Rail Transit Networks and Network Motifs: A Review and Research Agenda

Yunfang Ma, Jose M. Sallan * and Oriol Lordan

Department of Management, Universitat Politècnica de Catalunya, C/Colom 11, 08222 Terrassa, Spain; yunfang.ma@upc.edu (Y.M.); oriol.lordan@upc.edu (O.L.)
* Correspondence: jose.maria.sallan@upc.edu

Abstract: The railway plays an essential role in urban and intercity transport of goods and people. Intercity and urban rail transit infrastructures contribute to the economic and environmental sustainability of global economies. Those infrastructures can be modeled as complex networks, so that we can evaluate system properties of the network structure. This stream of research has focused on the topological analysis of global network structure, but little research exists that examines how local network structures affect system properties. The local structure of complex networks can be examined with network motif analysis, as those network motifs are the building blocks of networked systems. Nevertheless, there has been scarce attention given to local network properties in rail transit networks. We contribute to covering this gap in the literature with a literature review of motif analysis research and its application to weighted and unweighted rail transit networks, also covering the current state-of-the-art of network motif decomposition and analysis. We demonstrate that network motif analysis is not only applicable, but also beneficial for the design and planning of rail transit networks, enhancing their sustainability by improving efficiency, reducing environmental impact, and optimizing resource allocation. Based on our findings, we propose future research directions that involve applying motif analysis to enhance the sustainability features of both unweighted and weighted rail transit networks.

Keywords: rail transit; complex networks; motif analysis

1. Introduction

Railway systems are vital to the infrastructure networks that support human societies, akin to telecommunications, transportation, and electricity, with significant implications for sustainable urban development [1]. With the continuous and rapid development of the economy and constant urbanization, traffic volume within and between large and medium cities has increased sharply and railway systems have contributed to handle it [2]. However, railway systems often encounter challenges such as passenger flow stagnation and low operational efficiency due to their highly networked operations and the complex interplay of various factors [3,4]. These challenges highlight the need for robust and efficient rail transit infrastructures, like railway corridors, which are instrumental in the sustainable development of global economies by facilitating the movement of goods and people [5]. For instance, it is crucial to determine if connecting two stations with more than one path would lead to improved travel efficiency from a global standpoint, or if it would result in a wastage of resources. This challenge is also evident in subway network planning, where identifying the optimal location for a new station requires examining the network from a structural viewpoint. Furthermore, in the event of an emergency at a specific subway station, it is vital to swiftly and efficiently reconfigure the subway line, necessitating a structural approach to studying these issues. Against this backdrop, to enhance the overall operational capabilities of public transport and ensure its safety enhancing network robustness [6–9], it is essential to examine the efficiency and functional layout of rail transit networks.
The integration and forecasting of passenger flow in multi-level rail transit networks are critical for enhancing connectivity between different transport modes, thus contributing to the sustainability of urban transport systems [10]. Evaluating the importance of urban rail stations in a topology network considering traffic characteristics can further improve the reliability and performance of urban rail networks, essential for sustainable urban development [11]. This emphasizes the need for robust methodologies that can effectively assess and optimize the structural health and operational efficacy of railway systems.

The development of complex network theory has helped us better understand phenomena in multiple fields, such as biochemistry, neurology, ecology, sociology, engineering, and transportation [12–16]. A real-world network structure is the evolving result of the forces shaping it, and this structure certainly affects the network’s properties [17]. This relationship between network structure and properties has been acknowledged in railway systems research [18]. Some of the properties examined in rail transit networks are vulnerability [19–21], topology analysis [22–24], robustness [2], optimal scheduling [25], centrality [26], and community detection [27]. These studies form the basis of planning and operational strategies that directly influence sustainability outcomes in railway systems [28,29].

A common feature of the studies mentioned above is that they focus on the global properties of the network. Examples of these global properties are degree distribution, average shortest path length, and average clustering coefficient. However, networks that are similar in terms of global topological properties may noticeably differ at a local level [30]. In spite of this, few studies have analyzed how local structures affect network properties [31]. Local properties of complex networks can be analyzed through network motif analysis. The concept of network motifs, for instance, offers a powerful tool for uncovering local structural insights that could significantly enhance the resilience and sustainability of rail transit networks [14,32].

Though network motifs have gathered much attention as a concept to uncover structural design principles of complex networks in multiple fields in recent years, there is very limited research on its application to rail transit networks. The proposed study aims to apply network motif analysis to rail transit networks, bridging a critical research gap and contributing to the theory of rail transit networks by analyzing local properties through a sustainability lens.

2. Materials and Methods

To accomplish the research aims, we proceeded to undertake a systematic literature review. This study followed the original guidelines for systematic literature reviews proposed by [33]. General quality criteria for a systematic review include transparency, reproducibility, and systematic methods [34]. Our review specifically aims to integrate sustainability into the examination of network motifs within rail transit networks, a topic that holds substantial potential for advancing sustainable urban mobility. To initiate a concept-centric study, the first step is to conduct research on the problem and carefully choose relevant keywords before searching for literature across various databases.

Given the nascent stage of research on network motifs in the field of transportation, particularly from a sustainability perspective [35], our keyword strategy was divided into two main areas: “rail transit network and network structure” and “network motif and transportation”. This approach ensured a comprehensive search, capturing both direct and tangential literature relevant to our sustainability-centric research aims. We conducted searches across several databases renowned for their extensive coverage of sustainability and transportation research, including Science Direct, Web of Science, EBSCOhost, ProQuest, and Emerald.

Our search did not limit the temporal scope but was naturally inclined towards studies published after the pioneering work by [14] on network motifs. A systematic literature review involves identification, screening, eligibility, and inclusion processes [36]. The process adhered to the PRISMA framework [36], ensuring a rigorous and reproducible
methodology, as depicted in Figure 1. We included studies that specifically tackled themes around the optimization of rail transit networks through the lens of sustainability, such as energy efficiency, emission reductions, and enhancement of service quality. We conducted an initial search using broad search terms to identify relevant articles. After obtaining the full text of the relevant articles, we review their reference lists to identify additional sources. These are the ones under the group “Additional records identified through other sources”. It is worth noting that, apart from utilizing Science Direct, Web of Science, EBSCOhost, ProQuest, and Emerald databases, we gathered further records through both forward and backward search methods and through related references and keyword searches.

While screening the literature search, we adhered to specific inclusion and exclusion criteria. We focused on articles that not only discuss rail transit networks or related topics like urban transportation and network optimization, but also explicitly considered the sustainability impacts of these systems. Articles were required to discuss rail transit network motifs or topology and be peer-reviewed or published by reputable industry sources known for sustainability studies. Articles that did not meet these criteria were excluded from consideration.

The journals where the final sample of articles was published more frequently are listed in Figure 2. The journals with the most sample articles published were Physica A, Transportation Research Part E, and the Journal of Transport Geography. While the first journal is a frequent outlet for complex network theory contributions, the other two are focused on transportation analysis. This confirms the multidisciplinary role of this field of study.

Figure 1. Sequencing of the phases of the systematic literature review.
Figure 2. Main peer-reviewed journals where the selected articles are published.

Once the selected articles were identified, we grouped them into three large subgroups: articles dealing with intercity and urban rail transit (Section 3), articles describing advances in rail transit networks as complex networks (Section 4), and a final group presenting contributions related to network motif analysis (Section 5). The examination of these three streams of literature allowed us to define a research agenda on the application of network motif analysis in rail transit networks. By analyzing these motifs, our study aims to propose methods that enhance the operational efficiency, resilience, and environmental footprint of urban rail systems, thereby aligning with the goals of sustainable urban development.

3. Intercity and Urban Rail Transit

Rail transit consists of the transportation of passengers and goods using wheeled vehicles that travel along rail tracks. Rail transit systems include intercity and urban rail transit systems, with different characteristics according to transportation purpose and distance range [37]. Rail transit systems provide a remedy for urban traffic congestion, delivering frequent and secure journeys to a large number of passengers [38]. The rail transit network focuses on the topological structure formed by the interaction between stations in the entire complex system, which is the basis for understanding the complex system’s nature and function. Weighted and unweighted rail transit networks reflect different characteristics of rail transit. Different types of weight could provide different information, which could help understand the operation of the rail transit system.

3.1. Intercity Rail Transit

Intercity rail transit refers to the passenger rail transit with high speed, public transport, and large capacity between major central cities in economically developed and densely populated urban agglomerations or within a large city’s rail transit commuter circle [37,39].

The railway transportation system began in European countries [40]. Thus, intercity rail transit has developed early and matured in Europe. The rail transit system in continental Europe is closely connected, and passenger transportation efficiency is high. For example, intercity rail transit in Germany and France has precise functional positioning and division regarding line setting, operating speed, service objects, and application functions. In Japan, rail transit has formed a complete network system at all levels, and its operation mode and market scale are very mature [41]. In China, the development of high-speed railways and urban rail transit is becoming increasingly mature, while the intercity rail transit, which is connects a trunk line and urban rail transit, has just started [42]. The construction of intercity rail transit in many large urban agglomerations has achieved great success, and world-class metropolitan areas such as Tokyo, New York, and London have emerged. Intercity rail transit is the product of the continuous development and...
expansion of urban agglomerations and is also an essential part of the regional rail transit system of urban agglomerations. Since the layout of intercity rail transit is relatively modeled, it is also necessary to pay more attention to its coordination and connection with urban agglomeration planning and other transportation network planning.

Intercity rail transit has a large volume, high speed, high efficiency, safety, reliability, is fast and convenient, low-carbon, and energy-saving, which is the most suitable way of public transportation to connect all nodes and micro-centers in the metropolitan area [43]. It can connect network elements within the existing urban agglomeration, greatly change the accessibility of the regional space, and guide industrial spatial reorganization and rational population distribution [41]. It is conducive to promoting the regional transformation of economic development modes, optimizing resource allocation, improving land-use efficiency, and promoting regional integration and urbanization development [44]. The layout of intercity rail transit is closely related to the size of passenger flow, travel purpose, and travel distance [43].

Based on the differences in the development mode of the regional spatial structure of the urban agglomeration and the objectives and layout characteristics, intercity rail transit can be divided into four layout modes: radial type, pendulum type, bead type, and network type [45]. The structural design of intercity rail lines has commonly followed a hub-and-spoke model, linking a central hub city to secondary cities acting as spokes. This configuration aims to optimize track utilization and leverage network effects at the hub city, facilitating frequent service to various destinations [46]. Authors of [47] built a general equilibrium and showed that the construction of a subway system would not necessarily result in reversing the trend toward suburbanization. The studies [48,49] illustrated the relationship between spatial economy and transportation structure. The correlation between high-speed rail station location and urban spatial structure was investigated in [50]; the results show that the location of the stations has an important impact on the urban layout and households’ location choices.

Complex network theories have been used widely on intercity rail transit. In [51], a review of the literature on the application of complex network theory to high-speed railway systems at different spatial and temporal scales was undertaken. These systems present different configurations regarding the weight of the edges in a weighted network. Authors of [52] applied complex network theory to assess the structural vulnerability and intervention of high-speed rail transit networks, which are fragile to severe external disturbances, and they discovered that the Japanese high-speed rail transit network has better global connectivity than that of China and the US. However, the Chinese high-speed railway has better local connectivity than Japan and the US. The study [53] used complex network analysis to explore how the spatial structure of a high-speed rail transit network evolves at a local level. The findings illustrate the progression of station placements, community organization, and the interconnections among stations on a regional scale. Additionally, they demonstrate the local effects of high-speed rail transit network development within core cities on structures and connectivity. Additionally, authors of [54] studied the reliability and topology structure of the high-speed rail transit network and discovered that key stations play an essential role in improving the whole network’s reliability. Together, these studies indicate that structural principles greatly impact the intercity rail transit system.

3.2. Urban Rail Transit

The term “urban rail transit” encompasses a wide range of local rail systems that offer passenger services to urban or suburban areas. These systems include tram, commuter rail, light rail, monorail, funicular, and cable car. Occasionally, there is overlap, as certain systems or lines may incorporate aspects of multiple types [55]. Rapid transit, known as rail transit, has high passenger capacities and frequency of service in an urban area, fulfilling the needs of the growing population’s mobility [56].

Urban rail transit adapts to the urban structure where it is located and has played a considerable role in promoting the development of the urban economy and society [57]. Since
the world’s first subway was completed and opened to traffic in London, England, in 1863, the construction of rail transit has become essential for major cities to improve urban traffic problems [55]. After World War II, the construction of rail transit declined. However, with the excessive use of cars, urban traffic problems have become increasingly severe, so urban rail transit has developed again, and many new technologies and processes have emerged during rail transit construction [58]. Now, urban rail transit has developed from the single subway in the past to light rail, monorail, and other forms [59]. However, the commuter travel circle has expanded without restriction. The rail transit network expanded to the metropolitan area according to the traditional subway model, and many radial expressways support the space of the metropolitan area, making the transportation system in the peripheral areas inefficient [55]. The traffic corridor congestion problems are prominent, and the traffic organization and operation of the metropolitan area have hidden dangers [60]. Therefore, the rail transit rational layout is highly relevant to urban construction.

4. Rail Transit Networks

The application of complex network theory has proven to be a valuable approach in the study of transportation systems. The introduction of the small-world model by [61] triggered many studies on various transportation networks, such as rail transit networks [27,38,62], airline networks [63,64], and bus networks [22,65]. Focused on the rail transit network, many researchers have discussed the network topology characteristics in different cities, such as Beijing [2,27,66], Shanghai [67], Chengdu [68], Nanjing [19,69], Shenzhen [70,71], Xi’an [20], Chongqing [72], Boston [62], Amsterdam [73], Delhi [74], and Leicester [28]. Different cities’ rail transit networks show similar global properties: small-world and scale-free degree distribution [75]. There are also distinct types of rail transit network studies, including vulnerability analysis [21,76], topology analysis [22–24], robustness analysis [2], behavior analysis [77], node importance analysis [78–80], risk analysis [3,72], rail transit system planning [28,73,81], and community detection [27]. All of these studies shed light on the importance of the rail transit network.

As a complex system, urban rail transit can be conceptualized as a network involving the interaction of stations and lines, exerting a substantial impact on passenger and logistics circulation [82]. In this rail transit network model, stations are represented as nodes, and physical railway connections as links [65]. Various types of connectivity give rise to specific rail transit networks. For instance, the rail transit line network has nodes representing lines connected if there is at least one route between them. Other example is the rail transit station network, which features nodes representing stations connected if there are consecutive stops on a given route [70,83]. Some examples of networks have weighted links, such as operation timetable [38], passenger flows [27,71], and geographical distances between stations [84] dynamically influencing rail transit network. Unweighted and weighted representations offer different insights into its operation. The relationship between complex network topology, traffic behavior, and the dynamics of maximally connected networks was studied in [85]. The authors of Ref. [86] integrated topological properties with socio-economic data, such as population and income, to explore the correlation between traffic volume and topological structure in a weighted network. Optimal traffic network structures, considering congestion costs, were identified by [87] who found the star structure to be effective. The growth model of complex networks proposed in [88] includes a model of urban rail transit networks, connecting nodes with a preferential attachment rule, thus obtaining a scale-free network with node degree distribution following a power law. In article [89], the clustering coefficient, average path length, and node average degree of Boston and Vienna urban rail transit networks were compared, finding that both networks have the small-world property. Authors of [90] used the number of lines passing through each node as the node’s degree, conducted a network modeling analysis of the topological characteristics of thirty-three urban rail transit networks from different cities, finding that most of them are scale-free networks. These studies have
explored the correlation between urban rail transit network structure and system from different perspectives.

Analyzing the relationship between global and local structure and network properties is one of the main aims of complex network analysis. In this section, we introduce network efficiency and average path length as examples of global structure measures and then introduce the study of local structures on the rail transit network.

4.1. Global Rail Transit Network Structure

Various representations exist for a rail transit system within the framework of complex networks theory. The most direct approach involves nodes representing stations and links indicating physical connections [65]. Three approaches for defining the topology of transportation systems were extensively discussed in [62]. The space of changes disregards physical distance, linking stations when at least one vehicle stops at both. The space of stops connects two stations if they are consecutive stops on a route. Lastly, the space of stations links stations directly without any intermediary stops, reflecting real-life infrastructure. This categorization results in two main classes of rail transit network modeling: Space-P and Space-L [70,83]. In Space-P, nodes are connected if there is at least one route between them, while in Space-L, nodes are connected if they are consecutive stops on a given route. Many studies use either Space-L or Space-P to analyze rail transit networks. For example, Ref. [23] employed Space-P to construct a rail transit network, evaluating characteristics such as degree, clustering coefficient, and average path length. They also assessed the basic functional unit based on transfer times, coverage intensity of attraction zones, and load of transfer stations. Similarly, Ref. [66] modeled the unweighted Beijing rail transit using both Space-P and Space-L, demonstrating its small-world and scale-free network properties. A topological analysis based on Space-P and Space-L of the Shenzhen metro was carried out in [70], which also showed that both models have the properties of scale-free and small-world, and Space-P shows more evident small-world properties than Space-L. In the study [91], the topological network by Space-P was obtained and the improved local-world evolving model was also developed to reflect the real characteristics of the transportation network. The Space-wise methods are not only used in the rail transit network, but also in other transportation networks. For instance, in [92], an unweighted compound network of subway and bus transport networks was established by using Space-P and Space-L separately, and compared the topology characteristics of the compound network and each sub-network, which shows that the compound network also has properties of small-world and a scale-free network. Space-P and Space-L were also used in [68], building unweighted subway networks, bus networks, and subway-bus networks separately. Overall, the Space-wise method makes a major contribution to the analysis of transportation networks. However, most Space-wise studies only pay attention to the unweighted rail transit networks. Considering different kinds of weight, such as operation timetable [38], passenger flows [27,71], geographical distances between stations [84], average one-way fares [93], or other weighted elements, a research gap in the Space-wise rail transit network still exists.

Many studies of the rail transit network combine different networks, which can be mainly classified into multilayer networks and multiplex networks. A multilayer network is a network made up of multiple layers, each of which represents a given operation mode, like a rail transit network with a passenger flow layer and timetable layer [94]. The multiplex transportation network is a network with different layers representing different transportation modes, such as the air-transportation network and bus-subway network [95]. Authors of [92] built the multiplex network between a subway network and a bus network and comprehensively analyzed the characteristics of the multimodal network, paying special attention to transfer status. In the [24] study, the multiplex network of road and rail transport networks of the third-ring road of Beijing was constructed. Results show that the multiplex network, with a higher network capacity, has properties of both random and scale-free network models. A topology and invulnerability analysis of a
multiplex network of Chengdu’s rail transit and bus network was carried out by [68]. They found that the invulnerability of the subway–bus multiplex network is higher than both the subway and bus transit networks. A multilayer model of train and passenger flows was presented in [38], with the aim to analyze patterns of traffic flows in the rail transit network, which elucidates fundamental differences between the traffic flows. Previous studies offer important insights into the research in multilayer and multiplex networks. However, the universality of much-published research on this issue is problematic. Many studies mixed the definition of multilayer and multiplex. Besides, due to the limitation of data, especially weight data for a network, multiplex weighted networks still require further study, like topology analyses, robustness analyses, and other types of analyses.

Global and local network efficiency were firstly introduced by [84]. They used global efficiency to measure the transmission efficiency in urban rail transit networks and local efficiency to measure network fault tolerance. The study [96] evaluated the rail transit network performance with average path length, global efficiency, and local efficiency and studied the relationship among network efficiency and transfer nodes proportion, transfer nodes connecting lines, and transfer nodes distribution. Global and local efficiency of the subway network and bus networks were compared in the [92] study. While some research has been carried out on efficiency analysis, there is still little scientific understanding of rail transit network performance. Authors of [97] did a performance analysis of the Beijing rail transit network comparing characteristic values. However, as there is not a unique definition of network performance, systematic research on network performance is still necessary.

4.2. Local Rail Transit Network Structure

A complex network is an abstraction and simplification of a complex system, which supports several kind of studies, like topology analysis [22], robustness analysis [2], and risk analysis [3], among others. The common feature of these studies is that they pay attention to global properties. A fruitful stream of research in complex networks theory proposes the existence of a hierarchical structure of complex networks, presented in the pyramid theory of complex networks [98]. At the apex of this pyramid lays the large scale organization of the network. Global network properties like degree distribution, network efficiency, and the small-world property are examined at this level. At the second level of organization lay at the functional modules. The nodes of each module have a tighter connection between them than with nodes of other modules. Conversely, each of the modules contains small subsets of nodes with similar structure. Each of the possible structures of a small subset of nodes are the network motifs, known as the “building blocks” of the complex network [16]. Network motifs are the local structure of the complex network. Differences at the local structure level may explain why networks with a similar global structure have different properties [99]. Network motif analysis is an emerging method developed in recent years to uncover the structural principles of complex networks, which have been widely used in multiple fields [100].

While most of the research on rail transit networks has been carried out at the global level, some studies have examined local properties. The study [23] summarized and analyzed the two-, three-, and four-line units of the urban rail transit network, finding that the most frequents are ring, cross, and radiation. Triangles with a ring, triangles with triangles, and multiple triangles are relatively ideal complex forms in rail transit. The article [27] reported a study of network substructures by identifying sub-networks with dense internal connections and sparser external connections to the rest of the network based on distribution of travel patterns and demand. In a recent study, [101] examined the topological structure of the station and line networks of the Beijing metro, at the global and local subgraph levels.

Although there is a growing interest in discovering structural principles of complex networks analyzing network motifs, there is still little research on rail transit using this technique. Authors of [82] detected the characteristics of motifs and super-families for several
national critical transportation networks in China, and primarily analyzed the four-node subgraph concentrations, and the results showed that the concentration of subgraphs with low connectivity is less than the concentration of subgraphs with high connectivity. In their work, Ref. [102] characterized urban transportation networks through the examination of network motifs, focusing on describing and comparing the frequency and distributions of motifs of different sizes. In a related vein, Ref. [103] emphasized algorithms in motif analysis, introducing a convolution-inspired mechanism to vectorize nodes in multi-graphs (subway, bus, and road networks). Notably, they calculated link weights with attributes instead of directly extracting motifs from established networks. Additionally, Ref. [104] utilized large-scale smart card data to showcase temporal motifs for travel pattern analysis. The temporal motif variations among travelers using different public transportation modes unveiled distinctions in travel behavior, highlighting variations in functionality and service across transit modes. This underscores the significance of motifs in the analysis of public transportation networks, enabling a quantitative exploration of the network organization modes in transportation.

Up to now, there are still no systematic network motif studies on the rail transit network, and existing research has been descriptive in nature. According to the current studies, for unweighted rail transit networks, Space-L and Space-P methods could be considered to build a complex network and perform a Space-wise network motif analysis. For a weighted rail transit network, considering different types of weight could provide a new sight of the rail transit network and uncover local network principles.

5. Network Motif Detection and Analysis

In order to facilitate the exploration of local properties within complex networks, Ref. [14] introduced the concept of network motifs. These motifs refer to patterns of interconnections observed at frequencies significantly higher than those found in randomized networks.

A network motif is formed by connecting a few nodes: three- and four-node motifs are relatively more common. For example, a non-directed graph composed of three nodes has only two different connectivity motif structures (V and triangle), and a non-directed graph with four nodes has six different motif structures, as shown in Figure 3. Furthermore, there are 13 different connection modes in a directed graph composed of three nodes, as shown in Figure 4. As reported in [105], each network motif can perform particular information processing functions. Network motif analysis has been widely used to understand local network structures in multiple fields, such as biochemistry, neurology, ecology, sociology, engineering, and transportation [12–16,106]. Motif research studies can be categorized into three types: network motif detection, motifs in different fields, and motif patterns.

Figure 3. Three-node motifs and four-node motifs of non-directed networks. (Source: [14]).

Figure 4. Possible directed connected triads for networks.

Network motif detection is an essential step of motif research [14]. Several motif detection algorithms have been defined in recent years, such as Kavosh [107], G-Tries [108], QuateXelero (QX) [109], FSM (fast and scalable network motif discovery) [110], and MODA [111]. Currently, popular network motif detection and analysis tools include
Mfinder [112], MAVisto [113], Pajek [114], and Fanmod [115,116]. Mfinder supports three-node to eight-node motif detection using enumeration and random sampling. The output of the Mfinder is a text document, which can be displayed visually only with the work tool mDraw. Additionally, as the number of nodes increases, detection becomes slower. MAVisto (Motif analysis and visualization tool) visualizes the network motifs with the force-directed placement algorithm [117], which primarily works on three-node to five-node motifs. With the increase of subgraphs, the detecting speed becomes slower. Pajek is a tool for analyzing and visualizing large networks in different areas, but some motif analysis limitations still exist. It can explore three-node motifs, but it is insufficient to support subgraph enumeration and random graph analysis. Fanmod uses the RAND-ESU algorithm to enumerate and sample subgraphs, efficiently exploring eight-node motifs. Network motif detection algorithms and tools have been improved throughout the years, despite the limitation in the motif size, runtime, interface, and memories. Therefore, increasing the detectable motif size with a friendly interface and reducing memories is still worth studying. The details of current motif detection tools and motif detection algorithms are shown in Table 1.

Table 1. Motif detection methods.

<table>
<thead>
<tr>
<th>Motif Detection Methods</th>
<th>Motif Detection Size</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Motif detection tools</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pajek</td>
<td>Three-node motifs</td>
<td>[114]</td>
</tr>
<tr>
<td>Mfinder</td>
<td>3–7-node motifs</td>
<td>[112]</td>
</tr>
<tr>
<td>MAVisto</td>
<td>3–5-node motifs</td>
<td>[113]</td>
</tr>
<tr>
<td>Fanmod</td>
<td>3–8-node motifs</td>
<td>[115]</td>
</tr>
<tr>
<td><strong>Motif detection algorithms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NeMoFinder</td>
<td>3–12-node motifs</td>
<td>[118]</td>
</tr>
<tr>
<td>LaMoFinder</td>
<td>3–20-node motifs</td>
<td>[119]</td>
</tr>
<tr>
<td>Grochow-Kellis</td>
<td>3–15-node motifs</td>
<td>[120]</td>
</tr>
<tr>
<td>MODA</td>
<td>3–9-node motifs</td>
<td>[111]</td>
</tr>
<tr>
<td>Kavosh</td>
<td>No restrictions</td>
<td>[107]</td>
</tr>
<tr>
<td>G-Tries</td>
<td>3–9-node motifs</td>
<td>[108]</td>
</tr>
<tr>
<td>NetMODE</td>
<td>3–6-node motifs</td>
<td>[121]</td>
</tr>
<tr>
<td>Acc-MOTIF</td>
<td>3–4-node motifs</td>
<td>[122]</td>
</tr>
<tr>
<td>QuateXelero</td>
<td>3–13-node motifs</td>
<td>[109]</td>
</tr>
<tr>
<td>FSM</td>
<td>5–8-node motifs</td>
<td>[110]</td>
</tr>
</tbody>
</table>

Typical local subgraphs found in transportation networks are star, line, Y-shaped, ring, and fully connected subgraphs [123,124]. In the rail transit station network, a star subgraph appears when several unconnected stations converge at a central station. In the line network, star subgraphs appear when several unconnected lines are be linked via another central line for transfers. In the station network, a line subgraph represents sequentially connected stations without forming a ring. In line network, a line subgraph represents multiple lines connected in sequence, allowing transfers between them, although not all lines within the subgraph are interconnected. In the station network, a Y-shaped subgraph implies that three unconnected stations converge at a central station, with one of the three stations leading to several connected stations subsequently. In the line network, the Y-shaped subgraph consists of three unconnected lines linked to another central line, with one of the three lines gradually leading to several other lines. A ring subgraph in the station network appears when several stations are connected sequentially forming a ring. In the line network, the ring subgraph allows for the gradual transfer from the original line back to the starting line. Figure 5 presents the typical five-node subgraphs.
While network motif analysis has been applied in several fields for almost twenty years, it is only in the last decade that it has been considered in management science. Network motif analysis was introduced by [15], examining the Escherichia Coli transcriptional regulatory network. Network motifs were also detected by [125] in several large-scale scientific collaboration networks. They also analyzed the characteristics of different types of motifs. The multi-scale network collaboration patterns and behavior mechanisms of the scientist collaboration network were researched by [126] using network motif analysis. Authors of [13] examined the distribution of network motifs in several social networks like Twitter, Facebook, and Google Plus. In [127], the local structure properties of an automobile cooperative network were examined at the motif level, also analyzing the influence of the motif distribution on the whole network structure. The study [128] built the collaboration network of teams working in response to the emergency triggered by the Wenchua earthquake and detected the network motifs to analyze its microstructure and construction mechanism, which verified that the combination of motifs obeys specific rules from bottom to top. The local structures of national emergency organizational collaboration networks of China and America were compared in [18], and it was found that the building blocks are characterized by homogeneous types of motifs, as well as heterogeneous distribution of motifs. The study [129] examined networks of critical infrastructures the properties of superfamilies and motifs. In [130], two motif-based techniques were introduced to extract the functional backbones from complex networks. They validated these techniques on a transportation reachability network and on the US airport network. The hubs and network motifs of the global terrorism network were identified in [131]. Star structures were prevalent in the network, meaning that a single source can attack multiple targets or that a single target is possibly targeted by multiple sources. The study [132] conducted a motif analysis of China’s Passenger Airline Network, offering insights applicable to the motif analysis of rail transit networks. Through motif concentration curves, they categorized thirty-seven airline companies into three development stages: mono-centric divergence companies, transitional companies, and multi-centric and hierarchical. These groups of companies correspond with the low-level, intermediate, and advanced development stages, respectively. This classification can be used as a basis for optimizing airline networks by adjusting the appropriate number of network motifs. In [133], the local properties of the container shipping network were examined through network motif analysis. In [134], the impact of seasonality on systems like bike-sharing networks was analyzed through network motif analysis. The resilience of subgraph structures within the air traffic network were examined in [12]. They identified lower-connected, medium-connected and a few higher-connected subgraph structures, aimed at enhancing the overall network capacity. Additionally, Ref. [30] concentrated on motif-based analyses of network resilience and reliability under different intentional attacks, shedding light on the local dynamics and vulnerability of networks. In a recent study, authors of [135] use a clustering method to detect network motifs in attributed road networks. Most studies of the motif are about biochemistry and neurology, some studies focus on collaboration networks, and fewer studies draw on transportation networks. In fact, the network motif provides a new perspective for all of these studies, not only for air traffic networks, but also bus networks, subway networks, and multiplex networks.

There is relatively little literature concerned with motif patterns and properties in networks. To study the similarity in the local structures of networks, Ref. [136] illustrated the significance profile (SP) of small subgraphs of motifs in the network compared to
randomized networks and also proposed the definition of superfamily, which is used to group into the networks with similar characteristic profiles. The significance profile (SP) is usually a metric to evaluate the network motif. Besides, the concept of superfamily gives a new way to classify the different types of networks. The study [137] exemplified the interwoven structure of the network of communities and explored the overlapping community structures. Community is a local structure and property of a network, and a network motif could be used to decompose the community and analyze the community’s properties. In [123], group structures were discussed by dealing with the space consisting of collections of cells and distinguished patterns of communication within a structure into three types: the innermost region of a structure, the central region of a structure, and the peripheral region of a structure. The authors of [138] focused on the precise relationship between network motifs and the global structure and function of networks. They demonstrated that, in some real networks, the global structure is statistically influenced by the probability of connections within motifs of a size of three nodes or less. Structure communication patterns were grouped by [124] into four broad types in the order circle, chain, Y, and wheel. In [139], the family classification and characteristics of scientists’ cooperative networks of different scales and fields were systematically identified, which also studied the subgraph ratio profile method based on complex network family identification, the subgraph combination strength, and the subgraph concentration ranking method. Subgraphs were classified by [129] into three categories: necessary, unnecessary, and characteristic subgraphs, which could be used to have a better understanding of the network structure. In the [140] study, authors presented a mobility prediction model, where human behavior is modeled through motif-preserving paths. The [141] study detected networks motifs in urban mobility through a non-negative tensor decomposition. Although extensive research has been carried out on the importance of subgraphs and motif patterns, no single study exists that focuses on decomposing the motif based on typical subgraphs. Therefore, the study gap about a network motif decomposition still exists, which could be used to analyze both unweighted and weighted networks.

6. A Research Agenda of Network Motif Analysis in Rail Transit Networks

There are different advantages and disadvantages in the layout methods of rail transit hubs in different cities, but the existing methods are poor in subjectivity, freedom, and implementability [142]. Complex networks are indeed a mature research area that focuses on the study of complex systems as networks of interconnected elements [143]. Network motifs, which are recurring patterns of connections within a network, are considered to be important building blocks of complex networks and play a central role in shaping their structure and function [14]. The primitive structure can reasonably fill the research gaps in the current layout methods. Network motif analysis provides a new perspective for studying the local structure of complex networks. However, there is no systematic motif research on rail transit networks yet. On the one hand, network motif analysis can reveal the network structure from the perspective of primitives. On the other hand, it can also present the connection preferences of the sites and explore the rationality of the public transportation network structure. Due to the cost of station construction, technical requirements, and geographical constraints, rail transit stations have to be compromised on location in the initial stage of the construction. However, setting up rail transit network nodes reasonably and maximizing resource utilization is still a problem. The structure of rail transit networks at the global level adopted by different countries can be either centered in a large hub city, or can be designed as a grid network where the centrality of hubs is less salient. Spanish and French railway networks are highly centric, centered in Madrid and Paris, while countries like Germany and Poland have adopted grid railway network designs. Those differences are also present at the local level, where mono-centric and poly-centric agglomerations can appear. Network motif analysis can show the connection structure and mode between sites for each of these types of rail transit networks. This analysis can not only analyze the reliability and stability of the network but also help to
optimize the layout of the transportation network [136]. Therefore, it is necessary to assess the structural properties of public transportation network motifs.

The discussed studies offer significant insights into both the global and local structures of rail transit networks. Network motif analysis emerges as a practical and valuable approach to unveil the local structure and illustrate stations’ connectivity preferences. Through motif analysis, the specific relationships among stations are revealed, enabling the analysis of network resilience and reliability. Utilizing motif analysis aids in gaining a deeper understanding of existing rail transit networks, thereby facilitating optimization of network layout.

Due to the limitation of data availability, many rail transit studies focus on unweighted rail transit networks, yet weighted rail transit networks could show more accurate information. There is a significant research gap in the rail transit network on network motif. Additionally, the current motif research is mainly about motif detection algorithms, but seldom pays attention to motif decomposition, which could be used to analyze detected motifs and get the relationship between global and local structures, and reveal the connectivity of the local rail transit network. Therefore, we propose three lines of future research: unweighted rail transit motif analysis, weighted rail transit network motif analysis, and network motif decomposition and analysis.

6.1. Unweighted Rail Transit Network Motifs Analysis

Network motifs have an enriching application in different networks, but less research focuses on the transportation network. Based on previous studies on rail transit networks, we propose using Space-P to build the rail transit line network [70,83] and rail transit station network using Space-L. Though some studies have paid attention to the unweighted rail transit network, the space-wised network motif is still a research gap. Therefore, the motif detection algorithms could be used to discover the motifs and uncover the local structural properties of the rail transit network to analyze the resilience and reliability of rail transit networks, helping to improve the performance of the entire rail transit system.

In addition, multiplex networks make a significant contribution to research on transportation networks, in which each network is independent but interacts with the other. For unweighted multiplex networks, topological analysis [22–24] and robustness analysis [2] can be assessed through the global structure of a network. However, details could make a difference. The local structure has a non-negligible influence on the network properties. Therefore, it is necessary to get the local structure and analyze the building blocks of multiplex networks with network motifs. For example, for subway–bus multiplex networks, detected motifs include subway station nodes and bus stop nodes. Combined with the particularity and the connectivity of the motif, the structural principle can be obtained. With these results, the location of the bus or subway stations, route connectivity, node preference, and the rationality of the current layout can all be analyzed.

Additionally, network efficiency has been proposed to evaluate the network, and the significance profile is the metric to assess the network motif. What is the relationship between efficient networks and significant motifs? Are there any intersections and how do they affect each other? All of these are practical issues that need to be further researched.

To summarize, we propose to undertake new lines of research related to the examination through network motif analyses of resilience, reliability, and efficiency of the following:

- Rail transit station networks, built with the Space-L representation.
- Rail transit line networks, defined with the Space-P representation.
- Multiplex transportation networks including rail transit, examining specifically network motifs related to transfers between means of transportation.

6.2. Weighted Rail Transit Network Motif Analysis

The significance of the weighted rail transit network has yet to be examined in previous research. However, the analysis of local properties of the weighted rail transit network still is a significant research gap. Different weights show different characteristics of global
structure. For instance, the operation timetable [38] could reflect each origin and destination stations’ start time and end time. The cohesion of different line times is a critical topic. Analyzing the rail transit network with an operation timetable could help to understand the rationality of the existing schedule and provide more reasonable travel time advice. Passenger flows [27,71] are another significant weight for a rail transit network. Both rail transit congestion or passenger behavior can be analyzed from the weighted network. Similarly, geographical distances between stations [84] could show the layout of the current rail transit network, directly affecting travel time. Then, applying the network motif to weighted networks, we can get the details of the weighted rail transit local structure. For instance, if the weight is the operation timetable, the balance of the local structure needs to be considered. Based on the basic local structure patterns, whether the time between any two nodes in a local structure is too long or too short is a problem that needs attention. What kind of local structures does the passenger flow weighted network prefer? How do different types of local structures affect the resilience and reliability of the rail transit system? With different kinds of weights, there will be different results and conclusions. Hence, local structure analysis on weighted rail transit networks needs to gain more attention.

Weighted multilayer and multiplex networks can also be considered, symbolizing the superposition of weights to some degree. Analyzing the multiplex networks from multiple weight perspectives will help to understand transportation networks. Taking some weights as examples, based on the local structure, the relationship between the operation timetable [38,144], passenger flow [71,145], and transit fare [146] could be shown clearly. There are more practical research issues. For example, does the geographical distance [84] affect the passenger flow? How does the operation timetable influence passenger flow? Does the basic local structure satisfy the communication among these weights? From this point, “multi-motif” could be a new definition in the study for the weighted network local structure analysis on multilayer networks or multiplex networks.

Therefore, we propose to examine the following through network motif analysis:

- Weighted rail transit station network.
- Weighted rail transit line network.
- Weighted multiplex transportation networks.

Regarding network weights, we propose lines of research examining, through network motif analysis, the impact on passenger flow of the following:

- Operation timetable, including frequency of service and timestamps of departure and arrival.
- Geographical distance between stations.
- Communication between different means of transportation in multiplex transportation networks.

6.3. Network Motif Decomposition and Analysis

Complex network structures can be modeled as layered combinations of local structures of small scale. The node’s degree plays a crucial role in the connectivity of the local structure function. For a fixed number of nodes, higher degrees correspond to higher connectivity in the local structure. Previous research on the local structure of rail transit networks has progressed gradually, describing subgraphs from three to eight nodes using three-node motifs [128]. However, being a common subgraph in all subgraphs, the three-node subgraph possesses the same structure and functional connotation, which may not adequately capture the actual function of the local network.

In previous studies on network motifs, most tended to focus on the motif detection algorithm [14]. Two main problems are detecting larger-size motifs and speeding up the detection. However, few studies have investigated the relationship among different size motifs. Motifs are defined as subgraphs of a small number of nodes that occur at frequencies significantly higher than in random networks. Analyzing detected motifs and decomposing high and low motifs could help understand the motifs in depth.
A motif decomposition could be carried out based on the typical five-node network motifs presented in Figure 5. In rail transit networks, each of the network motifs represents various connections between the lines or stations. Therefore, the following research suggestions can arise:

- What is the impact of the potential prevalence of any of the five-node network motifs on resilience, reliability, and efficiency on rail transit networks?
- How does the prevalence of any of the five-node network motifs impact on passenger flows?
- What are the five-node network motifs that allow the most effective passenger flow on multiplex transportation networks?

7. Conclusions

The objective of this review is to bridge the gap between two significant areas of research: rail transit network analysis and the analysis of local network structures through network motifs. Our review has led to the proposal of several promising lines of research, each aimed at enhancing the sustainability of rail transit systems.

Unweighted rail transit networks can be constructed using either Space-P or Space-L methods. The topology analysis results can reveal these two networks’ global characteristics and differences. Besides, the motif detection algorithm helps discover the local structures, allowing analysis of both networks’ local structural differences. It also can present the specific relationship among stations. Therefore, we propose to analyze the resilience, reliability, and efficiency of rail transit station and line networks. Regarding multiplex transportation networks, we suggest focusing on network motifs present in transfers between means of transportation. Enhancing these aspects can lead to more integrated and sustainable urban mobility solutions.

For weighted rail transit networks, we have proposed several network weights, the most relevant of which is the flow of passengers between each pair of nodes. Passenger flow may depend on other network weights like geographical distance, timestamps, and frequencies coming from operation timetable. As a result, weighted rail transit networks evolve over time, and so does the relationship between pairs of stations; therefore, network properties evolve dynamically [147]. In this context, we propose to examine the impact of local structures detected with network motif analysis on passenger flows, considering the impact of operational timetables and geographical distance. Understanding these relationships can lead to more adaptive and sustainable network designs that respond better to real-time changes and user demands.

Regarding network motifs decomposition and analysis, previous research has identified on five typical network motifs in transportation networks: the star, chain, Y-shaped, ring, and fully connected subgraphs. In this field, we have proposed to undertake new lines of research examining the impact of their potential prevalence on relevant properties (resilience, reliability, and efficiency) for rail transit networks. The presence or prevalence of these five network motifs can also be used as an explanatory variable for passenger network flows in rail transit and in multiplex transportation networks.

Rail transit networks are transportation infrastructures that can significantly enhance economic sustainability as facilitators of movement of goods and people, also contributing to environmental sustainability using clean energy sources [148]. To deliver an effective, efficient, and robust service, rail transit infrastructures must be planned carefully [149,150]. We believe that this review of extant research on the examination of local properties of rail transit networks through network motif analysis can improve the planning of railway infrastructures and, therefore, contribute to the overall sustainability of the transportation system.

**Author Contributions:** Conceptualization, Y.M. and O.L.; methodology, J.M.S.; validation, Y.M., J.M.S., and O.L.; formal analysis, Y.M.; investigation, Y.M.; resources, J.M.S. and O.L.; data curation, Y.M.; writing—original draft preparation, Y.M. and O.L.; writing—review and editing, Y.M. and
J.M.S.; visualization, Y.M. and J.M.S. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research received no external funding.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** The list of references of the systematic literature review are available upon request.

**Conflicts of Interest:** The authors declare no conflicts of interest.

**References**


Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.