Dynamic Right-of-Way Allocation on Bus Priority Lanes Considering Traffic System Resilience

Jia Hu, Zhexi Lian, Xiaoxue Sun, Arno Eichberger, Zhen Zhang, and Jintao Lai

Abstract: Bus priority is an effective way to improve traffic efficiency and sustainability. To achieve this, the Bus Priority Lane (BPL) is adopted to provide exclusive right-of-way for buses. However, the BPL is underutilized if the frequency of buses is low. To address this issue, many studies focus on improving the BPL's utilization efficiency by intermittently allowing general vehicles to access it. However, these studies still have some shortcomings: (i) bus priority cannot be guaranteed if general vehicles run on the BPL; and (ii) the traffic system lacks resilience, especially when the traffic demand is unbalanced. This paper proposes a dynamic right-of-way allocation for the BPL, considering traffic system resilience. On the one hand, it ensures absolute bus priority by controlling Connected Automated Vehicles (CAVs), so as they do not interfere with buses. On the other hand, it can improve traffic system resilience by allocating right-of-way for CAVs with heavy turning-movement demand. To test the effectiveness, the proposed control strategy is compared with the non-control baseline. The experiments are conducted under seven unbalanced-traffic-demand levels, four congestion levels, and five CAV Penetration Rates. The results show that the proposed strategy can ensure absolute bus priority and improve traffic efficiency and traffic system resilience.

Keywords: bus priority lane; dynamic right-of-way allocation; traffic system resilience

1. Introduction

Congestion at signalized intersections poses significant challenges to urban transportation systems, necessitating effective strategies to mitigate traffic congestion. Ensuring bus priority has emerged as a crucial approach to alleviate congestion and improve the overall efficiency and sustainability of transportation systems [1–4]. Unfortunately, the transit bus priority cannot be guaranteed, especially when the traffic demand is heavy. The reason is that buses would easily be interfered with by general vehicles when approaching the signalized intersection. Hence, guaranteeing absolute bus priority is critical [5]. To achieve this goal, one commonly adopted approach is the implementation of a Bus Priority Lane (BPL), which provides exclusive right-of-way for buses, enabling them to bypass general traffic and improve traffic sustainability and efficiency [6,7].

However, an important concern arises when the utilization frequency of buses on the BPL is low, resulting in underutilization of this BPL. Such underutilization not only compromises the effectiveness of the transportation system in reducing congestion but also raises concerns about its overall efficiency and sustainability [8]. In response to this challenge, numerous studies have proposed strategies to improve the utilization efficiency of BPLs. Some of these studies focus on converting BPLs into intermittent BPLs [9–12]. Such intermittent BPLs allow general vehicles to access the BPL...
when the number of buses is low. By opening up intermittent BPLs for general vehicles, these studies can improve lane occupancy and reduce vehicle delay on the general-purpose lanes [13–15]. With the advance of Automated Vehicle (AV) technology, AV is emerging to revolutionize the transportation system [16–19]. AV technology can collect detailed information on individual vehicles in real time. AV technology enables vehicle information shared between AVs and AVs via Vehicle-to-Vehicle (V2V). Each AV can also obtain supplementary information from the road unit via Vehicle-to-Infrastructure (V2I). Using the collected real-time information, AV technology can predict traffic conditions accurately and plan optimized trajectories as control commands for AVs to implement. Unfortunately, it is still difficult for AVs at the current stage to cope with complex interactions with surrounding vehicles at the signalized intersection. Hence, other studies focus on providing exclusive right-of-way for AVs by converting BPLs into mixed-use AV/bus lanes [20,21]. Such mixed-use AV/bus lanes not only enhance AV safety but also improve the lane capacity of the BPL.

Although these studies can improve the lane capacity of the BPL, they still have some shortcomings. Firstly, converting BPLs into intermittent BPLs raises concerns about ensuring bus priority, as the intermittent availability of the BPLs may lead to delays and reduced reliability for bus services [22–25]. This is because the transit buses would possibly be disturbed by general vehicles when approaching the signalized intersection. Secondly, the existing studies lack consideration of traffic system resilience [26,27]. Traffic system resilience denotes the ability of a transportation network to withstand and recover from disruptions or challenges while maintaining its functionality and efficiency. Especially when the traffic demand is heavy and unbalanced, the traffic system cannot handle all vehicles, and it experiences collapse.

To overcome these shortcomings, an innovative control strategy is necessary to ensure bus priority while improving the resilience of the traffic system. This paper proposes a dynamic right-of-way allocation strategy for the bus priority lane, taking into account the concept of traffic system resilience. The proposed control strategy bears the following features:

- Improve traffic system efficiency at the signalized intersection.
- Enhance traffic system resilience under various traffic demand patterns.
- Guarantee absolute bus priority even when traffic is congested.

The remainder of this paper is structured as follows. The section “Problem Statement” describes scenarios and research problems. The section “Logic Structure of the Control Strategy” illustrates the logic of the proposed control strategy. The section “Mathematical Formulation” presents problem formulation and the associated solution. The section “Evaluation” shows the experimental design and related results. The section “Conclusions” contains the conclusions from the experiments.

2. Literature Review

For mitigating congestion at the signalized intersection, many studies focusing on guaranteeing bus priority have been proposed. Transit Signal Priority (TSP) is commonly adopted to grant transit buses priority at the signalized intersection [28,29]. The TSP can provide intersection priority for buses by switching the traffic signal to green when buses arrive at the stop bar [30–32]. To reduce the negative impact caused by frequent signal switching for buses, some studies proposed to reserve or re-allocate a green phase without sacrificing green time for other turning-movement vehicles [2,33].

However, these TSP strategies cannot prevent buses from being interfered with by surrounding general vehicles when approaching the intersection. Due to this, other studies are proposed to adopt a Bus Priority Lane (BPL) to separate buses from general vehicles [34–36]. With the help of BPL, the transit buses can approach the intersection without being interfered with by general vehicles [37,38]. However, the traditional BPL still has one shortcoming. This is because the BPL would cause a waste of road resources.

To overcome the aforementioned shortcoming, some studies present a strategy to convert the BPL into an intermittent BPL. This strategy allows general vehicles to share
the BPL when the utilization frequency of transit buses is low [39]. The typical strategy is that general vehicles are only permitted to access the BPL during the off-peak hours [10,13]. To be specific, this strategy can dynamically decide whether to open the BPL to general vehicles based on the utilization frequency of buses [40,41]. Nevertheless, such strategies cannot guarantee bus priority if general vehicles run with buses in the same lane. In other words, these strategies would sacrifice the benefits of transit buses during the off-peak hours. At the same time, these strategies cannot prevent the traffic system from collapsing when the traffic demand is heavy and unbalanced.

Realizing the research gap, an innovative control strategy should be proposed. This study presents a dynamic right-of-way allocation for the BPL, considering the traffic system resilience.

3. Notations and Problem Statement

3.1. Notations

The notations in this study are described in Table 1.

Table 1. Notations.

<table>
<thead>
<tr>
<th>General Notations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c, c'$</td>
<td>The index of CAV</td>
</tr>
<tr>
<td>$k$</td>
<td>The index of CHV</td>
</tr>
<tr>
<td>$b$</td>
<td>The index of the bus</td>
</tr>
<tr>
<td>$m$</td>
<td>Approach lane index</td>
</tr>
<tr>
<td>$t$</td>
<td>Time step index</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Time step interval</td>
</tr>
<tr>
<td>$C$</td>
<td>Set of CAV indices</td>
</tr>
<tr>
<td>$B$</td>
<td>Set of bus indices</td>
</tr>
<tr>
<td>$T$</td>
<td>Set of time-step indices</td>
</tr>
<tr>
<td>$M$</td>
<td>A sufficiently large positive number</td>
</tr>
<tr>
<td>$L$</td>
<td>The length of the control zone</td>
</tr>
<tr>
<td>$R$</td>
<td>The red time</td>
</tr>
<tr>
<td>$G$</td>
<td>The green time</td>
</tr>
<tr>
<td>$\phi_G$</td>
<td>The set of green times</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{d}^{f,n}$</td>
<td>Initial time prediction of vehicle $n$ arriving at the stop line</td>
</tr>
<tr>
<td>$t_{d}^{f,n-1}$</td>
<td>Initial time prediction of vehicle $n-1$ passing the intersection</td>
</tr>
<tr>
<td>$t_{e}^{f,n}$</td>
<td>Earliest time vehicle $n$ arrives at the stop line without considering the preceding vehicles</td>
</tr>
<tr>
<td>$t^0$</td>
<td>Current time</td>
</tr>
<tr>
<td>$t^p$</td>
<td>The passing time if the vehicle arrives at the stop line during the red time</td>
</tr>
<tr>
<td>$t_5^{f,k}$</td>
<td>The terminal time of vehicle $k$ passing the intersection</td>
</tr>
<tr>
<td>$v_0^{n,t}$</td>
<td>The current speed of vehicle $n$ at the time step $t^0$</td>
</tr>
<tr>
<td>$v^{k,t}$</td>
<td>The speed of CHV $k$ at the time step $t$</td>
</tr>
<tr>
<td>$a_{k,t}$</td>
<td>Acceleration of CHV $k$ at the time step $t$</td>
</tr>
<tr>
<td>$a_{p,k}^{f,t}$</td>
<td>The predicted acceleration of the vehicle $k$ if there is a preceding vehicle</td>
</tr>
<tr>
<td>$a_{p,k}^{f,t}$</td>
<td>The predicted acceleration of the vehicle $k$ if there is no preceding vehicle</td>
</tr>
<tr>
<td>$a_d^{p,k}$</td>
<td>The desired deceleration of the vehicle</td>
</tr>
<tr>
<td>$\Delta x_{d,pm,k}^{p,n}$</td>
<td>The desired minimum distance gap with the preceding vehicle $pn$</td>
</tr>
<tr>
<td>$\Delta x_{pm,k}$</td>
<td>The distance gap between the vehicle $k$‘s preceding vehicle and the vehicle $k$</td>
</tr>
<tr>
<td>$\Delta x_{d,pm,k}^{f}$</td>
<td>The desired minimum distance gap when there is no preceding vehicle</td>
</tr>
<tr>
<td>$\Delta x_{pm,k}^{f}$</td>
<td>The speed difference between the preceding vehicle and the CHV $k$</td>
</tr>
<tr>
<td>$\Omega_1$</td>
<td>The set of all left-turning vehicles on the road</td>
</tr>
<tr>
<td>$\Omega_2$</td>
<td>The set of all go-through vehicles on the road</td>
</tr>
<tr>
<td>$\Omega_3$</td>
<td>The set of all right-turning vehicles on the road</td>
</tr>
</tbody>
</table>
Table 1. Cont.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(Ω_i)$</td>
<td>The demand level of the different turning-traffic flow</td>
</tr>
<tr>
<td>$Ω^S_i$</td>
<td>The set of vehicles with turning $i$ that can pass through the intersection during the current signal cycle</td>
</tr>
<tr>
<td>$r^S_i$</td>
<td>The effective green time for the vehicles with turning movement $i$</td>
</tr>
<tr>
<td>$q^S_i$</td>
<td>The saturation flow rate for the turning movement</td>
</tr>
<tr>
<td>$τ^c$</td>
<td>Travel time of the CAV $c$</td>
</tr>
<tr>
<td>$x^c_{f,t}$</td>
<td>Position of the CAV $c$ at the end of the optimization horizon</td>
</tr>
<tr>
<td>$v^c_{f,t}$</td>
<td>Speed of the CAV $c$ at the end of the optimization horizon</td>
</tr>
<tr>
<td>$v_{L}$</td>
<td>Minimum speed in the control zone, m/s</td>
</tr>
<tr>
<td>$v_{U}$</td>
<td>Maximum speed in the control zone, m/s</td>
</tr>
<tr>
<td>$w^c_{m,t}$</td>
<td>Binary variable. $w^c_{m,t} = 1$ if CAV $c$ is on the general purpose lane $m$</td>
</tr>
<tr>
<td>$s_0$</td>
<td>Safety distance</td>
</tr>
<tr>
<td>$t_h$</td>
<td>Safety headway</td>
</tr>
<tr>
<td>$a_L$</td>
<td>Minimum acceleration in the control zone, m/s$^2$</td>
</tr>
<tr>
<td>$a_U$</td>
<td>Maximum acceleration in the control zone, m/s$^2$</td>
</tr>
<tr>
<td>$x^c_{p,t}$</td>
<td>Position of the CAV $c$’s preceding vehicle at time step $t$</td>
</tr>
<tr>
<td>$x^c_{f,t}$</td>
<td>Position of CAV $c$ at time step $t$</td>
</tr>
<tr>
<td>$v^c_{f,t}$</td>
<td>Instantaneous speed of CAV $c$ at time step $t$</td>
</tr>
<tr>
<td>$a^c_{f,t}$</td>
<td>Acceleration of CAV $c$ at time step $t$</td>
</tr>
<tr>
<td>$l^c$</td>
<td>Length of CAV</td>
</tr>
<tr>
<td>$l^b$</td>
<td>Length of the bus</td>
</tr>
<tr>
<td>$x^b_{p,t}$</td>
<td>Position of the bus $b$’s preceding vehicle at time step $t$</td>
</tr>
<tr>
<td>$x^b_{f,t}$</td>
<td>Position of the bus $b$’s following vehicle at time step $t$</td>
</tr>
<tr>
<td>$x^b_{f,t}$</td>
<td>Position of bus $b$ at time step $t$</td>
</tr>
<tr>
<td>$v^b_{f,t}$</td>
<td>Speed of bus $b$ at time step $t$</td>
</tr>
<tr>
<td>$t_{sop}$</td>
<td>The start time of the optimization horizon</td>
</tr>
<tr>
<td>$t_{eop}$</td>
<td>The end time of the optimization horizon</td>
</tr>
<tr>
<td>$x^c_{ve}$</td>
<td>Position of CAV $c$ at the end of the optimization horizon</td>
</tr>
<tr>
<td>$ξ^c$</td>
<td>Binary variable. $ξ^c = 1$ if CAV could get through the intersection during the given optimization horizon</td>
</tr>
<tr>
<td>$α$</td>
<td>Weight of total travel time in the objective</td>
</tr>
<tr>
<td>$β$</td>
<td>Weight of total fuel consumption in the objective</td>
</tr>
<tr>
<td>$µ$</td>
<td>Weight of the control precision</td>
</tr>
</tbody>
</table>

3.2. Problem Statement

The research scenario is a signalized intersection under a partially connected and automated traffic environment. As shown in Figure 1, a control zone is divided into an upstream section and a downstream section. The upstream section is named Section 1 and the downstream is named Section 2. In Section 1, there are three lanes. The left lane is a Bus Priority Lane (BPL). The other lanes are General Purpose Lanes (GPLs). Section 2 includes four lanes. One is the BPL and the other lanes are, respectively, for vehicles with a left-turning movement, going-straight movement, and right-turning movement. All vehicles can be divided into general vehicles and buses. The general vehicles include Connected Automated Vehicles (CAVs) and Connected Human-driven Vehicles (CHVs). All vehicles enable communication with other vehicles and the roadside unit via Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) in real time. CAVs’ trajectories can be accurately controlled by implementing optimized trajectory planning. As for CHVs, they can run on the drivers’ own decisions. To cope with the stochasticity of CHVs’ driving behavior, the optimized trajectory planning for CAVs should be updated at each time step by a rolling horizon. A cooperative relationship can be established between CAVs and CHVs if CHVs follow the guidance of surrounding CAVs. The BPL is adopted to guarantee absolute bus priority and is open to CAVs with heavy turning-movement demand. To be specific, the BPL is only open to CAVs when the traffic volume is more than the lane capacity. All CAVs accessing the BPL cannot interfere with transit buses.
3.2. Problem Statement

The research scenario is a signalized intersection under a partially connected and automated traffic environment. As shown in Figure 1, a control zone is divided into an upstream and a downstream. In the downstream, there are three lanes. The left lane is a Bus Priority Lane (BPL), and the other two lanes are General Purpose Lanes (GPLs). Section 2 includes four components, including traffic information collection, turning-movement demand-level determination, dynamic right-of-way allocation of the BPL, and trajectory planning and implementation. At each time step, the control logic would be activated. Some detailed information about these four components is described in the following sections.

4. Logic Structure of the Control Strategy

In this section, the logic structure of the proposed control strategy is illustrated in Figure 2. The proposed control strategy is designed to improve traffic system resilience when the traffic demand is heavy and unbalanced. The strategy can be divided into four components, including traffic information collection, turning-movement demand-level determination, dynamic right-of-way allocation of the BPL, and trajectory planning and implementation. At each time step, the control logic would be activated. Some detailed information about these four components is described in the following sections.

![Figure 1. Illustration of research scenario.](image1)

![Figure 2. Logic structure of the control strategy.](image2)

- Traffic Information Collection

  The traffic information collection is the first component in the logic structure of the proposed control strategy. It is designed to collect traffic information about vehicle states, signal schemes, and turning-movement intentions via V2X and V2V. All the collected information would be the input of the next component (turning-movement demand-level determination).

- Turning-movement Demand-Level Determination

  This section introduces the second component of the control logic. This component is the turning-movement demand-level determination. It is designed to determine whether
the current traffic demand is heavy and unbalanced. When the current signal scheme is fed into this component, this component will calculate the lane capacity for each turning movement. Compared with the current turning demand, this component would determine a demand level for each turning movement. Finally, the turning-movement demand-level results would output to the next component.

**Dynamic Right-of-way Allocation of the BPL**

This section introduces the third component of the control logic. This component is called the dynamic right-of-way allocation of the BPL. It is designed to determine whether and how to allocate the right-of-way of the BPL for CAVs. When the turning-movement demand level is high, this component is activated. Firstly, this component determines the opening of the BPL for turning-movement demand that is heavy and unbalanced. Next, this component would determine the sequence of CAV candidates that can access the BPL. Based on the determined sequence, this component would dynamically allocate the right-of-way of the BPL for CAV candidates. The information about the allocated right-of-way on the BPL would output to the next component.

**Trajectory Planning and Implementation**

Trajectory planning and implementation is the fourth component of the control logic. According to the allocated right-of-way, this component would optimize the trajectory plan for CAV candidates to utilize the objective right-of-way on the BPL. Then, the component would convert the trajectory plan into a control command for CAV candidates to implement.

5. **Mathematical Formulation**

5.1. **Terminal-Time Prediction**

The optimized trajectory plan for CAVs accessing the Bus Priority Lane (BPL) consists of trajectory planning for vehicles at every time step and determining the terminal time when vehicles reach the stop line. However, due to the stochasticity of CHVs’ driving behavior, the trajectory planning for CAV candidates needs to be updated at every time step. Consequently, the prediction of the terminal time when vehicles reach the stop line must also be recalculated at every time step. The terminal-time prediction can be calculated as below.

\[
t_{f,n}^{d} = \max \left( t_{f,n}^{d} + t_{f,n}^{e}, t_{f,n} \right) \quad \forall n \in N
\]

\[
t_{f,n}^{e} = t_{0} + \frac{L - \frac{v_{n}^{2} - v_{max}^{2}}{2v_{max}}}{v_{max}} + \frac{v_{n} - v_{0}}{a_{max}}
\]

\[
t_{f,n} = \begin{cases} 
t_{f,n}^{d} & \text{if } t_{f,n}^{d} \in \phi_{G} \\
\left( t_{f,n}^{d} \right)^{*} (R + G) + R & \text{ otherwise}
\end{cases}
\]

where \( n \) is the vehicle index. \( N \) is the vehicle-index set in the mixed traffic. \( t_{f,n}^{d} \) is the initial time prediction of vehicle \( n \) arriving at the stop line. \( t_{f,n}^{d} \) is the initial time prediction of vehicle \( n - 1 \) passing the intersection. \( t_{f,n}^{e} \) is the earliest time vehicle \( n \) arrives at the stop line without considering the preceding vehicles and traffic signal. \( t_{0} \) is the current time. \( L, v_{max}, a_{max} \), respectively, denote the length of the control zone, the maximum speed, and the acceleration. \( v_{n} \) is the speed of vehicle \( n \) at the current time. \( R \) and \( G \) are green time and red time. \( t_{f,n}^{d} \) is the terminal time of vehicle \( n \) passing the intersection.

5.2. **CHV Trajectory Prediction**

As for CHVs, they can run on their own decisions. The implementation of CHVs’ trajectory plans is influenced by factors such as the trajectory information of preceding vehicles and traffic signal schemes. Consequently, it becomes necessary to re-predict CHV trajectories at each time interval to accommodate the mixed traffic conditions. The
Intelligent Driver Model (IDM) is a widely accepted model for predicting CHV trajectories. The CHVs’ acceleration prediction can be mathematically formulated as follows.

\[
a^{pk}_i = a_U \left[ 1 - \left( \frac{v_i^p}{v_U} \right)^4 - \left( \frac{\Delta x^{d,s}_{pn,k}}{\Delta x^{d,s}_{pn,k}} \right)^2 \right] \quad \forall t \in T, n \in N, k \in N_k \tag{4}
\]

\[
a^{lk}_i = a_U \left[ 1 - \left( \frac{v_i^l}{v_U} \right)^4 - \left( \frac{\Delta x^{d,s}_{k}}{L-x_i^l} \right)^2 \right] \quad \forall t \in T, k \in N_k \tag{5}
\]

\[
\Delta x^{d,s}_{pn,k} = s_0 + \max \left( v_i^p \cdot t_h + \frac{v_i^p \cdot \Delta v_{l}^{pn,k}}{2\sqrt{a_U \cdot a_U^d}}, 0 \right)
\]

\[
\Delta x^{d,s}_{k} = s_0 + \max \left( v_i^l \cdot t_h + \frac{v_i^l \cdot \Delta v_{l}^{s}}{2\sqrt{a_U \cdot a_U^d}}, 0 \right)
\]

\[
t_\varphi = \left[ \frac{t_f^l}{R + G} \right] \cdot (R + G), \quad \forall t_f^l \notin \varphi_G, \forall k \in N_k
\]

\[
a^k_i = \begin{cases} a^{pk}_i & \forall t \notin \left[ t_0, \infty \right] \cap \left[ t_\varphi, t_\varphi + R \right] \\ a^{lk}_i & \forall t \in \left[ t_0, \infty \right] \cap \left[ t_\varphi, t_\varphi + R \right] \end{cases}
\]

where \( k \) is the vehicle index of CHVs. \( N_k \) is the vehicle-index set of CHVs. \( a^d \) is the desired deceleration of the vehicle. \( \Delta v_{l}^{pn,k} \) is the speed difference between the vehicle \( k \)'s preceding vehicle and the vehicle \( k \). \( t_h \) is the safety headway. \( \Delta x^{d,s}_{pn,k} \) is the desired minimum distance gap from the preceding vehicle \( pn \). \( \Delta x^{d,s}_{pn,k} \) is the distance gap between the vehicle \( k \)'s preceding vehicle and the vehicle \( k \). \( \Delta x^{d,s}_{k} \) is the desired minimum distance gap if no preceding vehicle exists. \( a^k_i \) is the vehicle \( k \)'s predicted acceleration if the preceding vehicle exists. \( a^{pk}_i \) is the vehicle \( k \)'s predicted acceleration if the preceding vehicle does not exist. \( t_\varphi \) denotes the terminal time of vehicle \( k \) arriving at the stop line.

5.3. Turning-Movement Demand-Level Calculation

To calculate the turning-movement demand level, the model analyzes the spatiotemporal occupancy of traffic and considers the vehicles’ turning-movement demand. To achieve this, we split the traffic flow as follows:

\[
\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3
\]

where \( \Omega \) represents the set of all vehicles on the continuous road, \( \Omega_1 \) represents the set of all left-turning vehicles on the road, \( \Omega_2 \) represents the set of all go-through vehicles on the road, and \( \Omega_3 \) represents the set of all right-turning vehicles on the road.

\[
D(\Omega_i) = \frac{\left| \Omega_i \right|}{r^q_i q^i_0}
\]

where \( D(\Omega_i) \) represents the demand level of the different turning-traffic flows, \( \Omega_i^q \) represents the set of vehicles with turning \( i \) that can pass through the intersection during the current signal cycle, \( r^q_i \) represents the effective green time for vehicles with turning movement \( i \) and \( q^i_0 \) represents the saturation flow rate for the turning movement. Therefore, the turning movement with the highest demand intensity is:

\[
i^* = \arg\max_i D(\Omega_i)
\]
If \( D(\Omega_f) = 0 \), it means that all vehicles can pass the signalized intersection during the current green time. Hence, the allocated right-of-way on the BPL is not needed for any vehicles. Otherwise, the allocated right-of-way is required to be provided for CAVs with heavy turning movement \( i^* \).

### 5.4. Dynamic Allocation of the Right-of-Way on the BPL

Right-of-way allocation means the road resource occupied by the vehicle at time \( t \). The right-of-way for buses and CAVs is determined by the current traffic demand level. The right-of-way determination should be divided into two kinds.

\[
s_c^t = \{ x | x_c^t - l_c^* \leq x \leq x_c^t + l_c^* \} \quad \forall t \in T, c \in N_c(13)
\]
\[
s_b^t = \{ x \mid x_b^t - l_b^* \leq x \leq x_b^t + l_b^* \} \quad \forall t \in T, b \in N_b(14)
\]

where \( s_c^t \) and \( s_b^t \), respectively, denote the right-of-way allocation for CAVs and buses.

### 5.5. Trajectory Planning for CAVs

#### 5.5.1. Assumption

Some assumptions are adopted to simplify the formulation of the mathematical model. These assumptions are as follows: (1) all vehicles’ turning intentions can be obtained via V2X and V2V; and (2) all CAVs accessing the BPL can be controlled accurately without communication delay.

#### 5.5.2. Objective Function

The objective function has three components: travel time, fuel consumption, and control precision. It is described as follows:

\[
J = \alpha \sum_c \tau_c + \beta \sum_c \sum_f |a_c^f| + \mu \left( |v_c^f - L| + |v_c^f - v_U| \right)
\]

The travel time \( \tau_c \) is designed to improve traffic efficiency. The definition of \( \tau_c \) is illustrated below:

\[
\tau_c = \sum_f \sum_m w_{m,t} \cdot \Delta t \quad \forall c \in N_c(16)
\]

If the vehicle \( c \) is in the control zone, the binary decision variable \( w_{m,t} \) must be updated at every time step. \( w_{m,t} \) is a binary decision variable to denote CAV \( c \) on the general-purpose lane or the BPL. Hence, the travel time can be illustrated as a constraint (16).

In this objective function, the second component describes fuel consumption using the acceleration \( a_c^f \). The last component describes control precision. To be specific, it is designed to improve intersection capacity. \( x_c^f \) and \( v_c^f \) are, respectively, the position and speed of the vehicle \( c \) at the end of the optimization horizon.

#### 5.5.3. Constraints

1. Vehicle kinematic constraints

   The movement of vehicles has to be subject to kinematic constraints, as described below.

   \[
v_L \leq v_c^f \leq v_U \quad \forall t \in T, c \in N_c(17)
\]
   \[
a_L \leq a_c^f \leq a_U \quad \forall t \in T, c \in N_c(18)
\]
   \[
   v_c^f = v_{c-1}^f + a_c^f \cdot \Delta t \quad \forall t \in T, c \in N_c(19)
\]
   \[
   x_c^f = x_{c-1}^f + v_{c-1}^f \cdot \Delta t + \frac{1}{2} a_c^f \cdot \Delta t^2 \quad \forall t \in T, c \in N_c(20)
   \]
where \( v_c^t \) is the speed of vehicle \( c \) at time \( t \). \( v_b \) and \( v_f \), respectively, denote the minimum and maximum speed limit. Constraint (18) denotes the acceleration limit. Constraints (19) and (20) depict the kinematic equation.

(2) Traffic signal constraints

To keep safe, the trajectory planning for CAVs should avoid conflict with surrounding vehicles. Hence, the following constraints should be satisfied.

\[
\begin{align*}
    x_{t}^{pc} - x_{t}^{c} &> s_0 + v_{c}^{t} - t_h + 2 \cdot t^c \quad \forall t, c \in N_c \\
    x_{t}^{pb} - x_{t}^{b} &> s_0 + v_{b}^{t} - t_h + 2 \cdot t^b \quad \forall t, b \in B \\
    x_{t}^{b} - x_{t}^{fb} &> s_0 + v_{f}^{t} - t_h + 2 \cdot t^c \quad \forall t, c \in N_c, b \in B
\end{align*}
\]

where \( x_{t}^{pc} \) is the position of vehicle \( c \)'s preceding vehicle at time \( t \). \( x_{t}^{pb} \) is the position of bus \( b \)'s preceding vehicle at time step \( t \). \( x_{t}^{fb} \) is the position of bus \( b \)'s following vehicle at time step \( t \). \( v_{c}^{t} \) and \( v_{b}^{t} \), respectively, denote the position and speed of bus \( b \) at time \( t \). \( v_{f}^{t} \) is the speed of bus \( b \)'s following vehicle at time step \( t \). \( l^c \) and \( l^b \), respectively, represent the CAV length and the bus length.

(3) Traffic signal constraints

The optimized vehicle trajectory plans should obey the traffic signal scheme. The following constraints are imposed:

\[
\begin{align*}
    t_{e}^{b} + \sum_{t} \sum_{m} w_{m,t} \cdot \Delta t &\geq t_{o}^{e} - G + (\xi^c - 1) \cdot M \quad \forall t, c \in N_c \\
    t_{o}^{e} + \sum_{t} \sum_{m} w_{m,t} \cdot \Delta t &\leq t_{e}^{o} + (1 - \xi^c) \cdot M \quad \forall t, c \in N_c \\
    x_{e}^{o} &\geq L + (\xi^c - 1) \cdot M \quad \forall t, c \in N_c \\
    x_{e}^{e} &\leq L + (1 - \xi^c) \cdot M \quad \forall t, c \in N_c
\end{align*}
\]

where the optimization horizon is defined by the start time \( t_{o}^{e} \) and end time \( t_{e}^{e} \). \( \xi^c \) is a binary variable, and \( \xi^c = 1 \) if vehicle \( c \) could pass the intersection during the given optimization horizon. Constraints (24) and (25) ensure vehicles pass the intersection only during the given green time. Constraints (26) and (27) ensure vehicles are in the control zone if they cannot pass the signalized intersection in the current optimization horizon.

6. Evaluation

In this section, we evaluate the proposed control strategy through simulation experiments, comparing it to a non-control baseline. The effectiveness of the control strategy is measured using throughput, delay, and fuel consumption as performance indicators. The simulation experiments are conducted under varying levels of congestion and CAV Penetration Rates, encompassing five different scenarios for each. The objective is to ensure a fair assessment of the control strategy’s ability to enhance traffic system resilience while maintaining absolute bus priority.

6.1. Experimental Design

6.1.1. Testbed

The simulation experiment’s testbed is depicted in Figure 3, representing a signalized intersection with a Bus Priority Lane (BPL). The control zone consists of two sections: Sections 1 and 2. Section 1 spans 350 m in length, while Section 2 is 150 m long. Section 2 accommodates four lanes, due to lane channelization for various turning movements.
Additionally, a roadside unit is installed to facilitate Vehicle-to-Infrastructure (V2I) communication with vehicles. All vehicles in the simulation enable communication through Vehicle-to-Vehicle (V2V) technology. The simulation platform utilized for the experiments is based on PTV-VISSIM [42].

Figure 3. Testbed.

6.1.2. Scenario

Two scenarios focusing on ensuring absolute bus priority are tested:

- Non-control baseline: in this scenario, a dedicated bus lane is implemented to separate buses from general vehicles. The purpose of this separation is to ensure that buses have exclusive access to their designated lane, while general vehicles are restricted to the General Purpose Lane (GPL). This arrangement aims to prioritize bus movement and provide an unobstructed route for buses, thereby enhancing efficiency and reliability in public transportation.

- The proposed strategy: in this scenario, the Bus Priority Lane (BPL) is made accessible to Connected and Autonomous Vehicles (CAVs) under specific circumstances. When there is a high and imbalanced demand for turning movements at the intersection, the BPL is opened for CAVs to utilize. During this period, CAVs are granted the allocated right-of-way on the BPL, ensuring their smooth passage without any interference with buses. This strategy aims to optimize traffic flow and accommodate the varying needs of different vehicle types during periods of increased turning-movement demand.

6.1.3. Measurements of Effectiveness

To validate the effectiveness of the proposed control strategy, three Measurements of Effectiveness (MOE) are selected, including throughput, delay, and traffic system resilience. The definition of traffic system resilience can be described as follows:

\[
\text{Resilience} = \frac{\text{Delay}_n - \text{Delay}_s}{\text{Delay}_n - \text{Delay}_o} \times 100\%
\]

where \(\text{Delay}_n\) denotes the average vehicle delay in the non-control baseline scenario. \(\text{Delay}_o\) represents the average vehicle delay in the proposed control strategy. \(\text{Delay}_s\) denotes the average vehicle delay when the congestion level is 1.0 in the non-control baseline.

6.1.4. Sensitivity Analysis

To ensure a fair and comprehensive validation of the proposed strategy, a sensitivity analysis is conducted. This analysis includes seven different unbalanced demand levels, ranging from low to high proportions of turning-movement demand. Additionally, five different congestion levels (0.6, 0.8, 1.0, 1.2 and 1.4) and five different CAV Penetration Rates (10%, 20%, 30%, 40% and 50%) are considered. The unbalanced demand level represents the varying proportion of turning-movement demand in each scenario. For example, proportion 4:4:2 denotes the left-turning demand proportion, go-straight demand proportion, and right-turning proportion, respectively. A higher unbalanced-demand level means that the
difference between the left-turning demand and go-straight demand is larger. Hence, the low unbalanced-demand level includes the proportions 3:5:2, 4:4:2 and 5:3:2. The high unbalanced-demand level contains the proportions 1:7:2, 2:6:2, 2:6:2 and 1:7:2.

6.2. Results

In this section, we present the simulation results that validate the effectiveness of the proposed control strategy in comparison to the non-control baseline. These results demonstrate the robust performance of the proposed strategy across different turning-movement demand patterns, congestion levels, and CAV Penetration Rates (CPR).

The simulation results clearly indicate the advantages offered by the proposed control strategy, including notable improvements in traffic efficiency and traffic system resilience. The strategy effectively enhances the overall flow of traffic, resulting in reduced delays and increased throughput. Furthermore, the simulation results affirm the credibility of the proposed strategy in ensuring absolute bus priority within the traffic system.

6.2.1. Traffic Efficiency Improvement Validation

Throughput Comparison Results

Figure 4 shows the results of the throughput comparison between the non-control baseline and the proposed control strategy under the condition of low unbalanced demand levels (3:5:2, 4:4:2 and 5:3:2). Compared with the non-control baseline, the proposed control strategy has no obvious benefits when the congestion level is less than 1.0. The reason is that the traffic system can easily handle all vehicles when the traffic demand is low. When the congestion level is more than 1.0, it means that the traffic system is in the condition of oversaturated traffic demand. The proposed control strategy has more obvious benefits. This is because the proposed control strategy can improve intersection capacity by controlling CAVs to utilize the right-of-way on the BPL. With the increment of CPRs, the proposed control strategy performs better. The reason is that more CAVs can be controlled to improve traffic efficiency by utilizing the right-of-way on the BPL.

Figure 5 illustrates the results of the throughput comparison between the non-control baseline and the proposed control strategy under high unbalanced demand levels (1:7:2, 2:6:2, 6:2:2 and 7:1:2). Remarkably, the proposed control strategy continues to perform well even when the congestion level exceeds 1.0. As congestion levels and CAV Penetration Rates (CPRs) increase, the benefits of throughput improvement become more prominent. This is attributed to the ability of the proposed control strategy to enhance intersection capacity by effectively allocating the unutilized right-of-way of the Bus Priority Lane (BPL) to CAVs. This allocation optimizes the utilization efficiency of road resources, resulting in improved throughput at the signalized intersection. It is worth noting that the proposed control strategy achieves substantial improvements in throughput, particularly in scenarios with higher congestion levels and CPRs. These findings provide strong evidence of the strategy’s capability to enhance traffic flow and maximize the utilization of available road resources.
obvious benefits. This is because the proposed control strategy can improve intersection capacity by controlling CAVs to utilize the right-of-way on the BPL. With the increment of CPRs, the proposed control strategy performs better. The reason is that more CAVs can be controlled to improve traffic efficiency by utilizing the right-of-way on the BPL.

Figure 5 illustrates the results of the throughput comparison between the non-control baseline and the proposed control strategy under high unbalanced demand levels (1:7:2, 2:6:2, 6:2:2 and 7:1:2). Remarkably, the proposed control strategy continues to perform well even when the congestion level exceeds 1.0. As congestion levels and CPRs increase, the benefits of throughput improvement become more prominent. This is attributed to the ability of the proposed control strategy to enhance intersection capacity by effectively allocating the unutilized right-of-way of the Bus Priority Lane (BPL) to CAVs. This allocation optimizes the utilization efficiency of road resources, resulting in improved throughput at the signalized intersection. It is worth noting that the proposed control strategy achieves substantial improvements in throughput, particularly in scenarios with higher congestion levels and CPRs. These findings provide strong evidence of the strategy’s capability to enhance traffic flow and maximize the utilization of available road resources.

Figure 4. Throughput comparison results under low unbalanced-traffic-demand levels.

Figure 5. Throughput comparison results under high unbalanced-traffic-demand levels.
Delay Comparison Results

Figure 6 shows the results of the delay comparison between the non-control baseline and the proposed control strategy under the condition of low unbalanced-demand levels (3:5:2, 4:4:2 and 5:3:2). Different from the throughput improvement, the proposed control strategy can effectively achieve delay reduction at any congestion levels. With the increment of congestion levels and CPRs, the benefits of delay reduction are more obvious.

6.2.2. Traffic-System-Resilience Improvement Validation

Figure 8 is the result of a traffic-system-resilience comparison under various unbalanced-demand levels. The proposed control strategy can improve traffic system resilience when the congestion level is 1.2 and 1.4. The reason for selecting high congestion levels is that the traffic system cannot easily experience system collapse without any control strategies. In Figure 8, the red dotted line denotes the resilience as 100%. To be specific, the proposed control strategy can effectively recover the traffic system from collapse to normal operation if the resilience is more than 1.0. Compared with the high unbalanced-demand levels, the benefits of traffic-system-resilience improvement are more obvious under the conditions of
low unbalanced-demand levels. With the increment of CPRs, the proposed control strategy can perform better in traffic-system-resilience improvement.

Figure 7. Delay comparison results under high unbalanced-traffic-demand levels.

Figure 8. Traffic-system-resilience comparison results.
6.2.3. Bus Priority Validation

The proposed control strategy can guarantee bus priority while improving traffic efficiency and traffic system resilience. To validate the capability of guaranteeing absolute bus priority, several experiments were conducted. As shown in Tables 2 and 3, bus delay has been an MOE for evaluating the capability of guaranteeing bus priority. The results demonstrate that the proposed strategy can guarantee that bus delay is less than 1 s under various congestion levels and unbalanced-demand levels.

Table 2. Bus delay under various congestion levels and unbalanced-demand levels (CPR = 0.3).

<table>
<thead>
<tr>
<th>Penetration Rate = 30%</th>
<th>Delay in sec/pcu</th>
<th>Proportion</th>
<th>1:7:2</th>
<th>3:5:2</th>
<th>4:4:2</th>
<th>6:2:2</th>
</tr>
</thead>
<tbody>
<tr>
<td>V/C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>0.8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.07</td>
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<td>1.0</td>
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<td>0</td>
<td>0.46</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>0</td>
<td>0</td>
<td>0.06</td>
<td>0.20</td>
<td></td>
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</tr>
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</table>

Table 3. Bus delay under various congestion levels and unbalanced-demand levels (CPR = 0.5).

<table>
<thead>
<tr>
<th>Penetration Rate = 50%</th>
<th>Delay in sec/pcu</th>
<th>Proportion</th>
<th>1:7:2</th>
<th>3:5:2</th>
<th>4:4:2</th>
<th>6:2:2</th>
</tr>
</thead>
<tbody>
<tr>
<td>V/C</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>0</td>
<td>0.04</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Discussion

This paper proposed a dynamic right-of-way allocation strategy for the BPL, considering traffic system resilience. This control strategy can guarantee absolute bus priority while improving traffic efficiency and enhancing traffic system resilience. Such a control strategy can perform well in improving traffic system reliability even when the traffic demand is heavy and unbalanced. It is beneficial for policy-makers and urban planners to adopt this control strategy. The reason is that the proposed control strategy can ensure absolute bus priority while maximizing traffic system efficiency and resilience.

However, the proposed control strategy still has two limitations. The first limitation is that the proposed control strategy lacks consideration of optimizing the traffic signal scheme for transit buses. The other limitation is that this study only optimizes trajectory planning for CAVs near the BPL to access it. With that being said, the traffic system resilience can be enhanced more by allowing CAVs that are not near the BPL to access it.

In future work, joint traffic-signal-scheme optimization and trajectory planning can be considered to improve traffic efficiency and enhance traffic system resilience.

8. Conclusions

This paper proposed a control strategy to improve traffic system resilience while ensuring absolute bus priority. The proposed control strategy can improve traffic efficiency in throughput improvement and delay reduction. At the same time, the proposed control strategy can enhance traffic system resilience. To be specific, the control strategy can effectively avoid system collapse even when the traffic demand is heavy and unbalanced.
To evaluate the proposed control strategy, simulation experiments have been conducted under the low unbalanced-traffic-demand levels and the high unbalanced-traffic-demand levels. Sensitivity analysis was performed for congestion levels and CPRs.

Compared to the non-control baseline, the proposed control strategy has been validated for its effectiveness in terms of throughput, delay, and traffic system resilience. The conclusions drawn from the results are as follows:

- No significant advantages in terms of throughput enhancement are observed under low congestion levels. The proposed control strategy demonstrates the potential for throughput improvement when congestion levels are high (1.2 and 1.4), with potential benefits ranging from 10% to 40%. In contrast to throughput enhancement, the proposed control strategy offers benefits in terms of reducing delays at all congestion levels.

- With the increment of the CPRs, the proposed control strategy can achieve more throughput improvement benefits and delay reduction benefits under high congestion levels. Especially when the congestion level is 1.4, the delay reduction benefits are more obvious.

- Compared with the non-control baseline, the proposed control strategy outperforms for traffic system resilience under high congestion levels. Especially when the left-turning demand proportion is high, the proposed control strategy can recover the traffic system to handle all vehicles, even if the congestion level is 1.4.

- Absolute bus priority can be guaranteed under various congestion levels and CPRs. The bus delay is less than one second, which means that the bus priority is not interfered with by general vehicles accessing the BPL.


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