two aspects, namely external financing \( K_E \) and internal financing \( K_M \). External financing \( K_E \) includes three aspects, which are loans provided by financial institutions \( K_C \), financing provided by the stock market \( K_S \) and government subsidies \( K_G \). Internal financing \( K_M \) includes two aspects which are firms’ own funds and reinvestment of retained earnings \( K_F \) and subsidies provided by firms and residents actively, consciously and self-disciplined practice green low-carbon behavior \( K_A \).
Here, loans provided by financial institutions $K_C$ is $K_C = [\theta_1 \varphi + \theta_2 (1 - \varphi)] \lambda \omega sY$, financing provided by the stock market $K_S$ is $K_S = (1 - \lambda) \theta_3 \omega sY$, government subsidies $K_G$ is $K_G = m_1 Y$, firms’ own funds and reinvestment of retained earnings $K_F$ is $K_F = (1 - \omega) sY$, subsidies provided by firms and residents actively, consciously and autonomously practice green low-carbon behavior $K_A$ is $K_A = m_2 Y$.

Where $\omega (0 \leq \omega \leq 1)$ is the income share obtained by residents in national income, $1 - \omega$ is the income share obtained by firms in national income. Firms and residents have $1 - s (0 \leq s \leq 1)$ of their income for consumption. $\lambda (0 \leq \lambda \leq 1)$ is the investment proportion of residents for savings in financial institutions, $1 - \lambda$ is the investment proportion of residents in stock investment. $\theta_1$ is the loan deposit ratio of state-owned banks, $\theta_2$ is the loan deposit ratio of small and medium sized financial institutions, $\theta_3 (\theta_1 \leq \theta_2 \leq \theta_3)$ is the capital conversion rate of the stock market, $\varphi (0 \leq \varphi \leq 1)$ is the loan market share of state-owned banks, $1 - \varphi$ is the loan market share of small and medium sized financial institutions, $m_1$ is the share of government subsidies, $m_2$ is the share of subsidies for firms and residents actively, consciously and autonomously practice green low-carbon behavior.

Therefore, the external financing of firms is:

$$K_E = K_C + K_S + K_G = [\theta_1 \varphi + \theta_2 (1 - \varphi)] \lambda \omega sY + (1 - \lambda) \theta_3 \omega sY + m_1 Y$$

The internal financing of firms is:

$$K_M = (1 - \omega) sY + m_2 Y$$

Therefore, combined with external financing and internal financing, the total financing scale of the firms is:

$$F = \{[\theta_1 \varphi + \theta_2 (1 - \varphi)] \lambda \omega +(1 - \lambda) \theta_3 \omega +(1 - \omega)\} sY + m_1 Y + m_2 Y \quad (12)$$

2.1.4. R&D Sector

According to Yu [34], this paper divides the technical knowledge of the R&D sector into domestic existing technical knowledge and new technology obtained through the transformation of advanced technology in developed countries, and divides the existing domestic technical knowledge into new technologies that have not been transformed by independent innovation and old technologies that have been transformed.

$$\tilde{D} = \mu_1 (n_1 H) S (\alpha D ) \{(1 - \omega)D \} + \mu_2 [(1 - n_1 - n) H] S [\eta_1 (1 - \omega) \frac{D}{D^*} (D^* - D)] \quad (13)$$

where $\mu_1$ is the production efficiency parameters of existing technologies in the R&D sector, $n_1 H$ is human capital of R&D sector for advanced technological transformation in developed countries, $S$ is social capital, $\alpha$ is the intensity of intellectual property protection, that is, the proportion of independent innovation technologies that have not been transformed, $\mu_2$ is the production efficiency parameters of R&D sector through advanced technological transformation in developed countries, $(1 - n_1 - n) H$ is remaining human capital of R&D sector, $\eta_1$ is technological transformation capacity parameters, $D$ is technological knowledge stock in developing countries, $D^*$ is technological knowledge stock in developed countries. Here, set $M = \frac{D}{D^*}$ as the technical level of developing countries relative to developed countries, also reflects the technological gap between the two countries. The level of financial development and the firms’ own funds are also important factors affecting the growth of knowledge and technology capital in the R&D sector. Suppose that the R&D sector has perfect competitiveness and the economic profit is zero.

No arbitrage conditions of R&D department sector is:

$$p_D \tilde{D} F = w_D L_D \quad (14)$$
where \( p_D \) is the price of technological innovation, \( w_D \) is the labor price of R&D sector, \( p_D DF \) is the value created by R&D sector, \( w_D \xi D \) is the cost pay for employing labor.

2.1.5. Capital

The motion equation of capital accumulation is:

\[
K = Y - C
\]

where \( C \) is consumption.

2.1.6. Investment in Reducing Carbon Emissions

The motion equation of investment in reducing carbon emissions is:

\[
\dot{e} = e^\xi Y - a e
\]

where \( \xi \) is the elasticity of investment in reducing carbon emissions to pollution, \( a \) is the environmental autonomy coefficient.

2.1.7. Environmental Quality

The motion equation of environmental quality is:

\[
N = TY - \delta Y + aN
\]

where \( T \) is the emission reduction expenditure rate, \( TY \) is the emission reduction expenditure, \( \delta \) is the emission output coefficient, \( \delta Y \) is pollution emission level.

2.1.8. Energy Sector

The motion equation of the energy stock is:

\[
E = zE - E^{\gamma_2}
\]

where \( \gamma_2 \) is elasticity coefficient of energy output, \( z \) is new energy resources R&D factors.

2.2. Household

Based on the Chu and Lai [37], the household utility function in this paper can be determined by consumption \( C \), environmental quality \( N \) and investment in reducing carbon emissions \( e \). Therefore, the household utility function is expressed as:

\[
U = \frac{(CN)^{1-\sigma} - 1}{1-\sigma} - \theta \frac{e^{1+\omega_1} - 1}{1+\omega_1}
\]

where \( U \) is utility function of household, \( C \) is the consumption, \( N \) is the environmental quality, \( e \) is the investment in reducing carbon emissions, \( \omega_1 \) is the awareness of firms and residents to reduce carbon emissions, \( \sigma \) is the coefficient of relative risk aversion, \( \eta \) is the weight of utility related to environment, \( \theta \) is the conversion coefficient, \( \sigma, \eta, \theta > 0, \omega_1 < 0, -\omega_1 > 0 \).

Maximize household utility:

\[
\text{Max } \int_0^\infty \exp(-\rho t) \left[ \frac{(CN)^{1-\sigma} - 1}{1-\sigma} - \theta \frac{e^{1+\omega_1} - 1}{1+\omega_1} \right] dt
\]

s.t.

\[
\begin{align*}
K &= Y - C \\
\dot{e} &= e^\xi Y - a e \\
N &= TY - \delta Y + aN \\
\dot{E} &= zE - E^{\gamma_2} \\
D &= \mu_1(n_1 H)S(\omega D)(1-\omega)D + \mu_2((1-n_1 - n)H)\sigma_1(1-\omega) \frac{D}{s} (D^* - D)
\end{align*}
\]

where \( \rho \) is the subjective time discount rate.
The Hamilton function is constructed and then the first-order condition and Euler equation are solved. Finally, the economic growth rate is obtained as follows:

\[ \dot{Y} = \frac{\rho - \frac{\gamma}{\eta(1-\sigma) - \sigma}}{\rho} \]  

(21)

### 3. Competitive Market Equilibrium of Model

According to Equation (5), this paper can obtain:

\[ w_D = \frac{\rho D}{(1-n)H} \]  

(22)

Because the labor market is balanced, labor wages and prices are equal, that is \( w_D = \dot{Y} \).

Then, combined with Equations (4), (6), (9) and (11), this paper can obtain:

\[ r = \frac{na(1-a) D}{1-n} \]  

(23)

Bring Equation (23) into Equation (21) and combine Equations (12) and (13) to obtain:

\[ \dot{Y} = \frac{\alpha + a}{\alpha H + a} \left\{ \frac{\mu_1 D}{1-\sigma} \right\} \]  

(24)

Combining Equation (24) and \( M = \frac{D + r}{\rho} \), this paper can obtain:

\[ \dot{Y} = \frac{-na(1-a)}{a \sqrt{1-\sigma} - \sigma} \left\{ \frac{\mu_1 D + \mu_2 H S_1}{1-\sigma} \right\} \]  

(25)

According to Chu and Lai [37] and Wan et al. [33], this paper can obtain that \( \frac{\alpha H}{\sqrt{1-\sigma} - \sigma} < 0 \). Therefore:

1. \( \frac{\partial \dot{Y}}{\partial H} > 0 \), this paper finds that the economic growth rate \( \dot{Y} \) increases with the increase of total human capital \( H \);
2. \( \frac{\partial \dot{Y}}{\partial \eta_1} > 0 \), this paper finds that the economic growth rate \( \dot{Y} \) increases with the increase of technological transformation capacity parameters \( \eta_1 \);
3. Considering the relationship between \( \dot{Y} \) and \( \mu_1, \mu_2 \), this paper finds that the economic growth rate \( \dot{Y} \) increases with the increase of the production efficiency parameters of existing technologies in the R&D sector \( \mu_1 \) and the production efficiency parameters of R&D sector through advanced technological transformation in developed countries \( \mu_2 \);
4. \( \frac{\partial \dot{Y}}{\partial a} > 0 \), this paper finds that the economic growth rate \( \dot{Y} \) increases with the increase of social capital \( S \);
5. \( \frac{\partial \dot{Y}}{\partial \nu_1} > 0 \), this paper finds that the economic growth rate \( \dot{Y} \) increases with the increase of the proportion of human capital of R&D sector used in advanced technological transformation of developed countries in total human capital \( \nu_1 \);
6. \( \frac{\partial \dot{Y}}{\partial M} < 0 \), this paper finds that the economic growth rate \( \dot{Y} \) decreases with the increase of the technical level of developing countries relative to developed countries \( M \);
7. \( \frac{\partial \dot{Y}}{\partial F} > 0 \), this paper finds that the economic growth rate \( \dot{Y} \) increases with the increase of the total financing scale of the firms \( F \);
8. Considering the relationship between \( \dot{Y} \) and \( m_1, m_2 \), this paper finds that the economic growth rate \( \dot{Y} \) increases with the increase of the share of government subsidies \( m_1 \) and the share of subsidies for firms and residents actively, consciously and autonomously practice green low-carbon behavior \( m_2 \).

To summarize, the economic growth rate \( \dot{Y} \) increases with the increase of total human capital \( H \), technological transformation capacity parameters \( \eta_1 \), production efficiency parameters \( \mu_1 \) and \( \mu_2 \), social capital \( S \), human capital allocation of R&D sector \( \nu_1 \), total financing scale of the firms \( F \), share of two types of subsidies \( m_1, m_2 \); the economic growth rate \( \dot{Y} \) increases with the increase of the technical level of developing countries relative to developed countries \( M \).
4. Improvement

If, considering the probability of success of intermediate goods improvement in the intermediate goods sector, then we need to add the parameter $A_{it}$ to indicate the technical level.

Therefore, Equation (1) can be written as:

$$Y = A_{it}A(N)[\int_0^D x_{it}^i(j) dj] (nH_{it})^\phi (nH_{it})^\beta e_{it}^{1-a-\beta-\gamma}$$

(26)

Equation (2) can be written as:

$$Y = A_{it}N^{\gamma}K_{it}^\alpha D^{\phi-\gamma} (nH_{it})^\beta e_{it}^{1-a-\beta-\gamma}$$

(27)

For intermediate goods, considering the improvement degree of intermediate goods in the intermediate goods sector:

$$A_{it} = \begin{cases} \gamma_i A_{it-1}, & \text{probability is } \mu_{it} \\ A_{it-1}, & \text{probability is } 1 - \mu_{it} \end{cases}$$

(28)

where $\gamma_i$ is the degree of improvement, $\mu_{it}$ is the probability that $A_{it}$ is improved, $1 - \mu_{it}$ is the probability that $A_{it}$ is not improved, if it is not improved, the intermediate goods will still maintain the original production efficiency $A_{it}$.

$$\mu_{it} = \lambda_i(\frac{R_{it}}{H_{it}A_{it}})$$

(29)

where $\lambda_i$ is the R&D efficiency parameter, $R_{it}$ is the R&D investment, $A_{it}$ is technical level.

The financial market is completely competitive. In equilibrium, the marginal cost of banks and other financial institutions is equal to the marginal income (the expected profit is zero).

$$\theta_iF = p_{it}f_{it}R_{it}$$

where $\theta_i$ is the probability that any R&D project in the $i$th industry can bring positive returns, $p_{it} = 1$ is the price of final goods, $f_{it}$ is financial frictional coefficient of the $i$th industry, $R_{it}$ is the R&D investment.

Therefore:

$$\theta_iF = f_{it}R_{it}$$

(30)

Similar to the calculations in parts 3 and 4, after the profit of the final goods manufacturer is maximized, this paper finds the first derivative of $x_{it}$ and $nH_{it}$, and gets:

$$w_Y = N^{\gamma} \beta A_{it}(nH_{it})^{\beta-1}D^{\phi}x_{it}^i(j)E_{it}^{1-a-\beta-\gamma}$$

(31)

$$p_x = A_{it}N^{\gamma} (nH_{it})^\beta D^{\phi-1} x_{it}^i(j)E_{it}^{1-a-\beta-\gamma}$$

(32)

To maximize the profits of intermediate goods manufacturers, this paper can get:

$$p_{it}(j) = \frac{r}{\alpha}$$

(33)

$$\pi_x = \frac{r(1 - \alpha)}{\alpha(r+1)} x_{it}(j)$$

(34)

$$p_D = \frac{r(1 - \alpha)}{\alpha(r + 1)} x_{it}(j)$$

(35)

According to Equations (32) and (33), this paper obtains:

$$x_{it}(j) = (\frac{\alpha^2 A_{it}N^{\gamma} (nH_{it})^\beta D^{\phi-1} E_{it}^{1-a-\beta-\gamma}}{r})^{\frac{1}{m}}$$

(36)

According to Equation (28), this paper obtains:

$$A_{it} = \gamma_i A_{it-1} \mu_{it} + A_{it-1}(1 - \mu_{it})$$

(37)

The productivity growth rate of the firm $i$ is:

$$g_{it} = (\gamma_i - 1) \mu_{it}$$

(38)
The intermediate goods sector needs to pay positive external financing costs to obtain R&D funds. The innovation cost of the intermediate goods sector, in addition to R&D investment $p_{it}R_{it}$, also includes the project financing cost paid to financial institutions $F$.

Therefore, the optimization objective function of the intermediate goods sector is:

$$\max \mu_{it} - p_{it}R_{it} - F$$

Then according to Equations (27), (29), (30), (34) and (36), this paper obtains:

$$\mu_{it} = \frac{1}{a(\frac{d}{A} + 1)} \frac{r_{it}^\alpha A_{it}^\alpha \gamma_{it}^{\beta \Lambda} E_{it}^{\gamma i} F_{it}^{\phi i}}{\int_{N_{it}^{\alpha}}^1 H_{it}^{\gamma i} e_{it}^{1+\beta - \gamma} D_{it}^{\phi i} \lambda_i^2}$$

According to Equations (38) and (39), this paper obtains:

$$g_{it} = (\gamma_{it} - 1) \frac{1}{a(\frac{d}{A} + 1)} \frac{r_{it}^\alpha A_{it}^\alpha \gamma_{it}^{\beta \Lambda} E_{it}^{\gamma i} F_{it}^{\phi i}}{\int_{N_{it}^{\alpha}}^1 H_{it}^{\gamma i} e_{it}^{1+\beta - \gamma} D_{it}^{\phi i} \lambda_i^2}$$

where $g_{it}$ is the technical advancement growth rate of industry $i$.

By analyzing Equation (40), this paper obtains:

1. $\frac{\partial g_{it}}{\partial \gamma_{it}} > 0$, this paper finds that the economic growth rate $g_Y$ increases with the increase of degree of improvement $\gamma_{it}$;
2. $\frac{\partial g_{it}}{\partial \lambda_{it}} > 0$, this paper finds that the economic growth rate $g_Y$ increases with the increase of R&D efficiency parameter $\lambda_{it}$;
3. $\frac{\partial g_{it}}{\partial \theta_{it}} > 0$, this paper finds that the economic growth rate $g_Y$ increases with the increase of the probability that any R&D project in the $i$th industry can bring positive returns $\theta_{it}$;
4. $\frac{\partial g_{it}}{\partial f_{it}} < 0$, this paper finds that the economic growth rate $g_Y$ decreases with the increase of the financial frictional coefficient of $i$th industry $f_{it}$;
5. $\frac{\partial g_{it}}{\partial A_{it}} > 0$, this paper finds that the economic growth rate $g_Y$ increases with the increase of technical level $A_{it}$;
6. $\frac{\partial g_{it}}{\partial e_{it}} > 0$, this paper finds that the economic growth rate $g_Y$ increases with the increase of the investment in reducing carbon emissions $e_{it}$;
7. $\frac{\partial g_{it}}{\partial E_{it}} > 0$, this paper finds that the economic growth rate $g_Y$ increases with the increase of the amount of energy invested in the final goods production sector $E_{it}$.

To sum up, the economic growth rate $g_Y$ increases with the increase of the degree of improvement $\gamma_{it}$, R&D efficiency parameter $\lambda_{it}$, the probability that any R&D project in the $i$th industry can bring positive returns $\theta_{it}$, technical level $A_{it}$, the investment in reducing carbon emissions $e_{it}$, the amount of energy invested in the final goods production sector $E_{it}$, decreases with the increase of the financial frictional coefficient of $i$th industry $f_{it}$.

5. Numerical Simulation and Discussion

In this section, this paper mainly gives some baseline parameters to be considered according to the China Statistical Yearbook 2021 [32], as shown in Table 1. This paper discusses: the relationship between economic growth rate and total human capital; technological transformation capacity parameters; production efficiency parameters; social capital; human capital allocation of R&D sector; total financing scale of the firms; the share of two types of subsidies; the technical level of developing countries relative to developed countries; the degree of improvement; R&D efficiency parameter; the probability that any R&D project in the $i$th industry can bring positive returns; technical level; the investment in reducing carbon emissions; the amount of energy invested in the final goods production sector; and the financial frictional coefficient of $i$th industry.

1. The relationship between economic growth rate and total human capital.
Table 1. Baseline value.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Data Sources</th>
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According to Equation (25) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and total human capital, as shown in Figure 1.
This is because investing more and more human capital in various sectors can produce more and more goods, improve output, and then improve the rate of economic growth.

When the critical state is exceeded, the economic growth rate is negative. This is because investing more and more advanced technology to produce advanced goods, they will lose their innovation ability. With the passage of time, the economic growth rate becomes negative with the increase of technological transformation. At this critical state, the economic growth rate is negative. This is because investing more and more advanced technology to produce advanced goods, they will lose their innovation ability. With the passage of time, the economic growth rate becomes negative with the increase of technological transformation. At this critical state, the economic growth rate is negative.

According to Equation (25) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and two kinds of production efficiency parameters, as shown in Figure 3.

According to Figure 1, this paper finds that the economic growth rate increases with the increase of total human capital $H$. This is because investing more and more human capital in various sectors can produce more and more goods, improve output, and then improve the rate of economic growth.

According to Figure 2, this paper finds that the economic growth rate increases with the increase of technological transformation capacity parameters $\eta_f$. However, if more and more people rely on the transformation of foreign advanced technology to produce advanced goods, they will lose their innovation ability. With the passage of time, the economic growth rate becomes negative with the increase of technological transformation. At this time, only by combining technological transformation with independent innovation of firms, with independent innovation as the mainstay and technological transformation as the supplement, can the economic growth rate be maximized.

(3) The relationship between economic growth rate and production efficiency parameters.
According to Equation (25) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and two kinds of production efficiency parameters, as shown in Figure 3.

![Figure 3](image-url)

**Figure 3.** Economic growth rate and production efficiency parameters.

According to Figure 3, this paper finds that the economic growth rate $g_Y$ increases with the increase of the production efficiency parameters of existing technologies in the R&D sector $\mu_1$ and the production efficiency parameters of R&D sector through advanced technological transformation in developed countries $\mu_2$. This is because the higher the production efficiency parameters of the R&D sector for the transformation of existing technologies and advanced technologies in developed countries, the more advanced goods will be produced, which will promote economic growth and the higher the economic growth rate.

(4) The relationship between economic growth rate and social capital.

According to Equation (25) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and social capital, as shown in Figure 4.

![Figure 4](image-url)

**Figure 4.** Economic growth rate and social capital.

According to Figure 4, this paper finds that the economic growth rate $g_Y$ increases with the increase of social capital $S$. This is because of the relationship between the generation of social capital and people. These relationships help people to achieve common goals and promote production. Higher social capital helps to reduce transaction costs, promote the completion of production and transactions, and thus improve the economic growth rate.


According to Equation (25) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and the proportion of human capital used for R&D
sector’s transformation of advanced technology in developed countries to total human capital, as shown in Figure 5.

![Figure 5](image_url)  
**Figure 5.** Economic growth rate and the proportion of human capital.

According to Figure 5, this paper finds that the economic growth rate \( g_Y \) increases with the increase of the proportion of human capital used for R&D sector’s transformation of advanced technology in developed countries to total human capital \( n_1 \).

(6) The relationship between economic growth rate and technical level of developing countries relative to developed countries.

According to Equation (25) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and the technical level of developing countries relative to developed countries, as shown in Figure 6.

![Figure 6](image_url)  
**Figure 6.** Economic growth rate and technical level of developing countries relative to developed countries.

According to Figure 6, this paper finds that the economic growth rate \( g_Y \) decreases with the increase of technical level of developing countries relative to developed countries \( M \). This is because the technical level of developing countries relative to developed countries reflects the technological gap between the two countries. The higher the technical level, the greater the technological gap between the two countries, and the lower the economic growth rate of developing countries.

(7) The relationship between economic growth rate and total financing scale of the firms.

According to Equation (25) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and total financing scale of the firms, as shown in Figure 7.
This is because the total financing of firms comes from the following aspects: loans provided by financial institutions; financing provided by the stock market; government subsidies; firms’ own funds; and reinvestment of retained earnings and subsidies provided by firms and residents actively, consciously and in a self-disciplined manner, practicing green low-carbon behavior. These funds will be invested in the final production. When more capital is invested, more foreign advanced technology can be absorbed. Similarly, the funds used for independent innovation of firms will also increase, and the firm’s production enthusiasm will be improved, so as to produce more advanced goods and thus increase the economic growth rate.

(8) The relationship between economic growth rate and the share of two types of subsidies.

According to Equation (25) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and the share of two types of subsidies, as shown in Figure 8.

According to Figure 8, this paper finds that the economic growth rate $g_Y$ increases with the increase of the share of government subsidies $m_1$ and the share of subsidies for firms and residents actively, consciously and autonomously practice green low-carbon behavior $m_2$. This is because with the increase of the share of government subsidies and the share of subsidies for firms and residents actively, consciously and autonomously practicing green low-carbon behavior, the capital is increased. Then, we can learn more advanced foreign technology, have more funds to support firms to innovate, produce more advanced goods, thus improving the economic growth rate.

(9) The relationship between economic growth rate and the degree of improvement.
According to Equation (40) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and the degree of improvement, as shown in Figure 9.

![Figure 9. Economic growth rate and the degree of improvement.](attachment:image9.png)

According to Figure 9, this paper finds that the economic growth rate $g_Y$ increases with the increase of the degree of improvement $\gamma_i$. This is because as the degree of improvement increases, the technology gets more and more improvements and becomes more mature. Therefore, more advanced goods can be produced and the economic growth rate can be improved.

(10) The relationship between economic growth rate and R&D efficiency parameter.

According to Equation (40) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and R&D efficiency parameter, as shown in Figure 10.

![Figure 10. Economic growth rate and R&D efficiency parameter.](attachment:image10.png)

According to Figure 10, this paper finds that the economic growth rate $g_Y$ increases with the increase of R&D efficiency parameter $\lambda_i$. This is because the higher the R&D efficiency, the more advanced technologies can be developed in a shorter time, so as to produce more goods and improve the economic growth rate.

(11) The relationship between economic growth rate and the probability that any R&D project can bring positive returns.

According to Equation (40) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and the probability that any R&D project can bring positive returns, as shown in Figure 11.
This is because as the technical level becomes higher, the demand is saturated, and the economic growth rate grows slowly.

Figure 13 shows the relationship between economic growth rate and financial frictional coefficient, as shown in Figure 12.

According to Figure 12, this paper finds that the economic growth rate decreases with the increase of the financial frictional coefficient of the industry. This is because the existence of financial friction, especially the asymmetry or incompleteness of financial market information, leads to many problems in capital liquidity and wealth distribution, and seriously restricts the redistribution of capital, which leads to the continuous reduction of economic growth rate.

(13) The relationship between economic growth rate and technical level.

According to Figure 13, this paper finds that the economic growth rate decreases with the increase of technical level. After reaching a critical value, the economic growth rate slowed down with the increase of technical level. This is because at first, the innovation continued to increase with the increase of technical level, which leads to the continuous reduction of economic growth rate.

According to Figure 11, this paper finds that the economic growth rate increases with the increase of the probability that any R&D project in the industry can bring positive returns. This is because the greater the probability that the R&D project can bring positive returns means that the project can improve economic output and thus promote economic growth.

(12) The relationship between economic growth rate and financial frictional coefficient.

According to Equation (40) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and financial frictional coefficient, as shown in Figure 12.

According to Equation (40) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and financial frictional coefficient. According to Figure 12, this paper finds that the economic growth rate decreases with the increase of the financial frictional coefficient of the industry. This is because the existence of financial friction, especially the asymmetry or incompleteness of financial market information, leads to many problems in capital liquidity and wealth distribution, and seriously restricts the redistribution of capital, which leads to the continuous reduction of economic growth rate.

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(12) The relationship between economic growth rate and financial frictional coefficient.

According to Equation (40) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and financial frictional coefficient, as shown in Figure 12.
According to Figure 13, this paper finds that the economic growth rate $g_Y$ increases with the increase of technical level $A_{tt}$. This is because as the technical level becomes higher, firms can produce more advanced goods in a shorter time, thus improving the economic growth rate. At first, the economic growth rate increased rapidly with the increase of technical level. After reaching a critical value, the economic growth rate slowed down with the increase of technical level. This is because at first, the innovation continued to increase, the technical level continued to increase, and the economic growth rate increased rapidly. However, with the passage of time, the innovation was exhausted. Although the technical level was still increasing, the demand was saturated, and the economic growth rate grew slowly.

(14) The relationship between economic growth rate and the investment in reducing carbon emissions.

According to Equation (40) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and the investment in reducing carbon emissions, as shown in Figure 14.

According to Figure 14, this paper finds that the economic growth rate $g_Y$ increases with the increase of the investment in reducing carbon emissions $e_{tt}$. This is because the investment in reducing carbon emissions will reduce the corresponding carbon emissions. The reduction of carbon emissions means the generation of clean technologies and clean goods, which promote economic growth. At first, the economic growth rate increased rapidly with the increase of the investment in reducing carbon emissions. After reaching a critical value, the economic growth rate slowed down with the increase of the investment in reducing carbon emissions.

(15) The relationship between economic growth rate and the amount of energy invested in the final goods production sector.
According to Equation (40) and the baseline parameters in Table 1, this paper discusses the relationship between economic growth rate and the amount of energy invested in the final goods production sector, as shown in Figure 15.

![Economic growth rate and the amount of energy invested in the final goods production sector.](image)

**Figure 15.** Economic growth rate and the amount of energy invested in the final goods production sector.

According to Figure 15, this paper finds that the economic growth rate $g_Y$ increases with the increase of the amount of energy invested in the final goods production sector $E_{it}$. This is because the more energy put into the final goods production sector, the more goods will be produced, thus improving the economic growth rate.

6. Conclusions

In this paper, the financial development sector and R&D sector are considered in the economic growth model. The total financing scale and financial frictional coefficient of firms are considered in the financial development sector, together with: the intensity of intellectual property protection; production efficiency parameters; and technological transformation capacity parameters. The technical level of developing countries relative to developed countries are considered in the R&D sector in order to discuss: the relationship between economic growth and total human capital; technological transformation capacity parameters; production efficiency parameters; social capital; the human capital allocation of R&D sector; the total financing scale of the firms; the share of government subsidies; the share of subsidies for firms and residents actively; consciously and autonomously practicing green low-carbon behavior; the technical level of developing countries relative to developed countries; the degree of improvement; the R&D efficiency parameter; the probability that any R&D project can bring positive returns; technical level; the investment in reducing carbon emissions; the amount of energy invested in the final goods production sector; and the financial frictional coefficient. Finally, the paper finds that the economic growth increases with: the increase of total human capital; technological transformation capacity parameters; two kinds of production efficiency parameters; social capital; the human capital allocation of R&D sector; the total financing scale of the firms; the share of two types of subsidies; the degree of improvement; the R&D efficiency parameter; the probability that any R&D project can bring positive returns; technical level; the investment in reducing carbon emissions; and the amount of energy invested in the final goods production sector. It decreases with the increase of technical level of developing countries relative to developed countries and the financial frictional coefficient.

7. Policy Recommendations

1. Only by increasing government subsidies and subsidies for firms and residents actively, consciously and autonomously practicing green low-carbon behaviors, and by expanding the total financing scale of firms, can economic growth be promoted.

2. Instead of relying solely on the transformation of foreign advanced technology to produce final goods, we should also make independent innovation, develop new technologies, and combine transformation and innovation to maximize output.

3. From one of the results of this manuscript, it is found that increasing investment in technological transformation, improving the level of technological transformation, and achieving the ability to enhance their own innovation can promote economic growth.
(4) According to the conclusion of this manuscript, increasing the proportion of human capital used by R&D sectors for advanced technological transformation in developed countries, increasing the proportion of social capital, and improving R&D efficiency, will promote economic growth.

(5) Efforts to obtain market information and reduce the risk of asymmetric market information are required to promote the normalization of economic operation.

(6) Increase investment in reducing carbon emissions, promote economic growth at a certain threshold, but slow down rapid economic growth after the certain threshold.

Author Contributions: Conceptualization, Y.Z. and B.W.; methodology, Y.Z.; software, Y.Z.; validation, Y.Z.; formal analysis, B.W.; investigation, Y.Z. and B.W.; resources, Y.Z.; data curation, Y.Z.; writing—original draft preparation, Y.Z. and B.W.; writing—review and editing, L.T. and B.W.; supervision, Y.Z. and B.W. All authors have read and agreed to the published version of the manuscript.

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References
1. Donou-Adonsou, F. The effects of health conditions on financial sector development. *Econ. Model.* 2022, 114, 105974. [CrossRef]
2. Wang, Z.H.; Pham, T.L.H.; Sun, K.; Wang, B.; Bui, Q.; Hashemizadeh, A. The moderating role of financial development in the renewable energy consumption—CO2 emissions linkage: The case study of Next-11 countries. *Energy* 2022, 254, 124386. [CrossRef]
23. Yang, L.S.; Ni, M.Y. Is financial development beneficial to improve the efficiency of green development? Evidence from the “Belt and Road” countries. *Energy Econ.* 2022, 105, 105734. [CrossRef]