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The Development of Modeling Shared Spaces to Support Sustainable Transport Systems: Introduction to the Integrated Pedestrian–Vehicle Model (IPVM)

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Abstract: The significance of developing shared road infrastructure in cities throughout the world is growing. Driven by the need to improve traffic management in ways that enhance multiple sustainability outcomes, developing the tools needed to test shared space proposals is becoming more sought after by responsible agencies. This paper reviews approaches to simulation modeling focused on representing and assessing shared spaces, culminating in a new approach presented here called the Integrated Pedestrian–Vehicle Model (IPVM)—a novel framework that combines social force models, car-following models and other algorithms from the robotics domain to better describe both mobility and activity within a shared space. The IPVM recognizes that while shared spaces are inherently multimodal, past efforts have tended to use pedestrian models as a starting point. Most consider the interaction of pedestrians with other pedestrians and static road infrastructure. Shared space models are generally microscopic models that integrate a social force model with a variety of car-following models to describe the interaction between vehicles and pedestrians. However, there is little research and few practical methodologies that address the long-range conflict avoidance between vehicles and pedestrians. This aspect is crucial for accurately representing the desire lines and pathways of pedestrians and active transport users in complex environments like shared spaces. The IPVM describes and visualizes shared road infrastructure with an absence of separating infrastructure between users and outputs. It generates metrics that can be used in conjunction with the latest evaluation approaches to gauge the sustainability credentials of shared space road proposals. Enhanced modeling of shared space solutions can lead to more effective implementation, which can potentially reduce the presence of cars, increase public and active transport use and lead to a more sustainable transport system.

Keywords: shared spaces; traffic modelling; sustainable transport systems; social force model; microsimulation model

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1. Introduction

In the context of road infrastructure, shared space designs aim to balance the mobility of all users of a space whilst creating a place for people to congregate, socialize and be productive [1–3]. This is achieved by removing separating features of the infrastructure (such as lane marking, signage, curb and guttering or street clutter) and relying on the instincts of users to move safely and avoid collisions, making more room for features that positively define a place (including street furniture, vegetation/tree canopy, street frontage dining). Examples of shared space designs are prevalent across Europe, displaying successful outcomes [4]. However, these examples are generally historic in nature (developed during the 1970s/1980s or modified squares from the 16th and 17th century), and modern-day adoption has faced significant challenges, especially in meeting the needs of vulnerable road users [5–7].

As shared spaces equalize priority for all users and remove separation, it has the possibility of challenging road users that require priority, such as a vision-impaired person attempting to cross the road. In addition, the 'shared' nature of a location, where there is limited separation, results in different interactions and conflicts between users that traverse the site and those that use the location for the purposes of socializing. Therefore, it is important to develop guidelines, standards and tools that can support the design and evaluation process to inform better decision making in future implementations of shared space designs.

Simulation models—in particular operational microsimulation models—are a key tool for practitioners to conduct optioneering and scenario testing of traffic management solutions such as the implementation of shared space designs [8]. Shared space models are underpinned by the social force model [9] and combined with car following models [8,10] to define integrated operations of the road space. However, these tools do not consider the attractiveness of the shared space or its component elements and the effect this has on the movement and distribution of pedestrians within the shared space. In addition, the conditions under which the application of long-range conflict-avoidance tactics and strategies to different classes of agents that yield good results, remains an active area of research.

The following paper builds upon the research presented in Slack-Smith et al. [11] by specifically detailing the advancement of the social force model to better incorporate realistic interactions of multiple agents in multiple modes. The next section details the related background literature to define the current state of modeling from a foundational perspective. Leveraging the state-of-the-art, the paper then describes the development of a novel modeling framework called the Integrated Pedestrian–Vehicle Model (IPVM) that supports multiple transport modes within a multimodal social force model which also integrates the velocity obstacle approach [12–15] for proactive interaction handling. Thus, the IPVM has the potential to advance and operationalize shared space modeling.

2. Literature Review

Slack-Smith et al. [11] present a detailed categorization of the shared space modeling literature to date and describe relationships between some of the main literature relevant to shared spaces, including pedestrian models, vehicle models, and shared space models. Categorization shows that traffic modeling can be separated into macroscopic (leveraging aggregate representations, e.g., [16]) and microscopic (depiction of individual agents, e.g., [8,10]) modeling.

The most well-known physical model used for microscopic modeling of pedestrians is the social force model (SFM) [9,17,18], which has previously been adapted to depict inter-vehicular interactions [19] and short-range intermodal interactions such as pedestrian–vehicle interactions [8,10]. Subsequent work has extended the SFM to consider longer range (timescale) collision avoidance through the application of game theory [8,20], cost-minimization [21,22], curve-fitting [23], and discrete choice models [24,25]. While the SFM supports multiple conflicts well at short ranges/timescales through linear superposition [9], multiple conflicts at longer ranges/timescales have not been investigated in sufficient detail and further research is necessary [26–28].

Another research field that can provide insights to future shared space modeling is the mobility of robots. Robot cooperative motion has made extensive use of motion models based on velocity obstacles [12,13,29]. A velocity obstacle is a concept used in robotics and computer science to help a moving object (a robot or a person), avoid collisions with other moving objects. It represents potential paths that an object can take to avoid collisions, and thus could be leveraged in modeling interactions within shared spaces. Interestingly, this work and the social force model [9,17] were both inspired by Reynolds' rule-based flocking model [30]. Fiorini and Shiller [12] proposed the velocity obstacle (VO) concept to enable robots to avoid objects moving at a constant velocity as well as static obstacles, but it did not attempt to describe mutual avoidance [29]. van den Berg et al. [13] extended the velocity obstacle concept to the reciprocal velocity obstacle (RVO) by distributing responsibility for avoidance between the agents. If both agents are assumed to be capable of making the same calculations, this yields an effective algorithm for collision avoidance. van den Berg et al. [31] extended the algorithm to handle multiple collisions with the optimal reciprocal collision avoidance (ORCA) method, in which each agent derives a half-plane constraint from each of its counteragents and then constructs a convex (usually polygonal) constraint for the agent's desired velocity. In addition, the method has been extended to wheeled robots and autonomous vehicles [14,15,32,33] and aerial autonomous vehicles [34–38].

The velocity obstacle approach has also been applied to pedestrians [39–43] and human-driven ground vehicles [44,45]. Previous work has added new features for modeling pedestrians such as virtual agents in the vicinity of pedestrian agents [39] to a velocity obstacle model to represent social phenomena such as priority/privilege arising from aggression, authority, and guidance; right-of-way to describe pedestrian priority [46]; potential fields for crowd simulation [41]; multiple timescales to support long-range collision avoidance [42]; and average group velocity to support grouping [47]. Other work has integrated motion constraints to support wheeled robots and vehicles such as a steering model [32], acceleration–velocity obstacles [14], and C^n (smooth) control obstacles [15]. Finally, other research has explored the use of more varied shapes based upon the medial axis transform (skeletonization) to allow diverse mixed traffic to be supported [44,45].

Figure 1 depicts the relationships between the literature identified in the review. The color coding of the rectangles represents the different fields that have investigated the modeling of shared spaces. Red represents models that are focused on vehicular movements. Orange represents models that describe interactions between vehicles and pedestrians. Yellow represents pedestrian movement and interaction models. Green represents animal behavior models, for example, models that mathematically define flocking, herding, and schooling. Brown represents "Activity Oriented Modelling" that focuses on more than just the movement of users from origin to destination but also takes into consideration activities and social interaction. Grey represents robotics models and blue represents aircraft movement models. The text coding presented within the smaller boxes of Figure 1 refers to the initials of the authors and the year of publication of each key paper, for example, "HT 98" refers to the paper published in 1998 by Helbing and Tilch [19], "HJT 05" refers to the paper published in 2005 by Helbing, Jiang and Treiber [48] and so on and so forth. Appendix A provides an exhaustive list of the coding presented in Figure 1. The arrows between boxes present the interconnected nature of the evolution of the literature concerning the modeling of shared spaces. An example of this is Zhang et al. (ZZGJ 23) who utilize the SFM from Helbing and Molnár (HM 95) as well as the RVO from van den Berg et al. (BLM 08) [9,13,49].

Leveraging the categorization of the literature provides insights into how robotics has shaped recent efforts in traffic modeling with the advantage that multiple conflict situations can be handled simply and efficiently [31], but the disadvantage of relying upon each agent being able to make the same calculations independently. Since humans do not calculate as quickly as robots, these methods are likely to be less applicable than the SFM to human interactions on shorter timescales. This leads to questions around the feasibility of amalgamating an SFM with a VO model for the purpose of modeling shared spaces. SFM and VO methods have been directly compared in the context of right-of-way [46]. An asymmetric SFM and an RVO algorithm (ORCA) have recently been combined to model unidirectional pedestrian flow [49].

In addition to the papers focused directly on pedestrians and vehicle traffic, several of the other velocity obstacle papers disclose techniques that could be useful for modeling shared spaces. Though modeling aircraft [34,36–38] may not be directly relevant, there are opportunities to use speed management and evasion methods to improve shared space modeling. Pathways could involve tailoring VO/RVO methods to better support vehicles with slow deceleration profiles by assuming constant speed during evasive maneuvers [36]; foreclosing an evasion direction [37]; or expecting non-reciprocal evasive maneuvers when time allows [38]. These could all be applicable to conflicts between pedestrians and vehicles in shared spaces.

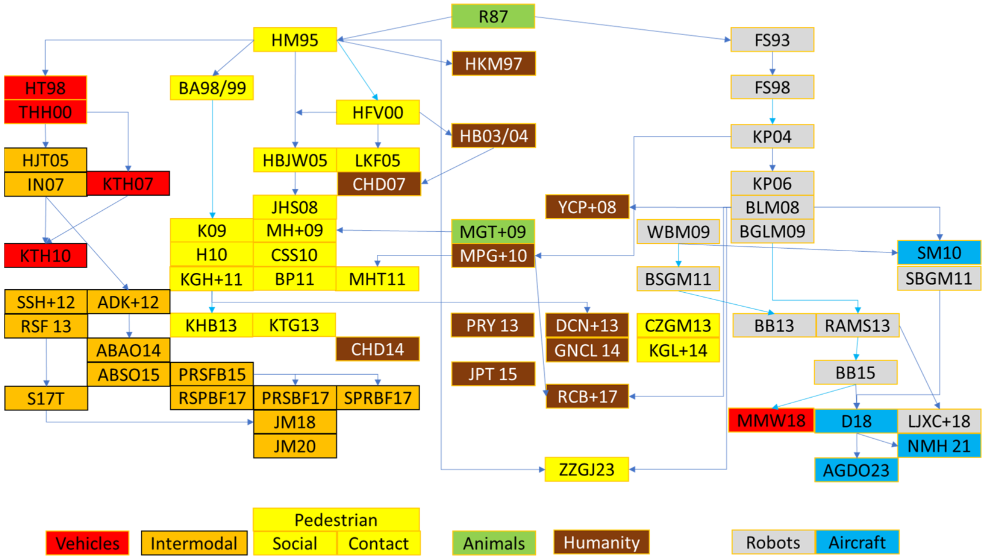

Figure 1. Connectivity of relevant modelling literature (q.v. Table A1 for detailed explanation, the figure is coded to represent initials of authors followed by year of publication).

The primary focus is on understanding the process of integrating these modeling frameworks. There is also a gap concerning the potential amalgamation of this framework with activity modeling to comprehensively address both the 'movement' and 'place' objectives within a shared space.

To address the gaps highlighted above, the next section of the paper describes an Integrated Pedestrian–Vehicle Model (IPVM) that will allow us to predict the performance of shared space designs before implementation. Extending the work of Slack-Smith et al. [11], it also details the theory underpinning it and the software design required to implement it. Finally, it integrates the pedestrian modeling equations proposed in [50] to model signalized crossings and attractions (activity areas). While activity models have been integrated into pedestrian models before [51,52], specific integrations to support shared space modeling have not been developed in the current literature.

3. Novel Shared Space Modelling Framework: The IPVM

The IPVM combines components of social force models, car following models and algorithms from the robotics domain to better describe both mobility and activity within a shared space. Figure 2 illustrates the IPVM that has been designed to conform to a Model View Controller (MVC) framework to create a user-friendly application that divides the model, visualization and the control into three interconnected but independent components.

The key functionality of the framework occurs within the Model component, which runs the model, keeps track of the current state of all agents, handles all interactions with the environment and between agents and reports relevant information to the View and Logger modules for display and logging. The Geometry component allows the modeled space to be subdivided into tiles and defines common two-dimensional shapes (circles, segments, and polygons). The tile subdivision enables algorithms dependent on proximity to be performed more efficiently. The Zone component defines the shape of zones and the rules that apply within them. A zone can be used to mark out an area which excludes or prioritizes types of agents (or all agents). Its rules can be static or dynamic, allowing signalized crossings or intersections to be represented. The Topology component defines nodes and connects them to construct graphs for agent routing. The nodes can be centroids, i.e., origins and/or destinations; approach nodes, which serve as waiting areas for agents while an associated zone excludes those agents; and activity nodes, which serve as waiting areas to represent attractive places such as cafés or gardens. The Agency component defines agents and interactions and determines what agents want and need to do and what they can do. Decomposing the model in this way facilitates an incremental extension of the model by integrating different techniques, enabling the model to integrate the best algorithms from diverse sources, including combinations of techniques from multiple disciplines.

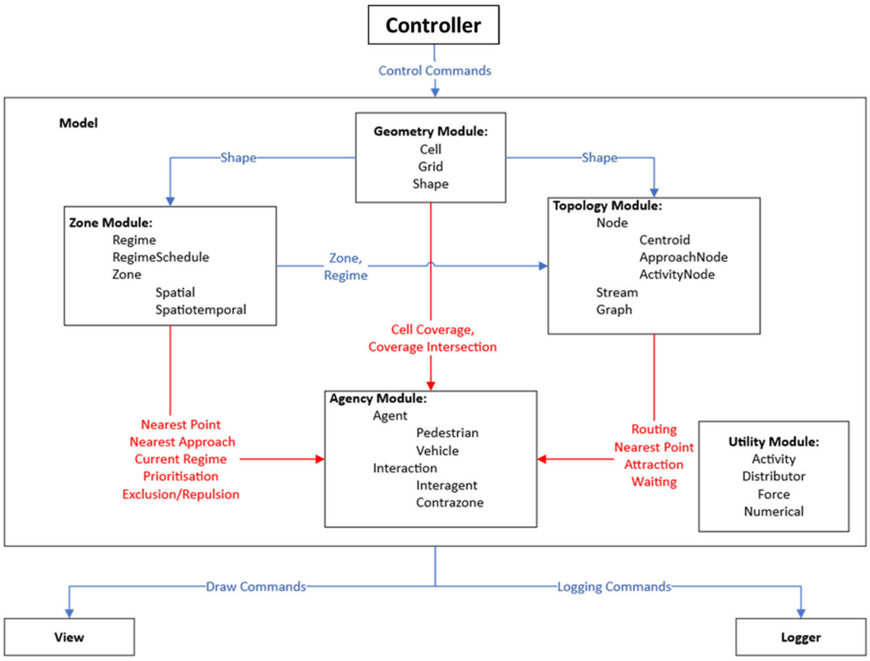

Figure 2. IPVM model structure.

Table 1 presents key mathematical notation that will be used across the remainder of the paper to discuss agents and the relationships between those agents and counteragents (i.e., other agents) and with zones and nodes.

Table 1. Mathematical notation.

Agent Properties	Description	Relationship Properties	Description
$\alpha \in A$	Agent	$\beta \in A - \{\alpha\}$	Counteragent
m_α	Mass	$\zeta \in Z$	Zone
T_α^{relax}	Relaxation time (duration)	$\omega \in \Omega \subseteq Z$	Obstacle
$\boldsymbol{x}_\alpha^*(t)$	Desired position	$\nu \in N$	Node
$\boldsymbol{x}_\alpha(t)$	Current position	$d(\boldsymbol{x},\boldsymbol{y}) = \lVert \boldsymbol{x}-\boldsymbol{y} \rVert = \sqrt{(\boldsymbol{x}-\boldsymbol{y})^2}$	Distance function
v_α^{pref}	Preferred speed	$\boldsymbol{x}_U(\boldsymbol{x}_\alpha) = \operatorname{argmin}_{x \in X_U}\left[d^2(\boldsymbol{x},\boldsymbol{x}_\alpha)\right] \exists U \in \{\zeta,\nu\}$	Nearest point
$v_\alpha^*(t)$	Desired speed	$\boldsymbol{x}_{\alpha U} = \boldsymbol{x}_U - \boldsymbol{x}_\alpha \exists U \in \{\beta,\zeta,\nu\}$	Relative position
$\hat{\boldsymbol{e}}_\alpha^*(t)$	Desired direction	$d_{\alpha U} = d(\boldsymbol{x}_U,\boldsymbol{x}_\alpha) \exists U \in \{\beta,\zeta,\nu\}$	Relative distance
$\boldsymbol{v}_\alpha^*(t) = v_\alpha^*(t)\hat{\boldsymbol{e}}_\alpha^*(t)$	Desired velocity	$\hat{\boldsymbol{e}}_{\alpha U} = \frac{\boldsymbol{x}_{\alpha U}}{\lVert \boldsymbol{x}_{\alpha U} \rVert}$	Relative direction
$\boldsymbol{v}_\alpha(t)$	Current velocity	$\hat{\boldsymbol{n}}_{\alpha U} = -\hat{\boldsymbol{e}}_{\alpha U}$	Unit normal
$\hat{\boldsymbol{e}}_\alpha = \frac{\boldsymbol{v}_\alpha}{\lVert \boldsymbol{v}_\alpha \rVert}$	Current direction	$\hat{\boldsymbol{t}}_{\alpha U} = \hat{\boldsymbol{n}}_{\alpha U}^{\perp}$	Unit tangent

Table 1. *Cont.*

Agent Properties	Description	Relationship Properties	Description
l_α, w_α	Length and width	$\boldsymbol{v}_{\alpha\beta} = \boldsymbol{v}_\beta - \boldsymbol{v}_\alpha$	Relative velocity
$s_\alpha^*(t)$	Preferred netto (bumper-to-bumper) distance	$r_\alpha\left(\boldsymbol{x}_{\alpha\beta}\right)$	Effective radius
$s_\alpha(t) = x_{\alpha-1} - x_\alpha - l_{\alpha-1}$	Current netto distance	$r_{\alpha\beta}\left(\boldsymbol{x}_{\alpha\beta}\right) = r_\alpha\left(\boldsymbol{x}_{\alpha\beta}\right) + r_\beta\left(-\boldsymbol{x}_{\alpha\beta}\right)$	Sum of radii
$\Delta v_\alpha = v_\alpha - v_{\alpha-1}$	Excess speed	$A_{\alpha U} \exists U \in \{\beta, \zeta\}$	Magnitude
$\Theta(h) = \begin{cases} 1\ \forall h > 0 \\ 0\ otherwise \end{cases}$	Heaviside function	$D_{\alpha U} \exists U \in \{\beta, \zeta\}$	Characteristic distance
$E[\boldsymbol{x}_\alpha(t)]$	Expected (future) position		

3.1. Integration of Models

To distinguish between social and physical forces, the paper uses $\boldsymbol{I}$ for the former and $\boldsymbol{F}$ for the latter. While the previous literature (e.g., [20,28]) has used $\boldsymbol{I}$ for interactions, it is herein used for all types of influences, both social and intrinsic. The model is based upon the following equations:

$$\boldsymbol{I}_\alpha^{\text{nav}} = \frac{\boldsymbol{v}_\alpha^* - \boldsymbol{v}_\alpha}{T_\alpha^{\text{relax}}} \tag{1}$$

$$\boldsymbol{I}_\alpha^{\mathbf{soc}} = \sum_{\beta \neq \alpha} \boldsymbol{I}_{\alpha\beta}^{\text{soc}} + \sum_{\zeta} \boldsymbol{I}_{\alpha\zeta}^{\text{soc}} \tag{2}$$

$$\boldsymbol{I}_\alpha^{\mathbf{soc}} = \sum_{\beta \neq \alpha} \boldsymbol{I}_{\alpha\beta}^{\text{soc}} + \sum_{\zeta} \boldsymbol{I}_{\alpha\zeta}^{\text{soc}} \tag{3}$$

$$\boldsymbol{F}_\alpha^{\text{int}} = \boldsymbol{f}(\boldsymbol{I}_\alpha^{nav}, \boldsymbol{I}_\alpha^{soc})\ \exists \boldsymbol{f} \approx m_\alpha(\boldsymbol{I}_\alpha^{nav} + \boldsymbol{I}_\alpha^{soc}) \tag{4}$$

where Equations (1) and (2) are almost identical in form to the original social force model (SFM) [9] without the attraction term, Equation (3) is identical in form to the physical terms of the SFM with contact forces [17] and the free-path seeking model of [53], and Equation (4) is similar to the social terms of [17] but allows an agent's intrinsic physical constraints to restrict the agent's will. Physical constraints could include a maximum speed [9,54] and a maximum angular velocity [55]. Note that none of these equations specify how the desired velocity $\boldsymbol{v}_\alpha^*$ should be determined.

This IPVM is designed to be able to integrate, reproduce, and extend:

1. A pedestrian SFM;
2. A car following model (CFM) such as the generalized force model (GFM) [19] or intelligent driver model (IDM) [56];
3. Waiting models such as those published in [50];
4. Velocity obstacle approaches such as those published in [14,15].

Figure 3 shows how the lower (non-strategic) layers work together. The tactical layer determines how each agent should resolve their goal and any conflicts, determining what their target position and/or velocity should be, and this is where car following [19,56], waiting [50], and any proactive collision avoidance [8,12,13,21,31] behaviors are decided and calculated. The operational layer uses the non-contact social force model [9] to combine the agent's desired behavior and any reactive collision avoidance behavior. The dynamic layer uses the contact social force model [17] to add unexpected or unavoidable (and uncontrolled) collisions [51,52]. Through this integration, the IPVM enables microscopic modeling of intersections, roads, and spaces such as signalized, marked and unmarked crossings as well as shared spaces and parks. The remaining sub-sections in Section 3 detail the specific mechanics of the modeling framework and its ability to capture the variety of modeling scenarios.

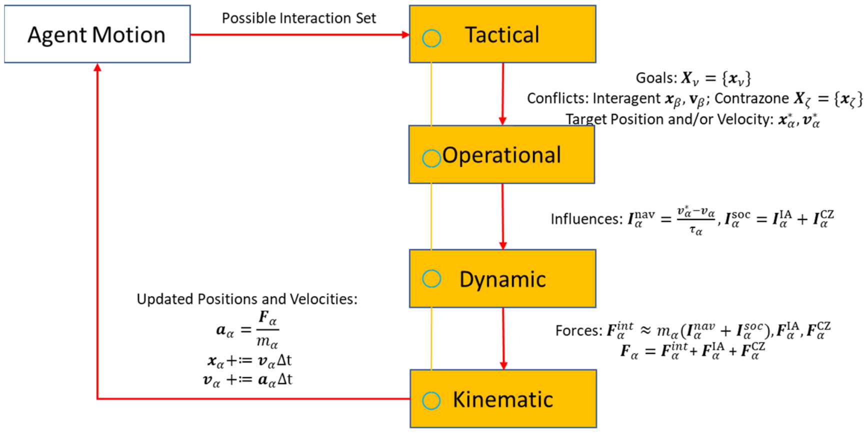

Figure 3. Lower behavioral layers.

3.2. Agent Interactions and Conflicts

The model is given a predefined description of a shared space and a specification of demand for different transport modes and executes the following steps in a loop, as shown in Figure 4.

1. Add agents beginning their journeys to the model scene. (Strategic)
2. Update model:
    a. Determine the set of relevant interactions; (Tactical)
    b. Resolve the influences on all agents; (Operational)
    c. Update agent kinematics. (Physical)
3. Remove agents who have completed their journeys. (Strategic)

Figure 4. Model iteration process.

The first of these steps determines how many agents must be added for each origin-destination pair for the current step with a stochastic method. An origin node may force agents to queue outside the model until there is space for it to enter the model scene. When the agent enters the scene, it is allocated a route in the form of a list of nodes it must pass through between the origin and destination. This can be achieved using a graph routing algorithm such as one of the Dijkstra algorithms [57] or A* [58]. Step 2 updates the model based on the location and kinematics of the agents within the network, and then step 3 removes agents whenever they are sufficiently close to the destination or have passed by or through the destination.

Step 2 is the most interesting and complex of these steps. Sub-step 2a determines which pairwise interactions should be considered by each agent. Because the set of all possible pairwise interactions scale quadratically with the number of agents, sub-step 2a aims to exclude interactions as early as possible in the process. To this end, the space is subdivided into a grid of cells, each agent is registered with the cell it is currently located within, and each agent registers a region of interest at each timestep. The process of detecting and dispatching relevant interactions is described in Figure 5.

Interactions are considered important if the agents are near, are converging, have just impacted each other, or are likely to impact each other soon. An interaction is considered perceptible to an agent if the agents are near each other or if the counteragent is currently within the agent's field of vision, to efficiently eliminate the majority of potential interactions from further consideration. This is consistent with the anisotropic perception factor present in most previous force models (e.g., [9,17,19]) and will result in greater efficiency by eliminating many irrelevant or negligible conflicts earlier.

All agents register their regions of interest.
Each cell determines sets of:
(pairwise) interagent interactions; and
contrazone interactions.
Set union operations are used to construct global sets of interagent interactions and contrazone interactions.
All interactions are analysed for:
importance;
perceptibility; and
urgency,
and then registered with agents who might be concerned.

Figure 5. Interaction detection and dispatch process.

Each agent then independently decides what course of action it should take. The interactions which have been registered with the agent may be resolved by minor actions (e.g., sidestepping) or major actions (e.g., swerving to avoid a counteragent or stopping and waiting for a counteragent to pass by). These different ways of reacting to interactions are associated with different agent states.

3.3. Agent States and Policies

The people within a real multimodal environment (such as a shared space) possess a diverse and often conflicting set of objectives, of varying urgency and importance, so a shared space model which supports a diverse set of objectives can achieve higher levels of realism, such as the representation of a lively area with cafés, kiosks, and other shops. Distinguishing between two main types of objectives: (1) achieving desired goals within the shared space and (2) avoiding collisions with other agents or obstacles, will help us decouple their analysis.

The primary objectives (goals) can encapsulate diverse aspects of human behavior and intentions within shared spaces, providing a nuanced representation that goes beyond traditional traffic simulations. Agents can represent not only pedestrians and vehicles seeking to reach a destination in minimum time but also those who wish to spend time within the shared space, socializing or otherwise enjoying being present within the space.

The secondary objectives (conflicts) encapsulate the undesirability of colliding with other agents and obstacles. They can encapsulate the relative desirability of colliding with vehicles, pedestrians, and inanimate obstacles.

We expect an agent's primary objectives (goals) to persist across the lifetimes of most if not all of their secondary objectives (conflicts), but they are likely to be less urgent and important. Each agent is expected to navigate between the locations of their goals while avoiding conflicts.

The IPVM therefore proposes an internal state machine subdivided into two independent parts: a goal state and a conflict state, presented in Figure 6. The goal state describes the agent's relationship with their current destination node, i.e., whether they are approaching it, converging to it, or waiting at the node. The conflict state describes the agent's relationships with other agents, i.e., whether it is largely ignoring/sidestepping them or whether the agent is taking active measures to avoid one or more agents. If an agent's current interactions are with other agents of the same agent type, then the agent's conflict state should be in the MODULATING state. If one or more of an agent's interactions are with agents of a different type, then the agent may need to take a more targeted approach to avoiding conflict. Currently, we have defined two avoidance states, VEERING and YIELDING. A VEERING agent will attempt to travel along a curve around the counteragent. A YIELDING agent will attempt to stop before impact occurs and wait until the counteragent has passed by. Once the actions defined by an avoidance state have been completed, the agent will return to the MODULATING state.

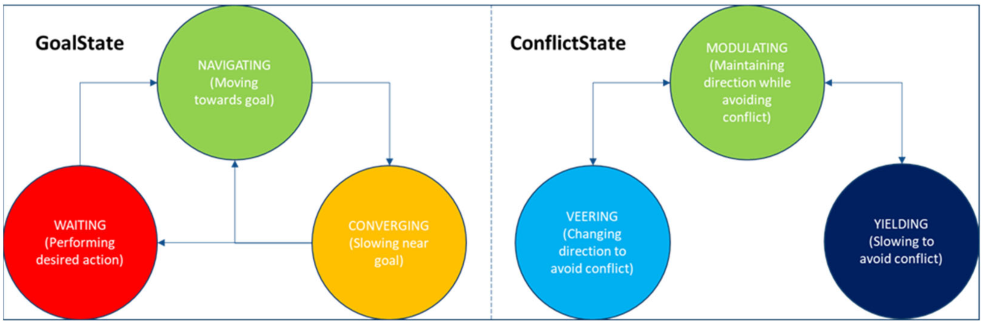

Figure 6. Definition of agent states within the IPVM framework.

An agent who is traveling between nodes with no prior intention of slowing down or stopping should be in the NAVIGATING state. An agent who intends to stop at the next node they visit will change their goal state to CONVERGING when they are close enough to the node and then to the WAITING state when they reach a focal area associated with the activity the agent wishes to perform at the node.

Each agent has the responsibility to resolve conflicts between these components of its internal state and between its internal state and external interactions/conflicts. One possible conflict resolution scheme is presented in Figure 7.

```
if agent's goal-state is not APPROACHING:
        I^nav = agent.apply_waiting_policy()
        I^soc = agent.accumulate_minor_influences()
else:
        if agent's conflict-state is not IGNORING:
                if registered counteragent is still relevant:
                        agent.clear_conflict_state()
        if agent's conflict-state is not IGNORING:
                I^nav = agent.apply_avoidance_policy()
                I^soc = agent.accumulate_minor_influences()
        else:
                Find the soonest most important conflict in the agent's interactions.
                if there is an important conflict requiring active measures:
                        agent.select_avoidance_policy()
                        I^nav = agent.apply_avoidance_policy()
                        I^soc = agent.accumulate_minor_influences()
                else:
                        I^nav = agent.apply_approaching_policy()
                        I^soc = agent.accumulate_minor_influences()
```

Figure 7. Agent conflict resolution process.

All policies determine the agent's desired velocity $\boldsymbol{v}_\alpha^*$ and then use Equation (1) to calculate $\boldsymbol{I}_\alpha^{\text{nav}}$, but different policies calculate $\boldsymbol{v}_\alpha^*$ differently, usually by selecting a pair of these equations to define desired positions and velocities:

$$\boldsymbol{x}_\alpha^* = \text{argmin}_{\boldsymbol{x}\in X_\nu} d^2(\boldsymbol{x},\, \boldsymbol{x}_\alpha) \tag{5}$$

$$\boldsymbol{v}_\alpha^* = \frac{\boldsymbol{x}_\alpha^* - \boldsymbol{x}_\alpha}{\|\boldsymbol{x}_\alpha^* - \boldsymbol{x}_\alpha\|} \tag{6}$$

$$\boldsymbol{x}_\alpha^* = \text{argmin}_{\boldsymbol{x}\in X_\phi} d^2(\boldsymbol{x},\, \boldsymbol{x}_\alpha) \tag{7}$$

$$\boldsymbol{v}_\alpha^* = \frac{\boldsymbol{x}_\alpha^* - \boldsymbol{x}_\alpha}{\text{d}_{\text{focal}}} \tag{8}$$

Equations (5) and (6) are equivalent to the driving force from the original social force model [9], and Equations (7) and (8) are equivalent to the Preferred Position waiting algorithm [50].

Agents who are APPROACHING a node will use Equations (5) and (6) to find the nearest point and velocity. Agents who are CONVERGING to a node will use Equations (7) and (8) to approach the nearest point of the focal shape. Agents who are WAITING at a node can also use Equations (7) and (8) but may prefer to use the Preferred Velocity algorithm from [50] to determine v_α^*. An agent who is YIELDING to a counteragent will determine where they can safely stop and will use that position as their desired position x_α^*, then use Equation (8) to determine the desired velocity v_α^*. An agent who is VEERING around a counteragent of a different mode will attempt to follow the locus of a curve [23,25], and will use this curve to determine their desired position x_α^*, and hence determine their desired velocity v_α^* with Equation (6). An agent in the CONVERGING or WAITING state should behave similarly to the agents described in [50], an agent in the VEERING or YIELDING state should behave similarly to the agents performing long-range conflict-avoidance manoeuvres (LRCA), as described in the previous shared space literature [23,59], and an agent in the MODULATING state should behave similarly to a multimodal social force model, as described in [10]. While pedestrians can avail themselves of all agent states, motor vehicles are currently not supported to use the VEERING state.

3.4. Positional Conflict Avoidance

Pascucci et al. [23] distinguished between lateral and longitudinal conflicts. This taxonomy has been extended to also include an oblique category. A longitudinal conflict can be solved by one or both agents moving laterally and a lateral conflict by one or both agents accelerating or decelerating so that one agent passes behind the other. An oblique conflict can be solved by one or both agents turning such that the agents are traveling parallel to one another or diverging transversely. Since cars do not currently swerve in the model, this responsibility for swerving obliquely currently belongs solely to pedestrians, so a pedestrian agent must choose between crossing the path of a vehicle or swerving or stopping to avoid doing so. Figure 8 illustrates examples of inter-agent interactions between vehicles and pedestrians. Each row of the diagram shows examples of conflicts (with X marking future impacts) on the left side and possible remedies on the right. While the first and third rows show cases also considered by Pascucci et al. [23], the second and fourth rows show the benefits of an extended taxonomy.

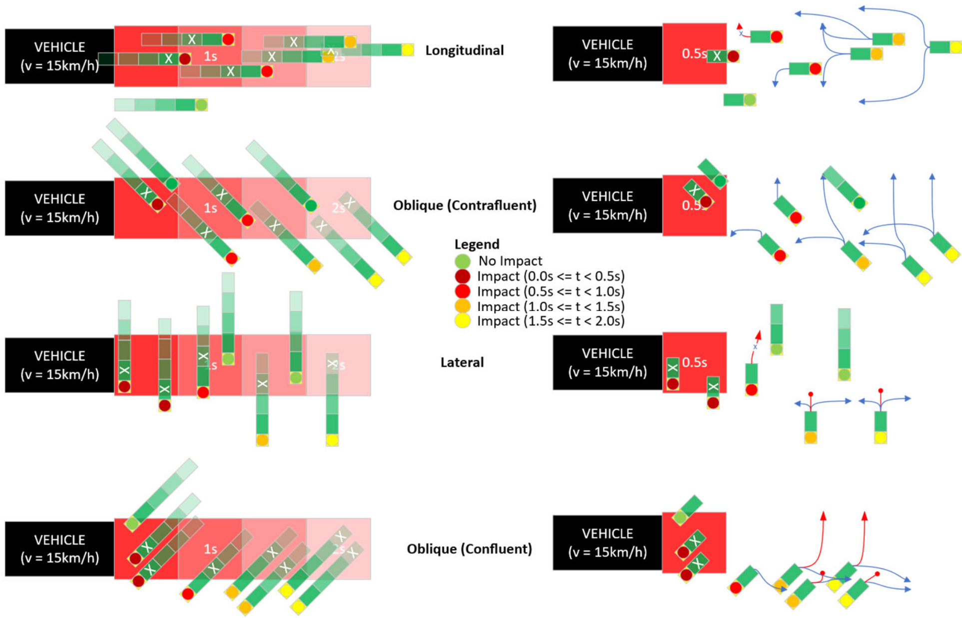

Figure 8. Longitudinal, oblique, and transverse conflicts between vehicles and pedestrians.

3.5. Motion Curves

Pascucci et al. [23] proposed the use of elasticas [60] for an agent's trajectory and for all modifications of that trajectory. We have chosen to approximate elastica-like behavior for agents' unimpeded trajectories using an angular velocity constraint (as proposed by [55]) to approximate a smooth transition when an agent is rerouted. When an avoidance policy for an agent to avoid a car needs to be decided, one or two smooth modified agent trajectories are constructed by:

- Choosing a limited number of target positions and velocities depending on the agents' positions and velocities;
- Constructing cubic splines which are close to elasticas using criteria derived from Brander et al. [61];
- Estimating the arc length and (curvature) energy of these splines using the algorithm of Gravesen [62];
- Estimating the time required to follow the splines from the estimated arc length, estimated energy, agent velocity, and agent maximum angular velocity;
- Rejecting any spline trajectory which cannot be completed within the time available until impact.

If no viable trajectories can be constructed, then the agent must stop instead. If multiple viable trajectories can be constructed, then the agent must choose one based on that agent's preferences. Minimum time, minimum energy, lateral bias (as discussed by Anvari et al. [21]), and other factors could all contribute to this decision. Note the potential for agents to experience dilemmas if multiple viable trajectories exist.

3.6. Veloci-Centric Conflict Avoidance

Without loss of generality, the following description assumes a pedestrian (ego) agent and vehicle counteragent. Modeling a vehicle as an oblong (i.e., a rectangle with rounded corners) and a pedestrian as a circle, the velocity obstacle [12] can be derived from a cone with its vertex at the pedestrian's position and two rays, each tangent to a corner of a shape defined by dilating the shape of the vehicle by the radius of the pedestrian. The reciprocal velocity obstacle [13] is based upon the truncation of the cone by the front face(s) of the dilated vehicle shape.

Figure 9 illustrates the use of nine vehicle-relative regions to simplify the calculation of reciprocal velocity obstacles, using regions constructed similarly to those previously used in computational geometry for line clipping algorithms [63]. If the pedestrian is in one of the corner regions, the RVO can be constructed from two opposing corners of the vehicle and an intermediate corner point. If the pedestrian is in one of the other exterior regions, the RVO can be constructed solely from the two corners nearest to the pedestrian's position. It is assumed that an RVO will not be useful for a pedestrian in the internal region of the vehicle.

Rufli et al. [15] extended the velocity obstacle concept to construct a reference line constructed by assuming instantaneous velocity change and a family of smooth motion curves that asymptotically converge to that reference line. We propose using the reference line to define a desired position and desired velocity, then constructing a cubic Bézier curve as a trial solution to the velocity obstacle problem. This offers significant advantages, including convergence to the reference line within a finite time and the relative ease of verifying the non-collision of a polynomial motion curve.

Figure 10 illustrates the process of generating such curves from a velocity obstacle when the pedestrian's current velocity lies within the RVO generated by a vehicle. Because a pedestrian crossing in front of a car is expected to be the most important/dangerous, the figure shows how a velocity obstacle can be used to compute minimal viable detour curves for a pedestrian approaching a car from the front.

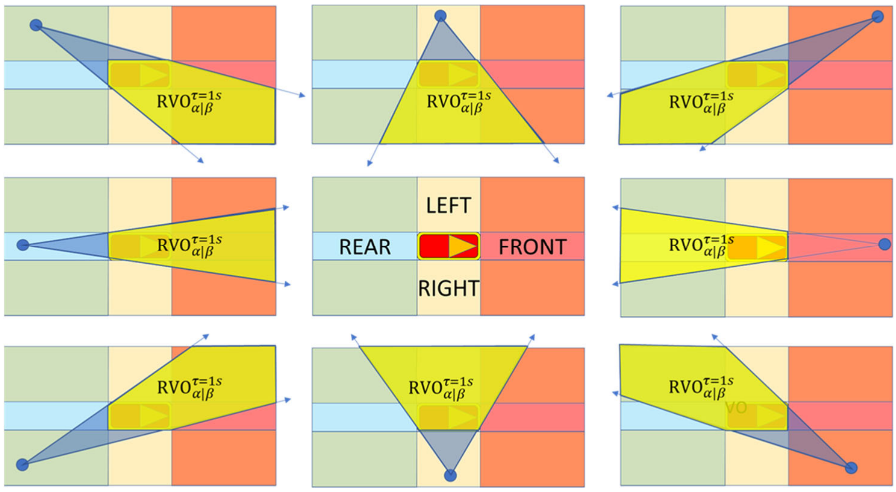

Figure 9. Reciprocal velocity obstacle constructions.

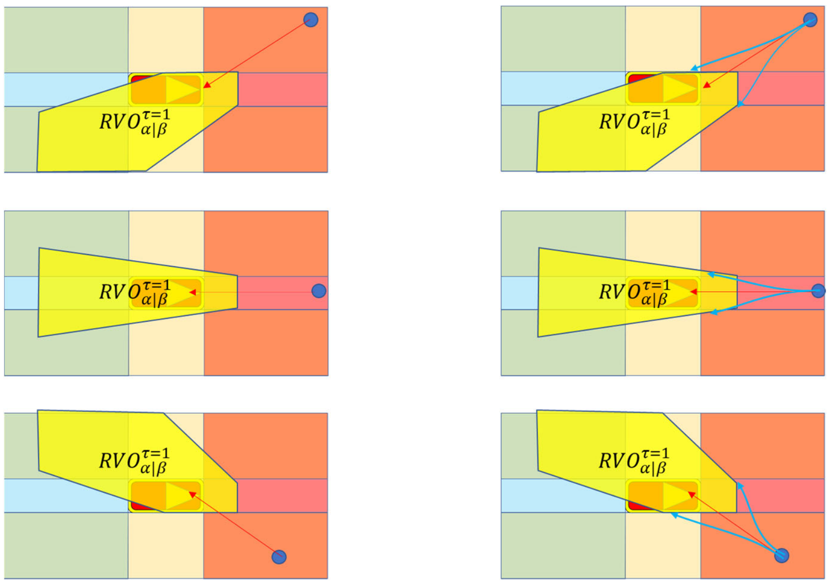

Figure 10. Curve generation from velocity obstacle.

3.7. Kinematic Constraints

Once an agent has resolved what it should do, its influences can be used to determine an intrinsic force (i.e., generated by the agent), constrained by the physical limitations of the agent. A simple way of doing this is to decompose the current velocity and the next velocity into speed and direction and then restrict the change in direction as described in [55] and restrict the speed as described in [9,64]. This should be appropriate for most wheeled agents (e.g., bicycles and cars) and sufficient for pedestrians. It is important to note that the maximum angular velocity may be velocity-dependent, as described in [21]. Possible pseudocode for this step is shown in Figure 11. It is expected that this approach will help to clarify the relationship between an agent's decisions and their motion, their desires limited by their physical capabilities.

A more complicated alternative would be to decouple the agent's facing direction from its movement direction, as discussed in the context of modeling an agent's motion with a model incorporating submicroscopic details such as an agent's stride length and stride width by Park et al. [65]. One way to adapt this approach to a microscopic force-based agent model would be to derive the agent's new facing direction only from the navigational influence, add the social influence to the navigational influence, and limit the agent's new velocity with an anisotropic (e.g., elliptical) constraint. Motion parallel to the agent's new direction would represent turning and acceleration, whereas motion perpendicular to it could represent sidestepping or slipping. This could be appropriate for agents capable of sidestepping or slipping, e.g., pedestrians, or cars with slick or bald tires.

Given a position $\boldsymbol{x}_\alpha$, a velocity $\boldsymbol{v}_\alpha$, a navigational influence $\boldsymbol{I}_\alpha^{\text{nav}}$ and a social influence $\boldsymbol{I}_\alpha^{\text{soc}}$, calculate $\boldsymbol{F}_\alpha^{\text{int}}$:

$$\boldsymbol{I}_\alpha := \boldsymbol{I}_\alpha^{\text{nav}} + \boldsymbol{I}_\alpha^{\text{soc}}$$
$$\boldsymbol{v}_\alpha^* := \boldsymbol{v}_\alpha + \boldsymbol{I}_\alpha \Delta t$$

\# Decompose current and next velocity into speed
\# and direction.

$$v_\alpha \hat{\boldsymbol{e}}_\alpha := \boldsymbol{v}_\alpha \; s.t. \, v_\alpha = \|\boldsymbol{v}_\alpha\|$$
$$v_\alpha^* \hat{\boldsymbol{e}}_\alpha^* := \boldsymbol{v}_\alpha \; s.t. \, v_\alpha^* = \|\boldsymbol{v}_\alpha^*\|$$

\# Constrain the change of direction.

if $\hat{\boldsymbol{e}}_\alpha \cdot \hat{\boldsymbol{e}}_\alpha^* < \cos(\omega_\alpha^{\max} \Delta t)$:

$$\hat{\boldsymbol{e}}_\alpha^\perp := R_{\pi/2} \hat{\boldsymbol{e}}_\alpha$$
$$\hat{\boldsymbol{e}}_\alpha^\perp := \operatorname{sgn}(\hat{\boldsymbol{e}}_\alpha^* \cdot \hat{\boldsymbol{e}}_\alpha^\perp)\, \hat{\boldsymbol{e}}_\alpha^\perp$$
$$\hat{\boldsymbol{e}}_\alpha^* := \cos(\omega_\alpha^{\max} \Delta t)\, \hat{\boldsymbol{e}}_\alpha + \sin(\omega_\alpha^{\max} \Delta t) \hat{\boldsymbol{e}}_\alpha^\perp$$

\# Constrain the future speed.

if $v_\alpha^* > v_\alpha^{\max}$:

$$v_\alpha^* := v_\alpha^{\max}$$

$$\boldsymbol{v}_\alpha^* := v_\alpha^* \hat{\boldsymbol{e}}_\alpha^*$$
$$\boldsymbol{I}_\alpha = (\boldsymbol{v}_\alpha^* - \boldsymbol{v}_\alpha)/\Delta t$$
$$\boldsymbol{F}_\alpha^{\text{int}} = m_\alpha \boldsymbol{I}_\alpha$$

Figure 11. Intrinsic force calculated by physically constraining total social influence.

3.8. Model Functionality

The IPVM is designed to allow for the visualization and measurement of multimodal traffic on a street or shared space to compare the performance of various configurations of crossings, obstacles, and activity areas. Figure 12 depicts a shared space application (T-intersection with left-hand traffic and consisting of adjacent land uses of a park, a bar and a café), illustrating the expected behavior of modeled agents, including cars and pedestrians interacting with each other. The cars are interacting with pedestrians who are crossing the intersection by slowing down and stopping whenever necessary, and the pedestrians are crossing freely in contrast to traditional crossing movements at marked and signalized crossings. At the intersection, pedestrians and vehicles must avoid collisions with other pedestrians and vehicles traveling in numerous directions. Vehicles C3 and C4 must avoid intermodal collisions by yielding to pedestrians already in the intersection such as P2, P4, and P5; pedestrian P6 must yield to avoid colliding with vehicle C1; and pedestrians P1 and P7 will veer to avoid colliding with vehicles C1, C2, and C3. As detailed in Section 2, current shared space modeling approaches have focused on bilateral conflicts in a movement-centric context, but multilateral conflict handling needs to be improved. In addition, the slower movement patterns characteristic of places such as activity nodes need to be integrated with other movement types, which is feasible with the IPVM framework.

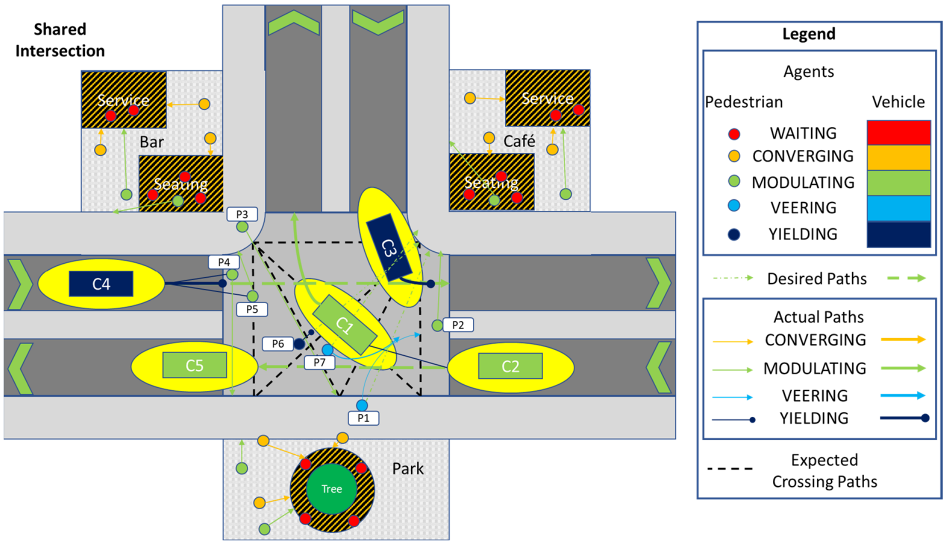

Figure 12. Shared space use case.

By deriving action states from conflict and goal states, the IPVM advances existing microsimulation approaches to modeling shared spaces. Specifically, the IPVM has the capacity to emulate the intricacies of user objectives, accommodating not only the dynamic movement of various transport modes but also the stationary activities which contribute to the vibrant and multifaceted nature of shared spaces. Agents can now interact with activities adjacent to the road network, and the behavior of agents will differ prior to the activity and following the activity (as described through the action states), thus having a different impact on the performance of the network. While activity models have previously been integrated into pedestrian models (e.g., [51]), their integration into shared space modeling is difficult.

Traditionally, these nuances in modeling required input from the modeler by specifying behavior pre and post engagement with an activity (either by modifying demand or the movement parameters). However, the inclusion of goal states and the novel conflict assessment technique means that the IPVM simply requires demand inputs for key activities, and the movements throughout the system are controlled using goal states. This results in more consistent modeling outputs as they are not dependent on user inputs and have the potential to evaluate the safety, efficiency, and comfort, fostering a more comprehensive understanding of the intricate dynamics inherent in shared space design and management.

While more complicated schemes are possible, we have opted to make the conflict state dominant. If the conflict state is anything besides MODULATING, the goal state will be temporarily ignored, otherwise the goal state will determine an agent's behavior. The states are the key to understanding the complexity of agent's behavior in this system. If an agent is in one of the WAITING or CONVERGING action states, then they will behave according to the Preferred Position algorithm from Johansson et al. [50]. If an agent is in the YIELDING action state, then they will halt and wait for the set of counteragents to whom they are yielding to pass by. If an agent is in the VEERING state, then they will follow a planar curve derived from the $C^1 - CO$ algorithm [15]. Finally, if an agent is in the MODULATING state, then their behavior will be dominated by a unimodal algorithm, the social force model for pedestrians and a car following model for vehicles. Therefore, by leveraging state-based definitions for movement and interaction, there will be implicit negotiation of traversal paths between agents, thus having the potential for greater realism in depicting multi-modal environments such as shared spaces. It is important to note that this paper introduces the IPVM framework, and further research and application of the framework is necessary to confirm the potential of the model.

4. Discussion

The IPVM presented in this paper has been developed in response to the increasing importance of shared road infrastructure to help communities meet their sustainability aspirations, as expressed in programs such as the United Nations Sustainable Development Goals 9 and 11 [66]. The IPVM is an augmented social force model that enables the evaluation of shared space designs by predicting the effects of both new shared spaces and the revitalization of existing shared spaces. Advancing previous microscopic models, the IPVM supports a streamlined interaction classification, reducing the number and increasing the relevance of interactions each user must consider. This enables a diversity of goal states, offering a more realistic representation of human decision making. This is particularly valuable when individuals are simultaneously working towards their goals while avoiding conflicts with other people and vehicles. It addresses scenarios where conflicting objectives cannot be easily resolved simultaneously, which is common when users navigate between areas of interest and wait within those areas. The inclusion of algorithms from the robotics literature has made it possible to better approximate collision avoidance interactions between vehicles and pedestrians.

Finally, the model incorporates a more complete conflict taxonomy that provides a detailed categorization of potential deviations pedestrians and vehicles can take to avoid conflicts with other users. This allows for a broader range of user choices, while keeping the computational load manageable by requiring consideration for only a limited number of possible deviations. The advantage of capturing the complexity of the taxonomy of conflicts is important to describe vulnerable users interacting with the road space. Though not currently detailed, the flexibility of the modeling framework would allow for the inclusion of users with mobility impairments and perceptual disabilities, including users who are using assistive technologies, enabling the analysis of the compliance of modeled infrastructure to any relevant sociolegal frameworks (e.g., the Americans with Disabilities Act).

Currently, the IPVM is a mathematical framework that is being developed into a simulation software platform. The next steps of the research will involve demonstrating an application of the model in a simulation environment and then eventually calibrating and validating the model outputs to real-world sites. The IPVM framework can provide a more nuanced and realistic understanding of shared road infrastructure in spaces where there is no separating infrastructure between users. It has the potential to facilitate advanced evaluations while also providing valuable metrics aligned with the latest approaches in the field aimed at enhancing the development of more sustainable transport systems.

5. Concluding Remarks

The research presented in this paper emphasizes the growing significance of shared road infrastructure in urban settings worldwide, aiming to enhance traffic management and promote sustainability. It underscores the necessity for tools to assess shared space concepts and introduces the Integrated Pedestrian–Vehicle Model (IPVM) as a novel approach. Unlike previous efforts primarily focused on pedestrian models, the IPVM integrates various simulation techniques to depict mobility and activity in shared spaces comprehensively. Particularly noteworthy is its attention to the interaction between vehicles and pedestrians, especially in terms of long-range conflict avoidance, crucial for accurately modeling pedestrian behavior in complex environments. By simulating behavior within shared road infrastructure and providing metrics for sustainability evaluation, the IPVM offers a valuable tool for assessing the viability of shared space road proposals. Future work will involve demonstrating the IPVM and examining the effectiveness of the proposed structure for the evaluation of real-world projects, which in turn can lead to a more sustainable transport network.

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Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. Explanation of Figure 1

Table A1. Key to chronological topology of the literature.

Reference Code	Authorship	Title	Numbered Citation
ABAO 14	Anvari, Bell, Angeloudis, Ochieng (2014)	Long-range Collision Avoidance for Shared Space Simulation based on Social Forces	[59]
ABSO 15	Anvari, Bell, Sivakumar, Ochieng (2015)	Modelling shared space users via rule-based social force model	[21]
ADK+ 13	Anvari, Daamen, Knoop, Hoogendoorn, Bell (2012)	Shared Space Modeling Based on Social Forces and Distance Potential Field	[67]
AGDO 23	Alligier, Gianazza, Durand, Olive (2023)	Dual-Horizon Reciprocal Collision Avoidance for Aircraft and Unmanned Aerial Systems	[38]
BB 13	Bareiss and van den Berg (2013)	Reciprocal collision avoidance for robots with linear dynamics using LQR-Obstacles	[68]
BB 15	Bareiss and van den Berg (2015)	Generalized reciprocal collision avoidance	[33]
BA 98	Blue and Adler (1998)	Emergent Fundamental Pedestrian Flows from Cellular Automata Microsimulation	[69]
BA 99	Blue and Adler (1999)	Cellular Automata Microsimulation of Bidirectional Pedestrian Flows	[70]
BLM 08	van den Berg, Lin, Manocha (2008)	Reciprocal Velocity Obstacles for real-time multi-agent navigation	[13]
BGLM 09	van den Berg, Guy, Lin, Manocha (2009)	Reciprocal n-Body Collision Avoidance	[31]
BSGM 11	van den Berg, Snape, Guy, Manocha. (2011)	Reciprocal collision avoidance with acceleration-velocity obstacles	[14]
BP 11	Baglietto and Parisi (2011)	Continuous-space automaton model for pedestrian dynamics	[71]
CHD 07	Campanella, Hoogendoorn, Daamen (2007)	Microsimulation model of hybrid time-based and event driven management of pedestrians	[72]
CHD 14	Campanella, Hoogendoorn, Daamen (2014)	The nomad model: Theory, developments and applications	[73]
CSS 10	Chraibi, Seyfried, Schadschneider (2010)	Generalized centrifugal-force model for pedestrian dynamics	[74]
CZGM 13	Curtis, Zafar, Gutub, Manocha (2013)	Right of way	[46]

Table A1. *Cont.*

Reference Code	Authorship	Title	Numbered Citation
D 18	Durand (2018)	Constant speed optimal reciprocal collision avoidance	[36]
DCN+ 13	Dutra, Cavalcante-Neto, Vidal, Musse (2013)	A Multipotential Field Model for Crowds with Scalable Behaviors	[41]
GNCL 13	Golas, Narain, Curtis, Lin (2014)	Hybrid Long-Range Collision Avoidancefor Crowd Simulation	[42]
H 10	Hartmann (2010)	-	[75]
HB 03	Hoogendoorn and Bovy (2003)	Simulation of pedestrian flows by optimal control and differential games	[51]
HB 04	Hoogendoorn and Bovy (2004ab)	Pedestrian route-choice and activity scheduling theory and models Dynamic user-optimal assignment in continuous time and space	[52,76]
HBJW 05	Helbing, Buzna, Johansson, Werner (2005)	Self-Organized Pedestrian Crowd Dynamics: Experiments, Simulations, and Design Solutions	[18]
HFV 00	Helbing, Farkas, Vicsek (2000)	Freezing by heating in a driven mesoscopic system	[17]
HKM 97	Helbing, Keltsch, Molnár (1997)	Modelling the evolution of human trail systems	[77]
HJT 05	Helbing, Jiang, Treiber (2005)	Analytical investigation of oscillations in intersecting flows of pedestrian and vehicle traffic	[48]
HM 95	Helbing and Molnár (1995)	Social force model for pedestrian dynamics	[9]
HT 98	Helbing and Tilch (1998)	Generalized force model of traffic dynamics	[19]
IN 07	Ishaque and Noland (2007)	Behavioural Issues in Pedestrian Speed Choice and Street Crossing Behaviour: A Review	[78]
JHS 08	Johansson, Helbing, Shukla (2008)	Specification of the social force pedestrian model by evolutionary adjustment to video tracking data	[79]
JM 18	Johora and Müller (2018)	GSFM	[20]
JM 20	Johora and Müller (2020)	Zone-Specific Interaction Modeling of Pedestrians and Cars in Shared Spaces	[28]
JPT 15	Johansson, Peterson, Trapani (2015)	Waiting pedestrians in the social force model	[50]
KTH 07	Kesting, Treiber, Helbing (2007)	General Lane-Changing Model MOBIL for Car-Following Models	[80]
KTH 10	Kesting, Treiber, Helbing (2010)	Enhanced intelligent driver model to access the impact of driving strategies on traffic capacity	[81]
KGL+ 14	Kim, Guy, Liu, Wilkie, Lau, Lin, Manocha (2014)	BRVO: Predicting pedestrian trajectories using velocity-space reasoning	[43]
KGH+ 11	Kretz, Große, Hengst, Kautzsch, Pohlmann, Vortisch (2011)	Quickest Paths in Simulations of Pedestrians	[82]
K 09	Kretz (2009)	Pedestrian traffic: on the quickest path	[83]
KHB 13	Kneidl, Hartmann, Borrmann (2013)	A hybrid multi-scale approach for simulation of pedestrian dynamics	[84]
KP 04	Kluge and Prässler (2004)	Reflective navigation: individual behaviors and group behaviors	[85]
KP 06	Kluge and Prässler (2006)	Recursive Probabilistic Velocity Obstacles for Reflective Navigation	[29]
KTG 13	Köster, Treml, Gödel (2013)	Avoiding numerical pitfalls in social force models	[64]

Table A1. *Cont.*

Reference Code	Authorship	Title	Numbered Citation
LJXC+ 18	Liu, Jiang, Xu, Cheng, Xie, Lin (2018)	Avoidance of High-Speed Obstacles Based on Velocity Obstacles	[86]
LKF 05	Lakoba, Kaup, Finkelstein (2005)	Modifications of the Helbing-Molnár-Farkas-Vicsek Social Force Model for Pedestrian Evolution	[87]
MH+ 09	Moussaïd, Helbing, Garnier, Johansson, Combe, Theraulaz (2009)	Experimental study of the behavioural mechanisms underlying self-organization in human crowds	[88]
MHT 11	Moussaïd, Helbing, Theraulaz (2011)	How simple rules determine pedestrian behavior and crowd disasters	[53]
MPGHT 10	Moussaïd, Perozo, Garnier, Helbing, Theraulaz (2010)	The walking behaviour of pedestrian social groups and its impact on crowd dynamics	[89]
MMW 18ab	Ma, Manocha, Wang (2018ab)	Efficient Reciprocal Collision Avoidance between Heterogeneous Agents Using CTMAT, AutoRVO: Local Navigation with Dynamic Constraints in Dense Heterogeneous Traffic	[44,45]
NMH 21	Niu, Ma, Han (2021)	Directional optimal reciprocal collision avoidance	[37]
PRSFB 15	Pascucci, Rinke, Schiermeyer, Friedrich, Berkhahn (2015)	Modeling of Shared Space with Multi-modal Traffic using a Multi-layer Social Force Approach	[23]
PRSBF 17	Pascucci, Rinke, Schiermeyer, Berkhahn, Friedrich (2017)	A discrete choice model for solving conflict situations between pedestrians and vehicles in shared space	[24]
PRY 13	Park Rojas, Yang (2013)	A collision avoidance behavior model for crowd simulation based on psychological findings	[65]
R 87	Reynolds (1987)	Flocks, herds and schools: A distributed behavioral model	[30]
RAMS 13	Rufli, Alonso-Mora, Siegwart (2013)	Reciprocal Collision Avoidance With Motion Continuity Constraints	[15]
RCB+ 17	Ren, Charambolous, Bruneau, Peng, Pettré (2017)	Analysis of the influence of detouring obstacle avoidance behavior on unidirectional flow	[47]
RSF 13	Rudloff, Schönauer, Fellendorf (2013)	Comparing Calibrated Shared Space Simulation Model with Real-Life Data	[26]
RSPBF 17	Rinke, Schiermeyer, Pascucci, Berkhahn, Friedrich (2017)	A multi-layer social force approach to model interactions in shared spaces using collision prediction	[25]
SBGM 11	Snape, van den Berg, Guy, Manocha (2011)	The Hybrid Reciprocal Velocity Obstacle	[35]
SM 10	Snape and Manocha (2010)	Navigating multiple simple-airplanes in 3D workspace	[34]
SSH+ 12	Schönauer, Stubenschrott, Huang, Rudloff, Fellendorf (2012)	Modeling Concepts for Mixed Traffic: Steps toward a Microscopic Simulation Tool for Shared Space Zones	[8]
SPRBF 17	Schiermeyer, Pascucci, Rinke, Berkhahn, Friedrich (2017)	Modeling and Solving of Multiple Conflict Situations in Shared Spaces	[27]
THH 00	Treiber Hennecke Helbing (2000)	Congested traffic states in empirical observations and microscopic simulations	[56]
WBM 09	Wilkie, van den Berg, Manocha (2009)	Generalized velocity obstacles	[32]
YCP+ 08	Yeh, Curtis, Patil, van den Berg, Manocha, Lin (2008)	Composite agents. in Symposium on Computer Animation	[39]
ZZGJ 23	Zhang, Zhang, Guo, Jiang (2023)	Analysis of the influence of detouring obstacle avoidance behavior on unidirectional flow	[49]

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