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Sustainable Shaping of Lightweight Structures Created According to Different Methods

Dominika Bysiec

Faculty of Civil Engineering and Architecture, Opole University of Technology, Katowicka 48, 45-061 Opole, Poland; d.bysiec@po.edu.pl

Abstract: This paper presents the problem of the efficient shaping of spherical structures of geodesic domes, which is the basis for creating a regular octahedron, in the aspect of sustainable development. The proposed two methods of shaping covered by this study differ in the way the dividing points of the initial edges of the regular octahedron are connected, and, therefore, in the way the sphere is shaped. Using different methods, two families of domes with different lengths of struts but with a similar number of them were obtained. The conducted comparative analysis leads to the indication of this method of shaping the topology, thanks to which it is possible to obtain structures with less consumption of construction material, and, consequently, with less weight. Both the geometry and weight indicate the advantages of geodesic domes created using the first subdivision method. The selection of the appropriate method of shaping geodesic domes is a consequence of a sustainable design strategy. The presented structures in the form of geodesic domes, the basis of which is a regular octahedron, can be original, innovative coverings, while the detailed analysis carried out is intended to provide design guidelines that will facilitate both architects and designers.

Keywords: single-layer geodesic dome; topology; geometry; static analysis; regular octahedron; sustainable design

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1. Introduction

Geodesic dome structures are innovative construction solutions that have been an inspiration for designers since the last century. Due to the multitude of advantages of geodesic domes, they are still used all over the world. The most important advantages are no need to use internal supports, stability, and durability, as well as economic considerations. These are structures erected in accordance with the principles of sustainable development, which allow you to make savings. Approaches to sustainable development are presented in many papers; for example, by Ma et al. [1], Kisku et al. [2], and Embaby et al. [3], as well as Maleska and Beben [4]. In the case of geodesic domes, the cost-effectiveness results from the possibility of approximating the initial face of the polyhedron to smaller ones and, consequently, obtaining straight struts with a slight variation in their length. As a consequence, we obtain structures that need very little structural materials. Assembling a geodesic dome takes very little time. Creating structures in the form of a geodesic dome is also a financial saving on heating and cooling, compared to objects in the shape of a rectangle. Considering the above-mentioned positive aspects of geodesic dome structures, the interest of engineers and researchers in dome structures is increasing.

Therefore, there are many different studies available in the literature on this subject. These are primarily studies on domes, the basis of which is an icosahedron, modelled on Fuller's patent [5] because, in the case of these structures, we obtain the smallest number of elements of different lengths. The considerations relate to geometry and shape topology, optimization of designed structures, financial savings on heating and cooling, structural durability and stability, and the impact of dynamic or seismic loads.

Waghmode and Kulkarni [6] studied modelling and validating single-layer geodesic domes generated from an icosahedron with various height to span ratios. Meshing of structures based on their topology was considered by Carbonell-Márquez et al. [7], as well as Moskaleva et al. [8]. Tarnai [9] presented the wide application of geodesic domes in fields of science other than engineering. Obrębski [10] presented a short review of the scientific and engineering activities led by him for 44 years. These achievements concern some new approaches to shaping and analyzing lightweight structures. The geometry of the icozadeltahedral structures was dealt with by Siber in [11].

In the paper [12], Guan et al. compared the experiment with the numerical analysis of the complete post-buckling behavior of shallow geodesic lattice domes. The obtained results from the experimental tests showed a close correlation with the numerical analysis performed using the finite element method. Barbieri et al. [13] used the finite element method to develop models of geodesic domes subjected to dynamic loads. Szaniec and Zielinska [14] used the existing reticulated dome and subjected it to wind loads. In this way, they conducted a dynamic analysis of this structure. Satria et al. [15] presented the dynamic behavior of a new type of two-way single-layer lattice dome with nodal eccentricity. Fu [16] showed both the new forms of tensegrity domes and analyzed them in terms of static and design.

Another aspect considered when designing geodetic domes is related to their optimization. The use of various optimization algorithms leads to lightweight structures, which are of great importance when deciding on the choice of the form of the covering.

Saka [17,18] used the harmony search algorithm to present the optimal solution to the structure geometry. A relatively small number of searches was enough to indicate the optimal height and sectional designations for members, using the harmony search algorithm. Saka [19] also presented another algorithm, that is a coupled genetic algorithm. Thanks to it, he obtained the optimum topological design of an analyzed dome. Another approach to the topic of optimization can be seen in the paper of Kaveh and Rezaei [20]. They used the enhanced colliding bodies optimization (ECBO) method for a lamella dome. The same optimization method was used by Kaveh et al. in the paper [21]. Kaveh and Rezaei also considered the optimal design of Schwedler and ribbed domes in their paper [22] by presenting the optimal cross-sections for the elements, the optimal height for the crown, the optimal number of the nodes in each ring, and the optimum number of rings under the determined loading conditions. Another work that takes into account the optimization of geodesic domes (Kaveh and Talatahari [23]) relates to finding the optimal geometry and topology using the CSS (charged system search) method. SHADE algorithm was used by Kaveh et al. [24] to investigate the efficiency of it in relation to large-scale truss optimization problems. Dede at al. presented the developed metaheuristic algorithms known as Rao's algorithms in [25], as well as the Jaya algorithm in [26]. Gholizadeh and Barati [27], in turn, presented the optimization process, the optimum number of rings, and the optimum height of the crown and tubular section of the member groups, taking into account the geometric nonlinear behavior of the domes. The optimization of domes was also considered by Ye and Lu in [28], as well as by Grzywinski et al. in [29].

The wind pressure coefficient (CP) in relation to various places of roofing in the form of a geodesic dome was indicated by an experiment by Faghih and Bahadori [30]. These authors extended their research in [31] by examining the thermal requirements for domed roofs. Another aspect that was analyzed was the underfloor heating system. This study was conducted by Khademinejad et al. [32]. A different approach is presented in [33] by Romero-Gómez et al., who assessed indoor microclimatic conditions in a naturally ventilated greenhouse in local Mexican climatic conditions. Using CFD modelling, Lu et al. [34] studied the performance of displacement ventilation in a dome cinema auditorium in summer. To investigate its ventilation and thermal performance in different climatic scenarios, Soleimani et al. [35] presented three-dimensional CFD modelling of a two-story geodesic dome house. The three-dimensional analysis using the finite element method was also used in [36].

The static properties and stability of structures were presented by Tan et al. [37]. It is worth noting that the dome structures were also tested under extreme loads, such as seismic loads or explosions. Papers taking into account the seismic loads of geodesic domes were presented by Pilarska and Maleska [38], Xu and Sun [39], and Yang et al. [40,41], as well as Nair et al. [42]. The research concerning the impact of the explosion on the structure of domes is presented in [43].

As demonstrated above, there are many scientific studies on geodesic dome structures which refer to different aspects. The topics related to the optimization of these structures, using various optimization algorithms, are analyzed, in particular, in depth. However, there are very few considerations for dome structures based on a regular octahedron. Therefore, it should be subjected to detailed comparisons and analyses. Initial studies confirming the use of the regular octahedron as the basis for shaping the spatial forms of geodesic domes were presented in the papers of Pilarska [44–46].

The main purpose of this study is to investigate the geometric and static properties of geodesic domes derived from a regular octahedron, created according to two different methods of shaping the topology of their grids, and then to indicate this method, thanks to which it is possible to obtain structures with less consumption of construction material and, consequently, with less weight. Each method consists of eight novel structures. In order to evaluate and indicate a behavior of domes created according to two different methods, a number of comparative analyses are carried out. The following parameters are compared: (i) the number of nodes, struts, and supports, (ii) the number of groups of struts of equal lengths, (iii) the minimum and maximum length of the struts, (iv) the total length and weight of all the struts, (v) the maximum values of the tensile and compressive forces, (vi) the maximum and minimum stress values, and (vii) the maximum vertical and horizontal displacements of the nodes. The results obtained in this study show the behavior of structures created according to the first and second methods, as well as allow to indicate a more accurate choice of the method of shaping geodesic domes created on the basis of a regular octahedron, in the aspect of sustainable development. These results are an introduction to more advanced future research, including experimental ones.

2. Geodesic Domes Shaped Based on the Regular Octahedron

The structures of geodesic domes are made of triangles, creating a self-supporting, durable structure, without the need to use internal supports. Their uniqueness is manifested in the wide possibilities of covering large areas, where the lack of disturbances caused by the use of additional internal structural elements is their great advantage.

The geodesic dome is created after projecting the edges of a polyhedron onto the sphere described on it. Consequently, a large number of faces of the polyhedron causes the multiplicity of its edges, which form the arcs of the circle of the sphere described on the polyhedron. The force lines in the domes are distributed along the geodetic lines, so the forces in the struts are axially acting. The morphology of construction clearly defines the elements of space and strictly shows the methods of combining them into an architectural shape [47]. Professor Fuliński developed methods for creating geodesic domes based on regular polyhedra. These methods were often used to create the topology of geodesic dome structures generated from a regular dodecahedron or icosahedron. However, the remaining Platonic solids, including, inter alia, the regular octahedron, have not been the basis for studies and analyses concerning the creation of geodesic domes so far.

This paper covers the research area of domes generated on the basis of regular octahedrons. Two of Professor Fuliński's methods [47] were used to design geodesic reticulated domes derived from the regular octahedron. Each of them is based on a subdivision of the edges of the triangle, which is the original face of the regular octahedron, into smaller parts. The appropriate different connection of these parts leads to the creation of grids reflecting the grids of the designed domes. The more subdividing parts on a given edge, the more structural elements (struts) characterize the dome. The principles of the two methods proposed by Professor Fuliński for subdividing the edges of the initial triangle that were used in the paper are shown in Figure 1.

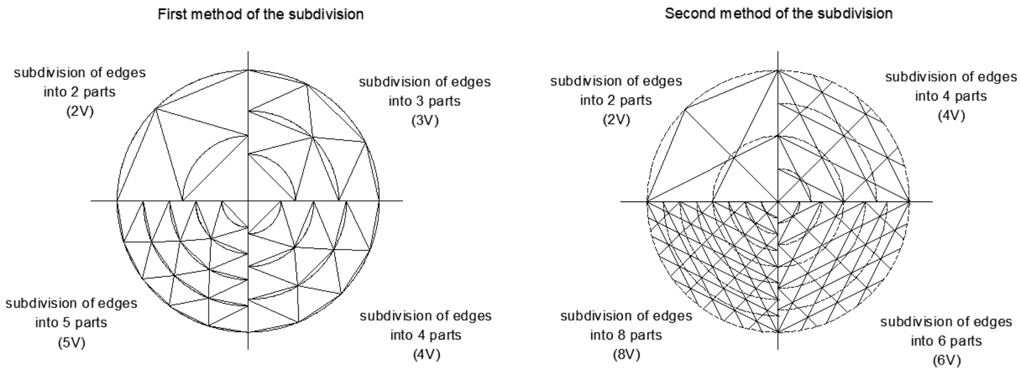

Figure 1. Methods of subdividing the initial triangle edge according to Professor Fuliński.

Using the two methods of subdividing the original octahedron face into smaller parts, two dome families were obtained, which were then subjected to a detailed comparative analysis. Each of the initial triangles of the octahedron is broken into smaller triangles through the tessellation of the initial triangle into smaller triangles. This is performed by frequency (V). The term "frequency" is identified by the letter "V". Therefore, "2 V" means "2 frequencies", that is, a 2-frequency dome. Using the presented division, as a consequence, it is possible to obtain a three-way mesh built on the walls of the original regular octahedron. The central projection of the vertices on the sphere describing the regular octahedron used in the mesh reflects the polyhedron approximating this sphere. The nodes of the resulting mesh are the only ones that lie on the surface of the sphere. As the frequency increases, we obtain a much smoother sphere. The first method used to create geodesic domes is to subdivide the initial edge of the triangular octahedron mesh into n frequencies, then draw three families of lines parallel to each edge. In this method, the subdivision number "n" can be both even and odd. The second method of subdivision also consists of subdividing the initial edge of the triangular mesh of the octahedron into n frequencies but drawing three families of lines parallel to the height line. In this case, the numbers of n frequencies are even. This method produces left and right triangle pairs. After connecting them, we obtain an entire second method triangle.

Each designed family consists of eight structures with a 50 m diameter, selected in terms of a similar number of nodes and struts, which had a decisive impact on further comparative analysis. The first family are domes formed according to the first subdivision method. It is 8 structures formed after subdividing the initial edge of the triangle, which is the basic face of the regular octahedron, into 19, 20, 21, 22, 23, 24, 25, and 26 frequencies. Thanks to this subdivision, geodesic reticulated domes were designed derived from a 2888-hedron, 3200-hedron, 3528-hedron, 3872-hedron, 4232-hedron, 4608-hedron, 5000-hedron, and 5408-hedron. By subdividing the initial edge of the triangle according to the second subdivision method into 22, 24, 26, 28, 30, 31, 34, and 36 frequencies, domes were created based on a 2904-hedron, 3456-hedron, 4056-hedron, 4704-hedron, 5400-hedron, 6144-hedron, 6936-hedron, and 7776-hedron. Example diagrams of the designed reticulated domes, created based on a 3200-hedron and 5000-hedron according to the first method and a 3456-hedron and 6936-hedron according to the second method, are presented in Figure 2.

The term "n-hedron" refers to the derivative of the regular octahedron resulting from successive subdivisions of the initial triangular face and indicating the number of faces obtained after subdividing it.

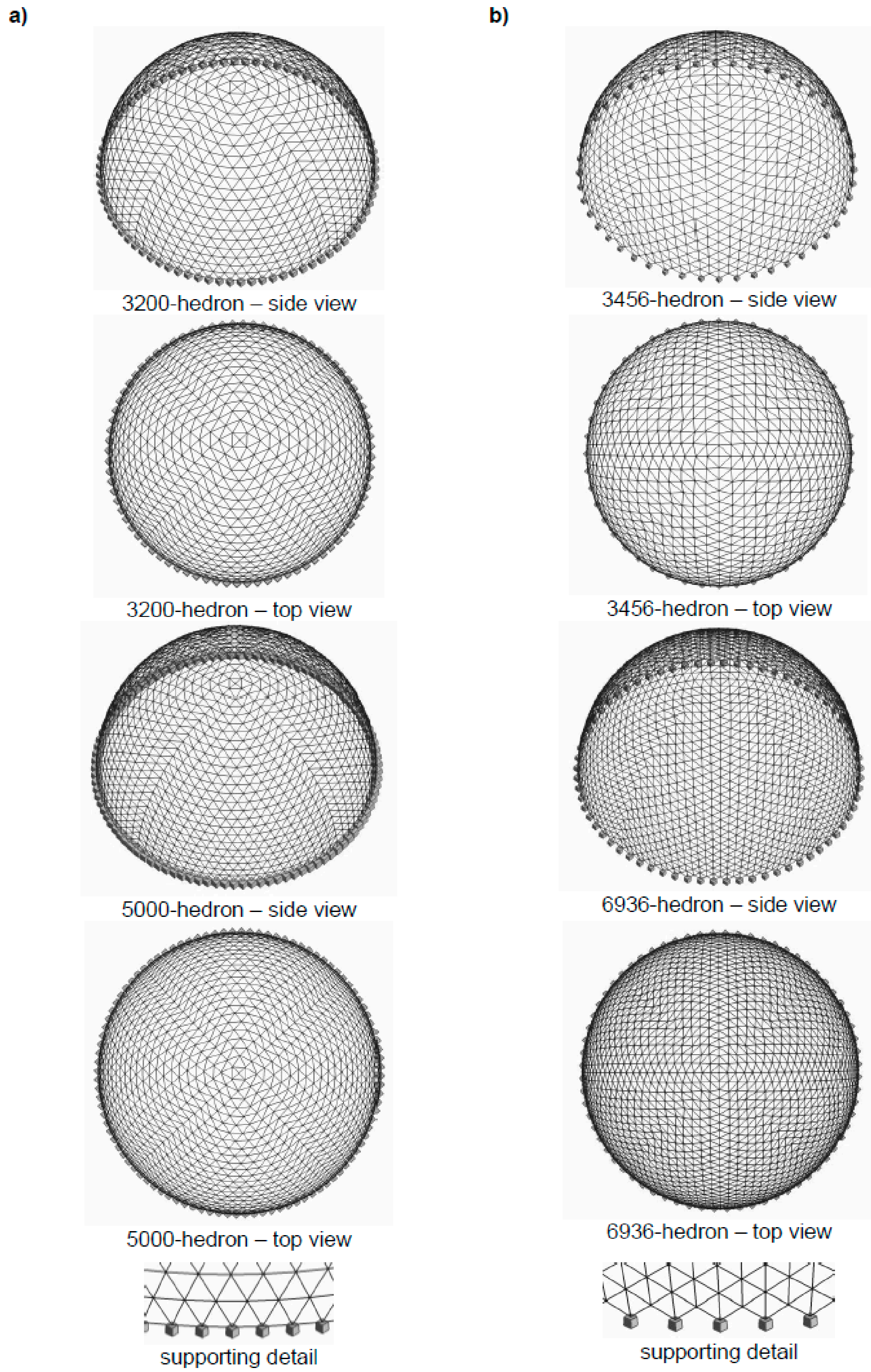

Figure 2. Example designed geodesic strut domes with their boundary conditions: (**a**) the first subdivision method, (**b**) the second subdivision method.

3. Analysis of Geometry Results

A comparative geometric analysis was carried out for the following calculations: the number of struts and nodes, the number of supports, the number of groups of struts of equal lengths, the minimum and maximum length of the strut, and the total length and weight of all the struts in a given dome.

First, the number of nodes and struts for each designed single-layer structure was compared. The first designed dome from both analyzed families consisted of a similar number of nodes and struts (the first dome formed from a 2888-hedron using the first method: 761 nodes and 2204 struts; the first dome created from a 2904-hedron using the second method: 749 nodes and 2156 struts). Each successive additional subdivision of the initial edge of the triangular octahedron face causes an increase in the number of nodes and struts. In the case of the domes generated using the first method, the increase in the number of nodes and struts is initially by approx. 10% and decreases linearly to approx. 7% in the case of the last, eighth dome from the first family. The subdivision of the initial edge of the regular octahedron using the second method also translates into a greater number of nodes and struts in each successive designed structure. Initially, this increase is by approx. 19% and decreases linearly with the increasing density of the dome grid to approx. 12% of the last, eighth dome from the second family. Geodesic reticulated domes formed based on the regular octahedron using the first subdivision method of the original edge of the triangle are characterized by a smaller number of structural elements, such as nodes and struts, compared to structures whose basis is the second subdivision method used. The linear upward trend in the number of nodes and struts for all 16 single-layer reticulated domes designed according to the first and the second method is shown in Figure 3.

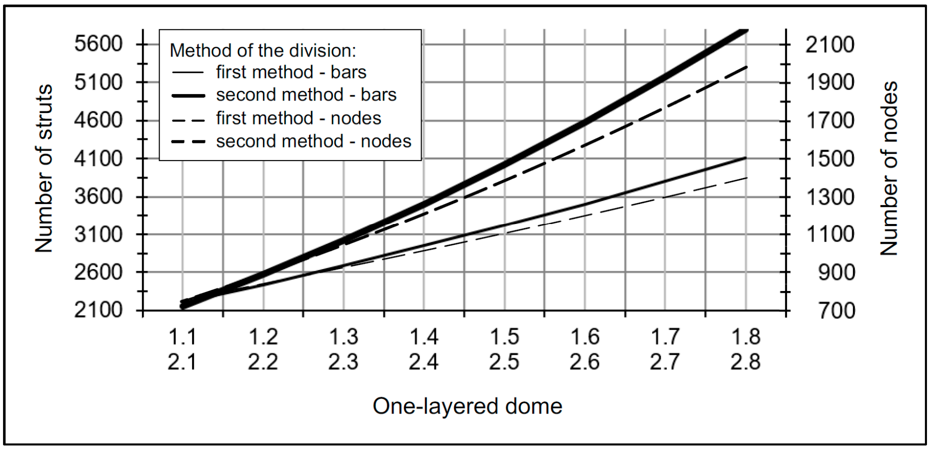

Figure 3. Number of struts and nodes in the designed domes according to the first and the second subdivision method used. Single-layer dome according to the first method of the subdivision: 1.1 (2888-hedron); 1.2 (3200-hedron); 1.3 (3528-hedron); 1.4 (3872-hedron); 1.5 (4232-hedron); 1.6 (4608-hedron); 1.7 (5000-hedron); 1.8 (5408-hedron). Single-layer dome according to the second method of the subdivision: 2.1 (2904-hedron); 2.2 (3456-hedron); 2.3 (4056-hedron); 2.4 (4704-hedron); 2.5 (5400-hedron); 2.6 (6144-hedron); 2.7 (6936-hedron); 2.8 (7776-hedron).

Another parameter for comparison is the number of supports present in each designed structure. The subdivision of the initial edge of a regular octahedron according to the first method is reflected in a much larger number of supports compared to the second method used, which is presented in Figure 4.

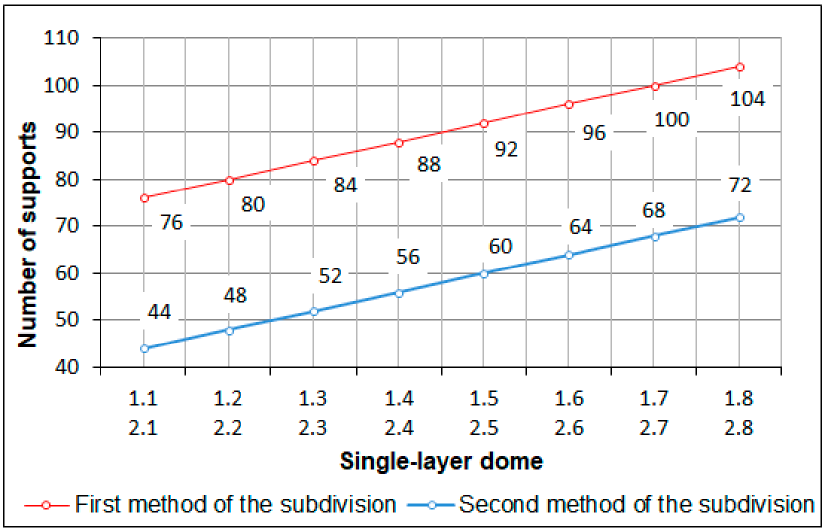

Figure 4. Number of supports in the designed domes according to the first and the second subdivision method used (characteristics of the individual designed domes as described in Figure 3).

One of the most important parameters taken into account when choosing a designed dome-shaped covering is undoubtedly the number of groups of struts of equal lengths. When creating structures with a large number of strut elements, it is important that they consist of the smallest number of groups of struts of equal lengths. Domes designed according to the first method show a 20–30% smaller number of groups of struts of equal lengths compared to domes created according to the second method. When analyzing individually designed domes in a given family, those that are characterized by a higher grid density by the strut elements deserve attention. Designing structures with an increasing number of struts does not increase the number of groups of struts of equal lengths. In fact, a downward trend can be observed; for example, in domes 1.5, 1.6, and 1.7 created using the first method or in domes 2.7 and 2.8 generated using the second method (see Figure 5).

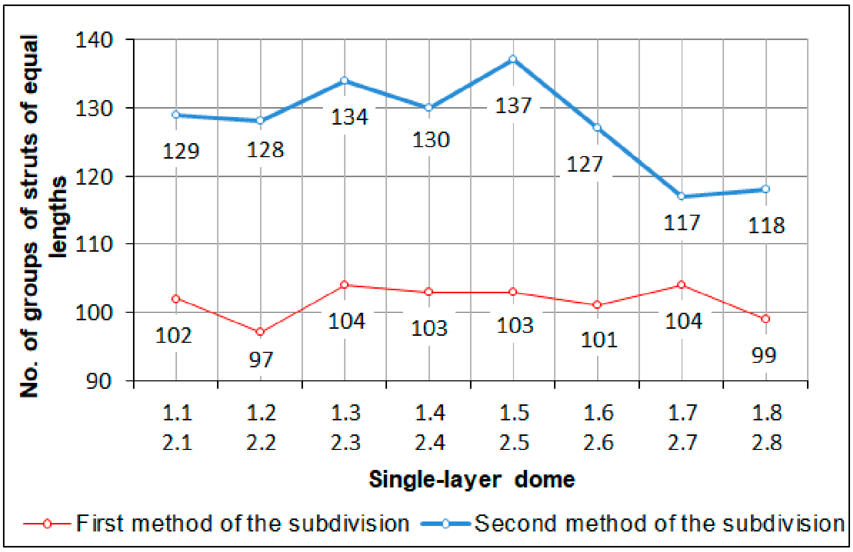

Figure 5. Number of groups of struts of equal lengths in the designed domes according to the first and the second subdivision method used (characteristics of the individual designed domes as described in Figure 3).

The minimum and maximum length of the struts in each designed single-layer dome is another parameter subjected to comparative analysis. The application of the second method is reflected in the obtained smaller minimum strut lengths compared to the first method used. Dome 2.1 created using the second method is described by the shortest element whose length is 21% shorter than dome 1.1, designed using the first method. The difference in the minimum length of the strut increases with the increase in the subdivisions of the initial edge of the regular octahedron and is 45% when comparing dome 2.8 to dome 1.8. Considering the maximum length of the strut occurring in each designed dome, we notice that dome 2.1 created using the second method is described by a longer maximum strut than dome 1.1 (about 3%). Then, after crossing point A, there is a change and each successive pair of compared domes shows a tendency similar to the minimum length of the struts. Structures created based on the second method have shorter maximum strut lengths than the domes generated using the first method. Finally, the difference in the maximum strut length between domes 2.8 and 1.8 is about 16%. The interpreted dependencies are presented in Figure 6.

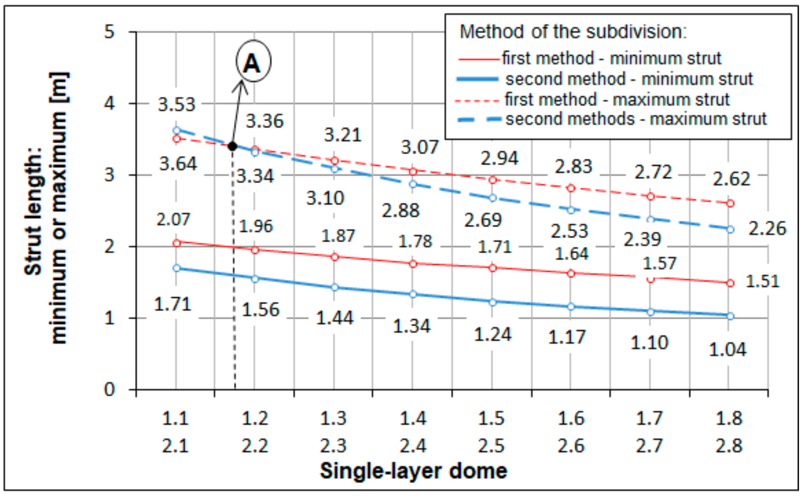

Figure 6. Minimum and maximum length of struts in the designed domes according to the first and the second subdivision method used (characteristics of the individual designed domes as described in Figure 3).

The comparative geometric analysis also covered the total length and weight of all the strut elements of the designed single-layer structures, and the results obtained are presented in Figure 7. The total length of all the struts depends on the subdivision method used. The first subdivision method used is reflected in the smaller total length of all member elements. This involves a smaller number of struts in the designed domes compared to the domes created using the second method (as shown in Figure 3). This undoubtedly has an impact on the lower consumption of construction material needed to produce all the strut elements in each analyzed dome. However, point A in Figure 7 indicates that this applies to domes 1.2 to 1.8. The first designed dome according to the first method (dome 1.1) is characterized by a greater total length of all the strut elements than dome 2.1, created using the second method. The last pair of designed domes, 1.8 and 2.8, differ in the total length of the struts by about 17% (dome 1.8—7686.96 m, dome 2.8—9065.96 m).

The weight of the strut elements of the designed domes is conditioned by the adopted cross-sections of the struts. The weight of the first two domes formed according to the first method (1.1 and 1.2) is greater than for domes 2.1 and 2.2, generated according to the second method. Then, after crossing point B, there is a change and the domes shaped according to the first method are lighter than the structures obtained according to the second method.

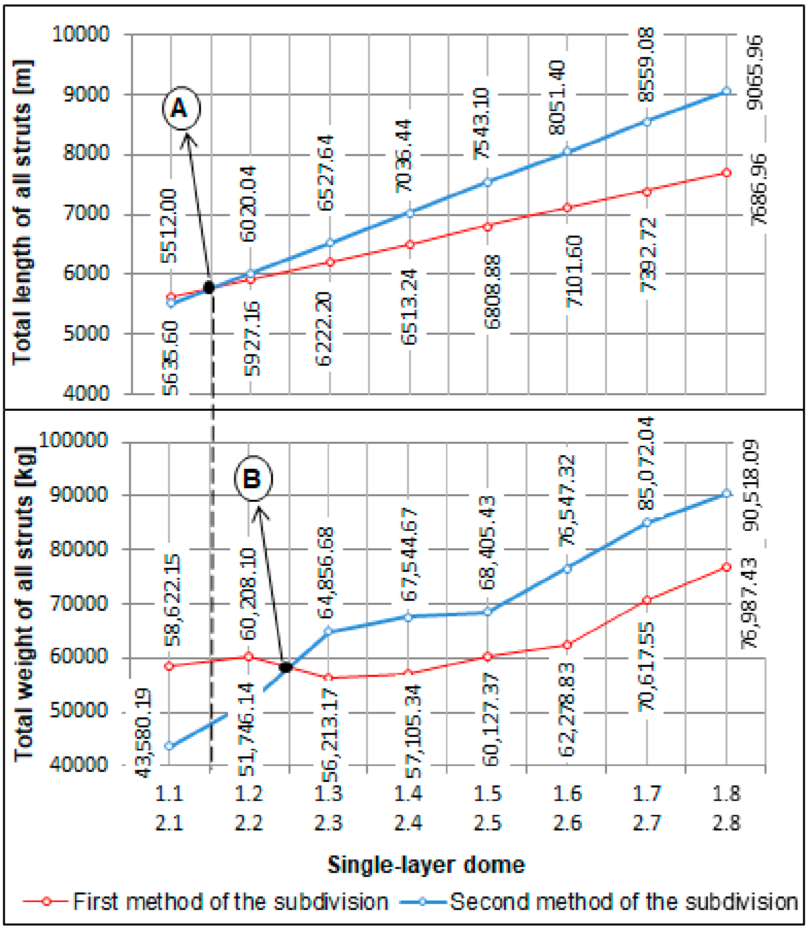

Figure 7. Total length and weight of all struts in the designed domes according to the first and the second subdivision method used (characteristics of the individual designed domes as described in Figure 3).

4. Analysis of Static Results

4.1. Assumptions

The simplest geometrically rigid polygons are triangles and tetrahedrons. If struts in the model of such figures are articulated in the corners, these figures will not change their shape; they will not fold. A structure made of such elements does not transfer the bending moment; there are only compressive or tensile forces along the struts. This phenomenon has been used since time immemorial in flat or spatial trusses as structural elements. A geodesic dome is an object of the mutual stabilization of tension and compression elements.

Using a computer program (Robot Structural Analysis), a numerical analysis of the statics of the modelled innovative 16 geodesic domes was performed. Fixed loads and variable loads were taken into account, from which load combinations were developed. The following loads were taken into account: the fixed load, i.e., own weight of construction and weight of cover constituting glass panes with a weight of 0.6 kN/m^2, as well as variable load, i.e., snow and wind for the first climate zone, where the value of the characteristic pressure of the wind speed was 0.3 kN/m^2. From the presented interactions, 12 load combinations were created. Combinations No. 1–4 include fixed influences as well as leading variable influences of the wind and accompanying variable influences of the snow. Combinations No. 5–8 consist of fixed influences as well as leading variable influences of the snow and accompanying variable influences of the wind. Combinations No. 9–12 were fixed influences as well as leading variable influences of the wind.

All the struts of the developed geodesic domes were assigned a tubular cross-section "R" made of S235 steel (yield strength of 235 MPa), which has the following properties:

(i) Young's modulus (E) 210 GPa, (ii) Kirchhoff module (G) 80.76 GPa, (iii) Poisson's ratio (ν) 0.3, (iv) volumetric weight (γ) 7850 kg/m^3, (v) thermal expansion coefficient (α) 1.2×10^{-5}, and (vi) partial safety factor (γ_M) 1.0. Supports were given as restraints, nodes were assumed as articulated.

4.2. Dimensioning

Taking into account the developed geometric parameters, as well as the topology of creating geodesic domes and the methods of subdividing the initial octahedron mesh, 16 innovative dome structures were modelled. The strut elements of these structures were grouped into four groups, taking into account the stress distribution in each strut element under the influence of their own weight. Finally, the struts were designed as round tubes with the cross-section assigned to the most stressed strut in a given group at the level of 80–90%. Table 1 presents the groups of the strut elements of the individual domes along with the assigned cross-section and the load capacity of the strut.

Table 1. The division into groups of bars with assigned cross-sections and load capacity of strut in designed bar domes.

No. of One-Layered Dome	Group 1		Group 2		Group 3		Group 4	
	Cross-Section	Load Capacity of Strut	Cross-Section	Load Capacity of Strut	Cross-Section	Load Capacity of Strut	Cross-Section	Load Capacity of Strut
First method of the division								
1.1	R 63.5 × 8.8	0.89	R 70.0 × 8.0	0.80	R 44.5 × 3.6	0.89	R 44.5 × 5.6	0.87
1.2	R 63.5 × 8.0	0.85	R 70.0 × 8.0	0.80	R 44.5 × 4.0	0.89	R 44.5 × 5.6	0.87
1.3	R 70.0 × 5.6	0.82	R 70.0 × 7.1	0.86	R 42.4 × 3.6	0.90	R 44.5 × 6.3	0.82
1.4	R 54.0 × 8.0	0.89	R 63.5 × 8.0	0.84	R 42.4 × 3.6	0.90	R 44.5 × 6.3	0.85
1.5	R 54.0 × 7.1	0.86	R 63.5 × 8.0	0.83	R 42.4 × 4.0	0.86	R 44.5 × 6.3	0.89
1.6	R 60.3 × 8.0	0.86	R 70.0 × 6.3	0.90	R 42.4 × 5.0	0.84	R 48.3 × 6.3	0.86
1.7	R 60.3 × 8.0	0.83	R 70.0 × 7.1	0.82	R 42.4 × 5.0	0.89	R 51.0 × 6.3	0.85
1.8	R 60.3 × 8.0	0.89	R 70.0 × 7.1	0.82	R 51.0 × 6.3	0.88	R 48.3 × 6.3	0.87
Second method of the division								
2.1	R 70.0 × 7.1	0.82	R 63.5 × 8.0	0.88	R 57.0 × 5.6	0.88	R 51.0 × 3.2	0.80
2.2	R 70.0 × 7.1	0.85	R 63.5 × 7.1	0.90	R 57.0 × 8.0	0.86	R 48.3 × 4.5	0.87
2.3	R 70.0 × 8.0	0.89	R 63.5 × 8.8	0.90	R 57.0 × 8.8	0.88	R 44.5 × 5.6	0.87
2.4	R 70.0 × 8.0	0.90	R 63.5 × 8.8	0.88	R 57.0 × 8.0	0.90	R 44.5 × 6.3	0.84
2.5	R 60.3 × 8.8	0.89	R 60.3 × 8.0	0.87	R 54.0 × 8.0	0.85	R 51.0 × 5.0	0.86
2.6	R 70.0 × 8.0	0.88	R 63.5 × 8.0	0.90	R 57.0 × 8.0	0.87	R 48.3 × 6.3	0.89
2.7	R 70.0 × 8.8	0.84	R 63.5 × 8.8	0.84	R 57.0 × 8.0	0.84	R 51.0 × 6.3	0.90
2.8	R 70.0 × 8.8	0.86	R 63.5 × 8.0	0.89	R 57.0 × 8.8	0.88	R 54.0 × 6.3	0.89

R—tubular cross-section.

The subdivision into four groups of struts for the four example-designed single-layer structures is shown in Figure 8 (two structures formed using the first subdivision method and two structures designed—based on the second subdivision method).

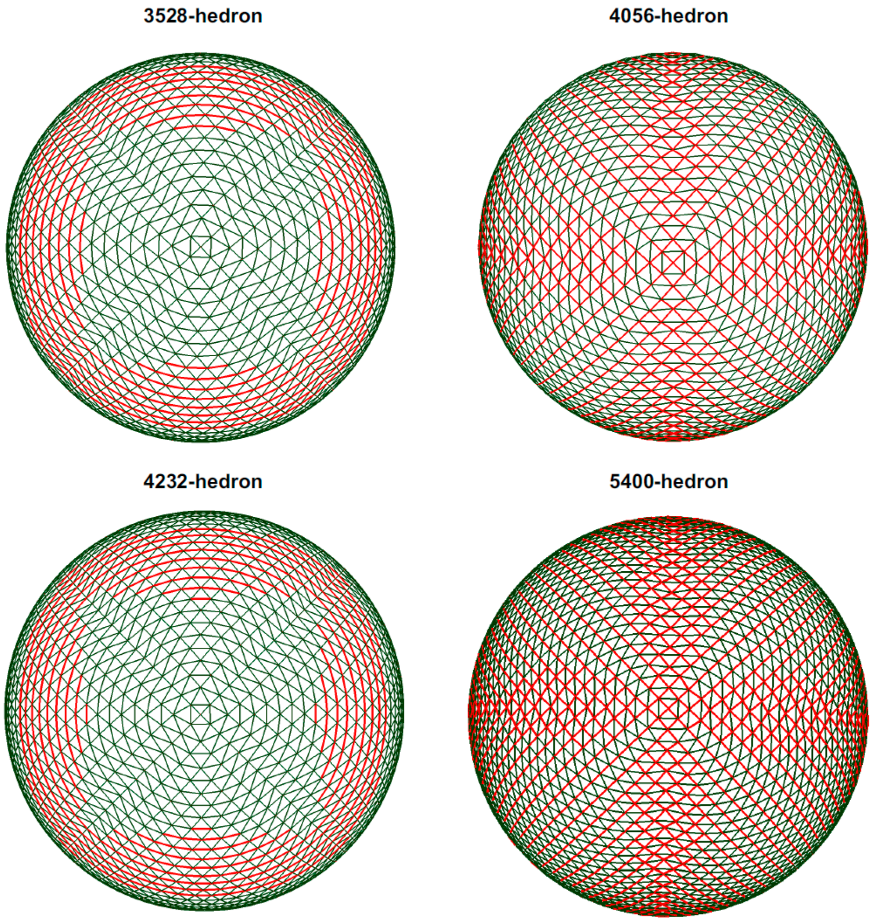

Figure 8. Subdivision into groups of struts in the sample structures created using the first subdivision method (3528-hedron and 4232-hedron) and the second subdivision method (4056-hedron and 5400-hedron).

4.3. Static Analysis

The maximum axial forces occurring in each designed single-layer dome, minimum and maximum stresses occurring in the struts, and the horizontal and vertical displacement of the nodes were subjected to static analysis. The maximum values of the tensile and compressive forces are shown in Figure 9.

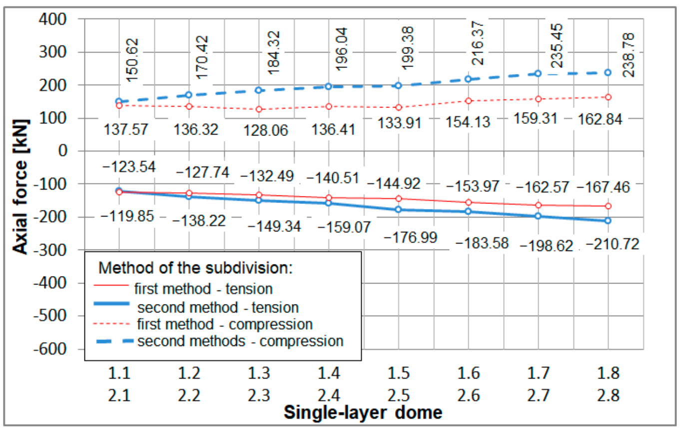

Figure 9. The maximum values of tensile and compressive forces occurring in struts of domes shaped using the first and the second subdivision method (characteristics of the individual designed domes as described in Figure 3).

The applied combination 1 of the loads, taking into account self-weight, as well as wind as a leading variable influence and snow as an accompanying variable influence, showed the highest values of axial internal forces. In the domes formed using the first method of the subdivision of the original face of the regular octahedron, the extreme values of the internal compression and tensile forces of the struts are similar to each other (the difference is only 2–8%). For example, the tensile force for dome 1.8 is 167.46 kN, while the compressive force for the same structure is 162.84 kN. In this case, the difference is about 3%. The maximum values of the tensile and compressive forces for the domes generated using the first method occur primarily in the support zone. In the structures shaped using the second method, the internal compressive forces are higher than the tensile ones. The difference is about 15–25%. As in method 1, here also the highest values of the axial forces occur in the support zone. The compressive forces in most cases are transmitted by the struts lying on a straight perpendicular line to the base of the original face of the regular octahedron. On the other hand, the tensile forces appear in the struts perpendicular to the side edges of the original triangular face.

Upon analyzing the maximum values of the tensile and compressive forces, it was noticed that both of them obtain higher values for the domes created using the second method. For the tensile forces, the difference is 8–25%. The greater difference occurred for the domes with a higher strut density (e.g., 167.46 kN for dome 1.8, compared to 210.72 kN for dome 2.8, which gives a difference of about 25%). For the compressive forces, the values differ by 25–47%, while the increase in difference is also noticeable for more advanced structures (e.g., 162.84 kN for dome 1.8, compared to 238.78 kN for dome 2.8, which gives a difference of approximately 47%). Interactions exerted on the domes with a strut topology using the second method of subdividing the original face of the regular octahedron result in higher values of the tensile and compressive forces in the struts than in the case of the domes designed using the first method.

The loads taken into account for combination 1 (self-weight, as well as wind as a leading variable influence and snow as an accompanying variable influence) cause the appearance of the highest stress values. The maximum stresses occurring in the struts of the domes designed using the second subdivision method are higher than in the struts of the structures formed using the first subdivision method by approx. 10–20%. The minimum stress values are similar for the systems designed using the first and second subdivision methods (see Figure 10).

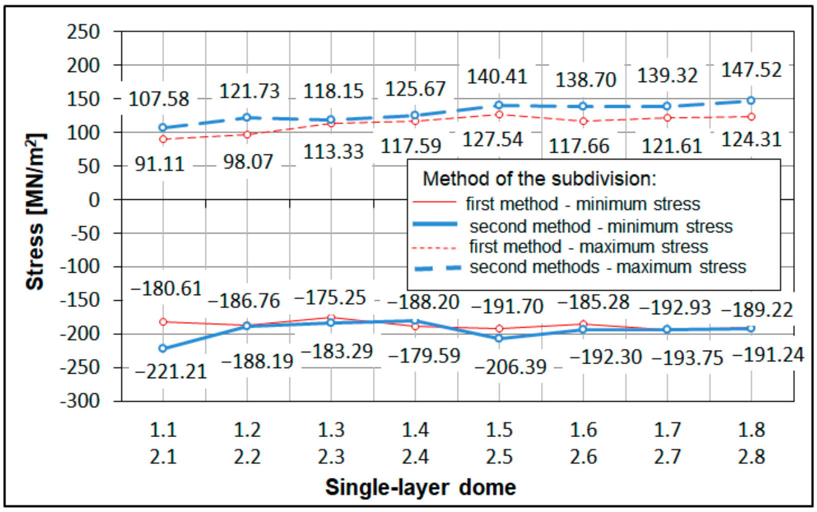

Figure 10. Maximum and minimum stress values occurring in struts of the domes shaped using the first and the second subdivision methods (characteristics of the individual designed domes as described in Figure 3).

The maximum node displacements occur at the same combination of loads (combination 1) as the maximum values of the axial forces and maximum stresses. The maximum vertical displacements of the nodes for the structures formed using the first method are 2.40–2.70 cm, while using the second method, they are 2.50–3.30 cm. Domes 1.7 and 2.7 have the same vertical displacement value, 2.70 cm, as shown in Figure 11 as point A. The maximum horizontal displacements of the nodes in the case of the domes formed based on the first method are 2.60–3.90 cm, while in the case of the structures generated using the second method, they are 3.70–5.60 cm. For the structures created using both the first and second methods, the maximum horizontal displacements are greater than the vertical. Thanks to the topology of the structure shaped using the first method, the maximum displacements of the nodes, both vertical and horizontal, occurring as a result of the loads are smaller than in the case of the systems designed using the second method.

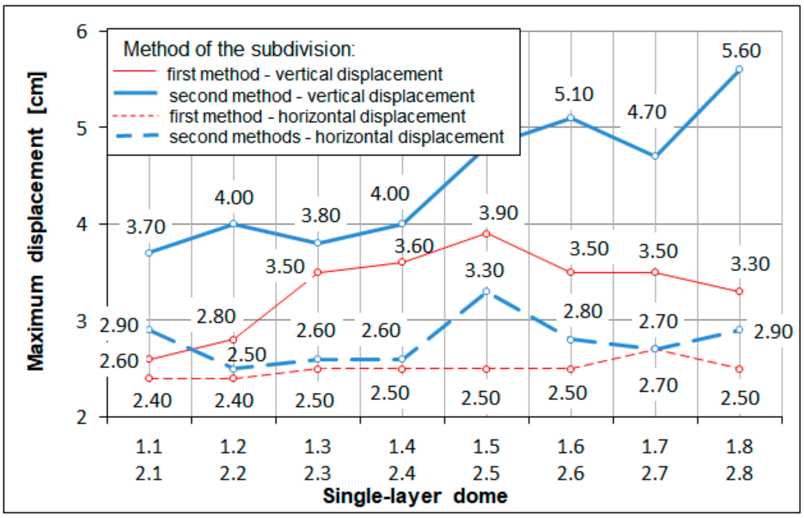

Figure 11. Maximum vertical and horizontal displacements of nodes of the domes shaped using the first and the second subdivision methods (characteristics of the individual designed domes as described in Figure 3).

5. Discussion

So far, many studies were carried out in the field of optimization of lightweight structures, which included various optimization algorithms [17–29]. In these studies, as in this article, the focus was on minimizing the weight of the structures through the selection of appropriate cross-sections of the strut elements. Indicated studies [17–29] were aimed at weight reduction using optimization algorithms, while the approach shown in this paper takes into account weight reduction by selecting the appropriate method of shaping the topology of these structures. To reduce the weight of innovative geodesic domes shaped on the basis of a regular octahedron, two methods of creating the topology of their grids were used. Thus, the goal of both approaches that are in earlier studies [17–29] as well as this paper is the same, i.e., to obtain the lowest possible weight of the structure, using a different way of solving the problem. In connection with the above, it is impossible to clearly refer to the results from previous studies.

In the paper [25], the authors refer to four dome structures, such as: 120 bar, 600 bar, 1180 bar, and 1410 bar. The obtained results using the Rao algorithm are compared with the previous results using other optimization algorithms, available in the literature. The authors indicated that the weight of the analyzed four domes was lower by a maximum of about 2%. In the paper [24], the weights of three domes (600 bar, 1180 bar, and 1410 bar) were optimized using a different optimization algorithm and also compared with previous studies. Mean weights were reduced by a maximum of about 9.2% for the 600-bar dome, 2.5% for the 1180-bar dome, and 7.6% for the 1410-bar dome. In turn, this paper shows that the weight of the domes generated from a regular octahedron according to the first method of shaping the topology of their structures is lower by about 17% compared to the weight of the analyzed domes created according to the second method. Thanks to this, it is possible to obtain lighter coverings, using less construction material, which has an impact on sustainable development.

The discussion presented above concerns the area of sustainability development in terms of reducing the construction material. However, the area of sustainability is very broad, which is confirmed by numerous studies on this subject. In papers [3,4], the authors contributed to the aspect of sustainable development by using a construction material in the form of a natural aggregate (backfilled sand). Thanks to this, a large span of the structure was obtained with a relatively low consumption of structural steel. The approach of the authors of papers [1,2], in turn, takes into account the use of a recycled aggregate as a sustainable construction material. As a result, the aggregate was reused, in line with the ideas of sustainable development.

6. Conclusions

As a result of the comparative analysis of the behavior of the geodesic dome structures obtained based on the two methods of subdividing the original face of the regular octahedron, the following conclusions can be drawn:

- The two subdivision methods used are characterized by a different subdivision of the edges of the triangle, which is the original face of the regular octahedron. The appropriate (different) connection of the points formed after the subdivision leads to the creation of grids reflecting the grids of the designed domes. The more subdividing points on a given edge, the more structural elements (struts, nodes) characterize the dome.
- The choice of the appropriate method of shaping the structure affects the final geometric and static properties of geodesic spherical domes, as well as the consumption of construction material and, consequently, the weight of the dome. In addition, the aspect of the sustainable design of steel structures was taken into account.
- The conducted geometric analysis of the original geodesic domes (eight domes according to the first method and eight domes according to the second method) showed that the structures designed according to the first method are characterized by (i) fewer numbers of nodes and struts, (ii) 45–70% more supports, (iii) a 20–30% reduction in the number of groups of struts of equal length, and (iv) a smaller total length of all the strut elements, which translates into a lower consumption of construction material needed to make all the strut elements.
- The static analysis of all the designed domes shaped according to the first and the second method showed that the structures generated due to the first method are characterized by (i) smaller values of the tensile forces (by approx. 8–25%) and compressive forces (by approx. 25–47%) in the struts, (ii) smaller (by approx. 10–20%) maximum stresses occurring in the struts, and (iii) smaller maximum vertical and horizontal displacements of the nodes.
- The spatial truss structures of the roof covering are the various groups of constructions of any structure and can satisfy the high requirements of architects and designers. In addition, they create a great aesthetic impression by their originality. The geometric-static analysis of single-layer reticulated domes created according to the first and second methods of initial octahedron face subdivision, presented in this study, may inform the choice of the best structure solution for the architect and the designer.

Generally, the behavior of these structures is very important given their size and strategic importance for the site. This study can be helpful to the architect and structural engineer during the design process. Based on this study, it can be concluded that the behavior of geodesic domes under dynamic loads can be very interesting; thus, this aspect is going to be taken into consideration in further studies.

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