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Optimal Population and Sustainable Growth Under Environmental Constraints

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Abstract

This paper develops a dynamic optimal growth model integrating population, economic activity, and environmental constraints to investigate sustainable long-run development. The model incorporates capital accumulation, consumption, pollution abatement, and an endogenous demographic equation in which population growth responds negatively to pollution. A critical environmental threshold is imposed beyond which population growth collapses. Calibrating the model with plausible parameter values indicates that a sustainable steady state can support a global population of approximately 5 billion, a level consistent with high per capita consumption and stable environmental conditions. The optimal policy entails devoting roughly one-third of output to pollution abatement, which is sufficient to stabilize pollution below the safe threshold without imposing excessive economic cost. In this equilibrium, the economy achieves high consumption per person, a stable capital stock, and environmental balance, thereby avoiding overshoot and collapsing scenarios. The results highlight the trade-off between economies of scale and environmental limits. Larger populations can stimulate production and innovation but risk unsustainable pollution levels, whereas smaller populations allow higher per capita welfare within ecological boundaries. These findings suggest that achieving global sustainability requires balancing population size, consumption, and ecological limits through effective pollution abatement.

Keywords: optimal population; sustainable development; environmental constraints; pollution abatement; population dynamics

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1. Introduction

Concerns about global sustainability have intensified as both population size and per capita resource use continue to grow. Recent projections indicate that the world population will rise from over eight billion in 2022 to nearly ten billion by 2050 [1,2]. Meanwhile, humanity's ecological footprint already exceeds the Earth's regenerative capacity, with current consumption being roughly 1.7 times the sustainable level [2,3].

These trends have prompted competing perspectives on how to achieve sustainability. Proponents of "degrowth" argue that high-consumption lifestyles and continued demographic expansion are incompatible with ecological sustainability, emphasizing the need for deliberate economic contraction [3–5]. In contrast, advocates of "green growth" maintain that technological and policy interventions can decouple economic growth from environmental degradation. For example, expanding renewable energy, improving resource efficiency, and enforcing stricter pollution controls could allow high living standards to be maintained even with a large population [6,7].

The working hypothesis of this study is that long-run human welfare is maximized at an intermediate global population: large enough to realize economies of scale and innovation benefits, yet small enough to remain within ecological limits. This hypothesis reflects the central mechanism of the model developed below. In the model, population dynamics respond endogenously to environmental conditions, and achieving sustainability requires balancing demographic scale with pollution abatement.

The Place of This Study in the Literature

Debates about population, resources, and sustainability have deep historical roots. Thomas Malthus argued that unchecked population growth would eventually outpace food supply, leading to poverty and famine [8]. Subsequent works reinforced these concerns, warning that exponential growth in population and consumption could breach ecological limits and cause societal collapse [4,5]. These perspectives underscore the role of environmental limits in shaping demographic outcomes, a theme that is central to the hypothesis of this study.

In contrast, some scholars have highlighted the benefits of larger populations. For instance, Michael Kremer showed that historically, larger populations coincided with faster technological progress, suggesting that scale itself can accelerate innovation [9]. Endogenous growth theory builds on this insight by treating knowledge creation as increasing with the number of people. These arguments imply that population should not be minimized but rather balanced against environmental costs, again consistent with the idea of an intermediate optimum.

Economists have long incorporated resource constraints into growth theory. Solow [10] and Stiglitz [11] analyzed optimal growth paths with exhaustible resources, while John Hartwick [12] formalized the principle that reinvesting resource rents can sustain consumption over time. More recent analyses questioned whether current growth trajectories are depleting natural capital too rapidly to be sustainable [13]. Together, these studies highlight the need to balance economic scale with ecological limits.

Other models demonstrate collapse risks when population interacts with finite resources. Brander and Taylor, inspired by Easter Island, show how resource overuse can trigger population decline [14]. The idea of the "tragedy of the commons" [15] makes a similar point: without collective management, overuse undermines long-term viability. Climate–economy models [16] extend this logic to global greenhouse gas accumulation, formalizing trade-offs between economic output and climate stability.

However, few sustainability models explicitly treat population as an endogenous variable. A notable exception is Lehmijoki and Rovenskaya [17], who introduced an optimal population decision into a pollution–growth framework (though their model abstracts from technological progress). The present paper builds on that contribution by integrating demographic dynamics, pollution thresholds, and capital accumulation in a unified model. In doing so, it investigates the hypothesis that long-run welfare is maximized at an intermediate population level, where innovation benefits and ecological limits are jointly balanced.

2. The Model

Consider a closed economy in continuous time with four key state variables: physical capital $K(t)$, population $N(t)$, pollution $P(t)$ (an index of environmental degradation), and technology $A(t)$. (Time arguments are omitted when context permits.) Output $Y(t)$ is produced using capital and labor (population) as inputs, augmented by technology. A benevolent social planner chooses how much output to consume versus invest, and how much effort to devote to pollution abatement, aiming to maximize social welfare. An environmental sustainability constraint is imposed such that pollution cannot exceed a safe threshold $\overline{P}$, beyond which population growth would collapse. Table 1 summarizes the model's variables and parameters.

Table 1. Model Variables and Parameters.

Symbol	Description	Units or Notes
$K(t)$	Physical capital stock	Monetary units
$N(t)$	Population (number of people)	Individuals
$P(t)$	Pollution stock	Pollution units
$A(t)$	Technology level	Productivity index
$Y(t)$	Economic output (GDP)	Monetary units per year
$C(t)$	Consumption	Monetary units per year
$\tau(t)$	Abatement effort (fraction of output)	Fraction
α	Output elasticity of capital	Dimensionless
θ	Pollution emitted per unit output	Pollution units per output
ϕ	Pollution natural decay rate	Per year
$\overline{P}$	Safe pollution threshold (collapse level)	Pollution units (e.g., ppm CO_2)
g_0	Baseline population growth rate	Per year
ψ	Population sensitivity to pollution	Per pollution unit per year
δ	Depreciation rate of capital	Per year
s	Investment (saving) rate	Fraction of output
ρ	Social discount rate	Per year
σ	Elasticity of marginal utility (CRRA)	Dimensionless

2.1. Production

Output is represented by a Cobb–Douglas production function with constant returns to scale in capital and labor:

$$Y(t) = A(t)K(t)^{\alpha}N(t)^{1-\alpha}, \tag{1}$$

where $\alpha < 1$. Here, α is the output elasticity of capital (and $1 - \alpha$ is that of labor), and $A(t)$ is a technology factor that augments the productivity of inputs. For simplicity, we treat $A(t)$ as exogenous: either growing at a constant rate or held fixed when analyzing the steady state at a given technological level.

2.2. Pollution and the Environmental Constraint

Economic activity generates pollution as a by-product. Let $P(t)$ represent the stock of pollution (e.g., atmospheric CO_2 or a composite environmental quality index). Pollution dynamics are given by

$$\dot{P}(t) = \theta\,[1 - \tau(t)]\,Y(t) \;-\; \phi P(t)\,, \tag{2}$$

where θ is pollution emitted per unit of output and τ is the abatement effort (the fraction of output devoted to pollution control). Thus, $(1 - \tau)Y$ represents the effective output generating emissions after abatement. If $\tau = 0$, no abatement is undertaken and emissions are at their maximum; if $\tau = 1$, emissions are fully eliminated (at a cost to output). The parameter ϕ is the natural decay (absorption) rate of pollution.

A pollution threshold $\overline{P}$ represents an upper limit for the pollution stock. If P were to exceed $\overline{P}$, the planet's population carrying capacity would collapse. This reflects the concept of planetary boundaries (e.g., a critical CO_2 concentration or biodiversity loss level) beyond which nonlinear damage or societal collapse could occur. In the model, we impose $P(t) < \overline{P}$ at all times as a hard constraint: the planner never allows pollution to cross this boundary. (Violating it would cause population to crash, yielding extremely low welfare, so it is never optimal to do so.)

2.3. Capital Accumulation and Consumption

Physical capital $K(t)$ accumulates through investment. We assume a standard capital accumulation equation:

$$\dot{K}(t) = s\,Y(t) \; - \; \delta\,K(t)\,, \tag{3}$$

where δ is the depreciation rate of capital and s is the investment rate (the fraction of output saved and invested). In this formulation, a fixed fraction s of output is invested, and the rest, $(1-s)$, is available for consumption, as well as covering any abatement costs or pollution damages. For the optimal control problem, let consumption be the decision variable and derive investment from it. Total consumption, $C(t)$, can be defined as the portion of output not invested in new capital and not used up by abatement or lost to pollution damages. That is,

$$C(t) = Y(t) \; - \; I(t) \; - \; \text{Abatement cost} \; - \; \text{Damage due to pollution}. \tag{4}$$

For simplicity, suppose abatement incurs a cost proportional to the square of the abatement rate (reflecting rising marginal abatement costs): Abatement cost $= a\,\tau(t)^2\,Y(t)$. Also, let us assume pollution causes direct damages equivalent to a fraction $b\,P(t)$ of output. (For small b, this could represent a few percent of output lost to climate-related damage, health costs, etc.) Under these assumptions, consumption is

$$C(t) = \left[1 - s - a\tau(t)^2\right] Y(t) - bP(t). \tag{5}$$

Per capita consumption is $c(t) = \frac{C(t)}{N(t)}$. This formulation captures the trade-off according to which higher output increases consumption and thus welfare, but it also generates more pollution, thus reducing welfare.

2.4. Population Dynamics

Population, $N(t)$, grows (or declines) endogenously depending on environmental conditions. We specify a reduced-form demographic equation linking population growth to pollution:

$$\dot{N}(t) = N(t)[g_0 - \psi P(t)], \tag{6}$$

where g_0 is the baseline net birth rate (the intrinsic population growth rate in the absence of pollution), and ψ measures how pollution reduces population growth through increased mortality or reduced fertility. As pollution P rises, the change in population, $\dot{N}(t)$, declines. If pollution becomes very high, it can turn population growth negative.

Thus, there exists a critical pollution level, $P = \frac{g_0}{\psi}$, at which the net population growth rate is zero. We can interpret $\frac{g_0}{\psi}$ as the pollution level that would stabilize population. In the absence of pollution ($P \to 0$), population would grow at rate g_0. We can think of g_0 as reflecting the net reproduction rate under ideal conditions. At the optimal solution, the planner avoids the collapse scenario by never allowing P to exceed a maximum level, $\overline{P}$.

2.5. Utility and Social Welfare

Assume a utilitarian social welfare function that accounts for both the number of people and their consumption. At time t, if each person consumes $c(t)$, assume per capita utility

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \tag{7}$$

which is a constant-relative-risk-aversion (CRRA) form with parameter $\sigma > 0$ (and $\sigma \neq 1$)

governing the curvature of utility. (If $\sigma = 1$, $u(c)$ is understood as $\ln c$.) Total instantaneous utility is then

$$\mathrm{U}(t) = N(t)\, u(c(t)). \tag{8}$$

This formulation implies that adding more people increases total utility only if those people have a positive level of consumption. Future utility is discounted at a rate $\rho > 0$. The social welfare objective is the present value of total utility:

$$\mathrm{V} = \int_0^\infty e^{-\rho t} U(t)\, dt = \int_0^\infty e^{-\rho t}\, N(t) \frac{c(t)^{1-\sigma} - 1}{1-\sigma}\, dt. \tag{9}$$

The planner's control variables over time are the consumption rate (equivalently, the saving/investment decision) and the abatement rate $\tau(t)$. Population is not directly controlled; it evolves according to the demographic equation determined by pollution.

This setup reflects the view that direct population control lies outside the immediate policy toolkit, for ethical, social, and practical reasons. Population outcomes are influenced indirectly through policies that affect health, education, and environmental quality rather than through any explicit population mandate. We take as given the initial values $K(0)$, $N(0)$, $P(0)$, $A(0)$. The planner faces the dynamic constraints for $\dot{K}$, $\dot{N}$, $\dot{P}$, and $\dot{A}$ as described above, plus the inequality constraint $P(t) < \overline{P}$ for all t.

3. Model Calibration and Sustainable Population Estimate

The model's key parameters are calibrated based on empirical data and findings in the literature, to gauge the implications for global sustainability.

3.1. Population Growth Parameters

We choose g_0 to represent the net natural population growth rate in the absence of environmental stress. Mid-20th century global population growth peaked around 2% per year, but in recent decades it has fallen to roughly 1% and is projected to continue declining [18]. We set $g_0 \approx 0.02$ (2% per year) as a historical baseline. The parameter ψ is set to ensure that if pollution reaches a certain critical level, it can fully negate this natural growth. Suppose ψ (population sensitivity to pollution) is chosen so that $\frac{g_0}{\psi} = \overline{P}$, meaning that once pollution exceeds the safe threshold $\overline{P}$, the net population growth rate becomes negative.

For instance, suppose $\overline{P} = 2$ (i.e., twice the current pollution level), and $g_0 = 0.02$. Then, ψ would be on the order of 0.01 (per unit of P). This ensures that once pollution approaches the threshold, population growth effectively halts.

3.2. Production and Technology Parameters

Let $\alpha = 0.3$, a typical value in economic growth models. Technological progress is assumed to be exogenous. For example, $A(t)$ grows at an annual rate of 1–2% per year, reflecting ongoing productivity improvements. In steady-state, we hold A constant (at A^*), to analyze the long-run equilibrium for a given technology.

3.3. Pollution and Abatement Parameters

The emission intensity θ is calibrated so that in the absence of abatement, the current level of output would increase P at a realistic rate. For example, global CO_2 emissions of around 40 Gt/year currently raise atmospheric CO_2 concentration by about 2 ppm per year [18]. If we measure P in ppm and Y in trillions of dollars, we could normalize θ accordingly. To do so, suppose current annual output is $Y \sim \$100$ trillion and produces a 2 ppm annual increase when $\tau \approx 0$ and $P \approx 420$ ppm (current level), and assume a small natural absorption rate $\phi \sim 0.01$ (about 1% of the stock absorbed per year). Then, according to (2), θ should be such that $\theta Y \approx \phi P + 2$.

While for a theoretical purpose a precise value of θ is not necessary, consistency between two other parameters, ψ and g_0, must be ensured. We choose ψ and g_0 to align with the chosen $\overline{P}$ as above (Equation (6)). We also assume that in an optimal scenario the planner will apply some $\tau > 0$ to keep P at or below $\overline{P}$ indefinitely, so we anticipate an interior solution for τ.

3.4. Optimal Control Solution

This dynamic problem can be solved using optimal control techniques. The state variables are K, N, P, and A; the control variables are C (consumption, which determines investment) and τ (abatement). The planner maximizes the discounted welfare objective V subject to the state equations and the pollution threshold constraint. We set up the current-value Hamiltonian and first-order conditions for an interior optimum.

Let $\lambda_K(t)$, $\lambda_N(t)$, $\lambda_P(t)$, and $\lambda_A(t)$ be the costate variables (shadow values) corresponding to the states $K(t)$, $N(t)$, $P(t)$, and $A(t)$, respectively. The costate variables represent the present-value marginal utility of an incremental unit of each state variable. The current-value Hamiltonian (excluding the inequality constraint, which would enter via a Karush–Kuhn–Tucker condition if binding) is formulated as

$$\begin{aligned} \mathcal{H} \; &= N(t)\,\frac{c(t)^{1-\sigma}-1}{1-\sigma} \\ &+\lambda_K(s\,Y(t) - \delta\,K(t)) \\ &+\lambda_N((g_0 - \psi\,P(t))\,N(t)) \\ &+\lambda_P(\theta\,[1-\tau(t)]\,Y(t) \;-\; \phi\,P(t)) \\ &+\lambda_A\Big(\dot{A}(t)\Big), \end{aligned} \tag{10}$$

where we include the $\dot{A}$ term for generality. (If A grows exogenously at rate g_A, then $\dot{A} = g_A A$ and the planner cannot influence it; λ_A would then follow from the costate equation for A but there is no control associated with A.) Both τ and the consumption/saving decision (which determines C or, equivalently, I) appear in this Hamiltonian, so we obtain first-order conditions with respect to those controls.

3.5. First-Order Conditions

3.5.1. Consumption Versus Investment

Because consumption $C(t)$ and investment $\mathrm{I}(t)$ are directly linked ($\mathrm{I} = Y - C -$ other uses), we can derive the FOC for optimal consumption. With per capita consumption $c(t) = \frac{C(t)}{N(t)}$, an increase in C at time t yields immediate utility $N\,u'(c)$ (since $\mathrm{U} = Nu(c)$). Meanwhile, $\frac{\partial c}{\partial C} = \frac{1}{N}$. Thus, the instantaneous marginal benefit of one more unit of consumption is

$$\frac{\partial \mathcal{H}}{\partial C} = N\,u'(c) \cdot \frac{1}{N} = u'(c)\,. \tag{11}$$

For CRRA, $u'(c) = c^{-\sigma}$, so this is the marginal utility per person. On the other hand, allocating one more unit of output to consumption means one less unit invested in capital. The lost investment reduces $\dot{K}$ by that unit (see $\dot{K} = sY - \delta K$). In the Hamiltonian, this effect appears via the term $\lambda_k[sY - \delta K]$; λ_K is the shadow value of capital, so reducing investment by 1 (and thus reducing $\dot{K}$ by 1) imposes a cost of λ_K in terms of foregone future welfare.

At an interior optimum, the planner chooses consumption such that the marginal utility of consumption equals the shadow value of capital (the marginal utility of saving that consumption for future use):

$$u'(c(t)) = \lambda_K(t)\,, \tag{12}$$

for all t along the optimal path. Intuitively, if $u'(c)$ were greater than λ_K, then consuming a little more now would yield more utility than the future gain from investment, so consumption should increase. If $u'(c)$ fell below λ_K, consuming less and investing more would increase welfare.

3.5.2. Pollution Abatement (τ)

For the abatement rate τ the planner balances the marginal benefit of reduced pollution against its marginal cost (output diverted from consumption/investment). Setting $\frac{\partial \mathcal{H}}{\partial \tau} = 0$, and noting that τ appears in the Hamiltonian only in the pollution term (and implicitly in C through abatement cost if we modeled it explicitly), we get

$$\lambda_P\,[-\theta\, Y(t)] + \frac{\partial \mathcal{H}}{\partial C}\frac{\partial C}{\partial \tau} = 0 \tag{13}$$

In an interior optimum with $0 < \tau < 1$, this condition can be interpreted as

$$\lambda_p\, \theta\, Y = \frac{\partial\,(\text{utility from consumption})}{\partial \tau} \tag{14}$$

The planner increases τ until the utility gain from reducing pollution (the left side, via λ_P) equals the utility loss from the output sacrificed to abatement (right side). If we specify an abatement cost function, we can derive a more concrete condition. For example, if abatement cost is $a\,\tau^2 Y$ and pollution damage is bP, the optimal τ would satisfy (in steady state):

$$2a\,\tau^*\,Y^* = b\,P^*\,, \tag{15}$$

meaning the marginal cost of abating an additional fraction of emissions ($2a\,\tau^* Y^*$) equals the marginal benefit in terms of avoided damage (b times the steady-state pollution level saved). The general implication is that the optimal abatement policy equates marginal abatement cost to marginal pollution damage, effectively a Pigouvian condition for emissions.

3.5.3. Population

The planner does not directly control $N(t)$, but the shadow price $\lambda_N(t)$ gives the value of an additional person. This value is nuanced as an extra person adds to utility directly (more people enjoying consumption) but also strains resources and creates additional pollution, which imposes future costs. At the optimum, $\lambda_N(t)$ adjusts such that if population growth is too high, the shadow cost of an additional person (through more pollution and capital dilution per capita) balances the direct utility benefit of having another person.

In steady state, N settles at a level where these forces are in balance, yielding the optimal population N^*. In fact, in the long-run steady state we have $\dot{N} = 0$, which (from the demographic equation) implies $P = P^* = \frac{g_0}{\psi}$. At the optimum steady state, we expect $\lambda_N = 0$; this reflects that N^* is chosen such that the planner is indifferent to a marginal change in population in the long run. If λ_N were positive, it would imply a benefit to adding an extra person (so we haven't reached the optimal population yet); if λ_N were negative, it would imply the population is beyond the optimum. Thus N^* is the point where an additional person has zero net impact on welfare at the margin.

Applying the first-order conditions together with the steady-state requirements ($\dot{K} = \dot{N} = \dot{P} = \dot{A} = 0$ and $P < \overline{P}$) allows us to solve for the balanced-growth steady state. In a sustainable steady state, by definition $\dot{N} = 0$ and $\dot{P} = 0$. From $\dot{N} = 0$, as noted, we immediately get

$$P^* = \frac{g_0}{\psi}, \tag{16}$$

which implies that the steady-state pollution level is determined entirely by the demographic parameters. For the optimum to be interior (with a positive long-run population), we must have $P^* < \overline{P}$. In our calibration we ensure this is the case. (If $P^* \geq \overline{P}$, the pollution constraint would bind first, meaning the planner would cap pollution at $\overline{P}$ to avoid collapse, and population would stabilize at some level at or below that corresponding $\overline{P}$.)

From $\dot{P} = 0$, we get the condition for a stable pollution stock:

$$\theta\,[1 - \tau^*]\,Y^* = \phi\,P^*\,. \tag{17}$$

In a steady state, the flow of new pollution emissions ($\theta(1 - \tau^*)Y^*$) equals the natural removal of pollution (ϕP^*). This gives the steady-state pollution level as

$$P^* = \frac{\theta(1 - \tau^*)Y^*}{\phi}, \tag{18}$$

consistent with the $P^* = \frac{g_0}{\psi}$ condition (the planner will choose τ^* and Y^* such that these conditions hold together). The economy must either abate sufficiently or be small enough that emissions do not exceed the assimilative capacity of the environment.

Finding a closed-form solution for N^* would require combining the conditions above with the economic optimum conditions for K^* and τ^*. Instead of deriving an explicit formula, it is more intuitive to evaluate sustainable consumption per capita c as a function of N to identify the optimum. In our model, $c^*(N)$ tends to have an interior maximum: at very low N, output (and thus consumption) is limited by insufficient labor, whereas at very high N output per person falls due to strained resources and high pollution. Using our calibrated parameter set, we find that c^* indeed peaks at an intermediate population size. A population N^* of about 5 billion maximizes long-run per capita consumption under sustainability constraints.

This estimate is consistent with empirical studies suggesting that current consumption overshoots Earth's biocapacity and that sustainable global populations may lie between 2 and 10 billion depending on lifestyle and technology [2–4].

4. Results

4.1. Steady State

Using the calibrated parameters, we solve the steady-state equations to obtain an internally consistent sustainable equilibrium. The resulting steady-state values are as follows:

- Population: $N^* \approx 5$ billion.
- Output (GDP): $Y^* \approx 132$ trillion per year (in 2025 U.S. dollars).
- Capital stock: $K^* \approx 636$ trillion (about 4.8 times annual output). This implies a capital–output ratio consistent with a 20–25% investment rate (since $\delta \approx 5\%$ and maintaining K^* requires about \$32 trillion of investment per year, which is roughly 24% of Y^*). (See Appendix A for details.)
- Consumption per capita: $c^* \approx 17,200$ per person annually. This represents a high average standard of living, slightly above the world's current average per capita consumption, which is made possible by improved productivity and a smaller population. It indicates that at the optimum, fewer people can enjoy higher consumption.

- Abatement effort: The optimal rate is $\tau^* \approx 0.33$, meaning one-third of emissions are abated at a cost of about 2.2% of GDP. The remaining two-thirds are released, causing damages of roughly 8.9% of output ($bP^* \approx 0.089$). Full abatement would be prohibitively costly, so the planner accepts some pollution while keeping it in check.
- Pollution stock: At $P^* \approx 8.87 \times 10^3$, natural absorption ($\phi P^*$) offsets the ongoing emissions, stabilizing the pollution stock. In this equilibrium, anthropogenic emissions are sharply reduced relative to a business-as-usual scenario but not eliminated. External studies suggest that stopping the growth of atmospheric CO_2 would require about 80% cuts in emissions [19], but in our model stability is achieved with a one-third reduction. This is because the elevated P^* allows natural sinks to absorb the remaining emissions.

Figure 1 illustrates how steady-state per capita consumption would vary with population in our model. In this simulation, per capita consumption peaks at approximately 4.8 billion people, indicating the population size that yields the highest average living standard under the given technological and environmental parameters.

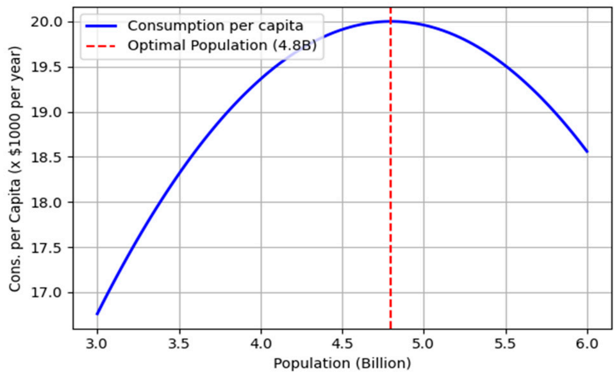

Figure 1. Optimal population versus per capita consumption derived from model simulation.

4.2. Stability Analysis

To analyze local stability, we linearize the system around the steady state. Let $K^*, N^*,$ and P^* denote the steady-state values of capital, population, and pollution, and let τ^* and c^* be the steady-state abatement fraction and per capita consumption. Including the consumption equation (via the Euler condition) for a full optimality analysis, our state vector is (K, N, P, c). The Jacobian matrix J can be written symbolically as

$$\mathrm{J} = \begin{pmatrix} \left(1 - a\tau^{*2} - bP^*\right)\alpha\frac{Y^*}{K^*} - \delta & \left(1 - a\tau^{*2} - bP^*\right)(1-\alpha)\frac{Y^*}{N^*} - c^* & -b\,Y^* & -N^* \\ 0 & 0 & -\psi N^* & 0 \\ \theta(1-\tau^*)\,\alpha\frac{Y^*}{K^*} & \theta(1-\tau^*)\,(1-\alpha)\frac{Y^*}{N^*} & -\phi & 0 \\ \frac{c^*\alpha(\alpha-1)}{\sigma}\frac{Y^*}{K^*}\left(1 - 2a\tau^* + a\tau^{*2} - bP^*\right) & \frac{c^*\alpha(1-\alpha)}{\sigma}\frac{Y^*}{N^*}\left(1 - 2a\tau^* + a\tau^{*2} - bP^*\right) & \frac{c^*}{\sigma}\left(-b\alpha\frac{Y^*}{K^*} - \psi\right) & 0 \end{pmatrix}. \quad (19)$$

Substituting the given parameter values and steady-state quantities into J yields a numerical Jacobian. The calibrated steady state is characterized by:

- $\alpha = 0.33$, $\delta = 0.05$, $\rho = 0.02$, $\sigma = 2.0$.
- $a = 0.20$, $b = 1 \times 10^{-5}$, $\phi = 0.01$, $\theta = 1.0$.
- $g_0 = 0.015$, and $\overline{P} \approx 8870$ (which gives $\psi = \frac{g_0}{P^*} \approx 1.69 \times 10^{-6}$).
- Steady-state technology $A^* = 5.36$ and population $N^* = 5.0 \times 10^9$.

Plugging these values into the Jacobian, we obtain the following numerical matrix (elements in per-year units):

$$J \approx \begin{pmatrix} 0.020 & -2.08 & -1.11 \times 10^{6} & -5.00 \times 10^{9} \\ 0 & 0 & -8.45 \times 10^{3} & 0 \\ 0 & 0 & -0.010 & 0 \\ -0.202 & 0 & -1.45 \times 10^{-5} & 0 \end{pmatrix}. \quad (20)$$

4.3. Eigenvalues and Discussion of Stability

Finally, we compute the eigenvalues of the Jacobian matrix to assess the local stability of the equilibrium. The eigenvalues λ_i of J are approximately $\lambda_1 \approx +3.18 \times 10^4$, $\lambda_2 \approx -3.18 \times 10^4$, $\lambda_3 \approx -0.01$, and $\lambda_4 \approx 0.00$.

The dynamic equilibrium is saddle-path stable. The economy–environment system will converge to a steady state only if the controls (c, τ) are adjusted along the unique stable manifold. Two eigenvalues (including the crucial large one associated with consumption) are negative, ensuring convergence along those directions, while one positive eigenvalue indicates an unstable direction (requiring a jump in controls to avoid divergence). The near-zero eigenvalue reflects marginal stability of population at the environmental threshold.

In practice, the planner's optimal policy would manage the system by adjusting τ to maintain the population close to N^*, ensuring that all relevant trajectories approach the steady state. The equilibrium can therefore be characterized as a saddle point with a one-dimensional unstable subspace (and a neutral/manifold direction due to population), which is typical for optimal growth models with a state constraint or threshold effect.

4.4. Sensitivity and Uncertainty Analysis

To test the robustness of our findings, we conduct an uncertainty analysis by varying key parameters within ±20% of their baseline values. The steady-state conditions linking the demographic equilibrium,

$$P^* = \frac{g_0}{\psi}, \quad (21)$$

with the pollution balance,

$$\theta(1 - \tau^*)Y^* = \phi P^*, \quad (22)$$

jointly determine the sustainable pollution level, population size, and abatement rate. For each set of parameter values, we solve these conditions (along with the optimality conditions for consumption and τ to find the new steady-state outcomes. Per capita consumption is then inferred accordingly.

Table 2 reports the steady-state results for different parameter scenarios. The baseline scenario corresponds to $g_0 = 0.02$, $\psi = 4.444 \times 10^{-5}$ (from (21), (22), with today's level of pollution $P^* = 450$ ppm for calibration), $\phi = 0.01$, $\theta = 0.062$ (calibrated from today's $Y \approx 100$ trillion USD, $P = 420$) ppm, $\dot{P} = 2$ and (2) with previously assumed ϕ. Other calibrated parameters are the savings rate $s = 0.24$, $a = 0.202$ and $b = 0$. The optimal consumption C^* is then determined by (2), and the optimal population by a numerical method to maximize C^*/N.

Table 2. Sensitivity of steady-state outcomes to ±20% variations in key parameters.

Scenario	N^* (Billion)	c^* (USD/Year)	τ^* (Share of Output)
Baseline	4.99	16,087	0.330
$g_0 + 20\%$	6.64	15,132	0.396
$g_0 - 20\%$	3.63	16,996	0.264
$\psi + 20\%$	3.84	16,847	0.275
$\psi - 20\%$	7.11	14,886	0.412
$\phi + 20\%$	5.99	16,087	0.330
$\phi - 20\%$	3.99	16,087	0.330

Table 2. *Cont.*

Scenario	N^* (Billion)	c^* (USD/Year)	τ^* (Share of Output)
$\theta + 20\%$	4.16	16,087	0.330
$\theta - 20\%$	6.24	16,087	0.330

Notes: Values are rounded.

4.5. Interpretation of Sensitivity Analysis

The sensitivity analysis indicates that demographic factors exert the strongest influence on long-run outcomes. Changes in the baseline population growth rate g_0 or in the pollution-related mortality parameter ψ lead to large shifts in the sustainable population size N^*. In fact, varying g_0 or ψ within plausible bounds produces a wide range of steady-state population outcomes (roughly between 3 and 8 billion). At the same time, per capita consumption c^* moves inversely with population: a smaller N^* allows higher consumption per person, whereas a larger population dilutes resources and lowers c^*.

In our model, demographic equilibrium requires births to equal deaths, hence the balance condition $P^* = \frac{g_0}{\psi}$. This equation implies that the steady-state pollution level P^* (the environmental stress determining mortality) is set by the ratio of the base growth rate to the pollution-sensitivity of mortality. A higher g_0 (or lower ψ) thus permits a larger equilibrium population before ecological constraints bind.

In contrast, ecological parameters have a weaker impact on the results. Variations in the natural pollution decay rate ϕ or the emissions intensity θ mainly affect the optimal pollution abatement fraction τ^*, while leaving the sustainable population N^* relatively stable. These parameters determine the steady-state pollution balance: $\theta\,(1-\tau^*)\,Y^* = \phi\,P^*$, which equates pollution generation (left side, given by effective output $(1-\tau^*)Y^*$ times emission factor θ) to pollution absorption (right side, given by ϕP^*). Altering ϕ or θ changes the required abatement effort τ^* needed to satisfy this equilibrium, but such changes do not markedly shift the feasible population N^* in our calibration. In essence, while ϕ and θ influence how much output must be diverted to pollution control, the long-run population level is less sensitive to these environmental parameters compared to the dominant demographic feedback.

Across all scenarios, the model's central finding remains robust. In each case examined, the welfare-maximizing population emerges at an intermediate scale rather than at the extremes. Quantitatively, the optimal long-run population N^* consistently lies in the mid-range of a few billion people (approximately between 3 and 8 billion, with a baseline around 5 billion). This consistency indicates that neither unbounded population growth nor radical depopulation would maximize welfare; instead, there is an interior optimum where ecological constraints and economies of scale are balanced.

5. Discussion

In our model, population is not a direct control variable but evolves endogenously in response to environmental feedback. The key policy instrument available to the planner is the abatement rate τ. According to the pollution Equation (2), higher τ lowers effective emissions and slows the increase in the pollution stock. Meanwhile, the demographic Equation (6) specifies that population growth declines as pollution rises. By linking these mechanisms, we see that greater pollution abatement mitigates pollution buildup, which in turn reduces environmental stress on the population and supports a higher long-run population. Conversely, if abatement is insufficient, pollution will climb toward the threshold $\overline{P}$, causing the net population growth rate to fall to zero or below, and eventually pushing N downward.

The analysis supports the hypothesis that long-run welfare peaks at an intermediate population: large enough to sustain economies of scale, yet small enough to respect ecological thresholds. The calibrated baseline suggests a sustainable range of the order of 5 billion people. Sensitivity analysis shows that under some alternative assumptions the sustainable level may be higher, up to about 8 billion. Thus, the qualitative conclusion is robust: unchecked demographic expansion is unsustainable, while a desired standard of living can be achieved within stable population limits.

A New Perspective on the Population–Climate Modeling

The pollution and population dynamics described in Equations (2) and (6) transform the familiar IPAT/Kaya decomposition into a dynamic system in which population $N(t)$ responds endogenously to accumulated impact $P(t)$. Integrating this approach into IPAT [4] or STIRPAT [20] frameworks means replacing an exogenous $N(t)$ with the demographic law (6) and imposing an explicit guardrail $P(t) \leq \overline{P}$. A straightforward policy implication follows from (2): choose the abatement share τ so that the steady state

$$P^* = \frac{\theta(1-\tau)Y}{\phi} \tag{23}$$

remains well below $\overline{P}$; equivalently, set τ such that $\theta(1-\tau)Y \leq \phi\underline{P}$ for a precautionary threshold $\underline{P} < \overline{P}$ Embedding this demographic–environmental feedback in integrated assessment models [16] would convert accounting identities into a closed system linking today's abatement to tomorrow's demographic path, while leaving policy levers recognizable: reducing θ (through cleaner technology and efficiency), raising τ (via carbon pricing or regulatory standards), and investing in health and education that lower ψ.

This study complements existing work that has introduced endogenous demographic responses into growth–environment frameworks [17]. Empirically, three advances would enable policy relevance: (i) regional estimation of ψ, g_0, and ϕ; (ii) incorporating endogenous technical change so that θ declines with learning; and (iii) carrying uncertainty forward through robust-control or guardrail approaches, where τ is chosen to keep P^* below $\overline{P}$ across various scenarios. In short, the contribution of the present study is to make the welfare and feasibility consequences of endogenous population dynamics explicit within the IPAT/Kaya/IAM toolkit [4,16], without invoking direct population controls.

6. Conclusions

This study develops a unified growth–environment model in which population evolves endogenously in response to ecological conditions. By imposing a pollution threshold and linking demographics to environmental stress, the analysis identifies an optimal policy mix that includes a feasible abatement effort of roughly one-third of potential emissions. This balance allows high levels of per capita welfare while maintaining ecological stability.

The results indicate that sustainability lies at an intermediate population level: neither in unbounded expansion nor in radical contraction, but where ecological integrity and economies of scale are jointly preserved. Welfare is also maximized at an intermediate global population. Sensitivity analysis suggests a range for the optimal population of 3 to 8 billion but centered around 5 billion. The study highlights the need for policies that manage at the same time both population dynamics and pollution abatement to ensure long-run prosperity.

The contribution of the study lies in quantifying the trade-off between economies of scale and ecological limits within a single analytical framework. Several limitations must be acknowledged. Technological change is treated as exogenous, the ecological damage function is simplified, and the model abstracts from distributional, regional, and political factors. Future research could address these limitations by endogenizing innovation, incorporating resource heterogeneity, and examining policy feasibility in real-world contexts, refining our understanding of sustainable population levels.

The population–climate debate is not only about modeling techniques, but about the role of population itself. As discussed in the introductory section, while some scholars emphasize consumption and technology as the principal drivers of emissions, others insist that population growth must be explicitly recognized as a core cause of climate pressure. The framework here takes the latter view seriously, while still situating population within a broader system of affluence, technology, and abatement. Perhaps our approach, in which population outcomes are shaped indirectly through the choice of an environmental policy instrument, may offer common ground in this debate, being more acceptable to both those who see population as a central driver of climate change and those who prefer to focus policy exclusively on consumption and technology.

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Appendix A. A Simplified Model for Analytical Illustration

We present a simplified version of the model, in which utility is logarithmic in consumption and includes a linear disutility from pollution. This tractable formulation makes the algebra more transparent while still capturing the essential trade-off between consumption and environmental quality. By contrast, the main text uses a more general CRRA utility function and models pollution damage as an output loss. The Appendix analysis should be viewed as an analytical complement rather than a literal restatement of the full model.

Consider a simplified planner model for the global economy with three state variables: population N, capital stock K, and pollution stock P. Time is continuous. The production function is Cobb–Douglas in labor (population) and capital. Pollution is generated as a by-product of output but can be partially abated. The planner maximizes the present value of utility, which in this simpler case depends on consumption per capita and includes a disutility from pollution. The model's structure is summarized by the following equations:

Production: $Y = K^{\alpha} N^{1-\alpha}$, where α is the capital share and total factor productivity is normalized to 1 (constant).

Capital accumulation: $\dot{K} = Y - C - R - \delta K$. Here C is total consumption, R is resources devoted to pollution abatement, and δ is the capital depreciation rate. In other words, output is allocated to consumption, abatement, and replacing depreciated capital.

Abatement and pollution: Let τ be the abatement effort, defined as the fraction of output devoted to abatement ($R = \tau Y$). The remaining fraction $(1 - \tau)$ of output is the effective output for consumption and investment. Pollution accumulates according to $\dot{P} = \theta(1 - \tau)Y - \phi P$, where θ is an emission factor (pollution produced per unit of output) and ϕ is the natural pollution absorption rate. In a steady-state, emissions $\theta(1 - \tau)Y$ equal natural absorption ϕP.

Population dynamics: $\dot{N} = [g_0 - d(P)]N$. Here g_0 is an exogenous birth rate and $d(P)$ is a mortality rate function assumed to increase with pollution. This specification captures a Malthusian "positive check" mechanism whereby higher pollution reduces population by increasing mortality. In a steady-state, $\dot{N} = 0$ requires $g_0 = d(P)$, so the steady-state pollution P^* is that which raises the death rate enough to offset births, yielding a stable population at $\dot{N} = 0$.

Utility: The planner's objective is

$$W = \int_0^\infty N(t)\, u(c(t), P(t))\, e^{-\rho t} \mathrm{d}t, \tag{A1}$$

where ρ is the rate of time preference. We assume per capita utility takes a form $u(c, P)$ that increases in consumption c and decreases in pollution P. A convenient specification (used for numerical examples) is logarithmic utility with linear disutility of pollution:

$$u(c, P) = \ln c - \beta P, \tag{A2}$$

where β is the weight on pollution disutility. This implies diminishing marginal utility of consumption and linear damage from pollution (each person suffers disutility β from the pollution stock). The total utilitarian framework (Nu) means that having more people can increase total utility (since more people enjoy life), subject to the environmental and capital constraints. The model thus captures the fundamental trade-off: adding an extra person provides utility benefits but also imposes costs by diluting capital and increasing pollution.

Sustainability: We define a path as sustainable if consumption never declines and population does not decline over time. We focus on a steady state that satisfies this sustainability criterion (non-declining c and N). In steady state, $\dot{K} = 0$, $\dot{N} = 0$, and $\dot{P} = 0$, so all variables remain constant. Below, we derive the conditions characterizing the optimal steady state (K^*, N^*, P^*) that maximizes welfare subject to the model constraints.

Appendix A.1. Steady-State Conditions and Derivations

In a steady state, population, capital, and pollution are constant. We denote steady-state values with asterisks. The steady-state conditions are:

Population: $\dot{N} = 0$ implies $g_0 = d(P)$. Thus, the steady-state pollution stock P^* is implicitly determined by the equation $d(P) = g_0$. (If $d(P)$ is an increasing function, this pins down a unique P^*.) The corresponding population N^* is whatever level is consistent with that pollution stock given the economic decisions. In other words, N will adjust until pollution-induced mortality equals the birth rate in steady state.

Pollution: $\dot{P} = 0$ implies $\theta(1-\tau)Y = \phi P^*$. This means the steady-state pollution stock satisfies $P = \frac{\theta(1-\tau)Y}{\phi}$, i.e., the ratio of sustained emissions to the natural absorption rate. Intuitively, higher abatement τ or lower output Y reduces the pollution stock needed to balance outflows.

Capital: $\dot{K} = 0$ implies $Y - C - R = \delta K$. The term $Y - R$ is output net of abatement costs, which is used for either consumption or investment. In steady state, net investment must equal depreciation δK just to maintain the capital stock. Equivalently, $C = Y - \delta K - R$. This equation will be used along with the optimality conditions to solve for C^* and τ^*.

Next, we use the optimality conditions for the planner's control variables in steady state. We set up the current-value Hamiltonian for this simplified problem:

$$\mathcal{H} = N[\ln c - \beta P] + \mu[Y - C - R - \delta K] + \nu[\theta(1-\tau)Y - \phi P] + \xi[(g_0 - d(P))N], \tag{A3}$$

with costate (shadow price) μ for capital, ν for pollution, and ξ for population. In steady state, the first-order conditions and costate equations yield the following conditions (for an interior solution with $N > 0$, $\tau > 0$):

Consumption (Euler) equation: $\frac{\partial \mathcal{H}}{\partial C} = 0$ gives $\mu = \frac{\partial(N \ln c)}{\partial C} = \frac{N}{C}$. Since $c = C/N$, this implies $\mu = 1/c$. Thus, the shadow price of capital μ equals the marginal utility of consumption. In steady state, the costate dynamic equation $\dot{\mu} = \mu(\rho + \delta - f'(k))$ must also hold at zero. Setting $\dot{\mu} = 0$ and solving yields the modified golden-rule condition:

$$f'(k^*) = \delta + \rho. \tag{A4}$$

Here $f'(k)$ is the marginal product of capital per worker. If $\rho = 0$ (often assumed for sustainability), this simplifies to $f'(k) = \delta$. This is the classic golden rule condition: at the optimum, capital per worker is chosen such that the marginal product of capital equals the depreciation rate. Intuitively, this maximizes consumption per capita in steady state. In our numerical simulations we take ρ to be small (e.g., 1–2%), so the difference between $f'(\mathrm{k}) = \delta + \rho$ and $f'(k^*) = \delta$ is minor. Abatement condition: $\partial\mathcal{H}/\partial\tau = 0$ gives $-\mu Y + \nu\theta Y = 0$, which equates the marginal utility cost of abating (lost output valued at μ utils per unit) to the marginal utility benefit (reduced pollution valued at ν utils). Substituting $\mu = 1/c$, we get $\nu = \frac{1}{\theta c}$. The costate ν can be interpreted as the shadow cost of pollution in utility terms. Meanwhile, the costate equation for P at steady state yields $\nu\phi = \beta N$ (because the marginal disutility of an extra unit of P is βN for the whole population, and in steady state this is balanced by the discounted future dilution of that pollution via natural decay ϕ). For small ρ, approximately $\nu \approx \frac{\beta N}{\phi}$. The shadow cost of pollution ν adjusts until the marginal disutility from an extra unit of P (suffered by N people) is balanced by the natural removal ϕ of pollution. Combining $\nu \approx \frac{\beta N}{\phi}$ with $\nu = \frac{1}{\theta c}$, we obtain an implicit formula for the optimal abatement effort τ^*:

$$\frac{\tau^* Y}{Y} = \frac{\lambda_c \tau^2 Y}{Y} \approx \frac{\beta N^*}{\phi}, \tag{A5}$$

which, after simplification and cancelling Y, can be interpreted as the fraction of output devoted to abatement in terms of population and pollution parameters. (This expression is schematic; more precise algebra would yield a specific function of N^*, β, θ, and ϕ.) In simpler terms, the planner equalizes the percentage of output spent on abatement to a function of population and environmental sensitivity. A larger population or a higher damage weight β calls for greater abatement effort, ceteris paribus. At the optimum, spending another unit of output on abatement yields the same utility gain (through reduced pollution) as spending it on consumption, which determines τ^*.

Population (fertility) condition: $\partial\mathcal{H}/\partial N = 0$ yields $\ln c - \beta P + \xi(g_0 - d(P)) = 0$, since N appears in both the utility term and the demographic term. The costate ξ for population evolves as $\dot{\xi} = \xi(\rho - (g_0 - d(P))) - [\ln c - \beta P]$. Setting $\dot{\xi} = 0$ in steady state and using $g_0 = d(P^*)$ gives $\xi = 0$. This is the optimal population condition: in steady state the shadow price of population ξ must be zero (otherwise N could increase or decrease to raise welfare). Expanding the first-order condition for N with $\xi = 0$ gives

$$\ln c^* - \beta P^* = 0. \tag{A6}$$

The planner adds people until the utility gain of an extra person ($\ln c^*$, the additional utility that person enjoys) equals the disutility they impose via pollution (βP^*, suffered by that person and others).

Putting the above steady-state conditions together, we have a system of equations that determine N^* and τ^* (and the other variables then follow). It is convenient to express some relationships in per capita terms using $\mathrm{y} = Y/N$ and $\mathrm{k} = K/N$. The results will change as follows:

Optimal capital per person: $f'(k) = \delta + \rho$. If ρ is near zero, this gives $f'(k) \approx \delta$. This is the modified golden rule condition that maximizes output per person net of depreciation. (Note that with constant returns and fixed A, k^* is independent of N.) The corresponding output per person is $\mathrm{y} = f(k^*)$.

Consumption and output: In steady state, $Y = C + R + \delta K$. Dividing by N gives $y = c + \tau y + \delta k$. Using $f'(k^*) = \delta$ and the Cobb–Douglas production form, one can show that $C^* = (1 - \tau^*)(1 - \alpha)y^*$. Intuitively, $1 - \alpha$ is labor's share of output in Cobb–Douglas, which in steady state (after replacing depreciated capital) is the share of net output available for consumption. Thus c^* is a fixed fraction of net output per person determined by τ^*.

Pollution stock: $P^* = \frac{\theta(1-\tau^*)Y^*}{\phi}$, as derived earlier.

Optimal abatement: The planner's first-order condition for abatement (A5) can be rearranged (using the expressions for c^* and y^* above) into an implicit equation for τ^* as a function of N^* (and other parameters). In general, one must solve this equation simultaneously with the population optimum condition to find N^* and τ^*.

Optimal population: Using the optimal population condition derived above and substituting $Y = Ny$ and P from the pollution equation, we obtain an equation that can be solved for N^* given the parameters g_0 and β. After cancelling common factors, one can rewrite it in a form that highlights key determinants: for example, it shows that the optimal N^* is smaller when the pollution damage weight β is larger or the natural absorption ϕ is lower, and it is larger when productivity (output per person) is higher. In other words, more productive economies can sustain a larger population, but higher environmental sensitivity or lower planetary resilience push the optimal population down.

Solving the above equations simultaneously determines the optimal N^* and τ^* (the other variables then follow). For simplicity, we turn to numerical simulation to find the optimal values for the baseline scenario.

Appendix A.2. Calibration and Results

We follow the simplified Appendix model: per capita utility $u(c, P) = \ln c - \beta P$; abatement uses a fraction of output $R = \tau Y$; pollution evolves as $\dot{P} = \theta(1 - \tau)Y - \phi P$. Output Y is measured in trillions of USD per year and pollution P in ppm, so θ has units ppm per (trillion USD · year). (For $\ln c$, consumption can be viewed as scaled by a fixed numeraire so c is dimensionless in the utility index; reported dollar levels below are for interpretation.) The following values are used for calibration:

Emissions intensity θ: Match today's observed net increase with no abatement:

$$\dot{P} = \theta(1 - \tau)Y - \phi P \tag{A7}$$

Using $Y \approx 100$, $P \approx 420$, $\tau \approx 0$, $\phi \approx 0.01\,\mathrm{yr}^{-1}$, and $\dot{P} \approx 2\,\mathrm{ppm/yr}$:

$$2 = \theta \cdot 100 - 0.01 \cdot 420 \Rightarrow \theta = \frac{2 + 4.2}{100} \approx 0.062\,\mathrm{ppm/(trillion\ USD \cdot yr)} \tag{A8}$$

Demography (g_0, ψ): Make the safe pollution threshold a demographic fixed point:

$$g_0 - \psi \overline{P} = 0 \quad \Rightarrow \quad \psi = \frac{g_0}{\overline{P}}. \tag{A9}$$

(E.g., with $g_0 = 0.02\,\mathrm{yr}^{-1}$ and $\overline{P} = 450$ ppm, $\psi \approx 4.44 \times 10^{-5}\,\mathrm{yr}^{-1}$ per ppm.)

Steady-state pollution and abatement: In steady state,

$$\dot{P} = 0 \Rightarrow \theta(1 - \tau^*)Y^* = \phi P^*. \tag{A10}$$

Using $P^* = \frac{g_0}{\psi}$, the implied interior abatement share is

$$\tau^* = 1 - \frac{\phi}{\theta}\frac{g_0}{\psi Y^*}. \tag{A11}$$

Baseline values (illustrative):

Depreciation $\delta = 0.05\,\mathrm{yr}^{-1}$; investment share $s \approx 0.24$; ρ small (e.g., 0.01).

Natural removal $\phi = 0.01\ \mathrm{yr}^{-1}$.
Emissions intensity $\theta \approx 0.062$ (from the calibration above).
Steady-state output $Y^* \approx 80$ (trillion USD/yr).
Safe/steady pollution $P^* \approx \overline{P}$ (e.g., ~ 450 ppm) with $g_0 - \psi P^* = 0$.
Steady-state population $N^* \approx 5.0 \times 10^9$.
Abatement share $\tau^* \approx 0.33$.
Per capita consumption c^* corresponds to roughly $16,000/year.

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