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The Efficiency of Value-at-Risk Models during Extreme Market Stress in Cryptocurrencies

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Abstract: In recent years, the cryptocurrency market has been experiencing extreme market stress due to unexpected extreme events such as the COVID-19 pandemic, the Russia and Ukraine war, monetary policy uncertainty, and a collapse in the speculative bubble of the cryptocurrencies market. These events cause cryptocurrencies to exhibit higher market risk. As a result, a risk model can lose its accuracy according to the rapid changes in risk levels. Value-at-risk (VaR) is a widely used risk measurement tool that can be applied to various types of assets. In this study, the efficacy of three value-at-risk (VaR) models—namely, Historical Simulation VaR, Delta Normal VaR, and Monte Carlo Simulation VaR—in predicting market stress in the cryptocurrency market was examined. The sample consisted of popular cryptocurrencies such as Bitcoin (BTC), Ethereum (ETH), Binance Coin (BNB), Cardano (ADA), and Ripple (XRP). Backtesting was performed using Kupiec’s POF test, Kupiec’s TUFF test, Independence test, and Christoffersen’s Interval Forecast test. The results indicate that the Historical Simulation VaR model was the most appropriate model for the cryptocurrency market, as it demonstrated the lowest rejections. Conversely, the Delta Normal VaR and Monte Carlo Simulation VaR models consistently overestimated risk at confidence levels of 95% and 90%, respectively. Despite these results, both models were found to exhibit comparable robustness to the Historical Simulation VaR model.

Keywords: market risk; value-at-risk; market stress; cryptocurrency; blockchain



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1. Introduction

The current interest in cryptocurrencies among both individual investors and major corporations is substantial. The birth of cryptocurrencies can be traced back to the creation of Bitcoin in 2008 by a developer operating under the pseudonym Satoshi Nakamoto [1]. Bitcoin was intended to facilitate peer-to-peer transactions and aimed to decentralize the financial system. The emergence of Bitcoin spurred the development of other cryptocurrencies, commonly referred to as “Altcoins,” which seek to address various limitations inherent in Bitcoin, such as transaction speed, feasibility, and functionality. Despite this, Bitcoin (BTC) still holds the largest market capitalization in the world, followed by Ethereum (ETH), Binance (BNB), Cardano (ADA), Tether (USDT), Ripple (XRP), and many more. These currencies are believed to have the potential to become the future of currencies [2]. However, studies have shown that cryptocurrencies are better treated as speculative assets or hedging assets rather than a medium of exchange due to their volatility, which is significantly higher than that of fiat currencies [3–5]. Another type of cryptocurrency, known as “Stablecoins,” is pegged to fiat currencies. Stablecoins allow users to use the blockchain technology without being exposed to the extreme market risk of cryptocurrencies [6–8].

Cryptocurrencies are known to possess a high level of market risk, but during the time frame of our study, the risk was even greater. The COVID-19 pandemic has had a significant impact on the global economy [9–12], resulting in a stock market crash [13,14],

leading to an influx of funds and increased attention from investors as they seek alternative investments [15,16]. There is evidence from BTC's market capitalization which has reached its peak of USD 1.28 trillion on 9 November 2021. It was approximately 10 times higher than its value on 31 March 2020 (USD 117.15 billion). Despite of receiving a large amount of fund flow, the study results from Lahmiri and Bekiros [17] and Vojtko and Cisár [4] suggest that the cryptocurrency market exhibits a higher level of risk due to the pandemic. Additionally, the escalating conflict between Russia and Ukraine has added further stress to the cryptocurrency market, resulting in more volatility [18]. Khalfaoui et al. [19] explained that there were sell-offs from large holders in response to the war which resulted in a decline in prices. Furthermore, there have been rapid changes in monetary policies in the developed economies, for instance, the FED interest rates raise from 0–0.25% in March 2020 to 4.25–4.5% in December 2022 [20], which have also had an impact on the cryptocurrency market [21,22]. Moreover, the study results of Yu and Chen [23] indicate that China's regulatory ban in cryptocurrency mining and transactions in May 2021 has resulted in an amplification of the volatility spillover effects among cryptocurrencies. Additionally, it has been suggested that there was a collapse of speculative bubbles in the cryptocurrency market during the period of our study [24–27]. Bazán-Palomino [24] discovered that the collapse of the bubble resulted in a higher market volatility and expected shortfall. During this period, the market stress has led to a maximum drawdown of BTC at −76.71%. In such period of market stress, risk modeling is crucial for making investment decisions; the value-at-risk (VaR) model is one of the risk measurement tools that can be applied to multiple types of assets. The concept of VaR has existed since 1888 [28] and has continually evolved to improve its performance when applied to more complex or specific investments [29].

The market stress of cryptocurrencies may result in the loss of accuracy of VaR models, which have been criticized for their usage in times of financial turmoil. Therefore, in this paper, we will evaluate the efficiency of the Historical Simulation VaR (HS VaR), Delta Normal VaR (DN VaR), and the Monte Carlo Simulation VaR (MC VaR) when they are applied to the top five cryptocurrencies: BTC, ETH, BNB, ADA, and XRP. The time horizon of analysis is between 31 March 2020 and 25 December 2022, a period during which the cryptocurrency market experienced a large inflow of funds from investors and market stress. It should be noted that USDT is not included in the study; we will focus on altcoins that are not stablecoins, as they exhibit a higher degree of volatility subjected to the U.S. dollar. We will compare the accuracy of the models by backtesting them with statistical significance levels of 99%, 95%, and 90%, respectively. Additionally, the VaR models will be applied at the same confidence levels of 99%, 95%, and 90%. We will use Kupiec's POF and Kupiec's TUFF tests to examine the accuracy of the models by considering the failure rates, and also test the robustness using the Independence test. Lastly, we will combine Kupiec's POF test and the Independence test for a conditional coverage test, which is Christoffersen's Interval Forecast test.

In the next section, we will provide a comprehensive overview of existing literature related to cryptocurrencies' market risk, the market stress during the time horizon of our research, and the underlying blockchain mechanisms of each cryptocurrency. Following, the next section, we provide a detailed explanation of the data which are used in this paper, the VaR models, and backtesting methodology. Lastly, we discuss and summarize the main findings and the managerial and theoretical implications of this paper.

2. Literature Review

2.1. Cryptocurrencies' Risk and VaR Model

It is well established that cryptocurrencies possess a high level of volatility. The value-at-risk (VaR) model is a widely utilized risk measurement tool in the financial industry. There have been previous studies on the application of VaR models as a risk measurement tool for cryptocurrencies. Likitratcharoen et al. [30] determined that the HS VaR and DN VaR can be applied to the cryptocurrency market. During the COVID-19 pandemic, Likitratcharoen et al. [31] examined the VaR models in the Bitcoin market and found that

these models effectively captured the potential adverse losses of BTC, particularly at a 99% VaR confidence level, despite the impact of COVID-19 on both stock markets and the cryptocurrency market [32,33]. However, the study results of Mavani [34] and Kourouma et al. [35] show that VaR models are unfit in times of financial crisis.

Jorion [36] and Holton [29] defined VaR as the worst expected loss over a given time horizon under normal market conditions at a given confidence level. It has become a commonly employed method for managing risks among financial institutions. Mathematically, VaR can be represented as:

$$\text{VaR}_\alpha(X) = -\inf\{x \in \mathbb{R} : F_X(x) > \alpha\} \quad (1)$$

where X is a distribution of profit and loss of investment.

2.2. A Cryptocurrency Market Stress

During the time frame of our examination, the COVID-19 pandemic led to a global economic recession and a decrease in the stock market [10,13,14]. Additionally, during this period, cryptocurrencies displayed heightened volatility while outperforming traditional assets [17,33,37]. These fluctuations were also influenced by spillover effects from other financial markets. Akylidirim et al. [38] found evidence of volatility spillover from financial markets during this time of market stress. Similarly, Özdemir [39] discovered that there was volatility spillover among the cryptocurrency market and that the VaR and cVaR values were greater than those of the SandP500 and SSE index during this period. Similar findings from Doumenis et al. [40] found that Bitcoin's volatility was significantly higher than SandP500, gold, and treasury. Furthermore, the uncertainty caused by the pandemic reduced the ability of cryptocurrencies to act as hedges [4,41–44]. The findings of this study are consistent with those of Beneki [45], who determined that high levels of uncertainty may decrease the hedging potential of cryptocurrencies. However, the outbreak also had a positive impact by increasing the market efficiency of cryptocurrencies, as they received more attention from investors during this period [15,16,46]. Following the pandemic, the conflict between Russia and Ukraine escalated. Khalfaoui et al. [19] indicated that the conflict had a causal effect on the cryptocurrency market. The conflict also impacts the liquidity of cryptocurrencies; Theiri et al. [47] and Appiah-Otoo [48] found that the conflict decreased the liquidity of the cryptocurrency market. Furthermore, uncertainties in government monetary policies also contributed to increased risks for cryptocurrencies. Haq et al. [41], Cheng and Yen [49], and Nguyen et al. [50] have determined that economic policy uncertainties have a significant impact on the cryptocurrency market. In this period, Aboura [21] found that changes in US interest rates also influence the cryptocurrency market. Jarboui and Mnif [22] have concluded that the FOMC meetings during this period of market stress had a negative effect on the cryptocurrency market. The results cohere with those of Mužić and Gržeta [51], which indicate that policies announcement effects the volatility of Bitcoin significantly. In addition to macroeconomic factors, Bazán-Palomino [24], Naeem et al. [25], Taskinsoy [26], and Haykir and Yagil [27] suggested that there was a collapse of the speculative bubbles in the cryptocurrency market. Furthermore, the collapse of Terra-Luna has caused further market stress by increasing negative sentiment in the market [52].

2.3. General Characteristics of Cryptocurrencies Market

The first cryptocurrency which was ever created to decentralize financial transactions is Bitcoin [53,54]. Despite its initial purpose of decentralizing financial transactions, cryptocurrencies do not exhibit characteristics of a currency due to its extreme volatility in comparison to other assets [55,56]. As a result, it is considered a speculative asset rather than a medium of exchange or a means of value storage such as precious metal commodities [3–5]. Additionally, Alshamsi and Andras [57] found that a lack of understanding among users regarding the mechanism of cryptocurrencies may discourage its use as a payment method. However, various research has also shown that cryptocurrencies can

serve as hedging assets. Bouri et al. [58] found that BTC can act as a hedge against stock market fluctuations. Similar findings from Bouri et al. [59] stated that BTC has safe-haven properties for Asian stocks. Fang et al. [60] discovered that the volatility of cryptocurrencies is negatively impacted by NVIX. Despite its potential as a portfolio diversifier, it should be noted that there is a positive correlation between cryptocurrencies [55,61].

Multiple studies have demonstrated that cryptocurrencies lack intrinsic value and are susceptible to speculative bubbles which contradicts the efficient market hypothesis [53,62–64]. Moreover, Almeida and Gonçalves [65] indicate that cryptocurrencies' risk is predictable using machine learning techniques. Jiang et al. [66] also found that the risk of cryptocurrencies is predictable using the ARFIMA-FIGARCH model. However, it has been argued that they can be valued. Liu and Tsyvinski [67] determined that the underlying blockchain technology could potentially play a role in determining the price of a cryptocurrency. Dey et al. [68] found that the number of transactions can affect the value of a cryptocurrency. Additionally, it has been suggested that cryptocurrencies can serve as a form of value storage, such as precious metals commodities. Bianchi [69] observed a positive correlation between gold and cryptocurrencies. Ciaian et al. [70] determined that the price formation of BTC can be explained through a standard economic model of currency price formation. Hayes [71] found that the production cost is a driving factor in the market price of cryptocurrencies.

2.4. Bitcoin: The New Order of the Financial World

Bitcoin (BTC) was first introduced to the world by Satoshi Nakamoto in 2008. It was designed with a decentralized concept in mind [1,54]. Blockchain technology plays a crucial role in its transactions, where financial institutions do not control the process but rather participants within the chains approve and validate the transactions and are rewarded with Bitcoin [72]. The value of Bitcoin is determined by factors such as demand, supply, and scarcity mechanisms. Unlike fiat currencies, which are backed by precious metal commodities or US dollars, the value of Bitcoin is not based on any assets; thus, its value should take into account the production cost [53,71]. Despite this, the valuation of Bitcoin remains unclear, and developers continue to introduce new cryptocurrencies to the market utilizing this concept.

2.5. Ethereum: The Smart Contract Technology Pioneer

Ethereum operates similarly to Bitcoin by utilizing a decentralized system of participants monitoring transactions rather than a centralized server. This network allows for the use of smart contract technology, enabling developers to create decentralized applications under their own rules within the blockchain [73]. The utilization of smart contracts, which are agreements written in programming code, ensures the safety, traceability, and irreversibility of transactions. Additionally, Ethereum has various other applications, such as the storage of non-fungible tokens and trading of cryptocurrencies [74–77]. However, it does have certain drawbacks, such as a high transaction fee resulting in limited scalability for both users and miners. Users of the Ethereum blockchain pay transaction fees in “Ether” or “ETH”, which is referred to as “Gas” [78]. In recent times, Ethereum has transitioned its mechanism from proof-of-work (PoW) to proof-of-stake (PoS) in order to remain competitive [73].

2.6. Binance Coin: Funding of The World's Largest Cryptocurrencies Platform

Binance is widely recognized as the leading cryptocurrency trading platform globally. The platform acquired its funding through the issuance of Binance Coin (BNB) via an initial coin offering (ICO) that was conducted in accordance with the Ethereum blockchain and the ERC-20 token standard, with a maximum supply of 200 million tokens [79,80].

2.7. Cardano: The World's First Proof of Stake Protocol

Cardano (ADA) was developed to be the third generation of blockchain projects. Cardano intends to solve the scalability problem of Bitcoin and Ethereum through a new

mechanism which is the proof-of-stake (PoS) mechanism that reduces energy costs while providing standard blockchain security. The mechanism solves the scalability issue by addressing mining power to the number of coins held by a miner [81–83].

2.8. Ripple: A New Global Decentralized Currency System

Ripple (XRP) was created with the goal of becoming a decentralized, cross-border currency for peer-to-peer transactions [84]. Currently, it also functions as an interbank payment service for financial institutions [85,86]. The network of Ripple is the XRP Ledger which is secured and verified by the participants of the network [85,87,88]. However, during the period of our study, Ripple Labs, the creator of XRP, was facing legal challenges. On 23 December 2020, the Securities and Exchange Commission (SEC) filed a lawsuit against Ripple Labs Inc. and its two executives, accusing them of conducting unregistered securities sales through the distribution of XRP [89]. This event resulted in a significant drop in XRP's market value [90,91]. This drastic decline can cause VaR models to lose their accuracy and robustness.

3. Data and Methodology

3.1. Descriptive Statistics

In this section, we provide descriptive statistics to better understand the behavior of these selected cryptocurrencies. The time horizon of this study is from 31 March 2020 to 25 December 2022, when the cryptocurrency market obtained a large fund flow from investors followed by an extreme bearish period of the cryptocurrency market. Therefore, the cryptocurrencies have sharply risen, as visualized in Figure 1. In this paper, we collected price data of BTC, ETH, BNB, ADA, and XRP. These prices will be calculated by a logarithmic return to examine their risks with VaR models.

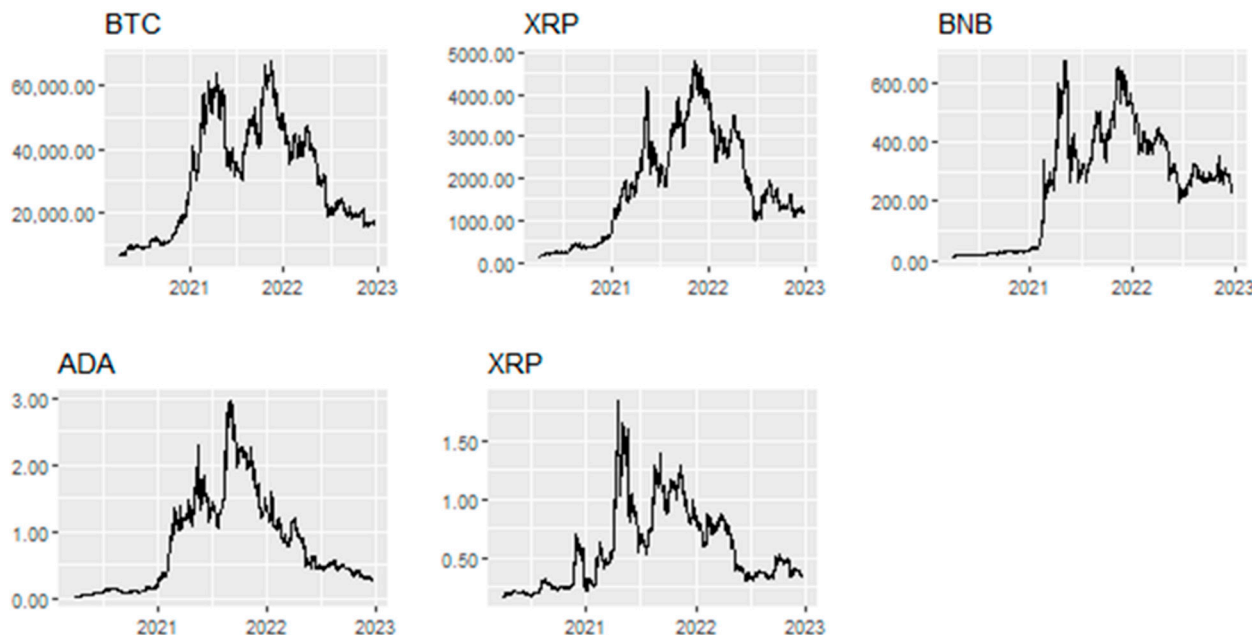


Figure 1. Price chart of BTC, ETH, BNB, ADA, and XRP 31, March 2020 to 25 December 2022.

Figure 2 and Table 1 show the histogram and the descriptive statistical information of each cryptocurrency's daily return. The shape of each return of cryptocurrency is described by Jarque-Bera statistics, which indicates that the data is not normally distributed. The daily returns were calculated using the logarithmic method which can be written as:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (2)$$

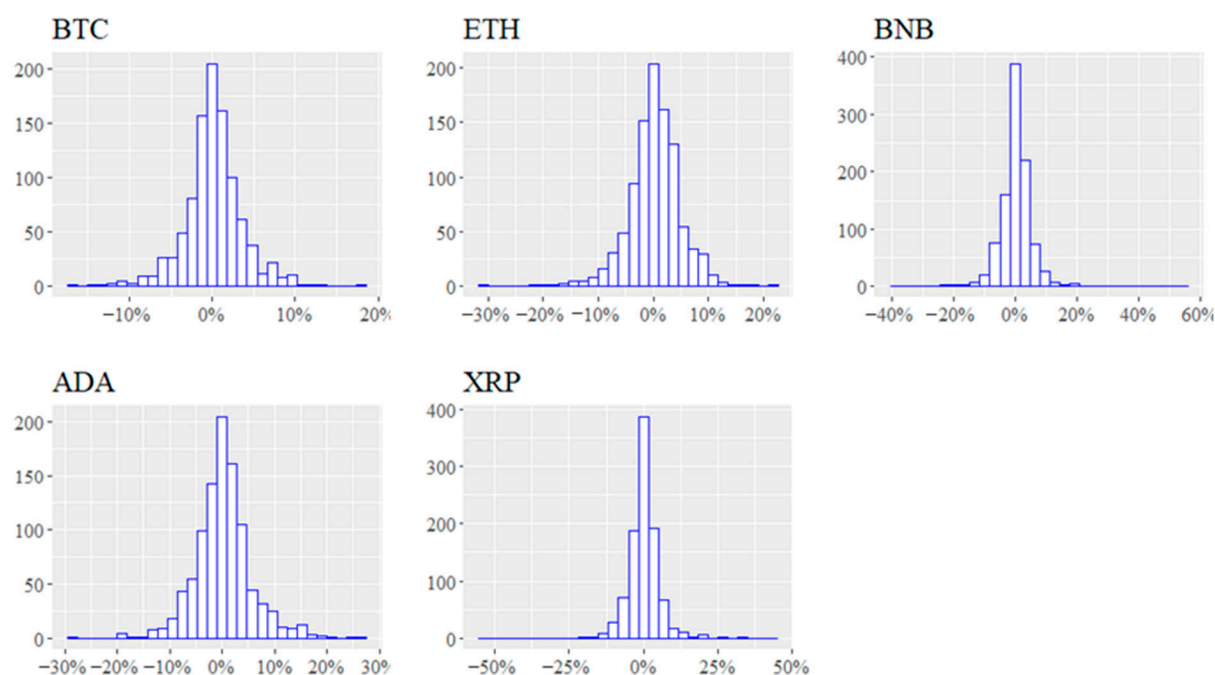


Figure 2. Histogram of daily returns between 31 March 2020 and 25 December 2022.

Table 1. Descriptive statistics of daily returns between 31 March 2020 and 25 December 2022.

Descriptive Statistics	BTC	ETH	BNB	ADA	XRP
Mean	0.10%	0.22%	0.31%	0.22%	0.08%
SD	3.61%	5.11%	6.27%	6.27%	6.21%
Skewness	−0.22	−0.38	0.83	0.27	0.07
Kurtosis	2.91	3.54	16.4	2.64	14.18
Jarque-Bera Test	800.46	771	30,545	597.35	33,330

In Equation (2), R_t denotes the daily logarithmic returns of an asset; P_t denotes the price in the day t .

3.2. Historical Simulation VaR Model

The Historical Simulation value-at-risk (HS VaR) model is a non-parametric approach that utilizes historical return data to calculate VaR values at a given confidence level. This model assumes that the behavior of future returns will be replicated by historical data without relying on any statistical parameters or assumptions about the distribution of returns [92]. The VaR value is defined by the equation:

$$\text{VaR}_{t+1}^{1-\alpha} = Q^{\alpha}(\{R_{it}\}_{t=1}^n) \quad (3)$$

where:

$Q^{\alpha}(\{R_{it}\}_{t=1}^n)$ = a quantile at α of $\{R_{it}\}_{t=1}^n$;

$\{R_{it}\}_{t=1}^n$ = a return of asset i when the time equals t between $t = 1$ to n .

This advantage of this model is that it is flexible through various shapes of distribution. However, it requires a large set of data to be accurate.

3.3. Delta Normal VaR Model

The Delta Normal value-at-risk (DN VaR) is a parametric method that relies on the assumption that the returns of a specific portfolio or asset are normally distributed. This approach is suitable for assessing the risk of linear instruments or portfolios composed of simple linear instruments, as the calculation is based on the normal distribution and

the exposure is linear. However, its accuracy may be limited when applied to non-linear instruments, such as options [93,94]. The formula for the DN VaR can be mathematically represented as [31]:

$$\text{VaR}_{1-\alpha} = m + Z_{\alpha} S \quad (4)$$

where:

m = the average logarithmic historical return given an interval;

Z_{α} = the standardized score of normal distribution at α ;

S = the standard deviation of logarithmic returns of an investment.

3.4. Monte Carlo Simulation VaR Model

The Monte Carlo Simulation value-at-risk (MC VaR) method, on the other hand, assumes that movements in risk factors are generated through drawings from a pre-specified distribution. This approach involves generating pseudo-random numbers from a specified distribution, which are then used to calculate VaR. The MC VaR method is the most flexible, but it also incurs a high computational burden [94]. In this paper, we perform Monte Carlo simulation $N = 1,000,000$ times to approximate the central limit theorem. It is assumed that the behavior of price, S_t , at time t follows a geometric Brownian motion with constant μ and σ , and the process is described as follows:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (5)$$

where μ is drift, σ is volatility, and $W_t \sim N(0, t)$ is a standard Wiener process which is normal distribution with mean 0 and variance t . This equation has been found to perform well under both real-world and risk-neutral measures. The choice between the two measures depends on the objective of the VaR calculation, whether to reflect real market conditions or a risk-neutral perspective. In this paper, the model will be presented using the risk-neutral measure. As a result, Equation (5) is suitable for use in the Monte Carlo simulation and a price return process can be derived using Itô's Lemma as follows:

$$d\ln S_t = \left(\mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t \quad (6)$$

where $W_t = \sqrt{t}Z_t$ and $Z_t \sim N(0, 1)$ is a standard normal distribution. We use the mathematical programming language MATLAB for the simulation.

Equation (5) presumes a fixed volatility, which does not correspond to the realities of financial markets. For instance, cryptocurrency markets are notorious for their high volatility and can experience rapid fluctuations due to a multitude of factors. To account for this, the constant volatility assumption in Equation (5) can be relaxed and replaced with models such as GARCH or stochastic volatility models, such as the Hull and White model [95] or the CIR model [96] introduced by Heston [97], which capture the underlying price's stochastic volatility. Nevertheless, the scope of this paper does not encompass such models and merely compares three models. As a result, this paper will assume a constant volatility in Equation (5).

3.5. Backtesting Methodology

An efficient VaR model must be capable of accurately predicting potential losses and maintain its robustness over varying periods of time. To assess the efficiency of a model, backtesting must be employed. Zhang and Nadarajah [98] have categorized backtesting methods into 4 categories, including unconditional tests, conditional tests, independence property tests, and other methods.

The unconditional method focuses on the violations to determine accuracy or adequacy of a VaR model, but it cannot detect patterns in its performance. Hence, testing the independence properties of a VaR model's performance is crucial as a robust model should perform equally throughout the time horizon. Therefore, in this study, both the

independence property approach and conditional approach are utilized to further evaluate the efficiency of VaR models.

This paper employs the unconditional test method including Kupiec's POF test and Kupiec's TUFF test. For independence property testing, the Independence test [99] is used to test the robustness of the VaR models. Finally, the Kupiec's POF test and the Independence test are combined for the Christoffersen's Interval Forecast test, which is a conditional coverage test that examines the accuracy and robustness of the VaR models simultaneously.

3.5.1. Kupiec's POF Test

Kupiec's POF test is also known as the proportion of failure test. It is applied to VaR models to test the equilibrium between the realized proportion of failure and the suggested proportion of failure for each confidence level. If the realized number of failures is significantly different from the suggested number of failures, the null hypothesis would be rejected. If the test results are accepted, it indicates that the model is accurate [100,101]. On the other hand, if the test is accepted, then it means that the VaR can predict losses accurately. The calculated likelihood ratio test statistics will be tested by Chi-squared values with a degree of freedom of one. The null hypothesis of the test can be written by

$$H_0 : p = \hat{p}$$

where p is the proportion of failures suggested by a certain confidence level and $\hat{p}$ is the observed failure rate. A likelihood ratio of the proportion of failure test statistics can be written as

$$LR_{POF} = -2 \ln \left(\frac{(1-p)^{n-x} p^x}{\left(1 - \left(\frac{x}{n}\right)\right)^{n-x} \left(\frac{x}{n}\right)^x} \right) \quad (7)$$

where n is the number of observations, and x is the number of realized exceptions.

3.5.2. Kupiec's TUFF Test

Kupiec's TUFF test, also known as the time until first failure test, measures the amount of time required for the first exception to occur. This test hypothesizes that the suggested failure rate is equal to the reciprocal of the time until the first exception [101]. If the first error occurs prematurely, the test would reject the model, indicating that the model underestimates risk. However, this test only considers the number of violations and ignores the temporal dynamics of violations. Furthermore, the test has limited ability in identifying poor VaR models [98]. The hypothesis is formulated as follows:

$$H_0 : p = \frac{1}{v}$$

where p is the suggested failure rate and v is the time until the first exception. The likelihood ratio of Kupiec's TUFF test failure can be written as

$$LR_{TUFF} = -2 \ln \left(\frac{p(1-p)^{v-1}}{\left(\frac{1}{v}\right) \left(1 - \frac{1}{v}\right)^{v-1}} \right). \quad (8)$$

3.5.3. Independence Test

The Independence test will detect the patterns of VaR violations. A robust VaR model should not possess any pattern of violations or a clustering violation. Instead, it should be able to adapt in a timely manner to the changes in risk levels and produce no time-dependent violations [99]. To perform the test, first, an indicator function is set up where if an exception occurs in the day, then the value will equal 1. If there is no exception, then the function will return 0. If the test resulted in a rejection, then it means these exceptions

depended on the previous occurrences. Haas [102] argued that the result of this test is weakly produced since it has limited power against general forms of time dependence in violations. The indicator function is expressed as:

$$LR_{ind} = \begin{cases} 0 & \text{if } VaR_{\alpha} \text{ is not breached,} \\ 1 & \text{otherwise} \end{cases} \quad (9)$$

Next, we construct the LR_{ind} table with the 2 consecutive days. Since there are only 0s and 1s for each day as shown in Equation (9), the results of 2 consecutive days will only include 00, 01, 10, and 11 (Table 2).

Table 2. Contingency Table of Independence Test Indicator [101].

	$LR_{ind,t-1}=0$	$LR_{ind,t-1}=1$	
$LR_{ind,t}=0$	n_{00}	n_{10}	$n_{00}+n_{10}$
$LR_{ind,t}=1$	n_{01}	n_{11}	$n_{01}+n_{11}$
	$n_{00}+n_{01}$	$n_{10}+n_{11}$	N

Where $n_{LR_{ind,t-1}, LR_{ind,t}}$ = numbers of days when two conditions are met for day t and day $t-1$.

The Independence test statistic can be written as

$$LR_M = -2 \ln \left(\frac{(1-\pi)^{n_{00}+n_{01}} \pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}} \pi_0^{n_{01}} (1-\pi_1)^{n_{10}} \pi_1^{n_{11}}} \right) \quad (10)$$

where $\pi_0 = \frac{n_{01}}{n_{00}+n_{01}}$, $\pi_1 = \frac{n_{11}}{n_{10}+n_{11}}$, and $\pi = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}}$.

3.5.4. Christoffersen's Interval Forecast Test

Christoffersen's Interval Forecast test, also referred to as the joint test, is a combination of Kupiec's POF test and the Independence test. This test not only measures the correct failure rate but also evaluates the independence of violations. The joint test conditionally assesses the accuracy of a VaR model. However, it has limited capacity to detect a VaR model that violates only one of the two properties. If one of the properties is satisfied, the joint test faces difficulty in identifying the inadequacy of the VaR measure. This test combines LR_{POF} and LR_M for LR_{CC} and tests it with Chi-squared critical value with two degrees of freedom [103].

$$LR_{CC} = LR_{POF} + LR_M \quad (11)$$

4. Study Results

4.1. HS VaR Model

The noise of each cryptocurrency's return and three of the VaR confidence levels can be observed in Figure 3. The chart illustrates that when the noise falls below the VaR lines, it will result in a VaR exception on that day. The number of exceptions is then used to determine a proportion, which is compared to a theoretical suggested number of failures. These calculations are reflected in the results of the Kupiec's POF test, presented in Table 3.

Table 3 illustrates that the null hypothesis of Kupiec's POF test was accepted for the HS VaR model for all the sample cryptocurrencies. This suggests that the HS VaR model can accurately predict extreme losses, as the observed failure rate is not statistically different from the suggested failure rate for each level of confidence. The results of the Kupiec's TUFF test, presented in Table 4, are consistent with those of the POF test and indicate that the HS VaR model is highly accurate in predicting losses. However, it should be noted that this test has limitations as it disregards a considerable amount of information. The Independence test results, displayed in Table 5, indicate that overall, the VaR model is robust during this market stress. However, there are seven rejections for XRP, which implies that the failures from the VaR model are dependent on each other and that the VaR model has produced clustering violations. Finally, the results of the Christoffersen's

interval forecast test, presented in Table 6, majorly imply that the model is accurate and robust for measuring the risk of cryptocurrencies. However, it should be noted that the model may lose its robustness when applied to XRP due to its extreme decline during the period under examination. Overall, the results from these tests suggest that the HS VaR model is appropriate during market stress, as it demonstrates efficiency in terms of accuracy and robustness.

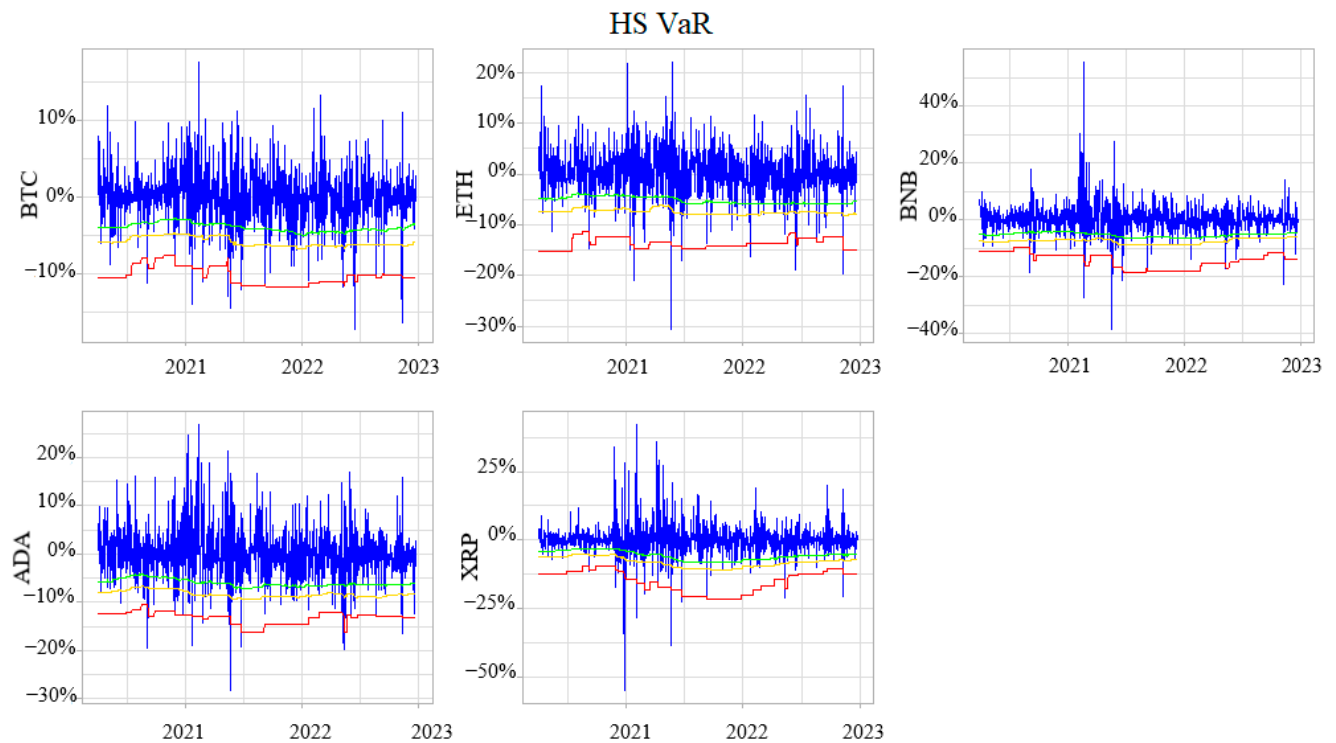


Figure 3. HS VaR of BTC, ETH, BNB, ADA, and XRP.

Table 3. Kupiec's POF test of HS VaR.

Kupiec's POF Test								
Cryptocurrency	Numbers of Observations	Confidence Level	Expected Number of Exceptions	Realized Number of Exceptions	Test Statistics LR_{POF}	Test Results		
						99% Significance	95% Significance	90% Significance
Bitcoin (BTC)	1000	99%	10	14	1.4374	Accept	Accept	Accept
	1000	95%	50	51	0.0209	Accept	Accept	Accept
	1000	90%	100	100	0.0000	Accept	Accept	Accept
Ethereum (ETH)	1000	99%	10	12	0.3798	Accept	Accept	Accept
	1000	95%	50	52	0.0832	Accept	Accept	Accept
	1000	90%	100	106	0.3931	Accept	Accept	Accept
Binance Coin (BNB)	1000	99%	10	11	0.0978	Accept	Accept	Accept
	1000	95%	50	45	0.5438	Accept	Accept	Accept
	1000	90%	100	96	0.1799	Accept	Accept	Accept
Cardano (ADA)	1000	99%	10	15	2.1892	Accept	Accept	Accept
	1000	95%	50	52	0.0832	Accept	Accept	Accept
	1000	90%	100	104	0.1757	Accept	Accept	Accept
Ripple (XRP)	1000	99%	10	14	1.4374	Accept	Accept	Accept
	1000	95%	50	51	0.0209	Accept	Accept	Accept
	1000	90%	100	106	0.3931	Accept	Accept	Accept

Table 4. Kupiec’s TUFF test of HS VaR.

Kupiec’s TUFF Test							
Cryptocurrency	Numbers of Observations	Confidence Level	Time Until First Failure	Test Statistics LR_{TUFF}	Test Results		
					99% Significance	95% Significance	90% Significance
Bitcoin (BTC)	1000	99%	46	0.4795	Accept	Accept	Accept
	1000	95%	46	0.9725	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept
Ethereum (ETH)	1000	99%	46	0.4795	Accept	Accept	Accept
	1000	95%	9	0.5332	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept
Binance Coin (BNB)	1000	99%	46	0.4795	Accept	Accept	Accept
	1000	95%	9	0.5332	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept
Cardano (ADA)	1000	99%	46	0.4795	Accept	Accept	Accept
	1000	95%	9	0.5332	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept
Ripple (XRP)	1000	99%	46	0.4795	Accept	Accept	Accept
	1000	95%	46	0.9725	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept

Table 5. Independence test of HS VaR.

Independence Test							
Cryptocurrency	Numbers of Observations	Confidence Level	Realized Number of Exceptions	Test Statistics LR_M	Test Results		
					99% Significance	95% Significance	90% Significance
Bitcoin (BTC)	1000	99%	14	1.7458	Accept	Accept	Accept
	1000	95%	51	0.7259	Accept	Accept	Accept
	1000	90%	100	0.0000	Accept	Accept	Accept
Ethereum (ETH)	1000	99%	12	2.2896	Accept	Accept	Accept
	1000	95%	52	0.6082	Accept	Accept	Accept
	1000	90%	106	0.7956	Accept	Accept	Accept
Binance Coin (BNB)	1000	99%	11	0.2449	Accept	Accept	Accept
	1000	95%	45	0.0004	Accept	Accept	Accept
	1000	90%	96	0.3984	Accept	Accept	Accept
Cardano (ADA)	1000	99%	15	0.4573	Accept	Accept	Accept
	1000	95%	52	0.0343	Accept	Accept	Accept
	1000	90%	104	0.1541	Accept	Accept	Accept
Ripple (XRP)	1000	99%	14	6.1763	Accept	Reject	Reject
	1000	95%	51	14.5673	Reject	Reject	Reject
	1000	90%	106	4.4666	Accept	Reject	Reject

4.2. DN VaR Model

Figure 4 visualizes the DN VaR and the daily returns of each cryptocurrency. Overall, the test results of DN VaR show that the model provided incorrect failure rates during the market stress. However, the model provides similar robustness to the HS VaR model.

Table 7 illustrates the results of the Kupiec’s POF test for the DN VaR model when applied to each cryptocurrency. It can be observed that the model largely provides inaccurate failure rates when predicting daily extreme losses in the cryptocurrency market, as evidenced by the 29 rejections and 16 acceptances. Based on the number of exceptions, it can be inferred that the model primarily overestimates the risk at 95% and 90% confidence levels. However, at 99%, the market risk is slightly underestimated for BTC and ETH. The next test applied in this study is the Kupiec’s TUFF test, which focuses on the time until the first failure. The results of this test can be viewed in Table 8. These results contradict those of the Kupiec’s POF test, suggesting that the model mostly has an accurate failure rate as the first failure from the model does not occur too prematurely or too delayed. The results of the Independence

test, found in Table 9, indicate that the violations of the model are independent to one another, indicating that the model still maintains its robustness. For further results, the Christoffersen's interval forecast test is applied, the results of which can be seen in Table 10. The null hypothesis of the test was mostly rejected during the market stress, indicating that the market stress has led to a lower efficiency for the VaR model. In conclusion, the DN VaR model loss some accuracy due to the market stress but the model still maintains its robustness. The overestimation of the risk provides adequate VaR measure, which can benefit conservative risk management strategies and capital reserves planning.

Table 6. Christoffersen's interval forecast test of HS VaR.

Cryptocurrency	Numbers of Observations	Confidence Level	Christoffersen's Interval Forecast Test					
			Test Statistics LR _{POF}	Test Statistics LR _M	Test Statistics LR _{CC}	99% Significance	Test Results 95% Significance	90% Significance
Bitcoin (BTC)	1000	99%	1.4374	1.7458	3.1832	Accept	Accept	Accept
	1000	95%	0.0209	0.7259	0.7468	Accept	Accept	Accept
	1000	90%	0.0000	0.0000	0.0000	Accept	Accept	Accept
Ethereum (ETH)	1000	99%	0.3798	2.2896	2.6693	Accept	Accept	Accept
	1000	95%	0.0832	0.6082	0.6914	Accept	Accept	Accept
	1000	90%	0.3931	0.7956	1.1887	Accept	Accept	Accept
Binance Coin (BNB)	1000	99%	0.0978	0.2449	0.3428	Accept	Accept	Accept
	1000	95%	0.5438	0.0004	0.5442	Accept	Accept	Accept
	1000	90%	0.1799	0.3984	0.5783	Accept	Accept	Accept
Cardano (ADA)	1000	99%	2.1892	0.4573	2.6466	Accept	Accept	Accept
	1000	95%	0.0832	0.0343	0.1175	Accept	Accept	Accept
	1000	90%	0.1757	0.1541	0.3299	Accept	Accept	Accept
Ripple (XRP)	1000	99%	1.4374	6.1763	7.6137	Accept	Reject	Reject
	1000	95%	0.0209	14.5673	14.5882	Reject	Reject	Reject
	1000	90%	0.3931	4.4666	4.8597	Accept	Accept	Reject

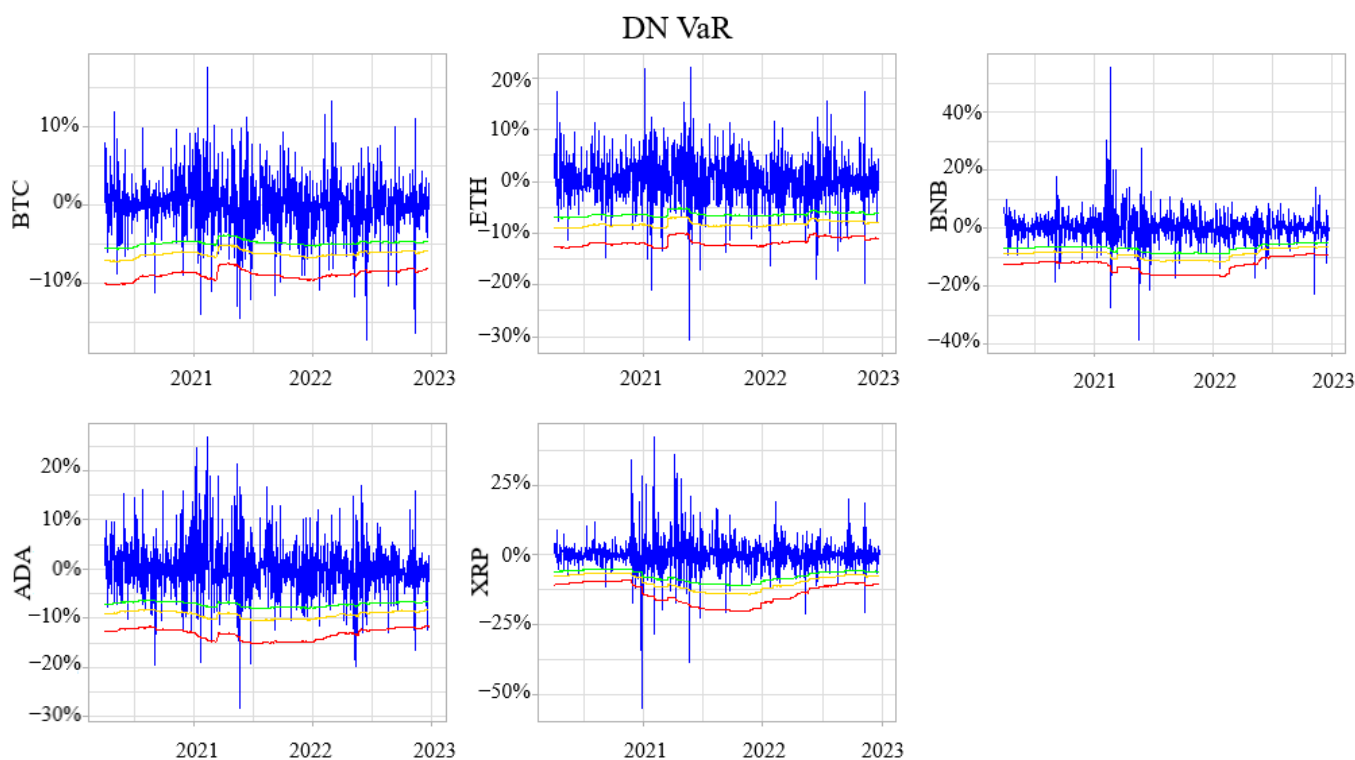


Figure 4. DN VaR of BTC, ETH, BNB, ADA, and XRP.

Table 7. Kupiec’s POF test of DN VaR.

Kupiec’s POF Test								
Cryptocurrency	Numbers of Observations	Confidence Level	Expected Number of Exceptions	Realized Number of Exceptions	Test Statistics LR _{POF}	Test Results		
						99% Significance	95% Significance	90% Significance
Bitcoin (BTC)	1000	99%	10	17	4.0910	Accept	Reject	Reject
	1000	95%	50	46	0.3457	Accept	Accept	Accept
	1000	90%	100	71	10.2909	Reject	Reject	Reject
Ethereum (ETH)	1000	99%	10	19	6.4725	Accept	Reject	Reject
	1000	95%	50	38	3.2937	Accept	Accept	Reject
	1000	90%	100	74	8.1804	Reject	Reject	Reject
Binance Coin (BNB)	1000	99%	10	14	1.4374	Accept	Accept	Accept
	1000	95%	50	27	13.2784	Reject	Reject	Reject
	1000	90%	100	49	34.9286	Reject	Reject	Reject
Cardano (ADA)	1000	99%	10	14	1.4374	Accept	Accept	Accept
	1000	95%	50	36	4.5530	Accept	Reject	Reject
	1000	90%	100	75	7.5358	Reject	Reject	Reject
Ripple (XRP)	1000	99%	10	16	3.0766	Accept	Accept	Reject
	1000	95%	50	31	8.7393	Reject	Reject	Reject
	1000	90%	100	59	21.5794	Reject	Reject	Reject

Table 8. Kupiec’s TUFF test of DN VaR.

Kupiec’s TUFF Test							
Cryptocurrency	Numbers of Observations	Confidence Level	Time Until First Failure	Test Statistics LR _{TUFF}	Test Results		
					99% Significance	95% Significance	90% Significance
Bitcoin (BTC)	1000	99%	46	0.4795	Accept	Accept	Accept
	1000	95%	46	0.9725	Accept	Accept	Accept
	1000	90%	46	4.4522	Accept	Reject	Reject
Ethereum (ETH)	1000	99%	46	0.4795	Accept	Accept	Accept
	1000	95%	9	0.5332	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept
Binance Coin (BNB)	1000	99%	9	3.0922	Accept	Accept	Reject
	1000	95%	9	0.5332	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept
Cardano (ADA)	1000	99%	9	3.0922	Accept	Accept	Reject
	1000	95%	9	0.5332	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept
Ripple (XRP)	1000	99%	46	0.4795	Accept	Accept	Accept
	1000	95%	46	0.9725	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept

4.3. MC VaR Model

This section shows the study results of the MC VaR model. Figure 5 shows a volatile return or noise of each cryptocurrency compared to the confidence levels of VaR. The number of violations from the MC VaR model is shown in Table 11. The violations can be visualized when the blue lines cross down the green, yellow, and red lines which represent 90%, 95%, and 99% VaR confidence levels.

Table 9. Independence test of DN VaR.

Independence Test							
Cryptocurrency	Numbers of Observations	Confidence Level	Realized Number of Exceptions	Test Statistics LR_M	Test Results		
					99% Significance	95% Significance	90% Significance
Bitcoin (BTC)	1000	99%	17	1.1211	Accept	Accept	Accept
	1000	95%	46	0.7931	Accept	Accept	Accept
	1000	90%	71	0.1987	Accept	Accept	Accept
Ethereum (ETH)	1000	99%	19	0.8032	Accept	Accept	Accept
	1000	95%	38	0.2071	Accept	Accept	Accept
	1000	90%	74	0.0506	Accept	Accept	Accept
Binance Coin (BNB)	1000	99%	14	0.3980	Accept	Accept	Accept
	1000	95%	27	1.5002	Accept	Accept	Accept
	1000	90%	49	0.0790	Accept	Accept	Accept
Cardano (ADA)	1000	99%	14	0.3980	Accept	Accept	Accept
	1000	95%	36	2.6922	Accept	Accept	Accept
	1000	90%	75	1.0511	Accept	Accept	Accept
Ripple (XRP)	1000	99%	16	5.1359	Accept	Reject	Reject
	1000	95%	31	9.5613	Reject	Reject	Reject
	1000	90%	59	3.2041	Accept	Accept	Reject

Table 10. Christoffersen's interval forecast test of DN VaR.

Christoffersen's Interval Forecast Test								
Cryptocurrency	Numbers of Observations	Confidence Level	Test Statistics LR_{POF}	Test Statistics LR_M	Test Statistics LR_{CC}	99% Significance	Test Results	
							95% Significance	90% Significance
Bitcoin (BTC)	1000	99%	4.0910	1.1211	5.2121	Accept	Accept	Reject
	1000	95%	0.3457	0.7931	1.1388	Accept	Accept	Accept
	1000	90%	10.2909	0.1987	10.4897	Reject	Reject	Reject
Ethereum (ETH)	1000	99%	6.4725	0.8032	7.2757	Accept	Reject	Reject
	1000	95%	3.2937	0.2071	3.5008	Accept	Accept	Accept
	1000	90%	8.1804	0.0506	8.2310	Accept	Reject	Reject
Binance Coin (BNB)	1000	99%	1.4374	0.3980	1.8354	Accept	Accept	Accept
	1000	95%	13.2784	1.5002	14.7785	Reject	Reject	Reject
	1000	90%	34.9286	0.0790	35.0076	Reject	Reject	Reject
Cardano (ADA)	1000	99%	1.4374	0.3980	1.8354	Accept	Accept	Accept
	1000	95%	4.5530	2.6922	7.2452	Accept	Reject	Reject
	1000	90%	7.5358	1.0511	8.5869	Accept	Reject	Reject
Ripple (XRP)	1000	99%	3.0766	5.1359	8.2125	Accept	Reject	Reject
	1000	95%	8.7393	9.5613	18.3006	Reject	Reject	Reject
	1000	90%	21.5794	3.2041	24.7835	Reject	Reject	Reject

Table 11 shows Kupiec's POF test of MC VaR for the five cryptocurrencies' daily logarithmic returns. Overall, the MC VaR model has been proven to be applicable with the number of rejections and acceptations of 29 and 16. The results are comparable to the DN VaR model since the daily returns were simulated following a geometric Brownian motion which was prespecified as a normal distribution. The model underestimates the risk for some cryptocurrencies at the 99% confidence level, while mostly overestimating the risk at 95% and 90% confidence levels. As a result, this leads to rejections for each confidence level. Next, we will investigate the VaR model with Kupiec's TUFF test in Table 12. The results of Kupiec's TUFF indicate that the model can predict losses accurately since it the first violation occurred expectedly. However, Kupiec's TUFF test can sometimes be misleading since it ignores a lot of relevant information.

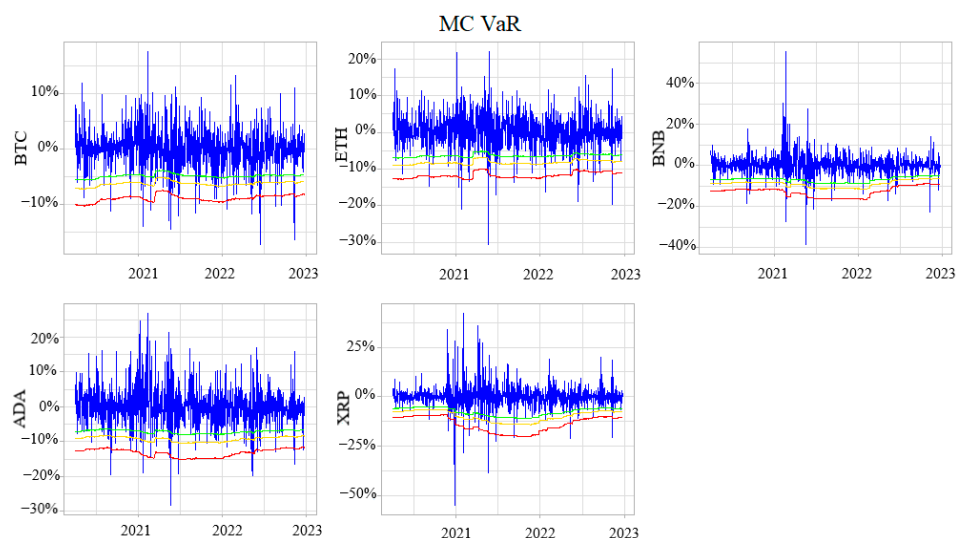


Figure 5. MC VaR of BTC, ETH, BNB, ADA, and XRP.

Table 11. Kupiec's POF Test for MC VaR.

Kupiec's POF Test								
Cryptocurrency	Numbers of Observations	Confidence Level	Expected Number of Exceptions	Realized Number of Exceptions	Test Statistics LR_{POF}	Test Results		
						99% Significance	95% Significance	90% Significance
Bitcoin (BTC)	1000	99%	10	17	4.0910	Accept	Reject	Reject
	1000	95%	50	46	0.3457	Accept	Accept	Accept
	1000	90%	100	71	10.2909	Reject	Reject	Reject
Ethereum (ETH)	1000	99%	10	19	6.4725	Accept	Reject	Reject
	1000	95%	50	38	3.2937	Accept	Accept	Reject
	1000	90%	100	74	8.1804	Reject	Reject	Reject
Binance Coin (BNB)	1000	99%	10	14	1.4374	Accept	Accept	Accept
	1000	95%	50	27	13.2784	Reject	Reject	Reject
	1000	90%	100	49	34.9286	Reject	Reject	Reject
Cardano (ADA)	1000	99%	10	13	0.8306	Accept	Accept	Accept
	1000	95%	50	36	4.5530	Accept	Reject	Reject
	1000	90%	100	75	7.5358	Reject	Reject	Reject
Ripple (XRP)	1000	99%	10	16	3.0766	Accept	Accept	Reject
	1000	95%	50	31	8.7393	Reject	Reject	Reject
	1000	90%	100	59	21.5794	Reject	Reject	Reject

The Independence test results, shown in Table 13, indicate that the failures of the MC VaR model are independent from each other. This suggests that the model still maintains its robustness despite the overestimation of risk. Finally, we applied the Christoffersen's interval forecast test, the results of which can be viewed in Table 14. The test results show that the null hypothesis was mostly rejected, indicating that the MC VaR model was not accurate during market stress and the shocks led to a lower accuracy for the model. In summary, the tests indicate that MC VaR model experiences a reduction in accuracy during market stress. However, the model still shows stability of its performance. Furthermore, it mainly provides overestimation, making it suitable for conservative risk management practices.

Table 12. Kupiec’s TUFF test of MC VaR.

Kupiec’s TUFF Test							
Cryptocurrency	Numbers of Observations	Confidence Level	Time Until First Failure	Test Statistics LR_{TUFF}	Test Results		
					99%Significance	95%Significance	90%Significance
Bitcoin (BTC)	1000	99%	46	0.4795	Accept	Accept	Accept
	1000	95%	46	0.9725	Accept	Accept	Accept
	1000	90%	46	4.4522	Accept	Reject	Reject
Ethereum (ETH)	1000	99%	46	0.4795	Accept	Accept	Accept
	1000	95%	9	0.5332	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept
Binance Coin (BNB)	1000	99%	9	3.0922	Accept	Accept	Reject
	1000	95%	9	0.5332	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept
Cardano (ADA)	1000	99%	9	3.0922	Accept	Accept	Reject
	1000	95%	9	0.5332	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept
Ripple (XRP)	1000	99%	46	0.4795	Accept	Accept	Accept
	1000	95%	46	0.9725	Accept	Accept	Accept
	1000	90%	9	0.0120	Accept	Accept	Accept

Table 13. Independence Test of MC VaR.

Independence Test							
Cryptocurrency	Numbers of Observations	Confidence Level	Realized Number of Exceptions	Test Statistics LR_M	Test Results		
					99% Significance	95% Significance	90% Significance
Bitcoin (BTC)	1000	99%	17	1.1211	Accept	Accept	Accept
	1000	95%	46	0.7931	Accept	Accept	Accept
	1000	90%	71	0.1987	Accept	Accept	Accept
Ethereum (ETH)	1000	99%	19	0.8032	Accept	Accept	Accept
	1000	95%	38	0.2071	Accept	Accept	Accept
	1000	90%	74	0.0506	Accept	Accept	Accept
Binance Coin (BNB)	1000	99%	14	0.3980	Accept	Accept	Accept
	1000	95%	27	1.5002	Accept	Accept	Accept
	1000	90%	49	0.0790	Accept	Accept	Accept
Cardano (ADA)	1000	99%	13	0.3428	Accept	Accept	Accept
	1000	95%	36	2.6922	Accept	Accept	Accept
	1000	90%	75	1.0511	Accept	Accept	Accept
Ripple (XRP)	1000	99%	16	5.1359	Accept	Reject	Reject
	1000	95%	31	9.5613	Reject	Reject	Reject
	1000	90%	59	3.2041	Accept	Accept	Reject

Table 14. Christoffersen’s Interval Forecast test of MC VaR.

Christoffersen’s Interval Forecast Test								
Cryptocurrency	Numbers of Observations	Confidence Level	Test Statistics LR_{POF}	Test Statistics LR_M	Test Statistics LR_{CC}	99% Significance	Test Results 95% Significance	90% Significance
Bitcoin (BTC)	1000	99%	4.0910	1.1211	5.2121	Accept	Accept	Reject
	1000	95%	0.3457	0.7931	1.1388	Accept	Accept	Accept
	1000	90%	10.2909	0.1987	10.4897	Reject	Reject	Reject
Ethereum (ETH)	1000	99%	6.4725	0.8032	7.2757	Accept	Reject	Reject
	1000	95%	3.2937	0.2071	3.5008	Accept	Accept	Accept
	1000	90%	8.1804	0.0506	8.2310	Accept	Reject	Reject
Binance Coin (BNB)	1000	99%	1.4374	0.3980	1.8354	Accept	Accept	Accept
	1000	95%	13.2784	1.5002	14.7785	Reject	Reject	Reject
	1000	90%	34.9286	0.0790	35.0076	Reject	Reject	Reject

Table 14. Cont.

Cryptocurrency	Numbers of Observations	Confidence Level	Christoffersen's Interval Forecast Test					
			Test Statistics LR _{POF}	Test Statistics LR _M	Test Statistics LR _{CC}	99% Significance	Test Results 95% Significance	90% Significance
Cardano (ADA)	1000	99%	0.8306	0.3428	1.1734	Accept	Accept	Accept
	1000	95%	4.5530	2.6922	7.2452	Accept	Reject	Reject
	1000	90%	7.5358	1.0511	8.5869	Accept	Reject	Reject
Ripple (XRP)	1000	99%	3.0766	5.1359	8.2125	Accept	Reject	Reject
	1000	95%	8.7393	9.5613	18.3006	Reject	Reject	Reject
	1000	90%	21.5794	3.2041	24.7835	Reject	Reject	Reject

5. Discussion

The results of our study found that the HS VaR model is the most appropriate model for predicting extreme losses in the cryptocurrency market during times of market stress. This is due to its distinct statistical assumptions compared to the DN VaR and MC VaR models, which assume that the daily returns of cryptocurrencies in our study are normally distributed. Our findings are consistent with previous research by Likitracharoen et al. [30] and Likitracharoen et al. [31], who found that the HS VaR model is more suitable for cryptocurrencies.

The results of the Kupiec's POF test indicate that the HS VaR model provides the most accurate failure rate, as it reflects the downside risk of cryptocurrencies without overestimating or underestimating it. On the other hand, the DN VaR and MC VaR models are found to mostly provide incorrect failure rates, tending to overestimate risks and sometimes underestimate risks at the 99% confidence level.

The results of the Kupiec's TUFF test suggest that all three VaR models are usable for risk measurement during market stress in terms of the time until the first failure. However, this test may be limited in its assessment, as it only focuses on the first failure and does not consider additional information.

The Independence test was conducted to analyze the independence properties and adaptability of the models. The results indicate that all three models maintain their robustness during the time horizon of our study. The Christoffersen's interval forecast test further confirms that the HS VaR model is the most appropriate in terms of accuracy and robustness.

6. Conclusions

The market risks of cryptocurrencies have rapidly increased in recent years due to events such as a significant influx of funds and attention, increased volatility, economic policy uncertainties, and the collapse of speculative bubbles. A VaR model may lose its efficiency as a result of the market stress.

Our results provide empirical evidence that the HS VaR model is the most appropriate model for predicting extreme losses in the cryptocurrency market during times of market stress. The results of the Kupiec's POF test, the Kupiec's TUFF test, the Independence test, and the Christoffersen's interval forecast test all indicate that the HS VaR model is more precise and robust compared to the DN VaR and MC VaR models.

For Managerial implications, it is recommended that the HS VaR model should be employed for active risk management strategies during market stress, as the results from unconditional tests indicate that the model consistently provides balanced predictions. On the other hand, the DN VaR and MC VaR models are recommended for conservative risk management strategies, as they mostly provide risk overestimations, but maintain a level of robustness comparable to the HS VaR.

In a theoretical perspective, our results suggest that VaR models should be designed for flexibility to accommodate various distribution shapes as our empirical findings indicate

that the DN VaR and MC VaR models, which are based on normal distributions, produce overestimated values.

Although our findings suggest that the HS VaR model is the most appropriate for cryptocurrency risk measurement during market stress, further research could improve the DN VaR and MC VaR models. It is important to continue evaluating and refining risk measurement models to ensure that they accurately reflect the risk associated with cryptocurrencies.

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