Joint Optimization of Battery Swapping Scheduling for Electric Taxis

Zilong Zhao 1,2, Daxin Tian 1,*, Xuting Duan 1 and Randong Xiao 3,4,*

1 School of Transportation Science and Engineering, Beihang University, Beijing 100083, China; zhaozilong@jtw.beijing.gov.cn (Z.Z.); duanxuting@buaa.edu.cn (X.D.)
2 Beijing Municipal Commission of Transport, Beijing 100161, China
3 Beijing Key Laboratory for Comprehensive Traffic Operation Monitoring and Service, Beijing 100161, China
4 Beijing Intelligent Transportation Development Center, Beijing 100161, China
* Correspondence: dtian@buaa.edu.cn (D.T.); xiaorandong@jtw.beijing.gov.cn (R.X.)

Abstract: Electric taxis are of great benefit to reduce urban polluting gas emissions. Currently, there is a problem of uneven utilization among charging stations in the battery swapping activities of electric taxis, resulting in long battery swapping times for some taxis, which seriously affects operational efficiency. This study examines the real-time joint scheduling of electric taxis for battery swapping. The expense due to battery swapping for each electric taxi was measured as the sum of driving duration before battery swapping, queuing and operating duration during battery swapping, and cruising duration after battery swapping; to our knowledge, these have not been combined in the existing literature. A mathematical model was proposed with the objective of minimizing the total expense of all electric taxis, subject to the battery state-of-charge (SoC) constraint. The proposed model was transformed into a mixed-integer linear program and solved to global optimality by CPLEX. Numerical results validated the performance of our methodology. The results indicate that our proposed methodology can save total expenses by up to 7.61%.

Keywords: battery swapping; cruising duration; electric taxis; queuing duration

1. Introduction

Taxis are an essential part of a comprehensive city transport system and complement urban public transport by providing personalised transport services to the public and meeting people’s “door-to-door” travel needs [1,2]. According to statistics on the city of Beijing, in the first half of 2022, there were more than 60,000 cruising taxis in Beijing, and the average daily number of cruising vehicle trips was more than 410,000 with a passenger volume of approximately 590,000, thus comprising a huge amount of travel. However, the existing taxis are mainly fuel vehicles, which generate a large amount of polluting gas emissions during operation [3,4]. In recent years, taxis have started to be gradually electrified due to the need for energy saving and emissions reduction. By rough estimate, a purely electric taxi replacing a traditional fuel taxi can reduce carbon emissions by 50,000 tonnes per year [3]. Therefore, governments around the world are stepping up efforts to promote electric taxis. On 23 March 2017, China issued the 2017 Air Pollution Prevention Work Programme for Beijing, Tianjin, Hebei and surrounding areas, which requires that all new taxis in Beijing be exchanged by electric vehicles, and other cities are actively promoting the replacement of taxis with electric vehicles or new energy vehicles. According to the “14th Five-Year Plan for Transport Development and Construction in Beijing”, it is necessary to strengthen the optimization of transport travel structure and energy structure, vigorously promote the new energy of transport means, and continuously reduce carbon emissions in the transport sector. In the 14th Five-Year Plan period, 100% new energy will be used for cruising taxis. By the first half of 2022, Beijing completed the addition and renewal of more than 20,000 pure electric taxis.
For electric vehicles (EVs), there are two modes of electric power replenishment: battery charging and battery swapping. However, the battery charging mode holds many problems in practical application, such as long charging time and difficult battery maintenance [5,6]. Generally speaking, it may take one or more hours for an EV to fully charge its battery, depending on factors such as the size of its battery and when and where it is charged. For EVs, the excessively long charging time shortens the operating time and indirectly reduces their operating income. This makes it difficult for the charging model to meet the demand for energy replenishment for large-scale EVs.

The battery swapping mode uses a replacement battery to replenish the energy of the EVs. Battery swapping stations would swap out a battery from a vehicle that is almost completely discharged. When the empty battery is returned to the station, it is fully charged to be reused later. Since the vehicle just needs to wait for the battery to be switched, not charged, this technique of refilling EVs has the benefit of being very quick for each vehicle. The separable nature of the taxi and the battery enables the battery to be charged outside of the taxi’s downtime limit, which is more likely to ensure the operational efficiency of EVs. Unlike the charging mode, the battery swapping mode can replenish energy quickly, the vehicle can be replaced within 1.5 to 3 min after arriving at the station, and the battery charging process is centrally managed and monitored by the battery swapping station (BSS), which is safer and more controllable. From the point of view of battery compatibility and vehicle unified management, the battery swapping mode is more suitable for vehicles with rapid electrical energy replenishment needs such as electric taxis.

At present, Beijing’s taxi cabs are switched at Aulton New Energy Vehicle Technology Company Limited (Aulton), which has more than 160 BSSs in operation in Beijing (as shown in Figure 1), forming a large-scale commercial battery swapping operation solution with 1000 daily changes. However, through analysis of drivers’ battery swapping at each BSS, we found that a serious imbalance between supply and demand occurs at the stations during peak hours, with some BSSs having long queues, which both increases the waiting cost of taxis and tends to cause traffic jams, while others have very few vehicles to exchange, failing to effectively share the demand for vehicle exchange during peak periods. This is mainly due to the fact that drivers can currently only find a BSS on their own in two ways: either by choosing a frequent BSS based on experience, or by checking the location and queues of nearby BSSs through the app. This experience-dependent, spontaneous switching behaviour increases the cost of battery swapping for drivers in both time and space, and inevitably results in demand-supply imbalance of BSSs and does not maximise the utilisation of the BSSs.

![Figure 1. The BSSs operated by Aulton company in Beijing.](image-url)
and lacks a large-scale synergy for the whole electric taxi system. In recent years, the emergence and construction of city-level Mobility-as-a-Service (MaaS) platforms have unlocked multiple sources of traffic data, enabling us to gain a comprehensive understanding of the state of traffic within cities and the associated use of resources, in real time. In this regard, this study uses the MaaS platform to generate a system-optimal recommendation scheme for BSSs based on the real-time location of electric taxis, queuing situation of BSSs and other data. The main contribution of this study is that it simultaneously takes into account various factors including driving duration before battery swapping, queuing duration, and operating duration during battery swapping, and cruising duration after battery swapping, so as to effectively reduce the total expense due to battery swapping of the electric taxi system.

The rest of this paper is structured as follows. The studies about the energy replenishment of EVs are reviewed in Section 2. A problem description and analysis about the battery swapping scheduling of electric taxis is provided in Section 3. Section 4 formulates the optimization model. Section 5 conducts numerical experiments. The work is concluded in Section 6, which also offers suggestions for future research.

2. Literature Review

Aiming at improving the efficiency of energy replenishment of EVs, many scholars have conducted research on the optimization of scheduling in charging mode or battery swapping mode. In the line of charging mode, Sun and Yang [7] and Yang et al. [8] provided a reverse inductive optimization algorithm for the selection of charging time slots for electric taxis based on time-sharing tariffs and additional charging costs. Based on customer booking demand and charging schedule, Lu et al. [9] proposed a taxi fleet scheduling strategy. Yang et al. [10] constructed a mixed-integer model to implement the charging allocation problem for electric taxis and the best charging sequence assigned to the same charging pile, thus minimizing the non-operating time of all taxis, considering the real-time charging demand of electric taxis and the state of charging piles. Guo et al. [11] studied the spatial scheduling problem for new energy vehicles, aiming to provide recommendations for geographically distributed charging stations for new energy vehicles to minimize travel and queuing times. An efficient game-theoretic strategy is suggested to guarantee the fairness of the recommendations. Numerical results show that the method is effective in saving time and reducing the idle rate of charging stations and the queuing rate of new energy vehicles. Alizadeh et al. [12] took into account the decision-making process faced by an individual EV owner who must choose a route, including the charging stations along the way and the corresponding charge amount, while taking into account changing traffic conditions and dynamic location-based power pricing. The authors demonstrated that the issue is analogous to locating the shortest path on a larger transportation graph. To depict the user’s charging possibilities, the study specifically used virtual linkages with negative energy requirements to extend the original transportation graph.

In the line of battery swapping mode, in order to provide a framework for BSSs with a local charging mode, Tan et al. [13] created a queueing network model with an open queue of EVs and a closed queue of batteries. The authors demonstrated that the queueing system’s equilibrium equations and the steady-state distribution is the result of solving these finite equilibrium equations. Meanwhile, they provided an alternative yet significantly simpler method to compute the steady-state distribution by utilizing the embedded Markov chain. Numerous significant performance metrics have been analytically developed based on the steady-state distribution. Adler and Mirchandani [14] modelled the online routing of EVs as a Markov chance-decision process to reduce the average delay of all vehicles. The temporal differencing with linear models approximate dynamic programming method was used to determine the best policy. When connected to a central computer, onboard vehicle software may quickly route vehicles using the solution algorithm. Gan et al. [15] provided a decentralized system to plan EV charging efficiently. In order to enforce a broad understanding of valley-filling and examine the characteristics of ideal charging profiles,
the EV charging scheduling problem was defined as an optimal control problem, which is then iteratively solved using a decentralized approach. The algorithm uses the elasticity of the loads on EVs to fill in the valleys in the profiles of electric load. By permitting the available stations to function as both recharging stations and battery swapping stations, Verma [16] provided a variation of the EV routing problem with time windows and recharging stations. A model and method for this problem were described, and conclusions about the circumstances in which taking into account BSSs is most helpful were offered. In Zheng et al. [17], a framework for optimal design of battery charging/swap stations in distribution systems based on life cycle cost (LCC) was presented. To compare the effects of battery swapping stations and rapid charging stations, battery charging/swap station models were created. The distribution network needs to be strengthened in the meanwhile to accommodate the higher power needs during the charging period. The right magnitude and location of the stations will help to control the cost of the reinforcement. The LCC criterion was utilized to evaluate the project for the best cost-benefit analysis and safety operation, and a modified differential evolution method was employed to resolve the issue. Wang et al. [18] modelled the fleet of electric taxis as a mobile sensor network, examined historical sensing data from taxi routes, assessed the demand profile for battery swapping, the power consumption of individual taxis, as well as the travel time between spots in the road network. After that, they provided an algorithm to determine an efficient battery swapping station design and then described a real-time algorithm to schedule a portion of the empty taxicabs to change batteries early by allowing them to escape congestion.

Based on the above studies, we can conclude that the existing research is deficient in two aspects. Firstly, the current research considers a single factor, while the battery swapping scheduling of electric taxis needs to consider many factors such as the order demand around BSSs after battery swapping [19,20]. Secondly, most of the existing research rarely considers the relevant influence between the internal battery swapping behaviour of electric taxis and lacks a large-scale synergy for the whole electric taxi system [21].

3. Problem Description and Analysis

Electric taxis constantly consume battery energy during operation. When the battery level of an electric taxi falls below a certain threshold, it will withdraw from service and generate a request for battery swapping to a BSS. For an electric taxi with a battery swapping request, there may be multiple options for a BSS, which requires station selection and route planning. After selecting and arriving at a BSS, the electric taxi has to wait in queue until performing battery swapping. Once the battery swapping is complete, the electric taxi leaves the BSS and resumes its service status.

We consider the potential expense of an electric taxi due to battery swapping. The factors that need to be considered in selecting BSSs include driving duration before battery swapping, queuing duration, and operating duration during battery swapping, and cruising duration after battery swapping. These are defined below.

- **Driving duration before battery swapping:** When an electric taxi goes to a BSS, the travel duration refers to the time it takes to reach the target BSS from its current location; this would be influenced by the real-time traffic condition on the road.
- **Queuing duration during battery swapping:** When an electric taxi reaches a BSS, the queuing duration refers to the queuing time it takes to wait the battery swapping.
- **Operating duration during battery swapping:** When an electric taxi begins battery swapping at the BSS, the operating duration refers to the time from the beginning to the end of the battery swapping operation.
- **Cruising duration after battery swapping:** The demand heat for rides near the BSS would affect the taxi’s cruising time after battery swapping. The cruising duration is defined as the time it takes to pick up an order after it leaves the BSS.

As can be seen, the driving duration before battery swapping mainly depends on the distance between the taxi and the BSS. The queuing duration during battery swapping depends mainly on the size of the battery swapping demand at the BSS. The size of the
battery swapping demand varies significantly from one BSS to another. Taking Beijing city as an example, Figure 2 counts the average daily number of completed swappings at each of the BSSs operated by Aulton company. Each BSS is represented by a pillar, and the higher the pillar, the redder the color, which means the average daily number of completed swappings is higher. As can be seen, the highest number of power changes is at Fengtai Beijing South Station, Chaoyang Suyuanqiao D Station, Fengtai Majiapu Station, Fengtai Liuliqiao Station and Chaoyang Jingwangjiayuan D Station, and the highest number of battery swapping is at Dongcheng Anzhenli Station, with 219 swappings per day.

![Figure 2](image-url) \(\text{Figure 2. The average daily number of completed swaps at each of the BSSs.}\)

The cruising duration after battery swapping is directly related to the demand heat for rides in the vicinity of the BSS, which affects the time it takes for a taxi to pick up orders after battery swapping. In practice, the demand for taxi rides varies greatly from BSS to BSS, so this can have a significant impact on the battery swapping scheduling of electric taxis. We counted the number of orders within a 3 km radius, centred on the BSSs operated by Aulton company in Beijing, as shown in Figure 3. In Figure 3, the larger the area of the circle indicates the higher the demand around the BSS. As can be seen, the demand for taxi rides varies greatly from BSS to BSS.

![Figure 3](image-url) \(\text{Figure 3. Demand heat map for rides around BSSs.}\)

This study considers the problem of battery swapping scheduling for an electric taxi system in a city. From the perspective of an urban electric taxi system, there will be multiple taxis generating a demand for battery swapping at the same moment in time, and the battery swapping scheduling between different taxis will affect each other. For example,
if multiple taxis visit the same BSS at the same moment, the queuing duration for these taxis could be significantly longer. This therefore requires a joint scheduling of all taxis requiring battery swapping within the whole system. The objective of this study is to jointly schedule all the electric taxis for battery swapping and to recommend the most suitable BSS for each electric taxi, so as to minimize the total expenses due to battery swapping of the whole system.

4. Battery Swapping Scheduling Optimization Model for Electric Taxis

A group of electric taxis is controlled by an operation center, which is responsible for providing advice on BSS for the taxis that are out of electricity. There is a total of $J$ BSSs, $j \in Z = \{1, 2, \cdots, J\}$. The position of BSS $j$ is $d_j$. Consider a discrete time horizon $t \in T = \{1, 2, \cdots, T\}$, and the operation center needs to assign BSSs for the taxis at each time. At each time $t$, there are $I(t), i \in \mathbb{R}(t) = \{1, 2, \cdots, I(t)\}$ electric taxis that need to be assigned to BSSs for battery swapping. Electric taxi $i$ has its current position and current state-of-charge (SoC) $b_i(t)$. The task of the operation center is to determine the BSS $j$ for each electric taxi $i$ to swap its battery at each time $t$.

The optimization objective in this study is to minimize the expenses due to battery swapping, i.e., to minimize the sum of driving duration before battery swapping, queuing and operating duration during battery swapping, and cruising duration after battery swapping. At time $t$, we assume that the decision variable for assigning each electric taxi to a single BSS is $x_{ij}(t) \in \{0, 1\}$, which is a binary variable that equals 1 if electric taxi $i$ is assigned to BSS $j$, otherwise 0. Then, for electric taxi $i$, the travel time to the BSS is

$$TT_i = \sum_{j \in Z} d_{ij}(t) x_{ij}(t)$$

in which $d_{ij}(t)$ represents the travel time needed from the position of electric taxi $i$ to the position of BSS $j$ at time $t$. In practice, we can obtain the fastest travel path using a map service in real time.

For an electric taxi, the queuing duration during battery swapping is mainly determined by the length of the queue before it. Therefore, an electric taxi may be assigned to a BSS a little further to avoid waiting a long time at a closer BSS. To this end, $n_j(t)$ is defined to be the number of electric taxis waiting for battery swapping at BSS $j$ at the beginning of time $t$. Then, the length of the queue before electric taxi $i$ is $n_j(t) + \text{Card}\{i' | d_{ij}(t) + M_1 \left(1 - x_{ij}(t)\right) < d_{ij}(t), i' \in \mathbb{R}(t)\}$, in which $M_1$ is a large enough number, and function Card is to count the number of taxis in the set $\mathbb{R}(t)$ that meet the given conditions $d_{ij}(t) + M_1 \left(1 - x_{ij}(t)\right) < d_{ij}(t), i' \in \mathbb{R}(t)$. Assume the operating time required to complete a battery swapping for each BSS is $\delta_j, j = 1, 2, \cdots, J$. Then, the queuing duration for battery swapping of electric taxi $i$ is

$$QT_i = \sum_{j \in Z} \left(n_j(t) + \text{Card}\{i' | d_{ij}(t) + M_1 \left(1 - x_{ij}(t)\right) < d_{ij}(t), i' \in \mathbb{R}(t)\}\right) \delta_j x_{ij}(t)$$

The operating duration for battery swapping of each BSS is determined by its level. For example, the BSSs operated by Aulton company in Beijing belong to two types: Type-3.0 and Type-4.0. For Type-3.0 BSSs, the operating time required to complete a battery swapping is 5 min, while the Type-4.0 BSSs only take approximately 3 min. Assume the operating time for each BSS is $\delta_j, j \in Z$. Then, for electric taxi $i$, the operating duration for the battery swapping is

$$OT_i = \sum_{j \in Z} \delta_j x_{ij}(t)$$

The cruising duration after leaving the BSS refers to the time a taxi takes to pick up an order after the swap is completed, which is related to the demand heat for rides in the vicinity of the BSS. Generally, the demand for taxi rides varies greatly from BSS to BSS and from time to time. Regarding this, we assume the cruising time needed to pick up an
order around BSS \( j \) at each time \( t \) is \( \rho_j(t) \). Then, for electric taxi \( i \), the cruising duration after battery swapping is

\[
PT_i = \sum_{j \in \mathbb{Z}} \rho_j(t)x_{ij}(t)
\]  

(4)

Summing up, the expense due to battery swapping of electric taxi \( i \) is \( TT_i + QT_i + OT_i + PT_i \). From the perspective of improving the battery swapping efficiency of the electric taxi system, the goal is to minimize the total expenses due to battery swapping of all electric taxis, formulated as follows

\[
\min \sum_{i \in \mathbb{R}} (TT_i + QT_i + OT_i + PT_i)
\]

(5)

When assigning the BSSs to electric taxis, some constraints must be satisfied. First, each electric taxi can only be assigned to a BSS, i.e.,

\[
\sum_{j \in \mathbb{Z}} x_{ij}(t) = 1, \quad \forall i \in \mathbb{R}(t)
\]

(6)

Second, for electric taxi \( i \), only BSSs within its driving range, which is depending on the SoC \( b_i(t) \), could be considered as its recommendation candidates. Assume the electricity consumption rate of electric taxis per unit time is \( \Delta \), so \( x_{ij}(t) \) must satisfy

\[
d_{ij}(t)\Delta - b_i(t) \leq M_2(1 - x_{ij}(t)), \quad \forall i \in \mathbb{R}(t), \ j \in \mathbb{Z}
\]

(7)

In Equation (7), \( M_2 \) is a large enough number. Equation (7) ensures that if \( d_{ij}(t)\Delta > b_i(t) \), i.e., BSS \( j \) is out of the driving range of electric taxi \( i \), then \( x_{ij}(t) \) must be equal to 0; if \( d_{ij}(t)\Delta \leq b_i(t) \), i.e., BSS \( j \) is within the driving range of electric taxi \( i \), then \( x_{ij}(t) \) can take any value.

The proposed model (1)–(7) is a binary program. In the proposed model, the objective function contains the following nonlinear terms

\[
\begin{align*}
&\left[ n_j(t) + \text{Card} \{i' | d_{ij}(t) + M_1(1 - x_{ij}(t)) < d_{ij}(t), i' \in \mathbb{R}(t)\} \right] \delta_{ij}(t) \\
&= n_j(t)\delta_{x_{ij}(t)} + \text{Card} \{i' | d_{ij}(t) + M_1(1 - x_{ij}(t)) < d_{ij}(t), i' \in \mathbb{R}(t)\} \delta_{x_{ij}(t)}
\end{align*}
\]

Introducing two sets of auxiliary variables \( y_{ij}(t) \geq 0, \ i \in \mathbb{R}(t), \ j \in \mathbb{Z} \) and \( z_{ij}(t) \in \{0, 1\}, \ i' \in \mathbb{R}(t), \ j \in \mathbb{Z} \), we transform the nonlinear term into a linear one

\[
n_j(t)\delta_{x_{ij}(t)} + \delta_{y_{ij}(t)} \]

by adding the following constraints:

\[
y_{ij}(t) \leq M_3x_{ij}(t), \quad \forall i \in \mathbb{R}(t), \ j \in \mathbb{Z}
\]

(8)

\[
z_{ij}(t) + M_4(x_{ij}(t) - 1) \leq y_{ij}(t), \quad \forall i, \ i' \in \mathbb{R}(t), \ j \in \mathbb{Z}
\]

(9)

\[
d_{ij}(t) - d_{ij}(t) - M_1(1 - x_{ij}(t)) < M_5z_{ij}(t), \quad \forall i, \ i' \in \mathbb{R}(t), \ j \in \mathbb{Z}
\]

(10)

in which \( M_3, M_4, M_5 \) are large enough numbers. Equation (8) ensures that when \( x_{ij}(t) = 0 \), then \( y_{ij}(t) = 0 \). When \( x_{ij}(t) = 1 \), Equation (9) leads to \( z_{ij}(t) \leq y_{ij}(t) \). In addition, if \( d_{ij}(t) + M_1(1 - x_{ij}(t)) < d_{ij}(t) \), Equation (10) constrains \( z_{ij}(t) = 1 \). Combining these, we have \( y_{ij}(t) \geq 1 \). Since the objective is minimization, \( y_{ij}(t) = 1 \) is optimal.

Through the above transformation, the proposed model becomes a mixed-integer linear programming. A mixed-integer linear programming can be solved to global optimality by the branch and bound method, branch and pricing method, or commercial solvers, such as CPLEX and Gurobi. Due to the fact that CPLEX is a flexible and high-performance solver for mixed-integer linear programming, we adopted CPLEX to solve the proposed model in this study.
5. Numerical Experiments

This section is devoted to verifying the effectiveness of the proposed methodology by numerical experiments. Consider there are 4 BSSs on a 10 km × 10 km plane, whose positions are (4.27, 4.96), (1.52, 7.59), (8.21, 7.63) and (7.79, 2.09), respectively. The operating time for completing a battery swapping, the order pick-up time and the initial queue length of each BSS are provided in Table 1. At a certain time $t$, there are 25 electric taxis requiring battery swapping. The position and SoC of each electric taxi are listed in Table 2. For simplicity, the Euclidean distance is used to represent the travel distances between electric taxis and BSSs, and the travel time is assumed to be proportional to the travel distance. The electricity consumption rate of electric taxis per unit time is $\Delta = 0.01$.

Table 1. The information of BSSs.

<table>
<thead>
<tr>
<th>No.</th>
<th>Position</th>
<th>Operating Time (Min)</th>
<th>Order Pick-Up Time (Min)</th>
<th>$n_j(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(4.27, 4.96)</td>
<td>3</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>(1.52, 7.59)</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>(8.21, 7.63)</td>
<td>5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>(7.79, 2.09)</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 2. The position and SoC of each electric taxi.

<table>
<thead>
<tr>
<th>No.</th>
<th>Position</th>
<th>SoC</th>
<th>No.</th>
<th>Position</th>
<th>SoC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(3.63, 3.53)</td>
<td>0.10</td>
<td>14</td>
<td>(5.01, 9.74)</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>(0.56, 5.70)</td>
<td>0.19</td>
<td>15</td>
<td>(2.81, 2.67)</td>
<td>0.09</td>
</tr>
<tr>
<td>3</td>
<td>(3.24, 2.41)</td>
<td>0.25</td>
<td>16</td>
<td>(6.29, 8.68)</td>
<td>0.21</td>
</tr>
<tr>
<td>4</td>
<td>(8.72, 7.01)</td>
<td>0.12</td>
<td>17</td>
<td>(4.57, 8.89)</td>
<td>0.07</td>
</tr>
<tr>
<td>5</td>
<td>(1.03, 7.74)</td>
<td>0.20</td>
<td>18</td>
<td>(7.14, 7.42)</td>
<td>0.06</td>
</tr>
<tr>
<td>6</td>
<td>(0.89, 4.34)</td>
<td>0.16</td>
<td>19</td>
<td>(3.05, 9.64)</td>
<td>0.07</td>
</tr>
<tr>
<td>7</td>
<td>(8.33, 7.29)</td>
<td>0.05</td>
<td>20</td>
<td>(7.75, 9.34)</td>
<td>0.10</td>
</tr>
<tr>
<td>8</td>
<td>(4.31, 7.30)</td>
<td>0.06</td>
<td>21</td>
<td>(8.27, 8.21)</td>
<td>0.08</td>
</tr>
<tr>
<td>9</td>
<td>(5.57, 3.04)</td>
<td>0.15</td>
<td>22</td>
<td>(1.58, 1.68)</td>
<td>0.21</td>
</tr>
<tr>
<td>10</td>
<td>(3.38, 8.78)</td>
<td>0.13</td>
<td>23</td>
<td>(6.67, 1.10)</td>
<td>0.10</td>
</tr>
<tr>
<td>11</td>
<td>(4.98, 4.38)</td>
<td>0.11</td>
<td>24</td>
<td>(9.60, 5.16)</td>
<td>0.11</td>
</tr>
<tr>
<td>12</td>
<td>(7.96, 2.37)</td>
<td>0.15</td>
<td>25</td>
<td>(9.58, 0.72)</td>
<td>0.24</td>
</tr>
<tr>
<td>13</td>
<td>(4.41, 4.11)</td>
<td>0.22</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We recorded all of our runs using the optimization software CPLEX 12.10.0. All the runs were solved to optimality. The optimal assignment scheme obtained by our proposed methodology is shown in Figure 4. Taxis 1, 3, 6, 8, 11, 13, 15, and 22 are assigned to BSS 1, taxis 2, 5, 10, 17, and 19 are assigned to BSS 2, taxis 4, 7, 14, 16, 18, 20, and 21 are assigned to BSS 3, and taxis 9, 12, 23, 24, and 25 are assigned to BSS 4. The corresponding total expense of the electric taxi system is 582.44.

Figure 4. The optimal assignment scheme.
In order to deeply analyse the effect of different factors, we considered five different optimization strategies, as follows:

- **Strategy 1**: Driving duration;
- **Strategy 2**: Driving duration + Operating duration;
- **Strategy 3**: Driving duration + Operating duration + Cruising duration;
- **Strategy 4**: Driving duration + Operating duration + Queuing duration;
- **Strategy 5**: Driving duration + Operating duration + Queuing duration + Cruising duration.

Strategy 1 only considers the factor of driving duration when assigning the taxis, while strategy 2 considers the factors of both driving duration and operating duration. On this basis, strategy 3 and strategy 4 further involve the factor of cruising duration and the factor of queuing duration, respectively. Strategy 5, i.e., our proposed methodology, considers all four factors. The comparative results are listed in Table 3. It can be seen that the corresponding total expense continuously decreases from strategy 1 to strategy 5. Comparing strategy 1 and strategy 5, the total expense decreases up to 7.61%. From the specific view, strategy 1 has the least driving duration, but its queuing duration is the largest, resulting in more expense than other strategies. Compared to strategy 1, strategy 2 shortens the operating duration and queuing duration, while slightly increasing the driving duration and cruising duration, resulting in an overall decrease in the total expense. This indicates the necessity of considering level differences of BSSs when assigning taxis. Furthermore, simultaneously considering the factor of cruising duration, strategy 3 significantly reduces the cruising duration of taxis. Simultaneously considering the busyness of BSSs, i.e., the waiting for battery swapping, strategy 4 significantly reduces the cruising duration and total expense of taxis. This indicates that each electric taxi should be assigned from the perspective of the whole electric taxi system, with consideration of the assignment of other taxis. At last, strategy 5 considers all four factors, and further reduces the total expense by balancing the different durations.

**Table 3.** The results of different strategies.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Total Expense</th>
<th>Driving Duration (Min)</th>
<th>Queuing Duration (Min)</th>
<th>Operating Duration (Min)</th>
<th>Cruising Duration (Min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>630.44</td>
<td>49.44</td>
<td>390</td>
<td>91</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>628.43</td>
<td>50.43</td>
<td>387</td>
<td>87</td>
<td>104</td>
</tr>
<tr>
<td>3</td>
<td>608.48</td>
<td>53.48</td>
<td>372</td>
<td>91</td>
<td>92</td>
</tr>
<tr>
<td>4</td>
<td>588.72</td>
<td>86.72</td>
<td>325</td>
<td>77</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>582.44</td>
<td>65.44</td>
<td>342</td>
<td>89</td>
<td>86</td>
</tr>
</tbody>
</table>

In order to further explore the performance of the proposed methodology, we conducted experiments on some key parameters. The first experiment is to investigate the effect of operating time for a battery swapping. We varied the value of operating time while keeping other parameters unchanged and obtained the corresponding total expense, as shown in Table 4. It can be seen that as the operating time continues to shorten, the total expense will significantly decrease. This reflects the importance of continuously improving battery swapping technology and further shortening the operating time for the BSSs.

**Table 4.** Effect of operating time for a battery swapping.

<table>
<thead>
<tr>
<th>BSS</th>
<th>Operating Time (Min)</th>
<th>Operating Time (Min)</th>
<th>Operating Time (Min)</th>
<th>Operating Time (Min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
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<tr>
<td>2</td>
<td>3</td>
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<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Total expense</td>
<td>582.44</td>
<td>537.44</td>
<td>461.44</td>
<td>360.44</td>
</tr>
</tbody>
</table>
The second experiment is to investigate the effect of order pick-up time for a ride order. We varied the value of order pick-up time while keeping other parameters unchanged and obtained the corresponding total expense, as shown in Table 5. It can be seen that as the order pick-up time continues to shorten, the total expense will significantly decrease. This suggests the importance of accurately predicting the ride demand.

**Table 5.** Effect of order pick-up time for a ride order.

<table>
<thead>
<tr>
<th>BSS</th>
<th>Order Pick-Up Time (Min)</th>
<th>Order Pick-Up Time (Min)</th>
<th>Order Pick-Up Time (Min)</th>
<th>Order Pick-Up Time (Min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>4</td>
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<tr>
<td>2</td>
<td>4</td>
<td>3</td>
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<td>2</td>
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<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Total expense</td>
<td>582.44</td>
<td>561.44</td>
<td>549.44</td>
</tr>
</tbody>
</table>

6. Conclusions and Future Research Directions

This paper presents the real-time joint scheduling of electric taxis for battery swapping at BSSs. The proposed model takes into account the driving duration before battery swapping, queuing and operating duration during battery swapping, cruising duration after battery swapping, and battery SoC constraint, with the objective of minimizing the total expenses due to battery swapping of the whole electric taxi system. The proposed model is transformed into a mixed-integer linear program, and the global optimality is obtained by CPLEX. Numerical results show the effectiveness of our proposed methodology.

This study can be extended in several directions. First, the electricity consumption rate per unit time is assumed as a constant value for all electric taxis. In fact, it may vary with different taxis, in relation to their own characteristics and the current SoC. A more in-depth description of the electricity consumption rate per unit time would be valuable. Second, as an integer program, the proposed model is hard to solve with a large number of electric taxis. Efficient heuristic algorithms are necessary for high-timeliness application.

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