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Multi-Objective Battery Coordination in Distribution Networks to Simultaneously Minimize CO₂ Emissions and Energy Losses

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Abstract: The techno–environmental analysis of distributed energy resources in electrical distribution networks is a complex optimization task due to the non-convexities of its nonlinear programming formulation. This research employs convex optimization to address this issue while minimizing the expected carbon dioxide emissions and daily energy losses of a distribution grid via the optimal dispatch of battery energy storage units (BESUs) and renewable energy units (REUs). The exact non-convex model is approximated via semi-definite programming in the complex variable domain. The optimal Pareto front is constructed using a weighting-based optimization approach. Numerical results using an IEEE 69-bus grid confirm the effectiveness of our proposal when considering unitary and variable power factor operation for the BESUs and the REUs. All numerical simulations were carried out using MATLAB software (version 2022b), a convex disciplined tool (CVX), and the semi-definite programming solvers SEDEUMI and SDPT3.

Keywords: battery energy storage units; semi-definite programming model; variable power factor operation; distribution networks; convex approximation; effective power coordination



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1. Introduction

1.1. General Context

Global warming, which is caused by massive carbon dioxide (CO₂) emissions, is a worldwide concern that has captured the attention of governments, researchers, industries, and society as a whole, since ensuring good living conditions for future human generations is the primary goal of our time [1]. Three human activities are the main contributors to global warming: (i) the use of transportation systems based on fossil fuels [2], (ii) the extensive production of meat [3], and (iii) the production of electricity through thermal plants using fossil fuels [4]. These activities have ushered the Earth into a new era known as the *Anthropocene*, which indicates that weather patterns have been altered by humans [5].

The Paris Agreement, adopted in 2015, set out the main goals for sustainable development, with one of the key pillars being the reduction in the total CO₂ emissions into the atmosphere [6]. In this context, most countries are promoting the widespread use of clean energy sources to meet the energy needs of industries without slowing down global economic growth, particularly in emerging economies [7]. Considering the significance of consistently mitigating the impacts of global warming, particularly in the electricity production sector, there is a need for further research on renewable generation and energy storage systems in order to enhance the quality of the services and to achieve the Sustainable Development Goals [8].

1.2. Motivation

Given the significance of gradually reducing CO₂ emissions from fossil-fuel-based energy sources, renewable energy resources have emerged as key drivers of global change. These sources utilize clean energy to meet energy demands [9]. In this context, renewable energy units (REUs) such as solar or wind technologies have been theoretically and experimentally validated as potential alternatives to diesel, coal, and natural gas [10,11]. The primary challenge posed by REUs is their dependence on weather patterns, which have been altered by the massive emissions of greenhouse gases. This unpredictability hinders their ability to provide consistent and dispatchable energy, limiting their potential to contribute to stable electrical markets [12].

To address these challenges regarding the use of REUs, energy storage technologies have emerged as the ideal complement. They can store energy from REUs when the availability is high and the electrical demand is low as well as release the stored energy to the grid when the demand increases and power generation decreases [13]. In electrical distribution applications, as is the case for medium-voltage networks, the most widely recognized energy storage systems are battery energy storage units (BESUs), which are based on lithium-ion technology. These devices are mature and reliable and have an adequate lifespan ranging from 6–30 years depending on the application [14]. They also offer excellent performance for stationary and mobile applications [15,16].

In light of the above, this research seeks to propose an effective coordination scheme for operating BESUs and REUs in electrical distribution networks. The aim is to reduce CO₂ emissions and daily energy losses through a practical multi-objective analysis.

1.3. Literature Review

Numerous research studies on the efficient operation of BESUs and REUs have been published in the scientific literature. Some examples are provided below.

The authors of [17] presented a two-state optimization approach for siting and sizing BESUs and dispersed generators in medium- and low-voltage distribution networks. The first stage utilized the classical simulated annealing optimization algorithm to determine the location of all of the nodes where the distributed energy resources (DERs) should be located. In the second stage, a convex approximation was used to linearize the energy loss function via a piecewise method. This was done to achieve effective power coordination between BESUs and REUs while minimizing both purchasing and operating costs.

The work by [18] proposed a mixed-integer, quadratically constrained programming approach to identify operation strategies for BESUs that can reduce energy losses and improve network efficiency from the distribution company's perspective. The proposed solution methodology employed classical backward/forward power flow to determine the grid operating conditions for each BESU's strategy. The main contribution of this work was the incorporation of batteries' reactive power capabilities. As is widely recognized in the specialized literature, managing reactive power through converters can also contribute to voltage improvement and can consequently reduce power losses.

The study by [19] presented a nonlinear programming model based on a nodal voltage formulation, which uses real-domain variables to operate BESUs in the face of large-scale REU integration. This research made two contributions: (i) the use of a recursive neural network to predict the expected renewable generation profile and (ii) the incorporation of BESUs' reactive power capabilities through decoupled control in power electronic converters.

The authors of [20] introduced a convex optimization approach based on second-order cone programming theory to locate and size BESUs in distribution networks while using an optimal power formulation. The optimization model considers variable market prices for feasible siting and sizing of BESUs, going beyond the classical approach, which only takes energy losses into account. By adapting an IEEE 69-bus grid system to the city of Nice, France, the authors validated the effectiveness of their convex formulation.

In [21], a stochastic-based optimization scenario was proposed in order to design an effective energy management system for active distribution networks (buildings across the

grid) considering heating, ventilation, and air conditioning systems. This proposal includes detailed modeling of thermal components in addition to considering solar photovoltaic (PV) generation and electric vehicles. The authors incorporate the uncertainties of the available REUs via stochastic analysis and propose a reduction method to select the most likely operating scenarios (from 1000 cases to only 5 by using the Monte Carlo approach). Numerical simulations considering the perspectives of the grid operator and the users confirmed the effectiveness of this optimization approach for applications involving active distribution networks.

The work by [22] proposed three optimization methodologies based on metaheuristic techniques for the optimal dispatch of BESUs in distribution networks. These methodologies included a continuous genetic algorithm (GA) and parallel versions of a particle swarm optimizer (PSO) and a vortex search algorithm (VSA). Numerical validations using IEEE 27- and 33-bus grids adapted to rural and urban networks of Colombia indicated that the VSA was the best metaheuristic optimizer for improving technical, economic, and environmental indices in this context. The primary limitations of this research lie in the fact that it considers a maximum power point tracking approach for the solar sources and that it requires unitary power factor operation for REUs.

The study by [23] introduced an optimal sizing and siting strategy for BESUs in distribution networks with a high penetration of renewable energy resources, with the aim of minimizing the expected energy losses. The solution methodology utilized a conventional GA, and numerical validations were conducted on an IEEE 33-bus system. However, no comparative analysis with other optimization methods was provided.

In [24], a new method for optimizing BESU coordination in distribution networks is proposed. This method considers electric vehicle substations and uses single- and multi-objective analysis. The authors employed a gorilla troop optimizer to determine the optimal power injection/absorption of electric vehicle charging substations while aiming to enhance voltage profiles and reduce energy losses. Numerical validation using a 108-bus grid confirmed the effectiveness of the proposed approach.

From this review of the state of the art, two main conclusions can be drawn: (i) multiple studies have been conducted to develop efficient strategies for operating REUs and BESUs in electrical systems from technical, economic, and environmental perspectives, and (ii) metaheuristics and convex-based approximations have been employed to solve optimization problems and address the complexities introduced by power balance constraints. This research identifies the relevance and timeliness of continuing to contribute with effective power coordination strategies for BESUs and REUs, and it seeks to propose a reliable operation strategy that can address the challenges posed by the new generation of active distribution networks.

1.4. Contributions and Scope

Based on the above, this research makes the following contributions:

- i. The proposal employs a convex-based optimization model based on semi-definite programming (SDP) theory to efficiently coordinate BESUs and REUs in distribution networks. This approach considers technical and environmental objective functions within a multi-objective optimization method.
- ii. This work assesses the impact of incorporating the reactive power control capabilities of the power electronic converters that connect BESUs and REUs. This allows operation with variable power factors, in contrast to conventional unitary power factor operation.

The numerical results obtained using the IEEE 33-bus grid, which was adapted to the operating conditions of an urban electrical system in Colombia, demonstrate the high solution quality offered by our proposal in comparison with those of the metaheuristic optimizers recently published in the literature. It is important to note that this research makes the following considerations: (i) the locations and sizes of the BESUs and REUs had already been determined via a technical–economic study conducted by the utility company,

so this research focuses solely on their efficient coordination, and (ii) the distribution company supplied the expected demand and power generation curves with daily resolution and without accounting for uncertainties. These curves are deterministic and serve as inputs for the proposed optimization model.

1.5. Document Structure

The remainder of this document is structured as follows. Section 2 provides the exact mathematical formulation of the effective power coordination problem for BESUs and renewable energy resources in distribution systems while considering technical and environmental objective functions. Section 3 describes the proposed convex approximation approach, where the SDP theory is applied to transform the exact nonlinear programming model into a convex one in the space of semi-definite Hermitian matrices. Section 4 outlines the main characteristics of the test feeder used for the analysis, which is an adaptation of an IEEE 33-bus grid to the characteristics of a Colombian urban distribution grid. Section 5 presents all numerical validations and their corresponding analysis, which includes a comparison with three population-based metaheuristics and Pareto front construction via a weighting-based method. Finally, Section 6 provides this work's main concluding remarks and proposals for future research.

2. Mathematical Modeling

Efficient energy management dispatch (EMD) of BESUs and REUs is a challenging problem in medium-voltage distribution networks [25] as two obstacles must be overcome: (i) solving for the time-coupling variables associated with the energy stored in or delivered by the BESUs and (ii) the nonlinear nature of the power balance equations given as a result of the products between complex voltages. The exact formulation of the EMD problem for BESUs in distribution grids is provided below using a complex variable domain representation. Note that the main objective functions under analysis correspond to the simultaneous minimization of daily carbon dioxide emissions (D_{CO_2}) and the expected daily energy losses (D_{loss}).

2.1. Objective Functions

The main goal of an effective EMD approach for BESUs and REUs is to improve the performance indices of interest for a distribution company, which may be economic, technical, environmental, social, or a combination of these [26]. This work aims for the simultaneous minimization of D_{loss} (the expected daily energy losses) and D_{CO_2} (the daily CO_2 emissions) [22]. These objective functions are defined in Equations (1) and (2).

$$\min D_{loss} = \operatorname{Re} \left\{ \sum_{h \in \mathbf{H}} \sum_{k \in \mathbf{N}} \sum_{m \in \mathbf{N}} \mathbb{Y}_{k,m} \mathbb{V}_{k,h}^* \mathbb{V}_{m,h} \Delta_h \right\}, \quad (1)$$

$$\min D_{CO_2} = \operatorname{Re} \left\{ \sum_{k \in \mathbf{N}} \sum_{h \in \mathbf{H}} \left(C_{CO_2,k}^g \mathbb{S}_{k,h}^g + C_{CO_2,k}^{dg} \mathbb{S}_{k,h}^{dg} \right) \Delta_h \right\}, \quad (2)$$

where $\operatorname{Re}\{\cdot\}$ is a function that returns the real part of its argument; $\mathbf{H}$ and $\mathbf{N}$ are the sets associated with the analyzed periods and the number of nodes, respectively; $\mathbb{V}_{k,h}$ and $\mathbb{V}_{m,h}$ correspond to the complex voltage variables associated with nodes k and m at time h ; $\mathbb{Y}_{k,m}$ is the nodal admittance parameter relating nodes k and m (complex value); Δ_h denotes the time discretization value; $\mathbb{S}_{k,h}^g$ and $\mathbb{S}_{k,h}^{dg}$ represent the complex power generation outputs of the conventional and renewable sources connected at bus k in period h , which have associated CO_2 emission rates given by $C_{CO_2,k}^g$ and $C_{CO_2,k}^{dg}$, respectively.

Remark 1. The objective functions defined in (1) and (2) have the following properties: (i) D_{loss} is a nonlinear function due to the product between voltage variables in the complex domain. Specifically, it is a quadratic objective function. However, it is strictly convex since the nodal admittance matrix

(i.e., $\mathbb{Y}_{n,n}$) is Hermitian semi-definite. (ii) D_{CO_2} is a linear function. Thus, applying Jensen's inequality [27] results in a convex function.

It is essential to highlight that fossil sources generating power at medium voltage levels for continuous energy consumption are not typically found in distribution networks, especially in urban ones. However, this does not imply that these grids are not responsible for any equivalent CO_2 emissions into the atmosphere. Distribution networks are typically interconnected with national power systems via transmission systems, which are the interface between large-scale generation systems and medium-voltage users. Depending on the country, most of these generation systems are composed of mixed matrices that include renewable and fossil energy sources [28]. In Colombia, the electricity generation matrix is composed of about 67% renewable sources (primarily hydraulic plants), and the remaining 33% is based on fossil fuels (coal, natural gas, and diesel), which implies that a third of the power consumption has a carbon footprint [29]. Considering the above, the $C_{CO_2,k}^g$ coefficient seeks to emulate the equivalent emissions caused by distribution networks [22].

Remark 2. In general terms, distribution networks for urban electricity service purposes can be analyzed as potential non-direct CO_2 emitters since a portion of the electrical energy consumed by the end-users was generated upstream of the substation and, in part, consisted of fossil or nonrenewable energy sources [22].

2.2. Set of Constraints

The hourly operation of BESUs and REUs in distribution networks is constrained by Kirchhoff's laws applied to electrical circuits with constant-power terminals, which yields a set of nonlinear and non-convex equations. The upper and lower operating bounds of the BESU and REU variables should be included, among other constraints. The constraints regarding the proposed EMD approach are listed from (3) to (15).

$$\mathbb{S}_{k,h}^{g,*} + \mathbb{S}_{k,h}^{dg,*} + \mathbb{S}_{k,h}^{b,*} - \mathbb{S}_{k,h}^{d,*} = \mathbb{V}_{k,h}^* \sum_{m \in \mathbf{N}} \mathbb{Y}_{k,m} \mathbb{V}_{m,h}, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (3)$$

$$SoC_{k,h+1}^b = SoC_{k,h}^b - \varphi_k^b \text{Re}\{\mathbb{S}_{k,h}^{b,*}\} \Delta_h, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (4)$$

$$\mathbb{Z}_{km} \mathbb{I}_{km,h} = \mathbb{V}_{k,h} - \mathbb{V}_{m,h}, \quad \{\forall h \in \mathbf{H}, \forall km \in \mathbf{L}\} \quad (5)$$

$$SoC_{k,h}^b = SoC_{k,i}^b, \quad \{\forall h = h_{\min}, \forall k \in \mathbf{N}\} \quad (6)$$

$$SoC_{k,h}^b = SoC_{k,f}^b, \quad \{\forall h = h_{\max}, \forall k \in \mathbf{N}\} \quad (7)$$

$$|\mathbb{S}_{k,h}^g| \leq S_{k,nom}^g, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (8)$$

$$\text{Re}\{\mathbb{S}_{k,h}^g\} \geq 0, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (9)$$

$$|\mathbb{S}_{k,h}^{gd}| \leq S_{k,nom}^{gd}, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (10)$$

$$|\mathbb{S}_{k,h}^b| \leq S_{k,nom}^b, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (11)$$

$$0 \leq \text{Re}\{\mathbb{S}_{k,h}^{dg}\} \leq S_{k,nom}^{dg} G_h^{dg}, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (12)$$

$$P_{k,\min}^b \leq \text{Re}\{\mathbb{S}_{k,h}^b\} \leq P_{k,\max}^b, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (13)$$

$$V_{\min} \leq |\mathbb{V}_{k,h}| \leq V_{\max}, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (14)$$

$$|\mathbb{I}_{km,h}| \leq I_{km,\max}, \quad \{\forall h \in \mathbf{H}, \forall km \in \mathbf{L}\} \quad (15)$$

Here, $\{\cdot\}^*$ is a function that returns the conjugate value of its argument; $\mathbb{S}_{k,h}^{b,*}$ corresponds to the power injected/absorbed by the BESU during period h when connected at node k ; $\mathbb{S}_{k,h}^{d,*}$ is the complex power consumption at bus k and time h ; $SoC_{k,h}^b$ and $SoC_{k,h+1}^b$ represent the state of charge of the BESU connected at node k during periods h and $h + 1$,

respectively; φ_k^b defines the average absorbed/injected power efficiency of the BESU connected at bus k ; $SoC_{k,i}^b$ and $SoC_{k,f}^b$ denote the initial and final states of charge of the BESU connected at bus k ; $\mathbb{Z}_{km}$ is the impedance parameter of the distribution branch connecting buses k and m , where the current flow is defined as $\mathbb{I}_{km,h}$ for each period h . The maximum complex power generation limits for conventional sources and REUs are defined by $S_{k,nom}^g$ and $S_{k,nom}^{dg}$, where $G_{k,h}^{dg}$ represents the renewable generation availability. Due to their dependence on weather conditions, these values depend on the hour during which they are analyzed. $S_{k,nom}^b$ is the complex power capacity of the power electronic converter interfacing with the BESU connected at bus k , where $P_{k,min}^b$ and $P_{k,max}^b$ represent the maximum power absorption and injection of the BESU, respectively. V_{min} and V_{max} represent the voltage regulation limits constraining the voltage variables at each node and period. $\mathbb{I}_{km}^{max}$ corresponds to the thermal capacity applicable to the branch that connects nodes k and m when transporting an electrical current at time h . Finally, h_{min} and h_{max} denote the initial and final data, respectively, in the set of times $\mathbf{H}$, and $\mathbf{L}$ represents the set containing all the network branches.

The mathematical interpretation of the constraints defined in (3)–(15) is as follows: Equality constraints (3)–(5) represent the complex power equilibrium for each node and time, the energy storage behavior of each BESU connected to the distribution network per period (which was formulated using the state of charge, which is equivalent to the energy parameter in percentage form), and Ohm's law applied to each network branch and period, respectively. Equalities (6) and (7) define the initial and final states of charge, respectively, of a BESU connected at bus k , i.e., the conditions defined by the distribution company to effectively operate BESUs. Box-type constraints (8) and (9) define the maximum complex power generation of the conventional source and imply that active power cannot be absorbed by these sources. Constraints (10) and (11) define the maximum complex power injection for the REUs and the complex power injection/absorption for the BESUs, respectively. Constraint (12) defines the time behavior of the primary renewable source's active power generation capabilities. Constraint (13) limits the active power absorption/injection of each BESU, which depends on the energy storage capabilities. Constraint (14) limits the voltage variables in accordance with regulatory policies. Finally, (15) defines the maximum current transference capacity applicable to the distribution branch connecting buses k and m for each period.

Remark 3. The set (3)–(15) generally represents a set of non-convex conditions even though most of these constraints are linear and norms. However, the product between voltages in (3) makes it non-convex due to the equality imposition, and the lower bound applicable to the norm (14) represents a hole in the solution space, i.e., a non-convexity.

Considering the non-convexities imposed by constraints (3) and (14) in the multi-objective EMD model (1)–(15), a global optimum cannot be ensured with conventional optimization methods (including metaheuristics). This constitutes a research opportunity that involves developing a multi-objective convex approximation via semi-definite programming theory.

3. Proposed Convexification Approach

To obtain an approximate model for coordinating batteries in distribution networks within the framework of a multi-objective analysis, this section proposes a convexification approach based on semi-definite programming theory to transform the power balance constraints (see Equation (3)). In addition, a weighting-based approach is presented to solve the multi-objective optimization problem.

3.1. Semi-Definite Programming Approximation

Semi-definite programming (SDP) is a subfield of convex optimization theory that works with the properties of semi-definite matrices to solve complex optimization problems.

This approach takes such matrices from the domain $\mathbf{R}^n$ to the domain $\mathbf{R}^{n \times n}$ (i.e., the matrix domain). Even though these transformations are counterintuitive, the geometrical properties of semi-definite matrices can efficiently solve complex nonlinear optimization problems [30]. This research proposes an SDP model to solve the multi-objective dispatch problem for BESUs in distribution networks, as modeled in (1)–(15).

To obtain an SDP equivalent in the complex domain, the following matrix must be defined:

$$\mathbb{W}_h = (\mathbb{V}_h^{\star, \top}) \mathbb{V}_h, \{h \in \mathbf{H}\} \quad (16)$$

$$\text{rank}\{\mathbb{W}_h\} = 1, \{h \in \mathbf{H}\} \quad (17)$$

where $\mathbb{W}_h \in \mathbf{C}^{n \times n}$ is a complex Hermitian semi-definite matrix. There is one such matrix for each period h . The main issue with the definition in (17) is that the rank constraint is non-convex. However, the authors of [31] recommend neglecting this condition in power flow studies since the nature of the solution space in this kind of problem allows the obtainment of the value of each vector $\mathbb{V}_h$ via the highest eigenvalue.

Now, considering the definition of the semi-definite matrix in (16), the power balance constraint in (3) can be convexified as follows:

$$\mathbb{S}_{k,h}^{g,\star} + \mathbb{S}_{k,h}^{dg,\star} + \mathbb{S}_{k,h}^{b,\star} - \mathbb{S}_{k,h}^{d,\star} = \sum_{m \in \mathbf{N}} \mathbb{Y}_{k,m} \mathbb{W}_{k,m,h}, \{ \forall h \in \mathbf{H}, \forall k \in \mathbf{N} \} \quad (18)$$

where $\mathbb{W}_{k,m,h}$ is equivalent to the product $\mathbb{V}_{k,h}^{\star} \mathbb{V}_{m,h}$.

Note that in the domain of SDP variables (i.e., $\mathbb{W}_h$), all the equations and constraints involving voltages in (1)–(15) must be transformed into a new set of variables. The current calculation shown in Equation (4) can be analyzed in this context. To obtain an equivalent equation as a function of the new variables, the auxiliary variable $\mathbb{J}_{km,h} = \|\mathbb{I}_{km,h}\|^2$, which yields

$$\|\mathbb{Z}_{km}\|^2 \mathbb{J}_{km,h} = \mathbb{W}_{k,k,h} - 2\text{Re}\{\mathbb{W}_{k,m,h}\} + \mathbb{W}_{m,m,h}, \{ \forall h \in \mathbf{H}, \forall km \in \mathbf{L} \} \quad (19)$$

Now the voltage regulation constraint, i.e., the box-type constraint (14), is rewritten as a function of the variable $\mathbb{W}_{k,k,h}$:

$$V_{\min}^2 \leq \mathbb{W}_{k,k,h} \leq V_{\max}^2, \{ \forall h \in \mathbf{H}, \forall k \in \mathbf{N} \} \quad (20)$$

which is now convex, as $\mathbb{W}_{k,k,h}$ is a real number, given that $\mathbb{W}_{k,k,h} = \mathbb{V}_{k,h}^{\star} \mathbb{V}_{k,h}$.

Finally, considering the definition of the new variable $\mathbb{J}_{km,h}$, the thermal bound shown in (15) is rewritten as (21):

$$\mathbb{J}_{km,h} \leq I_{km,\max}^2, \{ \forall h \in \mathbf{H}, \forall km \in \mathbf{L} \} \quad (21)$$

where it is convex since $\mathbb{J}_{km,h}$ a real-domain variable.

3.2. Approximated Convex Model

Now, in order to obtain a multi-objective convex optimization model, the objective function regarding the daily expected energy losses must also be written as a convex function in the set of new variables. This is done while following the same procedure shown in the previous subsection. Equations (22)–(37) present the proposed multi-objective convex optimization model for dealing with the effective operation of BESUs and REUs in distribution grids.

Objective functions:

$$\min D_{\text{loss}} = \text{Re} \left\{ \sum_{h \in \mathbf{H}} \sum_{k \in \mathbf{N}} \sum_{m \in \mathbf{N}} \mathbb{Y}_{k,m} \mathbb{W}_{k,m,h} \Delta_h \right\}, \quad (22)$$

$$\min D_{\text{CO}_2} = \text{Re} \left\{ \sum_{k \in \mathbf{N}} \sum_{h \in \mathbf{H}} \left(C_{\text{CO}_2,k}^g \mathbb{S}_{k,h}^g + C_{\text{CO}_2,k}^{dg} \mathbb{S}_{k,h}^{dg} \right) \Delta_h \right\}, \quad (23)$$

Set of constraints:

$$\mathbb{S}_{k,h}^{g,*} + \mathbb{S}_{k,h}^{dg,*} + \mathbb{S}_{k,h}^{b,*} - \mathbb{S}_{k,h}^{d,*} = \sum_{m \in \mathbf{N}} \mathbb{Y}_{k,m} \mathbb{W}_{k,m,h}, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (24)$$

$$\text{SoC}_{k,h+1}^b = \text{SoC}_{k,h}^b - \varphi_k^b \text{Re} \left\{ \mathbb{S}_{k,h}^{b,*} \right\} \Delta_h, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (25)$$

$$\|\mathbb{Z}_{km}\|^2 \mathbb{J}_{km,h} = \mathbb{W}_{k,k,h} - 2\text{Re} \left\{ \mathbb{W}_{k,m,h} \right\} + \mathbb{W}_{m,m,h}, \quad \{\forall h \in \mathbf{H}, \forall km \in \mathbf{L}\} \quad (26)$$

$$\text{SoC}_{k,h}^b = \text{SoC}_{k,i}^b, \quad \{\forall h = h_{\min}, \forall k \in \mathbf{N}\} \quad (27)$$

$$\text{SoC}_{k,h}^b = \text{SoC}_{k,f}^b, \quad \{\forall h = h_{\max}, \forall k \in \mathbf{N}\} \quad (28)$$

$$\left| \mathbb{S}_{k,h}^g \right| \leq S_{k,\text{nom}}^g, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (29)$$

$$\text{Re} \left\{ \mathbb{S}_{k,h}^g \right\} \geq 0, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (30)$$

$$\left| \mathbb{S}_{k,h}^{gd} \right| \leq S_{k,\text{nom}}^{gd}, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (31)$$

$$\left| \mathbb{S}_{k,h}^b \right| \leq S_{k,\text{nom}}^b, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (32)$$

$$0 \leq \text{Re} \left\{ \mathbb{S}_{k,h}^{dg} \right\} \leq S_{k,\text{nom}}^{dg} G_h^{dg}, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (33)$$

$$P_{k,\min}^b \leq \text{Re} \left\{ \mathbb{S}_{k,h}^b \right\} \leq P_{k,\max}^b, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (34)$$

$$V_{\min}^2 \leq \mathbb{W}_{k,k,h} \leq V_{\max}^2, \quad \{\forall h \in \mathbf{H}, \forall k \in \mathbf{N}\} \quad (35)$$

$$\mathbb{J}_{km,h} \leq I_{km,\max}^2, \quad \{\forall h \in \mathbf{H}, \forall km \in \mathbf{L}\} \quad (36)$$

$$\mathbb{W}_{s,s,h} = V_{\text{nom}}^2, \quad \{\forall h \in \mathbf{H}, s = \text{slack node}\} \quad (37)$$

$$\mathbb{W}_h \in \text{Hermitian semi-definite}, \quad \{h \in \mathbf{H}\} \quad (38)$$

where $\mathbb{W}_{s,s,h}$ represents the value of the variable associated with the equivalent voltage at the terminals of the substation. The remaining terms have already been defined.

Remark 4. Observe that the optimization model (22)–(38) is now a convex optimization model defined in the domain of the semi-definite matrices, i.e., it is an SDP approximation model to deal with the effective power coordination in BESUs and renewables for electrical medium-voltage distribution systems.

3.3. Weighting-Based Optimization Approach

To construct a Pareto front, i.e., to solve the SDP model (22)–(38), a weighting-based multi-objective approach is presented. The main characteristic of the weighting-based method for dealing with multi-objective optimization problems is that if the solution space (i.e., the set of constraints) is convex, then each point obtained in the Pareto front is optimal. In this vein, the proposed objective function Z_f is defined as shown in (39).

$$\min z_f = \omega \alpha D_{\text{loss}} + (1 - \omega) \beta D_{\text{CO}_2}, \quad (39)$$

where ω is a weighting factor that varies from 0 to 1, and α and β are normalization coefficients to allow for the summation of both objective functions with different units.

Note that the objective function in (39) is subject to the set of constraints (24)–(38) and defines a convex multi-objective optimization model.

Remark 5. Selection of the α and β coefficients is a critical factor in the weighting-based optimization method; inadequate selection can skew the final Pareto front solution. To avoid this issue, these parameters can be set as the extreme solution values (i.e., the worst solution) when performing a single-objective function analysis, thus allowing for normalization near the unitary objective function value. Thus, the weighting-based optimization approach can yield the optimal Pareto front.

4. Studied Test System

Our proposal was validated on an IEEE 33-bus grid that was adapted to a typical Colombian distribution network [22]. This system was designed to accommodate the expected generation and demand curves of Medellín, which is the second-largest city in Colombia. An IEEE 33-bus grid consists of 32 branches and 33 nodes that are operated from the substation bus at 12.66 kV. The system is a 3 kV DC microgrid equipped with three BESUs and three solar PV sources, as depicted in Figure 1.

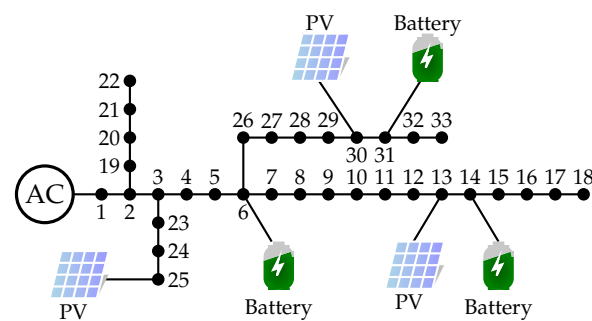


Figure 1. Nodal connection scheme for the IEEE 33-bus grid.

Figure 2 illustrates this system's expected demand and resource availability on a daily basis. In addition, its peak load consumption per node and its electrical branch parameters are listed in Table 1. It is worth mentioning that the expected demand curve presented in Figure 2 was measured at the terminals of the substation bus. This curve is multiplied by the peak load consumption at each node to evaluate the effects of daily demand variations.

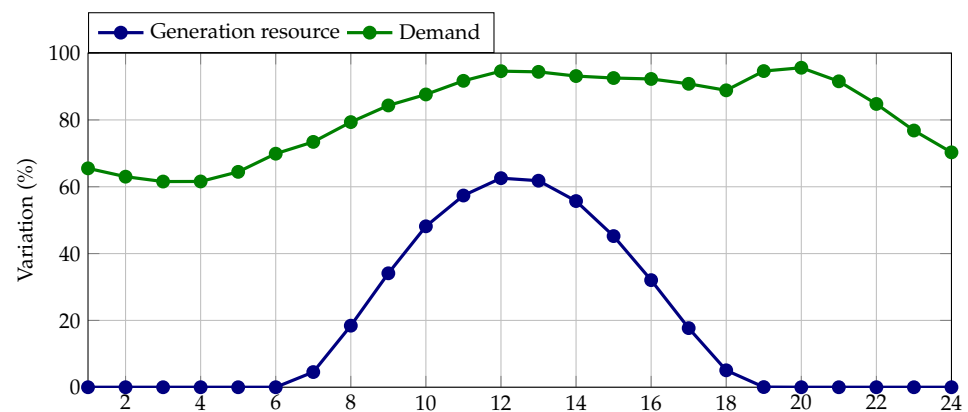


Figure 2. Daily generation and demand curves for the IEEE 33-bus grid in the studied scenario (Medellín, Colombia).

Table 1. Parameters of the IEEE 33-bus network.

Line l	Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)	$I_l^{\max}$ (A)
1	1	2	0.0922	0.0477	100	60	385
2	2	3	0.4930	0.2511	90	40	355
3	3	4	0.3660	0.1864	120	80	240
4	4	5	0.3811	0.1941	60	30	240
5	5	6	0.8190	0.7070	60	20	240
6	6	7	0.1872	0.6188	200	100	110
7	7	8	1.7114	1.2351	200	100	85
8	8	9	1.0300	0.7400	60	20	70
9	9	10	1.0400	0.7400	60	20	70
10	10	11	0.1966	0.0650	45	30	55
11	11	12	0.3744	0.1238	60	35	55
12	12	13	1.4680	1.1550	60	35	55
13	13	14	0.5416	0.7129	120	80	40
14	14	15	0.5910	0.5260	60	10	25
15	15	16	0.7463	0.5450	60	20	20
16	16	17	1.2890	1.7210	60	20	20
17	17	18	0.7320	0.5740	90	40	20
18	2	19	0.1640	0.1565	90	40	40
19	19	20	1.5042	1.3554	90	40	25
20	20	21	0.4095	0.4784	90	40	20
21	21	22	0.7089	0.9373	90	40	20
22	3	23	0.4512	0.3083	90	50	85
23	23	24	0.8980	0.7091	420	200	85
24	24	25	0.8960	0.7011	420	200	40
25	6	26	0.2030	0.1034	60	25	125
26	26	27	0.2842	0.1447	60	25	110
27	27	28	1.0590	0.9337	60	20	110
28	28	29	0.8042	0.7006	120	70	110
29	29	30	0.5075	0.2585	200	600	95
30	30	31	0.9744	0.9630	150	70	55
31	31	32	0.3105	0.3619	210	100	30
32	32	33	0.3410	0.5302	60	40	20

Regarding characterization of the BESUs and solar sources (see Figure 1), the following was taken into account [22]:

- The BESUs are located at buses 6, 14, and 31 and have nominal sizes of 2000, 1000, and 1500 kWh, respectively. In addition, the BESU connected to bus 6 has charging and discharging times of 5 h, whereas the remaining BESUs have times of 4 h.
- The PV sources are located at nodes 13, 25, and 30 and have installed capacities of about 1125, 1320, and 999 kW, respectively.

5. Numerical Validations

For the computational implementation of our proposal, the MATLAB programming environment (version 2022b) was used on a PC with an AMD Ryzen 7 3700 2.3 GHz processor and 16.0 GB RAM running on a 64-bit version of Microsoft Windows 10 Single Language. The solution of the proposed SDP model was reached via the CVX optimization environment using the SDPT3 and SEDUMI solvers [32,33].

5.1. Computational Validation against Literature Reports

A comparative analysis with a recent approach using metaheuristics demonstrated that the proposed SDP method efficiently solves the problem regarding the effective coordination of BESUs and solar renewable generation systems. These comparisons were made (i) considering unitary power factor operation for the BESUs and the REUs and (ii) per-

forming single-objective function analysis, i.e., the solution of the objective function (39) for the extreme values of the ω factor. The algorithms used for comparison were the continuous genetic algorithm (CGA) and the parallel versions of the particle swarm optimizer (PPSO) and the vortex search algorithm (PVSA). As for the emission coefficients, $C_{CO_2,k}^g$ was defined as 0.16438 kg/kWh, and $C_{CO_2,k}^{dg}$ was assumed to be zero since solar generation is regarded as a green energy generation technology in its operation stage.

The numerical results in Table 2 demonstrate that:

- i. The proposed SDP approximation outperforms the proposal made by [22] that uses the PVSA for both objective functions. In the case of the expected daily energy losses, the improvement is about 4.3975 kWh/day. As for the CO₂ emissions, the improvement is about 0.4283 kgh/day. Even though both reductions may seem like minor improvements, they demonstrate that the proposed SDP approximation effectively finds the best possible solution for the studied problem. At the same time, metaheuristic optimizers may become trapped in local optima.
- ii. With respect to the benchmark case (i.e., operation with solar sources and without BESUs), the improvement in daily energy losses is approximately 4.4743%, while the reduction in the expected CO₂ emissions is about 0.1848%. This confirms that the optimal coordination of BESUs in electrical distribution networks can improve both technical and environmental objective functions, implying that efficient energy management systems must be designed for utilities to take full advantage of their potential.

Table 2. Comparative results regarding both objective functions.

Method	D_{CO_2} (kg/day)	D_{loss} (kWh/day)
Benchmark case	9887.4082	2484.5747
CGA	9878.0207	2431.4745
PPSO	9869.6348	2380.8336
PVSA	9869.5623	2377.8028
SDP	9869.1339	2373.4053

5.2. Multi-Objective Analysis

Based on the numerical results reported above, the size of the PV generators may appear small since the use of BESUs, even though they improve the objective function, does not provide large improvements. In this vein, all the solar generators were assumed to have nominal power rates of about 2400 kW in our simulations. Two simulation cases were evaluated: (i) operating the BESUs and REUs with a unitary power factor (having defined the values of the α and β coefficients) and (ii) dispatching the BESUs and REUs with a variable power factor. Note that α was defined as 2031.7983 kWh/day, and β was defined as 7693.5172 kgh/day.

Remark 6. The scenario involving a variable power factor for the REUs and BESUs has no practical limitations since for these active power devices, the power factor can vary from 0 to 1 depending on the grid's operating conditions. An example of this is nighttime operation of PV plants, during which time the renewable energy resource is null; by controlling the power electronic converter interfacing this source, reactive power can be injected, absorbed, or set as null as a function of the grid requirements [34].

Figure 3 depicts the behavior of the Pareto front for both simulation scenarios.

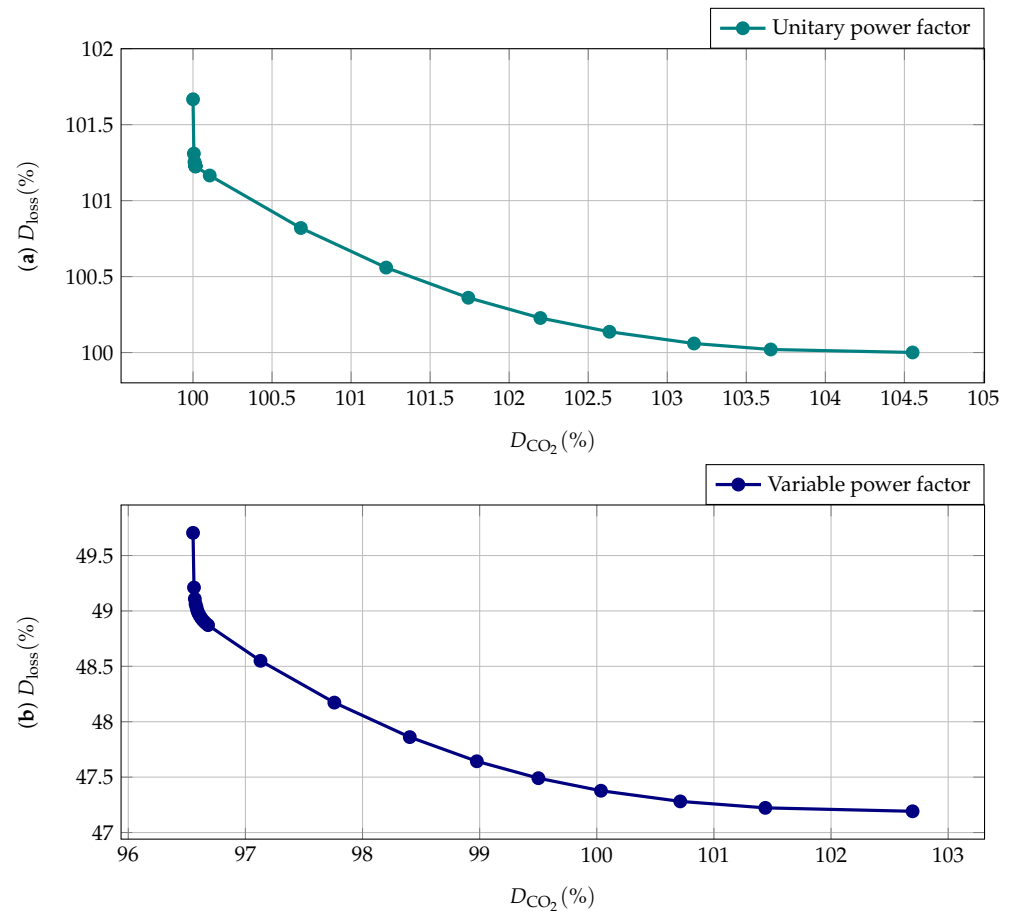


Figure 3. Pareto fronts for the daily operation of BESUs and solar sources in distribution networks: (a) unitary power factor operation and (b) variable power factor operation.

The numerical results in Figure 3 show that:

- i Regarding unitary power factor operation (see Figure 3a), when the optimal solution is reached for the objective function D_{loss} (see vertical axis with 100% of the value), the emissions level is shown to be higher than 104.5%, i.e., exactly 104.5518%. This means that, concerning the reference value assigned to the β coefficient, the CO₂ emissions level is 4.5518% higher, which corresponds to about 350.1935 kgh/day of additional emissions when minimizing the energy losses. In the case of CO₂ emissions minimization (see horizontal axis with 100% of the value), the energy losses increase by about 101.667% with respect to the reference value in α . This means that minimizing D_{CO_2} generates about 1.667% of additional energy losses in a typical day of operation.
- ii The results regarding variable power factor operation (see Figure 3b) show the behavior of the energy losses to be between 47.1917% and 49.7048% with respect to the reference case, i.e., the α value for unitary power factor operation. In other words, consideration of the reactive power control capabilities of the BESUs and solar generation sources allows for improvements greater than 50% for the energy losses indicator in comparison with the unitary power factor scenario. Moreover, the Pareto front in Figure 3b shows that the CO₂ emissions index varies between 96.5529% and 102.6962% with respect to the reference case associated with the β coefficient. This implies that when considering variable power factor operation, a wider range of possibilities is obtained. This means that the distribution company has more options to decide on for the daily operation of its DERs as a function of its interests and policies.

5.3. Complementary Analysis

To demonstrate the positive effect of a variable power factor on the daily operation of BESUs and REUs, Figure 4a,b compare the energy output of the slack source and the expected daily energy losses for both studied scenarios. Note that these results were obtained from the extremes of the Pareto fronts, i.e., for values of 0 and 1 assigned to the ω weighting factor.

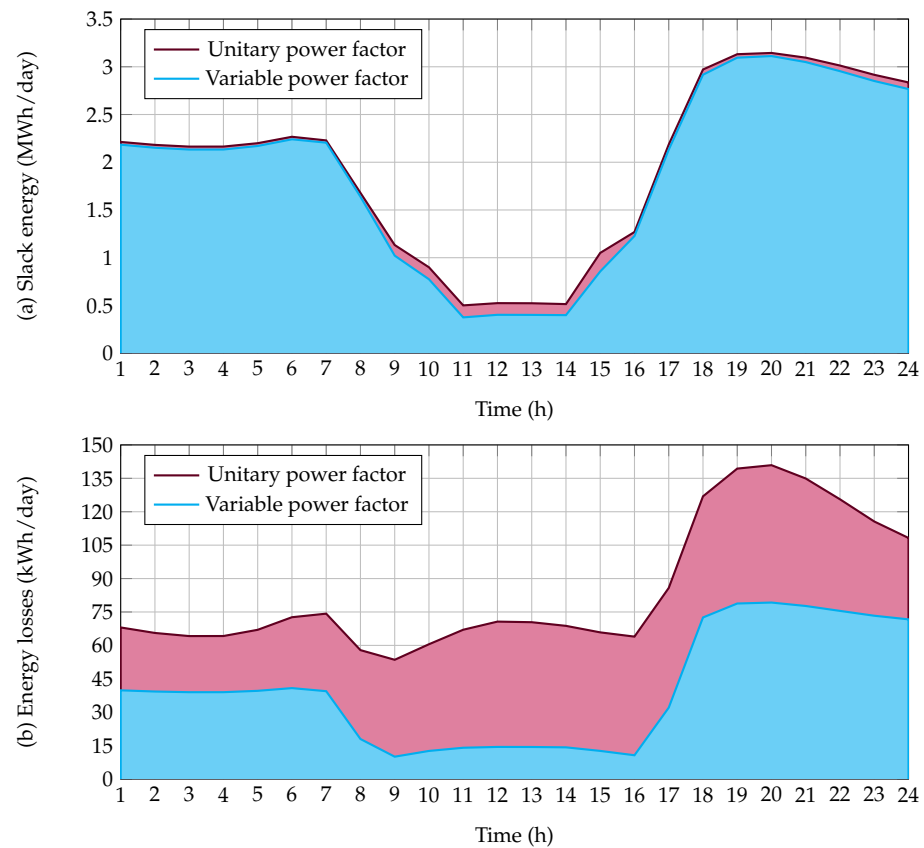


Figure 4. Daily power generation variations in the (a) slack source and (b) energy losses.

As can be seen in Figure 4a, a variable power factor allows for reducing the amount of power injected by the slack source in all periods; this is due to the fact that reactive power generation reduces the energy losses (see Figure 4b). This implies that lower power generation is required to supply the power consumed by the substation, which translates into a reduction in the total expected CO₂ emissions. As for the reduction in energy losses, Figure 4b shows the positive impact of dynamic reactive power injection as a function of the grid's requirements; as the voltage profile improves, the line current flow decreases, reducing the energy losses.

On the other hand, to illustrate the variations in the active and reactive power injections for the extremes of the Pareto front, Figure 5 shows the aggregate (complex power summation) of active and reactive power generation from all the PV sources when minimizing power losses or CO₂ emissions independently.

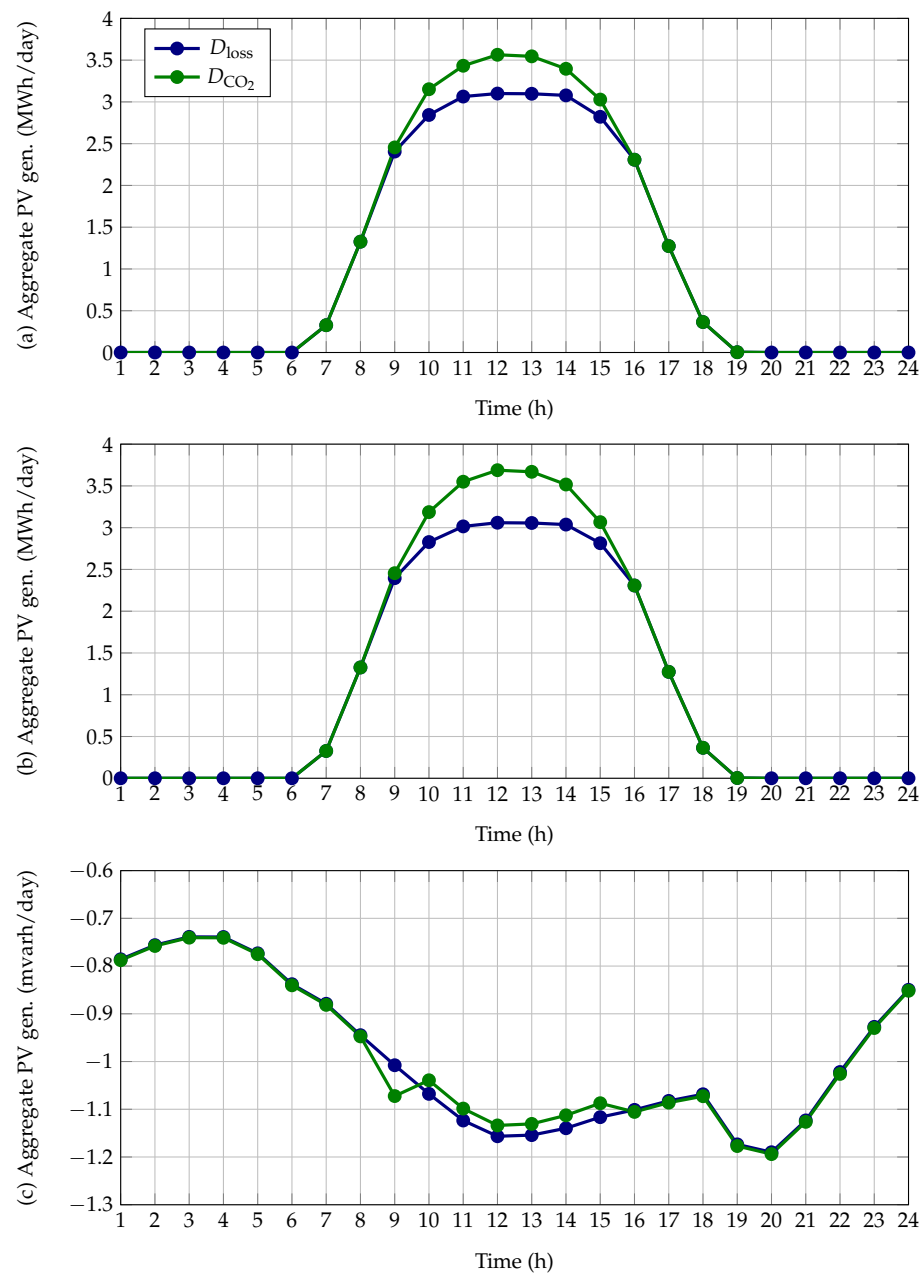


Figure 5. Aggregate PV generation when minimizing each single-objective function: (a) unitary power factor, (b) active power injection with variable power factor, and (c) reactive power injection with variable power factor.

In general terms, Figure 5 shows that:

- i. Active power generation in unitary and variable power factor operation is always greater in the case of minimizing CO₂ emissions, which is an expected result since carbon dioxide emissions are directly a function of the power injections in the equivalent substation bus, which implies that, if possible, all the available renewable generation will be injected into the grid to minimize the total power generation in the substation terminals.
- ii. In the case of variable power factor operation, the main result, as expected, is reactive power injection in all operating periods for both objective functions, which implies that with reactive power, energy losses are reduced. In the case of energy loss minimization, the expected results are evident. On the other hand, in the case of CO₂ emissions, the reduction in energy losses implies a reduction in the power injections of the

substations, i.e., a reduction in the total CO₂ emissions. In both objective function analyses, the expected level of reactive power injection shows similar demand behavior (compared to Figure 2), which is the expected behavior since energy losses are a quadratic function of the demanded current variations.

5.4. Stochastic Analysis

This section analyzes the performance of the proposed technical–environmental optimization model using stochastic optimization scenarios. This analysis was conducted to identify potential errors in predicting PV generation or demand, as it is essential to work towards minimizing them. To this effect, 1000 predictions were employed for PV generation and demand using an instance of Monte Carlo simulation with a normal distribution function [35]. The number of PV generation and demand scenarios were each reduced to five (i.e., the most representative ones) using the K-means algorithm. This reduction enhances computational efficiency and solution quality by eliminating scenarios that can introduce noise in the optimization process [36,37]. Figure 6 illustrates the 1000 predictions and the reduced PV generation or demand scenarios.

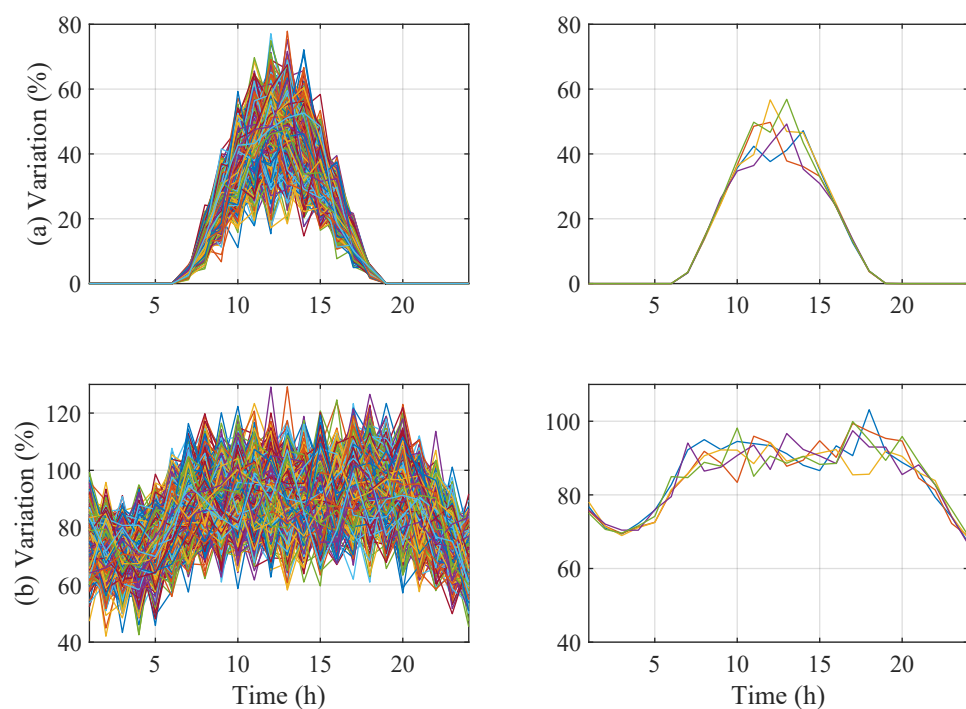


Figure 6. Predictions (left) and representative scenarios (right): (a) PV generation and (b) demand.

It is essential to mention that, as generated by five generation and demand scenarios, there are 25 possible combinations to be evaluated. For the sake of simplicity, only each objective function's minimum, mean, maximum, and standard deviation values are presented in Table 3.

Table 3. Results obtained for stochastic scenarios while considering a variable power factor.

Objective Function	Minimum	Average	Maximum	Standard Deviation
D_{CO_2} (kg/day)	8539.1511	8808.8502	9053.1053	13.49% (1188.7166)
D_{loss} (kWh/day)	1089.8598	1135.8386	1163.5758	2.13% (24.2202)

The numerical results in Table 3 show that:

- i. When considering the reduction of the expected daily emissions of CO₂ as the objective function, the minimum possible value corresponds to 8539.1511 kg/day under the most favorable demand and generation conditions, i.e., the high availability and low

demand profile. In contrast, the worst case exhibits a value of about 9053.1053 kg/day and is associated with a high demand value and low PV generation potential. In addition, the most likely value for a daily operation scenario corresponds to the average emissions profile, with about 9053.1053 kg/day and a standard deviation of about 13.49%.

- ii. The stochastic scenario has reduced effects on the expected energy losses profile. Under the worst operating conditions, the value of the expected losses is about 1163.5758 kWh/day. In contrast, in the most favorable scenario, the minimum possible value is about 1089.8598 kWh/day. In this case, the standard deviation is only 2.13%, and the mean value is 1135.8386 kWh/day.
- iii. In a general analysis, for the IEEE 33-bus network under the studied operating conditions, there are possible variations of about 73.7160 kWh/day and 513.9542 kg/day regarding energy losses and CO₂ emissions, respectively. This shows the potential effect of variations in the expected demand and PV generation curves with respect to the deterministic case, allowing distribution companies to define the adequate dispatch of REUs and BESUs as a function of their operating plans and interests.

It is worth mentioning that the stochastic analysis presented in this section considers that the REUs and BESUs are dispatched while considering variable power factor operation, as their effectiveness in comparison to unitary factor operation has already been demonstrated.

6. Conclusions and Future Work

This research addressed the problem regarding the simultaneous coordination of BESUs and solar REUs in medium-voltage distribution networks from a convex approximation perspective, transforming the exact nonlinear non-convex optimization model via SDP theory. simultaneous minimization of the expected daily CO₂ emissions and energy losses was reached via a weighting-based multi-objective optimization approach. Numerical validations carried out on an adapted version of an IEEE 33-bus grid allow stating that:

- i. Single-objective function analysis considering unitary power factor operation for BESUs and REUs shows that the proposed SDP approach outperforms the metaheuristic methods reported in the literature regarding optimal values for D_{loss} and D_{CO_2} . It is worth mentioning that the optimizers used for comparison were the CGA, the PPSO, and the PVSA.
- ii. The results obtained with multi-objective operation that takes into consideration the expected daily energy losses and CO₂ emissions shows that with high renewable energy availability, the objective functions are effectively in conflict, i.e., minimizing one of them implies compromising the other. In addition, when comparing the variable and unitary power factor approaches, it is observed that the expected energy losses can be improved by more than 50%. At the same time, the Pareto front for the CO₂ emissions level varies by about 6%, which means that the distribution company has more options to define the operation of its electrical network and its DERs.

The main limitation of the proposed SDP approach is that it considers deterministically expected demand and solar generation availability curves, i.e., probable variations are not taken into account via stochastic or robust optimization. However, this is a research opportunity that can be addressed in future work. Future work could (i) propose a mixed-integer convex approach to define the optimal location/reallocation points for BESUs while aiming for technical, economic, or environmental improvements; (ii) consider the self-discharging effects of BESUs and the energy losses of power electronic converters interfacing with DERs; (iii) consider new methodologies for analyzing uncertainties in renewable energy sources and demand profiles using scenario-based stochastic programming model approaches or robust optimization methods; or (iv) perform a complementary analysis including the costs associated with energy losses and CO₂ emissions.

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Nomenclature

Indices

b	Superscript associated with battery.
g	Superscript associated with conventional source.
dg	Superscript associated with renewable source.
k, m	Subscript associated with nodes.
h	Subscript associated with period.

Parameters and Variables

D_{loss}	Objective function value regarding the expected daily energy losses (Wh/day).
D_{CO_2}	Objective function value regarding the daily CO ₂ emissions (kg/day).
$C_{CO_2,k}^g$	CO ₂ emission rates associated with conventional source (kg/W).
$C_{CO_2,k}^{gd}$	CO ₂ emission rates associated with renewable source (kg/W).
$V_{k,h}$	Complex voltage variables associated with node k at time h (V).
$Y_{k,m}$	Nodal admittance parameter relating nodes k and m (S).
Δ_h	Length of time discretization (h).
$S_{k,h}^g$	Complex power generation of the conventional source connected at bus k during period h (VA).
$S_{k,h}^{gd}$	Complex power generation of the renewable source connected at bus k during period h .
$S_{k,h}^b$	Complex power injected/absorbed by the BESU connected at node k during period h (VA).
$S_{k,h}^{dr}$	Complex power consumption at bus k and time h (VA).
$SoC_{k,h}^b$	State of charge of the BESU connected at node k during period h (%).
$SoC_{k,h+1}^b$	State of charge of the BESU connected at node k during period $h + 1$ (%).
ϕ_k^b	Efficiency of the absorbed/injected power of the BESU connected at bus k (%/Wh).
$SoC_{k,i}^b$	Initial state of charge of the BESU connected at bus k (%).
$SoC_{k,f}^b$	Final state of charge of the BESU connected at bus k (%).
Z_{km}	Impedance of the branch connected between nodes k and m (Ω).
$I_{km,h}$	Complex current flow through the branch connected between nodes k and m during period h (A).
$S_{k,nom}^g$	Maximum complex power generation limits for a conventional source connected at node k (VA).
$S_{k,nom}^{dg}$	Maximum complex power generation limits for a renewable source connected at node k (VA).
$S_{k,nom}^b$	Complex power capacity of the BESU connected at bus k (VA).
$P_{k,min}^b$	Maximum active power absorption of the BESU connected at bus k (W).
$P_{k,max}^b$	Maximum active power injection of the BESU connected at bus k (W).
V_{min}	Minimum allowable voltage (V).
V_{max}	Maximum allowable voltage (V).
I_{km}^{max}	Maximum current flow through the branch connected between nodes k and m (A).
h_{min}	Initial analysis period (h).
h_{max}	Final analysis period (h).
W_h	Complex Hermitian semi-definite matrix during period h (V ²).
$W_{k,m,h}$	Complex square voltage results product between $V_{k,h}^* V_{m,h}$ during period h (V ²).
$J_{km,h}$	Magnitude of current flow squared through the branch between nodes k and m during period h (A ²).
ω	Weighting factor.

Sets

L	Set that contains all branches of the network.
N	Set that contains all nodes in the network.
H	Set that contains all analysis periods.

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