A Study of Electronic Product Supply Chain Decisions Considering Quality Control and Cross-Channel Returns

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Abstract: This article focuses on the return problem of electronic products caused by the low level of quality control in the production process. Based on previous research, a centralized decision model (DM) and four kinds of decentralized DMs are constructed according to the differences in the supply chain decision order and the return loss bearers. Moreover, the impact of quality control on the return rate and market demand is also considered in the model research to achieve the optimization of the model. Using Stackelberg game knowledge to compare and analyze supply chain decisions, profits and influencing factors in different models, the final conclusions are drawn: (1) the decision is optimal and the profit is the highest in the centralized DM; (2) when the manufacturer dominates the supply chain decision, the difference in return loss bearers does not affect the supply chain decision and profit; (3) compared with other decentralized DMs, when the retailer dominates the supply chain decision and the manufacturer bears the return loss, the supply chain decision is better and the profit is higher; (4) the quality control level and the total profit in the five supply chain models are proportional to the quality control effect and the consumer perception of quality control. In addition, this paper introduces a joint contract to coordinate the supply chain under the decentralized DM and achieves the desired results. Finally, the relevant research findings and the validity of the contract are verified in the arithmetic analysis.

Keywords: quality control level; supply chain decision; cross-channel returns; loss of return

1. Introduction

In recent years, rapid advances in technology have driven the rapid growth of electronics retailing, and with it, changes and updates to the electronics supply chain model, creating new challenges for the partnership between supply chain members. The production process of electronic products is complex, and the requirements and dependence on quality control are high. However, due to the imbalance of supply chain cooperation, the quality control of many products in the production process often fails to reach the ideal level, and some products have more or less quality problems when they leave the factory. Defective products entering the market can trigger consumer returns, which in turn can lead to certain losses in the supply chain. Therefore, it is important to improve quality control during the production of electronic products and reduce the return rate of products to promote the coordination and sustainable development of the electronic product supply chain.

Quality control plays a significant role in improving product quality, and many scholars have conducted research on how to reduce the rate of defective products around quality control. Azmi et al. [1] described strategies and methods for quality control, focusing on the importance of quality control in improving product quality and meeting customer needs. Purbawati et al. [2] used such determinations in quality improvement and assessment.
to reduce the production of defective products. Benbarrad et al. [3] proposed to use a machine vision model to predict the production process so as to reduce the defect rate of products. Peng et al. [4] used the exponential weighted moving average method to control product quality in the production process. Shirai et al. [5] and Zhang et al. [6] both used the least squares method to improve quality control in the production process. Lu et al. [7] conducted a systematic study on the improvement of product edge-sealing quality using statistical process control quality control methods. In addition, references [8–11] used the PDCA cycle method in the manufacturing process to enhance product quality control to reduce the rate of defective products.

Product quality problems can lead to consumer returns, and many studies on supply chains have explored the impact of returns on the supply chain [12,13]. Wang et al. [14] emphasized that online retailers are facing increasingly severe product return problems, and the high average return rate leads to the loss of performance of online retailers. Lin et al. [15] constructed a supply chain model consisting of a manufacturer and a retailer with corporate social responsibility to address the issue of manufacturer’s quality improvement strategies in response to consumer returns due to quality issues. Pei [16] considered the impact of product quality on the return rate when studying product pricing and product quality decisions. Ni [17] discussed three return channels for product heterogeneity returns and found that when the liquidator was introduced into the return processing channel, the supply chain profit was higher than the other two return channels. Zhang et al. [18] studied the impact of returned incomplete products on the vertical dual-channel management of a closed-loop supply chain. In addition, some scholars have conducted research on consumers’ cross-channel returns. Zhang et al. [19] introduced the revenue sharing coordination contract strategy under the online and offline cooperation mechanism when discussing the impact of cross-channel return service on supply chain enterprises. Yang et al. [20] proposed a benchmark strategy, an original channel return strategy and a cross-channel return strategy and concluded that the cross-channel return strategy is more conducive to retailers. Pan et al. [21] pointed out that there is a significant impact on the pricing strategy and profit distribution of a closed-loop supply chain between the cross-channel return rate and consumer channel preference. Zhang et al. [22] proposed that in the case of providing not providing cross-channel return services, when the spillover effect is greater than the threshold, retailers will cooperate with manufacturers to adopt cross-channel return policies. In summary, consumer return behavior and its impact cannot be ignored in supply chain optimization.

Who bears the loss of returned products is a debatable topic, and the party bearing the loss of returned products may differ in different supply chain models [23]. Xu et al. [24] considered the manufacturer to bear the loss of return in the study of the return of agricultural products. In addition, Farahani et al. [25] and Borenich et al. [26,27] also considered the loss borne by the manufacturer in the study of electronic product supply chain optimization. Zhang et al. [28] used the Stackelberg game method to study the supply chain optimization problem in which the supplier bears the return loss and proposed a “preservation technology + cost sharing + benefit sharing” coordination contract. In addition, there are also studies that consider the retailer’s loss of return. Liu et al. [29] took the loss of customer return to the retailer as the starting point and analyzed the impact of return loss on the retailer’s optimal pricing and profit in different channels from three different return methods. From the perspective of profit maximization, Kim et al. [30] pointed out that online retailers will be motivated to provide refund guarantees only when the return loss is low enough. In summary, most of the current studies consider a single supply chain member to bear the return loss, and there are few studies on the impact of different return loss bearers on supply chain decisions.

How to reduce the return rate is the key to coordinating the supply chain with return behavior. Liu et al. [31] pointed out that online channels should provide more detailed product descriptions to reduce the uncertainty of products, thereby reducing the return rate. Rao et al. [32] and Zhao et al. [33], from the perspective of logistics delivery time, found that
if the goods cannot be delivered on time, it will lead to consumer return behavior, and they make recommendations to shorten the logistics delivery time. Liu et al. [34] studied the manufacturer’s return processing strategy by establishing a newsboy model. Fu et al. [35] introduced consumer online reviews into the fresh e-commerce supply chain in the study of reducing the return rate, forming the optimal pricing and preservation effort decision of the supply chain. Kim et al. [36] found that introducing online retailers into the experience channel can expand the scale of potential market demand and reduce the product return rate. In addition, Ding et al. [37] pointed out that the revenue sharing–cost sharing contract can improve the service input level of online retailers, reduce the defect-free return rate of customers and achieve supply chain coordination.

At present, the research system of supply chain optimization is relatively perfect, and different studies adopt different methods to optimize the supply chain from different angles. Yılmaz et al. [38] used disassembly line balance knowledge to study the multi-objective optimization problem, and finally realized the optimization of the supply chain operation process. This method is usually used for optimization in the production process. Zhang et al. [39] used genetic algorithms to study closed-loop supply chains with uncertain demand and recovery. Genetic algorithms are often used to search for optimal solutions in large-scale solution spaces. In addition, the Stackelberg game theory is widely used in supply chain optimization. This theory is mainly used to solve the supply chain decision optimization problem considering the decision order of the supply chain members and explore the factors affecting the supply chain decision. Liu et al. [40] established a Stackelberg model composed of a recycler and a manufacturer to study the impact of recycling waste products on sales. Shen et al. [41] constructed a variety of subsidy recovery models and analyzed the impact of different subsidy objects on supply chain decisions using the Stackelberg game. Peng et al. [42] used Stackelberg game knowledge to build a low-carbon supply chain model and verified the correlation between supply chain decision and government low-carbon subsidies. Lv et al. [43], Wu et al. [44] and Han et al. [45] also used this method to explore the decision differences under different supply chain recycling models. The Stackelberg game theory is also widely used in the study of agricultural supply chains. Chen et al. [46], Dong et al. [47] and Fang et al. [48] used this method to solve and analyze the model when studying the supply chain of fresh agricultural products and introduced contracts to realize the coordination of the supply chain. In addition, Jiao et al. [49] also used the Stackelberg game method to coordinate the water supply chain, providing a theoretical basis for managers to make correct water supply decisions. To sum up, the Stackelberg game theory is mature and can be applied to supply chain optimization research in many fields.

Table 1 summarizes the previous research from the aspects of research perspective, method and deficiency.

In this paper, the cross-channel return behavior of consumers in the retail industry of electronic products is studied. Previous studies mainly have the following shortcomings: (1) In the research on the return of electronic products, the comparison of supply chain models is lacking, and the impact of return losses and decision order on supply chains is not highlighted. (2) The impact of product quality control on market demand and the return rate is not considered. Compared with previous studies, the innovations of this study are as follows: (1) According to the differences in decision order of decision makers and the differences in return loss bearers, a centralized decision model and four decentralized decision models are constructed, and the optimal decision of different models and the supply chain profit are compared and analyzed. The comparison of the centralized decision model and decentralized decision models provides theoretical support for the coordination of the electronic product supply chain. Through the comparison of different decentralized decision models, the influence mechanism of differences in decision order and the return loss undertaken on supply chain decisions is revealed, which provides theoretical guidance for managers to make correct decisions in different supply chain models. (2) Based on the cross-channel returns of consumers, this paper incorporates the positive impact of quality
control on the market demand for electronic products and the negative impact on the return rate into the model research, which makes up for the shortcomings of previous studies in this aspect and makes the final research conclusion more realistic.

Table 1. Review and summary of previous studies.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Goal and Perspective</th>
<th>Method</th>
<th>Deficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2,3]</td>
<td>Reduce the rate of defective products by studying how to improve quality control.</td>
<td>This paper puts forward the use of intelligent equipment to improve and measure the production process and improve the production efficiency.</td>
<td>Product quality control is not taken as the main factor of supply chain decision to study supply chain coordination.</td>
</tr>
<tr>
<td>[16]</td>
<td>Consider the impact of product quality on return rates in supply chain optimization.</td>
<td>Based on the situation that product quality affects the return rate, this paper uses the game theory method to analyze the supply chain decision in the two models of unified pricing and autonomous pricing.</td>
<td>Does not consider the impact of product quality on market demand, that is, consumers have a certain degree of perception of product quality control.</td>
</tr>
<tr>
<td>[21]</td>
<td>The influence of consumers’ cross-channel return behavior on supply chain decision is studied.</td>
<td>Based on Stackelberg game and Nash equilibrium game theory, centralized and decentralized supply chain models are analyzed.</td>
<td>The impact of product quality control on consumer return rate is not considered.</td>
</tr>
<tr>
<td>[23,28]</td>
<td>In the study of return supply chain, the loss of consumer return to supply chain is considered.</td>
<td>A Stackelberg game model was established for the return of defective products. In the model, a single member of the supply chain was considered to bear the return loss.</td>
<td>Only a single supply chain member is considered to bear the return loss, and the impact of different return loss bearers on the supply chain is not considered.</td>
</tr>
<tr>
<td>[31]</td>
<td>Explore ways to reduce product return rates from different perspectives and give the final strategy.</td>
<td>Based on whether retailers open return service, several models are established, and game theory is used to solve and analyze the models.</td>
<td>The research on product quality is not carried out, and the influence of product quality on supply chain coordination is ignored.</td>
</tr>
</tbody>
</table>

The full text consists of seven sections: the first section is related background information and a full-text introduction. The second section is the construction and assumption of the model. In this section, five supply chain models are introduced in detail. The third section is the comparative analysis of the models. In this section, the paper compares the final decision of different supply chain models and the profits of all parties and discusses how the quality control effect and consumers’ perception of quality control affect the supply chain decision. The fourth section is the coordination of a contract. In this section, the article designs a joint contract to coordinate the supply chain in the decentralized decision model and focuses on the internal logic of contract coordination. The sixth section is the analysis of numerical examples. In this section, the paper verifies the conclusions of the previous research and the effect of contract coordination through the analysis of numerical examples, analyzes the effectiveness of the model results, and finally puts forward suggestions from different perspectives. The seventh section is the research conclusion.

In summary, this study can answer the following questions:

1. What are the outcomes of decisions made by supply chain members in different types of supply chains?
2. What is the influence of the decision order of supply chain members on supply chain decision and profit?
3. What is the impact of different return loss bearers on supply chain decisions and profits?
4. How do quality control effects and consumers’ perception of quality control affect supply chain decisions and profits?
5. How can the internal logic of contract coordination be explored and how can the coordination effect be verified?

2. Description and Assumptions

This paper mainly studies an electronic product supply chain system composed of a manufacturer and an offline retailer, in which the manufacturer is responsible for the production of products and decides the wholesale price and quality control level in the
production process, while the retailer is responsible for the offline sales of products and decides the retail price of products. The supply chain relationship is shown in Figure 1.

![Figure 1. Supply chain relationship description.](image)

According to Figure 1, after purchasing electronic products, if consumers find quality problems in the products within the effective time, they can choose to return them online (to the manufacturer) or offline (to the retailer), and the proportion of offline return is $\epsilon$. After returning the product, consumers will contact the manufacturer or retailer to re-issue it, and the residual value of the returned product is $v$. Upon receipt of the returned product, the retailer is required to return the product to the manufacturer at a return cost of $l$.

This paper constructs five supply chain models: one centralized decision model and four decentralized decision models. Among them, the centralized decision model (SS model) means that members of the supply chain make joint decisions and bear the return loss of electronic products together. The decentralized decision model refers to the supply chain members making decisions successively, and one member alone bears the return loss of electronic products. The classification and description of the decentralized decision model are shown in Table 2.

Table 2. Classification and specification of decentralized decision models.

<table>
<thead>
<tr>
<th>Manufacturer-First Decision (Manufacturer-Led)</th>
<th>Retailer-First Decision (Retailer-Led)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The manufacturer bears the loss of return</td>
<td>MM model</td>
</tr>
<tr>
<td>The retailer bears the loss of return</td>
<td>RM model</td>
</tr>
<tr>
<td></td>
<td>MR model</td>
</tr>
<tr>
<td></td>
<td>RR model</td>
</tr>
</tbody>
</table>

This research requires the following assumptions:

Assumption 1. Referring to Ha et al. [50], when the quality control level is $f$, the input cost is $c(e)$, $c(e) = \frac{1}{2}uf^2$.

Assumption 2. The defective product rate is affected by quality control level, referring to Zhang et al. [28], expressed as $k = k_0 - nf$, where $k$ is the defective product rate, $k_0$ is the initial defective product rate and $n$ is the quality control effect. The return rate of products can be expressed as $h = \phi f$, and $\phi$ is the consumer’s perception of defective products.

Assumption 3. The market for electronic products is influenced by both price and quality control level. This refers to the study by Ren et al. [51], denoted as $D = a - bp + gf$, where $a$ is the initial market size of the product, $b$ is the price sensitivity coefficient and $g$ is the sensitivity coefficient of the quality control level.
Assumption 4. The manufacturer and retailer are rational decision makers and do not make extreme decisions.

According to the above assumptions, when the manufacturer bears the loss of return, the profit function of each member of the supply chain is as follows (m represents the manufacturer, r represents the retailer):

$$\Pi_m(\frac{MM}{MR}) = (w - c)D - (c - v)hD - \epsilon l h D - \frac{1}{2} u f^2,$$ (1)

$$\Pi_r(\frac{MM}{MR}) = (p - w)D.$$ (2)

Formula (1) consists of four parts: the first part is the profit obtained by the manufacturer from selling the product to the retailer; the second part is the loss caused by the returned products to the manufacturer; the third part is the return cost that the manufacturer needs to pay when consumers choose offline return; and the fourth part is the input cost paid by the manufacturer to improve the quality control level.

Formula (2) is the profit made by the retailer from the sale of the products.

When the retailer bears the loss of return, the profit function of each member of the supply chain is as follows:

$$\Pi_m(\frac{RM}{RR}) = (w - c)D - \frac{1}{2} u f^2,$$ (3)

$$\Pi_r(\frac{RM}{RR}) = (p - w)D - (c - v)hD - \epsilon l h D.$$ (4)

Formula (3) consists of two parts: the first part is the profit obtained by the manufacturer from selling the product to the retailer, and the second part is the input cost paid by the manufacturer to improve the quality control level.

Formula (4) consists of three parts: the first part is the retailer’s profit from selling products; the second part is the loss caused by the returned products to the retailer; and the third part is the cost that the retailer needs to pay when consumers choose offline channels to return goods.

The symbol descriptions in the model are shown in Table 3.

Table 3. Symbol description table.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>p</td>
<td>Retail price</td>
</tr>
<tr>
<td>w</td>
<td>Wholesale price</td>
</tr>
<tr>
<td>v</td>
<td>Average salvage value of defective goods</td>
</tr>
<tr>
<td>c</td>
<td>Production cost</td>
</tr>
<tr>
<td>f</td>
<td>Quality control level</td>
</tr>
<tr>
<td>u</td>
<td>Cost coefficient of quality control</td>
</tr>
<tr>
<td>h</td>
<td>Rate of return</td>
</tr>
<tr>
<td>ε</td>
<td>The proportion of consumers who choose to return goods offline</td>
</tr>
<tr>
<td>k</td>
<td>Rate of defective goods</td>
</tr>
<tr>
<td>k₀</td>
<td>Initial rate of defective goods</td>
</tr>
<tr>
<td>n</td>
<td>Effect of quality control</td>
</tr>
<tr>
<td>φ</td>
<td>Consumer perception of defective goods</td>
</tr>
<tr>
<td>a</td>
<td>Initial market size</td>
</tr>
<tr>
<td>b</td>
<td>Price sensitivity coefficient</td>
</tr>
<tr>
<td>g</td>
<td>Sensitivity coefficient of quality control level</td>
</tr>
<tr>
<td>l</td>
<td>Additional return costs incurred during offline return</td>
</tr>
<tr>
<td>D</td>
<td>Market demand</td>
</tr>
<tr>
<td>Πₘ</td>
<td>Manufacturer’s profit</td>
</tr>
<tr>
<td>Πᵣ</td>
<td>The retailer’s profit</td>
</tr>
<tr>
<td>Πₛ</td>
<td>Total supply chain profit</td>
</tr>
</tbody>
</table>
3. Solution and Analysis of the Models

In this paper, we discuss the game of supply chain members under the condition of information symmetry, which meets the conditions of the Stackelberg game. According to Stackelberg game knowledge, the supply chain members make decisions successively, and the member who makes the decision first can predict the decision information of the member who makes the decision later. Backward induction is used to solve the model, that is, the reasoning and induction are carried out step by step according to the decision order. In the process of calculation, the Hessian matrix is used to judge whether the objective equation has an optimal solution, and the conditions of the existence of the optimal solution are found. Five decision models are solved in this section, and the solution results can provide a theoretical reference for scientific decisions under different supply chain models.

3.1. MM Model

MM model: The manufacturer leads the supply chain decision and the manufacturer bears the loss of return.

**Proposition 1.** Under the MM model, the optimal decision and profit of the supply chain are

\[ p_{(MM)}^* = \frac{cg^2 - 3au - bcu + g^2k_0\phi_t + agn\phi_t - bk_0\phi_t u + abn^2\phi^2t^2 + bgk_0n\phi^2t^2 + bcgn\phi_t}{b^2n^2\phi^2t^2 + 2bggn\phi_t - 4ub + g^2}. \]  

(5)

\[ w_{(MM)}^* = \frac{cg^2 - 2au - 2bcu + g^2k_0\phi_t + agn\phi_t - 2bk_0\phi_t u + abn^2\phi^2t^2 + bgk_0n\phi^2t^2 + bcgn\phi_t}{b^2n^2\phi^2t^2 + 2bggn\phi_t - 4ub + g^2}. \]  

(6)

\[ f_{(MM)}^* = \frac{(g + bgn\phi)(bc - a + bk_0\phi t)}{b^2n^2\phi^2t^2 + 2bggn\phi_t - 4ub + g^2}. \]  

(7)

\[ \Pi_{m(MM)}^* = -\frac{u(bc - a + bk_0\phi t)^2}{2(b^2n^2\phi^2t^2 + 2bggn\phi_t - 4ub + g^2)}. \]  

(8)

\[ \Pi_{r(MM)}^* = \frac{bu^2(bc - a + bk_0\phi t)^2}{(b^2n^2\phi^2t^2 + 2bggn\phi_t - 4ub + g^2)^2}. \]  

(9)

\[ \Pi_{c(MM)}^* = -\frac{u(bc - a + bk_0\phi t)^2 (b^2n^2\phi^2t^2 + 2bggn\phi - 6ub + g^2)}{2(b^2n^2\phi^2t^2 + 2bggn\phi_t - 4ub + g^2)^2}. \]  

(10)

Here, \( t = c - v + el \), and the same expression is adopted below.

**Proof of Proposition 1.** In the MM model, the manufacturer decides the quality control level and the wholesale price first, and the retailer decides the retail price later. According to backward induction, first of all we can easily obtain the following:

\[ \frac{\partial \Pi_{r(MM)}}{\partial p_{(MM)}} = a + f_{(MM)}^* 2 - 2bp_{(MM)}^* + bw_{(MM)}. \]

Since \(-2b < 0\), let \( \frac{\partial \Pi_{r(MM)}}{\partial p_{(MM)}} = 0 \), and we can obtain

\[ p_{(MM)} = \frac{a + fg + bw_{(MM)}}{2b}. \]
By substituting $p_{(MM)}$ into $\Pi_{m(MM)}$, we can obtain
\[ \frac{\partial \Pi_{m(MM)}}{\partial w_{(MM)}} = \frac{a}{2} + f_{(MM)}w_{(MM)} - \frac{baw_{(MM)}}{2} + \frac{b(c - w_{(MM)})}{2} + \frac{b\phi f(t_0 - f_{(MM)}u)}{2}, \]
\[ \frac{\partial \Pi_{m(MM)}}{\partial f_{(MM)}} = n\phi f\left(\frac{a}{2} + f_{(MM)}w_{(MM)} - \frac{baw_{(MM)}}{2} - \frac{g(c - w_{(MM)})}{2} - f_{(MM)}u\right) - \frac{g\phi f(t_0 - f_{(MM)}u)}{2}. \]

Thus, the second-order Hessian matrix of $\Pi_{m(MM)}$ is
\[ H_1 = \begin{pmatrix} -b & \frac{g}{2} - \frac{b\phi t}{2} \\ \frac{g}{2} - \frac{b\phi t}{2} & g\phi t - u \end{pmatrix}. \]

The condition for the negative definite of the Hessian matrix is that its second order principal sub-formula is greater than 0, that is, $bu - 2b\phi t g - b^2 n^2 g^2 \phi^2 = -\lambda^2 > 0$. At this point, $\Pi_{m(MM)}$ is a concave function and there is an optimal solution. Let $\frac{\partial \Pi_{m(MM)}}{\partial w_{(MM)}} = 0$ and $\frac{\partial \Pi_{m(MM)}}{\partial f_{(MM)}} = 0$, and we can obtain the optimal solutions:
\[ w^*_{(MM)} = \frac{c\phi^2 - 2au - 2cu + g^2 k_0 \phi f + a g \phi f - 2b k_0 \phi f t + abn^2 g^2 \phi^2 t^2 + bgk_0 g^2 n^2 t^2 + bg \phi f t}{b^2 n^2 g^2 t^2 + 2bg \phi f t - 4ub + g^2}, \]
\[ f^*_{(MM)} = \frac{(g + bn \phi f)(bc - a + bk_0 \phi f t)}{b^2 n^2 g^2 t^2 + 2bg \phi f t - 4ub + g^2}. \]

By substituting $w^*_{(MM)}$ and $f^*_{(MM)}$ back into $p_{(MM)}$, we obtain
\[ p^*_{(MM)} = \frac{c\phi^2 - 3au - 2bcu + g^2 k_0 \phi f + ag \phi f - bk_0 \phi f t + abn^2 g^2 \phi^2 t^2 + bgk_0 g^2 n^2 t^2 + bg \phi f t}{b^2 n^2 g^2 t^2 + 2bg \phi f t - 4ub + g^2}. \]

Finally, by substituting $w^*_{(MM)}$, $f^*_{(MM)}$ and $p^*_{(MM)}$ into $\Pi_{m(MM)}$, $\Pi_{r(MM)}$ and $\Pi_{c(MM)}$, we can obtain $\Pi^*_{m(MM)}$, $\Pi^*_{r(MM)}$ and $\Pi^*_{c(MM)}$. □

3.2. RM Model

RM model: The manufacturer leads the supply chain decision and the retailer bears the loss of return.

Proposition 2. Under the RM model, the optimal decision and profit of the supply chain are
\[ p^*_{(RM)} = \frac{c\phi^2 - 3au - 2bcu + g^2 k_0 \phi f + ag \phi f - bk_0 \phi f t + abn^2 g^2 \phi^2 t^2 + bgk_0 g^2 n^2 t^2 + bg \phi f t}{b^2 n^2 g^2 t^2 + 2bg \phi f t - 4ub + g^2}, \]
\[ w^*_{(RM)} = \frac{c\phi^2 n^2 g^2 t^2 + 2bg \phi f t + 2k_0 u \phi f t - 2cu + c g^2 - 2au}{b^2 n^2 g^2 t^2 + 2bg \phi f t - 4ub + g^2}, \]
\[ f^*_{(RM)} = \frac{(g + bn \phi f)(bc - a + bk_0 \phi f t)}{b^2 n^2 g^2 t^2 + 2bg \phi f t - 4ub + g^2}, \]
\[ \Pi^*_{m(MM)} = \frac{u(bc - a + bk_0 \phi f t)^2}{2(b^2 n^2 g^2 t^2 + 2bg \phi f t - 4ub + g^2)}. \]
the manufacturer then decides the wholesale price and the quality control level. Let

\[ \Pi^*_r(RM) = \frac{bu^2(bc - a + bk_0\phi t)^2}{(b^2n^2\phi^2t^2 + 2bgn\phi t - 4ub + g^2)^2} \]

\[ \Pi^*_c(RM) = -\frac{u(bc - a + bk_0\phi t)^2(2b^2n^2\phi^2t^2 + 2bgn\phi t - 6ub + g^2)}{2(b^2n^2\phi^2t^2 + 2bgn\phi t - 4ub + g^2)^2}. \]

**Proof of Proposition 2.** The decision order of supply chain members in the RM model is consistent with that in the MM model, and the solution process of the two models is similar.

The Hesse matrix of \( \Pi_m(RM) \) about \( w(RM) \) and \( f(RM) \) is

\[ H_2 = \begin{pmatrix} -b & \frac{bn\phi t}{2} \\ \frac{bn\phi t}{2} & -u \end{pmatrix}. \]

When \( 4bu - g^2 - 2bgn\phi t - b^2n^2\phi^2t^2 > 0 \), \( \Pi_m(RM) \) has a maximum value with respect to \( w(RM) \) and \( f(RM) \). It is easy to find \( \Pi^*_m(RM), \Pi^*_r(RM), \Pi^*_c(RM) \) and \( p^*_m(RM), f^*_r(RM) \) by solving \( w^*_m(RM), f^*_r(RM) \).

### 3.3. MR Model

**MR Model:** The retailer leads the supply chain decision and the manufacturer bears the loss of return.

**Proposition 3.** Under the MR model, the optimal decision and profit of the supply chain, respectively, are

\[ p^*_m(RM) = \frac{k_0b^2gn\phi^2t^2 + cb^2gn\phi t + 2ab^2n^2\phi^2t^2 - k_0ub^2\phi t}{2b(b^2n^2\phi^2t^2 + 2bgn\phi t - 4ub + g^2)^2}, \]

\[ w^*_m(RM) = \frac{k_0b^2n^2\phi^2t^2 + cb^2n^2\phi^2t^2 + 3k_0b^2gn\phi^2t^2 + 3c^2n^2\phi^2t^2 + 3c^2b^2gn\phi t + 3c^2b^2gn\phi t + 3c^2b^2gn\phi t + 3c^2b^2gn\phi t + 3c^2g^2 + ag^2 + au}{2b^2n^2\phi^2t^2 + 4bgn\phi t - 4ub + 2g^2}, \]

\[ f^*_m(RM) = \frac{(g + bn\phi t)(bc - a + bk_0\phi t)}{2b^2n^2\phi^2t^2 + 4bgn\phi t - 4ub + 2g^2}, \]

\[ \Pi^*_m(RM) = -\frac{u(bc - a + bk_0\phi t)^2}{8(b^2n^2\phi^2t^2 + 2bgn\phi t - 2ub + g^2)^2}, \]

\[ \Pi^*_r(RM) = -\frac{u(bc - a + bk_0\phi t)^2}{4(b^2n^2\phi^2t^2 + 2bgn\phi t - 2ub + g^2)^2}, \]

\[ \Pi^*_c(RM) = -\frac{3u(bc - a + bk_0\phi t)^2}{8(b^2n^2\phi^2t^2 + 2bgn\phi t - 2ub + g^2)^2}. \]

**Proof of Proposition 3:** In the MR model, the retailer first decides the retail price and the manufacturer then decides the wholesale price and the quality control level. Let
\[ p_{(MR)} = w_{(MR)} + y_{(MR)} \], where \( y_{(MR)} \) denotes the retailer’s profit from selling a single item. In this case, the manufacturer’s profit and retailer’s profit, respectively, are

\[ \Pi_{m(MR)} = (w_{(MR)} - c)[a - b(w_{(MR)} + y_{(MR)})] - th[a - b(w_{(MR)} + y_{(MR)})] - \frac{1}{2} w^2 f^2_{(MR)}, \]

\[ \Pi_r(MR) = y_{(MR)}[a - b(w_{(MR)} + y_{(MR)}) + g f_{(MR)}]. \]

According to the above formulas, we can easily obtain:

\[ \frac{\partial \Pi_{m(MR)}}{\partial w_{(MR)}} = a - b(w_{(MR)} + y_{(MR)}) + f_{(MR)} b + (c - w_{(MR)}) + b \phi t(k_0 - f_{(MR)} n), \]

\[ \frac{\partial \Pi_{m(MR)}}{\partial f_{(MR)}} = n \phi t(a - b(w_{(MR)} + y_{(MR)}) + f_{(MR)} b) - g_{(MR)}(c - w_{(MR)}) - g \phi t(k_0 - f_{(MR)} n) - f_{(MR)} u. \]

From \( \frac{\partial \Pi_{m(MR)}}{\partial w_{(MR)}} \) and \( \frac{\partial \Pi_{m(MR)}}{\partial f_{(MR)}} \), it is easy to obtain the Hessian matrix of \( \Pi_{m(MR)} \) with respect to \( w_{(MR)} \) and \( f_{(MR)} \):

\[ H_3 = \begin{pmatrix} -2b & g - bn \phi t \\ g - bn \phi t & 2\phi t n u - b \end{pmatrix}. \]

The condition for this matrix to be negative definite is that \( 2bu - 2bn^2\phi^2t^2 - g^2 > 0 \). Under this condition, there is an optimal solution for \( \Pi_{m(MR)} \). If \( \frac{\partial \Pi_{m(MR)}}{\partial w_{(MR)}} = 0 \) and \( \frac{\partial \Pi_{m(MR)}}{\partial f_{(MR)}} = 0 \), we can obtain

\[ w_{(MR)} = \frac{c g - au - bcw_{(MR)} + bnu_{(MR)} + g^2 k_0 \phi t - b^2 n^2 \phi^2 t^2 y_{(MR)} + ag \phi t}{b^2 n^2 \phi^2 t^2 + 2b \phi t n - 2ub + g^2}, \]

\[ f_{(MR)} = \frac{(g + bn \phi t)(bc - a + by + bk_0 \phi t)}{b^2 n^2 \phi^2 t^2 + 2b \phi t n - 2ub + g^2}. \]

By substituting \( w_{(MR)} \) and \( f_{(MR)} \) into \( \Pi_r(MR) \) and taking the first derivative of \( \Pi_r(MR) \) with respect to \( p_{(MR)} \), we can obtain

\[ \frac{\partial \Pi_r(MR)}{\partial y_{(MR)}} = \frac{bu(b c - a + 2y_{(MR)} + bk_0 \phi t)}{b^2 n^2 \phi^2 t^2 + 2b \phi t n - 2ub + g^2}. \]

Let \( \frac{\partial \Pi_r(MR)}{\partial y_{(MR)}} = 0 \), and we can obtain

\[ y^*_{(MR)} = \frac{bc - a + bk_0 \phi t}{2b}. \]

By substituting \( y^*_{(MR)} \) back into \( w_{(MR)} \) and \( f_{(MR)} \), we can obtain

\[ w^*_{(MR)} = \frac{k_0 b^2 n^2 \phi^3 t^3 + cb^2 n^2 \phi^2 t^2 + 3k_0 b^2 \phi^2 t^2 + 3c b^2 \phi^2 t^2 + abn^2 \phi^2 t^2}{2b^2 n^2 \phi^2 t^2 + 4b \phi t n - 4ub + 2g^2}, \]

\[ f^*_{(MR)} = \frac{(g + bn \phi t)(bc - a + bk_0 \phi t)}{2b^2 n^2 \phi^2 t^2 + 4b \phi t n - 4ub + 2g^2}. \]
3.5. SS Model

Proof of Proposition 4: The process of solving the RR model is similar to that of the MR game among the members of the supply chain, the manufacturer and retailer are regarded decentralized decision models. Under the centralized decision model, there is no interest in the sales of the manufacturer and retailer. Finally, we can solve for the values of $\Pi_{m(MR)}^*, \Pi_{r(MR)}^*$ and $\Pi_{c(MR)}^*$.

3.4. RR Model

RR model: the retailer dominates the supply chain decision and the retailer bears the loss of return.

Proposition 4. Under the RR model, the optimal decision and profit of the supply chain are

\begin{align*}
p_{(RR)}^* &= \frac{k_0b^2g_{nft}t^2 + cb^2g_{nft}t + 2ab^2n^2\phi t^2 - k_0ub^2\phi t}{2b(b^2n^2d_{nft}t^2 + 2b_{nft}\phi - 2ub + g^2)}.
\end{align*}

Finally, we can solve for the values of $\Pi_{m(RR)}^*$, $\Pi_{r(RR)}^*$ and $\Pi_{c(RR)}^*$. □

Proof of Proposition 4: The process of solving the RR model is similar to that of the MR model. Let $p_{(RR)} = w_{(RR)} + y_{(RR)}$, and we obtain

\begin{align*}
\Pi_{m(RR)} &= \frac{u(2bu - g^2)(bc - a + bk_{o\phi}t)^2}{8(g^2 + b_{nft}g - 2bu)^2},
\end{align*}

\begin{align*}
\Pi_{r(RR)} &= -\frac{u(bc - a + bk_{o\phi}t)^2}{4(g^2 + b_{nft}g - 2bu)^2},
\end{align*}

\begin{align*}
\Pi_{c(RR)} &= -\frac{u(3g^2 + 2bnftg - 6bu)(bc - a + bk_{o\phi}t)^2}{8(g^2 + b_{nft}g - 2bu)^2}.
\end{align*}

The condition that satisfies this Hesse matrix is negative definite is $2bu - g^2 > 0$. The subsequent solution process is consistent with the RR model solution process, and finally $p_{(RR)}, w_{(RR)}', y_{(RR)}', f_{(RR)}, \Pi_{m(RR)}, \Pi_{r(RR)}, \Pi_{c(RR)}$ can be solved. □
as a whole, and the decision goal of both sides is to realize the optimal interests of the whole supply chain. In this model, the supply chain profit function is

$$\Pi_c(SS) = (p_{(SS)} - c)(a - bp_{(SS)} + gf_{(SS)}) - th(a - bp_{(SS)} + gf_{(SS)}) - \frac{1}{2}uf_{(SS)}^2. \quad (29)$$

**Proposition 5.** Under the SS model, the optimal decision and profit of the supply chain are

$$p^*_c(SS) = \frac{cS^2 - au - bcu + g^2k_0\phi t + agn\phi t - bk_0\phi t u + abm^2\phi^2t^2 + bgk_0n\phi^2t^2 + bck_0n\phi t}{b^2n^2\phi^2t^2 + 2bgn\phi t - 2ub + g^2},$$

$$f^*_c(SS) = \frac{(g + b\phi t)(bc - a + bk_0\phi t)}{b^2n^2\phi^2t^2 + 2bgn\phi t - 2ub + g^2},$$

$$\Pi^*_c(SS) = -\frac{u(bc - a + bk_0\phi t)^2}{2(b^2n^2\phi^2t^2 + 2bgn\phi t - 2ub + g^2)}. \quad (32)$$

**Proof of Proposition 5.** The first partial derivatives of $$\Pi_c(SS)$$ with respect to $$p_{(SS)}$$ and $$f_{(SS)}$$ are as follows:

$$\frac{\partial \Pi_c(SS)}{\partial p_{(SS)}} = a + f_{(SS)}g - 2bp_{(SS)} + bc + b\phi t(k_0 - f_{(SS)}n),$$

$$\frac{\partial \Pi_c(SS)}{\partial f_{(SS)}} = gP_{(SS)} - gc - f_{(SS)}u - g\phi t(k_0 - f_{(SS)}n) + n\phi t(a + f_{(SS)}g - bp_{(SS)}).$$

According to $$\frac{\partial \Pi_{c(SS)}}{\partial p_{(SS)}}$$ and $$\frac{\partial \Pi_{c(SS)}}{\partial f_{(SS)}}$$, it is easy to obtain the Hesse matrix of $$\Pi_c(SS)$$ with respect to $$p_{(SS)}$$ and $$f_{(SS)}$$ as follows:

$$H_5 = \begin{pmatrix} -2b & g - b\phi t \\ g - b\phi t & 2g\phi t - u \end{pmatrix}.$$

Under the conditions mentioned above, this second-order Hessian matrix is negative definite, which implies that $$\Pi_c(SS)$$ is a concave function. By setting $$\frac{\partial \Pi_{c(SS)}}{\partial p_{(SS)}} = 0$$ and $$\frac{\partial \Pi_{c(SS)}}{\partial f_{(SS)}} = 0$$, the following optimal solutions can be obtained:

$$p^*_c(SS) = \frac{cS^2 - au - bcu + g^2k_0\phi t + agn\phi t - bk_0\phi t u + abm^2\phi^2t^2 + bgk_0n\phi^2t^2 + bck_0n\phi t}{b^2n^2\phi^2t^2 + 2bgn\phi t - 2ub + g^2},$$

$$f^*_c(SS) = \frac{(g + b\phi t)(bc - a + bk_0\phi t)}{b^2n^2\phi^2t^2 + 2bgn\phi t - 2ub + g^2}.$$

By substituting $$p^*_c(SS)$$ and $$f^*_c(SS)$$ into $$\Pi_c(SS)$$, we can obtain $$\Pi^*_c(SS)\. \Box$$

**4. Comparison and Analysis of Models**

This section compares decision and profit in different supply chain models and analyzes the impact of important factors on the supply chain. The main contents include the following: by comparing the total profit, quality control level and price of the five supply chain models, Proposition 6, Proposition 8 and Proposition 9 are obtained; by comparing the profit of the manufacturer and retailer in different supply chain models, Proposition 7 is
It is easy to obtain Proof of Proposition 6.

\[
\frac{\Pi^*_c(MM)}{\Pi^*_c(MR)} = \frac{4}{3},
\]

\[
\frac{\Pi^*_c(MM)}{\Pi^*_c(RM)} = 1,
\]

\[
\frac{\Pi^*_c(RR)}{\Pi^*_c(MR)} = \frac{3(b^2n^2\phi^2t^2 + 2b\phi gft - 4ub + g^2)^2}{4(b^2n^2\phi^2t^2 + 2b\phi gft - 2ub + g^2)(b^2n^2\phi^2t^2 + 2b\phi gft - 6ub + g^2)} > 1,
\]

and

\[
\frac{\Pi^*_c(RR)}{\Pi^*_c(MR)} = \frac{(3g^2 + 2b\phi gtg - 6bu)(b^2n^2\phi^2t^2 + 2b\phi gft - 2ub + g^2)}{3(g^2 + b\phi gtg - 2bu)^2} < 1.
\]

□

Remark 1. Proposition 7 shows that in the decentralized decision model, the profit of the supply chain members who make decisions first is higher than that of other supply chain members.

Proposition 7. \(\Pi^*_m(MM) > \Pi^*_r(MM), \Pi^*_m(RM) > \Pi^*_r(RM), \Pi^*_m(MR) < \Pi^*_r(MR), \Pi^*_m(RR) < \Pi^*_r(RR).\)

Proof of Proposition 7. It is easy to obtain

\[
\Pi^*_m(MM) - \Pi^*_r(MM) = \frac{u(bc - a + bk\phi t)^2(b^2n^2\phi^2t^2 + 2b\phi gft - 4ub + g^2)}{2(b^2n^2\phi^2t^2 + 2b\phi gft - 4ub + g^2)^2} > 0,
\]

\[
\Pi^*_m(RM) - \Pi^*_r(RM) = \frac{u(bc - a + bk\phi t)^2(b^2n^2\phi^2t^2 + 2b\phi gft - 4ub + g^2)}{2(b^2n^2\phi^2t^2 + 2b\phi gft - 4ub + g^2)^2} > 0,
\]

\[
\Pi^*_m(MR) - \Pi^*_r(MR) = \frac{u(bc - a + bk\phi t)^2}{8(b^2n^2\phi^2t^2 + 2b\phi gft - 4ub + g^2)} < 0,
\]

\[
\Pi^*_m(RR) - \Pi^*_r(RR) = \frac{u(bc - a + bk\phi t)^2(g^2 + b\phi gtg - 2bu)}{8(g^2 + b\phi gtg - 2bu)^2} < 0.
\]

□

Remark 2. Proposition 7 shows that in the decentralized decision model, the leader of the supply chain decision has an advantage in the interest game, so its profit is always higher than that of the follower.

Proposition 8. \(f^*_r(RM) = f^*_r(MM) < f^*_r(MR) < f^*_r(SS) < f^*_r(RR) < f^*_r(MR).\)

Proof of Proposition 8. It is easy to obtain

\[
\frac{f^*_r(SS)}{f^*_r(MR)} = 2 > 1, \frac{f^*_r(MM)}{f^*_r(RM)} = 1,
\]
Proof of Proposition 9. When $bc - a + bk_0 \phi t < 0$ and $g < bnft$, it is easy to obtain

$$p^{*}_{(SS)} - p^{*}_{(RM)} = \frac{(bc - a + bk_0 \phi t) (g^2 + bnftg - bu)}{2b(b^2n^2 \phi t^2 + 2bgnft - 2ub + g^2)} < 0,$$

and

$$p^{*}_{(MM)} - p^{*}_{(RM)} = \frac{(g + bnft)^2 (bc - a + bk_0 \phi t) (g^2 + bnftg - bu)}{2b\left(b^4n^4 \phi t^4 + 4b^3gn^2 \phi t^3 - 6b^2n^2 \phi t^2u + 6b^1gn^2 \phi t^2 - 12b^2gnftu + 8b^2u^2 + 4b^3nft - 6bgnftu + 3b^3g^2nt - 4b^2g^2u + g^4\right)} < 0,$$

and

$$p^{*}_{(MR)} - p^{*}_{(RR)} = \frac{bnftu(g - bnft)(bc - a + bk_0 \phi t)}{2\left(b^3gn^3 \phi t^4 - 2b^2n^2 \phi t^2u + 3b^2g^2 \phi t^2 - 6b^2gnftu + 4b^2u^2 + 3bgnftu - 4b^2g^2u + g^4\right)} < 0.$$

Remark 3. Proposition 8 shows that the quality control level in the centralized decision model is the highest. In the decentralized decision model, the quality control level in the MR model is higher than that in the RR model and MM model, and the quality control level in the MM model is equal to that in the RM model.

Proposition 10. (1) $\frac{\partial \Gamma^{*}_{(SS)}}{\partial \delta} > 0$, $\frac{\partial \Gamma^{*}_{(MM)}}{\partial \delta} > 0$, $\frac{\partial \Gamma^{*}_{(MR)}}{\partial \delta} > 0$, $\frac{\partial \Gamma^{*}_{(RM)}}{\partial \delta} > 0$, $\frac{\partial \Gamma^{*}_{(RR)}}{\partial \delta} > 0$;

(2) $\frac{\partial f^{*}_{(SS)}}{\partial g}$, $\frac{\partial f^{*}_{(MM)}}{\partial g}$, $\frac{\partial f^{*}_{(MR)}}{\partial g}$, $\frac{\partial f^{*}_{(RM)}}{\partial g}$, $\frac{\partial f^{*}_{(RR)}}{\partial g}$.

Proof of Proposition 10. It is easy to obtain

$$\frac{\partial \Gamma^{*}_{(SS)}}{\partial \delta} = \frac{u(g + bnft)(bc - a + bk_0 \phi t)^2 (b^2n^2 \phi t^2 + 2bgnft - 8ub + g^2)}{(b^2n^2 \phi t^2 + 2bgnft - 4ub + g^2)^3} > 0,$$

and

$$\frac{\partial f^{*}_{(SS)}}{\partial g} = \frac{(bc - a + bk_0 \phi t)(b^2n^2 \phi t^2 + 2bgnft + 2ub + g^2)}{(b^2n^2 \phi t^2 + 2bgnft - 2ub + g^2)^2} > 0,$$

the proof process of other items is similar, so it is omitted. 

Remark 4. Proposition 9 shows that the selling price in the MM model is equal to the retail price in the RM model, and that both the retail price in the MM model and the retail price in the RR model are larger than the retail price in the MR model. The retail price is the lowest in the centralized decision model.
Remark 5. Proposition 10 shows that the total profit of the supply chain and the quality control level in different supply chain models are proportional to the sensitivity coefficient of the quality control level, that is, consumers’ perception of quality control has an impact on the supply chain decision.

Proposition 11. (1) \( \frac{\partial \Pi^*_{(SS)}}{\partial n} > 0, \frac{\partial \Pi^*_{(MM)}}{\partial n} > 0, \frac{\partial \Pi^*_{(MR)}}{\partial n} > 0, \frac{\partial \Pi^*_{(SK)}}{\partial n} > 0; \)

(2) \( \frac{\partial f^*_{(SS)}}{\partial n}, \frac{\partial f^*_{(MM)}}{\partial n}, \frac{\partial f^*_{(MR)}}{\partial n}, \frac{\partial f^*_{(SK)}}{\partial n} \).

Proof of Proposition 11. It is easy to obtain

\[
\frac{\partial \Pi^*_{(SS)}}{\partial n} = \frac{b \phi^2 t (g + b \phi t) (bc - a + b \phi t)^2}{(b^2 \phi^2 t^2 + 2 b g \phi t - 2 a b + g^2)^2} > 0,
\]

\[
\frac{\partial f^*_{(SS)}}{\partial n} = -\frac{b \phi (bc - a + b \phi t) (b^2 \phi^2 t^2 + 2 b g \phi t + 2 a b + g^2)^2}{(b^2 \phi^2 t^2 + 2 b g \phi t - 2 a b + g^2)^2} > 0,
\]

the proof process of other items is similar, so it is omitted. □

Remark 6. Proposition 11 shows that the total profit of the supply chain and the quality control level in different supply chain models are proportional to the quality control effect. This means that quality control effects can influence supply chain decisions.

5. Joint Contract Coordination

According to the research conclusion in the above section, the supply chain decision and profit of the centralized decision model are better than those of the four decentralized decision models. We introduce here joint contracts to coordinate the decentralized decision model (MM model, for example). Joint contracts mainly include cost sharing, revenue sharing and discounted wholesale prices. The purpose of contract coordination is (1) to improve the overall profit of the supply chain; (2) to improve the quality control level in the production process of electronic products; (3) to improve the profits of all members of the supply chain. The contract requires that the retailer shares part of the revenue with the manufacturer and helps the manufacturer to share part of the input cost of quality control, with the share ratio of \( \varphi \) and the share ratio of \( \rho \). In addition, the manufacturer needs to reduce wholesale prices, which indirectly brings profits to the retailer. The proof of the manufacturer and the profit of the retailer are as follows (the contract coordination model is marked with “C”):

\[
\Pi_m(C) = (w(C) - c)D(C) - (x - v)hD(C) - \epsilon hD(C) - \frac{1}{2}(1 - \rho)\mu f(C)^2 + \varphi p(C)D(C), \quad (33)
\]

\[
\Pi_r(C) = ((1 - \varphi)p(C) - w(C))D(C) - \frac{1}{2}\rho \mu f(C)^2. \quad (34)
\]

Proposition 12. Under contract coordination, the decisions of supply chain are

\[
f^*_{(C)} = \frac{g\left(c - w(C)\right)}{2} + \frac{\varphi^2 \phi t}{2} - n \phi t \left(a + \frac{b w(C) - a(\varphi - 1)}{2\varphi - 2}\right) - \frac{g\left(a + \frac{b w(C) - a(\varphi - 1)}{2\varphi - 2}\right)}{2b} + \frac{g\left(b w(C) - a(\varphi - 1)\right)}{4b(\varphi - 1)}
\]

\[
2u\left(\frac{\varphi}{2} - \frac{1}{2}\right) + \frac{\varphi^2 \phi t}{2b} + \varphi \phi t
\]

\[
p^*_{(C)} = -\frac{b w(C) - \left(a + f^*_{(C)}\right)(\varphi - 1)}{2b(\varphi - 1)}. \quad (35)
\]
Proof of Proposition 12. Let \( \frac{\partial \Pi_m(C)}{\partial p(C)} = 0 \), and we obtain

\[
p(C) = \frac{(q - 1)(a + f(C)g) - bw(C)}{2b(q - 1)}.
\]

We can solve for \( f^*_C \) and \( p^*_C \) by substituting \( p(C) \) into \( \Pi_m(C) \) and setting \( \frac{\partial \Pi_m(C)}{\partial p(C)} = 0 \).

\[ \square \]

Proposition 13. If \( q, \rho \text{ and } w(C) \) satisfy

\[
\begin{align*}
q &= \frac{a + \phi_fSS \delta - 2b p^*_SS + bw(C)}{a + \phi_fSS \delta - 2b p^*_SS} \\
\rho &= \frac{a^2 g + f^*_SS \delta^2 + 2a f^*_SS \delta^2 + 2abg w(C) - bcf^*_SS \delta^2 + 2b c g p^*_SS - 2b f^*_SS \delta^2 p^*_SS}{ab f^*_SS \delta^3 + 2b f^*_SS \delta u + 2b f^*_SS \delta^2 w(C) + 2b f^*_SS \delta^2 p^*_SS w(C) - abc g - 2abg p^*_SS}, \\
\end{align*}
\]

then the supply chain profit can reach the optimum.

Proof of Proposition 13. If \( f^*_C = f^*_SS \) and \( p^*_C = p^*_SS \), then the relationship among \( q, \rho \text{ and } w(C) \) can be obtained. \( \square \)

Proposition 14. According to the incentive compatibility principle, when the value range of wholesale price satisfies the following inequality, the profits of supply chain members under contract coordination can be optimized, and at this time, all members of the supply chain are willing to accept the contract.

\[
2 \left( \frac{f^*_SS F^4}{4b F^2} - bu^2 F^2 \right) \left( a + f^*_SS \delta - 2b p^*_SS \right) < w(C) < 2 \left( \frac{f^*_SS F^4}{4b F^2} - u F^2 \right) \left( a + f^*_SS \delta - 2b p^*_SS \right),
\]

where

\[
F1 = f^*_SS \delta^2 + 4b^2 p^*_SS^2 + 4abc + af^*_SS \delta - 4ab p^*_SS - 4b^2 cp^*_SS + 3bc f^*_SS \delta - 4b f^*_SS \delta p^*_SS, \\
F2 = 4abk \delta + 4b^2 k \delta p^*_SS t - b f^*_SS \delta^2 \delta n p^*_SS t - 2ab f^*_SS \delta n p^*_SS t + 3b f^*_SS \delta k \delta p^*_SS t,
\]

\[
F3 = \frac{(bc - a + bk \delta)}{b^2 n^2 \delta^2 t + 2b g \delta n p^*_SS t - 4ub + \delta^2},
\]

\[
F4 = ag + f^*_SS \delta^2 - b c g - 2b f^*_SS \delta u + 2ab n p^*_SS t - 2b g \delta k \delta p^*_SS t - b f^*_SS \delta g n p^*_SS t, \\
F5 = 2a^2 - 4ab p^*_SS + 3a f^*_SS \delta + 2b^2 p^*_SS^2 - 3b f^*_SS \delta p^*_SS + f^*_SS \delta^2.
\]

Proof Proposition 14. From \( \Pi^*_{m(C)} > \Pi^*_{m(MM)} \) and \( \Pi^*_{r(C)} > \Pi^*_{r(MM)} \), we can find the range of values of \( w(C) \). \( \square \)
6. Numerical Example

This section uses numerical simulation to verify the above conclusions and further analyzes the differences between different supply chain models. The model parameter settings in this paper refer to Zhang et al. [28]. All parameters should satisfy the following relationships:

(i) The parameter value conforms to the actual market sales of electronic products;
(ii) The parameter setting conforms to all the assumptions above;
(iii) \( bu - 2bnftg - b^2n^2\phi l^2 - g^2 > 0, g < bnot, k = k_0 - nf > 0. \)

After the above conditions are met, the parameters of this study are set in Table 4.

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6.1. Comparison of Models

From Table 5, the following conclusions can be drawn:

(1) Compared with the other four supply chain models, the SS model is a centralized decision model, that is, all members of the supply chain make decisions together, and the goal of the decisions is to achieve the optimal overall profit of the supply chain. The total profit of the supply chain is positively correlated with the level of quality control and negatively correlated with the retail price. Therefore, the SS model has the highest level of quality control, the lowest retail price and the highest total profit of the supply chain.

(2) When the manufacturer is the leader: the manufacturer gives priority to the decision of the wholesale price and quality control level. In the MM model, considering the input cost of quality control, the manufacturer is more inclined to adjust the wholesale price to transfer its return losses. Therefore, the quality control level, retail price and profit in the RM model are the same as those in the MM model, and the wholesale price of the MM model is higher than that of the RM model.

(3) When the retailer is the leader: in the RR model, the retailer can only compensate for the loss of profit caused by the return by increasing the retail price; in the MR model, the manufacturer will improve the quality control level, reduce the rate of defective products, and then reduce the return loss. Therefore, compared with the RR model, the quality control level in the MR model is higher and the retail price is lower. This will cause the total profit of the MR model to be higher than the total profit of the RR model.

(4) When the manufacturer bears the loss: under the premise of considering the input cost, the manufacturer will appropriately improve the quality control level and adjust the wholesale price to reduce the return loss. However, in the MR model, in order to obtain greater market demand, the retailer will indirectly guide the manufacturer to improve the quality control level of products through decisions, while reducing retail prices. Therefore, compared with the RR model, the manufacturer in the MR model has a higher level of quality control, a lower rate of defective products, and a greater total profit of the supply chain.

(5) In the MM model and RM model, as the leader of supply chain decisions, the manufacturer obtains more profits than the retailer in the decision game. In the MR model and the RR model, the retailer’s profit is higher than the manufacturer’s. This is because the leader can predict the follower’s decision when making priority decisions, which can make it dominant in the interest game.

In this section, Propositions 6–9 are tested. From the above conclusions and analysis, it can be seen that the supply chain leader with the priority decision can obtain greater game advantages, and the supply chain members should actively strive for the priority decision to obtain greater profits. In addition, when different members of the supply chain bear the loss of returned goods, there are differences in the decisions of the supply chain.
on price and quality control level, but when the manufacturer and retailer share the loss, the profit and decision of the supply chain are the best. Therefore, from the perspective of the optimal overall interests of the supply chain, the supply chain members should be encouraged to cooperate closely and make joint decisions.

Table 5. Decision and profit numerical tables under different supply chain models.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>$p^*$</th>
<th>$f^*$</th>
<th>$w^*$</th>
<th>$\Pi_m^*$</th>
<th>$\Pi_r^*$</th>
<th>$\Pi_c^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MM model</td>
<td>80.3029</td>
<td>0.3137</td>
<td>60.4490</td>
<td>783.4352</td>
<td>394.1777</td>
<td>1177.6129</td>
</tr>
<tr>
<td>RM model</td>
<td>80.3029</td>
<td>0.3137</td>
<td>59.7078</td>
<td>783.4352</td>
<td>394.1777</td>
<td>1177.6129</td>
</tr>
<tr>
<td>M R model</td>
<td>80.1785</td>
<td>0.1357</td>
<td>40.7185</td>
<td>394.1932</td>
<td>788.3864</td>
<td>1182.5796</td>
</tr>
<tr>
<td>RR model</td>
<td>80.2413</td>
<td>0.0990</td>
<td>39.8082</td>
<td>391.8760</td>
<td>781.6333</td>
<td>1173.5093</td>
</tr>
<tr>
<td>SS model</td>
<td>60.3569</td>
<td>0.6313</td>
<td>——</td>
<td>——</td>
<td>——</td>
<td>1576.7729</td>
</tr>
</tbody>
</table>

6.2. Sensitivity Analysis

It can be seen from Figure 2 that the total profit of the supply chain in the five models is positively correlated with $g$ and $n$.

![Figure 2. The trend chart of the impact of $g$ and $n$ changes on the supply chain profit.](image)

The better the quality control effect is, the higher the return on investment of the quality control level is. At this time, the manufacturer tends to improve the quality control level to reduce the return rate of products: the greater the sensitivity coefficient of the quality control level, the higher the consumer’s preference for the quality control level and the greater the market demand that will be obtained by improving the quality control level. Therefore, both $g$ and $n$ can positively promote the improvement in the quality control level and the total profit of the supply chain.

In this section, Propositions 10 and 11 are tested. Quality control is of great significance to the supply chain. While improving the quality control level, supply chain managers should also pay attention to the improvement of the quality control effect to obtain greater benefits. In addition, supply chain managers should also strengthen the publicity of product quality control, improve consumers’ preference and perception regarding product quality control, and then increase the positive impact of quality control on the supply chain.

Figure 3 shows that in the five models, the change trend of the quality control level is positively influenced by $g$ and $n$. 

![Figure 3. The trend chart of the impact of $g$ and $n$ changes on the quality control level of the supply chain.](image)
In this section, Propositions 10 and 11 are tested. Quality control is of great significance to the supply chain. While improving the quality control level, supply chain managers should also pay attention to the improvement of the quality control effect to obtain greater benefits. In addition, supply chain managers should also strengthen the publicity

6.3. Verification of Contract Coordination Effect

According to the parameter values in this section, the wholesale price ranges from 5.0557 to 10.1752. From Figure 4, it can be seen that contract coordination has a significant effect on the improvement of supply chain profits; the profits of the manufacturer and retailer after contract coordination are higher than those before contract coordination. As wholesale prices rise, manufacturers’ profits are gradually shrinking, while retailers’ profits are increasing. According to the graph trend in Figure 4, when the wholesale price value is not within [5.0557, 10.1752], the contract coordination fails, which verifies the rationality of Proposition 14.

The wholesale price change within [5.0557, 10.1752] does not affect the coordination effect of the supply chain but affects the distribution of benefits between the manufacturer and retailer. The specific value of the wholesale price depends on the bargaining power
between the manufacturer and the retailer. Members with a strong voice can obtain greater profits through the adjustment of wholesale prices.

Contract coordination can promote the common improvement of the profits of all parties in the supply chain. As a member of the supply chain, on the one hand, the contract should be strictly performed to ensure the coordination effect of the contract; on the other hand, the member should pursue the maximization of their own interests as much as possible through excellent negotiation abilities under the conditions of contract constraints.

6.4. Optimization Verification of the Solutions

Taking the centralized decision model as an example, the optimality of the solution is verified. The validity of the research method in this paper is verified. As can be seen from Figure 5, there is a unique optimal value for the total profit of the supply chain, and the optimal solution corresponding to the optimal value is consistent with the statements in Table 5 and Proposition 5. This proves the validity of the research method and verifies the proposition of decision result in the paper.

![Figure 5. The influence of decision change on the total profit of supply chain.](image)

7. Conclusions

Based on the impact of the level of quality control on the return rate and market demand of electronic products, this paper constructs five supply chain models that consider the cross-channel return behavior of consumers, with differences in the order of decision and return loss bearers among supply chain members in different models. Stackelberg knowledge is used to solve the optimal decision under different supply chain models. Through the analysis, the following conclusions were reached:

1. The supply chain decision under the centralized decision model is optimal, which makes the total profit of the supply chain reach the ideal level;
2. The total profit and quality control level of the supply chain under the RM model and the MM model are the same, and the manufacturer transfers its own return loss by adjusting the wholesale price;
3. The supply chain decision under the MR model is better than the MM model and the RR model;
4. The supply chain leader’s profit is always higher than its followers;
5. The quality control level and total profit under different supply chain models are positively correlated with \( g \) and \( n \). In addition, in order to solve the problem of supply chain imbalance in the decentralized decision model, this paper designs a joint contract to coordinate the cooperative relations of supply chain members. It is found that under the coordination contract, the supply chain members realize the transfer of benefits through the adjustment of the wholesale price, and the change in the wholesale price in a given interval will not affect the contract coordination effect. Finally, this paper verifies the validity of the relevant conclusions and contracts through an
example analysis. The research conclusions provide a theoretical basis for supply chain decisions under different modes. Among them, contract coordination plays an important role in improving the quality of electronic products and promoting the stability of supply chains. According to the relevant research conclusions, this paper puts forward some suggestions to supply chain members and managers: supply chain members should strengthen cooperation and actively strive for decision leadership; under contract constraints, supply chain members should take the initiative to exert their negotiating advantages to obtain greater profits; and supply chain managers should pay attention to the effect of quality control and strengthen the publicity of product quality control.

There are some limitations in this paper. For example, in this paper, manufacturers and retailers play games in the case of information symmetry, and in the actual supply chain of electronic products, information asymmetry also exists. In future research, the supply chain with asymmetric information and the possible false information of supply chain members will be studied. In addition, the decision problem of multi-type electronic product supply chains will be explored.

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