

Article

An Integrated Two-Step Optimization Model and Aggregative Multi-Criteria Approach for Establishing Sustainable Tram Transportation Plan

Svetla Stoilova * and Ivan Pulevski

Faculty of Transport, Technical University of Sofia, 8 Kl. Ohridski Blvd., 1000 Sofia, Bulgaria

* Correspondence: stoilova@tu-sofia.bg

Abstract: The choice of the most appropriate sustainable scheme for the organization of tram transportation in cities is of great importance for tram operators, for users of transportation services, and for the protection of the environment from harmful emissions. This study aims to propose a methodology for formulating a tram transportation plan considering technological, environmental, economic, and social indicators. The variant schemes represent the routes of a tram in the tram network. The methodology includes four stages. The first stage involves the determination of variant schemes for a transportation plan of service with trams. In the second stage, a two-step optimization model is proposed to determine the number and trams by types for each tram route for each variant scheme, and also to establish the distribution of trams by depots. The third stage includes ranking the variant schemes by applying the sequential interactive model for urban systems (SIMUS) multi-criteria method. Eleven quantitative and qualitative criteria for evaluating the tram transportation plan were introduced. A verification of the results is performed in the fourth stage. For this purpose, a comparison of the preference ranking organization method for enrichment of evaluations (PROMETHEE) method and the technique for order of preference by similarity to ideal solution (TOPSIS) method is made. Both methods have different approaches for decision making and differ from the SIMUS method. Two strategies were proposed to determine the criteria weights. One is based on the Shannon entropy method and the other uses the objective weights obtained through the SIMUS method. Finally, in the fifth stage, the results obtained through the SIMUS, PROMETHEE and TOPSIS methods are combined using the expected value obtained by applying the program evaluation and review technique method (PERT). The proposed methodology was applied to study tram transportation in Sofia, Bulgaria. Five variant schemes were considered. The schemes are optimized through the criterion of minimum energy consumption. The number of trams by routes and by type was determined. An improved scheme for tram transportation in Sofia was proposed. The scheme makes it possible to increase the frequency of the trams by 13%, to reduce the zero mileage of rolling stock, and to reduce carbon dioxide pollution by 11%.

Keywords: tram; urban transportation; sustainable transportation plan; optimization; MCDM; SIMUS; PROMETHEE; TOPSIS; Shannon entropy; aggregation

Academic Editors: Yiming Bie, Hu Zhang and Shidong Liang

Received: 5 December 2024

Revised: 3 January 2025

Accepted: 10 January 2025

Published: 12 January 2025

Citation: Stoilova, S.; Pulevski, I. An Integrated Two-Step Optimization Model and Aggregative Multi-Criteria Approach for Establishing Sustainable Tram Transportation Plan. *Sustainability* **2025**, *17*, 543. <https://doi.org/10.3390/su17020543>

Copyright: © 2025 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

The sustainability of transportation depends on the implementation of convenient, regular, safe and secure, energy-efficient, and environmentally friendly carriage. Tram transportation is one of the main means of public transportation in cities around the world. It is environmentally friendly because it uses electrical energy and has a large carrying capacity, which makes it useful for society. The growing importance of sustainable transportation development in cities with tram transportation has created a need for scientific methods for assessing transportation services. Despite extensive discussion in the scientific community, there is no general model for their measurement. The aim of this study is to present a methodological approach for determining the most suitable sustainable tram transportation scheme.

On a tram network, the tram transportation plan includes the tram routes, number of trams, and the type of trams on each route. A tram route is the organization of movement between the first stop and the final stop. Tram scheme planning is related to drawing up a timetable, tram rolling stock planning, crew scheduling, and other operational parameters. The assessment of the limiting conditions of the tram infrastructure is important for changes in the tram route scheme, as failure to comply with the capacity of the sections of the tram network would lead to tram congestion, failure to keep to the timetable, low speed and, as a result, service degradation.

The tram transportation scheme has to meet the current transportation needs of the population in order to make public transportation attractive to citizens. An important task for tram transportation in the cities is to determine the optimal scheme for the movement of trams. This is a multi-component task, as the capabilities of the carrier and the needs of citizens for transportation must be considered. The quantity and condition of tram rolling stock is of particular importance for any transportation system. It is of particular importance that trams are serviceable and safe. It is also important for passengers that they are clean, comfortable, air-conditioned, and accessible.

The hypothesis of this research is that the choice of a tram transportation plan depends on different quantitative and qualitative criteria. It is necessary to evaluate and compare different alternative schemes of tram service routes in order to be able to choose the most appropriate one. This could help the tram operator to make decisions about the improvement of the tram transportation service in the city.

This paper aims to expand scientific knowledge on the selection of a tram transportation scheme.

The focus of this study is to solve the following problems:

- Optimization of the parameters of the transportation plan with respect to the number and the type of trams for each route, and the distribution of trams by depots.
- Assessment of the tram transportation schemes based on different multi-criteria approaches considering the quantitative and qualitative criteria important for the carrier and the passengers.
- Choosing the most appropriate tram transportation scheme taking into account the ranking produced by using the different multi-criteria approaches.

The present study aims to propose an integrated approach, based on optimization methods and multi-criteria analysis, for the development of a transportation plan for tram services in cities that considers the capabilities of tram operators and the needs of passengers.

Two-step optimization was applied. Nonlinear optimization was used to determine the number of trams of various types to serve each tram scheme based on an optimization criterion. In this study, energy consumption was used as an optimization criterion. Linear optimization was applied for location of the different types of trams to depots. Multi-

criteria analysis was applied for ranking the variant schemes and for choosing the most appropriate one.

The advantages of the integration of the optimization methods and multi-criteria analysis for evaluating a tram transportation plan can be summarized as follows:

- The variant schemes of the tram transportation plan, pre-optimized according to the criterion of minimum electricity consumption, are compared. On the one hand, this criterion is directly related to the pollution of the environment with carbon dioxide in the production of electricity; on the other hand, it is related to the operational costs of traffic management. Thus, environmental and economic benefits are considered in the optimization criterion.
- Additional quantitative and qualitative criteria are defined for the optimized variant schemes, including technological, environmental, economic, and social indicators.
- The choice of the most appropriate tram variant scheme is performed based on the complex impact of all the defined criteria.
- The aggregation of multi-criteria methods, such as SIMUS, PROMETHEE, and TOPSIS, makes it possible to consider three different types of ranking. The first is based on linear optimization, the second uses outranking, and the third is a distance-based approach.

The contributions of this paper to scientific knowledge and to practice can be summarized as follows:

- (1) A two-step optimization for a tram transportation scheme was proposed. It includes a nonlinear optimization model based on energy consumption to determine the number and type of trams in the tram transportation scheme, and a linear model for the distribution of trams by depots.
- (2) This methodology includes integration between a two-stage optimization model and multi-criteria analysis. This allows for the selection of the most appropriate, optimized tram transportation scheme taking into account both quantitative and qualitative criteria. An aggregation approach is proposed, based on three multi-criteria methods—SIMUS, PROMETHEE, and TOPSIS.

This paper is structured in five sections. The first section is the introduction. The second section presents a review of the literature. The third section introduces the materials and methods where the novel integrated nonlinear optimization–multi-criteria analysis approach is explained. The fourth section presents the results obtained for the case of the Sofia tram network by applying the new methodology. The obtained results are discussed and the advantages of the new methodology are presented in the fifth section. The conclusions are presented at the end of the paper.

2. Literature Review

Tram transportation has been studied in terms of its efficiency, tram routes, rolling stock, timetable, and organization in various works. The authors have applied the methods of multi-criteria analysis to perform their research.

In [1], the authors investigated the specifics of tram transportation in the Czech Republic, France, Germany, Hungary, Poland, and Turkey. The following criteria for assessing the characteristics of tram transportation were proposed: comfort of travel, reliability, durability, safety, traction-operational characteristics, operational costs, price, environmental friendliness, utility, and functionality. The research was carried out using the elimination and choice translating reality (ELECTRE) III/IV and the analytic hierarchy process (AHP) methods.

The efficiency of tram transportation in an urban transportation system was studied in [2]. Based on the literature analysis, the authors determined six criteria for an

assessment of the efficiency of a tram route. They studied traffic safety, economic feasibility, security, exploitation by citizens, accessibility, and environmental preservation. The AHP method and GIS approach were applied to evaluate the criteria and to determine their weights. The methodology was applied for an urban transportation system in Setif, Algeria.

An integrated multi-criteria approach based on the use of the evaluation based on distance from the average solution (EDAS) method and the criteria importance through inter-criteria correlation (CRITIC) method were elaborated on in [3]. The methodology was applied for determining trams and metro vehicles. The author proposed 22 criteria to assess urban rail vehicles operating in a public transportation system. The model was applied for ranking 10 tram types in Turkey. It was found that the most important criteria are maximum grade slopes, emissions (CO₂), and the number of seats.

A linear optimization model for a tram timetable was developed in [4]. The minimization of the travel time of the tram and the utility of the signal priority were chosen as criteria for optimization.

A methodology for determining the best tram route was elaborated on in [5]. The authors established six main criteria for assessment of tram routes, such as accessibility, safety, economy, environment, population density and trips, and security. The methodology was applied for the tram system in city of Ramadi in Iraq. Four tram routes were investigated and ranked by using GIS and multi-criteria analysis.

In [6], the authors investigated the possibility of developing an urban line without a catenary. The study was made for Brescia, Italy. It discussed the use of batteries and the main parameters of the new technology.

Tram rolling stock in Poland was analyzed in [7]. The authors calculated the energy consumption for different types of trams in Warsaw.

New types of sustainable energy trams were studied in [8]. The authors studied trams with different energy sources.

In [9], the authors examined the impact of the number of passengers and the frequency of the trams on the operational cost of the trams.

A multi-objective optimization model based on energy consumption of trams was elaborated on in [10]. The model was tested for the Haizhu tramway in Guangzhou China.

In [11], the best and worst method (BWM) and the power Heronian weighted aggregated sum product assessment method (WASPAS'PH) were used for selecting tram vehicles. The authors studied fourteen types of tramcars. The tramcars were ranked by using seventeen criteria selected through a literature review of previous studies. It was found that the cost per tram, the number of seats in a vehicle, and energy consumption are important criteria for choosing the type of tramcar.

In [12], the AHP method and grey relationship analysis (GRA) were used for tramway selection in Turkey. The authors assessed eight alternative tramway cars based on criteria related to technical parameters, such as length of the vehicle, height, empty weight, number of doors, maximum speed, capacity, and comfort. Multi-objective optimization based on ratio analysis (MOORA) was used to verify the results.

In [13], the AHP and ELECTRE III/IV multi-criteria methods were applied to assess the tram systems in the city of Gorzów in Poland. The authors proposed social, technical, economic, and environmental criteria to assess tram systems.

Alternatives for an urban transportation system including trams were presented in [14]. The authors proposed and evaluated criteria, such as effectiveness of fleet use, travel time, environmental friendliness, integration and reliability, safety and security, availability, and investment costs. The AHP and ELECTRE III methods were applied to assess the criteria and rank the alternatives. The Polish city of Krakow was the object of study.

Tram stops were examined with respect to safety and functionality in [15]. The authors identified the criteria, which included equipment and technical parameters. The criteria were assessed by experts using a rating scale defined by authors. Three groups of criteria according to the impact of functionality and safety were defined.

Multi-criteria analysis was used for transportation mode selection in a traffic corridor of Wrocław, Poland [16]. Six alternatives for transportation by trams and buses were studied. The alternatives were assessed using criteria, such as time, comfort, accessibility to the stop, and cost of rolling stock. The authors applied ELECTRE III/IV and AHP to make decisions.

In [17], the authors studied transportation risks with buses and trams in Gdansk, Poland. The sources of hazards for accidents were determined.

Tram vehicles selected from a worldwide market were analyzed with their main technical parameters in [18].

Tram transportation in Wrocław, Poland was analyzed in [19]. Changes in the organization of traffic in order to carry out reconstruction of the network were considered.

Tram catenary-free systems were studied in [20]. The authors used microsimulations. Three scenarios of tramway systems were studied using cost–benefit analysis.

In [21], simulations were made to investigate different scenarios of traffic priority for trams at intersections in Warsaw, Poland. The authors found that traffic light priority affected electricity consumption, time, and pollutant emissions.

In [22], the authors produced an optimization model to determine energy-efficient tram speed considering the impact of traffic lights.

In [23], the tram network and tram routes in Melbourne were analyzed by using Lorenz curves and accounting for the volume of passengers, length, and link speed.

On the basis of previous research, it can be said that the most used criteria for the evaluation of tram services were operating costs [1,5,12,20], comfort of travel [1,12,16], environmental friendliness [1,2,5,14], emissions (CO₂) [3,21], number of seats [3,11], energy consumption [7,10,11,21], and travel time [4,14,16,22,23]. Different methods of multi-criteria analysis were used. The AHP [1,2,12–14,16], CRITIC [3], and BWM [11] methods were applied to assess the criteria, while the AHP [2], ELECTRE [1,13,14,16], EDAS [3], WASPASP [11], CRA [12], and MOORA [12] methods were used to analyze tram services. Some authors used mathematics optimization, such as linear programming [4], and metaheuristics, such as PSO [10], to study tram service.

In previous studies, however, the methods of optimization and multi-criteria analysis, used for choosing the most appropriate scheme of tram transport, were not integrated. This study proposes an innovative approach that combines optimization of tram service by type and number of trams and their distribution by depot, with multi-criteria analysis for selecting the most appropriate tram route scheme, considering both quantitative and qualitative criteria. This approach makes it possible to compare pre-optimized tram transportation schemes and choose the most appropriate scheme.

This paper differs from previous studies in the methodology allowing us to optimize the variant schemes of tram transportation on the basis of energy consumption, and to rank them on the basis of quantitative and qualitative criteria related to the transportation process by using multi-criteria analysis and an aggregative approach.

3. Materials and Methods

The methodology of this study includes the following steps (Figure 1):

- Stage 1. Determine variant schemes of a transportation plan of service with trams. The variant schemes represent the routes of the tram in the tram network.
- Stage 2. Two-step optimization model. This stage includes two optimization models: (1) optimization of the tram transportation plan depending on the number and types

of trams for each tram route for each variant scheme; (2) optimization of the distribution of trams by depots for each optimized variant scheme considering the number and types of trams:

- Step 1. Optimize the tram transportation plan using the nonlinear generalized reduced gradient method (GRG). In this step a nonlinear optimization model for minimizing energy consumption is proposed.
- Step 2. Optimize the distribution of trams by depots using linear programming methods.
- Stage 3. Multi-criteria analysis for ranking variant schemes. This stage covers the following steps:
 - Step 1. Form the parameters of the multi-criteria model. This step determines the quantitative and qualitative criteria for evaluating the variant schemes and compiling the decision matrix. The parameters of the decision matrix are determined on the basis of the results obtained in Stage 2.
 - Step 2. Rank the variant schemes using the SIMUS multi-criteria method.
- Stage 4. Verification. It is carried out by making a comparison with the use of other multi-criteria analysis methods. This study proposed the PROMETHEE and TOPSIS methods for verification of the results. Both methods have different approaches for decision making and differ from the SIMUS method. PROMETHEE is an outranking method, whereas TOPSIS is a distance-based method. Both methods require setting criteria weights. Two approaches are proposed to determine the weights. The first approach uses the Shannon entropy method. It is an objective method that is based on information from the data used. The second approach uses the objective weights obtained through the SIMUS method. The following verification options were created: (1) PROMETHEE–Shannon entropy; (2) TOPSIS–Shannon entropy; (3) PROMETHEE–SIMUS objectives; (4) TOPSIS–SIMUS objectives.

Stage 4 includes the following steps:

- Step 1. Determination of the criteria weights using the Shannon entropy method.
- Step 2. PROMETHEE–Shannon entropy approach for ranking variant schemes.
- Step 3. TOPSIS–Shannon entropy approach for ranking variant schemes.
- Step 4. PROMETHEE–SIMUS objectives weights for ranking variant schemes.
- Step 5. TOPSIS–SIMUS objectives weights for ranking variant schemes.

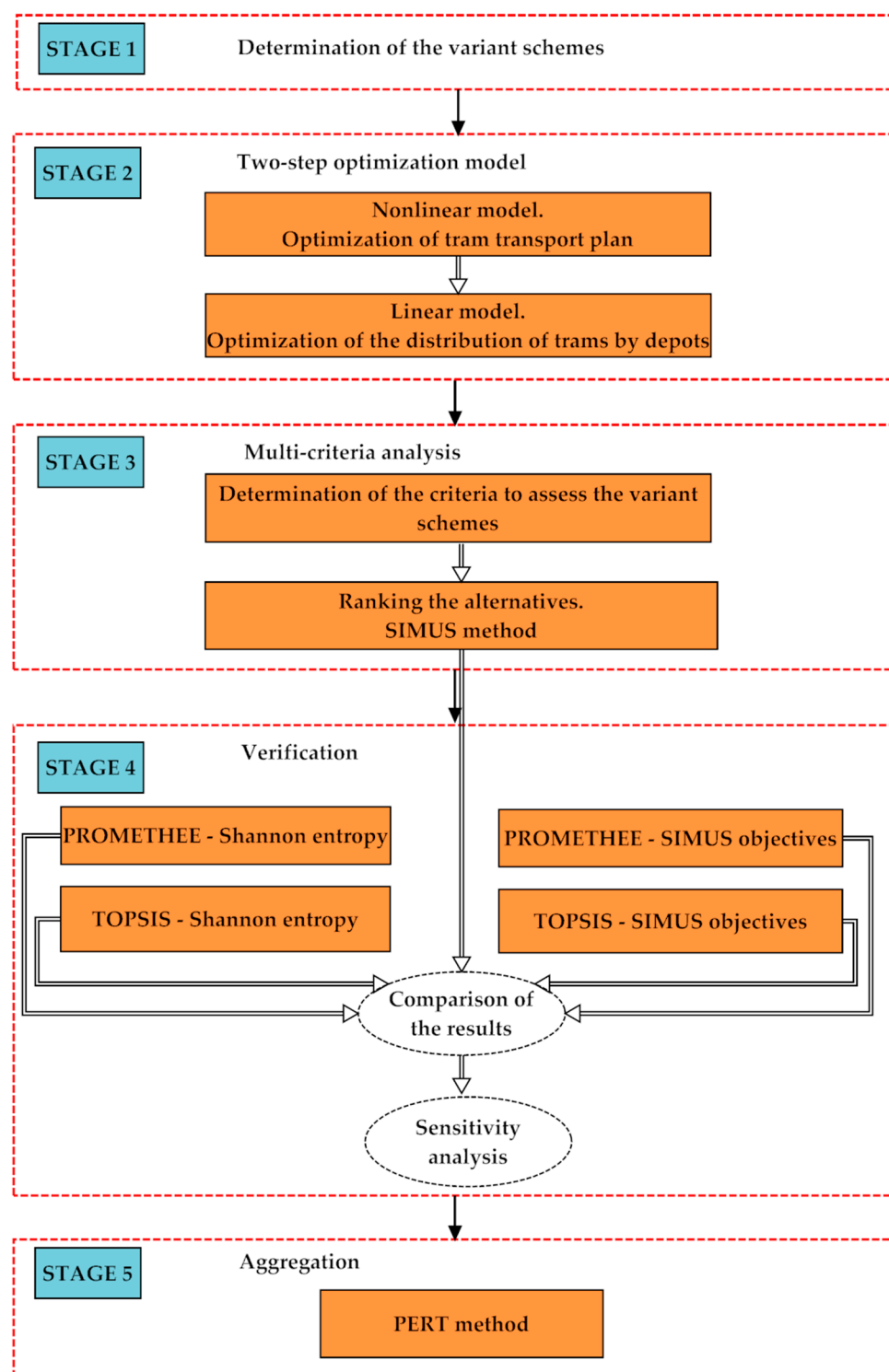


Figure 1. Scheme of methodology.

- Stage 5. Aggregation. Finally, the results obtained by means of the SIMUS, PROMETHEE, and TOPSIS methods are combined using the expected values obtained by using the program evaluation and review technique (PERT) method. It uses three estimates corresponding to the three methods considered. Thus, the three studied methods are combined in a general assessment.

The main advantages of the proposed multi-criteria methods are as follows:

(1) The SIMUS method is based on linear optimization; it is not a subjective method; it does not apply weights of the criteria; it permits us to evaluate the sensitivity of input data.

(2) The PROMETHEE method uses a preference function for each criterion; two flows and net flow for each alternative are calculated, which represents the preferences between alternatives; the calculation process is simple; it is a popular outranking method for decision making.

(3) The TOPSIS method has simple mathematics; it is a popular destination-based method for decision making.

Both the PROMETHEE and TOPSIS methods require that the weights of the criteria be set. For this purpose, this research proposes an integration between the Shannon entropy method and each of the proposed methods.

3.1. Stage 1. Determination of Variant Schemes of a Tram Transportation Plan

The tram transportation plan includes the routes in the tram network. The routes depend on the tram infrastructure. At this stage, the variant schemes are drawn up. The variant schemes differ according to the tram service routes, the number of routes, and the number of trams passing through the routes in the tram network.

3.2. Stage 2. Two Step Optimization Model

3.2.1. Step 1. Optimizing the Tram Transportation Plan Using the Nonlinear Generalized Reduced Gradient (GRG) Method

In this stage, for each variant scheme a nonlinear optimization model is applied to determine the number and types of trams for each tram route. The optimization criterion is minimum energy consumption for peak period. Determining the number of tram services based on electricity consumption is important as it is directly related to ecology, since carbon dioxide emissions are released during the production of electricity.

For each variant scheme an optimization model is compiled. The goal is to minimize energy consumption for each variant scheme.

$$Z_s = \sum_{r=1}^R \sum_{u=1}^U w_u \cdot L_{sr} \cdot \frac{k_{sru}}{k_{sr}} \rightarrow \min, kWh \text{ per day}, \quad (1)$$

where: Z_s is the energy consumption for variant scheme s ; k_{sru} is the number of trams of type u , running on route r , in scheme s ; $s = 1, \dots, S$ is the number of variant schemes; $u = 1, \dots, U$ is the number of tram types; $r = 1, \dots, R$ is the number of tram routes; k_{sr} is the total number of trams of any type, running on route r , variant scheme s ; w_u is the energy consumption of type u trams (kWh/km); L_{sr} is the daily running distance (both directions) for route r of scheme s (km).

The unknown values in the nonlinear optimization model are the number of trams of type u , running on route r , in scheme s (k_{sru}).

The daily running distance (both directions) L_{sr} for route r of scheme s is determined as follows:

$$L_{sr} = l_{sr} \cdot d_{sr}, \text{ km}, \quad (2)$$

where: l_{sr} is the distance (both directions) of route r of scheme s (km); d_{sr} is the number of daily departures of trams on route r of scheme s .

The number of daily departures on route r of scheme s is determined as follows:

$$d_{sr} = \sum_{g=1}^G d_{srg}, \quad (3)$$

where: d_{srg} —number of departures on route r of scheme s during period g (e.g., morning peak) of the day.

$$d_{srg} = \frac{t_g \cdot k_{srg}}{t_{sr}}, \quad (4)$$

where: t_g is the duration of the period g (e.g., for morning peak: 06:30–09:30, 180 min); k_{srg} is the number of trams of any type, running on route r in scheme s during period g ; t_{sr} is the duration of the route (both directions) (min).

The number of trams of any type, running on route r in scheme s during period g is determined as follows:

$$k_{srg} = c_{srg} \cdot k_{sr}, \quad (5)$$

where: k_{sr} is the total number of trams of any type, running on route r , in scheme s ; c_{srg} is an adjustment factor for the period g on route r , in scheme s .

$$k_{sr} = \sum_{u=1}^U k_{sru}, \quad (6)$$

Since the optimization was performed for an hour in a peak period with the highest passenger load, an adjustment factor is applied to obtain results for another period of the day:

$$c_{srg} = \frac{I_{srg}^{w/h}}{\max_{g=0}^{24} (I_{srg}^w)}, \quad (7)$$

where: $I_{srg}^{w/h}$ is the value for the passenger flow on route r of variant scheme s at time g of the day on a weekday (w) or holiday (h) day, taken by the average value of passenger flows by hour. In the case of a missing route, the value of the passenger flow is selected for a route in the active tram scheme, which to a great extent coincides with the r -th route in the s -th variant scheme.

The methodology consists of the following restrictive conditions.

1. Number of trams. This number must be a non-negative integer number:

$$k_{sru} \in \mathbb{Z}; k_{sru} \geq 0, \quad (8)$$

2. Infrastructure capacity. The total number of trams on route r of scheme s should be within the specified limits based on infrastructure capacity and operational intervals:

$$k_{sru} \in \mathbb{Z}; k_{sru} \geq 0, \quad (9)$$

The maximum and the minimum number of trams on a route r of variant scheme s depends on the intervals between trams.

$$k_{max\ sr} = \frac{t_{sr}}{f_{min\ sr}}; k_{min\ sr} = \frac{t_{sr}}{f_{max\ sr}}, \quad (10)$$

where: $k_{max\ sr}$ is the maximum number of trams on route r of scheme s ; $k_{min\ sr}$ is the minimum number of trams on route r of scheme s ; $f_{min\ sr}$ is the minimum interval for running trams on route r of variant scheme s (min); $f_{max\ sr}$ is the maximum interval for running trams on route r of variant scheme s (min); t_{sr} is the duration for one turn on every route r of variant scheme s (min).

The maximum interval $f_{max\ sr}$ is determined in relation to the set goals for service quality.

The minimum interval $f_{min sr}$ is determined according to the number of tram routes serving a section and the intersections with switches.

The tram network has the following limiting conditions regarding capacity:

- Traffic lights at intersections without tram signaling—the trams run cyclically over a period of time.
- Traffic lights at intersections with tram signals—the tram cycle for separate tram traffic directions over a period of time.
- Availability of single or double stops in close proximity to intersections—in certain situations, the throughput determined by the traffic light can be changed.
- Crossing signals—they provide distance between trams.
- Single-track sections—they require that trams wait on a busy section.

These limiting conditions have a direct impact on the capacity of individual sections, respectively, on the minimum possible movement intervals.

The minimum interval $f_{min sr}$ is determined as follows:

$$f_{min sr} = \max \left(\max_{k=1}^K \left(\frac{f_{min k} \cdot r_{sk}}{60} \right); \max_{i=1}^I \left(\frac{f_{min ij} \cdot r_{sij}}{60} \right) \right), \min, \quad (11)$$

where: $k = 1, \dots, K$ is the number of sections; $i = 1, \dots, I$ is the number of intersections with tram switches; $j = 1, \dots, J$ is the number of directions in the intersection; r_{sk} is the number of tram routes in section k for s variant scheme; $r_{sk} = 1$ is the number of sections where the tram route is independent; r_{sij} is the number of routes passing through the intersection i in direction j ; $f_{min k}$ is the minimum interval in section k for s variant scheme; $f_{min ij}$ is the minimum interval for j -direction in intersection i .

The first part of Equation (11) shows the minimum interval according to tram routes in a section. The second part of Equation (11) represents the minimum interval when the route passes through intersections with tram switches. The minimum interval $f_{min sr}$ is determined on the basis of the larger value of the two minimum intervals given in Equation (11).

3. Passenger demand. The capacity of the tram must meet the needs of passenger flow.

$$\sum_{r=1}^R \sum_{u=1}^U k_{sr uk} \cdot p_u \geq I_{hk}, \quad (12)$$

where: p_u is passenger capacity on u type trams (pass.); I_{hk} is the maximum passenger flow intensity in section k (pass./h); $k = 1, \dots, K$ is the number of sections.

4. Availability. Transportation must perform with the available rolling stock of trams.

$\sum_{r=1}^R k_{sr u} \leq K_u$ —the total number of trams of type u , used in scheme s must be less than or equal to the available number of such trams (K_u).

The presented optimization model is nonlinear. The generalized reduced gradient (GRG) method is one of the most popular methods for solving problems of nonlinear optimization [24,25]. It was proposed in [24]. This research uses the GRG nonlinear solver in Excel, which is an optimization algorithm used to find the maximum or minimum of a function subject to constraints.

3.2.2. Step 2. Optimization of the Distribution of Trams by Depots Using the Linear Programming Method

The optimal number and type of trams for each route for each variant scheme is determined after performing the optimization according to the nonlinear model. A second

optimization is carried out for the distribution of trams by depots. The main criterion is the mileage of the tram from the depot to its inclusion in the route, called “zero mileage”.

For each variant scheme a linear optimization model is compiled. The goal is to minimize the total tram zero mileage included in the routes.

The objective function F_s for each variant scheme s is determined from the results obtained from the nonlinear optimization model about the tram routes

$$F_s = \sum_{q=1}^Q \sum_{r=1}^R l_{qrs} \cdot n_{qrs} \rightarrow \min, \text{ km}, \quad (13)$$

where: l_{qrs} is the tram zero mileage for getting to r route in s variant scheme from depot q (km); n_{qrs} is the number of trams serving r route in s variant scheme from depot q ; $q = 1, \dots, Q$ is the number of tram depots; $r = 1, \dots, R$ is the number of tram routes in s variant scheme.

The following restrictive conditions are set:

$$\sum_{q=1}^Q n_{qrs} = a_{rs}, \quad q = 1, \dots, Q, \quad (14)$$

i.e., the number of trams delivered from the depots should be equal to the number of trams serving each r route in s variant scheme.

$$\sum_{r=1}^R n_{qrs} \leq b_{qs}, \quad r = 1, \dots, R, \quad (15)$$

where: a_{rs} is the number of trams serving r route in s variant scheme; b_{qs} is the tram in depot q in s variant scheme.

$$n_{qrs} \geq 0, \quad n_{qrs} \text{ is integer.} \quad (16)$$

Condition (14) means that the number of trams delivered from the depots should be equal to the number of trams serving each r route.

Condition (15) means that the number of trams operating on a given route served by the q depot is less than or equal to the depot capacity.

Condition (16) means that the number of trams has to be positive and also an integer.

The optimization model presented herein is linear and can be solved through linear programming. This research uses the Simplex algorithm solver in Excel, which is a linear optimization algorithm used to find the maximum or minimum of a function subject to constraints.

3.3. Stage 3. Multi-Criteria Analysis for Ranking Variant Schemes

The selection of an appropriate variant scheme is carried out through a multi-criteria analysis. For this purpose, the results obtained from the two-step optimization model presented in Stage 2 are applied.

3.3.1. Step 1. Forming the Parameters of the Multi-Criteria Model

On the basis of the literature review, it was found that the following criteria for evaluating the tram routes are most commonly used: operating costs, comfort of travel, environmental friendliness, emissions (CO₂), seat number, energy consumption, and travel time. In this study, the criteria for assessment of the variant schemes are proposed taking into account the indicators used in previous research as well as an analysis of tram organization.

The criteria are as follows:

- C1—run per hour peak period (km). This indicator represents satisfying the needs of passengers of tram transportation during peak hours.

- $C2$ —annual run (km). This indicator represents satisfying the needs of passengers of tram transportation during the year.
- $C3$ —transportation capacity (number of seats for the year). This indicator represents the need of tram transportation for passengers.
- $C4$ —average travel time of a course (min). This indicator is a measure of the time spent travelling between the start and end stop of the tram route.
- $C5$ —zero tram mileage to include in the route (km). The value of this criterion is determined by the results obtained from the linear optimization model for each variant scheme.
- $C6$ —intensity (number of trams/day). This criterion represents the service level of tram transportation.
- $C7$ —average travel distance (km). This criterion represents the capability of the tram operator to offer different routes in the city in the transportation plan.
- $C8$ —transportation costs (euro/year). This criterion is an economic assessment of tram transportation.
- $C9$ —percentage of kilometers covered by low-floor trams. This indicator is a measure of the comfort offered during travel in the city.
- $C10$ —percentage of kilometers traveled by trams equipped with air conditioning. This indicator is a measure of the comfort offered during travel in the city.
- $C11$ —carbon dioxide emissions, CO_2 (kgCO_2/kWh). The value of this criterion is determined based on results for electricity consumption received by means of the non-linear optimization model.

Criteria $C1$ – $C7$ are technological criteria. Criterion $C8$ is an economic criterion. Criteria $C9$ and $C10$ are social criteria related to travel comfort. Criterion $C11$ is an ecological criterion.

The values of the criteria are determined for each variant scheme after performing the two-step optimization in Stage 2.

Transportation costs ($C8$) are determined based on the annual run for each variant scheme and the costs per kilometer run as follows:

$$C8_s = (250 \cdot \sum_{g=0}^{24} (k_{srg}^w \cdot l_{sr}) + 115 \cdot \sum_{g=0}^{24} (k_{srg}^n \cdot l_{sr})) \cdot p_{km}, \text{ euro/year} \quad (17)$$

where: $k_{srg}^{w/n}$ is the number of tram cars on the route r of variant scheme s at time g of the day on a weekday (w) or holiday (h) day; 250 and 115 are the average number of weekdays and holidays, respectively, in the year; p_{km} is the transportation cost per kilometer run (euro/km):

The values of the number of tram cars $k_{srg}^{w/n}$ are determined based on the results obtained in Stage 2.

The values of the carbon dioxide emissions ($C11$) criterion for each variant scheme are determined based on the data for the greenhouse gas emission intensity of electricity generation for each EU country [26].

$$C11_s = Z_s \cdot c^{CO_2}; s = 1, \dots, S, \text{ KgCO}_2, \quad (18)$$

where: Z_s is the energy consumption for each variant scheme (KWh/day); c^{CO_2} is CO_2 emission intensity (kgCO_2/kWh) from public electricity production. $c^{CO_2} = 0.455 \text{ kgCO}_2/\text{kWh}$ for Bulgaria [26].

The results of the criteria values for each variant scheme are used to form the decision matrix $(x_{st})_{S \times T}$ of the multi-criteria model; $s = 1, \dots, S$ is the number of variant schemes; $t = 1, \dots, T$ is the number of criteria.

The decision matrix provides the input data for conducting a multi-criteria analysis and ranking of the variant schemes.

3.3.2. Step 2. Ranking Variant Schemes by Applying the SIMUS Multi-Criteria Method

This study uses the SIMUS method for ranking the variant schemes of the tram transportation plan. This method is based on linear programming, and it does not use weights of criteria or experts' assessments [27]. The procedure of the SIMUS method is presented in [27]. It is necessary for each criterion to set the type of optimization—maximum or minimum. The criteria are presented in the rows of the decision matrix, the variant schemes are given in the columns. Decision matrix normalization is performed by summing the values of the rows. Another method of normalization could also be used. Linear optimization models are compiled for each of the criteria by applying the normalized matrix. For this purpose, the row with criterion is considered as an objective function, and the other rows are taken as restrictive conditions. The elements of the right-hand side (RHS) of the restrictive conditions are determined according to the maximum normalized value of the row when the optimization is maximum; in the case of minimum optimization, the minimum value of the row is used. The number of criteria is equal to the number of optimization linear models. The scores of each alternative are determined as results of optimizations. They form an efficient result matrix (ERM), which has to be normalized. The normalized efficient result matrix (NERM) is used to rank alternatives based on the weighted sum method.

The weights of w_t objectives can be determined by using the results from the rows of the NERM.

$$w_t = \frac{\max_s NERM_{ts}}{\sum_{t=1}^T NERM_{ts}}, \quad (19)$$

$$0 \leq w_t \leq 1, \sum_{t=1}^T w_t = 1, \quad (20)$$

where: $t = 1, \dots, T$ is the number of criteria; $s = 1, \dots, S$ is the number of variant schemes.

Thus, after performing the optimizations, the importance of each criterion can be determined, if necessary. This makes the SIMUS method different from other multi-criteria methods that use criteria weights as input information to perform ranking procedures.

3.4. Stage 4. Verification

The verification is carried out by making a comparison with the use of other methods of multi-criteria analysis. This study proposes the use of the PROMETHEE and TOPSIS methods for verification of the results. The first is an outranking method, the second is a distance-based method. Both methods require setting the criteria weights. This study proposes two approaches for the application of both multi-criteria methods. The first approach employs Shannon entropy. The second approach uses the objective weights obtained through the SIMUS method. Both approaches are non-subjective. The Shannon entropy method is based on the information entropy for each criterion. The SIMUS method makes it possible to determine the objective weights as the result of linear optimizations, as was noted above.

The following variants of verification were deployed:

- PROMETHEE–Shannon entropy. In this case, the weights of criteria are determined by means of the Shannon entropy method. The ranking is obtained using the PROMETHEE method.
- TOPSIS–Shannon entropy. In this case, the weights of criteria are determined by means of the Shannon entropy method. The ranking is obtained using the TOPSIS method.

- PROMETHEE–SIMUS objectives. In this case, the weights of criteria are equal to the weights of objectives determined by using the SIMUS method. The ranking is obtained through the PROMETHEE method.
- TOPSIS–SIMUS objectives. In this case, the weights of criteria are equal to the weights of objectives determined by using the SIMUS method. The ranking is obtained using the TOPSIS method.

3.4.1. Shannon Entropy Method

Shannon entropy is a non-subjective method for determining criteria weights. It is based on the information entropy for each criterion.

The information entropy is determined as follows [28]:

$$E_t = -\frac{\sum_{s=1}^S p_{st} \ln p_{st}}{\ln S}, 0 \leq E_t \leq 1, \quad (21)$$

where: E_t is the information entropy; t is the number of criteria, $t = 1, \dots, T$; p_{ij} is the normalized values of decision matrix $(x_{st})_{S \times T}$; s is the number of alternatives, $s = 1, \dots, S$.

The normalized values p_{st} are calculated as follows:

$$p_{st} = \frac{x_{st}}{\sum_{s=1}^S x_{st}}, \quad (22)$$

The weights of the criteria according to Shannon entropy are as follows:

$$w_t^E = \frac{1 - E_t}{\sum_{t=1}^T (1 - E_t)}, \quad (23)$$

3.4.2. PROMETHEE Method

The PROMETHEE method is an outranking method of multi-criteria analysis [29–31]. The type of optimization (maximum or minimum) and the preference function is set for each criterion. The method uses six types of preference functions. It uses pre-set criteria weights.

The methodology of the PROMETHEE method includes the calculation of the positive and negative outranking flow for each alternative.

The net outranking flow $\varphi(a_s)$ of alternative a_s is determined as the difference between the positive outranking flow and the negative outranking flow.

$$\varphi(a_s) = \varphi^+(a_s) - \varphi^-(a_s) \quad ; \quad s = 1, \dots, S \quad (24)$$

where: $\varphi^+(a_s)$ is the positive outranking flow for alternative a_s ; $\varphi^-(a_s)$ is the negative outranking flow for alternative a_s ; $s = 1, \dots, S$ is the number of alternatives (variant schemes).

For the net outranking flow to be valid:

$$\varphi(a_s) \in [-1; 1]; \quad \sum_{s=1}^S \varphi(a_s) = 0 \quad (25)$$

The ranking of the alternatives is based on the maximum value of the net outranking flows.

3.4.3. TOPSIS Method

The TOPSIS method is a distance based multi-criteria approach [32,33]. It uses pre-set criteria weights.

The alternatives are ranked on the basis of the highest performance score. It depends on two distances: the shortest geometric distance from the positive ideal solution and the farthest geometric distance from the negative ideal solution.

The performance score is determined as follows:

$$C_s = \frac{D_s^-}{D_s^+ + D_s^-}; 0 \leq C_s \leq 1 \quad (26)$$

$$D_s^+ = \sqrt{\sum_{t=1}^T (v_{st} - v_t^+)^2}; D_s^- = \sqrt{\sum_{t=1}^T (v_{st} - v_t^-)^2} \quad (27)$$

$$v_{st} = r_{st} \cdot w_t, \quad \sum_{t=1}^T w_t = 1, \quad w_t \in \{0,1\}, \quad (28)$$

$$r_{st} = \frac{x_{st}}{\sqrt{\sum_{s=1}^S x_{st}^2}}, s = 1, \dots, S; t = 1, \dots, T. \quad (29)$$

where: D_s^+ is the Euclidean distance from the ideal best solution; D_s^- is the Euclidean distance from the ideal worst solution; $v_t^+ = \min_s v_{st}$ is the ideal best solution; $v_t^- = \max_s v_{st}$ is the ideal worst solution; r_{st} are the elements of the normalized matrix; w_t are the weights of the criteria.

3.5. Stage 5. Aggregation

Finally, the results obtained through the SIMUS, PROMETHEE, and TOPSIS methods are aggregated using the PERT method. Thus, the results of all those methods are combined and the ranking of the variant schemes is obtained.

The aggregation procedure includes the following steps:

Step 1: Determination of the normalization scores of each variant of multi-criteria assessment.

$$NS_{sv} = \frac{SC_{sv}}{\sum_{s=1}^S SC_{sv}}. \quad (30)$$

where: S_{sv} is the score of s variant scheme by v multi-criteria assessment; $s = 1, \dots, S$ is the number of alternatives (variant schemes); $v = 1, \dots, V$ is the number of variants of multi-criteria assessment.

It is necessary for the scores of the multi-criteria assessments to be positive:

$$SC_{sv} \geq 0, \quad (31)$$

The PROMETHEE method has positive and negative values of the net outranking flows, which are the scores of multi-criteria assessments. For this reason, a transformation is carried out in order to determine the normalized scores, as follows:

$$\varphi(a_s)^* = \beta + \varphi(a_s); s = 1, \dots, S, \quad (32)$$

where: β is the correction parameter; $\varphi(s)$ is the net outranking flow for s variant scheme.

$$\beta = \text{integer} \left[\max_s \varphi(a_s) \right]. \quad (33)$$

Step 2. The PERT (program evaluation and review technique) method to determine the expected value

The PERT method uses three scores—optimistic, pessimistic, and most likely [34].

The aggregation value for each variant scheme is determined by applying the expected value used in the PERT method as follows:

$$PERT_s = \frac{\min SC_{sv} + 4 \frac{\sum_{v=1}^V SC_{sv}}{V} + \max SC_{sv}}{6}. \quad (34)$$

The best variant scheme is determined on the basis of the maximum value of the PERT assessment. Thus, the results of all examined multi-criteria approaches are considered in the final assessment.

4. Results

This study is addressed to the tram transportation in Sofia city, the capital of Bulgaria. The peculiarities of tram transportation in Sofia can be summarized as follows:

- In the city of Sofia, two autonomous tram networks operate with different gauges—standard (1435 mm) and narrow gauge (1009 mm). Currently, the total length of the network is over 75 km (150 km single track), there are around 340 stops, 3 operating tram depots, and over 300 trams. The network is served by 15 routes, 12 of which are on lines with a narrow gauge and 3 are on lines with a standard gauge.
- The tram network in Sofia is served by tram rolling stock of different types. The narrow-gauge tram network is served by modernized and reconstructed trams manufactured at the former tram factory in Sofia, second-hand trams manufactured in Switzerland, trams produced in the Czech Republic, and new trams delivered from Poland (Pesa Swing- PESA Bydgoszcz SA.). Trams produced in the Czech Republic and Germany run along the standard tram network. The different types of trams are assigned to different depots. In addition, the zero mileage (the distance that trams travel from the depots to the end route and back) is not rationally distributed.
- The current tram route scheme in Sofia has not been updated for decades. The possibilities of compiling diverse tram route schemes are limited due to the specificities of tram transportation regarding infrastructure limitations. The studied variant schemes were developed entirely on the basis of the current situation with the tram infrastructure (railway, catenary network, and technical and turnover points) and rolling stock. Optimization of the transportation scheme for the trams would improve the overall public transportation service and increase revenue.

This research was carried out under the following limitations:

- Three types of trams rolling stock are considered: Pesa Swing (PT), single tram cars (ST), and double-articulated tram cars (AT). The first type refers to new trams that have operated on Sofia tram networks since 2020, when they were delivered from Poland. They run along the narrow-gauge tram network. The last two types include modernized and reconstructed trams manufactured at the former tram factory in Sofia, second-hand trams manufactured in Switzerland, and trams produced in the Czech Republic.
- The day is divided into the following periods: morning peak (MP), off-peak periods during the day (DP), evening peak (EP), and late evening (NP).
- The total intervals (when duplicating tram lines) in peak hours should not exceed 8 min—otherwise the line would be unattractive; the intervals on any line at peak hours should not be more than 10 min for the same reason.
- The passenger flows were determined on the basis of a census in the month of May 2023. Passenger flows are used in the compilation of the variant schemes, as they show the most congested and least congested sections of the network, as well as the change in passenger flows when passing from section to section.
- This study does not include a determination of the number of drivers required to operate the tramcars.

- Each tram line should have a turnaround point where there is an opportunity for breaks for the Whatmans from the tram line.

Five tram transportation schemes, which differ slightly in the tram routes, have been considered. Figure 3 illustrates the variant schemes.

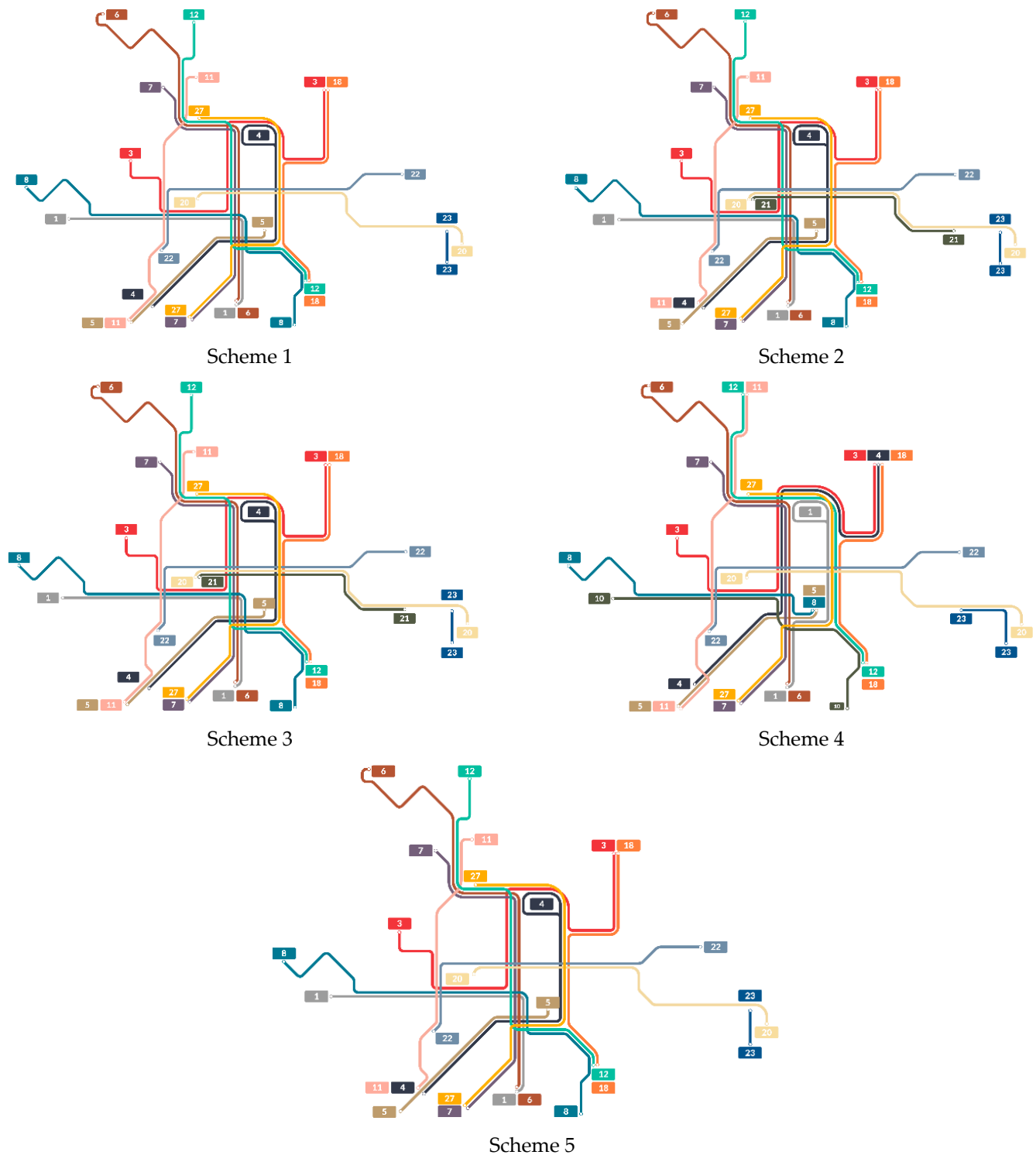


Figure 3. Tram transportation variant schemes.

- Scheme 1. The tram routes are as is in the existing tram scheme.
- Scheme 2–Scheme 5. There are changes in the routes of tram lines 1, 4, 8, 10, 11, 12, 21, and 23 compared to the existing scheme. Scheme 2 and Scheme 3 do not include

tram route 27 or tram route 10. Scheme 4 and Scheme 5 do not have tram route 21 or tram route 10. In Scheme 4, compared to Scheme 5, there are no tram lines 1 or 3. Tram route 11 in these schemes is of a different length; tram route 1, tram route 8, and tram route 23 are with changed lengths compared to the current situation.

The tram routes are shown in different colors. The number of tram lines corresponds to the actual tram lines in Sofia. When drawing up the variant schemes, the passenger flows in the tram network, infrastructure limitations, and duplication of tram routes in the final sections of the tram lines are considered.

4.2. Stage 2. Two Step Optimization Model

4.2.1. Nonlinear Optimization Model

The first step of the two-step optimization model includes an optimization of the tram transportation plan in view of the number and types of trams for each tram route for each variant scheme. For each k section of the tram network, the theoretical minimum intervals were calculated by applying Equation (11) in order to determine the parameters of the optimization model. Table 1 shows the values of the minimum and maximum intervals, as well as the values of the determined intervals as a result of the optimization for each variant scheme. The results for the correction coefficients by periods of the day are shown in Table 2.

Table 1. Minimum intervals and maximum intervals obtained by the optimization model.

Interval	Tram Route															
	1	3	4	5	6	7	8	10	11	12	18	20	21	22	23	27
min	8	6	6	6	8	8	4	4	6	6	6	3	5	3	10	8
max	10	8	10	8	8	10	8	8	8	8	10	8	10	8	25	10
Scheme 1	10	7.5	10	7.9	8	8.8	7.5	8	8	7.7	9.3	7.7	-	6.8	25	8.6
Scheme 2	7.7	7.5	10	6.9	8	9	8	-	7	7.7	9.3	8	9.4	6.8	10	-
Scheme 3	7.7	8.2	10	7.9	8	9	8	-	8	7.7	9.3	8	9.4	6.4	10	-
Scheme 4	-	-	10	8	8	8.2	8	-	7	7.7	7.7	8	-	5	10	7.9
Scheme 5	7.7	8.2	9	7.9	8	9	8	-	8	7.7	7.7	8	-	6.4	10	7.9

Table 2. Correction coefficients by periods of the day.

Scheme	Period	Tram Route															
		1	3	4	5	6	7	8	10	11	12	18	20	21	22	23	27
Correction coefficient	MP	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	DP	0.67	0.75	0.71	0.67	0.67	0.63	0.82	0.82	0.60	0.78	0.80	1.00	0.80	0.80	0.75	0.63
	EP	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	NP	0.33	0.33	0.43	0.33	0.33	0.25	0.32	0.32	0.25	0.33	0.36	0.49	0.30	0.30	0.30	0.23

Figure 4 illustrates the values of the optimization function for each variant scheme—the energy consumption for a day.

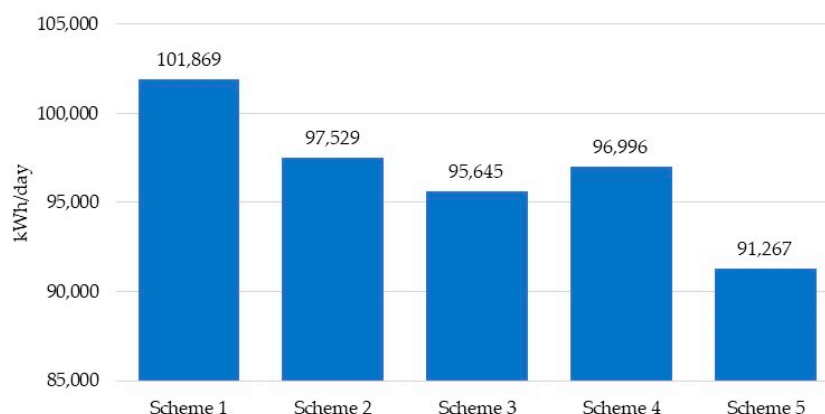


Figure 4. Energy consumption for a day.

Table 3 shows the results of the nonlinear optimization model—the number of trams by type for each route for peak periods. Scheme 4 has the least number of trams in peak periods. It could be seen that Scheme 5 has the minimum value for energy consumption. The number of trams for the other periods of the day were determined by correction coefficients according to Equation (7).

Table 3. Number of trams by type and route for peak period.

Tram Route	Scheme 1				Scheme 2				Scheme 3				Scheme 4				Scheme 5			
	ST	AT	PT	Total	ST	AT	PT	Total	ST	AT	PT	Total	ST	AT	PT	Total	ST	AT	PT	Total
1	7	0	0	7	0	0	9	9	9	0	0	9	-	-	-	-	0	0	9	9
3	4	1	7	12	0	0	11	11	11	0	0	11	-	-	-	-	0	0	11	11
4	0	0	10	10	0	0	10	10	0	0	10	10	0	0	10	10	0	0	11	11
5	3	0	4	7	5	0	3	8	0	0	7	7	4	0	4	8	5	0	2	7
6	9	0	6	15	14	0	1	15	0	0	15	15	12	1	2	15	15	0	0	15
7	0	0	11	11	0	1	9	10	0	0	10	10	0	0	11	11	0	0	10	10
8	7	0	3	10	16	0	2	18	16	0	2	18	12	1	5	18	18	0	0	18
10	0	4	11	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	14	0	0	14	11	0	1	12	0	0	10	10	11	1	0	12	9	0	1	10
12	12	0	1	13	4	0	9	13	13	0	0	13	13	0	0	13	2	0	11	13
18	0	4	3	7	6	0	1	7	7	0	0	7	2	0	5	7	7	0	0	7
27	0	0	11	11	0	1	11	12	0	0	12	12	1	0	11	12	0	0	12	12
20	15	0	0	15	14	0	0	14	14	0	0	14	14	0	0	14	14	0	0	14
21	-	-	-	-	9	0	0	9	9	0	0	9	-	-	-	-	-	-	-	9
22	16	1	0	17	16	1	0	17	18	0	0	18	23	0	0	23	18	0	0	18
23	0	2	0	2	0	2	0	2	0	2	0	2	0	2	0	2	0	2	0	2
Total	87	12	67	166	95	5	67	167	97	2	66	165	92	5	48	145	88	2	67	157

Pesa Swing (PT), single tram cars (ST), and double-articulated tram cars (AT).

4.2.2. Linear Optimization Model

The second step of the two-step optimization model includes optimization of the distribution of trams by depots for each optimized variant scheme according to the number and types of trams. There are three tram depots serving the tram network in Sofia: tram depot “Krasna Poliana” (Depot K), tram depot “Banishora” (Depot B), and tram depot “Iskar”. As noted above, there are two types of tram networks in Sofia, narrow-gauge and standard. Tram depot “Iskar” is specialized for servicing trams with standard gauge. The other two depots only serve narrow-gauge trams. This research deals with the

optimization of tram rolling stock distribution for narrow-gauge trams. Table 4 shows the tram zero mileage.

Table 4. Tram zero mileage by depots.

	Tram Route	1	3	4	5	6	7	8	10	11	12	18	27
Scheme 1	Depot K, km	5.3	2.3	2.8	2.8	0.7	4.7	2.2	2.2	0	4.7	4.6	4.7
	Depot B, km	1.2	1.4	1.9	3.1	0.7	0.7	2	2	0	0.7	1.9	0.7
Scheme 2–5	Depot K, km	2.2	2.3	2.8	2.8	0.7	4.7	2.2	2.2	0	4.7	4.6	4.7
	Depot B, km	2	1.4	1.2	3.1	0.7	0.7	2	2	0	0.7	1.9	0.7

Table 5 shows the results of the linear optimization models for each variant scheme. It can be seen that Scheme 3 has the minimum (zero) mileage. Scheme 5, which is optimal in terms of the criterion minimum energy consumption, holds the second position regarding this criterion. The last row in Table 5 shows tram zero mileage for the year, considering the number of the trams on the routes.

Table 5. Tram zero mileage by variant schemes.

Scheme	Current	1	2	3	4	5
Tram mileage (km)	216	164	153	150	154	152
Tram mileage (km/year)	184,285	128,062	114,069	118,913	114,169	119,013

4.3. Stage 3. Multi-Criteria Analysis for Ranking Variant Schemes

The results of the two-step optimization for each variant scheme were used to form a decision matrix for the multi-criteria model. In this stage the SIMUS method was used for ranking the variant schemes. Table 6 shows the criteria values for each variant scheme. The last column of the table shows the type of optimization for each criterion. The decision matrix is normalized by applying the sum of the row method. Table 7 shows the normalized decision matrix. The last columns of the table give the type of operator and the right-hand side of the restrictive conditions. Table 8 is the basis for constructing the linear optimization models. The number of optimization models is equal to the number of criteria. For example, when the first row is taken as an optimization function, the other rows show the restrictive conditions. The results of the linear optimization are given in an efficient results matrix (ERM) (Table 8). Table 8 consists of the efficient results matrix (ERM) and the value of the objective function (LHS) received through the SIMUS optimization. The objective is satisfied 100% when there is equality of the LHS and RHS. In the other cases, the results have to be analyzed.

Table 6. Initial Decision Matrix with the SIMUS method.

Criteria	Dimension	Scheme 1	Scheme 2	Scheme 3	Scheme 4	Scheme 5	Type
C1	km	2030	2023	1971	2009	1885	min
C2	km	9,960,364	9,862,495	9,688,950	9,762,590	9,278,747	min
C3	seats/day	33,327,813	34,828,227	33,852,137	34,300,828	32,963,862	max
C4	min	1045	1019	1008	961	981	min
C5	km	164	153	150	154	152	min
C6	trams/day	166	167	165	145	157	max
C7	km	164.24	162.66	161.52	163.52	162.31	max
C8	euro/year	46,343,144	45,887,785	45,080,322	45,422,949	43,171,749	min
C9	coef.	0.43	0.38	0.41	0.40	0.39	max
C10	coef.	0.39	0.37	0.41	0.38	0.39	max
C11	kgCO ₂ e/kWh	46,350	44,376	43,519	44,133	41,527	min

Table 7. Normalized Decision Matrix.

Criteria/ Targets	Alternatives					Type	Operator	RHS Val- ues
	Scheme 1	Scheme 2	Scheme 3	Scheme 4	Scheme 5			
C1	0.20	0.20	0.20	0.20	0.19	min	$\geq$	0.19
C2	0.21	0.20	0.20	0.20	0.19	min	$\geq$	0.19
C3	0.20	0.21	0.20	0.20	0.19	max	$\leq$	0.21
C4	0.21	0.20	0.20	0.19	0.20	min	$\geq$	0.19
C5	0.21	0.20	0.19	0.20	0.20	min	$\geq$	0.19
C6	0.20	0.20	0.20	0.20	0.19	max	$\leq$	0.20
C7	0.20	0.20	0.20	0.20	0.20	max	$\leq$	0.20
C8	0.21	0.20	0.20	0.20	0.19	min	$\geq$	0.19
C9	0.21	0.19	0.20	0.20	0.19	max	$\leq$	0.21
C10	0.20	0.19	0.21	0.20	0.20	max	$\leq$	0.21
C11	0.21	0.20	0.20	0.20	0.19	min	$\geq$	0.19

Table 8. Efficient Results Matrix (ERM).

Objective	Scheme					Objective Function Val- ues Per Target (LHS)
	1	2	3	4	5	
Z1	0.00	0.00	0.00	0.00	1.00	0.19
Z2	0.00	0.00	0.00	0.00	1.00	0.19
Z3	0.00	0.11	0.00	0.00	0.95	0.21
Z4	0.00	0.00	0.00	0.99	0.00	0.19
Z5	0.00	0.00	1.00	0.00	0.00	0.19
Z6	0.75	0.28	0.00	0.00	0.00	0.21
Z7	0.93	0.00	0.00	0.00	0.00	0.19
Z8	0.00	0.00	0.00	0.00	1.00	0.19
Z9	1.01	0.00	0.00	0.00	0.00	0.22
Z10	0.00	0.00	1.01	0.00	0.00	0.21
Z11	0.00	0.00	0.00	0.00	1.00	0.19

Objective Z6 refers to the frequency of trams, which naturally must be a maximum. The computed value for LHS (0.21) is greater than the maximum value established for RHS (0.20) in Table 7. This means that the frequency is higher than expected. This is positive for passengers, but it is related to a strain on operating costs. Objective Z9 refers to the low-floor trams for the convenience of mothers with prams and people with reduced mobility. The computed value for LHS (0.22) is greater than the maximum value established for RHS (0.21) in Table 7. This is positive for the convenience of passengers, but it is related to a strain on operating costs. Objective Z7 refers to the average transportation distance. This objective shows very good performance for the tram routes whose value of 0.20 is higher than the RHS value of 0.19.

The final part of the SIMUS method includes ranking the alternatives. For this purpose, the efficient results matrix is normalized. The weighted sum method is applied to the normalized matrix (NERM). Table 9 shows the normalized efficient results matrix. The second part of the table shows the SIMUS score for alternatives. First, the sum of the total of the columns is calculated. The participation factor shows the number of non-zero values in each column. The participation factor is normalized by dividing with the number of criteria. The SIMUS score is determined by multiplying the sum of the columns (SC) with the normalized participation factor (NPF). The maximum value of the score shows the best variant scheme. It can be seen that Scheme 5 is the most appropriate one. The second part of Table 9 shows the SIMUS weights. These are the weights of objectives determined

when considering the results of the optimization models. It could be concluded that the criteria for the weights are almost equal.

Table 9. Normalized Efficient Results Matrix (NERM).

Objective	NERM					SIMUS Weights	
	Scheme 1	Scheme 2	Scheme 3	Scheme 4	Scheme 5	Max	SIMUS Weight
Z1	0.00	0.00	0.00	0.00	1.00	1.00	0.094
Z2	0.00	0.00	0.00	0.00	1.00	1.00	0.094
Z3	0.00	0.10	0.00	0.00	0.90	0.90	0.084
Z4	0.00	0.00	0.00	1.00	0.00	1.00	0.094
Z5	0.00	0.00	1.00	0.00	0.00	1.00	0.094
Z6	0.73	0.27	0.00	0.00	0.00	0.73	0.069
Z7	1.00	0.00	0.00	0.00	0.00	1.00	0.094
Z8	0.00	0.00	0.00	0.00	1.00	1.00	0.094
Z9	1.00	0.00	0.00	0.00	0.00	1.00	0.094
Z10	0.00	0.00	1.00	0.00	0.00	1.00	0.094
Z11	0.00	0.00	0.00	0.00	1.00	1.00	0.094
SC	2.73	0.37	2.00	1.00	4.90	10.63	1.000
PF	3	2	2	1	5	-	-
NPF	0.27	0.18	0.18	0.09	0.45	-	-
SC × NPF	0.74	0.07	0.36	0.09	2.23	-	-
ERM Ranking	Scheme 5–Scheme 1–Scheme 3–Scheme 4–Scheme 2						

SC—Sum of Column; PF—Participation Factor; NPF—Normalized Participation Factor; SC × NPF—Final Result; Number of targets = 11.

4.4. Stage 4. Verification

The PROMETHEE and TOPSIS methods were used to verify the results given by the SIMUS method. Two types of weights were used for the ranking with both methods. The first is the Shannon entropy method, and the second is the weights of the objectives given by the SIMUS method.

The Shannon entropy method was used to determine criteria weights. Table 10 shows the results of Equation (23). It can be seen that the most important criteria are (C9) percentage of kilometers covered by low-floor trams, (C11) carbon dioxide emissions, CO₂ (kgCO₂e/kWh), (C5) zero tram mileage (km), and (C10) percentage of kilometers traveled by trams equipped with air conditioning. Criterion (C7), the average travel distance (km), is of small importance.

Figure 5 demonstrates the comparison between the Shannon entropy weights and the SIMUS objective weights. The difference in weights is due to the way the criteria are defined. The SIMUS estimates are a result of the individual optimizations. The Shannon entropy estimates are a result of the information from the criteria data.

Table 10. Parameters of the Shannon entropy method.

$-p_{st} \ln p_{st}$	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
Scheme 1	0.325	0.325	0.320	0.327	0.329	0.323	0.323	0.325	0.331	0.321	0.328
Scheme 2	0.324	0.324	0.325	0.324	0.321	0.324	0.322	0.324	0.316	0.317	0.323
Scheme 3	0.321	0.322	0.322	0.323	0.318	0.323	0.321	0.322	0.323	0.327	0.321
Scheme 4	0.323	0.323	0.323	0.317	0.321	0.323	0.322	0.323	0.320	0.318	0.322
Scheme 5	0.316	0.316	0.319	0.319	0.320	0.317	0.321	0.316	0.319	0.324	0.315
$-\sum_{s=1}^S p_{st} \ln p_{st}$	1.609	1.609	1.609	1.609	1.609	1.609	1.609	1.609	1.609	1.609	1.609
E_i	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
w_j^E	0.086	0.069	0.045	0.100	0.115	0.056	0.004	0.069	0.200	0.110	0.146

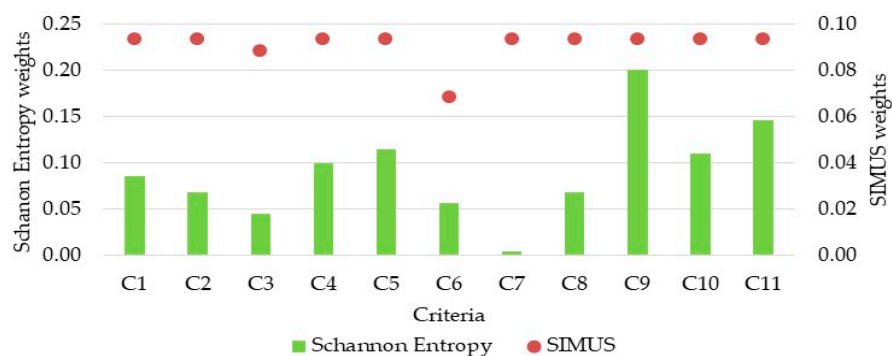


Figure 5. Comparison between the Shannon entropy weights and the SIMUS objective weights.

Figure 6 illustrates the results obtained by using the PROMETHEE method. The first part of Figure 6a,b shows the ranking of the variant schemes, the second part shows the criteria weights.

There are small differences between the results obtained by both approaches (Shannon entropy and SIMUS objectives). Scheme 5 is the best scheme under the PROMETHEE–Shannon entropy ranking. Scheme 3 and Scheme 5 have close results under the PROMETHEE–SIMUS ranking. Scheme 3 has a small lead over Scheme 5.

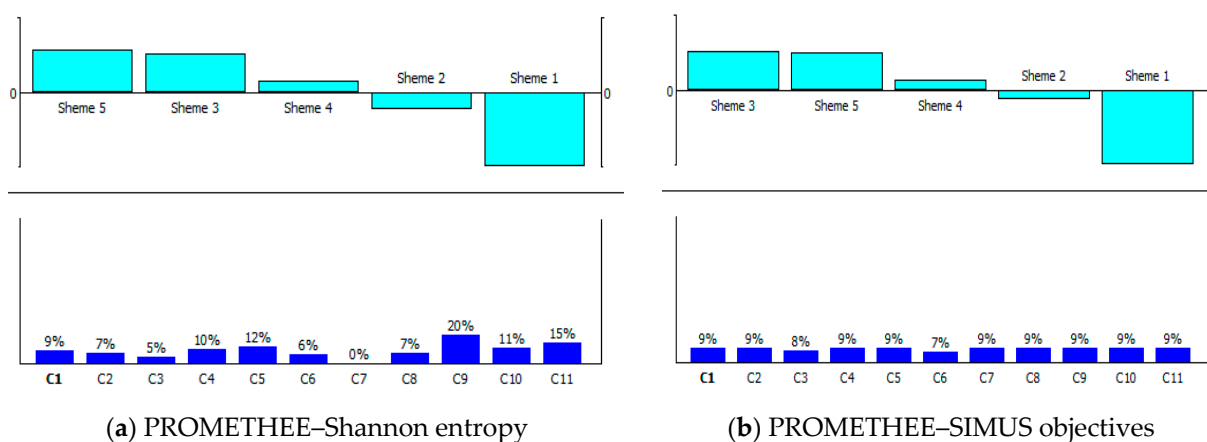


Figure 6. PROMETHEE Ranking.

The second multi-criteria method used for verification of the results in this research is the TOPSIS method. Table 11 shows the parameters and results when using the TOPSIS method. Scheme 3 is the best scheme under the PROMETHEE–Shannon entropy ranking. Scheme 5 is the best scheme under the PROMETHEE–SIMUS objectives.

It can be concluded that both the PROMETHEE and TOPSIS multi-criteria methods give similar results to those of the SIMUS method, which confirms the ranking and selection of the most appropriate variant scheme.

Table 11. TOPSIS Ranking.

	TOPSIS–Shannon entropy				TOPSIS–SIMUS Objectives			
	di+	di–	ci	Rank	di+	di–	ci	Rank
Scheme 1	0.011	0.011	0.514	3	0.009	0.006	0.380	4
Scheme 2	0.013	0.005	0.282	5	0.009	0.005	0.355	5
Scheme 3	0.008	0.009	0.551	1	0.005	0.007	0.576	2
Scheme 4	0.010	0.007	0.395	4	0.007	0.006	0.449	3
Scheme 5	0.009	0.010	0.533	2	0.005	0.009	0.621	1

4.5. Stage 5. Aggregation

The results obtained by using the SIMUS, PROMETHEE, and TOPSIS methods are aggregated using the PERT method. Table 12 shows the results.

Table 12. Aggregation.

Scheme	SIMUS			PROMETHEE– Shannon entropy				TOPSIS– Shannon entropy			PROMETHEE– SIMUS				TOPSIS– SIMUS			Aggregation	
	Score	NS	Rank	ϕ	ϕ^*	NS	Rank	Score	NS	Rank	ϕ	ϕ^*	NS	Rank	Score	NS	Rank	Score	Rank
1	0.74	0.21	2	−0.60	0.40	0.08	5	0.51	0.23	3	−0.65	0.35	0.07	5	0.38	0.16	4	0.15	4
2	0.07	0.02	5	−0.14	0.86	0.17	4	0.28	0.12	5	−0.09	0.91	0.18	4	0.36	0.15	5	0.12	5
3	0.36	0.10	3	0.30	1.30	0.26	2	0.55	0.24	1	0.33	1.33	0.27	1	0.58	0.24	2	0.21	2
4	0.09	0.03	4	0.09	1.09	0.22	3	0.40	0.17	4	0.08	1.08	0.22	3	0.45	0.19	3	0.15	3
5	2.23	0.64	1	0.34	1.34	0.27	1	0.53	0.23	2	0.32	1.32	0.26	2	0.62	0.26	1	0.37	1
sum	3.49	1		0.000	5.00	1.00		2.27	1.000		0.000	5.000	1		2.39	1.000		1.00	

Normalized Score—NS.

Scheme 5 and Scheme 3 have close results when using the PROMETHEE method and the TOPSIS method. It can be summarized that the normalized scores and the PERT method give similar results. Scheme 5 is the most appropriate scheme.

5. Discussion

The SIMUS method uses linear optimization. This allows us to perform a post-optimization analysis, i.e., a sensitivity analysis.

Linear optimization makes it possible also to determine the allowable increase and the allowable decrease of the objective coefficients. In this way, considering the results of the linear optimization models, the limits of variation of the input data can be determined at which the ranking of the alternatives is preserved.

Table 13 presents the results of the upper and lower values of the data obtained through sensitivity analysis. Each objective was investigated regarding the limits of the objective coefficients. The allowable range of variations of the criteria was determined. There are some values that are equal to zero or are unlimited. These values are only theoretical, and they are larger or smaller than practically possible. The limits of the criteria depend on the infrastructure conditions and the type of rolling stock.

The upper limits for criteria $C1$, $C2$, $C3$, $C6$, and $C7$ depend on the length and the maximum capacity of tram infrastructure. The upper value of the $C4$ criterion depends on tram infrastructure. The upper value of the $C5$ criterion depends on tram infrastructure and the location of the tram depots. The upper limits for the $C8$ criterion depend on the capabilities of the tram operator, the capacity of the tram lines, and the available rolling stock. The upper limits for criteria $C9$ and $C10$ are 1, as these criteria are the coefficients.

The upper limits for the $C11$ criterion depend on the type of trams and their energy consumption, the available rolling stock, and the capacity of the tram lines. The results presented in Table 13 show that most criteria have large limits of variation. Scheme 5 is close to the upper value of the $C1$ criterion. It is the same with the $C6$ and $C9$ criteria. This means that Scheme 5 could improve the intensity ($C6$) of tram transportation (proposed—157 tram cars, upper—159 tram cars). There is no possibility of increasing the mileage in peak periods. The emissions of CO_2 are close to the lower limit. The operating costs of the $C8$ criterion hold little possibility for variation.

The most suitable tram transportation scheme determined is valid not only for the specified input data, but for the input data change intervals determined by the SIMUS method.

Table 13. Sensitivity analysis SIMUS.

		Scheme 1	Scheme 2	Scheme 3	Scheme 4	Scheme 5
C1	U	∞	∞	∞	∞	1888
		2030	2023	1971	2009	1885
	L	2023	2004	1968	1983	0
C2	U	∞	∞	∞	∞	∞
		9,960,364	9,862,495	9,688,950	9,762,590	9,278,747
	L	0	0	0	0	0
C3	U	34,736,222	35,063,471	34,644,218	34,486,831	36,710,834
		33,327,813	34,828,227	33,852,137	34,300,828	32,963,862
	L	0	34,549,024	0	0	32,742,704
C4	U	∞	∞	∞	981	∞
		1045	1019	1008	961	981
	L	961	961	961	0	961
C5	U	∞	∞	152	∞	∞
		164	153	150	154	152
	L	150	150	0	150	150
C6	U	168	171	167	166	159
		166	167	165	165	157
	L	165	157	0	164	0
C7	U	164	∞	162	∞	∞
		164	163	162	164	162
	L	162	163	148	163	156
C8	U	∞	∞	45,422,949	∞	45,080,322
		46,343,144	45,887,785	45,080,322	45,422,949	43,171,749
	L	45,080,322	45,080,322	43,171,749	45,080,322	42,826,306
C9	U	1	0.43	0.43	0.43	0.41
		0.43	0.38	0.41	0.40	0.39
	L	0.41	0.00	0.00	0.00	0.00
C10	U	0.41	0.41	0.41	0.40	1
		0.39	0.37	0.41	0.38	0.39
	L	0	0	0	0	0
C11	U	∞	∞	43,713	∞	43,518
		46,350	44,376	43,519	44,133	41,527
	L	43,518	43,519	41,528	43,518	41,343

6. Conclusions

This research proposes an integrated approach based on optimization methods and multi-criteria analysis to develop a sustainability transportation plan for trams in the city. Two-step optimization, including nonlinear and linear optimization, was carried out to determine the number and the type of trams for each route, and the distribution of trams by depots. Multi-criteria analysis was made to select the most appropriate tram transportation scheme. The SIMUS method was selected as the main tool for making decisions. The results were verified on the basis of the PROMETHEE, TOPSIS, and Shannon entropy methods, which are outranking and distance-based methods. An aggregation approach for combining the results of the different multi-criteria methods was presented. Three methods of multi-criteria analysis, based on the SIMUS, PROMETHEE, and TOPSIS

methods, were aggregated to assess the variant schemes, taking into account the quantitative and qualitative criteria that are important for the carrier and the passengers. Eleven criteria for assessment of the tram transportation plan were defined, including technological and economic parameters, ecology, and comfort of travel.

The proposed methodology makes it possible to increase the sustainability of the tram transportation plan for economic, technological, social, and environmental benefits.

The methodology was applied for tram transportation in the capital of Bulgaria, the city of Sofia. Five variant schemes were investigated. An appropriate optimized tram transportation plan was proposed by employing the newly elaborated methodology.

The weights of the criteria used were calculated. Two approaches were applied, one based on the SIMUS objectives and the other using the Shannon entropy method. It was found that, based on the Shannon entropy method, the most important criteria are (C9) percentage of kilometers covered by low-floor trams, (C11) carbon dioxide emissions, CO₂ (kgCO_{2e}/kWh), (C5) tram zero mileage (km), and (C10) percentage of kilometers covered by trams equipped with air conditioning. The importance of the criteria determined through the SIMUS objectives is close. A sensitivity analysis was performed to establish the limits of criteria weights and the values of data where the ranking results are preserved.

The novelty of this research is the proposed method for selecting a tram transportation scheme, including two-stage optimization for determining the number and type of rolling stock by routes and distribution by depots, with an aggregated multi-criteria approach.

This research is a contribution to the scientific research for tram operators and for city transportation in Sofia. It proposes a novel methodological approach which integrates optimization and MCDM methods to choose the most appropriate tram transportation scheme.

Regarding scientific research, the new integrated approach permits us to select the most appropriate tram transportation organization scheme by examining pre-optimized schemes in terms of electricity consumption and depot distribution.

Regarding tram operators, the proposed methodology allows for analysis and comparison of previously optimized schemes of organization of tram transportation. The selected optimization criterion for determining the number of trams by type and by routes is the minimum electricity consumption for tram movement. In this way, energy-efficient schemes are studied. Electricity consumption is directly related to the amount of CO₂ released during the production of electricity by power plants. This determines the ecological nature of this criterion and the benefit for society of choosing an energy-efficient scheme. The integration of optimization models with multi-criteria analysis methods allows for the selection of the most appropriate scheme according to a set of criteria that are important not only for carriers but for tram transportation users. Thus, the goal is to satisfy the needs of both parties.

Regarding the case of Sofia, the research makes it possible to determine an improved scheme for tram transportation. It permits an increase the benefits as follows: for the users, in terms of service frequency, which increases by 13%; for the carrier, by reducing the zero mileage of the rolling stock; for the environment, by reducing carbon dioxide pollution, which decreases by 11%. The operating costs increase by 16% as compared to the current situation, the energy consumption for a day decreases by 11%.

The advantages of this study can be summarized as follows:

- The parameters of the tram transportation schemes are determined based on two-step optimization. The number and type of trams on the routes are calculated through optimization according to the criterion of minimum electricity consumption. This criterion is related to both operating costs and environmental pollution with

harmful emissions from electricity production. The distribution of trams by depots is also optimized.

- The optimized schemes for the tram transportation system are compared by using additional economic, technological, and social criteria related to the transportation process.
- The most appropriate scheme of tram transportation is determined by applying and aggregating three, different by their nature, multi-criteria methods. This allows the influence of different decision-making approaches to be considered.

Author Contributions: Conceptualization, S.S.; methodology, S.S.; software, S.S. and I.P.; validation, S.S.; formal analysis, S.S.; investigation, S.S. and I.P.; data curation, I.P.; writing—original draft preparation, S.S.; writing—review and editing, S.S.; supervision, S.S. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Bulgarian National Science Fund—BNSF of the Ministry of Education and Science of the Republic of Bulgaria [project number No. KP-06-H77/11 of 14 December 2023, “Modeling and development of a complex system for environmental and energy efficiency of urban transport”].

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in this study are included in the article, further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Žak, J.; Kurek, D. Multiple Criteria Evaluation of Trams based on Customers' Specifications (Expectations) in Selected Countries. *Transp. Res. Procedia* **2020**, *47*, 696–703.
2. Djouani, I.; Dehimi, S.; Redjem, A. Evaluation of the efficiency and quality of the tram route of Setif city, Algeria: Combining AHP and GIS approaches. *J. Geogr. Inst. Jovan Cvijic SAsA* **2022**, *72*, 85–102. <https://doi.org/10.2298/ijgi2201085d>.
3. Omer, G. Evaluation of the selection of proper metro and tram vehicle for urban transportation by using a novel integrated MCDM approach. *Sci. Prog.* **2021**, *104*, 0036850420950120.
4. He, J.; Xu, Y.; Li, Y.; Yang, J.; Long, S. An optimization model of tram timetables considering various signal priority strategies. *Sci. Rep.* **2022**, *12*, 16564. <https://doi.org/10.1038/s41598-022-19762-9>.
5. Alkubaisi, M.I.T. Predefined Evaluating Criteria to Select the Best Tramway Route. *J. Traffic Logist. Eng.* **2014**, *2*, 211–217. <https://doi.org/10.12720/jtle.2.3.211-217>.
6. Borghetti, F.; Colombo, C.G.; Longo, M.; Mazzoncini, R.; Somaschini, C. Development of a new urban line with innovative trams. *Urban Marit. Transp.* **2021**, *204*, 167–178.
7. Kulesz, B.; Sikora, A. Comparison of different tram cars in Poland basing on drive type, rated power and energy consumption. *MATEC Web Conf.* **2018**, *180*, 02006. <https://doi.org/10.1051/mateconf/201818002006>.
8. Cano, A.; Arjvalo, P.; Benavides, D.; Jurado, F. Sustainable tramway, techno-economic analysis and environmental effects in an urban public transport. A comparative study. *Sustain. Energy Grids Netw.* **2021**, *26*, 100462. <https://doi.org/10.1016/j.segan.2021.100462>.
9. Zhao, X.; Sun, Q.; Liu, L.; Ding, Y.; Shi, R. Analysis on Operational Cost of Modern Tram. In Proceedings of the 5th International Conference on Civil Engineering and Transportation (ICCET 2015), Guangzhou, China, 28–29 November 2015.
10. Xing, Z.; Zhu, J.; Zhang, Z.; Qin, Y.; Jia, L. Energy consumption optimization of tramway operation based on improved PSO algorithm. *Energy* **2022**, *258*, 124848. <https://doi.org/10.1016/j.energy.2022.124848>.
11. Görçün, F.; Pamucar, D.; Küçükönder, H. Selection of tramcars for sustainable urban transportation by using the modified WASPAS approach based on Heronian operators. *Appl. Soft Comput.* **2024**, *151*, 111127. <https://doi.org/10.1016/j.asoc.2023.111127>.
12. Canbulut, G.; Köse, E.; Arik, O.A. Public transportation vehicle selection by the grey relational analysis method. *Public Transp.* **2022**, *14*, 367–384. <https://doi.org/10.1007/s12469-021-00271-3>.

13. Janiak, M.K.; Zak, J. Multiple criteria evaluation of different redesign variants of the public tram system. *Transp. Res. Procedia* **2014**, *3*, 690–699. <https://doi.org/10.1016/j.trpro.2014.10.048>.
14. Kiciński, M.; Solecka, K. Application of MCDA/MCDM methods for an integrated urban public transportation system—Case study, city of Cracow. *Arch. Transp.* **2018**, *46*, 71–84. <https://doi.org/10.5604/01.3001.0012.2107>.
15. Rydlewski, M.; Tubis, A. Criteria for Assessing the Safety and Functionality of Tram Stops. *Sustainability* **2022**, *14*, 13162. <https://doi.org/10.3390/su142013162>.
16. Dudek, M.; Solecka, K.; Richter, M. A multi-criteria appraisal of the selection of means of urban passenger transport using the Electre and AHP methods. *Tech. Trans.* **2018**, *6*, 79–93. <https://doi.org/10.4467/2353737xct.18.088.8693>.
17. Budzynski, M.; Tubis, A.; Jamroz, K. Identifying Selected Tram Transport Risks. *IOP Conf. Series Mater. Sci. Eng.* **2019**, *603*, 042053. <https://doi.org/10.1088/1757-899x/603/4/042053>.
18. Megna, G.; Bracciali, A. Technical Comparison of Commercially Available Trams and Review of Standardization Frame and Design Principles. *Urban Rail Transit* **2022**, *8*, 16–31. <https://doi.org/10.1007/s40864-021-00163-6>.
19. Skiba, M. A selection analysis of solutions for temporary tram traffic organisation employed in wroclaw. *Transp. Probl.* **2023**, *18*, 169–178. <https://doi.org/10.20858/tp.2023.18.1.14>.
20. Guerrieri, M. Catenary-Free Tramway Systems: Functional and Cost-Benefit Analysis for a Metropolitan Area. *Urban Rail Transit* **2019**, *5*, 289–309. <https://doi.org/10.1007/s40864-019-00118-y>.
21. Czerepicky, A.; Krukowicz, T.; Górka, A.; Szustek, J. Traffic Light Priority for Trams in Warsaw as a Tool for Transport Policy and Reduction of Energy Consumption. *Sustainability* **2021**, *13*, 4180. <https://doi.org/10.3390/su13084180>.
22. He, J.; Li, Y.; Long, S.; Xu, Y.; Chen, J. Energy-efficient tram speed trajectory optimization considering the influence of the traffic light. *Front. Energy Res.* **2022**, *10*, 963275. <https://doi.org/10.3389/fenrg.2022.963275>.
23. Pavkova, K.; Currie, G.; Delbosc, A.; Sarvi, M. Selecting tram links for priority treatments—The Lorenz Curve approach. *J. Transp. Geogr.* **2016**, *55*, 101–109. <https://doi.org/10.1016/j.jtrangeo.2016.07.011>.
24. Lasdon, L.S.; Waren, A.D.; Jain, A.; Ratner, M. Design and Testing of a Generalized Reduced Gradient Code for Nonlinear Programming. *ACM Trans. Math. Softw.* **1978**, *4*, 34–50. <https://doi.org/10.1145/355769.355773>.
25. Chapra, S.; Canale, R. *Numerical Methods for Engineers*, 6th ed.; McGraw-Hill: New York, NY, USA, 2009; p. 968.
26. Greenhouse Gas Emission Intensity of Electricity Generation in Europe, European Environment Agency. 14 June 2024. Available online: <https://www.eea.europa.eu/en/analysis/indicators/greenhouse-gas-emission-intensity-of-1> (accessed on 1 July 2024).
27. Munier, N. *A Strategy for Using Multi? Criteria Analysis in Decision? Making*; Springer: Berlin/Heidelberg, Germany, 2011.
28. Hwang, C.L.; Yoon, K. *Multiple Attribute Decision Making: Methods and Applications, a State-of-the-Art Survey*; Springer: Berlin, Germany, 1981.
29. Brans, J.P.; Mareschal, B.; Vincke, P. PROMETHEE: A new family of outranking methods in MCDM. In *Operational Research'84 (IFORS'84)*; Brans J.P., Ed.; Springer: New York, USA, 1984; Volume 84, pp. 477–490.
30. Brans, J.P.; Vincke, P.; Mareschal, B. How to select and how to rank projects: The PROMETHEE method. *Eur. J. Oper. Res.* **1986**, *24*, 228–238. [https://doi.org/10.1016/0377-2217\(86\)90044-5](https://doi.org/10.1016/0377-2217(86)90044-5).
31. Brans, J.P.; Mareschal, B. PROMETHEE Methods. In *Multiple Criteria Decision Analysis: State of the Art Surveys International Series in Operations Research & Management Science Book Series (ISOR)*; Springer: New York, NY, USA, 2005; Volume 78, pp. 163–186.
32. Hwang, C.-L.; Lai, Y.-J.; Liu, T.-Y. A new approach for multiple objective decision making. *Comput. Oper. Res.* **1993**, *20*, 889–899. [https://doi.org/10.1016/0305-0548\(93\)90109-v](https://doi.org/10.1016/0305-0548(93)90109-v).
33. Yoon, K. A Reconciliation Among Discrete Compromise Solutions. *J. Oper. Res. Soc.* **1987**, *38*, 277–286. <https://doi.org/10.1057/jors.1987.44>.
34. Kerzner, H. *Project Management: A Systems Approach to Planning, Scheduling, and Controlling*, 10th ed.; Wiley: New York, NY, USA, 2009; p. 1112, ISBN 978-0-470-27870-3.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.