

Article

A Study of Electronic Product Supply Chain Decisions Considering Extended Warranty Services and Manufacturer Misreporting Behavior

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Abstract: In the supply chain management of electronic products, asymmetric cost information is a prevalent issue that can lead manufacturer to misreport costs, thereby exacerbating supply chain imbalances. This study focuses on the electronic product supply chain with an extended warranty service, where the manufacturer bears the after-sales responsibility during the extended warranty period. It explores the decision-making (DM) issues within the supply chain under different information environments and power structures. The Stackelberg game theory is employed to solve and analyze these models, and the main findings are as follows: (1) When supply chain information is symmetrical, centralized DM is the best choice. However, in cases where the supply chain adopts decentralized DM, it is more beneficial for the retailer and the supply chain if the retailer assumes the role of DM leader. Additionally, when the retail price sensitivity coefficient is low, the manufacturer will compete with the retailer for DM priority. Conversely, when the retail price sensitivity coefficient is higher, the manufacturer is better off as a follower in DM; (2) When the supply chain information is asymmetric, the manufacturer may engage in misreporting, which benefits the manufacturer but is detrimental to both the supply chain and the retailer. Moreover, if the price sensitivity coefficient is low, the manufacturer should lead the supply chain DM. Otherwise, the retailer should take the lead in supply chain DM. Adopting such a flexible strategy will prove advantageous for all parties involved in the supply chain. (3) The strategy of “reducing the retail price and increasing the extended warranty price” is a favorable strategy for the supply chain.

Keywords: Stackelberg game; extended warranty service; misreporting behavior; after-sales responsibility



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1. Introduction

With the continuous advancement of the new wave of the technological revolution, electronic products have achieved new breakthroughs in functionality and design. However, amidst such rapid innovation, quality issues arising from cost constraints and technical difficulties pose significant challenges to the supply chain management of electronic products. In fact, the disparity between the actual product quality experienced by consumers and the expected reference quality can adversely affect the product demand and brand reputation [1,2]. For instance, in 2022, Huawei faced a significant number of consumer complaints and returns due to battery overheating issues in some mobile phone models, adversely affecting the company’s brand image. Similarly, Xiaomi encountered widespread user dissatisfaction and increased after-sales demands in the same year due to software defects causing mobile phone system crashes. This issue is not confined to Chinese companies alone; global electronic product manufacturers are also grappling with similar

challenges. Notable examples include Apple's iPhone experiencing a battery performance decline and Samsung's Galaxy Note 7 suffering from battery overheating, both of which led to substantial user dissatisfaction and brand impacts. Cases of electronic product quality issues are common [3]; consequently, the quality control of electronic products has garnered considerable attention from scholars. Zhao et al. [4], in their research on product quality control, further showed that enhancing the level of product quality control would promote the overall performance of the supply chain. Liu et al. [5], Fan et al. [6], and Xiao et al. [7] all analyzed the impact of product quality on supply chain decisions in their research on supply chain coordination. Therefore, coordinating the supply chain concerning product quality issues has become the focal point of scholarly inquiry. For instance, Sher et al. [8] devised a supply chain coordination mechanism that considers using common replenishment times to effectively reduce system costs and ensure minimum standards for product quality. Alongside supply chain coordination, the improvement and optimization of the supply chain service system are equally important. To address consumers' concerns about quality issues, supply chains often provide extended warranty services, allowing consumers to extend the after-sales warranty service by paying a certain fee after the base warranty period ends. Providing an extended warranty service also has a positive impact on the supply chain [9–11]. However, Zhang et al. [12] pointed out that when the supply chain provides extended warranty services, attention should be paid to cost control of the extended warranty service. Debrina et al. [13] achieved a win–win outcome for supply chain members by coordinating a supply chain with extended warranty services. Additionally, Zhang et al. [14] and Wang et al. [15] examined the relationship between quality and extended warranty services in their respective studies, finding that the extended warranty service can not only improve consumers' purchasing experiences but can also strengthen cooperation among supply chain members.

In the ongoing quest for the continuous optimization of the electronic product supply chain, product recycling and reuse have become hot topics in industry research. From the perspective of government regulation, Hou et al. [16] and Wei et al. [17] have respectively explored how the government can optimize the recycling decisions of manufacturers, retailers, and third parties through incentive measures under different recycling participation models. Concerning the specific mechanisms for closed-loop supply chains, Chen et al. [18] underscored the critical role of collaboration between manufacturers and retailers in boosting recycling rates and total profits. He [19] focused on analyzing how to motivate all parties to engage in recycling through appropriate profit distribution. Considering the effects of the product quality and reuse value, Chen et al. [20] explored how the residual value of products influences manufacturers' pricing and recycling strategies under trade-in policies. Huang et al. [21] analyzed the impact of the quality grading of waste products on closed-loop supply chain decisions, presenting optimal strategies under different decision-making (DM) models. Additionally, Singh et al. [22] proposed a comprehensive reverse logistics network model, highlighting the potential to reduce the total costs and improve the environmental benefits through meticulous management. Most of the aforementioned studies focused solely on the recycling and reuse of products, with some neglecting the classified after-sales support of electronic products. Nonetheless, the findings from these studies still offer a solid theoretical foundation for this research.

Information asymmetry is a critical and common issue in supply chain management [23], garnering significant research attention, particularly concerning cost information asymmetry. Zhao et al. [24] constructed a non-cooperative game model revealing that bilateral cost information asymmetry can enhance participants' profits while simultaneously harming the interests of honest parties. Liu et al. [25] studied the construction of effective buyback contracts for supply chain disruptions under conditions of asymmetric sales cost information and risk-averse suppliers. Lin et al. [26] devised supply chain models under the two modes involving the platform and self-operation, considering asymmetric production cost information and uncertain detection issues, and analyzed the effects of production and freshness preservation efforts on product quality. Wen et al. [27] investigated the DM

difficulties brought to e-commerce platforms by manufacturer cost information asymmetry when designing optimal service contracts for e-commerce platforms. In supply chain management, companies located downstream in the supply chain often have superior access to consumer information, enabling more accurate market demand forecasts. Consequently, the demand information frequently becomes proprietary to companies at the end of the supply chain. In studies considering demand information asymmetry, Zhao et al. [28] analyzed the retailer's advantage in demand information and explored the impact of information asymmetry on supply chain DM. Chen et al. [29] examined information-sharing DM between supply chain members using a Stackelberg model, identifying risk aversion and platform commission ratios as key factors affecting the benefits of information sharing. Zhao et al. [30] also pointed out that demand information asymmetry exacerbates fairness concerns, which further affects supply chain pricing and profit distribution issues. Hu et al. [31] developed a dual-channel supply chain model in which e-commerce enterprises have private demand information, as part of their study on preferences for power structure.

In the existing literature, research on other types of information asymmetry is not uncommon. For instance, Zheng et al. [32] investigated the coordinated development of community group-buying supply chains, highlighting that supply chain entities may obscure the true freshness level of products. Du et al. [33] employed a signal game model to examine the impacts of retailer overconfidence on innovation investments and returns, considering the asymmetry of efficiency information. Wang et al. [34] explored the supply chain optimization path using blockchain technology, focusing on quality information asymmetry. Jian et al. [35] addressed the potential adverse selection issue in the supply chain arising from information asymmetry, particularly when the manufacturer possesses private information on process innovation efficiency. Zhang et al. [36] analyzed supply chain coordination strategies against the backdrop of carbon quota trading policies, taking into account information asymmetry in unit carbon emissions. Ding et al. [37] explored issues concerning bank loans and prepayments in the supply chain, suggesting that suppliers might have incentives to conceal their actual funding needs, potentially disrupting the supply chain. In addition, Cheng et al. [38] and Pinakhi et al. [39] examined the impact of asymmetric information on supply chains in the contexts of low-carbon supply chains and recycling supply chains, respectively. In conclusion, the research on information asymmetry has reached a considerable level of maturity, covering various aspects such as cost information and demand information. These studies offer a valuable theoretical framework and research insights for this paper to delve into the cost information asymmetry related to electronic products.

Table 1 summarizes the existing research literature, covering the perspectives, methods, and shortcomings. To make the table more compact, abbreviations are employed whereby EWD represents the extended warranty demand and CAS represents the classification of after-sales services.

In summary, while there is a plethora of studies on supply chain models concerning information asymmetry, few scholars have taken information asymmetry into consideration in their research on extended warranty services. Particularly scarce are studies focusing on after-sales services based on product quality grading during the extended warranty period. In contrast, the primary innovation of this paper lies in examining how different information environments and power structures influence the electronic product supply chain DM. Additionally, the model construction process considers the relationship between the extended warranty period and after-sales rates, and includes a classification of after-sales services based on the quality of defective products.

This study utilizes Stackelberg game theory to construct five supply chain decision models. By comparing and analyzing these models, it explores the DM problems of electronic product supply chains when providing extended warranty and after-sales services under various information environments and power structures. The article is divided into six sections. Section 1 introduces the research background, significance, methods, content, and relevant literature. Section 2 describes the models and necessary assumptions.

Section 3 addresses the solutions for the different models. Section 4 analyzes the model results, including the timing and impacts of misreporting. Section 5 presents a simulation case analysis, primarily to verify the propositions mentioned earlier and to explore further conclusions. Section 6 provides a summary of the entire article, including the research findings and limitations.

Table 1. Research review of some of the literature.

References	Research Contents	EWD	CAS	Information Asymmetry	Misreporting
[9–11]	Exploring the impact of extended warranty services on supply chain performance	✓	×	×	×
[15]		✓	×	×	×
[21]	Exploring pricing and incentive mechanisms in reverse supply chains	×	✓	×	×
[24]	Exploring the impact of bilateral cost information asymmetry on supply chain DM	×	×	✓	✓
[28]	Exploring the impact of asymmetric demand information on supply chain DM	×	×	✓	×
[35]	Exploring the potential adverse selection problem in the supply chain to overcome information asymmetry	×	×	✓	✓

2. Model Description and Assumptions

This article mainly studies the DM problem of an electronic product supply chain composed of a single manufacturer and a single retailer. The structure of the supply chain model is shown in Figure 1.

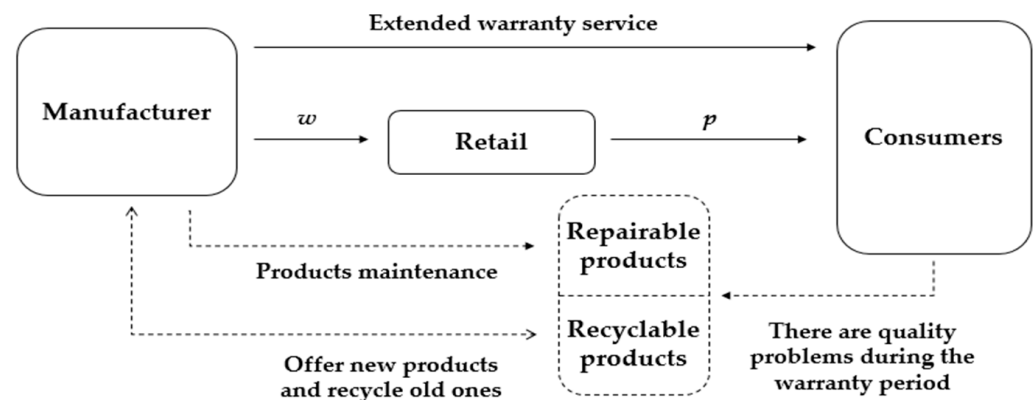


Figure 1. Electronic product supply chain with extended warranty service.

In this supply chain, the manufacturer offers extended warranty services to consumers. During the base warranty period, consumers can file claims for any quality problems with the product. However, after the basic warranty period ends, only consumers who have purchased the extended warranty service can receive after-sales support from the manufacturer.

The manufacturer can provide the following service solutions according to the specific situation of the products with quality problems: (1) for repairable products, the manufacturer provides repair service; (2) for non-repairable products, the manufacturer provides new product replacements and recovers the defective products. Among the products with quality problems, the proportion of non-repairable products is h .

The market demand for electronic products is negatively affected by the retail price. Referring to Chen et al. [18], the product demand function is expressed as $D_1 = a - fp$, where a represents the market potential of the product, p represents the retail price, f

represents the retail price sensitivity coefficient, and $f > 0$. When considering the extended warranty situation in the supply chain model, since the extended warranty demand is based on the product demand and the extended warranty demand is negatively affected by the extended warranty price, the extended warranty demand function can be set as $D_2 = D_1 - ub$, where b is the extended warranty price and u is the sensitivity coefficient of the extended warranty price, $u > 0$.

The main assumptions of the study are as follows:

Assumption 1. A non-reparable product after recycling can be resold by the manufacturer after reprocessing, and there will be no quality defects.

Assumption 2. To streamline the model, we assume the basic warranty period for the product is 0, which does not affect the research conclusions of this paper.

Assumption 3. Let r represents the extended warranty period and d denote the influence coefficient of after-sales service probability. Referring to Dan et al. [40], during the extended warranty period, dr^2 signifies the probability of product quality issues occurring, dr^2D_2 represents the number of products experiencing quality issues, hdr^2D_2 represents the number of products needing replacement, and $(1 - h)dr^2D_2$ denotes the number of products needing repair. The cost of repairing the product is denoted as m , while the cost of replacing the product is denoted as n (which is the total cost of replacement minus the salvage value of the defective product).

Assumption 4. The probability that a product will experience repeated quality problems during the extended warranty period is very low; therefore, this paper considers that each product will encounter at most one quality problem.

When there is asymmetric supply chain information, the manufacturer is motivated to misreport cost information. Based on differences in the power structure and information environment within the supply chain, we constructed five supply chain DM models. The classification and naming of the models are shown in Figure 2.

Centralized decision	C Model		
Decentralized decision		Manufacturer-led (M)	Retailer-led(R)
	Information transparency (A)	AM Model	AR Model
	Information opacity (B)	Misrepresent manufacturing cost information: BM Model	Misrepresent manufacturing cost information: BR Model

Figure 2. Classification and naming of the models discussed in this paper.

In Figure 2, the C model is a centralized DM model, in which supply chain members do not have a dominant advantage. The other four models are decentralized DM models, where the manufacturer and the retailer engage in a Stackelberg game, making decisions sequentially, as shown in Figure 3.

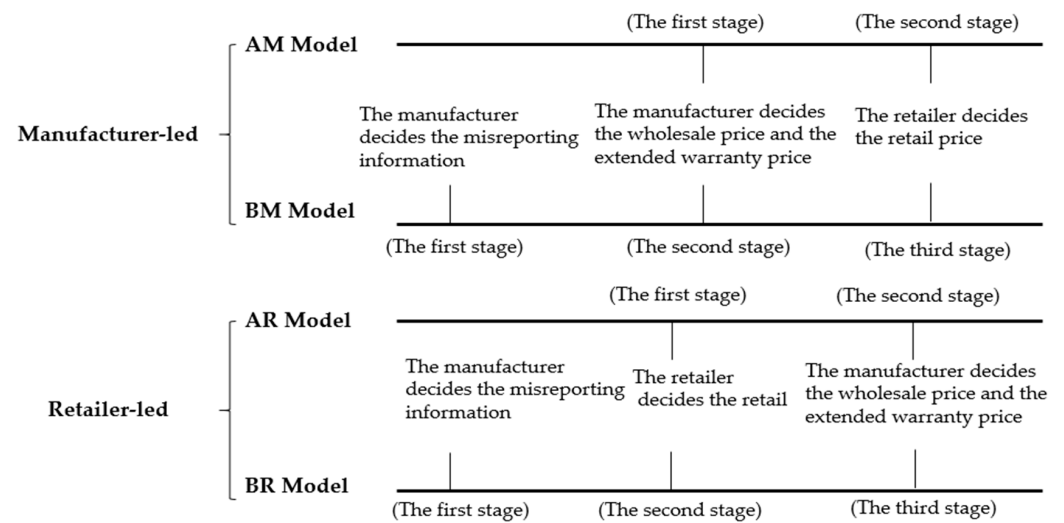


Figure 3. The DM sequences of supply chain members under different models.

Table 2 describes the mathematical symbols used in this article and their meanings.

Table 2. Symbols and their meanings.

Symbol	Meanings
p	Product retail price
f	Retail price sensitivity coefficient
b	Extended warranty price
u	The sensitivity coefficient of the extended warranty price
D_1	Product demand
D_2	Extended warranty demand
w	Wholesale price
c	Manufacturing cost
d	The influence coefficient of after-sales service probability
n	Replacement cost
m	Repair cost
r	Extended warranty period
h	Proportion of non-repairable products
v	Misreporting proportion
Π_r	Retailer's profit
Π_m	Manufacturer's profit
Π_c	The total supply chain profit

3. Supply Chain DM Models

3.1. Centralized DM Model (C Model)

When supply chain members adopt centralized DM, the total profit function of the supply chain is:

$$\Pi_c^C = (p^C - c)D_1 + b^C D_2 - nhdr^2 D_2 - m(1 - h)dr^2 D_2. \quad (1)$$

Here, Π_c^C is composed of four components—the product sales profit, extended warranty service sales profit, product replacement cost, and product repair cost. Note that the Hesse matrix of Π_c^C with respect to p^C and b^C is $H_1 = \begin{pmatrix} -2f & -f \\ -f & -2u \end{pmatrix}$.

Obviously, H_1 is a negative definite matrix if the condition (A): $4u > f$ is satisfied. Then, under condition (A), Π_c^C can reach the maximum value, and the maximum value

points p^{C*} and b^{C*} are the solutions of the equations $\frac{\partial \Pi_c^C}{\partial p^C} = 0$ and $\frac{\partial \Pi_c^C}{\partial b^C} = 0$, respectively. Then, we can easily get the following results:

$$p^{C*} = -\frac{dftur^2 - af + 2au + 2cfu}{f(f - 4u)}, \quad (2)$$

$$b^{C*} = -\frac{a - cf - dfr^2t + 2dr^2tu}{f - 4u}. \quad (3)$$

where $t = nh + m(1 - h)$. Then, the maximum profit value of the supply chain will be:

$$\Pi_c^{C*} = -\frac{u(a^2 - 2acf - adfr^2t + c^2f^2 + cdf^2r^2t + ud^2fr^4t^2)}{f(f - 4u)}. \quad (4)$$

From the above discussion, it can be inferred that in the C model, under condition (A), the optimal decisions of the supply chain are p^{C*} and b^{C*} and the total supply chain profit is Π_c^{C*} .

3.2. Decentralized DM Models under Information Symmetry

In the case of symmetric supply chain information, the manufacturer lacks the opportunity for misreporting. Following Stackelberg game theory, the leader in the supply chain makes decision first, while the follower makes decision based on the leader's DM results. Both parties' objectives in DM center on maximizing their individual interests.

3.2.1. AM Model

In the AM model, illustrated in Figure 2, the manufacturer prioritizes setting the wholesale price and extended warranty price, after which the retailer determines the retail price. Within this model, the profit functions of the manufacturer and the retailer are as follows:

$$\Pi_m^{AM} = (w^{AM} - c)D_1 + b^{AM}D_2 - n(1 - h)hdr^2D_2 - m(1 - h)dr^2D_2, \quad (5)$$

$$\Pi_r^{AM} = (p^{AM} - w^{AM})D_1. \quad (6)$$

When solving the Stackelberg game model, we adopt the reverse induction method. Obviously, $\frac{\partial^2 \Pi_r^{AM}}{\partial p^{AM2}} = -2f < 0$. Then Π_r^{AM} can reach the maximum value, and the maximum value point is the solution of the equation $\frac{\partial \Pi_r^{AM}}{\partial p^{AM}} = 0$. Then, we can easily get the following solution:

$$p^{AM} = \frac{a + fw^{AM}}{2f}. \quad (7)$$

By substituting Equation (7) into Equation (5), we get the Hesse matrix of Π_m^{AM} with respect to w^{AM} and b^{AM} as follows:

$$H_2 = \begin{pmatrix} -f & -\frac{f}{2} \\ -\frac{f}{2} & -2u \end{pmatrix}.$$

Obviously, H_2 is a negative definite matrix if the condition (B): $8u > f$ is satisfied. Then, under condition (B), Π_m^{AM} can reach the maximum value, and the maximum value points w^{AM*} and b^{AM*} are the solutions of the equations $\frac{\partial \Pi_m^{AM}}{\partial w^{AM}} = 0$ and $\frac{\partial \Pi_m^{AM}}{\partial b^{AM}} = 0$, respectively. Then, we can easily get the following results:

$$w^{AM*} = -\frac{2dftur^2 - af + 4au + 4cfu}{f(f - 8u)}, \quad (8)$$

$$b^{AM*} = -\frac{a - cf - dfr^2t + 4dr^2tu}{f - 8u}. \quad (9)$$

Substituting Equations (8) and (9) into Equation (7) yields:

$$p^{AM*} = -\frac{dftur^2 - af + 6au + 2cfu}{f(f - 8u)}. \quad (10)$$

Therefore, the manufacturer's profit, retailer's profit, and total supply chain profit are:

$$\Pi_m^{AM*} = -\frac{u(a^2 - 2acf - adfr^2t + c^2f^2 + cdf^2r^2t + 2ud^2fr^4t^2)}{f(f - 8u)}, \quad (11)$$

$$\Pi_r^{AM*} = \frac{u^2(dfr^2 - 2a + 2cf)^2}{f(f - 8u)^2}, \quad (12)$$

$$\Pi_c^{AM*} = -\frac{u \left(\begin{aligned} &a^2f - 12a^2u - 2acf^2 + 24acfu - adf^2r^2t \\ &+ 12adfr^2tu + c^2f^3 - 12c^2f^2u + cdf^3r^2t \\ &- 12cdf^2r^2tu + d^2f^2r^4t^2u - 16d^2fr^4t^2u^2 \end{aligned} \right)}{f(f - 8u)^2}. \quad (13)$$

From the above discussion, it can be inferred that in the AM model, under condition (B), the optimal decisions of the supply chain are p^{AM*} , w^{AM*} , and b^{AM*} . The manufacturer's profit is Π_m^{AM*} , the retailer's profit is Π_r^{AM*} and the total supply chain profit is Π_c^{AM*} .

3.2.2. AR Model

In the AR model, illustrated in Figure 2, the retailer prioritizes the retail price of the product and the manufacturer then decides the wholesale price and extended warranty price of the product. Considering the peculiarity of the reverse solution under this model, let $p = w + x$, while x represents the unit product profit of the retailer. The profit functions of the manufacturer and retailer in this model are:

$$\Pi_m^{AR} = (w^{AR} - c)(a - f(w^{AR} + x^{AR})) + (b - tdr^2)(a - f(w^{AR} + x^{AR}) - ub), \quad (14)$$

$$\Pi_r^{AR} = x^{AR}(a - f(w^{AR} + x^{AR})). \quad (15)$$

Obviously, the Hessian matrix of Π_m^{AR} with respect to w^{AR} and b^{AR} is equal to H_1 . Then, under condition (A), Π_m^{AR} can reach the maximum value, and the maximum value points are the solutions of the equations $\frac{\partial \Pi_m^{AR}}{\partial w^{AR}} = 0$ and $\frac{\partial \Pi_m^{AR}}{\partial b^{AR}} = 0$, respectively. Then, we can easily get the following results:

$$w^{AR} = -\frac{2au - af + f^2x_{(AR)} + 2cfu - 2fux_{(AR)} + dfr^2tu}{f(f - 4u)}, \quad (16)$$

$$b^{AR} = \frac{cf - a + fx_{(AR)} + dfr^2t - 2dr^2tu}{f - 4u}. \quad (17)$$

By substituting Equations (16) and (17) into the mathematical formula of Π_r^{AR} , as shown in Equation (15), we can see that $\frac{\partial^2 \Pi_r^{AR}}{\partial x^2} = \frac{4fu}{f-4u} < 0$ under condition (A). Therefore, Π_r^{AR} is a concave function, which can reach the maximum value, while the maximum value point, denoted by x^{AR*} , is the solution of the equation $\frac{\partial \Pi_r^{AR}}{\partial p^{AR}} = 0$. Then, we can easily get the following solution:

$$x^{AR*} = -\frac{dftur^2 - 2a + 2cf}{4f}. \quad (18)$$

Substituting Equation (18) back into Equations (16) and (17) yields:

$$w^{AR*} = \frac{2af - 4au + 2cf^2 - 12cfu + df^2r^2t - 6dfr^2tu}{4f(f - 4u)}, \quad (19)$$

$$b^{AR*} = -\frac{2a - 2cf - 3dfr^2t + 8dr^2tu}{4f - 16u}. \quad (20)$$

According the relationship formula $p^{AR*} = w^{AR*} + x^{AR*}$, we can get:

$$p^{AR*} = -\frac{dftur^2 - 2af + 6au + 2cfu}{2f(f - 4u)}. \quad (21)$$

Therefore, the manufacturer's profit, retailer's profit, and total supply chain profit are:

$$\Pi_m^{AR*} = -\frac{u(4a^2 - 8acf - 4adfr^2t + 4c^2f^2 + 4cdf^2r^2t - 3d^2f^2r^4t^2 + 16ud^2fr^4t^2)}{16f(f - 4u)}, \quad (22)$$

$$\Pi_r^{AR*} = -\frac{u(dftur^2 - 2a + 2cf)^2}{8f(f - 4u)}, \quad (23)$$

$$\Pi_c^{AR*} = -\frac{u(4a^2 - 8acf - 4adfr^2t + 4c^2f^2 + 4cdf^2r^2t - 3d^2f^2r^4t^2 + 16ud^2fr^4t^2)}{16f(f - 4u)}. \quad (24)$$

From the above discussion, it can be inferred that in the AR model, under condition (A), the optimal decisions of the supply chain are p^{AR*} , w^{AR*} , and b^{AR*} . The manufacturer's profit is Π_m^{AR*} , the retailer's profit is Π_r^{AR*} and the total supply chain profit is Π_c^{AR*} .

3.3. Decentralized DM Model under Information Asymmetry

When the supply chain information is asymmetric, the manufacturer has the opportunity to misreport the costs. In this scenario, the manufacturer prioritizes the decision of whether to misreport and then supply chain members make decision based on the DM order. If the production cost of an electronic product is c , then the manufacturer misreports the production cost as vc , where v is the misreporting proportion and $v > 0$. Specifically, when $v > 1$, the cost misreported by the manufacturer is higher than the actual cost. When $0 < v < 1$, the cost misreported by the manufacturer is lower than the actual cost. When $v = 1$, the manufacturer does not misreport the cost information.

3.3.1. BM Model

In the BM model, the manufacturer first determines the misreporting proportion, then the retailer determines the retail price and finally the manufacturer determines the wholesale price and extended warranty price. In this model, the manufacturer's publicly disclosed profit report is:

$$\Pi_m^{BM} = (w^{BM} - vc)D_1 + b^{BM}D_2 - tdr^2D_2. \quad (25)$$

The true profits for the manufacturer and retailer are presented in Equation (5) and Equation (6), respectively. The process of solving this model is similar to that of the AM model. Through calculations, we can ascertain the optimal strategies for each member in this model as follows:

$$w^{BM*} = -\frac{2dftur^2 - af + 4au + 4cfuv}{f(f - 8u)}, \quad (26)$$

$$p^{BM*} = -\frac{dftur^2 - af + 6au + 2cfuv}{f(f - 8u)}, \quad (27)$$

$$b^{BM*} = -\frac{a - cfv - dfr^2t + 4dr^2tu}{f - 8u}. \quad (28)$$

At this point, the manufacturer's profit, retailer's profit, and total supply chain profit are:

$$\Pi_m^{BM*} = -\frac{u(a^2 - 2acf - adfr^2t - c^2f^2v^2 + 2c^2f^2v + cdf^2r^2t + 2ud^2fr^4t^2)}{f(f - 8u)}, \quad (29)$$

$$\Pi_r^{BM*} = \frac{u^2(dft^2r^2 - 2a + 2cfv)^2}{f(f - 8u)^2}, \quad (30)$$

$$\Pi_c^{BM*} = \frac{u \left(\begin{aligned} &a^2f - 12a^2u - 2acf^2 + 8acfu + 16acfu - adf^2r^2t \\ &+ 12adfr^2tu - c^2f^3v^2 + 2c^2f^3v + 4c^2f^2uv^2 - 16c^2f^2uv \\ &+ cdf^3r^2t - 4cdf^2r^2tuv - 8cdf^2r^2tu + d^2f^2r^4t^2u - 16d^2fr^4t^2u^2 \end{aligned} \right)}{-f(f - 8u)^2}. \quad (31)$$

3.3.2. BR Model

In the BR model, the manufacturer prioritizes determining the misreporting proportion, followed by the retailer determining the retail price and finally the manufacturer determining the wholesale price and extended warranty price. Let $p^{BR} = w^{BR} + x^{BR}$. At this time, the manufacturer's publicly disclosed profit is:

$$\Pi_m^{BR} = (w^{BR} - vc)(a - f(w^{BR} + x^{BR})) + (b - tdr^2)(a - f(w^{BR} + x^{BR}) - ub). \quad (32)$$

The true profits of the manufacturer and the retailer are shown in Equation (14) and Equation (15), respectively. The solution process for this model is similar to that of the AR model. Through calculations, we can ascertain the optimal strategies for each member in this model as follows:

$$w^{BR*} = \frac{2af - 4au + 2vcf^2 - 12vcfu + df^2r^2t - 6dfr^2tu}{4f(f - 4u)}. \quad (33)$$

$$b^{BR*} = -\frac{2a - 2vcf - 3dfr^2t + 8dr^2tu}{4f - 16u}. \quad (34)$$

$$p^{BR*} = w^{BR*} + x^{BR*} = -\frac{dftur^2 - 2af + 6au + 2vcfu}{2f(f - 4u)}. \quad (35)$$

Therefore, the manufacturer's profit, retailer's profit, and total supply chain profit are:

$$\Pi_m^{BR*} = -\frac{u \left(\begin{aligned} &4a^2 + 8acfv - 16acf - 4adfr^2t - 12c^2f^2v^2 + 16c^2f^2v \\ &- 4cdf^2r^2tv + 8cdf^2r^2t - 3d^2f^2r^4t^2 + 16ud^2fr^4t^2 \end{aligned} \right)}{16f(f - 4u)}, \quad (36)$$

$$\Pi_r^{BR*} = -\frac{u(dft^2r^2 - 2a + 2vcf)^2}{8f(f - 4u)}, \quad (37)$$

$$\Pi_c^{BR*} = -\frac{u \left(\begin{aligned} &12a^2 - 8acfv - 16acf - 12adfr^2t - 4c^2f^2v^2 + 16c^2f^2v \\ &+ 4cdf^2r^2tv + 8cdf^2r^2t - d^2f^2r^4t^2 + 16ud^2fr^4t^2 \end{aligned} \right)}{16f(f - 4u)}. \quad (38)$$

4. Model Results Analysis

This section analyzes and discusses the solution results of the previous models, in order to better analyze and explain the underlying reasons behind real business phenomena and to explore certain innovative and valuable management insights. In the

following discussion, we always suppose that conditions (A) and (B) are satisfied and let $v_1 = (-dftr^2 + 2a + 4cf)/(6cf)$ and $v_2 = (dftr^2 - 2a + 4cf)/(2cf)$.

Proposition 1. *In the BM model, the manufacturer's optimal misreporting proportion is $v = 1$. This means that the manufacturer does not misreport costs in the BM model, and the DM results in this model align with those in the AM model. In this case, $p^{AM*} = p^{BM*}$, $b^{AM*} = b^{BM*}$, $w^{AM*} = w^{BM*}$, $\Pi_m^{AM*} = \Pi_m^{BM*}$, $\Pi_r^{AM*} = \Pi_r^{BM*}$, and $\Pi_c^{AM*} = \Pi_c^{BM*}$.*

Proof of Proposition 1. Due to $\frac{\partial^2 \Pi_m^{BM*}}{\partial v^2} = \frac{2c^2 fu}{f-8u} < 0$, the existence of $v = v^*$ optimizes the manufacturer's profit in the BM model. If we let $\frac{\partial \Pi_m^{BM*}}{\partial v} = \frac{2c^2 fu(v-1)}{f-8u} = 0$, we can solve the misreporting proportion $v^* = 1$. This indicates that the manufacturer's profit reaches its optimal level at this time. Therefore, the proof is complete. $\square$

Proposition 2. *In the BR model, the manufacturer's optimal misreporting proportion is v_1 and $v_1 > 1$. This means that when the retailer dominates the supply chain DM, the manufacturer will misreport the cost, and the misreported cost will be higher than the actual cost.*

Proof of Proposition 2. Obviously, $\frac{\partial^2 \Pi_m^{BR*}}{\partial v^2} = \frac{3c^2 fu}{2(f-4u)} < 0$. Then, $v = v^*$ optimizes the manufacturer's profit in the BM model. If we let $\frac{\partial \Pi_m^{BR*}}{\partial v} = 0$, we can solve the misreporting proportion $v^* = v_1$. Due to the DM result being non-negative according to Equation (18), we can see that $dftr^2 - 2a + 2cf < 0$. Therefore, $v_1 > 1$. Therefore, the proof is complete. $\square$

Proposition 3. (1) *In the BR model, there exists a misreporting proportion (i.e., $v = v_2$) that maximizes the total supply chain profit.* (2) *When $v = v_1$, the manufacturer's misreporting behavior is detrimental to the supply chain.*

Proof of Proposition 3. (1) Obviously, $\frac{\partial^2 \Pi_c^{BR*}}{\partial v^2} = \frac{c^2 fu}{2(f-4u)} < 0$. Then, $v = v^*$ optimizes the total supply chain profit in the BR model. Let $\frac{\partial \Pi_c^{BR*}}{\partial v} = 0$ yield $v^* = v_2$.

(2) Due to $dftr^2 - 2a + 2cf < 0$, we know that $v_2 < 1 < v_1$. If $\Pi_c^{BR*} = \Pi_c^{BR*}(v)$, it is obvious that $\Pi_c^{BR*}(v_2) > \Pi_c^{BR*}(1) > \Pi_c^{BR*}(v_1)$. Considering that $\Pi_c^{BR*} = \Pi_c^{BR*}(v)$ is a quadratic function of v , the trends of the total profits in the supply chain under different misreporting proportions are shown in Figure 4. $\square$

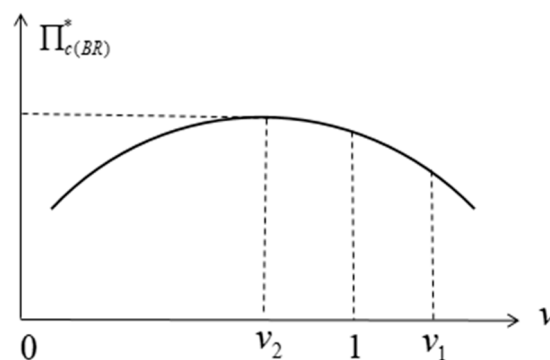


Figure 4. Supply chain profits under different misreporting proportions.

From Figure 4, it can be seen that when the misreporting proportion is v_1 , the total supply chain profit is lower than the total supply chain profit when the manufacturer does not misreport the cost.

Remark 1. Proposition 3 suggests that the manufacturer and supply chain cannot achieve optimal profits simultaneously. If the misreporting proportion maximizes the manufacturer's profit,

the benefits of the supply chain will be compromised. Further calculations indicate that when $(dftr^2 - 2a + 3cf)/(cf) < v < 1$, the misreporting proportion is beneficial to the supply chain, while when $0 < v < (dftr^2 - 2a + 3cf)/(cf)$, the misreporting proportion is detrimental to the supply chain.

According to Proposition 2, the manufacturer is prone to misreporting in the BM model, with the optimal misreporting proportion being v_1 . Consequently, v_1 is substituted into the BM model as the final misreporting proportion for the manufacturer. Propositions 4 and 5 were derived through a comprehensive comparison and analysis across different models. Specifically, Proposition 4 compares the supply chain DM results and total profits in different models from a supply chain management perspective, while Proposition 5 examines the profits of supply chain members in different models from the viewpoint of individual members within the supply chain.

Proposition 4. *The comparison of the supply chain DM results and total profits under different models is shown in Table 3.*

Table 3. Comparison of DM results and total profits under different models.

Result	Retail Price Sensitivity Factor (f)	
	$0 < f < 2u$	$2u < f < 4u$
(1)	$\Pi_c^{C*} > \Pi_c^{AR*} > \Pi_c^{AM/BM*} > \Pi_c^{BR*}$	$\Pi_c^{C*} > \Pi_c^{AR*} > \Pi_c^{BR*} > \Pi_c^{AM/BM*}$
(2)	$p^{BR*} > p^{AM/BM*} > p^{AR*} > p^{C*}$	$p^{AM/BM*} > p^{BR*} > p^{AR*} > p^{C*}$
(3)	$b^{BR*} < b^{AM/BM*} < b^{AR*} < b^{C*}$	$b^{AM/BM*} < b^{BR*} < b^{AR*} < b^{C*}$
(4)	$D_1^{BR*} < D_1^{AM/BM*} < D_1^{AR*} < D_1^{C*}$	$D_1^{AM/BM*} < D_1^{BR*} < D_1^{AR*} < D_1^{C*}$
(5)	$D_2^{BR*} < D_2^{AM/BM*} < D_2^{AR*} < D_2^{C*}$	$D_2^{AM/BM*} < D_2^{BR*} < D_2^{AR*} < D_2^{C*}$

Proof of Proposition 4. (1) Obviously,

$$\Pi_c^{C*} - \Pi_c^{AR*} = -u(dftr^2 - 2a + 2cf)^2 / (16f(f - 4u)) > 0,$$

$$\Pi_c^{AR*} - \Pi_c^{AM/BM*} = \frac{u(f - 16u)(dftr^2 - 2a + 2cf)^2}{16(f - 4u)(f - 8u)^2} > 0.$$

According to Proposition 3, we know that $\Pi_c^{AR*} > \Pi_c^{BR*}$. Obviously,

$$\Pi_c^{AM/BM*} - \Pi_c^{BR*} = -\frac{u(f - 2u)(f - 14u)(dftr^2 - 2a + 2cf)^2}{9f(f - 4u)(f - 8u)^2}.$$

Then, $\Pi_c^{AM/BM*} > \Pi_c^{BR*}$ under the condition $0 < f < 2u$ and $\Pi_c^{AM/BM*} < \Pi_c^{BR*}$ under the condition $2u < f < 4u$. Based on the above discussion, we can directly draw conclusion (1).

(2) Obviously,

$$p^{C*} - p^{AR*} = -\frac{u(dftr^2 - 2a + 2cf)}{2f(f - 4u)} < 0 \text{ indicates } p^{C*} < p^{AR*}.$$

$$p^{AR*} - p^{AM/BM*} = \frac{u(dftr^2 - 2a + 2cf)}{2(f^2 - 12fu + 32u^2)} < 0 \text{ indicates } p^{AR*} < p^{AM/BM*}.$$

$$p^{AR*} - p^{BR*} = -\frac{u(dftr^2 - 2a + 2cf)}{6f(f - 4u)} < 0 \text{ indicates } p^{AR*} < p^{BR*}.$$

Due to $p^{BR*} - p^{AM/BM*} = \frac{2u(f-2u)(dftr^2-2a+2cf)}{3f(f^2-12fu+32u^2)}$, $p^{BR*} > p^{AM/BM*}$ under the condition $0 < f < 2u$ and $p_{(BR)}^* < p_{(AM/BM)}^*$ under the condition $2u < f < 4u$. Based on the above discussion, we can directly draw conclusion (2).

(3) Obviously,

$$b^{C*} - b^{AR*} = \frac{dftr^2 - 2a + 2cf}{4(f-4u)} > 0 \text{ indicates } b^{C*} > b^{AR*}.$$

$$b^{AR*} - b^{AM/BM*} = -\frac{f(dftr^2 - 2a + 2cf)}{4(f^2 - 12fu + 32u^2)} > 0 \text{ indicates } b^{AR*} > b^{AM/BM*}.$$

$$b^{AR*} - b^{BR*} = \frac{dftr^2 - 2a + 2cf}{12(f-4u)} > 0 \text{ indicates } b^{AR*} > b^{BR*},$$

Due to $b^{BR*} - b^{AM/BM*} = -\frac{(f-2u)(dftr^2-2a+2cf)}{3(f^2-12fu+32u^2)}$, $b^{BR*} < b^{AM/BM*}$ under the condition $0 < f < 2u$ and $b^{BR*} > b^{AM/BM*}$ under the condition $2u < f < 4u$. Based on the above discussion, we can directly draw conclusion (3).

(4)

$$D_1^{C*} - D_1^{AR*} = \frac{u(dftr^2 - 2a + 2cf)}{2(f-4u)} > 0 \text{ indicates } D_1^{AR*} < D_1^{C*}.$$

$$D_1^{BR*} - D_1^{AR*} = -\frac{u(dftr^2 - 2a + 2cf)}{6(f-4u)} < 0 \text{ indicates } D_1^{BR*} < D_1^{AR*}.$$

$$D_1^{AR*} - D_1^{AM/BM*} = -\frac{fu(dftr^2 - 2a + 2cf)}{2(f-4u)(f-8u)} > 0 \text{ indicates } D_1^{AM/BM*} < D_1^{AR*}.$$

Due to $D_1^{BR*} - D_1^{AM/BM*} = -\frac{2u(f-2u)(dftr^2-2a+2cf)}{3(f-4u)(f-8u)} > 0$, $D_1^{AM/BM*} > D_1^{BR*}$ under the condition $0 < f < 2u$ and $D_1^{AM/BM*} < D_1^{BR*}$ under the condition $2u < f < 4u$. Based on the above discussion, we can directly draw conclusion (4).

(5)

$$D_2^{C*} - D_2^{AR*} = \frac{u(dftr^2 - 2a + 2cf)}{4(f-4u)} > 0 \text{ indicates } D_2^{C*} > D_2^{AR*}.$$

$$D_2^{AR*} - D_2^{AM/BM*} = -\frac{fu(dftr^2 - 2a + 2cf)}{4(f-4u)(f-8u)} > 0 \text{ indicates } D_2^{AR*} > D_2^{AM/BM*}.$$

$$D_2^{BR*} - D_2^{AR*} = -\frac{u(dftr^2 - 2a + 2cf)}{12(f-4u)} < 0 \text{ indicates } D_2^{AR*} > D_2^{BR*}.$$

Obviously,

$$D_2^{BR*} - D_2^{AM/BM*} = -\frac{u(f-2u)(dftr^2 - 2a + 2cf)}{3(f^2 - 12fu + 32u^2)}.$$

Then, $D_2^{AM/BM*} > D_2^{BR*}$ under the condition $0 < f < 2u$ and $D_2^{AM/BM*} < D_2^{BR*}$ under the condition $2u < f < 4u$. Based on the above discussion, we can directly draw conclusion (5). $\square$

Remark 2. Proposition 4 delineates several key findings: (1) It is evident that a lower retail price and higher extended warranty price lead to greater product demand and extended warranty demand, resulting in a higher total supply chain profit. (2) As a centralized DM model, the C model has the highest total supply chain profit. (3) The comparison between the retailer-led models (AR model and BR model) reveals that the decision under AR model is superior and the total supply chain profit is higher. This illustrates that manufacturer misreporting is detrimental to the supply chain. (4) By comparing the two models (AM model and AR model), it can be found that the supply chain decision

under the AR model is superior, resulting in a higher total supply chain profit. This suggests that it is better for the supply chain to let the retailer take the lead in the DM when the information is symmetrical. (5) When the retail price sensitivity coefficient is low (i.e., $0 < f < 2u$), the BR model demonstrates superior decision-making over the BM model. Conversely, when the coefficient is high (i.e., $2u < f < 4u$), the situation is reversed. This shows that when the information is asymmetric, whether the supply chain being dominated by the manufacturer or the retailer is beneficial to the supply chain depends on the retail price sensitivity coefficient.

Proposition 5. The comparison of individual supply chain members' profits under different models is shown in Table 4.

Table 4. Comparison of individual supply chain members' profits under different models.

Result	Retail Price Sensitivity Factor (f)		
	$0 < f < 2u$	$2u < f < \frac{8u}{3}$	$\frac{8u}{3} < f < 4u$
(1)	$\Pi_r^{AR*} > \Pi_r^{AM/BM*} > \Pi_r^{BR*}$	$\Pi_r^{AR*} > \Pi_r^{BR*} > \Pi_r^{AM/BM*}$	
(2)	$\Pi_m^{AM/BM*} > \Pi_m^{BR*} > \Pi_m^{AR*}$	$\Pi_m^{BR*} > \Pi_m^{AM/BM*} > \Pi_m^{AR*}$	$\Pi_m^{BR*} > \Pi_m^{AR*} > \Pi_m^{AM/BM*}$

Proof of Proposition 5. The proof process for Proposition 5 is similar to that of Proposition 4, so it will not be repeated. $\square$

Remark 3. Proposition 5 outlines several crucial discoveries: (1) The comparative analysis between the AR model and the BR model reveals that the manufacturer's misreporting behavior consistently benefits itself while disadvantaging the retailer. (2) Comparing the AR and AM models shows that under symmetric information, the retailer's profit is higher when acting as the DM leader rather than the follower. Moreover, when $0 < f < \frac{8u}{3}$, the manufacturer is more likely to be the DM leader, while when $\frac{8u}{3} < f < 4u$, the manufacturer is more inclined to follow the retailer's DM. (3) The comparative analysis between the BM and BR models reveals that under asymmetric information conditions, when $0 < f < 2u$, it is advantageous for both the manufacturer and the retailer for the manufacturer to assume the role of DM leader. Conversely, when $2u < f < 4u$, it is beneficial for both the manufacturer and the retailer for the retailer to assume the role of DM leader.

5. Simulation Case Analysis

Numerical examples are used to discuss the various models proposed in this paper. Based on the actual market situation for electronic products and referring to Ren et al. [41], the parameter assignments in this paper are $a = 100$, $c = 50$, $f = 1$, $u = 0.6$, $r = 1$, $d = 0.4$, $n = 5$, $m = 3$, and $h = 0.4$.

5.1. DM Results under Each Model

The comparison results for different models are closely tied to the magnitude of the retail price sensitivity coefficient. To enhance the rigor of the numerical analysis, f was set to three different values, and the DM results and profits for each model were calculated, which are presented in Table 5.

According to Table 5, there is a clear negative correlation between the retail price and the extended warranty price. The main reason for this correlation is that the extended warranty service is only provided to consumers who have purchased the product, meaning the extended warranty demand is based on the product demand. High retail prices tend to suppress product demand, thereby reducing the potential market for extended warranty services. As a result, decision-makers often opt to lower the extended warranty price to stimulate demand. Conversely, lower retail prices imply a larger potential market for extended warranty services, prompting decision-makers to increase the extended warranty price to maximize profitability. This pricing strategy allows decision-makers to better align with market demands and enhance the overall product profitability.

Table 5. DM results and profits for each model.

		p^*	b^*	D_1^*	D_2^*	Π_m^*	Π_r^*	Π_c^*
$f = 1$ ($0 < f < 2u$)	C Model	57.80	35.93	42.21	20.65	— —	— —	1039.46
	AM/BM Model	84.45	13.72	15.55	7.32	383.18	241.79	624.96
	AR Model	78.90	18.35	21.10	10.10	260.12	519.55	779.68
	BR Model	85.93	12.48	14.07	6.58	346.72	230.91	577.63
$f = 1.3$ ($2u < f < \frac{8}{3}u$)	C Model	48.38	31.68	37.10	18.10	— —	— —	485.72
	AM/BM Model	67.95	10.48	11.66	5.37	152.89	104.60	257.49
	AR Model	62.65	16.22	18.55	8.82	121.69	242.69	364.38
	BR Model	67.41	11.07	12.37	5.73	162.14	107.86	270.00
$f = 1.7$ ($\frac{8}{3}u < f < 4u$)	C Model	45.00	20.34	23.50	11.29	— —	— —	95.09
	AM/BM Model	55.70	5.18	5.30	2.20	21.74	16.56	38.30
	AR Model	51.91	10.55	11.75	5.42	24.03	47.37	71.40
	BR Model	54.22	7.29	7.83	3.46	31.93	21.05	52.98

A further analysis led to the following conclusions and managerial insights:

- (1) The supply chain profit in the C model is significantly better than other models. When the supply chain information is symmetric, supply chain members should make joint decisions and use the decision results of the C model (reducing the retail price while increasing the extended warranty price) as a reference to optimize supply chain DM. However, although centralized DM can improve the total supply chain profits, it cannot take into account the individual benefits of each member. To ensure the feasibility of optimal decisions within the supply chain, the manufacturer and retailer can employ profit transfer methods to balance their profits and promote the sustainable development of the supply chain. Conversely, if upstream and downstream enterprises fail to reach cooperative DM, the competition for DM priority is the main way for supply chain members to benefit.
- (2) In the case of asymmetric information, it is more beneficial for the retailer and the entire supply chain if the retailer assumes the role of DM leader. However, only when the retail price sensitivity coefficient is low can the manufacturer become the DM leader, which is beneficial to itself. Conversely, the manufacturer is better off being a DM follower. Therefore, to improve the total supply chain profit, the retailer should take on the role of DM leader. In this scenario, the retailer can achieve a win–win outcome by moderately yielding profits to the manufacture. From the perspective of the supply chain game, when the retail price sensitivity coefficient is high, the manufacturer should delegate the leading rights to the retailer, allowing it to prioritize DM. Conversely, when the retail price sensitivity coefficient is low, the manufacturer should actively compete for DM dominance to achieve greater profits.
- (3) Under the background of asymmetric information, when the retail price sensitivity coefficient is low, the manufacturer should assume DM leadership, leading to increased profits for all supply chain members. Conversely, when the retail price sensitivity coefficient is high, the retailer should become the DM leader, thereby benefiting the supply chain members more. Therefore, the supplier and retailer should determine the sequence of DM in the supply chain based on the consumers' price preferences.
- (4) The DM results and profits in the AM and BM models are the same, consistent with Proposition 1. This shows that when the manufacturer holds DM power, it is already in an advantageous position in the supply chain game. Therefore, even in the presence of information isolation, the manufacturer is unlikely to misreport information. Consequently, the retailer does not need to invest a lot of resources in eliminating information gaps. When the retailer assumes the leadership role, the

manufacturer engages in misreporting under information symmetry, which adversely affects the retailer and the supply chain.

5.2. The Impact of the Misreporting Proportion on the Profit

In the BM model, the manufacturer engages in misreporting behavior. Figure 5 shows the changes in the total supply chain profit (TP) and manufacturer's profit (MP) under different misreporting proportions.

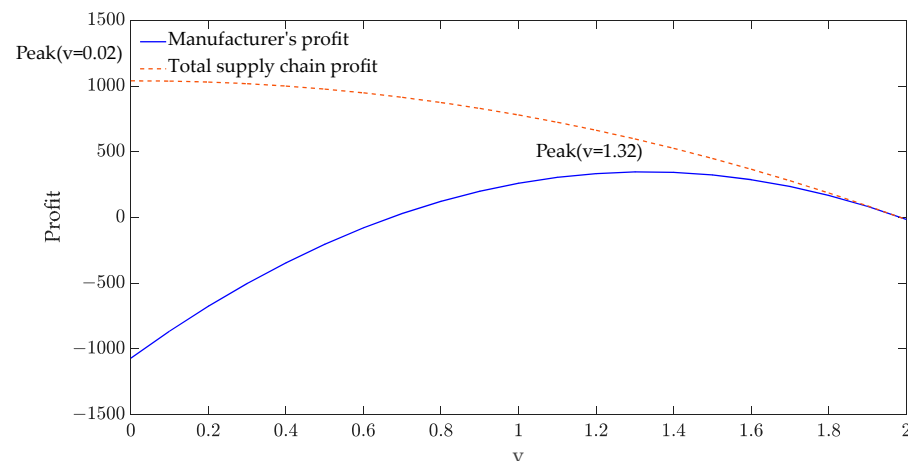


Figure 5. The impacts of the misreporting proportion on the TP and MP.

From Figure 5, the below conclusions can be drawn:

- (1) As the misreporting proportion changes, both the TP and MP show peak profits. When the misreporting proportion approaches 0.02, the TP reaches its maximum value (in this case, the MP is negative), and the manufacturer underreports the cost. When the misreporting proportion approaches 1.32, the MP reaches its maximum value, at which point the manufacturer over-reports the cost;
- (2) If the manufacturer over-reports the cost, the total supply chain profit will be adversely affected. The higher the misreporting proportion, the greater the loss of the supply chain profit. When the over-reporting proportion is small, the supply chain profit flows to the manufacturer, making the manufacturer's profit increase. However, when the over-reporting proportion is large, the manufacturer's profit also shows a downward trend.
- (3) If the manufacturer under-reports the cost, this misreporting can benefit the supply chain. However, because under-reporting can harm the manufacturer's own interests, the manufacturer often does not engage in this practice.

Based on the above research, the following suggestions can be given.

When there is information asymmetry in the supply chain, if the retailer holds DM power, the manufacturer may be incentivized to misreport the cost. Such misreporting can negatively impact the profits of both the supply chain and retailer. Consequently, as the DM leader in supply chain, the retailer should actively encourage other partners to participate in or establish blockchain platforms. Blockchain platforms offer characteristics such as information transparency and immutability, which can create a robust supply chain information environment, thereby eliminating misreporting in the supply chain. This approach not only helps ensure the accuracy and timely sharing of information but also effectively reduces potential profit losses and enhances the stability and sustainability of supply chain collaboration efforts.

5.3. Sensitivity Analysis of f and u

The impact of the retail price sensitivity coefficient (f) and extended warranty price sensitivity coefficient (u) on the total supply chain profit is shown in Figure 6.

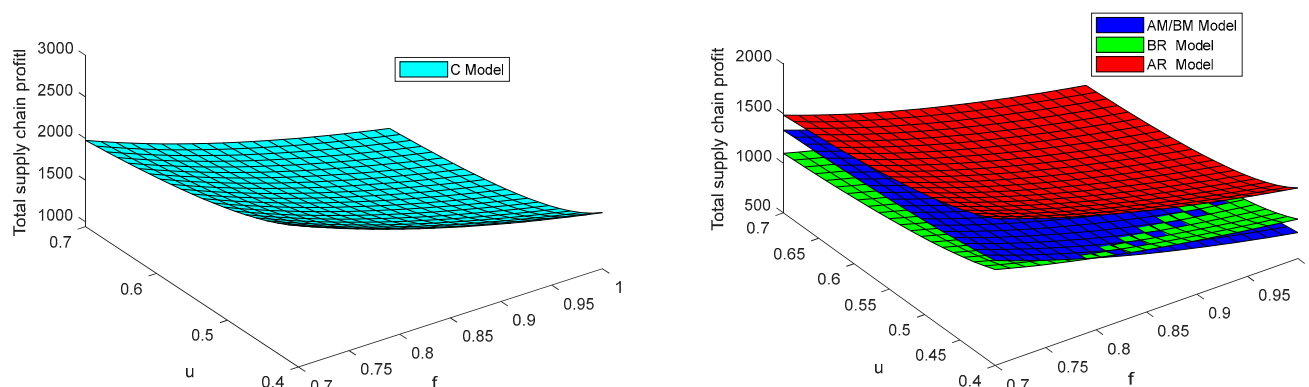


Figure 6. The impact of f and u on the total supply chain profit.

In Figure 6, the total supply chain profit in each model is negatively correlated with coefficients f and u . This is because both the product demand and extended warranty demand decrease as the prices increase, and this negative impact becomes more pronounced as the price sensitivity coefficients rise. Given that the total supply chain profit is positively influenced by demand, the negative correlation between the total supply chain profit and the coefficients f and u is evident.

Furthermore, Proposition 4 is validated in Figure 6; that is, when $0 < f < 2u$, $\Pi_{c(AM/BM)}^* > \Pi_{c(BR)}^*$, and when $2u < f < 4u$, $\Pi_{c(AM/BM)}^* < \Pi_{c(BR)}^*$. The main reasons for this are as follows.

Retailer-led DM is more beneficial to the supply chain, meaning $\Pi_{c(AM/BM)}^* < \Pi_{c(AR)}^*$. However, if the manufacturer misreports information, the situation will be different, whereby the higher the cost of misreporting, the higher the wholesale and retail prices. When the retail price sensitivity coefficient is high, considering product demand, the manufacturer's optimal misreporting proportion will not be too high; thus, the retail price increase will be less pronounced. In this scenario, the impact of misreporting on the total supply chain profit will be minimal, meaning $\Pi_{c(AM/BM)}^* < \Pi_{c(BR)}^*$. However, when the retail price sensitivity coefficient is low and has less impact on the product demand, the manufacturer's optimal misreporting proportion will significantly increase, leading to a substantial decrease in the total supply chain profit, meaning $\Pi_{c(AM/BM)}^* > \Pi_{c(BR)}^*$.

From the above findings, it is clear that reducing the consumer focus on price is beneficial to the supply chain. Supply chain managers can divert consumer attention away from price in various ways, such as by promoting the importance of product quality, appearance, and other aspects, thereby reducing their focus on the retail price. Additionally, supply chain managers can enhance the extended warranty service system to attract consumers' attention to the content of the extended warranty service, thereby reducing their sensitivity to the extended warranty price.

6. Conclusions

This paper, which focuses on electronic product supply chains with extended warranty services, has presented five supply chain DM models based on various power structures and information environments. Using Stackelberg game knowledge, these models were analyzed and compared, and the following conclusions were drawn:

- (1) When comparing the supply chain DM results under different information environments, it can be observed that when the manufacturer has a DM advantage, the manufacturer does not engage in misreporting, regardless of whether the information is symmetrical. At this point, the manufacturer's profit reaches an optimal state. Conversely, when the retailer has a DM advantage, the manufacturer tends to misreport the costs under information asymmetry, which adversely affects the retailer and supply chain.

- (2) Our comparison of supply chain DM models under information symmetry indicates that retailer-led DM is beneficial for itself and the supply chain. When the retail price sensitivity coefficient is low, it is advantageous for the manufacturer to become the DM leader; otherwise, being a DM follower is preferable for the manufacturer. Our comparison of supply chain DM models under asymmetric information shows when considering the benefits of all supply chain members, if the retail price sensitivity coefficient is low, the manufacturer should take on the role of DM leader; otherwise, the lead should be taken by the retailer.
- (3) Centralized DM is more ideal, and the results can serve as a reference for supply chain coordination. The extended warranty market depends on the product market, showing a negative correlation between the extended warranty price and the retail price. Additionally, a lower retail price combined with a higher extended warranty price is advantageous for this supply chain.
- (4) According to the sensitivity analysis results, it is evident that among the five supply chain models, the sensitivity coefficients of the extended warranty price and retail price negatively affect the total supply chain profits.

Compared with the existing research, this paper's notable findings include advantageous pricing strategies for the supply chain, the timing and impact of misreporting, and the influence of the power structure on the supply chain DM and price sensitivity coefficient. Based on the relevant research conclusions, this paper offers several suggestions from the perspectives of supply chain managers and participants in supply chain games. For instance, we recommend appropriately raising the extended warranty price while lowering the retail price and shifting consumers' attention away from the price, which can be more beneficial to the supply chain. Retailer should be concerned about potential misreporting by manufacturers only when there is information asymmetry and the retailer holds the leading DM power.

The research findings of this paper provide theoretical support for the coordinated development of electronic product supply chains. However, this study has some limitations. Here, we only considered the asymmetric cost information. In reality, the information asymmetry in the supply chain is varied and frequent. For example, demand information is the private information of downstream enterprises, and companies responsible for product recycling may conceal recovery information. Future research will further explore the impacts of multiple information asymmetries on the supply chain.

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