**Article**

**Path Optimization of Low-Carbon Container Multimodal Transport under Uncertain Conditions**

Meiyan Li and Xiaoni Sun *

School of Energy and Mining Engineering, Shandong University of Science and Technology, Qingdao 266000, China * Correspondence: 202082010065@sdust.edu.cn

**Abstract:** The development of multimodal transport has had a significant impact on China's transportation industry. Due to the variability of the market environment, in this study, based on the context of the official launch of the national carbon emission trading market, the uncertainty of the demand and the randomness of carbon trading prices were considered. Taking minimum total transportation cost as the objective function, a robust stochastic optimization model of container multimodal transport was constructed, and a hybrid fireworks algorithm with gravitational search operator (FAGSO) was designed to solve and verify the effectiveness of the algorithm. Using a 35-node multimodal transportation network as an example, the multimodal transportation costs and schemes under three different modes were compared and analyzed, and the influence of parameter uncertainty was determined. The results show that the randomness of carbon trading prices will lead to an increase or decrease in the total transport cost, while robust optimization with uncertain demand will be affected by the regret value constraint, resulting in an increase in the total transport cost. Multimodal carriers can reduce transportation costs, reduce carbon emissions, and improve the transportation efficiency of multimodal transportation by comprehensively weighing the randomness of carbon trading prices, the nondeterminism of demand, and the relationship between the selection of maximum regret values and transportation costs.

**Keywords:** container; multimodal transport; stochastic; carbon trading price; robust optimization; fireworks algorithm with gravitational search operator (FAGSO)

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**1. Introduction**

Global warming has become a serious problem facing the whole society. Therefore, the international community has paid increasing attention to energy conservation, emission reduction, and improving the quality of the ecological environment. As a major carbon emission country, in order to alleviate the increasingly tense carbon emission situation, China promoted the pilot carbon trading market during the “14th Five-Year Plan” period and officially opened the carbon trading emission trading market in July 2021. According to statistics, transportation carbon emissions are the second largest source of carbon emissions after energy generation. With the rapid development of the modern logistics industry, a single mode of transportation is no longer enough to adapt to the changing market environment. As a new mode of transportation that can effectively improve transportation efficiency and reduce carbon emissions, container multimodal transportation has been studied. The path optimization problem has practical significance in assisting China to achieve the goal of “carbon neutrality and peak carbon dioxide emissions”.

Scholars at home and abroad have conducted in-depth research on the optimization of container multimodal transport routes. Cho [1] constructed a weighted constrained shortest path problem (WCSPP) model with the goal of minimizing time and cost and took Busan to Rotterdam as an example to solve it. Liang Xiaokang [2] considered customer requirements for transportation timeliness and constructed a multimodal transport linear programming model with minimum transportation cost of container multimodal transport as the optimization goal. Based on the background of the “Belt and Road Initiative”
initiative, Xie Chuchu [3] took China–Europe multimodal transport as a case study, considered the environmental costs and environmental penalties, constructed a multimodal transport route optimization model with the smallest total cost, and used the improved NSGA-II to solve it. Providing a reference for the selection of China–Europe multimodal transport routes, Mu Baisong [4] considered container multimodal transport as well as the economics of intermodal transport and researched and determined a multimodal transport route scheme with the goal of minimizing the total cost in single origin and destination pair container multimodal transport. Based on the development status of China–Europe multimodal transportation routes, Dai Weidong [5] constructed a container multiobjective multimodal transportation route optimization model with the least transportation cost and transportation time, used the NSGA-II algorithm to solve it, and constructed a hierarchical evaluation model. The fuzzy AHP–TOPSIS method has been used to evaluate and make decisions on the container multimodal transportation route so as to select the best transportation route. Feng Fenling [6] constructed an international container multimodal transport route selection model with the goal of minimizing transportation cost, transportation time, comprehensive energy consumption, and transportation risk under the consideration of the risk in the international container multimodal transport route selection problem and took Changsha–Berlin as an example for verification analysis. Aiming at the path optimization problem of container–rail intermodal transportation and taking the total transportation cost as the objective function, Xu Jian [7] established a path optimization model for container multimodal transportation by considering transportation time limit, shift limit, and node storage cost and used a calculation example to verify the effectiveness of the model. Qi Panan [8] comprehensively considered the influencing factors of transportation mode schedule restrictions and capacity constraints and established a container multimodal transportation path planning model with the goal of minimizing the total cost, which was solved by an improved genetic algorithm with elite retention strategy.

With the increase in peak carbon dioxide emissions and carbon neutrality, researchers have increasingly taken carbon emissions into account in the selection and optimization of container intermodal transport routes. Craig [9] verified the advantages of multimodal transport in reducing carbon emissions compared to a single mode of transport. Chen Lei [10] introduced a carbon tax mechanism in the cost calculation of multimodal transport, considered the cost of carbon emissions generated during transportation, and established a mathematical programming model with the goal of minimizing the transport assembly to solve the route selection problem of multimodal transport. Based on the perspective of multimodal transport operators, Cheng Yaorong [11] considered the impact of carbon tax policies, established a comprehensive optimization model for multitask multimodal transport routes considering carbon emissions, and designed a cultural gene algorithm with a variable-length symbol coding mechanism to solve the model. Wan Jie [12] took the transportation routes and modes in the Sino–Russian trade interval as the research object; established a multiobjective optimization model with transportation cost, transportation time, and carbon emissions as the optimization objectives; and designed an improved fireworks algorithm to solve the model. Based on the assumption of transportation carbon tax collection, Chen Weiyi [13] considered transportation carbon tax and service quality commitment, constructed a multimodal path optimization model with the smallest total cost, and designed a genetic algorithm to solve it. Considering road congestion and different carbon emission policies, Cheng Xingqun [14] constructed a multimodal transport route selection model and a genetic algorithm solution model based on an optimization and immigration strategy.

The above studies all concern optimization problems of container multimodal transport routes under certain conditions, but the actual transportation situation is easily affected by uncertain factors, such as weather, market demand, policy requirements, and transportation equipment failures. Based on this, Zhang Dezhi [15], Jiang Qiwei [16], and Peng Yong [17] studied the optimization problem of container multimodal transport paths under the condition of time uncertainty; Alumur [18], Wang Hui [19], and Zou Gaoxiang [20]
studied the multimodal transport path optimization problem under the condition of uncertain demand; a smaller number of scholars, namely, Yang Jing [21], Lu [22], and Sun [23], considered the uncertainty of transportation capacity in their research; and a few scholars, such as Xu Zhang [24], Hu Hui [25], Li Jun [26], and Zhang Xu [27], studied the multimodal transport path optimization problem under multidimensional uncertain conditions. In particular, in the environment where the carbon trading market has just opened in China, as shown in Figure 1, carbon trading prices as a market trading mechanism is random. Few scholars have considered low-carbon containers under the conditions of multidimensional uncertainty of market demand and carbon trading prices regarding the intermodal transport route optimization problem.

Based on this, from the perspective of multimodal transport operators and considering the uncertainty of demand and the random price of carbon trading, this study established a hybrid robust stochastic optimization model of container multimodal transport and designed a fireworks algorithm with an attractive force search operator. The attractive force effect improved the dimensional information between particles, expanded the global search range, verified the effectiveness of the model, improved the algorithm through example analysis, and solved the multimodal transport path optimization problem under double uncertainty.

2. Model Building
2.1. Problem Description

Container multimodal transport operators plan to transport a batch of goods from the point of origin to the destination through a mixed road, rail, and water multimodal transport network. The adjacent nodes have \( K \) transportation modes to choose from, and each transportation mode has different transportation costs, time costs, and carbon emissions. Due to the randomness of carbon trading prices and the uncertain demand caused by the changeable market environment, we studied the path optimization problem of container multimodal transportation under the condition of double uncertainty, obtained a transportation scheme that meets the requirements of enterprises for transportation costs and time, and developed suggestions for transportation schemes formulated by container multimodal transportation carriers.

In China, based on the background of the official opening of carbon emission trading venues and due to the influence of market mechanisms, carbon-emission-related carbon
trading prices are random. Based on this, this study focused on the optimization problem of container multimodal transport routes under double uncertainty conditions and devised suggestions for container multimodal transport carriers to formulate reasonable transport plans.

2.2. The Following Assumptions Were Made:

(1) The same batch of container goods cannot be split or transported separately during transportation; each batch can only be transported as a whole in containers and cannot be transported with other goods in LCL.

(2) The variety and type of container goods and transportation means are not considered.

(3) The railway and waterway schedules are not considered.

(4) The loading capacity can satisfy the requirements of the freight volume of the enterprise.

(5) Transshipment is only possible at the transportation node; at this node, each batch of goods has at most one change in the mode of transportation.

(6) Each transfer node has sufficient cargo transfer capacity, ignoring the waiting time and waiting costs during the transfer of goods.

(7) The influence of weather, transportation equipment failure, cargo damage, and other factors during transportation is not considered.

2.3. Mathematical Model

2.3.1. Constructing Objective Functions under Certain Conditions

Based on the above analysis, a low-carbon multimodal transport path optimization model was constructed as the basic model in which both demand and carbon trading price are determined values, as follows:

\[ \min F(x) = \sum_{i \in M, j \in M} \sum_{k \in K} (F^{1,ijk}_{m} + F^{2,ijk}_{d})q_{ij} X^{k}_{ij} + \sum_{i \in M, k_{1} \in K} \sum_{k_{2} \in K} q_{ij} F^{k_{1}k_{2}}_{i} Y^{k_{1}k_{2}}_{i} + \sum_{i \in M, j \in M} \sum_{k \in K} F_{max}[(T^{a} - T), 0]q_{ij} + \sum_{i \in M, j \in M} F_{max}[(T - T^{b}), 0]q_{ij} + w \left( \sum_{i \in M, j \in M} \sum_{k \in K} e_{ijk} q_{ij} d_{ij}^{k} X^{k}_{ij} + \sum_{i \in M} \sum_{k_{1} \in K} \sum_{k_{2} \in K} \mu^{k_{1}k_{2}}_{i} q_{ij} Y^{k_{1}k_{2}}_{i} - E \right) \]

s.t.

\[ \sum_{k \in K} X^{k}_{ij} \leq 1 \ \forall i, j \in M, \forall k \in K \]  
\[ \sum_{k_{1} \in K} Y^{k_{1}}_{i} \leq 1 \ \forall i \in M, \forall k_{1}, k_{2} \in K \]  
\[ X^{k_{1}}_{ij}, X^{k_{2}}_{jl} = Y^{k_{1}k_{2}}_{i} \ \forall i, j, l \in M, \forall k_{1}, k_{2} \in K \]  
\[ X^{k}_{ij} \in \{0, 1\}, Y^{k}_{i} \in \{0, 1\} \]

Formula (1) is the objective function of the model, including the direct transportation cost, transshipment cost, time cost, and carbon emission cost. Formula (2) indicates that only one mode of transportation can be selected between nodes \(i\) and \(j\). Formula (3) indicates that, at node \(i\), only one transport mode can occur at most. Formula (4) indicates that when the transportation mode is changed at the node, it should correspond to the transportation mode before and after the node. Formula (5) indicates that the decision variable has a value of 0 or 1.

2.3.2. Constructing Objective Functions under Double Uncertainties

As an effective method to solve uncertain optimization problems, the idea of robust optimization has been widely used in various fields. This study applied the idea of robust optimization to the path optimization model of multimodal transportation [28].
Considering the uncertainty of product demand, the robust optimization model was constructed based on the scenario division method in the regret model in the robust optimization model. The uncertainty of the parameter $S$ is represented by a discrete set of scenarios with known probabilities. The uncertain requirement under each scenario is $q_{ijs}$, and the probability of occurrence is $P_s$. Considering the randomness of carbon trading prices, the interval robust optimization method was applied to the multimodal transport route optimization modeling under uncertain carbon trading prices, and the randomness of carbon trading prices was represented by the interval method [29–31].

Time series distribution characteristic and carbon trading price can be any value within the range, where the carbon trading price is $\tilde{\omega} \sim W(\tilde{\omega}, \omega + \tilde{\omega})$. The value range of $\tilde{\omega}$ is determined by the maximum allowable variation of the benchmark $\tilde{\omega}$ carbon trading price and the stochastic carbon trading price.

In summary, the robust stochastic optimization model under double uncertainty conditions is as follows:

$$
\min F'(x) = \sum_{s=1}^{S} p_s F_s(x)
$$

$$
= \sum_{s=1}^{S} p_s \left( \sum_{i\in I} \sum_{j\in M} \sum_{k\in K} \left( F_{m1}^{ij} + F_{m2}^{ij} \right) q_{ijs} X_{ij}^{k} + \sum_{i\in I} \sum_{j\in M} \sum_{k1\in K} \sum_{k2\in K} q_{ijs} F_{ik1}^{k2} Y_{ik1}^{k2} \right) + \
\sum_{i\in I} \max \left( T - T_{ij}, 0 \right) q_{ijs} + \sum_{i\in I} \sum_{j\in M} F_{pmax} \left( T - T_{ij}, 0 \right) q_{ijs} + \
\tilde{\omega} \left( \sum_{i\in I} \sum_{j\in M} \sum_{k\in K} e_{ijk} q_{ijs} X_{ij}^{k} + \sum_{i\in I} \sum_{j\in M} \sum_{k1\in K} \sum_{k2\in K} \mu_{ik1}^{k2} q_{ijs} Y_{ik1}^{k2} - E \right) \right)
$$

(6)

$$
\tilde{\omega} \sim W(\tilde{\omega}, \omega + \tilde{\omega}),
$$

(7)

$$
F_s(x) \leq (1 + \gamma) F^*_s,
$$

(8)

$$
\sum_{s=1}^{S} p_s = 1.
$$

(9)

Simultaneous constraints (2)–(5) still hold.

Formula (6) is an objective function based on robust optimization under the double uncertainty of demand and carbon trading prices. It consists of the corresponding direct transportation costs, transfer costs, time costs, and carbon emission costs affected by carbon trading prices under different demand scenarios. Formula (7) restricts the value range of carbon trading prices and obeys a uniform distribution within the value range. Formula (8) restricts the correlation of demand uncertainties. Formula (9) expresses the probability of the occurrence of each scenario $s$ and is 1.

3. Algorithm Design

A hybrid fireworks algorithm with gravitational search operator (FAGSO) is a new algorithm proposed by Zhu Qibing [32] that combines the hybrid fireworks algorithm and the gravitational search algorithm. By adding attractive force search operators to the standard fireworks algorithm (FA), due to the mutual attractive force between particles and the superior particle set for information interaction, new spark particles are generated, the diversity of the spark particles increases, and the search accuracy and convergence rate of the algorithm improve, thereby decreasing the adverse effects brought by the standard fireworks algorithm map rules and Gaussian mutation operators.

3.1. Fireworks Algorithm

(1) Initialization parameters: Let $X(m) = [x_1, x_2, \ldots, x_i, \ldots, x_N]$ be the initial set of fireworks of the $m$-th iteration, where $N$ is the number of fireworks. $x_j \in D^{ij}$ is the position of the $i$-th firework in the solution space, and the fitness value is represented by $f_i$. 
(2) Explosion operation: Explode each firework particle to generate a collection of spark particles. \( M_i(m) = [y_{i,1}, \cdots, y_{i,j}, \cdots, y_{i,\lambda_i}] \) indicates that the following formula describes the generation process of the spark particles:

\[
y_{i,j} = x_i + A_i Brand(-1,1), \quad 1 \leq i \leq N, 1 \leq j \leq \lambda_i
\]

(10)

The explosion of each firework within a certain explosion amplitude will generate a certain number of spark particles. The calculation formula for the explosion amplitude \( A_i \) of the exploding fireworks and the number \( \lambda_i \) of spark particles generated by the explosion is as follows:

\[
A_i = A - \frac{f_{\text{max}} - f_i + \epsilon}{N} \sum_{i=1}^{N} (f_{\text{max}} - f_i) + \epsilon
\]

(11)

\[
\lambda_i = \lambda - \frac{f_i - f_{\text{min}} + \epsilon}{N} \sum_{i=1}^{N} (f_i - f_{\text{min}}) + \epsilon
\]

(12)

where \( f_{\text{max}} \) and \( f_{\text{min}} \) are the maximum and minimum fitness values of the firework population, respectively; \( f_i \) is the fitness value of the firework \( i \); \( \epsilon \) is a very small value, which can avoid dividing by 0; and \( \lambda \) is a constant that controls the magnitude of the explosion. In order to avoid too many or too few sparks generated during the algorithm process, a constraint formula is set for the number of sparks generated for each firework as follows:

\[
\lambda_i = \begin{cases} 
\lambda_{\text{max}}, & \lambda_i > \lambda_{\text{max}} \\
\lambda_i, & \lambda_{\text{min}} < \lambda_i < \lambda_{\text{max}} \\
\lambda_{\text{min}}, & \lambda_i < \lambda_{\text{min}}
\end{cases}
\]

(13)

where \( \lambda_{\text{max}} \) and \( \lambda_{\text{min}} \) represent the maximum and minimum explosion spark numbers, respectively.

(3) Gaussian mutation: In order to increase the population diversity of the fireworks explosion sparks, Gaussian mutation sparks \( L_i \) are introduced here, and \( P \) fireworks are randomly selected in the firework population for the Gaussian mutation operation. The resulting Gaussian mutation set is as follows:

\[
L(m) = [L_1, L_2, \cdots, L_i, \cdots, L_p]
\]

(14)

\[
L_i = x_i Be \quad 1 \leq i \leq p
\]

(15)

The randomly selected fireworks are represented by \( x_i \). \( B \) is a \( 1 \times D \) dimensional random matrix, where the matrix elements are 0 or 1. \( \epsilon \) is a random value with a mean of 1 and a variance of 1 Gaussian distribution.

(4) Mapping rule: In order to prevent the two newly generated spark particles from exceeding the search range, the mapping rules of the modulo operation are used to calculate the sparks that may exceed the search range:

\[
y_{i,j}^U = [y_{i,j}^U, \cdots, y_{i,j}^U]_\% (y_{\text{max}}^U - y_{\text{min}}^U)
\]

(16)

where \( y_{i,j}^U \) represents the spark particles’ position in the Kth dimension; \( \% \) is the modulo operation; and \( y_{\text{max}}^U \) and \( y_{\text{min}}^U \) represent the upper and lower search boundaries in the Uth dimension, respectively.

(5) Selection strategy: In order to transfer the best individual to the next iteration, the best \( n \) individuals are selected from the explosion sparks and the Gaussian mutation sparks as the next generation of initial fireworks, while the other \( n - 1 \) firework particles
use roulette betting. A random selection is made. The probability $P(L_i)$ of each firework being selected is calculated by the following formula:

$$P(L_i) = \frac{U_y \sum_{j=1}^{U_y} \|L_i - L_j\|}{\sum_{j \in L_i} L_j}$$

(17)

where $\sum_{j=1}^{U_y} \|L_i - L_j\|$ is the sum of the distances between individual fireworks $L_i$ and other individual fireworks.

3.2. Adding the Gravitational Search Operator

The attractive force search operator’s operation process shifts the particles via the attraction between the particles, thus changing the dimensional information of the particles, and the movement process conforms to the dynamics law. Due to the attractive force, optimal information sharing between particles is achieved. The particles in the solution space receive the global information and improve the poor particle latitude values, attracting particle swarms to search for the optimal solution area. The specific operations are as follows:

Let $H(m) = \{X(m) \cup M(m) \cup L(m)\}$ be the moment $m$, in which all the particle sets are obtained after $N$ fireworks explode based on Gaussian mutation, wherein the total number of particles is $N_h$. Let the $i$-th particle in the set be $h_i \in D^U$, and the inertial mass $M_i$ is calculated by the following formula:

$$m_i = \frac{f_i - f_{\text{max}}}{f_{\text{min}} - f_{\text{max}}}$$

(18)

$$M_i = \frac{m_i}{\sum_{h_j \in H(m)} m_j}$$

(19)

The above two formulas are thus deduced. For a single spark particle, the higher the fitness value of the particle, the smaller the mass. In other words, a spark particle with a superior position has a larger inertial mass. Select the first $N_h$ particles with smaller fitness values from the total number of particle swarms to form a superior particle set $D$ and use the new set as the attractive force search operator to attract the particle $H(m)$ set. The attractive force calculation formula is as follows:

$$F^U_i = \sum_{j \in D, j \neq i} \text{rand}(0, 1) F^U_{ij}$$

(20)

$$F^U_{ij} = C \frac{M_i \times M_j}{r_{ij} + \epsilon} \left(h^U_j - h^U_i\right)$$

(21)

where $h^U_i$ and $h^U_j$ are the coordinates of particles $i$ and $j$, respectively, in the set $H(m)$ in the $U$-th dimension; $C$ is the attractive force constant; $F^U_i$ is the sum of the forces of the spark particles in the $U$-th dimension space; and $r_{ij}$ is the Euclidean distance between the particles.

Under the attractive force, the displacement space position of particle $h_i$ in each dimension is $v_i$, and the calculation formula is as follows:

$$v^U_i = h^U_i + \frac{F^U_i}{M_i}$$

(22)
The updated particle position set is \( V(m) = [v_1, v_2, \ldots, v_l, \ldots, v_{N_{\text{f}}} ] \), and, according to the size of the particle fitness value, \( N \) firework particles are selected from \( \{H(m) \cup V(m)\} \) as the initial firework of the next iteration, denoted as \( X(m + 1) \).

### 3.3. Fireworks Algorithm Flow with Attractive Force Search

Step 1: In the initialization algorithm, basic parameters, such as the number of fireworks \( N \), the explosion amplitude \( A_i \) of the explosive fireworks, the number of explosive spark particles \( \lambda_i \), and the number of Gaussian mutation sparks \( L_i \), are used.

Step 2: Randomly initialize the positions of \( N \) firework particles in the \( n \)-dimensional search space; set \( m = 1 \).

Step 3: Calculate the number of explosion sparks, explosion amplitude, and Gaussian variation sparks according to Formulas (10)–(12) and (14) and generate explosion sparks and Gaussian mutation sparks through \( N \) original fireworks.

Step 4: For spark particles that may be out of the search range, the mapping operation is performed according to Equation (15) to bring them into the searchable range.

Step 5: Calculate the inertial mass of each particle in the search space and select 2 \( \times \) \( N \) particles with large inertial mass to form a superior particle set \( D \).

Step 6: Calculate the Euclidean distance between the particles in set \( D \), calculate the attractive force value between each particle dimension and set \( D \) according to Equations (20) and (21), and apply the new spark particles generated by Equation (15) to the spark particles beyond the search range according to the formula mapping rules to make them searchable. Arrange all the firework and spark particles according to the inertial mass, select the first \( N \) spark particles as the initial fireworks of the next iteration, and let \( m = m + 1 \).

Step 7: If \( m < M \), return to Step 3; otherwise, stop running the algorithm and output the objective function value.

### 4. Case Analysis

#### 4.1. Case Data

Taking the multimodal transport system in the work by Xiong [33] as the research background, 35 network nodes (starting point of 1, ending point of 35) and 69 transport sections were selected. The network involved three transport modes: road, railway, and water transport. The transport network is shown in Figure 2, and the transport distance between nodes is shown in Table 1.

![Figure 2](image_url)

**Figure 2.** Schematic diagram of transport node network structure.
Table 1. Road, rail, and water transportation distances for each transportation section.

<table>
<thead>
<tr>
<th>Transport Section</th>
<th>Highway (km)</th>
<th>Railway (km)</th>
<th>Waterway (km)</th>
<th>Transport Section</th>
<th>Highway (km)</th>
<th>Railway (km)</th>
<th>Waterway (km)</th>
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<tr>
<td>(12, 16)</td>
<td>90</td>
<td>74</td>
<td>90</td>
<td>(27, 28)</td>
<td>132</td>
<td>120</td>
<td>150</td>
</tr>
<tr>
<td>(13, 18)</td>
<td>55</td>
<td>-</td>
<td>-</td>
<td>(28, 29)</td>
<td>137</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(13, 19)</td>
<td>103</td>
<td>-</td>
<td>-</td>
<td>(28, 33)</td>
<td>110</td>
<td>104</td>
<td>-</td>
</tr>
<tr>
<td>(13, 24)</td>
<td>128</td>
<td>138</td>
<td>132</td>
<td>(28, 35)</td>
<td>117</td>
<td>129</td>
<td>143</td>
</tr>
<tr>
<td>(14, 15)</td>
<td>143</td>
<td>146</td>
<td>146</td>
<td>(29, 30)</td>
<td>76</td>
<td>-</td>
<td>-</td>
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<tr>
<td>(14, 17)</td>
<td>63</td>
<td>-</td>
<td>-</td>
<td>(29, 32)</td>
<td>130</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(14, 18)</td>
<td>107</td>
<td>-</td>
<td>-</td>
<td>(29, 33)</td>
<td>104</td>
<td>96</td>
<td>-</td>
</tr>
<tr>
<td>(14, 20)</td>
<td>130</td>
<td>136</td>
<td>145</td>
<td>(30, 31)</td>
<td>141</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(15, 16)</td>
<td>103</td>
<td>87</td>
<td>111</td>
<td>(30, 32)</td>
<td>129</td>
<td>140</td>
<td>-</td>
</tr>
<tr>
<td>(15, 17)</td>
<td>132</td>
<td>141</td>
<td>-</td>
<td>(31, 32)</td>
<td>95</td>
<td>89</td>
<td>118</td>
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<tr>
<td>(16, 21)</td>
<td>100</td>
<td>73</td>
<td>114</td>
<td>(32, 34)</td>
<td>106</td>
<td>106</td>
<td>114</td>
</tr>
<tr>
<td>(17, 20)</td>
<td>129</td>
<td>-</td>
<td>-</td>
<td>(33, 34)</td>
<td>120</td>
<td>125</td>
<td>-</td>
</tr>
<tr>
<td>(17, 21)</td>
<td>81</td>
<td>-</td>
<td>-</td>
<td>(33, 35)</td>
<td>117</td>
<td>84</td>
<td>-</td>
</tr>
<tr>
<td>(17, 22)</td>
<td>-</td>
<td>8</td>
<td>0</td>
<td>(34, 35)</td>
<td>108</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: "-" means that the nodes are not connected.

This study assumed the total transportation time was controlled at 55–65 h. The unit storage fee for early arrival was CNY 15/h·TEU, the unit penalty fee for late arrival was CNY 30/h·TEU, and the carbon emission share of the transportation task was 8000 kg.

For the uncertainty of cargo demand, considering the daily data of the container cargo volume, the design in the existing literature [12,13], and the situation of highlighting high and low freight volumes, the freight transport volume and the corresponding scenario probabilities were as follows: 180 TEU (0.36), 90 TEU (0.5), and 45 TEU (0.14).

As a market mechanism, carbon trading prices have randomness. According to the relevant data of national carbon emission trading released by Shanghai Environment Energy Exchange, we selected the benchmark carbon trading price = 52.6, the maximum allowable change of carbon trading price = 11, and then any value in the range [41.6, 63.6].

According to the "International Container Vehicle Transportation Fee Collection Rules" and "Domestic Container Vehicle Transportation Fee Collection Rules", we referred to the relevant tariff information published by the 12306 China Railway Customer Service Center and queried the relevant literature to determine the transportation base price 1 and the transportation base price 2 of the three modes of transportation and the transportation speed of each mode of transportation [16]. See Table 2 for details.
Table 2. Transport base price and transport speed of different transport modes.

<table>
<thead>
<tr>
<th>Mode of Transport</th>
<th>Transport Speed (km/h)</th>
<th>Transport Base Price 1 (CNY/TEU)</th>
<th>Transport Base Price 2 (CNY/TEU km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highway</td>
<td>60</td>
<td>15</td>
<td>8</td>
</tr>
<tr>
<td>Railway</td>
<td>50</td>
<td>500</td>
<td>2.03</td>
</tr>
<tr>
<td>Water transport</td>
<td>25</td>
<td>-</td>
<td>1.85</td>
</tr>
</tbody>
</table>

By researching and consulting the relevant literature [34], the transshipment cost, transit time, and carbon emission coefficient of the different transportation modes were obtained, as shown in Table 3. The unit carbon emission of each transportation mode was obtained, as shown in Table 4. The unit carbon emissions of different transportation modes were obtained by calculating the energy consumption, carbon emission coefficient, and cargo turnover corresponding to the transportation modes [35].

Table 3. Transit costs, transit times, and carbon emission factors of different modes of transportation.

<table>
<thead>
<tr>
<th>Transport Mode Conversion</th>
<th>Transit Cost (CNY/TEU)</th>
<th>Transit Time (h/TEU)</th>
<th>Carbon Emission Coefficient (kg/TEU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male–iron</td>
<td>100</td>
<td>0.05</td>
<td>2.17</td>
</tr>
<tr>
<td>Iron–water</td>
<td>200</td>
<td>0.08</td>
<td>1.92</td>
</tr>
<tr>
<td>Water–male</td>
<td>150</td>
<td>0.06</td>
<td>1.98</td>
</tr>
</tbody>
</table>

Table 4. Unit energy consumption and unit carbon emissions of different transportation modes.

<table>
<thead>
<tr>
<th>Mode of Transport</th>
<th>Unit Fuel Consumption (kg/TEU·km)</th>
<th>Unit Carbon Emissions (kg/TEU·km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highway</td>
<td>0.2610</td>
<td>0.889</td>
</tr>
<tr>
<td>Railway</td>
<td>0.0462</td>
<td>0.156</td>
</tr>
<tr>
<td>Water transport</td>
<td>0.0952</td>
<td>0.322</td>
</tr>
</tbody>
</table>

4.2. Model Solution

According to the model constructed above, the fireworks algorithm with attractive force search operator was used, and the program was programmed using the MATLAB R2021b program. According to the design research analysis in the existing literature [36–38], the algorithm parameters in this study were set as follows: the population size was 80, the number of iterations was 200, the explosion amplitude of fireworks was 5, the total number of sparks generated was 100, the gravity parameter was 100, and the gravity parameter update factor was 20.

4.2.1. Algorithm Validity Test

Taking the container multimodal transport model under certain conditions as an example, fireworks with attractive force search operator were used to solve the problem. The transport scheme obtained was as follows: 1-4-5-12-16-21-27-28-35. The transportation mode was railway transportation, and the total transportation cost at this time was as low as CNY 666,776. The fireworks algorithm and the fireworks algorithm with gravitational search operator were used to solve the model. By comparing Figure 3 with Figure 4 (the abscissa represents the number of iterations, and the ordinate represents the total cost/CNY), it can be seen that fireworks algorithm with attractive force search operator had a faster convergence rate and higher solution efficiency than the fireworks algorithm.
4.2.2. Analysis of Results

The fireworks algorithm with gravitational search operator was used to determine the demand and carbon trading price (the demand at this time was the average value of the three scenarios; Mode 1). The demand was determined, and the carbon trading price was uncertain (allowable change in carbon trading price was 11, and the benchmark carbon trading price was 52.6; Mode 2) the uncertain demand and carbon trading price (the maximum regret value was 0.2; Mode 3) were solved to obtain the transportation route, route mode, and total transportation costs under each mode, as shown in Table 5 below.

### Table 5. Comparison of results of different modes.

<table>
<thead>
<tr>
<th>Transport Path</th>
<th>Mode of Transport</th>
<th>Transportation Costs (CNY)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1 1-4-5-12-16-21-27-28-35</td>
<td>3-3-3-3-3-3-3-3-3</td>
<td>666,776</td>
</tr>
<tr>
<td>Mode 2 1-4-5-12-16-21-27-28-35</td>
<td>3-3-3-2-3-3-3-3-3</td>
<td>469,627</td>
</tr>
<tr>
<td>Mode 3 1-4-5-12-16-21-27-28-35</td>
<td>3-3-2-2-2-3-3-3-3</td>
<td>568,344</td>
</tr>
</tbody>
</table>

According to the comparison in Table 5, it can be seen that the transportation routes under the three modes did not change (all were 1-4-5-12-16-21-27-28-35), but the transportation modes under the three modes were different. From the perspective of transportation costs and transportation methods, the lowest was CNY 469,627 for Mode 2, and the transportation method was “water-water-water-iron-water-water-water-water-water”. The cost of Mode 1 was CNY 666,776, and the transportation method was “water-water-water-water-water-water-water-water-water-water-water-water-water”. Compared to Mode 2, the cost increased by CNY
197,149. The transportation method also changed, which was due to the randomness of carbon trading prices. The cost of Mode 3 was CNY 568,344, and the transportation method was “water-water-iron-iron-water-water-water-water-water”, which was an increase of CNY 98,717 compared to Mode 2. The transportation method also changed, which was due to the increase in transportation costs caused by the pursuit of stability in the robust optimization problem under different demand scenarios.

4.2.3. Sensitivity Analysis of Double Uncertainty Parameters
1. \( \hat{w} \) Analysis of the impact of changes on the total cost

In order to explore the impact of carbon trading price randomness on decision-making, the parameter sensitivity in Mode 2 (carbon trading price uncertainty) was analyzed with \( \hat{w} \), as shown in Figure 5:

![Figure 5. Trend chart of total transportation costs with \( \hat{w} \).](image)

In Figure 5, it can be seen that the total transportation cost of container multimodal transportation in Mode 2 had obvious changes, which were caused by the randomness of carbon trading prices, and the changes led to increase or decrease in total transportation cost. Due to the influence of carbon trading policies, the carbon emission costs in the process of container multimodal transport will appear negative. The total transportation costs are reduced, incentivizing companies to reduce carbon emissions.

2. Analysis of the impact of \( \alpha \) on the maximum regret value

In order to verify the characteristics of robust optimization in the pursuit of stability, the analysis of the influence of maximum regret value \( \alpha \) on total transportation cost is shown in Figure 6. When \( 0 < \alpha < 0.2 \) was used, the total transportation cost decreased rapidly; when \( 0.2 < \alpha < 0.5 \) was used, the total transportation cost decreased more gently. Multimodal transport operators can maximize their benefits and improve the operational efficiency of multimodal transport by coordinating the relationship between the maximum regret value and the cost.
On this basis, we explored the maximum allowable change of carbon trading price under Mode 3, the total transportation cost, and the transportation cost changes under different demand scenarios, as shown in Figure 7.

In Figure 7, we can see that when $\hat{w}$ was randomly valued between 5 and 15, and the total transportation cost and the transportation cost under different demand scenarios decreased with the increase in $\hat{w}$. When $\hat{w}$ was randomly valued between 15 and 25, the transportation cost and the cost under different demand scenarios increased with the increase in $\hat{w}$, and the transportation cost under high demand scenarios changed significantly. In the case of uncertain demand, the random transportation cost of the carbon trading price was less than the transportation cost when the carbon trading price was determined. That is to say, under the conditions of uncertain demand and carbon trading prices, the ability of multimodal transport carriers to bear the randomness of carbon trading prices was reduced.

5. Conclusions

Considering the uncertainty of carbon trading prices and demand, in this study, a robust optimization stochastic model for container multimodal transportation was con-
constructed using a fireworks algorithm with attractive force search operator to solve this problem. The feasibility of the model and the effectiveness of the algorithm were verified through example calculations. The results of the verification show that changes in demand and carbon transaction prices have a significant impact on the optimization of container multimodal transport routes. The comparison of the three different modes shows that, in terms of transportation costs, the randomness of carbon trading prices will lead to an increase or decrease in the total transportation cost; there are obvious differences in the cost of different carbon trading price randomness; and robust optimization with uncertain demand will be affected by the regret value constraint, resulting in an increase in the total transportation cost. There will be different effects on transportation methods and transportation times.

By reading a large number of domestic and foreign literatures on research methods, model construction, and algorithms to solve multimodal transport-related issues, it was found that most of the previous studies were based on determining demand and time, and some scholars had studied improvement of random or fuzzy transport volume on transport schemes. Due to the double uncertainty caused by the actual transportation demand of complex goods and the transportation process, the impact of carbon trading policies on the optimization of multimodal transport routes was rarely considered, especially in our country’s national carbon emission rights trading market. However, research has been conducted after the carbon trading market was officially launched. Based on this, the research problem of this paper can provide a certain reference significance for the relationship between the cost and the maximum regret value under uncertain coordination demand and offer a choice to multimodal transport carriers that accounts for the randomness of the carbon trading price. At the same time, it has certain reference value for coordinating the relationship between environmental protection and the economy with regard to multimodal transport and improving the ability of transportation to cope with demand. It can promote low-carbon management in a market with demand uncertainty in the container multimodal transport sector.

The research content of this paper has certain theoretical and practical significance for container multimodal transport, but there are still deficiencies in the research. For example, the influence of practical factors, such as traffic congestion [14], partial node failure [39,40], and shift schedule restrictions [41], was not considered. In the future, the above-mentioned influencing factors can be added on the basis of this research to establish a more realistic container multimodal transport optimization model.

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**Abbreviations**

The main symbols in this work are defined as follows:
M set of transport nodes
K set of transport modes
T total transit time (h)
Z total carbon emissions (kg (CO$_2$)/km)
$F_z$ total carbon emission fee (CNY) corresponding to total carbon emission
$E_i$ carbon emission allowances under the carbon trading policy (kg)
w carbon trading price (CNY)
$T^a$ the minimum time window required to complete the entire transportation process (h)
$T^d$ the maximum time window required to complete the entire transportation process (h)
$F_c$ the unit storage costs for goods due to early arrival (CNY)
$F_p$ the unit penalty cost of goods due to late arrival (CNY)
$q_{ij}$, $j$ the freight volume between nodes $i$ and $j$, that is, the number of containers (TEU)
$D_{ij}$ the transportation distance between transportation nodes $i$ and $j$ (km)
$v_k$ average travel speed of mode $k$, $k \in K$(km/h)
$t_{ij}^{k_1,k_2}$ transit time required for conversion from transport mode $k_1$ to $k_2$ at transport node $i$ (h)
$t_{i}^{k_1,k_2}$ transit time per unit at transport node $i$ when switching from transport mode $k_1$ to $k_2$ (h)
$F_{m_{ij}^1}$ transport base price 1 when goods are transported by transport mode $k$ between transport nodes $i$ and $j$ (CNY)
$F_{m_{ij}^2}$ transport base price 2 when goods are transported by transport mode $k$ between transport nodes $i$ and $j$ (CNY)
$F_{k_1,k_2}$ the unit transshipment costs required for conversion from transport mode $k_1$ to $k_2$ at transport node $i$ (CNY)
$t_{ij}^{k_1,k_2}$ in-transit transportation costs incurred when mode $k$ is used for transportation between transport nodes $i$ and $j$ (CNY)
$P_{k_1,k_2}$ transshipment costs incurred when switching from transport mode $k_1$ to $k_2$ at transport node $i$ (CNY)
$F_{k_1}$ transport carbon emission coefficients for conversion from transport mode $k_1$ to $k_2$ at transport node $i$
$\beta_{ij}$ carbon emissions generated by transport mode $k$ when transporting goods between transport nodes $i$ and $j$
$\xi_{ij}^k$ carbon emissions per unit volume and per unit distance for mode of transport
$s$ set of all scenarios, where $P_s$ is the probability of the occurrence of scenario $s$
$X$ set of robust feasible solutions for all scenarios
$F_s(x)$ the objective function value of scenario $s$, where $x$ is the decision variable. For each individual scenario, $s \in S$, $F^*_s$ is the optimal objective function value of the deterministic problem under the scenario; let $F^*_s > 0$ be constant
$\gamma$ the maximum allowable regret value under scenario $s$, that is, the maximum deviation between the allowable objective function value of the scenarios and its corresponding optimal objective function value

Decision variables

$X_{ij}^k$ whether $k$ is used for transportation between transportation nodes $i$ and $j$. If it is used, $X_{ij}^k = 1$; otherwise, $X_{ij}^k = 0$

$Y_{i}^{k_1,k_2}$ whether the transport mode converts from $k_1$ to $k_2$ at the transport node $i$. If there is conversion, $Y_{i}^{k_1,k_2} = 1$; otherwise $Y_{i}^{k_1,k_2} = 0$.

References

5. Dai, W. Research on Route Selection of China-EU Container Multimodal Transport. Master’s Thesis, Southwest Jiaotong University, Chengdu, China, 2022. [CrossRef]

41. Liu, S.; Peng, Y.; Song, Q.; Yi, Y. The robust shortest path problem for multimodal transportation considering timetable with interval data. *Syst. Sci. Control. Eng.* 2018, 6, 68–78. [CrossRef]