Two-Level Full Factorial Design Approach for the Analysis of Multi-Lane Highway Section under Saturated and Unsaturated Traffic Flow Conditions

Hamad Almujibah 1,*, Afq Khattak 2, Saleh Alotaibi 3, Raed Alahmadi 4, Adil Elhassan 1,5, Abdullah Alshahri 1 and Caroline Mongina Matara 6,7

1 Department of Civil Engineering, College of Engineering, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia
2 College of Transportation Engineering, Tongji University, 4800 Cao’an Highway, Shanghai 201804, China
3 Department of Civil and Environmental Engineering, Faculty of Engineering—Rabigh Branch, King Abdulaziz University, Jeddah 21589, Saudi Arabia
4 Department of Civil Engineering, College of Engineering, Al-Baha University, Al-Baha P.O.Box 1988, Saudi Arabia
5 Department of Architecture Design, College of Architecture and Planning, Sudan University of Science and Technology (SUST), P.O. Box 407, Khartoum 11111, Sudan
6 Department of Civil and Resource Engineering, Technical University of Kenya, Haile Sellasie Avenue, Nairobi P.O. Box 52428-00200, Kenya
7 Department of Civil and Construction Engineering, University of Nairobi, Harry Thuku Road, Nairobi P.O. Box 30197-00100, Kenya

* Correspondence: hmujibah@tu.edu.sa

Abstract: Oversaturation of highways occurs due to their inadequate assessment and design. In this paper, we propose both a mathematical queuing model and a Discrete-Event Simulation (DES) framework based on Newell’s triangular flow-density relationship for the performance analysis of a multi-lane highway section. The proposed framework is a finite capacity queuing system, which captures an increase in the flow with the vehicle density up to the capacity of the section in an unsaturated condition and a decrease in the flow in the case of a saturated condition, depicting the actual traffic conditions on the highway section. First, the Birth–Death Process is used to build the mathematical queuing model (BDP), and the average number of vehicles (average queue length) and blocking probability on the highway section are estimated. Then, the accuracy of the mathematical queuing model is verified by the proposed DES framework. The “significance and effects” of different design factors are evaluated using the two-level full factorial design technique. The analysis of the experimental results reveals that the length of the highway section and the number of lanes are the most significant factors affecting the average queue length and blocking probability, while the jam density only has a significant effect on the average queue length and does not affect the blocking probability. In case of a two-way interaction, the combined effect of the “length-lanes” significantly affects the average queue length. In the end, a multiple-factor linear regression model is also developed for the prediction of the average number of vehicles on the highway section based on the design factors.

Keywords: multi-lane highway; mathematical queuing model; Discrete-Event Simulation; flow–density relationship; two-level full factorial design

1. Introduction

Traffic congestion on a highway network is a condition that appears when the volume of traffic on a highway reaches or exceeds its capacity at a particular segment. The issue of traffic congestion is a real problem with the ever-increasing population and use of vehicles, and it draws the interest of academic researchers and traffic engineers. Due to long traffic
jams and queues, the lack of productive time has an adverse impact on socioeconomic costs. Studying the interactions between cars, drivers, and the environment (roads, traffic control systems, etc.), with the goal of designing the ideal highway network and ensuring efficient traffic flow as well as minimal traffic congestion issues, is a highly challenging problem for researchers. The smooth movements of vehicles, smooth operation of traffic, and safety on the highways are the main concerns affecting the recognition of traffic and the highway system.

Scientific studies started in the 1930s to study traffic issues in order to understand and help in the prevention and resolution of traffic congestion problems. Several speed–density–flow models were developed over the years and categorized as single-regime or multi-regime models. Single-regime models assume a continuous relationship between speed, density, and traffic flow \([1–3]\), while the relationship is discontinuous in multi-regime models depending on the level of density. Similarly, Lighthill and Whitham presented one of the most popular macroscopic traffic models, which was based on the fluid dynamic theory \([4]\). Treating the traffic flow as a 1-D compressible fluid, they studied the traffic jam as a shockwave. Prigogine presented the gas-kinetic model based on the Boltzmann equation \([5]\). Using the premise of a delayed adaptation of velocity, Newell proposed the microscopic, optimum velocity model \([6]\).

Several queuing models were developed by different researchers for the analysis and design of transportation systems due to the inherent characteristics of urban transit systems, namely, the connection between the service facilities (walkways, stairs, ticket machines at transit stations, etc.) and the flow of entities (pedestrians) \([7–15]\). Similarly, Jain and MacGregor Smith \([16]\) established state-dependent queuing models for simulating vehicle traffic flows and revealed that they are more realistic in extremely congested conditions. Afterwards, several other researchers carried out performance assessments of multi-lane highways based on the queuing theory and simulation models \([17–21]\). However, these queuing and simulation modeling models neglected the bottleneck and spillback phenomenon on highways and in transit terminal facilities. In the spillback effect, traffic congestion propagates upstream, which leads to the delay of and speed reduction by passengers as well as vehicular traffic, which is the true saturation condition. Therefore, in this research, we describe the realistic propagation of vehicular traffic congestion by the insertion of different phenomena such as queuing, the spillback effect from downstream traffic, and dissipation along the highway. Here, we develop both a mathematical queuing model and a simulation model, which incorporates Newell’s kinetic wave model \([22]\). Newell’s flow model is basically based on a triangular flow–density \((q – K)\) relationship, which is shown by the fundamental diagram in Figure 1.

![Figure 1. Newell’s flow–density relationship.](image-url)
This basic diagram highlights a few key features, which are the highway capacity \( C_h \) based on the jam density (density occurring at zero speed) \( K_j \), number of lanes \( N \) and length of the highway \( L \), critical density \( K_{cr} \) (the density and speed experienced during peak operations), and maximum flow \( q_{max} \). Within this triangular relationship, there are two traffic conditions, i.e., a congested traffic flow condition and an uncongested traffic flow condition. In the case of the congested traffic condition \( (K < K_{cr}) \), the vehicles’ speed (wave pace) \( V \) is the slope of \( (q - K) \). When the traffic is not congested, low flow densities result in a monotonic increase in the traffic flow. The vehicles continue to move freely until the maximum flow is reached. At a critical density, the traffic state changes from free flow traffic to congested traffic. Beyond that point, in the case of the congested condition \( (K > K_{cr}) \), the space between vehicles becomes less as the density rises, which ultimately slows down the traffic. Since queues take up space, there is spillback in this case, and a bottleneck queue propagates upstream on the highway.

Based on the above analysis, our research is divided into three stages. First, we propose a mathematical queuing model for the vehicular traffic flow on a multi-lane highway section based on Newell’s triangular flow–density relationship to consider both the free-flow and congested conditions. The proposed queuing model is used to estimate the average number of vehicles on the multi-lane highway section. Secondly, for the verification of the accuracy of the proposed mathematical queuing model, a Discrete-Event Simulation (DES) framework is also developed in SimEvents®. Third, in order to assess the “significance and effect” of various design factors, a statistical two-level full factorial design approach is implemented in MINTAB software [23,24]. A regression model is also developed in the end to predict the average number of vehicles on the highway section based on various design factors.

2. Description of Multi-Lane Highway Section as a Queuing System

A section of highway is shown in Figure 2, in which vehicles occupy spaces as they enter the section. The available spaces on the highway section act as “servers”, and they become busy as vehicles enter the multi-lane highway segment.

The occupancy of these spaces causes an increase in the lane density \( K \). When all the vacant spaces are occupied, the vehicles’ movement ceases, and the traffic flow stops. Jam density refers to the lane density associated with the jam condition on a highway segment \( K_j \). If a highway segment has a length \( L \) and number of lanes \( N \), and the jam density

Figure 2. Queuing system representation of multi-lane highway section.
(\(K_i\)) of the section is known, the capacity (\(C_i\)) of the multi-lane highway section can be calculated by using Equation (1). \[ C_h = K_i LN \] (1)

Due to the limited capacity (\(C_i\)) of the highway section, the number of empty spaces reduces as the number of vehicles (\(n\)) increases. The density (\(K\)) increases with the ultimate reduction in vehicular speed (\(V\)). Thus, a multi-lane highway section is described as a state-dependent multi-server finite-capacity queuing system. The vehicular arrival rate can be described by \('X', and the state-dependent service rate of the multi-lane highway section based on the flow–density model is represented by \('Y(n)'\). The available spaces on the multi-lane highway section act as servers. The capacity of the multi-lane highway section is the sum of all the servers, i.e., the number of available spaces. Therefore, the general form to represent a finite-capacity multi-lane highway section is a \(X/Y\) queuing system.


In this section, we discuss the mathematical queuing model’s formulation for a highway section in which the arrival process of vehicles is assumed to be based on Poisson’s distribution, and the state-dependent service process of the highway section is based on exponential distribution. The uni-directional flow of vehicular traffic is considered in this research. To formulate the \(M/M(n)\)\(q-k\)/\(C_h/C_h\) model for the performance analysis of the highway section based on the state-dependent Newell’s flow–density model, we employ the Birth–Death (BD) Process [25].

In the case of Newell’s flow–density model, there are two conditions, i.e., a saturated condition and an unsaturated condition. Therefore, it is necessary to determine a transition point that demarcates the unsaturated condition from the saturated condition. Let \(n_{cr}\) be the traffic volume below which the condition is unsaturated \((n < n_{cr})\) and above which the condition is saturated \((n \geq n_{cr})\). For a finite capacity \(M/M(n)\)\(q-k\)/\(C_h/C_h\) queuing system, the state transition diagram is shown in Figure 3.

![State transition diagram for \(M/M(n)\)\(q-k\)/\(C_h/C_h\) queuing model.](image)

The vertical dotted line demarcates the saturated condition and the unsaturated condition. An infinitesimal generator matrix with birth rate \(\lambda\) and death rate \(\mu(n)\) can be created from the state transition diagram, based on the BDP, which is given by Equation (2). \[
\begin{pmatrix}
-\lambda_0 & \lambda_0 & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_n \\
\mu_1 & -\lambda_1 - (\lambda_1 + \mu_1) & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_n & \lambda_c \\
2\mu_1 & \lambda_1 + 2\mu_1 & -\lambda_2 - (\lambda_2 + 3\mu_1) & \lambda_3 & \lambda_4 & \lambda_n & \lambda_c \\
3\mu_1 & \lambda_1 + 2\mu_1 & \lambda_2 + 3\mu_1 & -\lambda_3 - (\lambda_3 + 4\mu_1) & \lambda_4 & \lambda_n & \lambda_c \\
4\mu_1 & \lambda_1 + 2\mu_1 & \lambda_2 + 3\mu_1 & \lambda_3 + 4\mu_1 & -\lambda_4 - (\lambda_4 + n\mu_1) & \lambda_n & \lambda_c \\
\vspace{-1cm} \\
\end{pmatrix}
\] (2)

Let ‘\(\pi'\)’ be the state probability, i.e., the number of vehicles on the highway section. \[ \pi Q = 0 \] (3)
Using Equations (2) and (3), the steady-state (equilibrium) equations can be obtained as

\[ \lambda_0 \pi_0 + \mu_1 \pi_1 = 0 \quad (4) \]

\[ \pi_1 = (\lambda_0 / \mu_1) \pi_0 \quad (5) \]

\[ \lambda_{i-1} \pi_{i-1} - (\lambda_i + \mu_i) \pi_i + \mu_{i+1} \pi_{i+1} = 0 \quad (6) \]

\[ \pi_{i+1} = \left( \frac{\lambda_i + \mu_i}{\mu_{i+1}} \right) \pi_i - \left( \frac{\lambda_{i-1}}{\mu_{i+1}} \right) \pi_{i-1} \quad (7) \]

Using the principle equation, making appropriate substitutions, and using the mean of recursion, all the state probabilities can be expressed in terms of zero-state probability. The zero-state probability can be obtained by the normalization condition, i.e., \( \sum_{n=0}^{\infty} \pi_n = 1 \).

\[ \pi_0 = \left[ 1 + \sum_{i=1}^{\infty} \frac{\lambda^n}{i! \prod_{j=1}^{n} u_j} \right]^{-1} \quad (8) \]

The vehicles’ flow rate on any highway section is related to the speed of the vehicles on the section, the number of vehicles on the section, and the length of the section, which is given by Equation (9):

\[ \mu = \frac{nV}{L} \quad (9) \]

However, there are two conditions, i.e., the unsaturated and saturated traffic conditions. The traffic flow rate can be expressed in terms of both the unsaturated and saturated flow rates, as shown by Equation (10).

\[ \mu_n = \begin{cases} \frac{nV}{L} & n < n_{cr} \\ \frac{nV}{L(C-n)} & n \geq n_{cr} \end{cases} \quad (10) \]

With the substitution shown by Equation (10), we obtain all the state probabilities for both the saturated and unsaturated conditions, as shown by Equation (11).

\[ \pi_n = \begin{cases} \frac{\lambda^n}{n! \prod_{j=1}^{n} u_j} \pi_0 & n < n_{cr} \\ \frac{n^C}{n_{cr} C \prod_{j=n_{cr}}^{n} u_j} \pi_0 & n \geq n_{cr} \end{cases} \quad (11) \]

In order to obtain the performance measures, such as the blocking probability and the average number of vehicles on the highway section, we can use Equations (12) and (13), which are based on the state probabilities from Equation (11).

\[ \pi_C = \frac{\lambda^C}{\prod_{n=1}^{n_{cr}-1} u_q \prod_{n=n_{cr}}^{C} u_n} \pi_0 \quad (12) \]

\[ E[N] = \sum_{i=1}^{C} i \pi_i \quad (13) \]
4. Discrete-Event Simulation Architecture of Multi-Lane Highway Section

To ensure that our suggested queuing model is accurate for the multi-lane highway section, the DES model is developed in the SimEvents® toolbox of the MATLAB programming environment, as shown in Figure 4.

![Figure 4. Discrete-Event Simulation Architecture of Multi-Lane Highway Section.](image)

The DES model consists of two phases, i.e., (1) average vehicles’ arrival phase; (2) flow–density based on the service phase of the highway. The major blocksets of the SimEvents® toolbox for the DES model are the FIFO_Queue blocks, which assign queueing spaces to the vehicles, the Server blocks, which temporarily store the passengers, and the Start and Read Timers, which display the average dwelling time of the vehicles on the highway section. The Level-2 S-function block computes and updates the state-dependent speed of the vehicles as the number of vehicles increases on the section. The Constant blocks are used to input values for various parameters in the DES model, such as arrival rate, length of the section, number of lanes, etc. The Display block displays the output, i.e., average number of vehicles on the highway section.

As discussed earlier, the highway section has a finite capacity (shown in Figure 2) in which the arriving demand cannot overcome the overall capacity of the highway section. Therefore, during the vehicles’ arriving phases, it must be assured that the number of arrivals stays less than or equal to the capacity \( C_h = K_j L N \) of the highway section. To execute this scenario in the DES environment, the vehicles’ arrival from the upstream section stays in the FIFO_Queue block before entering the Server block. The vehicles are subsequently sent to the downstream section. The MATLAB® Function block is used to perform the following functions:

- It assesses the number of vehicles arriving at the highway section with its capacity \( C_h = K_j L N \). This block prevents the entry of entities into the FIFO_Queue block when the highway section is at capacity \( n = C_h \).
- This block computes the blocking probability as the highway jams, i.e., operates at capacity. When the entrance to the FIFO_Queue block is blocked for vehicles, the blocked vehicles are simultaneously activated and registered through the output switch block’s second entity port (OUT2). Based on the blocking probability, the average number of vehicles is estimated from the FIFO_Queue block of the DES model.
5. Computational Experiments

In this section, first the accuracy of the mathematical queuing model is verified by the proposed simulation model. The two-level full factorial design approach is used to statistically analyze the effect of various factors on the average number of vehicles on the highway section.

5.1. Verification of Proposed \( M/M(n)_{q-k}/C_h/C_h \) Model

Before conducting the two-level full factorial design of the experiment for analyzing the significance of various factors, it is necessary to first verify the accuracy proposed by \( M/M(n)_{q-k}/C_h/C_h \). For verification purposes, we perform the experiments on a simple highway section, where the number of lanes is equal to one and two and the length of the section is equal to 0.1 mi and 0.2 mi with different arrival rates, which are according to the actual conditions on the collector road sections in Islamabad, Pakistan. The average number of vehicles (average queue length) \((EN)\) on the highway section, blocking probability \((PC)\), and average vehicles’ dwelling time \((EW)\) are obtained from both the mathematical queuing and DES models. The results are presented in Table 1.

Table 1. Verification of proposed mathematical queuing model.

<table>
<thead>
<tr>
<th>Length: 0.2 mi</th>
<th>Length: 0.1 mi</th>
<th>Length: 0.2 mi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lanes: 1</td>
<td>Lanes: 1</td>
<td>Lanes: 2</td>
</tr>
<tr>
<td>Jam Density: 80 veh/mi/ln</td>
<td>Jam Density: 90 veh/mi/ln</td>
<td>Jam Density: 100 veh/mi/ln</td>
</tr>
<tr>
<td>Speed: 60 mi/h</td>
<td>Speed: 60 mi/h</td>
<td>Speed: 90 mi/h</td>
</tr>
<tr>
<td>Proposed M/M(n)_{q-k}/C_h/C_h Model</td>
<td>Proposed M/M(n)_{q-k}/C_h/C_h Model</td>
<td>Proposed M/M(n)_{q-k}/C_h/C_h Model</td>
</tr>
<tr>
<td>Proposed DES Model (95% Confidence Interval)</td>
<td>Proposed DES Model (95% Confidence Interval)</td>
<td>Proposed DES Model (95% Confidence Interval)</td>
</tr>
<tr>
<td>(\lambda = 2100) veh/h</td>
<td>(\lambda = 1525) veh/h</td>
<td>(\lambda = 1720) veh/h</td>
</tr>
<tr>
<td>EN (veh)</td>
<td>EN (veh)</td>
<td>EN (veh)</td>
</tr>
<tr>
<td>15.85 (15.98, 16.85)</td>
<td>7.25 (6.11, 7.22)</td>
<td>39.92 (36.61, 38.22)</td>
</tr>
<tr>
<td>PC</td>
<td>PC</td>
<td>PC</td>
</tr>
<tr>
<td>0.88 (0.853, 0.871)</td>
<td>0.54 (0.511, 0.604)</td>
<td>0.93 (0.923, 0.947)</td>
</tr>
<tr>
<td>EW (s)</td>
<td>EW (s)</td>
<td>EW (s)</td>
</tr>
<tr>
<td>28.91 (29.11, 31.87)</td>
<td>17.12 (15.98, 17.25)</td>
<td>83.54 (81.97, 84.88)</td>
</tr>
</tbody>
</table>

During simulation experiments, the mean values of 30 replications are recorded and tabulated. Each simulation test is performed with a simulation time of 50,000 time units to achieve outputs at steady-state conditions. The 95% confidence interval approximations for the distribution of the mean, which follow the normal distribution, are also recorded in Table 1. All the computational experiments are carried out on a PC with Intel® Core™ i5-4570 CPU@ 3.20 GHz and 8 GB of RAM under a Windows® operating system. The mean computational times (CPU times in minutes) is also recorded for all the experiments. As shown in Table 1, both the mathematical and simulation models show a clear consistency. Therefore, the models can be used for the performance assessment of a multi-lane highway section under saturated and unsaturated conditions.

5.2. Factorial Design Approach

The two-level full factorial design approach is conducted to evaluate the significance and effect of four different factors such as the length of the highway section, the number of lanes of the highway section, the average vehicles’ arrival rate, and the jam density on the average number of vehicles (mean queue length) \((EN)\) on the highway section. The average number of vehicles on the highway section is obtained from our proposed mathematical queuing and simulation models. Table 2 displays the four factors along with
Table 2. Levels of factors studied.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Abb.</th>
<th>Units</th>
<th>Low Level</th>
<th>High Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>A</td>
<td>Miles</td>
<td>0.1 (−1)</td>
<td>0.2 (+1)</td>
</tr>
<tr>
<td>Number of Lanes</td>
<td>B</td>
<td>-</td>
<td>1 (−1)</td>
<td>2 (+1)</td>
</tr>
<tr>
<td>Vehicles’ Arrival Rate</td>
<td>C</td>
<td>Veh/hr</td>
<td>1500 (−1)</td>
<td>2000 (+1)</td>
</tr>
<tr>
<td>Jam Density</td>
<td>D</td>
<td>Veh/mile/lane</td>
<td>90 (−1)</td>
<td>120 (+1)</td>
</tr>
</tbody>
</table>

Table 3 displays the main effect estimates as well as the interaction effect estimates. The difference between the average response of a factor at a high level and that of a factor at a low level is referred to as the effect. The interaction effect between two factors (say, A*B) is defined as the mean difference between the effects of the “length of the highway section” at a high level for the “number of lanes” and the effect of the “length of the highway section” at a low level for the “number of lanes”.

Table 3. Coefficients and main and interaction effects’ estimates.

<table>
<thead>
<tr>
<th>Term</th>
<th>Effect</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T-Value</th>
<th>p-Value</th>
<th>VIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>22.668</td>
<td>0.855</td>
<td>26.50</td>
<td>0.000</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>• Length</td>
<td>17.484</td>
<td>8.742</td>
<td>0.855</td>
<td>10.22</td>
<td>0.000</td>
<td>1.00</td>
</tr>
<tr>
<td>• Number of Lanes</td>
<td>14.441</td>
<td>7.221</td>
<td>0.855</td>
<td>8.44</td>
<td>0.000</td>
<td>1.00</td>
</tr>
<tr>
<td>• Arrival Rate</td>
<td>−1.364</td>
<td>−0.682</td>
<td>0.855</td>
<td>−0.80</td>
<td>0.461</td>
<td>1.00</td>
</tr>
<tr>
<td>• Jam Density</td>
<td>5.156</td>
<td>2.578</td>
<td>0.855</td>
<td>3.01</td>
<td>0.030</td>
<td>1.00</td>
</tr>
</tbody>
</table>

MINITAB-19 software was used to calculate the full model regression coefficients in terms of their coded values [14]. The effect estimates of the factors can be obtained by multiplying the regression coefficient by a factor of 2. Table 4 shows the analysis of variance (ANOVA) of up to two factors’ interactions. An ANOVA with higher order interactions of three or four variables results in an ill model, so it is deserted due to the sparsity of effect principle.
Table 4. Analysis of variance.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F-Value</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODEL</td>
<td>10</td>
<td>2434.86</td>
<td>243.49</td>
<td>20.80</td>
<td>0.002</td>
</tr>
<tr>
<td>One-Way Interaction</td>
<td>4</td>
<td>2170.71</td>
<td>542.68</td>
<td>46.37</td>
<td>0.000</td>
</tr>
<tr>
<td>• Length</td>
<td>1</td>
<td>1222.73</td>
<td>1222.73</td>
<td>104.47</td>
<td>0.000</td>
</tr>
<tr>
<td>• Number of Lanes</td>
<td>1</td>
<td>834.20</td>
<td>834.20</td>
<td>71.28</td>
<td>0.000</td>
</tr>
<tr>
<td>• Arrival Rate</td>
<td>1</td>
<td>7.44</td>
<td>7.44</td>
<td>0.64</td>
<td>0.461</td>
</tr>
<tr>
<td>• Jam Density</td>
<td>1</td>
<td>106.35</td>
<td>106.35</td>
<td>9.09</td>
<td>0.030</td>
</tr>
<tr>
<td>Two-Way Interactions</td>
<td>6</td>
<td>264.15</td>
<td>44.03</td>
<td>3.76</td>
<td>0.084</td>
</tr>
<tr>
<td>• Length*Number of Lanes</td>
<td>1</td>
<td>175.23</td>
<td>175.23</td>
<td>14.97</td>
<td>0.012</td>
</tr>
<tr>
<td>• Length*Arrival Rate</td>
<td>1</td>
<td>7.94</td>
<td>7.94</td>
<td>0.68</td>
<td>0.448</td>
</tr>
<tr>
<td>• Length*Jam Density</td>
<td>1</td>
<td>59.25</td>
<td>59.25</td>
<td>5.06</td>
<td>0.074</td>
</tr>
<tr>
<td>• Number of Lanes*Arrival Rate</td>
<td>1</td>
<td>10.68</td>
<td>10.68</td>
<td>0.91</td>
<td>0.383</td>
</tr>
<tr>
<td>• Number of Lanes*Jam Density</td>
<td>1</td>
<td>2.60</td>
<td>2.60</td>
<td>0.22</td>
<td>0.657</td>
</tr>
<tr>
<td>• Arrival Rate*Jam Density</td>
<td>1</td>
<td>8.45</td>
<td>8.45</td>
<td>0.72</td>
<td>0.434</td>
</tr>
<tr>
<td>• Error</td>
<td>5</td>
<td>58.52</td>
<td>11.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>2493.38</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It asserts that the majority of processes are regulated by a few major factors and a few low-order interactions.

5.2.1. Effects of the Factors

From Figure 5, it can be observed that the effect estimates of the length of the highway section, number of lanes, and jam density are positive (positive slope), which means that they have a positive effect on the average number of vehicles (average queue length) on the highway section.
The length plot shows a relatively high positive slope, but the jam density plot has a somewhat positive slope. This suggests that increasing the amounts of these parameters reduces the average queue length. This shows that with a slight increase in the length of the highway section, i.e., from 0.1 mi to 0.2 mi, the average queue length also increases, when keeping all other factors constant.

Similarly, the average vehicles’ arrival rate is negative, which means that it has a negative impact on the queue length and the increase in this factor causes an increase in the average queue length, which is quite obvious in all cases. A cumulative normal probability plot of the main and interaction effects of the factors at $\alpha = 0.05$ is shown in Figure 6.

![Normal Plot of the Standardized Effects](image)

Figure 6. Normal plot of the standardized effects.

It is obvious that none of the components are in a straight line. Outlier variables, shown by dark red squares, mostly concern estimating the average queue length and are thought to be the most dominating elements. The factors that are insignificant tend to fall along a straight line on this plot, whereas the significant main effects and interaction have nonzero means and do not lie along the straight line. We can note that the average vehicle arrival rate is not a potential outlier; consequently, the influence of the average vehicle arrival rate explored in this study (for this specific highway section) is not significant compared to the length of the highway section, which is a potential outlier. This could be explained by the fact that the average vehicle arrival rate range (1500–2000 veh/h) covered in this study is rather small. The significance of factors can also be assessed by using a Pareto chart, as shown in Figure 7.

The factors with a horizontal bar extended beyond the dashed vertical line illustrate the significance of the factors.

5.2.2. Interaction Effects of the Factors

An interaction is the failure of one factor to produce the same effect on the response at the different levels of another factor. The interaction between two factors is said to occur when a change in the values of one variable alters the effect on another factor. This implies that insignificant factor interactions produce similar trends in response to the different
levels of another factor. The two-way interaction (two-factor interaction) effect is shown in Figure 8, and an insignificant two-factor interaction ($B*D$, $p$-value = 0.657) and one significant two-factor interaction ($A*B$, $p$-value = 0.012) at $\alpha = 0.05$ are presented.

Figure 7. Pareto chart of the standardized effects.

![Pareto Chart of the Standardized Effects](image)

**Factor** | **Name**
--- | ---
A | Length
B | Number of Lanes
C | Arrival Rate
D | Jam Density

Figure 8. Interaction plot for EN.

![Interaction Plot for EN](image)

**Interaction Plot for EN**

Fitted Means

Length * Number of La
Length * Arrival Rate
Length * Jam Density
Number of La * Length
Number of La * Arrival Rate
Number of La * Jam Density
Arrival Rate * Length
Arrival Rate * Number of La
Arrival Rate * Jam Density
Jam Density * Length
Jam Density * Number of La
Jam Density * Arrival Rate

**Mean of EN**

Length
Number of La
Arrival Rate
Jam Density

**Length**

-1.0
1.0

**Number of La**

-1.0
1.0

**Arrival Rate**

-1.0
1.0

**Jam Density**

-1.0
1.0
It is clear that when the length of the highway section is increased, while keeping the number of lanes at a low level, the average queue length (average number of vehicles) increases. Similarly, with an increase in the length of the highway section, while keeping the number of lanes at a higher level, the queue length again increases.

5.2.3. Regression Model

In order to predict future observations, the regression analysis shows the statistical relationship between one or more independent factors and the response factor. In this research, by considering only the main (linear) and interaction terms (two-factor interaction), a multiple-factor linear regression model is obtained for the prediction of the average vehicles’ queue length on the highway section, which is given by the following equation:

\[
EN = 22.668 + 8.742 \text{Length} + 7.221 \text{Number of Lanes} - 0.682 \text{Arrival Rate} + 2.578 \text{Jam Density} + 3.309 \text{Length} \times \text{Number of Lanes} + 0.704 \text{Length} \times \text{Arrival Rate} + 1.924 \text{Length} \times \text{Jam Density} - 0.817 \text{Number of Lanes} \times \text{Arrival Rate} + 0.403 \text{Number of Lanes} \times \text{Jam Density} - 0.727 \text{Arrival Rate} \times \text{Jam Density}
\]

The coefficient of the determination \( R^2 \) of the regression model is 0.976, and the adjusted \( R^2 \) is 0.930. \( R^2 = 93\% \) in the model shows that more than 93% of the variability of the responses can be explained by the factors of regression model.

6. Conclusions and Recommendations

In this research, we proposed a mathematical queuing and simulation model for the performance evaluation of a multi-lane highway section under congested and uncontested conditions using Newell’s triangular flow–density relationship. This model is more realistic as it considered different traffic flow conditions. The two-level-full-factorial design approach is used to determine the significant and insignificant design parameters such as the length of the highway section, number of lanes, average vehicles’ arrival rate, and jam density. It was observed that the length of the highway section, number of lanes, and jam density have a positive effect, which shows that increasing the level of these factors results in an increased value for the average queue length. The jam density has a mild positive slope, which shows that it has a slight significance compared to the other highly significant factors. Additionally, it was found that the majority of the highest-order interactions are insignificant, which supports the sparsity of effect principle. Only the two-way interaction of the “length-lanes” is a significant interaction affecting the average queue length on the highway section. A regression model to predict the average queue length from the two-level full factorial design is developed, which takes into account various design factors and their interactions.

The effects of the four factors studied are limited to a specific range of values. More research with a broader range at both low and high levels is recommended. Other different sites can be considered for the performance evaluation based on the full factorial design approach. Additionally, the proposed model could be modified slightly and used for the analysis of freeways. With a few minor adjustments, the proposed model may also be used to analyze passenger flow in airport and metro station corridors.

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