Assessment of Mountain Tourism Sustainability Using Integrated Fuzzy MCDM Model

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Abstract: The sustainable development of mountain tourism is crucial for preserving the delicate ecosystems and resources found in these unique landscapes. This research paper investigates the sustainability of mountain lodges, which serve as essential facilities for delivering mountain tourism services. To assess sustainability, expert decision making involving eight selected experts was employed. A hybrid approach combining the IMF SWARA (IMproved Fuzzy Step-wise Weight Assessment Ratio Analysis) method with Fuzzy Dombi Aggregation Operators was utilized to determine the weights of various sustainability criteria. The IMF SWARA method assigned initial weights based on expert input, which were subsequently adjusted using Fuzzy Dombi Aggregation Operators. The findings highlight the significance of two key criteria as per expert evaluations: the quality of the services offered (C21) and the preservation of natural resources (C15). To rank and evaluate the mountain lodges, the fuzzy CRADIS (Compromise Ranking of Alternatives from Distance to Ideal Solution) method was employed, ultimately identifying Zabrana (ML6) as the top-ranked mountain lodge. The validity of these results was confirmed through result validation and sensitivity analysis. This research contributes by providing insights into the current state of mountain tourism and offering guidelines for enhancing the overall mountain tourism experience through the integration of fuzzy methods.

Keywords: mountain tourism; mountain lodges; IMF SWARA method; Fuzzy Dombi Aggregation Operators; fuzzy CRADIS method

1. Introduction

The global tourism industry has witnessed a paradigm shift in its focus, with an increasing emphasis on sustainability and responsible practices. One particular niche within this context that has garnered significant attention is mountain tourism, driven by the allure of breathtaking landscapes, outdoor adventures, and cultural experiences. As this sector continues to grow, the need for a comprehensive assessment framework becomes imperative to ensure its long-term viability while preserving the delicate balance of the natural environment, local communities, and economic interests [1–3]. This paper introduces an innovative approach to tackle this challenge.

Mountainous regions have always captivated travelers with their stunning vistas, unique flora and fauna, and rich cultural heritage. The surge in mountain tourism, however, has led to concerns regarding its impact on these fragile ecosystems and local...
communities [4–6]. The notion of sustainability comes to the forefront, calling for a holistic evaluation of mountain tourism’s economic, ecological, and social dimensions.

Engaging in a comprehensive evaluation, the economic facet scrutinizes the financial viability of mountain tourism operations, encompassing factors such as revenue generation, employment opportunities, and local economic growth [7,8]. Simultaneously, the ecological dimension delves into the assessment of environmental impacts, placing emphasis on conservation efforts, carbon footprint management, waste handling strategies, and the preservation of biodiversity [9,10]. Lastly, the social aspect evaluates the dynamic interaction between tourism and local communities, encompassing the preservation of cultural heritage, fostering community engagement, and ensuring the equitable distribution of benefits [11,12]. These dimensions collectively form the carefully curated set of sustainability criteria that lies at the heart of the assessment framework proposed in this paper. By encompassing economic, ecological, and social aspects, this framework provides a comprehensive lens through which the sustainability of mountain tourism can be thoroughly examined and evaluated.

To illustrate the applicability and effectiveness of the proposed methodology, the paper undertakes the evaluation of six distinct mountain lodges. Each lodge represents a unique set of challenges and opportunities, contributing to the diversification of the research’s findings. By analyzing these case studies, the research aims to uncover trends, best practices, and potential areas of improvement in each property’s sustainability practices.

This research is dedicated to evaluating the current utilization of mountain lodges in the context of tourism. The primary aim is to discern both the strengths and weaknesses associated with these lodgings, ultimately striving to enhance the mountain tourism experience. To accomplish this objective, a specialized methodology, designed to assess these lodges, will be applied to a specific sample. Furthermore, this adaptable methodology can be extended to other mountain lodges, allowing a comparative analysis to determine their deviation from the observed ones. This, in turn, will facilitate the formulation of recommendations geared toward improving mountain tourism through the effective utilization of mountain lodges.

To delve into this undertaking, the research will employ the principles of multi-criteria decision making (MCDM). MCDM offers a structured approach to decision making by evaluating various alternatives against specific criteria. Additionally, expert decision making will play a pivotal role in this research, relying on the expertise of professionals to assess the observed alternatives using linguistic values. The application of these values will be facilitated through the utilization of a fuzzy set. The IMF SWARA (Improved Fuzzy Step-wise Weight Assessment Ratio Analysis) method will be employed to determine the weighting of the criteria, while the fuzzy CRADIS (Compromise Ranking of Alternatives from Distance to Ideal Solution) method will ascertain the ranking order of the observed mountain lodges.

The overarching objectives of this paper hold immense significance for diverse stakeholders within the mountain tourism industry. Policymakers and destination managers stand to gain from the insights presented, enabling them to formulate precision-oriented strategies that harmonize economic advancement, environmental conservation, and the well-being of local communities [13]. The framework discussed herein ensures not only the active engagement of local communities in decision making processes but also safeguards their vested interests against potential negative impacts.

Building upon this foundation, research presented in this paper sets the following objectives:

- Conduct a sustainable assessment of mountain tourism and provide comprehensive guidelines for its enhancement;
- Establish a unique methodological framework by integrating various multi-criteria analysis methods tailored to the specific requirements of mountain tourism assessment;
• Offer practical directives for the development of mountain tourism, informed by expert assessments and multi-criteria analysis methodologies.

The contribution of this paper extends to the academic arena by introducing a methodical approach to evaluating sustainability, effectively bridging the gap that often exists between theoretical discourse and practical implementation. This research thus serves to uncover and address significant research gaps in the field:

• The absence of Expert Evaluation. Previous studies on mountain tourism in Bosnia and Herzegovina have not incorporated expert evaluations using multi-criteria analysis methods.

• Novel Methodological Fusion. The amalgamation of these methods with the application of Fuzzy Dombi Aggregation Operators within the IMF SWAVE method represents a novel approach to assessing sustainable mountain tourism.

• The underexplored Role of Mountain Lodges. The role of mountain lodges as integral components of sustainable tourism development in Bosnia and Herzegovina has not been thoroughly investigated in this manner before.

The subsequent sections of this paper are organized as follows: Section 2 delves into the existing literature, shedding light on the evolving landscape of mountain tourism and sustainability assessments. Section 3 outlines the conceptual framework, providing comprehensive details on the methodology and the integration of the Fuzzy MCDM model. Section 4.1 presents the research results based on a variety of analyses. Section 4.2 engages in a discussion of the research conducted and the outcomes. Finally, Section 5 encapsulates the conclusions, practical implications, and avenues for future research.

2. Literature Review

The integration of sustainable practices in the tourism industry has emerged as a critical concern in recent years, driven by the growing recognition of the environmental and social impacts of tourism activities [14–16]. Every form of tourist offering, including sustainable mountain tourism, should contribute to the development of local communities, enabling them to become self-sufficient and sustainable [17]. To achieve this goal, the application of sustainable mountain tourism is imperative. Within this overarching context, mountain tourism holds a unique position, offering unparalleled natural beauty and cultural experiences [18]. However, the delicate balance between attracting tourists and preserving the pristine mountain ecosystems has become a paramount challenge [19]. This literature review delves into the existing body of knowledge surrounding mountain tourism sustainability, as well as on the use of MCDM frameworks as a powerful tool for assessment.

Mountainous regions have long been cherished as attractive tourist destinations, promising breathtaking landscapes, diverse flora and fauna, and opportunities for outdoor recreation [20].

The appeal of these areas is undeniable, but it has led to increased visitation rates, raising concerns about ecological degradation, overexploitation, and cultural disruptions [21,22]. Addressing these challenges requires a comprehensive understanding of the intricate relationships between tourism activities, the environment, and local communities [23].

The notion of sustainability is at the core of efforts to mitigate the negative impacts of tourism in mountainous areas. Researchers and practitioners emphasize the need to balance economic development with ecological conservation and social well-being [24–26]. Sustainability criteria, comprising economic, ecological, and social dimensions, are fundamental for assessing the overall performance of mountain tourism operations. The economic aspect evaluates the financial viability of these operations [27], while the ecological dimension focuses on minimizing the environmental footprint [28,29]. Simultaneously, the social dimension aims to ensure the engagement of local communities and equitable distribution of benefits [30].
Several assessment frameworks have been developed to evaluate the sustainability of mountain tourism. These frameworks often rely on expert opinions [31,32], stakeholder engagement [33], and the integration of various criteria [34]. The challenge lies in addressing the inherent uncertainty and subjectivity involved in such assessments. Several studies have delved into this notion. Dax and Tamme [1] explored sustainable mountain tourism and advocated for the utilization of mountain villages in the development of mountain tourism initiatives. Paunović and Jovanović [35] assessed sustainable mountain tourism in Slovenia and Croatia, emphasizing the constant need for information collection and updates regarding the tourist offerings in these regions. Barthod-Prothade and Leroux [36] investigated the impact of mountaineering activities on the sustainability of mountain tourism. Their findings revealed that the sustainability of mountain tourism is negatively affected by activities such as wild hunting and fishing, livestock wandering, and the absence of water purification devices. Martini et al. [37] researched the influence of female entrepreneurship on the development of sustainable mountain tourism, demonstrating that it can exert both direct and indirect effects in strengthening this type of tourism. Upadhyay [38] examined how policy development affects sustainable mountain tourism in Nepal and identified a lack of driving force to fortify this form of tourism in the country. These examples and others underscore the multifaceted nature of sustainable mountain tourism. However, achieving sustainability in mountain tourism necessitates responsible resource management to yield optimal outcomes in tourism [39]. Furthermore, these studies highlight the importance of engaging a broader spectrum of community members to foster a balanced and sustainable approach to mountain tourism [40].

Traditional MCDM techniques have been employed to address intricate decision-making challenges, yet they frequently grapple with managing the fuzzy nature of real-world evaluations [24,41]. Numerous studies have adopted these methods, some of which are detailed hereinafter. Krishnan et al. [42] tackled the issue of waste management in mountain tourism, employing the DEMATEL (Decision Making Trial and Evaluation Laboratory) method in their research. Fan et al. [43] utilized the SWAT and AHP (Analytic Hierarchy Process) methodology to rank sustainable mountain tourism strategies on Changbai Mountain. Cristache et al. [34] employed these same methods to identify avenues for enhancing tourism development and growth in Romania, with a focus on mountain tourism. Pan et al. [44] conducted an investigation into the feasibility of implementing wellness tourism in Chinese mountain regions, employing the Delphi and ANP (Analytic Network Process) methods. Georgios [45] leveraged the Delphi and AHP methodology in evaluating sustainable mountain tourism in Greece. Ren and Lie [46] used the AHP method to assess ecotourism resources in a mountain tourism context, utilizing the Qinling Mountains as an exemplar. Uyan et al. [47] harnessed Geographical Information System (GIS) and the Best Worst Method (BWM) to identify the most suitable locations for constructing facilities in mountain winter tourism.

The integration of Fuzzy MCDM models into the assessment of mountain tourism sustainability has shown promising results [48,49]. Fuzzy MCDM extends traditional MCDM techniques by incorporating fuzzy logic, allowing for the representation of uncertain and imprecise information [50]. This is particularly valuable in the context of expert assessments, where judgments may not always be precise or quantifiable. By incorporating fuzziness, the Fuzzy MCDM model provides a more realistic representation of the decision making process, capturing the nuanced nature of sustainability evaluations.

In previous research, the IMF SWARA method was employed in tourism-related publications, as evidenced in the work by Zorlu et al. [51]. In this study, they applied the IMF SWARA method to conduct a quantitative assessment of geotourism resources. On the other hand, the fuzzy CRADIS method is relatively new and, to date, has not found application in papers pertaining to the field of tourism.
3. Methodology and Methods

The research for the purposes of this paper was carried out in two phases: the phase of data collection and the phase of data analysis. Each of these phases is divided into additional phases (Figure 1).

Phase 1, the data collection phase, commences with the selection of experts. Eight experts, each holding a doctoral degree in tourism with extensive experience, were chosen. Care was exercised during the selection process to ensure that these experts possess a deep understanding of sustainability issues in tourism, particularly in the context of mountain tourism. These experts were required to provide specific references substantiating their expertise. Some of these experts engage in teaching and scholarly activities at academic institutions, where they impart knowledge on these concepts to students. Meanwhile, other experts contribute to the field of tourism through their research endeavors. Attention was paid to ensure that all selected experts were well acquainted with the prevailing challenges in the tourism industry. Collaboratively, sustainability criteria were selected to evaluate sustainable mountain tourism. These criteria were categorized into three primary sustainability dimensions: ecological, economic, and social (Table 1). These main criteria align with common practices in the evaluation of sustainable tourism. Sub-criteria were determined by adapting criteria from previous studies conducted by Lin [52], Puška et al. [53], Stecyk et al. [54], Hosseini et al. [55], Nasser et al. [56], Lopes et al. [57], Islam et al. [58] and Nedeljković et al. [59] (Table 1). In this manner, while drawing inspiration from and adapting some criteria from these studies, not all identical sub-criteria were incorporated. Moreover, new criteria were introduced that were not explicitly utilized in these previous studies. The strength of this model lies in its systematic adjustment and structured organization of criteria used to assess the sustainability of mountain tourism. As a result, these criteria represent a fresh and innovative approach to evaluating sustainability across diverse tourist regions.

![Table 1. Research criteria.](image)

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>Ecological criteria</td>
</tr>
<tr>
<td>C11</td>
<td>Geographical Characteristics</td>
</tr>
<tr>
<td>C12</td>
<td>Natural Resource Availability</td>
</tr>
<tr>
<td>C13</td>
<td>Natural Resource Conservation</td>
</tr>
<tr>
<td>C14</td>
<td>Aesthetic Landscape Quality</td>
</tr>
<tr>
<td>C15</td>
<td>Natural Resource Quality</td>
</tr>
</tbody>
</table>
With these criteria, an attempt had been made to thoroughly explain the fundamental aspects of sustainability. Each of these main criteria had been assigned with five sub-criteria in order to ensure that equal importance would have been given to each of these main criteria. If one of the main criteria had been assigned more auxiliary criteria than the others, then that criterion would have had a greater influence on the final order of the observed alternatives.

Once the criteria have been established, the next step involves the selection of suitable alternatives that exemplify sustainable mountain tourism. A logical choice in this context is the inclusion of mountain lodges that cater to the needs of tourists. These lodges serve as crucial refuge points, providing shelter from adverse weather conditions commonly encountered in mountainous regions, such as rain, snow, wind, and cold temperatures. Moreover, these facilities offer accommodations and dining services, making them integral to the mountain tourism experience. Based on information gathered from the internet portal dedicated to mountaineering and the promotion of mountain tourism [60], 40 mountain lodges were identified within the region of Bosnia and Herzegovina. Employing a random number generator to ensure a representative sample, a subset of six mountain lodges was selected for observation within this research. The reason only six mountain lodges were chosen is that this is a conceptual paper illustrating how mountain lodges can be evaluated for sustainable tourism needs. In future works, this methodology can be applied to a much larger sample. In this way, the following mountain lodges were chosen:

- Mountain Lodge Jedinci (ML1) is situated near the Hum mountain peak, offering 50 beds. It is located at an elevation of 858 m above sea level.
- Mountain Lodge Nedim Pilav Jogi (ML2) is positioned on the slopes of Mount Jahorina, providing accommodations for up to 40 guests. It is situated at an altitude of 1460 m above sea level.
- Mountain Lodge Igman—Malo polje (ML3) is located on Igman Mountain and offers lodgings for 50 guests. It is situated at an elevation of 1246 m above sea level.
- Mountain Lodge Lopata (ML4) is nestled amidst the slopes of Mount Bitovnja, boasting a capacity of 60 beds. It rests at an altitude of 1324 m above sea level.
- Mountain Lodge Ibrahim Fezić (ML5) graces the slopes of the Bjelašnica mountain, providing accommodations for a maximum of 32 guests. It is positioned at an elevation of 965 m above sea level.
- Mountain Lodge Zabranâ (ML6) is situated along the slopes of Crni Vrh hill, offering a cozy retreat with 18 available beds. It is positioned at an elevation of 620 m above sea level.

The altitudes of these mountain lodges were measured using a hypsometer instrument combined with barometer and thermometer instruments to determine the exact al-
titude. With the assistance of the Association Planinarenje, the actual height of all these mountain lodges, available on their website, has been determined.

Once the criteria and alternatives have been identified, a survey questionnaire is developed, comprising two distinct sections. The initial segment of the survey questionnaire is designed to assess the significance of the criteria as perceived by experts. In this section, experts are tasked with first evaluating the importance of the primary criteria and subsequently assessing the auxiliary criteria. They are required to gauge the significance of these criteria, establish a hierarchy of importance, and provide evaluations through linguistic ratings (Table 2). These linguistic assessments are tailored to align with the IMF SWARA method. These linguistic values are applied in a way that allows each expert to assess and rank the importance of each criterion. First, experts determine the significance of each criterion and arrange them in order of importance. The most crucial criterion is labeled as “Equal significant”, serving as the baseline. Subsequently, experts compare each criterion’s importance to this primary criterion. In this process, experts gauge how much more important the most significant criterion is compared to the next one, progressing through each criterion until they have evaluated all of them. If two criteria are equally important, they are both assigned the value, “Equal significant”. In completing the survey questionnaire, experts start by ranking the criteria and then proceed to order them by importance, while also determining each criterion’s significance relative to the next-lowest-ranked criterion.

Table 2. Membership function and fuzzy numbers.

<table>
<thead>
<tr>
<th>Linguistic Variable</th>
<th>TFN</th>
<th>Linguistic Terms</th>
<th>TFN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolutely less significant (ALS)</td>
<td>(1, 1, 1)</td>
<td>Absolutely bad (AB)</td>
<td>(1, 1, 2)</td>
</tr>
<tr>
<td>Dominantly less significant (DLS)</td>
<td>(1/2, 2/3, 1)</td>
<td>Very bad (VB)</td>
<td>(2, 3, 4)</td>
</tr>
<tr>
<td>Much less significant (MLS)</td>
<td>(2/5, 1/2, 2/3)</td>
<td>Bad (B)</td>
<td>(3, 4, 5)</td>
</tr>
<tr>
<td>Really less significant (RLS)</td>
<td>(1/3, 2/5, 1/2)</td>
<td>Medium bad (MB)</td>
<td>(4, 5, 6)</td>
</tr>
<tr>
<td>Less significant (LS)</td>
<td>(2/7, 1/3, 2/5)</td>
<td>Equal (E)</td>
<td>(5, 6, 7)</td>
</tr>
<tr>
<td>Moderately less significant (MDLS)</td>
<td>(1/4, 2/7, 1/3)</td>
<td>Medium good (MG)</td>
<td>(6, 7, 8)</td>
</tr>
<tr>
<td>Weakly less significant (WLS)</td>
<td>(2/9, 1/4, 2/7)</td>
<td>Good (G)</td>
<td>(7, 8, 9)</td>
</tr>
<tr>
<td>Equal significant (ES)</td>
<td>(0, 0, 0)</td>
<td>Extremely good (EG)</td>
<td>(8, 9, 10)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Absolutely good (AG)</td>
<td>(9, 10, 10)</td>
</tr>
</tbody>
</table>

The second segment of the survey questionnaire assumes the responsibility of evaluating the mountain lodge alternatives based on auxiliary criteria. For each expert, the task involves assessing the observed lodges utilizing linguistic ratings spanning from “Absolutely Bad (AB)” to “Absolutely Good (AG)” (Table 2). To this end, a nine-level value scale is employed. This scale is strategically adopted to obviate the need for experts to discern between benefit-type and cost-type criteria, as its applicability remains consistent across criteria of various natures. This scale is chosen to eliminate the need for experts to distinguish between benefit-type and cost-type criteria, as its applicability remains consistent across criteria of various natures. Thus, experts do not have to contemplate which criterion they are evaluating, but rather assess all criteria globally as if they were all benefit-type criteria. Consequently, only one type of normalization is used, preventing the error of specifying the criterion type incorrectly. One notable advantage of this approach is that experts are not required to provide a specific assessment; instead, they offer an approximate assessment in the form of linguistic values. This is advantageous as it aligns more closely with human thought processes, making the assessment process more intuitive. This approach is especially valuable when dealing with qualitative criteria, as it aligns well with the nature of the criteria employed in this research. Since qualitative criteria were used in this study, the use of linguistic values is logical. It is important to note that while these ratings provide approximate values for the criteria and
alternatives rather than specific numerical ratings, they serve a valuable purpose. Once the questionnaire was meticulously formulated, it was disseminated to the experts for their input. Subsequent to completion by the experts, these questionnaires were forwarded for processing, thereby paving the way for ensuing analysis.

Subsequently, the research proceeds to its second phase, characterized by data analysis. During the data analysis process, criterion weights are initially computed based on data derived from the first part of the survey questionnaire. When data is collected in linguistic values, these values are transformed into fuzzy numbers. Each of these values is transformed using a membership function to determine the value of the fuzzy number (Table 2). For example, the linguistic value, ‘Absolutely bad’, is transformed into a fuzzy number (1, 1, 2), while the linguistic value, ‘Equal’, is transformed into a fuzzy number (5, 6, 7). These membership functions are determined based on previous analyses of similar works. Eventually, they are adapted to specific needs. For instance, the linguistic scales and membership functions of the IMF SWARA method are defined in the paper by Vrtagić et al. [61].

Within this section, experts ranked the criteria and assigned them linguistic values tailored to facilitate the IMF SWARA method. This method differs from the fuzzy SWARA method only in the linguistic scale used for criterion evaluation; the other steps are identical in both methods. In the fuzzy SWARA method, all criteria are essentially evaluated in the same way, and no criteria are compared directly. Based on the aggregated scores, the criteria are ranked, and their weights are determined. In contrast, with IMF SWARA, the criteria are initially ranked, and then they are compared, with corresponding linguistic values assigned to each criterion. This approach allows for the possibility that two criteria may have the same weight, rather than doubling their values as in the classic fuzzy SWARA method [61]. Additionally, as the number of criteria increases in the IMF SWARA method, the worst criterion can receive negligible values and may not significantly affect the decision making process. Thus, using the IMF SWARA method offers advantages over the classic SWARA method. This analytical framework was employed to deduce criterion weights. Post computation of criterion weights for each expert, Fuzzy Dombi Aggregation Operators are computed independently. This operator is designed to harmonize individual expert weights into a unified set.

Upon calculating the weights of both criteria and auxiliary criteria, the ranking of mountain lodges is established in alignment with expert assessments. The ranking is determined by employing the fuzzy CRADIS method. The fuzzy CRADIS method has shown success in various fields [62], yet its application in tourism has been relatively unexplored. Therefore, the utilization of this method in the context of tourism represents a contribution of this research. Following this, the validation of the derived results ensues. This validation process involves applying the same criterion weights and alternative values, while altering the ranking procedure via an alternative fuzzy method. This step aims to corroborate or challenge the outcomes of the fuzzy CRADIS method. Subsequently, an inquiry into the significance of auxiliary criteria in shaping the ranking roster is pursued through sensitivity analysis.

3.1. IMF SWARA Method

IMF SWARA represents a modification of the fuzzy SWARA method [61,63]. The method comprises the following steps:

Step 1. Identification and selection of criteria.

Step 2. Criteria arrangement by importance, ranking from highest to lowest.

Step 3. Determination of relative criterion importance. Relative importance values are determined for each criterion, leveraging linguistic evaluations (Table 2). The criterion of utmost importance is attributed a value of zero (0), while the remaining criteria are assigned values based on these linguistic assessments.

Step 4. Computation of coefficient $k_f$ by summing the relative importance value of the criterion with one (1). This computation is formulated as follows:
\[ \tilde{k}_j = \begin{cases} 1 & \text{if } j = 1 \\ \tilde{s}_j + 1 & \text{if } j > 1 \end{cases} \]  

(1)

Step 5. Computation of importance values \( \tilde{q}_j \) by the most pivotal criterion receiving a value of one (1) and for the remaining criteria using the following expression:

\[ \tilde{q}_j = \begin{cases} 1 & \text{if } j = 1 \\ \frac{\tilde{q}_j - 1}{\tilde{k}_j} & \text{if } j > 1 \end{cases} \]  

(2)

Step 6. Calculation of criterion weights \( \tilde{w}_j \) by dividing the individual importance value by the total importance value, as indicated by the following expression:

\[ \tilde{w}_j = \frac{\tilde{q}_j}{\sum_{j=1}^{n} \tilde{q}_k} \]  

(3)

3.2. Fuzzy Dombi Aggregation Operators

The Fuzzy Dombi aggregator is mathematically expressed through Equations (4) and (5) [64,65]. These equations are articulated as follows:

\[ FDWGA(\tilde{q}) = (\tilde{q}^l_j, \tilde{q}^m_j, \tilde{q}^u_j) = \begin{cases} \tilde{q}^l_j = \frac{\sum_{j=1}^{n} (\tilde{q}^l_j)}{1 + \left\{ \sum_{j=1}^{n} \tilde{w}_j \left( \frac{1 - f(\tilde{q}^l_j)}{f(\tilde{q}^l_j)} \right)^p \right\}^{1/p}} \\ \tilde{q}^m_j = \frac{\sum_{j=1}^{n} (\tilde{q}^m_j)}{1 + \left\{ \sum_{j=1}^{n} \tilde{w}_j \left( \frac{1 - f(\tilde{q}^m_j)}{f(\tilde{q}^m_j)} \right)^p \right\}^{1/p}} \\ \tilde{q}^u_j = \frac{\sum_{j=1}^{n} (\tilde{q}^u_j)}{1 + \left\{ \sum_{j=1}^{n} \tilde{w}_j \left( \frac{1 - f(\tilde{q}^u_j)}{f(\tilde{q}^u_j)} \right)^p \right\}^{1/p}} \end{cases} \]  

(4)

where \( \tilde{w}_j \) represents the weights attributed to the decision makers involved in the research. Additionally, \( p (p \geq 0) \) denotes a non-negative parameter, \( \tilde{q}^l_j \)—lower bound of TFN, \( \tilde{q}^m_j \)—the mean value of TFN and \( \tilde{q}^u_j \)—the upper bound of TFN.

\[ f(\tilde{q}^l_j, \tilde{q}^m_j, \tilde{q}^u_j) = \begin{cases} f(\tilde{q}^l_j) = \frac{\sum_{j=1}^{n} (\tilde{q}^l_j)}{\sum_{j=1}^{n} (\tilde{q}^l_j)} \\ f(\tilde{q}^m_j) = \frac{\sum_{j=1}^{n} (\tilde{q}^m_j)}{\sum_{j=1}^{n} (\tilde{q}^m_j)} \\ f(\tilde{q}^u_j) = \frac{\sum_{j=1}^{n} (\tilde{q}^u_j)}{\sum_{j=1}^{n} (\tilde{q}^u_j)} \end{cases} \]  

(5)

3.3. Fuzzy CRADIS Method

The fuzzy CRADIS method, initially introduced in fuzzy form by Puška et al. [66], involves a series of procedural steps:

Step 1. Formation of the Decision Matrix. The method commences with the creation of a linguistic decision making matrix, wherein decisions are articulated through linguistic evaluations. Subsequently, these linguistic ratings undergo transformation into fuzzy numbers via the application of membership functions (Table 2).

Step 2. Normalization of the Decision Matrix. This step involves the classification of criteria into benefit-type and cost-type categories. Appropriate normalization procedures are then applied accordingly, leading to the formation of a normalized decision making matrix. For benefit criteria, the normalization is represented as
For cost criteria, the normalization takes the form of
\[
\text{n}_{ij} = \frac{x_{ij}^l}{\max x_{ij}^l} \frac{x_{ij}^m}{\max x_{ij}^m} \frac{x_{ij}^u}{\max x_{ij}^u}
\]

Step 3. Weighting of Decision Matrix. Weighting is achieved by performing matrix multiplication between the normalized decision matrix and the corresponding weight vector.
\[
\bar{v}_{ij} = (v_{ij}^l, v_{ij}^m, v_{ij}^u) = \vec{w}_j \times \vec{w}_j
\]

Step 4. Identification of Ideal and Anti-Ideal Values. The ideal value corresponds to the maximum value within the weighted matrix \(v_{ij}\), while the anti-ideal value corresponds to the minimum value within the same matrix \(v_{ij}\).
\[
t_i = \max \bar{v}_{ij}, \text{ where } \bar{v}_{ij} = (v_{ij}^l, v_{ij}^m, v_{ij}^u)
\]
\[
t_{ai} = \min \bar{v}_{ij}, \text{ where } \bar{v}_{ij} = (v_{ij}^l, v_{ij}^m, v_{ij}^u)
\]

Step 5. Computation of Deviations from Ideal and Anti-Ideal Values. In this step, deviations from the ideal and anti-ideal values are computed for all data elements in the matrix \(v_{ij}\) as follows:
\[
d^+ = t_i - \bar{v}_{ij}
\]
\[
d^- = \bar{v}_{ij} - t_{ai}
\]

This process results in the creation of two decision making matrices, representing deviations from ideal and anti-ideal values.

Step 6. Formation of optimal alternatives in the case of deviations from the ideal and anti-ideal value. When deviating from the ideal value, the lowest value among the alternatives for all criteria is sought, forming the optimal alternative. When deviating from the anti-ideal value, the optimal alternative comprises all values of the alternatives that have the highest values for each observed criterion.

Step 7. Computation of Cumulative Deviations from Ideal and Anti-Ideal Values.
\[
s_i^+ = \sum_{j=1}^{n} d^+
\]
\[
s_i^- = \sum_{j=1}^{n} d^-
\]

Step 8. Defuzzification of Deviations from Ideal and Anti-Ideal Values.
\[
s_{i, \text{def}}^\pm = \frac{d_i^+ + 4d_i^m + d_i^u}{6}
\]

Step 9. Computation of the Utility Function. In this step, the utility function is calculated for each alternative concerning the optimal alternatives. The objective is for each alternative to approach the optimal ideal alternative as closely as possible while distancing itself from the optimal anti-ideal alternative.
\[
K_i^+ = \frac{s_i^+}{s_0^+}
\]
\[
K_i^- = \frac{s_i^-}{s_0^-}
\]

Here, \(s_0^+\) represents the optimal ideal alternative, while \(s_0^-\) signifies the anti-ideal optimal alternative.
Step 10. Ranking of Alternatives. The final value of the fuzzy CRADIS method is computed as the average deviation of the alternatives from the utility function.

\[
Q_i = \frac{K_i^+ + K_i^+}{2} \tag{18}
\]

The best alternative is determined as the one with the highest value of the fuzzy CRADIS method \((Q_i)\), while the worst alternative is identified as the one with the lowest value of the fuzzy CRADIS method \((Q_i)\).

4. Results and Discussion

4.1. Results

In the preliminary stage, the assessment of criterion significance was conducted through the integration of the IMF SWARA method with the fuzzy Dombi operator. Given the participation of eight experts in this collective decision making endeavor, 32 models were computed for the main and sub-criteria. At each decision level, encompassing both main and sub-criteria within respective principal categories, the fuzzy Dombi operator was employed to establish the ultimate criterion weights, which are subsequently incorporated into the subsequent phase of the model. The calculation models using the IMF SWARA method for each individual expert are delineated in Table 3.

| Table 3. Calculation of coefficients for main criteria using the IMF SWARA method. |
|--------------------|--------------------|--------------------|--------------------|--------------------|
| E1                | s_j                | k_i                | q_j                | w_j                |
| C1                | 1.000              | 1.000              | 1.000              | 1.000              |
| C2                | 0.222              | 0.250              | 0.286              | 1.222              |
| C3                | 0.222              | 0.250              | 0.286              | 1.222              |
| SUM               | 2.383              | 2.440              | 2.488              |

| E2                | s_j                | k_i                | q_j                | w_j                |
| C1                | 1.000              | 1.000              | 1.000              | 1.000              |
| C2                | 0.000              | 0.000              | 0.000              | 1.000              |
| C3                | 0.222              | 0.250              | 0.286              | 1.222              |
| SUM               | 2.778              | 2.800              | 2.818              |

| E3                | s_j                | k_i                | q_j                | w_j                |
| C1                | 1.000              | 1.000              | 1.000              | 1.000              |
| C2                | 0.250              | 0.286              | 0.333              | 1.250              |
| C3                | 0.222              | 0.250              | 0.286              | 1.222              |
| SUM               | 2.333              | 2.400              | 2.455              |

| E4                | s_j                | k_i                | q_j                | w_j                |
| C1                | 1.000              | 1.000              | 1.000              | 1.000              |
| C2                | 0.250              | 0.286              | 0.333              | 1.250              |
| C3                | 0.250              | 0.286              | 0.333              | 1.250              |
| SUM               | 2.313              | 2.383              | 2.440              |

| E5                | s_j                | k_i                | q_j                | w_j                |
| C2                | 1.000              | 1.000              | 1.000              | 1.000              |
| C1                | 0.222              | 0.250              | 0.286              | 1.222              |
| C3                | 0.000              | 0.000              | 0.000              | 1.000              |
| SUM               | 2.556              | 2.600              | 2.636              |

| E6                | s_j                | k_i                | q_j                | w_j                |
| C1                | 1.000              | 1.000              | 1.000              | 1.000              |
| C3                | 0.000              | 0.000              | 0.000              | 1.000              |
| C2                | 0.222              | 0.250              | 0.286              | 1.222              |
When determining the weights of the main criteria, the experts first had to assess the importance of these criteria and rank them. Consequently, experts E1, E2, E3, E4, and E8 ranked the criteria in the following order: C1 > C2 > C3. In contrast, experts E5 and E7 ranked these criteria as C2 > C1 > C3, while expert E6 ranked the main criteria as C1 > C3 > C2. This discrepancy highlights the differing opinions among the experts and their varying levels of importance assigned to the criteria. Subsequently, the importance of criterion C1 in comparison to criterion C2 was determined, taking expert 1 (E1) as an example. According to E1’s perspective, C1 holds greater significance than C2. Consequently, E1 assigned the value, “Moderately less significant”, to criterion C2. Similarly, E1 regarded C2 as more important than C3 and assigned it the same value of, “Moderately less significant”. The most important criterion was then assigned the fuzzy number value, (0.332, 0.375) based on the membership function, while the other criteria were assigned the fuzzy number value, (0.222, 0.250, 0.286). This procedure led to the determination of the values in column $s_i$. Column $k_i$ values were obtained by adding one (1) to each of the individual fuzzy numbers. Criterion $q_j$ values were determined by copying the value of the best criterion (C1). To calculate the value for criterion C2, it involved dividing the value of C1 for $q_j$ by the value of $k_i$ for criterion C2 ($\frac{1.000}{1.286} = 0.778$, $\frac{1.000}{1.250} = 0.800$, $\frac{1.000}{1.222} = 0.818$). Similarly, the value of $q_j$ for criterion C3 was computed by taking the value of criterion C2 for $q_j$ and dividing it by the value of criterion C3 for $k_i$. The resulting $q_j$ values were then summed. To determine the $w_j$ values for the criteria, the individual values of $q_j$ for each criterion were divided by the aggregate value $q_j$. Using criterion C1 as an example, the calculation is as follows: $\frac{1.000}{2.488} = 0.402$, $\frac{1.000}{2.440} = 0.410$, $\frac{1.000}{2.383} = 0.420$. The same procedure was applied to the other criteria. This process was consistently applied to all expert evaluations, including auxiliary criteria.

Subsequently, the acquired $w_j$ values were subjected to aggregation using the Fuzzy Dombi operator, leading to the determination of final weights for the principal criteria as follows:

$w_1$ (ecological) = (0.357, 0.365, 0.374)
$w_2$ (economic) = (0.332, 0.343, 0.354)
$w_3$ (social) = (0.364, 0.280, 0.296)

The analogous process was repeated for the sub-criteria within each grouping, yielding the final weights presented in Table 4.

Table 4. Final criterion weights attained via IMF SWARA and fuzzy Dombi method.

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<th>$w_j$ Local</th>
<th>$w_j$ Global (Final)</th>
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<td>C13</td>
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<td>0.181</td>
</tr>
<tr>
<td>C15</td>
<td>0.200</td>
<td>0.210</td>
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</table>
Following the criterion weight determination, the task of evaluating the best sustainability rating among the observed lodges emerged. This was approached through expert-based decision making, wherein experts evaluated the lodges using linguistic expressions. The linguistic scale spanned from “Absolutely bad (AB)” to “Absolutely good (AG)”, as outlined in Table 5. To facilitate rank-based decisions, linguistic values were translated into fuzzy numbers using a decision function (Table 2). Each linguistic expression was corresponded with a distinct fuzzy number. For instance, “Absolutely bad (AB)” was assigned a fuzzy number of (1, 1), while “Very bad (VB)” corresponded to (2, 3, 4), and so forth, as shown in Table 2. These fuzzy numbers are derived with careful consideration to the possibility of partial overlap among individual numerical values. Additionally, it is ensured that the upper limit of a specific fuzzy number exceeds the preceding fuzzy number while remaining lower than the subsequent one.

Table 5. Linguistic initial decision matrix.

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The transition from linguistic values to fuzzy numbers established the initial fuzzy decision matrix. Given the involvement of eight experts, a harmonization process was undertaken by applying an arithmetic mean, wherein all experts held equal importance, each influencing 12.5% of the overall decision. This approach ensures that each expert's opinion is taken into account individually, without prioritizing any of the experts. The aggregated fuzzy decision matrix then served as the foundation for executing subsequent stages of the fuzzy CRADIS method.

The initial step in implementing the fuzzy CRADIS method entailed the normalization of the initial fuzzy decision matrix. This normalization was conducted by determining the maximum value for each criterion's alternatives and subsequently dividing the alternatives' values by this maximal value (Expression 6). Notably, normalization solely encompassed benefit criteria, given the uniform evaluation potential facilitated by the linguistic scale, eliminating the necessity to differentiate between cost and benefit criteria. It is worth noting that while fuzzy CRADIS allows for the use of other normalization methods, this research adhered to the standard normalization procedure prescribed by the method. The exploration of different normalization techniques and their impact on ranking is a subject that can be addressed in future research, particularly in the context of methodological studies.

For instance, in the context of criterion C11, a maximum value of 9.87 is identified. Consequently, each value corresponding to the alternatives within this criterion undergoes division by this identified maximum. To illustrate, consider the first alternative,
which is associated with fuzzy numbers (7.2, 8.3, 8.9). When divided by this maximum value, the result is a set of normalized values, exemplified by \( \frac{7.2}{9.87} = 0.73, \frac{8.2}{9.87} = 0.84, \frac{8.9}{9.87} = 0.91 \). This process is systematically applied to all criteria, thereby establishing the maximum value for each.

The subsequent step involves the weighting of the normalized decision matrix. During this phase, the normalized values are multiplied by their respective weights. Take, for instance, criterion C11, where the weights are specified as (0.06, 0.06, 0.07). In this scenario, for the first mountain lodge, the normalized values are subjected to multiplication with these designated weights, resulting in calculations such as (0.73 \times 0.06 = 0.04, 0.84 \times 0.06 = 0.05, 0.91 \times 0.07 = 0.06). Each distinct fuzzy number is subjected to multiplication by its corresponding weight, generating weighted normalized decision matrix.

Following the formation of the weighted normalized decision matrix, the process proceeds to calculate the maximum and minimum values within this matrix, denoted as the ideal and anti-ideal values, respectively. The quest for the maximum value for an individual criterion entails extracting the highest value among all corresponding alternatives. This procedure yields the ideal value, represented as a fuzzy number (0.07, 0.08, 0.09). Conversely, in the pursuit of the anti-ideal value, the minimum value is identified for each individual criterion, drawing from the collection of minimal values. This process results in the acquisition of an anti-ideal value in the form of the following fuzzy number (0.02, 0.03, 0.04).

Subsequent to this determination, the next stage in the application of the fuzzy CRADIS method involves the computation of deviations from the ideal and anti-ideal values. Within this phase, the deviation of all weighted values from both the ideal and anti-ideal values is meticulously determined. Initially, the values of the weighted decision matrix undergo subtraction from the ideal value, followed by subtraction of the anti-ideal value from the weighted value. This dual computation yields two matrices, one representing deviation from the ideal value, and the other from the anti-ideal value. The overarching objective for each alternative is to approach the ideal value as closely as possible while distancing itself from the anti-ideal value. Consequently, optimal values are computed based on these criteria. Given the existence of two decision matrices, two distinct optimal values are derived. In cases where deviation is from the ideal value, the optimal value comprises the alternatives with the lowest value for each criterion. Conversely, when deviating from the anti-ideal value, the optimal value entails the alternatives with the highest value for each criterion.

Following the calculation of these optimal values, the subsequent step involves quantifying the collective deviation of alternatives, encompassing all deviations for specific alternatives, including those designated as optimal (Table 4). Subsequently, the process proceeds to defuzzify these aggregated deviations, thereby converting fuzzy numbers into crisp numerical values. Standard defuzzification is employed here, as defined in the original fuzzy CRADIS method, where the central fuzzy number is given four times higher priority in relation to the other two numbers. In practice, various types of defuzzification exist; thus, future research should investigate whether this phase affects the final ranking of alternatives or not. Upon completing this defuzzification, the utility function is computed. In instances where deviations from the ideal value are in question, the optimal alternative is divided by the value of the aggregate deviations among alternatives. Conversely, when deviations are from the anti-ideal value, the value of the aggregate deviations among alternatives is divided by the value of the optimal alternative.

Illustrated using the example of the first mountain lodge, ML1, the calculations yield the following results: \( K^+_1 = \frac{0.537}{0.463} = 0.728, K^-_1 = \frac{0.248}{0.474} = 0.734 \). The culmination of the fuzzy CRADIS method entails computing the final value by determining the average utility function across individual alternatives. In the case of ML1, this computation appears as follows: \( Q_1 = \frac{0.728 + 0.734}{2} = 0.731 \). This process is replicated for all mountain lodges, leading to the formation of the final results, as outlined in Table 6.
Table 6. Aggregated deviations of alternatives, utility functions, and final rankings.

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<th>$s^-$</th>
<th>Def $s^+$</th>
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<td>6</td>
</tr>
<tr>
<td>ML2</td>
<td>(0.38, 0.44, 0.43)</td>
<td>(0.33, 0.40, 0.39)</td>
<td>0.426</td>
<td>0.385</td>
<td>0.791</td>
<td>0.812</td>
<td>0.801</td>
<td>5</td>
</tr>
<tr>
<td>ML3</td>
<td>(0.36, 0.41, 0.41)</td>
<td>(0.35, 0.42, 0.41)</td>
<td>0.405</td>
<td>0.406</td>
<td>0.832</td>
<td>0.857</td>
<td>0.844</td>
<td>2</td>
</tr>
<tr>
<td>ML4</td>
<td>(0.38, 0.43, 0.43)</td>
<td>(0.33, 0.40, 0.39)</td>
<td>0.422</td>
<td>0.389</td>
<td>0.799</td>
<td>0.821</td>
<td>0.810</td>
<td>4</td>
</tr>
<tr>
<td>ML5</td>
<td>(0.37, 0.42, 0.42)</td>
<td>(0.34, 0.41, 0.39)</td>
<td>0.416</td>
<td>0.395</td>
<td>0.811</td>
<td>0.834</td>
<td>0.822</td>
<td>3</td>
</tr>
<tr>
<td>ML6</td>
<td>(0.31, 0.35, 0.35)</td>
<td>(0.41, 0.48, 0.47)</td>
<td>0.343</td>
<td>0.467</td>
<td>0.982</td>
<td>0.987</td>
<td>0.984</td>
<td>1</td>
</tr>
<tr>
<td>ML0</td>
<td>(0.30, 0.34, 0.35)</td>
<td>(0.41, 0.49, 0.47)</td>
<td>0.337</td>
<td>0.474</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The analytical assessment based on expert evaluations indicates that mountain lodge ML6 exhibits the most favorable sustainability outcomes, followed by mountain lodge ML3, while mountain lodge ML1 exhibits the least favorable results. These findings will undergo validation and sensitivity analysis to ensure their robustness.

Result validation has become an integral step in the application of MCDM methodologies [67,68]. The goal of this analysis is to examine the robustness of the fuzzy CRADIS method, in such a way that its results are compared with the results obtained by other fuzzy methods. In this validation, the initial fuzzy decision making matrix, along with the derived weights, is employed. The sole objective is ranking alternatives using various methods. In this context, apart from the fuzzy CRADIS method, six additional fuzzy methods will be employed: fuzzy MABAC (Multi-Attribute Border Approximation Area Comparison), fuzzy WASPAS (Weighted Aggregated Sum Product Assessment), fuzzy SAW (Simple Additive Weighting), fuzzy MARCOS (Measurement Alternatives and Ranking according to the Compromise Solution), fuzzy ARAS (Additive Ratio Assessment), and fuzzy TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution). The results of this analysis reveal that the fuzzy TOPSIS method is the only one with a distinct ranking order of alternatives, whereas the remaining methods yield congruent alternative rankings, affirming the consistency of the results obtained through the fuzzy CRADIS method (Figure 2). Based on the results of this analysis, which demonstrated that the rankings generated by the fuzzy CRADIS method align with those produced by other fuzzy methods, it is evident that this method can be reliably employed in future research as the results are consistent with those obtained from various other fuzzy methods. In subsequent studies, there is no obligation to exclusively use the fuzzy CRADIS method; other approaches can also be considered. However, it is worth noting that the application of the fuzzy TOPSIS method should be approached with caution due to the observed disparity in rankings compared to other methods.

![Figure 2. Result validation.](image-url)
A sensitivity analysis will subsequently investigate the influence of specific criteria on the ultimate ranking of mountain lodges. Within this analysis, individual criteria weights will be varied by designated percentages, i.e., 30%, 60%, and 90%, while the weights of other criteria remain unaltered. This process serves to gauge the diminished importance of a given criterion within the final decision [69–71]. Given the existence of 15 auxiliary criteria, and with their weights adjusted threefold, 45 distinct scenarios are established. The application of these scenarios (Figure 3) reveals that two mountain lodges, ML6 and ML1, maintain their respective rankings throughout all scenarios. Specifically, ML1 consistently occupies the lowest rank, while ML6 consistently secures the highest rank. These two lodges demonstrated independence regarding changes in the individual weights of the criteria, and therefore the rankings of these lodges were not changed. However, for other mountain lodges, their ranking order varies with the modification of specific criteria. For instance, ML3 temporarily ascends to third place in the ranking in certain scenarios, a shift attributed to the fact that ML5 exhibits less favorable indicators than ML3 concerning tourist accessibility. The diminishing importance of this criterion results in ML5 surpassing ML3 in the ranking. Furthermore, there was a noticeable shift in the ranking of mountain lodge ML5 when the importance of the criterion related to the availability of natural resources was reduced in the scenario. In this particular instance, this lodge transitioned from the third position to the fifth. This observation underscores that compared to lodges ML2 and ML4, ML5 boasts superior access to natural resources. A similar change was observed in scenario 27, where the significance of the capacity criterion was reduced. In response to an alteration in the tourist accessibility criterion, this change propelled ML5 to the second position. This indicates that the ML2 mountain lodge offers better tourist accessibility in comparison to the ML5 mountain lodge. Analogously, all other shifts in lodge rankings can be elucidated using similar rationales. Further adjustments in rankings, compared to the initial ranking (Scenario S0), can be analyzed in a similar manner. These outcomes provide valuable insights for mountain lodges, identifying specific areas where they may lag behind others. Armed with this analysis, they can subsequently strategize improvements to enhance their sustainability. Every mountain lodge can utilize this sensitivity analysis to assess the strengths and weaknesses specific to their establishment. This assessment opens the door to the possibility of implementing tailored strategies to enhance the performance of individual lodges when engaging in mountain tourism.

![Figure 3. Sensitivity analysis results.](image-url)
4.2. Discussion

The allure of mountain regions is progressively drawing diverse tourists, playing a pivotal role in the advancement of tourism development [72]. The assessment of sustainability within mountain tourism constitutes a multifaceted undertaking that necessitates a holistic framework capable of effectively addressing the multifaceted dimensions inherent to this industry. Sustainability within the realm of tourism holds profound significance, primarily due to the utilization of a nation’s resources in the tourism sector. Consequently, safeguarding these resources for posterity becomes an imperative endeavor, achieved through the practical application of sustainability principles. Sustainability, in this context, is attained through the harmonization of three pivotal dimensions, namely environmental, economic, and social facets [73]. In the provisioning of mountain tourism services, it becomes essential to harness the ecological resources inherent to a specific region, facilitating economic outcomes while concurrently drawing upon social resources. Furthermore, the integration of mountain tourism with other forms of tourism becomes indispensable in order to furnish tourists with a comprehensive experiential package [74]. Tourists exhibit a heightened proclivity towards visiting specific mountain lodges when the destination boasts an array of tourism resources. Thus, it is incumbent upon destinations to leverage the entirety of their assets to offer tourists diverse experiences while ensuring the conservation of natural resources.

In this research, a comprehensive sustainability model for mountain tourism is developed. This model introduces a novel approach for assessing the sustainability of mountain lodges with the aim of enhancing the overall quality of mountain tourism experiences. The model is designed to accommodate the inherent uncertainties and subjectivity often encountered in expert assessments, making it a robust tool for evaluating the sustainability of mountain tourism. Each sustainability factor under consideration is further subdivided into five distinct criteria, ensuring that none of the examined sustainability factors is afforded undue preference, particularly when assessing the relative importance of these criteria. The determination of criterion importance is carried out systematically, commencing with the prioritization of the main criteria, followed by the auxiliary criteria. The final weights used for ranking the alternatives are obtained by multiplying the weights of the main criteria with their corresponding sub-criteria weights. In cases where one of the main criteria features a smaller number of associated auxiliary criteria, these criteria are given higher weighting, and vice versa. Consequently, adjustments to the weights are deemed necessary in such scenarios. To mitigate this, it is advisable to employ an equal number of sub-criteria within each main criterion, thus ensuring a more equitable weight distribution. Consequently, the results obtained through this approach cannot be directly compared with similar research, as it differs fundamentally in terms of methodology. Unlike previous studies, MCDM methods have been uniquely leveraged to first determine criterion weights and subsequently ascertain the ranking of mountain lodges. This innovative approach allows the identification of which of these lodges exhibits the most favorable performance in sustainable mountain tourism. Moreover, the specific evaluation of mountain lodges in this manner as a function of sustainable mountain tourism is a novel contribution to the field, further differentiating this research.

The assessment of criteria and auxiliary criteria significance, as well as the evaluation of how these criteria are realized by the observed alternatives, was accomplished through expert decision making. The insights and judgments of eight experts, all holding Doctor of Science degrees and possessing substantial experience in the field of mountain tourism, were solicited for this purpose. This group comprised university professors and seasoned scholars within the domain of tourism. The experts evaluated both the criteria and the alternatives employing linguistic values. To employ these values effectively in the assessment of mountain lodges’ sustainability, a transformation into fuzzy numbers was necessitated. In the process of determining criterion weights, the experts initially ranked these criteria based on their perceived importance. Subsequently, they employed
linguistic evaluations adapted to the IMF SWARA method. It is noteworthy that this method diverges from the conventional fuzzy SWARA method primarily in its utilization of specific linguistic values [66]. Initially, weights were assigned for each individual expert, followed by the harmonization of these weights using Fuzzy Dombi Aggregation Operators. Within this research, this aggregator was strategically employed to attain a final set of harmonized weights as assessed by these experts [75]. The outcomes of this weight assignment exercise revealed a remarkable consistency, indicating that none of the observed criteria held a significantly different level of importance relative to the others. Consequently, it can be inferred that all these criteria exert an equal influence on the final evaluation.

In the context of this research’s examination of sustainable mountain tourism, particular attention is directed towards mountain lodges. This choice is informed by the pivotal role that mountain lodges play in mountain tourism, offering visitors accommodations and gastronomic services. This transformation of visitors into tourists is a well-established concept [76], allowing individuals to extend their stay in these regions, providing ample time to explore the various attractions on offer. Consequently, mountain lodges are required to adapt by significantly enhancing their service offerings to cater to tourists’ needs. Typically, mountain lodges are owned and operated by specific mountaineering societies [77], with access traditionally granted to members of these organizations. However, to leverage mountain resources for tourism purposes effectively, it is imperative to open access to these lodges for all. This inclusive approach is essential for optimizing the utilization of mountain resources for tourism development.

To investigate and assess the utilization of mountain lodges for tourism purposes, the first step was to identify the specific mountain lodges in Bosnia and Herzegovina suitable for this research. This selection process was carried out using a random number generator, resulting in the choice of six mountain lodges. These lodges were then evaluated against sustainability criteria. The research aimed to determine the extent to which these lodges fulfilled the sustainability criteria. The expert ratings obtained through this assessment indicated that these mountain lodges performed exceptionally well in meeting the sustainability criteria.

However, the primary objective of this research was to identify which of these mountain lodges best achieved the sustainability goals. To achieve this, the fuzzy CRA-DIS method was employed to rank these lodges and ascertain which one most effectively met the sustainability objectives. This method revealed that, based on expert assessments, mountain lodge ML6 exhibited the most favorable indicators. The dominant factor contributing to the superior performance of this lodge was criteria C24—capacities and C25—accessibility to tourists. It is noteworthy that while this lodge may not boast extensive sleeping accommodations, it does offer two substantial halls where tourists can spend their daytime hours. Hence, this lodge was selected as the most effective in meeting the sustainability objectives. This particular mountain lodge should serve as a benchmark for comparison with other lodges, motivating them to strive for improved ratings in order to enhance sustainability within mountain tourism.

This model, encompassing economic, ecological, and social criteria, serves as a valuable tool for facilitating well-informed decision making among stakeholders. It encourages the prioritization of sustainable practices, thereby ensuring the enduring appeal of mountainous regions to tourists while concurrently safeguarding their natural splendor and fostering the welfare of local communities. Furthermore, this model offers an avenue for individual mountain lodges to enhance their business operations. This is achieved through a comprehensive analysis that compares these lodges with their counterparts. Consequently, insights are derived regarding areas where experts believe certain lodges excel or fall short in comparison. Building upon this analysis, mountain lodges can refine their offerings and align them more effectively with sustainability objectives. As a result, they are better positioned to harness the abundant resources that mountainous regions offer for their benefit in the future. Consequently, the outcomes of
this research extend beyond the borders of BiH, finding relevance in the enhancement of mountain tourism worldwide.

To initiate this process, a preliminary evaluation of these mountain lodges is imperative to identify areas requiring improvement in order to boost mountain tourism. Key areas of focus include streamlining accessibility to these lodges for tourists, as the current requirement of approval from mountaineering societies often hinders accessibility. Furthermore, efforts should prioritize the enhancement of surrounding infrastructure and the expansion of tourist offerings [78].

5. Conclusions

The research presented in this paper has evaluated mountain tourism establishments through a multi-criteria expert decision making approach, involving the participation of eight experts who provided assessments in the form of linguistic values. The results obtained from these evaluations were derived through the application of two distinct fuzzy methodologies: the IMF SWARA method and the fuzzy CRADIS method. Initiated with the IMF SWARA method, the weights of sustainability criteria and their associated sub-criteria were determined for individual experts. Subsequently, these individual weightings were harmonized through the application of fuzzy Dombi aggregation operators. Notable findings were yielded by this approach, with an emphasis on the prominence attributed to criteria such as the service quality (C21) and the natural resources quality (C15), while the tradition criterion (C31) was considered relatively less significant. Following this, the fuzzy CRADIS method was employed to rank the observed mountain lodges, critical facilities for mountain tourism. The outcomes of this application indicated that the most favorable evaluations from the experts were garnered by mountain lodge ML6, while mountain lodge ML1 received the lowest ratings. These assessments were further substantiated through result validation and sensitivity analysis.

Nonetheless, it is crucial to acknowledge certain limitations in this research. Notably, not all existing mountain lodges within Bosnia and Herzegovina were considered. This limitation arose from the absence of a comprehensive register encompassing all such lodges. Furthermore, this research represents a preliminary exploration into the evaluation of mountain lodges using sustainability criteria, serving as an initial foray into their role within the development of mountain tourism. Future research endeavors should encompass a broader spectrum of mountain lodges and incorporate an assessment of tourist satisfaction, a facet not addressed in this expert-driven approach. The results obtained through the IMF SWARA and fuzzy CRADIS methods highlight their ease of use and flexibility, suggesting their potential applicability in various domains beyond just mountain tourism. This methodology can be adapted for future research where the research problem hinges on determining criteria weights and ranking alternatives. Additionally, future research should delve into group decision making processes and how to effectively process group data, a dimension not explored within the scope of this paper.

The Assessment of Mountain Tourism Sustainability using an Integrated Fuzzy MCDM model represents a pioneering effort to harmonize the multifaceted dimensions of mountain tourism into a coherent framework that prioritizes sustainability. By integrating expert assessment and advanced decision making techniques, this research aims to equip stakeholders with a tool that enables informed and responsible decision making, fostering the continued growth of mountain tourism while safeguarding the environments and communities that make these destinations so special.

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