

Article

Dynamic Subsidy Design for Sustainable Fresh Agricultural Supply Chains: A Differential Game Approach

Linrong Zhou ^{1,2}, Guangxing Wei ¹, Mengying Feng ^{3,4,*} and Yiwei Duan ^{5,*}

¹ School of Economics & Management, Chongqing Jiaotong University, Chongqing 400074, China; linrongpj@163.com (L.Z.); wx777@126.com (G.W.)

² School of Management, Zunyi Normal University, Zunyi 563006, China

³ School of Economics & Management, Guangdong University of Petrochemical Technology, Maoming 525000, China

⁴ Faculty of Business & Law, Roehampton University, London SW15 5PJ, UK

⁵ Andrew Yong School of Policy Studies, Georgia State University, Atlanta, GA 30324, USA

* Correspondence: mengying.feng@roehampton.ac.uk (M.F.); yduan7@gsu.edu (Y.D.)

Abstract

Fresh agricultural products are highly perishable, and inadequate preservation leads to food loss and supply chain inefficiency, undermining sustainability. This study develops a continuous-time differential game model to analyze dynamic pricing and cold-chain investment decisions in a two-echelon fresh agricultural produce supply chain under government intervention. Two subsidy regimes are examined: one targeting suppliers' cold-chain investments and another supporting the retailer based on sales volume. By explicitly modeling the dynamic evolution of product freshness, we analyze how subsidy intensity and allocation influence firms' strategies, market outcomes, and social welfare over time. The results show that when initial freshness is low, firms consistently adopt a penetration pricing strategy and increase cold-chain investment irrespective of subsidy intensity. In contrast, when initial freshness is high, a critical subsidy threshold emerges: Below this threshold, firms employ skimming pricing and reduce investment, whereas above it, they switch to penetration pricing and raise investment. Under equal government expenditure, supplier subsidies achieve higher product freshness but raise retail prices, while retailer subsidies lower prices and stimulate demand, albeit with more modest freshness improvements. Welfare effects are non-linear: supplier subsidies are more effective at low intensities, whereas retailer subsidies become superior beyond a specific threshold. These findings provide actionable insights for designing sustainable, targeted subsidy policies in fresh agricultural supply chains.

Keywords: sustainable fresh agricultural product supply chain; government subsidies; dynamic pricing; cold-chain investment; differential game

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1. Introduction

With the continuous improvement in living standards, consumers have become increasingly concerned about the quality of agricultural products. Among various quality attributes, freshness is widely regarded as a critical indicator of agricultural product quality and plays a central role in shaping consumers' evaluations and purchasing decisions [1,2]. In general, consumers exhibit a strong preference for products with higher freshness levels. Empirical evidence supports this observation: according to Bizrate Insights,

approximately 55% of consumers identify freshness as their primary concern when purchasing agricultural products.

Existing literature establishes that consumer utility in the context of fresh agricultural products is jointly determined by price and freshness [3–5]. Specifically, utility functions are negatively correlated with price and positively correlated with freshness [3]. While the academic literature lacks a unified definition of freshness, it is widely operationalized through sensory attributes directly perceived by the consumer, such as appearance, color, texture, flavor, and vitality. For example, consumers typically evaluate vegetables based on visual cues, aquatic products by their vitality, and meat products by their texture and color. Regardless of subjective variations in perception, consumer utility consistently declines as these physical attributes deteriorate. Consequently, freshness commands a market premium, with consumers demonstrating a willingness to pay higher prices for superior quality [1,6]. These findings underscore the critical role of freshness not only in driving economic demand but also in enhancing the sustainability of agricultural supply chains through reduced product loss and improved resource efficiency.

However, the perishable and damage-prone nature of fresh agricultural products leads to substantial economic losses throughout the supply chain, posing a critical operational and sustainable challenge [7]. China, for instance, is the world's largest producer of fresh agricultural products. Despite this advantage, the sector continues to experience severe post-harvest losses due to the wide variety of products, non-standardized specifications, heterogeneous preservation requirements, and insufficient cold-chain infrastructure. Inadequate freezing and refrigeration equipment, together with underdeveloped preservation technologies, result in annual post-harvest loss rates of 15–25% for products such as fruits, vegetables, and tubers across all supply chain stages. These losses amount to nearly 200 million tons per year and correspond to economic losses of approximately 300 billion RMB [6]. Such substantial losses not only erode the profitability of suppliers and retailers but also constrain the expansion and sustainability of fresh agricultural product markets. Reducing post-harvest losses and stimulating consumption, therefore, requires greater investment in cold-chain infrastructure to enhance product freshness and quality [8]. In practice, suppliers must adopt dynamic preservation strategies—such as temperature-control management, pre-cooling, and insulated packaging—based on the evolving freshness of agricultural products. Retailers, in turn, need to actively promote fresh products to stimulate market demand. However, for suppliers, the rapidly rising costs associated with cold-chain investments are often difficult to recover, weakening incentives to maintain high freshness levels. Similarly, retailers face higher operating costs when engaging in promotional activities, which may discourage demand expansion.

Recognizing these challenges, the Chinese government has introduced a series of policy initiatives to reduce agricultural losses and accelerate the circulation of fresh products from rural production areas to urban markets. China's Central Document No. 1 (2021) explicitly called for accelerating the construction of a preservation and cold-chain logistics system to comprehensively promote the consumption of fresh agricultural products. More recently, the 2025 Implementation Plan for Promoting Agricultural Product Consumption emphasized strengthening regional capacities for pre-cooling, drying, storage, and preservation on the supply side, while supporting the standardized renovation of wet markets and fresh supermarkets on the sales side to expand the consumption. These policies aim to provide financial support for supply chain participants to invest in cold-chain preservation technologies and enhance market demand. At the regional level, targeted subsidy programs further illustrate the diversity of government interventions. For example, in 2024, Hunan Province introduced the *Implementation Plan for the Construction of Cold Storage and Preservation Facilities in Agricultural Production Areas*, offering suppliers a 30% subsidy for investments in cold storage and preservation facilities. In 2025, Ningxia

implemented the *Agricultural Industry Support Policy*, providing rewards and subsidies equal to 2% of product sales to enterprises engaging in online live-stream sales through e-commerce platforms such as Taobao and JD.com. These differentiated policy instruments highlight the coexistence of supply-side and demand-side subsidies, underscoring the need to systematically evaluate their relative effectiveness in promoting sustainable fresh agricultural product supply chains.

Against this background, this paper investigates dynamic pricing and cold-chain investment decisions in a fresh agricultural product supply chain under two alternative government subsidy schemes: subsidies directly toward suppliers' cold-chain investments and subsidies provided to retailers based on sales volume. Because market demand depends jointly on freshness and price, supply chain members must determine optimal dynamic pricing and preservation investment strategies to maximize profits under government interventions. Although existing studies have examined pricing and cold-chain investment in fresh agricultural product supply chains, several important research gaps remain.

First, prior research often overlooks the lagged effects of preservation investment on market demand and the dynamic evolution of freshness, which is jointly driven by cold-chain investment and natural deterioration over time. Second, the influence of government subsidies on suppliers' cold-chain investment decisions and retailers' pricing strategies has not been sufficiently incorporated into existing analytical frameworks. Third, relatively few studies systematically compare the differential impacts of alternative subsidy schemes within a dynamic setting, despite the long-term nature of freshness preservation and policy intervention.

Accordingly, this paper addresses the following research questions: (1) What are the optimal pricing and preservation investment strategies under supplier-subsidy and retailer-subsidy schemes? (2) How do government subsidies influence firms' pricing and cold-chain investment decisions over time? (3) Which subsidy scheme is more effective in improving supply-chain performance and social welfare?

To address these questions, we develop a two-echelon fresh agricultural product supply chain model in which a supplier acts as the leader and a retailer as the follower. The supplier, as the dominant party, determines the wholesale price and the level of cold-chain investment, while the retailer sets the retail price in response to the supplier's decisions. In the model, the supplier maintains product freshness through continuous cold-chain investment, and the retailer purchases products from the supplier and sells them to end consumers. We focus on how government subsidies reshape firms' pricing and investment strategies and compare the effectiveness of the two subsidy schemes in terms of supply chain performance and social welfare. The main findings show that when product initial freshness is low, firms consistently adopt a penetration pricing strategy and increase cold-chain investment regardless of subsidy intensity. In contrast, when initial freshness is high, a critical subsidy threshold emerges. Below this threshold, firms employ skimming pricing alongside lower investment levels; once subsidy intensity exceeds the threshold, they switch to penetration pricing and significantly increase cold-chain investment. Under an equal level of government expenditure, two subsidy regimes generate distinct outcomes. Supplier subsidies lead to higher product freshness but also result in higher retail prices, whereas retailer subsidies reduce prices and expand market demand, though with more modest improvements in freshness. Importantly, the welfare effects of these subsidy schemes are non-linear: supplier subsidies dominate at low subsidy intensities, while retailer subsidies become more welfare-enhancing once subsidy intensity surpasses a critical level.

This study makes four main contributions to the literature. First, it develops a dynamic framework for joint pricing and cold-chain investment decisions in fresh produce

supply chains under both supplier-subsidy and retailer-subsidy regimes. By explicitly modelling the dynamic interactions among supply chain participants, the framework captures how pricing strategies and preservation investments co-evolve over time. Second, the study systematically analyzes the intertemporal effects of government subsidies on firms' pricing and cold-chain investment decisions. It demonstrates how subsidy policies influence the strategic choice between penetration and skimming pricing, and their corresponding investment levels. Furthermore, it reveals that these strategies pivot sharply at a critical subsidy threshold when initial freshness is high. This finding offers actionable insights for coordinated decision-making in fresh produce supply chains. Third, the paper elucidates the differential impacts of subsidy allocation on operational decisions. By contrasting supplier-targeted and retailer-targeted subsidies, it highlights an inherent trade-off: supplier subsidies enhance product freshness at the expense of higher consumer prices, whereas retailer subsidies lower prices and stimulate demand but deliver more modest freshness improvements. Finally, the study derives clear policy implications for subsidy design based on social welfare analysis. It demonstrates the non-linear effectiveness of both subsidy schemes and establishes a pivotal policy criterion. Specifically, supplier subsidies are more effective for foundational quality enhancement under limited budgets, whereas retailer subsidies become superior for welfare maximization once subsidy intensity exceeds a critical threshold. These results provide concrete guidance for designing targeted and efficient agricultural support and cold-chain development policies.

The remainder of this paper is organized as follows: Section 2 reviews the relevant literature. Section 3 presents the model assumptions and problem formulation. Section 4 develops and solves the differential game-theoretic models under alternative subsidy schemes. Section 5 discusses the analytic results. Section 6 provides a case study, and Section 7 concludes the paper and outlines directions for future research.

2. Literature Review

This study reviews the relevant literature from two complementary perspectives: (i) modeling approaches for perishable supply chains, particularly fresh agricultural product supply chains, and (ii) government subsidy policies supporting cold-chain investment and sustainability in such supply chains.

2.1. Modeling Research in Fresh Agricultural Product Supply Chains

Fresh agricultural products are essential to human well-being, and their markets continue to expand rapidly. The inherent perishability of these products, together with increasing demand for high-quality and sustainable food systems, has spurred extensive methodological innovation in fresh supply chain research.

A growing body of literature employs diverse analytical and empirical approaches to address key operational and strategic challenges. For example, Cai & Ye [9] develop a two-stage heuristic framework to optimize equitable and efficient distribution of fresh produce under rigid institutional constraints, validated using a post-pandemic lockdown case study. Beltramo et al. [10] utilize a choice experiment with latent class analysis to examine Dutch consumers' willingness to pay for white asparagus featuring health, nutrition, and origin claims, identifying three distinct consumer segments and their determining factors. Han et al. [11] review how emerging digital technologies, such as IoT, AI, and blockchain, enable smart closed-loop cold chains, analyzing advances in sensing, control, and collaboration while also highlighting critical integration challenges. Yuchi et al. [12] propose a customized heuristic algorithm to optimize logistics network design and vehicle departure schedules, thereby reducing waiting times, quality deterioration, and distribution costs in fresh e-commerce contexts. Huang [13] constructs a tripartite evolutionary game model to analyze collaboration among e-commerce platforms, logistics

providers, and regulators, demonstrating how regulatory incentives shape cooperation stability. Cao & Sun [14] employ a game-theoretic framework and further explore the interaction between sourcing strategies and blockchain adoption under competitive dynamics. Collectively, these studies advance fresh supply chain research methodologies across logistics optimization, digital transformation, consumer behavior, and strategic modeling, laying a solid conceptual and analytical foundation for industry progress.

Nevertheless, important limitations persist. While insightful, empirical studies like Cai & Ye [9] and Beltramo et al. [10] are inherently constrained in capturing real-time strategic interactions under disruptive conditions. Algorithm-based logistics optimization models (e.g., Yuchi et al. [12]) often overlook the decentralized strategic inter-dependencies among supply chain decision-makers. Similarly, game-theoretic frameworks such as Stackelberg or evolutionary games (e.g., Huang [13]; Cao & Sun [14]) typically focus on discrete operational stages and lack the adaptability to model continuously evolving environments. Consequently, existing research provides limited insight into dynamic coordination and joint optimization between suppliers and retailers when product freshness deteriorates over time, and operational decisions require continuous adjustment.

Given that product freshness evolves continuously over time, an increasing number of studies adopt explicit dynamic analytical frameworks. Two main methodological streams dominate this literature. The first stream applies multi-stage dynamic programming to examine operational decisions such as dynamic pricing, inventory control, and replenishment policies under perishability and uncertainty [15–20]. Although these models generate valuable intertemporal insights, they often assume a single decision-maker or centralized optimization, abstracting away from strategic interactions among upstream and downstream firms. As a result, they may not fully capture the decentralized and strategic nature of fresh agricultural product supply chains, particularly when multiple stakeholders pursue conflicting objectives.

The second stream employs differential game theory, which offers a powerful framework for modelling long-term, continuous-time strategic interactions among multiple agents. Differential games are particularly suitable for supply chains characterized by long-term interdependence, conflict, and cooperation [21]. They allow decision-makers to adjust strategies dynamically over time and facilitate equilibrium characterization in evolving environments [22]. Consequently, differential game models have been increasingly applied to study long-term operational and sustainability-related decisions in fresh agricultural product supply chains [23].

Recent differential game studies examine a wide range of dynamic issues, including preservation investment, emission reduction, and technology adoption. For instance, Ma et al. [24], Wang et al. [25], Wang & Xu [26], and Liu et al. [27] develop differential game models to analyze how supply chain members invest in fresh-preservation and low-carbon technologies, and their impact on overall performance. Jiang et al. [28] formulate a differential game model to devise timely inventory control strategies that address advertising delay effects in the fresh produce supply chain. Luo et al. [29] develop a Stackelberg differential game to examine how shadow prices influence optimal decisions of both governments and enterprises. Liu et al. [30] employ a differential game framework to investigate blockchain adoption, showing how it affects freshness-preservation efforts, platform advertising, and freshness outcomes, with goodwill and reference freshness serving as moderating factors. Contractual coordination mechanisms with such dynamic settings have also been explored, including revenue-sharing to achieve coordination in dual-channel fresh product supply chains [31] and a “freshness-keeping cost-sharing-two-part tariff” contract designed to incentivize all parties to increase preservation efforts and achieve improvements [32]. Furthermore, Luo et al. [6,33] apply differential game models to

explore dynamic pricing and coordination mechanisms under traceability-based reputation effects.

Overall, the application of differential game models in fresh supply chain research primarily focuses on internal coordination, sustainability investment for emission reduction, or technology adoption issues such as blockchain implementation and traceability. Although Luo et al. [6,33] examine differential games to analyze pricing dynamics, their emphasis lies on contractual price differentials rather than the joint and strategic interaction between dynamic pricing and preservation investment decisions. More importantly, government subsidy interventions, which play a critical role in cold-chain development, remain insufficiently integrated into these dynamic frameworks that jointly consider pricing, cold-chain investment, and supply chain performance in the fresh agricultural product supply chain. This identified gap motivates the present study.

2.2. Government Subsidies in Fresh Agricultural Product Supply Chains

Government subsidies constitute a pivotal instrument for promoting cold-chain development and enhancing sustainability in fresh agricultural product supply chains [34]. Existing studies demonstrate that subsidy schemes can significantly influence firms' preservation investment and pricing behaviors, thereby affecting product freshness, market demand, and overall supply chain performance.

A substantial stream of research investigates subsidy effectiveness in stimulating preservation and cold-chain investment, and pricing. Zi et al. [35] show that subsidies for cold-chain facilities can significantly improve product freshness. Wu et al. [36] and Zhang et al. [37] further confirm that government subsidies can encourage cold-chain investment and enhance supply chain profitability, although such interventions may lead to higher retail prices of fresh agricultural products. Dan et al. [38] examine how subsidies on wholesale market transaction fees can mitigate price volatility and improve market stability. In digital sales channels, Su and Zheng [39] reveal that subsidizing live-streaming hosts on fresh-produce e-commerce platforms benefits upstream suppliers and induces greater preservation efforts. Ren et al. [34] compare centralized decision-making with supplier myopia and farsightedness, highlighting implications for quality stability.

Beyond investment incentives and subsidy effectiveness, policy outcomes depend critically on subsidy targets and policy design. Zhang et al. [40] and Qin et al. [41] show that different subsidy schemes may generate heterogeneous impacts across supply chain members. Zhang et al. [40] compare subsidies directed toward suppliers, retailers, and consumers, concluding that retailer and consumer subsidies yield similar outcomes, while supplier subsidies improve supply chain efficiency when per-unit subsidy levels are relatively low. Guo et al. [42] examine the effects of government cost subsidies on encouraging the adoption of green and efficient raw materials. Their findings indicate that when the effectiveness of raw materials is relatively high, subsidizing farmers directly is more effective than subsidizing downstream firms. Mardiana & Mahata [43] show that government subsidies for manufacturers' emission-reduction investment can generate Pareto improvements in both economic and environmental performance in deteriorating product supply chains.

Other studies evaluate subsidy regimes from a broader performance perspective. Zhong et al. [44] contrast scenarios without subsidies, subsidies to cooperatives, and subsidies to consumers, finding that subsidizing cooperatives is most effective in improving preservation effort, sales volume, and supply chain profits. Cao et al. [45] show that retailer subsidies consistently benefit retailers and total supply chain performance, whereas manufacturer subsidies primarily improve manufacturers' outcomes but may not improve retailers' outcomes or total supply chain performance. Qin et al. [41] incorporate fiscal budget constraints and demonstrate that, in the absence of budget limitations,

procurement subsidies outperform production subsidies in enhancing consumer surplus and social welfare, while under budget constraints, subsidizing enterprises achieves higher fiscal efficiency than directly subsidizing farmers. Shi et al. [46] find that subsidies supporting green technology investment outperform operational subsidies in promoting environmentally sustainable agricultural production. Shi et al. [47] show that, compared with green investment subsidies, agricultural insurance subsidies generate higher profits for both farmers and agro-dealers and consistently motivate farmers to expand their planting acreage. He & Yang [48] demonstrate that non-specific one-off government subsidies may stimulate firms' preservation investment more effectively than targeted emission-reduction subsidies.

Despite these valuable contributions, the majority of existing studies adopt static or comparative-static analytical frameworks [35–42,44–47], which are inherently limited in capturing the long-term dynamics of freshness deterioration, preservation investment, and pricing decisions. While some recent work incorporates dynamic modelling, key gaps persist. For instance, Ren et al. [34] and Mardiyana & Mahata [43] integrate government subsidies into a differential game setting, yet they typically focus on a single subsidy scheme and emphasize emission reduction or aggregate supply chain performance, rather than the joint evolution of pricing and freshness. He & Yang [48] employ a differential game approach to analyze profits, emission-reduction investment, and preservation investment under alternative subsidy schemes; however, their model abstracts from pricing decisions—a critical omission given that product freshness directly influences pricing and, consequently, market demand and supply-chain sustainability. As cold-chain investments generate delayed effects and product freshness evolves continuously over time, an integrated dynamic framework that simultaneously incorporates pricing, preservation investment, and government intervention remains underdeveloped. In contrast, our study examines dynamic pricing and cold-chain investment decisions under distinct government intervention schemes, including subsidies directed toward suppliers' preservation efforts and subsidies provided to retailers based on sales performance, within a continuous-time dynamic setting. By embedding these policies into a unified analytical framework, our analysis fills an important gap in the sustainability-oriented perishable supply chain literature.

Accordingly, this paper models a bilateral fresh produce supply chain consisting of one supplier and one retailer. Employing a differential game framework, we investigate dynamic pricing strategies and cold-chain investment decisions under distinct government subsidy schemes. Specifically, we analyze how key parameters—such as subsidy intensity and freshness characteristics—influence optimal strategic trajectories. Finally, we compare the policy efficacy of subsidizing different supply chain members to identify the most effective intervention strategies.

3. Problem Description and Basic Assumptions

3.1. Problem Description

This study considers a fresh agricultural product supply chain consisting of one supplier and one retailer operating under government subsidy intervention. To promote the sustainable development of the fresh produce market and mitigate post-harvest losses and food waste, the government implements two distinct subsidy schemes, extending financial support to either the supplier or the retailer. These subsidies are intended to incentivize investment in cold-chain preservation technologies and to stimulate market demand for fresh agricultural products.

Driven by both government policy incentives and consumers' demand preferences for freshness, supply chain members are motivated to improve product freshness levels. In the modeled supply chain, the supplier is responsible for investing in preservation and

cold-chain technologies, such as pre-cooling, refrigeration, and temperature-controlled transportation. These investments directly determine the level of product freshness, which in turn affects market demand and pricing decisions. Because cold-chain investment involves long-term costs and strategic considerations, the supplier bears a greater decision-making burden in determining the evolution of product freshness over time. Accordingly, we model the supplier as the leader and the retailer as the follower within a differential game framework. In this setting, the supplier determines both the level of freshness effort through cold-chain investment and the wholesale price of the product, while the retailer observes these decisions and sets the retail price in response. Both parties seek to maximize their respective discounted profit streams over an infinite planning horizon.

To examine how alternative subsidy schemes reshape firms' dynamic pricing and investment strategies, this framework explicitly incorporates government subsidies into the decision-making process. Two specific schemes are considered: subsidies directed toward suppliers' preservation investments and sales-based subsidies provided to retailers. The model captures the intertemporal trade-offs between preservation effort, pricing decisions, and market demand, thereby providing a dynamic perspective on the effectiveness of government intervention in promoting sustainable fresh agricultural product supply chains. The variables and parameters used in the model are defined in Table 1.

Table 1. List of notations.

Notation	Definition
Decision variables	
$u(t)$	Supplier's cold-chain investment at time t
$w(t)$	Wholesale price at time t
$p(t)$	Retail price at time t
Parameters and other variables	
$g(t)$	Freshness level at time t , $g(0) = g_0$
$Q(t)$	Demand for fresh products at time t
$c(t)$	Unit cost for fresh product suppliers at time t
$C_{u(t)}$	Cost for fresh product supplier's cold-chain investment at time t
$GS(t)$	Expenditure on government subsidies at time t
$CS(t)$	Consumer surplus at time t
$SW(t)$	Social welfare at time t
α	Potential market size
β	Price elasticity coefficient
δ	Coefficient of Consumer Freshness Preference
ε	Coefficient of the cold-chain investment
σ	Attenuation coefficient of the fresh degree
θ	Freshness input cost factor
ρ	The time discount factor is the discount rate
c_0, c_1	Supplier initial unit costs and marginal operating cost
J_s, J_r	Supplier and retailer profits
s_1, s_2	Subsidy coefficient for the supplier and retailer
Superscript	
S	The situation of subsidizing suppliers
R	The situation of subsidizing retailers
Subscript	
s	The supplier
r	The retailer

3.2. Relevant Assumptions

To capture the dynamic characteristics of freshness preservation and government intervention in fresh agricultural product supply chains, we introduce the following assumptions, which are grounded in industry practice and consistent with existing literature.

Assumption 1. *Freshness dynamics: the freshness of agricultural products depends both on the investment in preservation technologies and the initial freshness level.*

Consumers exhibit a strong preference for agricultural products with high freshness, which motivates suppliers to actively invest in preservation technologies to enhance product quality and attract demand [33,49]. Due to advancements in preservation technology, which can continuously ensure the freshness of agricultural products, dynamically describing changes in freshness is more realistic. However, agricultural products are inherently perishable, and their freshness naturally deteriorates due to biological decay. In short, the freshness of agricultural products increases not only with greater investment in cold-chain (preservation) technologies but also depends on the initial freshness level when the products arrive at the sales terminals (such as supermarkets and community convenience stores). Therefore, over the life cycle of fresh agricultural products $[0, T]$, which encompasses the entire process from production to final sale, the evolution of product freshness can be expressed as:

$$\dot{g}(t) = \varepsilon u(t) - \sigma g(t), \quad g(0) = g_0 \quad (1)$$

where g_0 is the initial level of product freshness. This formulation captures the combined effects of preservation investment and natural deterioration on freshness dynamics.

Assumption 2. *Variable production cost and preservation-related operating cost: The unit cost consists of basic production and marginal operating expenses to improve freshness, whereas the cold-chain input cost follows a quadratic function of the investment level.*

Investment in fresh-keeping and cold-chain technologies improves product freshness but also generates additional costs, such as product handling, transportation, and temperature-control expenses [6,15]. In particular, higher freshness levels require greater preservation efforts, resulting in higher unit production and operating costs for suppliers. Therefore, the variable unit cost of fresh produce suppliers can be expressed as:

$$c(t) = c_0 + c_1 g(t) \quad (2)$$

where c_0 denotes the supplier's basic unit production cost and c_1 represents the marginal operating cost associated with cold-chain investment to improve freshness. For example, Liu et al. [16] highlight that refrigerated transport is often characterized by high empty backhaul rates, resulting in resource underutilization and escalated marginal cold-chain costs. To ensure economically meaningful equilibrium outcomes, the parameters in Equation (2) must satisfy standard regularity conditions: $0 < c_1 < \delta / \beta$. That is, the marginal operating cost must remain below the ratio of the consumer freshness preference coefficient to the price elasticity coefficient. A larger value of c_1 indicates lower efficiency in the preservation process and higher marginal costs of freshness improvement.

In the field of operation management, the cold-chain input cost is assumed to be a quadratic function [1,16,30,50], i.e., the supplier's cold-chain input cost function can be expressed as:

$$C_{u(t)} = \frac{1}{2} \theta u^2(t) \quad (3)$$

where is the supplier's input cost factor for preservation. Equation (3) is a quadratic function relationship between the cold-chain cost and the cold-chain input of the fresh agricultural product supply chain operation enterprise. The convex cost function describes the diminishing marginal returns of the operating enterprise's freshness input [4,30,32], indicating that in order to achieve a certain level of freshness, the operating enterprise needs to bear lower costs initially, but in order to achieve a higher level of freshness, fresh agricultural product supply chain operators need to bear more costs.

Assumption 3. *The effect of agri-food freshness on market demand is positive.*

Considering that the agricultural product market demand is jointly influenced by the retail price $p(t)$ and the freshness $g(t)$ of agricultural products, and consumers' preference for freshness increases their willingness to purchase high-freshness agricultural products, there is a positive relationship between freshness and demand for fresh agricultural products. The higher the freshness, the higher the demand. Therefore, the demand function for fresh agricultural products can be expressed as:

$$Q(t) = a - \beta p(t) + \delta g(t) \quad (4)$$

To ensure positive equilibrium solutions, the model parameters of Equation (4) should satisfy the standard condition proposed by Luo et al. [33], i.e., that $a > \beta c_0$. Linear demand functions of this form are widely used in fresh agricultural product supply chain studies, such as Luo et al. [6], Liu et al. [49], and Yang & Tang [51].

Assumption 4. *Government subsidy schemes: supplier subsidy and retailer subsidy.*

The government implements two alternative subsidy policies (S and R) to promote fresh agricultural product circulation and preservation investment. Supplier subsidy (S) refers to the fact that the government subsidizes a fixed proportion of the supplier's cold-chain investment cost. This approach has been adopted in studies such as those by Zhang et al. [37] and Shi et al. [46]. The government subsidy expenditures at time t are $GS^S(t) = s_1 \theta u^2(t) / 2$, where s_1 denotes the subsidy coefficient for supplier investment. Retailer subsidy (R) means that the government subsidizes the retailer based on sales volume, consistent with Qin et al. [41] and Guo et al. [42]. At time t , government subsidy expenditure is $GS^R(t) = s_2 g(t)[a - \beta p(t) + \delta g(t)]$, where s_2 denotes the per-unit consumption subsidy coefficient. These two policies represent supply-side and demand-side interventions commonly observed in practice.

Assumption 5. *Social welfare depends on four components.*

Following Atasu et al. [52] and Zhang et al. [53], social welfare is defined as the sum of four key components: the profit of the supplier $V_s(t)$, the profit of the retailer $V_r(t)$, consumer surplus $CS(t)$, and government subsidy expenditures $GS(t)$. That is $SW(t) = V_s(t) + V_r(t) + CS(t) - GS(t)$, where, based on the research by Panda et al. [54], the consumer surplus is $CS(t) = (\alpha - \beta p(t) + \delta g(t))^2 / (2\beta)$.

Assumption 6. *Profit.*

The total profit of the fresh agricultural supply chain in the fresh produce lifecycle $[0, T]$ is discounted by the discount factor ρ ($\rho > 0$).

Based on the above assumptions, under the condition that the government does not provide subsidies, the profit functions of suppliers and sellers throughout the fresh produce lifecycle $[0, T]$ are:

$$\max_{u,w} J_s = \int_0^{\infty} e^{-\rho t} \left([w(t) - c_0 - c_1 g(t)] [\alpha - \beta p(t) + \delta g(t)] - \frac{1}{2} \theta u^2(t) \right) dt \quad (5)$$

$$\max_p J_r = \int_0^{\infty} e^{-\rho t} ([p(t) - w(t)] [\alpha - \beta p(t) + \delta g(t)]) dt \quad (6)$$

Based on Equations (5) and (6), the profit functions of supply chain members under the scenarios of subsidizing suppliers and subsidizing retailers throughout the entire operation cycle are given in Sections 4.1 and 4.2, respectively. For convenience of writing, parts of the modelling are omitted.

4. Model Construction and Solution

This section develops Stackelberg differential game models for a fresh agricultural product supply chain under two alternative government subsidy policies: supplier subsidies (scenario S) and retailer subsidies (scenario R). In both scenarios, the supplier acts as the dominant decision-maker and the retailer behaves as a follower. Specifically, the supplier first determines the wholesale price and the level of freshness effort through cold-chain investment over time t . Given the supplier's decisions, the retailer determines the retail price at time t . The dynamic evolution of product freshness follows the state equation specified in Equation (2), with the initial freshness level at the initial time denoted by g_0 . In the following subsections, we formulate and solve the differential game models under the two subsidy regimes and derive the equilibrium pricing and investment strategies.

4.1. The Scenario of Subsidizing Suppliers (S)

In scenario S, the government provides subsidies to suppliers by covering a proportion of their cold-chain investment costs. This policy is designed to incentivize suppliers to increase investment in cold chain infrastructure, thereby enhancing product freshness and quality and reducing post-harvest losses. Taking government subsidies into account, the suppliers' and retailers' objective functions under this scenario are formulated as follows.

$$\max_{u,w} J_s = \int_0^{\infty} e^{-\rho t} \left([w(t) - c_0 - c_1 g(t)] [\alpha - \beta p(t) + \delta g(t)] - \frac{1}{2} \theta (1 - s_1) u^2(t) \right) dt, \quad (7)$$

$$\max_p J_r = \int_0^{\infty} e^{-\rho t} ([p(t) - w(t)] [\alpha - \beta p(t) + \delta g(t)]) dt. \quad (8)$$

Theorem 1. *The equilibrium results (Appendix A) under the government subsidizing suppliers scenario are*

(1) *The optimal cold-chain investment, wholesaling price, and retailing price are as follows for the fresh agricultural supply chain members, respectively:*

$$u_{\infty}^{S*} = \frac{\varepsilon(A_2 + 2A_1 g^{S*})}{\theta(1 - s_1)}, \quad (9)$$

$$w^{S*} = \frac{\alpha + \beta c_0}{2\beta} + \frac{(\delta + \beta c_1)g^{S*}}{2\beta}, \quad (10)$$

$$p^{S*} = \frac{3\alpha + \beta c_0}{4\beta} + \frac{(3\delta + \beta c_1)g^{S*}}{4\beta}. \quad (11)$$

(2) The optimal trajectory for product freshness retention is

$$g^{S*} = g_{\infty}^{S*} + e^{-v^S t} (g_0 - g_{\infty}^{S*}) \quad (12)$$

among the formulas, $v^S = \frac{1}{2} \left[\sqrt{(\rho + 2\sigma)^2 - \frac{\varepsilon^2 (\delta - \beta c_1)^2}{\beta \theta (1 - s_1)}} - \rho \right] > 0$ denotes the evolution rate under the supplier subsidy scenario, $\Delta_1 = \sqrt{\beta \theta (1 - s_1) [\rho \theta (1 - s_1) (\rho + 2\sigma)^2 - \varepsilon^2 (\delta - \beta c_1)^2]}$, $g_{\infty}^{S*} = \frac{\beta \theta \varepsilon^2 (\alpha - \beta c_0) (\delta - \beta c_1) (1 - s_1)}{\Delta_1^2 - \beta^2 \theta^2 \rho^2 (1 - s_1)^2}$ denotes the steady-state level of agricultural product freshness under the supplier subsidy policy.

(3) The profits of fresh agricultural product suppliers and retailers are as follows, respectively.

$$\begin{aligned} J_s^S &= A_1 g^{S*2} + A_2 g^{S*} + A_3 \\ J_r^S &= B_1 g^{S*2} + B_2 g^{S*} + B_3 \end{aligned} \quad (13)$$

$$\begin{aligned} \text{where } A_1 &= \frac{\beta \theta (1 - s_1) (\rho + 2\sigma) - \Delta_1}{4\beta \varepsilon^2}, & A_2 &= \frac{\theta (\alpha - \beta c_0) (\delta - \beta c_1) (1 - s_1)}{2[\rho \beta \theta (1 - s_1) + \Delta_1]}, \\ A_3 &= \frac{(\alpha - \beta c_0)^2}{8\rho} \left\{ \frac{1}{\beta} + \frac{\theta \varepsilon^2 (\delta - \beta c_1)^2 (1 - s_1)}{[\rho \beta \theta (1 - s_1) + \Delta_1]^2} \right\}, & B_1 &= \frac{\theta (\delta - \beta c_1)^2 (1 - s_1)}{16\Delta_1}, \\ B_2 &= \frac{(\alpha - \beta c_0) (\delta - \beta c_1) \{ \beta \theta^2 \varepsilon^2 (\delta - \beta c_1)^2 (1 - s_1)^2 + 2\Delta_1 \theta (1 - s_1) [\rho \beta \theta (1 - s_1) + \Delta_1] \}}{8\Delta_1 [\rho \beta \theta (1 - s_1) + \Delta_1]^2}, & \text{and} \\ B_3 &= \frac{(\alpha - \beta c_0)^2}{\rho} \left\{ \frac{1}{16\beta} + \frac{\theta \varepsilon^2 (1 - s_1) (\delta - \beta c_1)^2}{8[\rho \beta \theta (1 - s_1) + \Delta_1]^2} + \frac{\beta \theta^2 \varepsilon^4 (1 - s_1)^2 (\delta - \beta c_1)^4}{16\Delta_1 [\rho \beta \theta (1 - s_1) + \Delta_1]^3} \right\}. \end{aligned}$$

An examination of Theorem 1 reveals that, under the supplier subsidy scenario, the cold-chain investment level, wholesale price, and retail price all exhibit linear relationships with product freshness. This indicates that both suppliers and retailers take changes in freshness into account when determining wholesale and retail prices. Moreover, the profits of both the supplier and the retailer are convex functions of product freshness, implying that their marginal profits increase as freshness improves. These results further suggest that the implementation of fresh supply chain management practices enhances the ability of supply chain members to generate economic returns.

Theorem 2. Under the supplier subsidy scenario, the optimal evolution trajectories (Appendix A) of the supply chain's cold-chain investment level, wholesale price, and retail price are given by:

$$u^{S*}(t) = \frac{\varepsilon A_2 + 2\varepsilon A_1 g_{\infty}^{S*}}{\theta(1 - s_1)} + \frac{2\varepsilon A_1 e^{-v^S t} (g_0 - g_{\infty}^{S*})}{\theta(1 - s_1)}, \quad (14)$$

$$w^{s*}(t) = \frac{\alpha + \beta c_0 + (\delta + \beta c_1)g_\infty^{s*}}{2\beta} + \frac{\delta + \beta c_1}{2\beta}(g_0 - g_\infty^{s*})e^{-v^\delta t}, \quad (15)$$

$$p^{s*}(t) = \frac{3\alpha + \beta c_0 + (3\delta + \beta c_1)g_\infty^{s*}}{4\beta} + \frac{3\delta + \beta c_1}{4\beta}(g_0 - g_\infty^{s*})e^{-v^\delta t}. \quad (16)$$

An examination of the optimal evolution trajectories of cold-chain investment, wholesale price, and retail price reveals that suppliers and retailers in the supply chain may adopt either skimming pricing or penetration pricing strategies. This is consistent with research findings by Lu et al. [3] and Luo et al. [33]. The selection of pricing strategy, however, depends on the degree of freshness improvement, specifically defined as the relationship between the initial freshness condition (g_0) and the steady-state equilibrium (g_∞^{s*}). When the initial freshness level exceeds the steady state freshness level ($g_0 > g_\infty^{s*}$), cold-chain investment, wholesale price, and retail price decrease monotonically, indicating that suppliers and retailers should adopt a skimming pricing strategy and reduce cold-chain investment. Conversely, when the initial freshness level is lower than the steady-state freshness level ($g_0 < g_\infty^{s*}$), cold-chain investment, wholesale price, and retail price increase monotonically over time, suggesting that suppliers and retailers should adopt a penetration pricing strategy and increase cold-chain investment. In addition, when the initial freshness level equals the steady-state freshness level ($g_0 = g_\infty^{s*}$), the levels of cold-chain investment, wholesale price, and retail price remain constant over time.

4.2. The Scenario of Subsidizing Retailers (R)

In scenario R, the government will provide subsidies to retailers based on the sales volume of fresh agricultural products. Therefore, the objective function in this case is represented as follows.

$$\max_{u,w} J_s = \int_0^\infty e^{-\rho t} \left([w(t) - c_0 - c_1 g(t)][\alpha - \beta p(t) + \delta g(t)] - \frac{1}{2} \theta u^2(t) \right) dt, \quad (17)$$

$$\max_p J_r = \int_0^\infty e^{-\rho t} ([p(t) - w(t) + \lambda g(t)][\alpha - \beta p(t) + \delta g(t)]) dt. \quad (18)$$

Theorem 3. *The equilibrium results (Appendix A) under the government-subsidizing-retailers scenario are*

(1) *The optimal cold-chain investment level, wholesale price, and retail price in the fresh agricultural product supply chain are, respectively, given by:*

$$u_\infty^{R*} = \frac{\varepsilon(X_2 + 2X_1 g^{R*})}{\theta}, \quad (19)$$

$$u_\infty^{R*} = \frac{\varepsilon(X_2 + 2X_1 g^{R*})}{\theta}, \quad (20)$$

$$p^{R*} = \frac{3\alpha + \beta c_0}{4\beta} + \frac{(3\delta + \beta c_1 - \beta s_2)g^{R*}}{4\beta}. \quad (21)$$

(2) *The optimal trajectory for product freshness retention is*

$$g^{R*} = g_{\infty}^{R*} + e^{-v^R t} (g_0 - g_{\infty}^{R*}). \quad (22)$$

Among the formulas,
$$v^R = \frac{1}{2} \left[\sqrt{(\rho + 2\sigma)^2 - \frac{\varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2}{\beta \theta}} - \rho \right] > 0$$
 denotes the evolution rate under the supplier subsidy scenario
$$\Delta_2 = \sqrt{\beta \theta [\beta \theta (\rho + 2\sigma)^2 - \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]},$$
 and
$$g_{\infty}^{R*} = \frac{\beta \theta \varepsilon^2 (\alpha - \beta c_0) (\delta - \beta c_1 + \beta s_2)}{\Delta_2^2 - \beta^2 \theta^2 \rho^2}$$
 denotes the steady-state level of agricultural product freshness under the supplier subsidy policy.

(3) The profits of fresh agricultural product suppliers and retailers are as follows, respectively.

$$\begin{aligned} J_s^R &= X_1 g^{R*2} + X_2 g^{R*} + X_3 \\ J_r^R &= Y_1 g^{R*2} + Y_2 g^{R*} + Y_3 \end{aligned} \quad (23)$$

where

$$\begin{aligned} X_1 &= \frac{\beta \theta (\rho + 2\sigma) - \Delta_2}{4\beta \varepsilon^2}, & X_2 &= \frac{\theta (\alpha - \beta c_0) (\delta - \beta c_1 + \beta s_2)}{2(\rho \beta \theta + \Delta_2)}, \\ X_3 &= \frac{(\alpha - \beta c_0)^2}{8\rho} \left[\frac{1}{\beta} + \frac{\theta \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2}{(\rho \beta \theta + \Delta_2)^2} \right], & Y_1 &= \frac{\theta (\delta - \beta c_1 + \beta s_2)^2}{16\Delta_2}, \\ Y_2 &= \frac{(\alpha - \beta c_0) (\delta - \beta c_1 + \beta s_2) \{ \beta \theta^2 \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2 + 2\Delta_2 \theta (\rho \beta \theta + \Delta_2) \}}{8\Delta_2 (\rho \beta \theta + \Delta_2)^2}, & & \\ Y_3 &= \frac{(\alpha - \beta c_0)^2}{\rho} \left\{ \frac{1}{16\beta} + \frac{\theta \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2}{8(\rho \beta \theta + \Delta_2)^2} + \frac{\beta \theta^2 \varepsilon^4 (\delta - \beta c_1 + \beta s_2)^4}{16\Delta_2 (\rho \beta \theta + \Delta_2)^3} \right\}. \end{aligned}$$

An examination of Theorem 3 shows that, under the retailer subsidy scenario, the cold-chain investment level, wholesale price, and retail price are also positively correlated with product freshness. This indicates that improvements in product freshness incentivize suppliers to increase cold-chain investment, which in turn supports higher product pricing. Moreover, the optimal profit functions of both the supplier and the retailer are convex with respect to product freshness, suggesting that increases in freshness enhance the economic profits of supply chain members.

Theorem 4. Under the retailer subsidy scenario, the optimal evolution trajectories (Appendix A) of the supply chain's cold-chain investment level, wholesale price, and retail price are given by:

$$u^{R*}(t) = \frac{\varepsilon X_2 + 2\varepsilon X_1 g_{\infty}^{R*}}{\theta} + \frac{2\varepsilon X_1 e^{-v^R t} (g_0 - g_{\infty}^{R*})}{\theta}, \quad (24)$$

$$w^{R*}(t) = \frac{\alpha + \beta c_0 + (\delta + \beta c_1 + \beta s_2) g_{\infty}^{R*}}{2\beta} + \frac{\delta + \beta c_1 + \beta s_2}{2\beta} (g_0 - g_{\infty}^{R*}) e^{-v^R t}, \quad (25)$$

$$p^{R*}(t) = \frac{3\alpha + \beta c_0 + (3\delta + \beta c_1 - \beta s_2) g_{\infty}^{R*}}{4\beta} + \frac{3\delta + \beta c_1 - \beta s_2}{4\beta} (g_0 - g_{\infty}^{R*}) e^{-v^R t}. \quad (26)$$

An examination of Theorem 4 shows that, similar to the supplier-subsidy scenario, two pricing strategies—skimming pricing and penetration pricing—also arise under the retailer-subsidy scenario. However, the choice of pricing strategy depends on the relative magnitude of the product's initial freshness and its steady-state freshness. When the initial

freshness exceeds the steady-state freshness ($g_0 > g_\infty^{R*}$), supply chain members should adopt a skimming pricing strategy. In this case, as product freshness declines over time, firms should gradually reduce prices and decrease cold-chain investment. Conversely, when the initial freshness is lower than the steady-state freshness ($g_0 < g_\infty^{R*}$), supply chain members should adopt a penetration pricing strategy. Under this condition, as product freshness increases over time, firms should progressively raise prices and expand cold-chain investment. Furthermore, when the initial freshness equals the steady-state freshness ($g_0 = g_\infty^{R*}$), the cold-chain investment level, wholesale price, and retail price remain constant over time.

5. Comparative Analysis and Discussion

The previous section has established the optimal pricing and preservation investment strategies under both supplier-subsidy and retailer-subsidy schemes. This section addresses the remaining two research questions of this study. This section first presents a sensitivity analysis of the equilibrium strategies under different government subsidy schemes. Specifically, it examines how key parameters—including consumer preference for fresh agricultural products, government subsidy intensity, and marginal operating costs—affect product freshness, cold-chain investment, and pricing. It then investigates the influence of government subsidies on the supply chain's pricing and cold chain investment adjustment strategies. Finally, it provides a comparative analysis of the equilibrium strategies, supplier and retailer profits, consumer surplus, and social welfare under equal government expenditure.

Proposition 1. Under different government subsidy scenarios, the steady-state freshness of agricultural products g_∞^{S*} and g_∞^{R*} has the following relationships with consumer preference for fresh products δ , government subsidy intensity s_1 and s_2 , and marginal operating costs c_1

$$(Appendix A): \frac{\partial g_\infty^{S*}}{\partial \delta} > 0, \frac{\partial g_\infty^{S*}}{\partial s_1} > 0, \frac{\partial g_\infty^{S*}}{\partial c_1} < 0; \frac{\partial g_\infty^{R*}}{\partial \delta} > 0, \frac{\partial g_\infty^{R*}}{\partial s_2} > 0, \frac{\partial g_\infty^{R*}}{\partial c_1} < 0$$

Proposition 1 indicates that under different government subsidy scenarios, the freshness of agricultural products is positively correlated with both the strength of consumer preference for fresh products and the level of government subsidies, but negatively correlated with the firm's marginal operating costs. Enhancing consumer preference for freshness and increasing government subsidies can incentivize firms to strengthen supply chain management, thereby improving product freshness. However, when increasing freshness significantly raises unit production costs, firms with lower operational efficiency may choose to moderately reduce freshness standards to control costs. Among these factors, consumer preference and government subsidies are external drivers, while operational efficiency is an internal constraint. Therefore, decision-makers in agricultural product supply chains need to consider both internal and external factors holistically to optimize the operation of fresh product supply chains.

Proposition 2. Under different government subsidy scenarios, the supply chain's cold chain investment u_∞^{S*} and u_∞^{R*} has the following relationships with consumer preference for fresh products δ , government subsidy intensity s_1 and s_2 , and marginal operating costs c_1 (Appendix A):

$$\frac{\partial u_\infty^{S*}}{\partial \delta} > 0, \frac{\partial u_\infty^{S*}}{\partial s_1} > 0, \frac{\partial u_\infty^{S*}}{\partial c_1} < 0; \frac{\partial u_\infty^{R*}}{\partial \delta} > 0, \frac{\partial u_\infty^{R*}}{\partial s_2} > 0, \frac{\partial u_\infty^{R*}}{\partial c_1} < 0$$

Proposition 2 indicates that under different government subsidy scenarios, the level of cold chain investment in the supply chain is positively correlated with both the strength of consumer preference for fresh products and the level of government subsidies, but negatively correlated with the firm's marginal operating costs. As consumers' demand for product freshness increases and government subsidies intensify, firms are more incentivized to invest in cold chain infrastructure. However, if a firm has lower operational efficiency, its willingness to invest in the cold chain may weaken. This is because, on one hand, the firm must bear higher cold chain preservation costs; on the other hand, this also raises the unit production cost of agricultural products, thereby increasing operational management pressure.

Proposition 3. Under different government subsidy scenarios, the steady-state sales price p_{∞}^{S*} and p_{∞}^{R*} has the following relationships with consumer preference for fresh products δ , government subsidy intensity s_1 and s_2 , and operational inefficiency c_1 (Appendix A): $\frac{\partial p_{\infty}^{S*}}{\partial \delta} > 0$, $\frac{\partial p_{\infty}^{S*}}{\partial s_1} > 0$, $\frac{\partial p_{\infty}^{S*}}{\partial c_1} < 0$; $\frac{\partial p_{\infty}^{R*}}{\partial \delta} > 0$, $\frac{\partial p_{\infty}^{R*}}{\partial s_2} > 0$, $\frac{\partial p_{\infty}^{R*}}{\partial c_1} < 0$.

Proposition 3 indicates that under different government subsidy scenarios, the sales price of fresh agricultural products is positively correlated with both the strength of consumer preference for freshness and the level of government subsidies, but negatively correlated with the firm's marginal operating costs. This is because, in order to better meet consumers' freshness preferences, firms increase cold chain investment to enhance product freshness, which raises their costs. To maintain profitability, suppliers need to increase their wholesale prices, which are then transmitted along the supply chain, prompting retailers to raise their sales prices accordingly. Similarly, an increase in government subsidies encourages greater cold chain investment by suppliers, but the subsidies are insufficient to fully cover the increased investment costs. As a result, suppliers raise wholesale prices, and retailers correspondingly increase sales prices. Conversely, if a firm has lower operational efficiency, it may reduce cold chain investment; the resulting decrease in supplier costs leads to lower wholesale prices, and retailers will adjust their sales prices downward accordingly.

Proposition 4. Under the scenario where the supplier receives subsidies, the supplier and retailer adopt the following pricing and cold chain investment strategies: When $g_0 \leq \bar{g}_0$, for any $s_1 \in (0, \bar{s}_1)$, both the supplier and retailer adopt a penetration pricing strategy and increase cold chain investment. When $g_0 > \bar{g}_0$, there exists a threshold $\hat{s}_1$ that triggers a shift in pricing strategy: Specifically, if $s_1 \in (0, \hat{s}_1)$ is bound to hold, supply chain members should adopt a skimming pricing strategy and reduce cold chain investment; whereas if $s_1 \in (\hat{s}_1, \bar{s}_1)$ is bound to hold, supply chain members should adopt a penetration pricing strategy and increase cold chain investment (Appendix A). Here, $\bar{s}_1 = \frac{4\beta\theta\sigma(\rho + \sigma) - \varepsilon^2(\delta - \beta c_1)^2}{4\beta\theta\sigma(\rho + \sigma)}$, $\bar{g}_0 = \frac{\varepsilon^2(\alpha - \beta c_0)(\delta - \beta c_1)}{4\beta\theta\sigma(\rho + \sigma) - \varepsilon^2(\delta - \beta c_1)^2}$, $\hat{s}_1 = \frac{2\{g_0[4\beta\theta\sigma(\rho + \sigma) - \varepsilon^2(\delta - \beta c_1)^2] - \varepsilon^2(\alpha - \beta c_0)(\delta - \beta c_1)\}}{g_0[8\beta\theta\sigma(\rho + \sigma) - \varepsilon^2(\delta - \beta c_1)^2] + \varepsilon\sqrt{g_0(\delta - \beta c_1)[g_0\varepsilon^2(\delta - \beta c_1)^3 + 16\beta\theta\sigma(\rho + \sigma)(\alpha - \beta c_0)]}}$.

Proposition 5. Under the scenario where the retailer receives subsidies, suppliers and retailers will adopt the following pricing and cold chain investment strategies: When $g_0 \leq \bar{g}_0$, for any $s_2 \in (0, \bar{s}_2)$, both suppliers and retailers adopt a penetration pricing strategy and increase their cold

chain investment levels. When $g_0 < \bar{g}_0$, there exists a threshold $\hat{s}_2$ that triggers a shift in pricing strategy: if $s_2 \in (0, \hat{s}_2)$ is bound to hold, supply chain members should adopt a skimming pricing strategy and reduce cold chain investment levels. If $s_2 \in (\hat{s}_2, \bar{s}_2)$ is bound to hold, supply chain members should adopt a penetration pricing strategy and increase cold chain investment levels (Appendix A). Here, $\bar{s}_2 = \frac{2\sqrt{\beta\theta\sigma(\rho+\sigma)} - \varepsilon(\delta - \beta c_1)}{\beta\varepsilon}$, $\bar{g}_0 = \frac{\varepsilon^2(\alpha - \beta c_0)(\delta - \beta c_1)}{4\beta\theta\sigma(\rho+\sigma) - \varepsilon^2(\delta - \beta c_1)^2}$, and $\hat{s}_2 = \frac{4\{g_0[4\beta\theta\sigma(\rho+\sigma) - \varepsilon^2(\delta - \beta c_1)^2] - \varepsilon^2(\alpha - \beta c_0)(\delta - \beta c_1)\}}{2\beta\varepsilon[\varepsilon(\alpha - \beta c_0) + 2g_0\varepsilon(\delta - \beta c_1) + \sqrt{16g_0^2\beta\theta\sigma(\rho+\sigma) + \varepsilon^2(\alpha - \beta c_0)^2}]}$.

By integrating Propositions 4 and 5, it can be seen that under different government subsidy scenarios, changes in the subsidy intensity lead supply chain members of the agricultural products to adopt different pricing and cold chain investment strategies. These strategies are related to the threshold of agricultural product freshness, where $\bar{s}_0$ serves as a critical value. When the baseline freshness of agricultural products is lower than $\bar{s}_0$, even modest cold chain investments by firms yield significant freshness gains, regardless of the magnitude of government subsidies s_1 and s_2 . Government subsidies effectively offset investment costs, encouraging supply chain members to increase cold chain investment and adopt a penetration pricing strategy to quickly capture market share, thereby increasing their profits. However, when the initial freshness of agricultural products is higher than $\bar{s}_0$, an increase in the intensity of government subsidies will initially lead supply chain members to reduce cold chain investments and adopt a skimming pricing strategy. Subsequently, they will increase cold chain investments and switch to a penetration pricing strategy. This is because when initial freshness is already high, further improvements require disproportionately larger investments for diminishing returns. In this case, as subsidy intensity increases, firms initially reduce cold chain investment and adopt a skimming pricing strategy to capitalize on existing freshness and maximize short-term profits. Once subsidies exceed a critical threshold that compensates for the high marginal cost of further freshness enhancement, firms find it profitable to reinvest in cold chains and switch to a penetration pricing strategy to expand the market.

This indicates that when the initial freshness of agricultural products is high, the pricing strategies of supply chain members are influenced by the threshold of government subsidy intensity, transitioning from a skimming strategy to a penetration strategy. It is noteworthy that while increased government subsidies can incentivize firms to enhance cold chain investments and improve the freshness of agricultural products, this may also lead to a corresponding increase in the selling price of agricultural products.

Proposition 6. *The comparative analyses of product freshness, cold-chain investment, and pricing across different subsidy schemes under equal government expenditure are presented below (Appendix A): $u_\infty^{S*} > u_\infty^{R*}$, $g_\infty^{S*} > g_\infty^{R*}$, $p_\infty^{S*} > p_\infty^{R*}$.*

Proposition 6 indicates that subsidizing suppliers is more conducive than subsidizing retailers in promoting cold chain investment in the supply chain and enhancing the freshness of agricultural products. Consequently, under supplier subsidies, suppliers raise wholesale prices to offset the increased cold-chain investment costs. These higher wholesale prices are transmitted along the supply chain, ultimately resulting in higher retail prices compared with the retailer-subsidy case. From the consumer perspective, although subsidizing suppliers implies higher prices, the greater improvement in agricultural product freshness results in a more favorable freshness-adjusted price-performance ratio than under retailer subsidies.

Proposition 7. *The comparative analyses of supplier profits, retailer profits, consumer surplus, and social welfare across different subsidy schemes under equal government expenditure are presented below (Appendix A): when government subsidy intensity is relatively low ($s_1 < 0.25$), $V_{s\infty}^{S*} > V_{s\infty}^{R*}$, $V_{r\infty}^{S*} > V_{r\infty}^{R*}$, $CS_{\infty}^{S*} > CS_{\infty}^{R*}$, and $SW_{\infty}^{S*} > SW_{\infty}^{R*}$; when government subsidy intensity is relatively high ($s_1 > 0.25$), $V_{s\infty}^{S*} < V_{s\infty}^{R*}$, $V_{r\infty}^{S*} < V_{r\infty}^{R*}$, $CS_{\infty}^{S*} < CS_{\infty}^{R*}$, and $SW_{\infty}^{S*} < SW_{\infty}^{R*}$.*

Proposition 7 indicates that when government subsidy intensity is relatively low, subsidizing suppliers is more effective than subsidizing retailers in enhancing supplier and retailer profits, consumer surplus, and social welfare. When government subsidy intensity is relatively high, subsidizing retailers outperforms subsidizing suppliers in terms of enhancing supplier and retailer profits, consumer surplus, and social welfare. This demonstrates that the effectiveness of subsidy schemes differs across subsidy intensity levels.

This occurs because supplier subsidies specifically incentivize cold-chain investment, manifesting primarily as improved freshness. Conversely, sales-based retailer subsidies exert a broader influence, affecting both pricing decisions and freshness outcomes. When subsidy intensity is low, the price differential between the two subsidy schemes is modest, while the freshness level is markedly higher under the supplier-subsidy scenario. In this case, consumers exhibit a willingness to pay a slight premium for the superior freshness, leading to larger overall market demand under supplier subsidies. Consequently, supplier profits, retailer profits, consumer surplus, and social welfare all exceed those achieved under retailer subsidies. Conversely, under high subsidy intensity, the supplier subsidy results in a significantly higher increase in retail prices compared to the retailer subsidy. Although freshness remains higher, consumers' marginal willingness to pay for the incremental freshness diminishes. This results in lower market demand for the supplier-subsidy case relative to the retailer-subsidy alternative. As a result, supplier profits, retailer profits, consumer surplus, and social welfare under supplier subsidies fall below the corresponding levels attained under retailer subsidies.

6. Case Study

To verify the validity and logical consistency of the theoretical propositions, this section presents a case study of a Chinese agricultural firm, organized into four subsections. The Section 6.1 offers a comprehensive overview of the case company and its operational context. The Section 6.2 examines the supply chain's pricing and cold-chain investment decisions under the two distinct government subsidy schemes. The Section 6.3 investigates how subsidy intensity influences retail prices, investment levels, and product freshness. Finally, based on Assumption 5, we construct a multidimensional evaluation framework to compare the effectiveness of the two subsidy approaches across three dimensions: firm-level profit, consumer surplus, and overall social welfare.

6.1. XWA Cold-Chain Investment Strategies

XWA (Xue Wang Agriculture (Chongqing) Co., Ltd.), located in Chongqing Municipality, China, is primarily engaged in the cultivation, procurement, sorting, and cold-chain distribution of fresh lemons. Since 2021, in response to growing consumer demand for freshness, the company has continuously upgraded its cold-chain infrastructure to enhance product quality and extend shelf life. Under ambient conditions, lemons typically have a shelf life of approximately three months. To address this limitation, XWA invested CNY 300 million in the construction of a 120,000 m² cold storage facility. By adopting advanced storage and bagging technologies and equipping its logistics system with

refrigerated trucks and containers, the company established an integrated cold-chain storage and transportation network, extending the shelf life of lemons to up to one year.

Through sustained technological innovation in cold-chain preservation, AWA has gained widespread recognition among consumers for product quality and freshness. The company supplies nearly 100,000 tons of fresh lemons annually, achieving an annual output value of approximately CNY 550 million [55]. XWA has also developed close partnerships with downstream retailers, enabling delivery of fresh lemons to domestic markets within 48 h. A proportion of its production is also exported to international markets, including Indonesia and Russia. In 2024, the company was officially included in the Chongqing Municipality's list of key agricultural cold-chain logistics base construction projects [56]. In the same year, the Chongqing municipal government provided a subsidy covering 50% of the company's total cold-chain investment.

To align with our theoretical framework developed in this study, empirical data from XWA were standardized following the methodology adopted by Wei et al. [57]. The key parameters are as follows: potential market size $\alpha = 10$, price elasticity coefficient $\beta = 2$, freshness input cost factor $\theta = 3$, supplier initial unit costs $c_0 = 1$, coefficient of consumer freshness preference $\delta = 1$, coefficient of the cold-chain investment $\varepsilon = 1$, discount rate $\rho = 0.1$, and attenuation coefficient of the fresh degree $\sigma = 0.4$. The planning horizon for the fresh produce life cycle is set as $[0, T]$, with $T = 30$.

To ensure the generality of the study, c_1 is deemed to satisfy: $0 \leq c_1 < \delta / \beta$, namely $c_1 \in [0, 0.5)$. According to Proposition 4, we can obtain $s_1 \in [0, 0.8]$. To address the comparability issue between the two subsidy scenarios, it is assumed that the government subsidy expenditures are equal under different subsidy policies, namely, $s_1 \theta u^{s^*2} / 2 = s_2 g^{R^*} Q^{R^*}$. Under the condition, by varying the value of s_1 over the interval of $s_1 \in [0, 0.8]$ in steps of 0.1, the converted values of the corresponding subsidy coefficients are shown in Table 2.

Table 2. The converted values of subsidy coefficients under equal expenditures.

s_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
s_2	0	0.0047	0.0113	0.0204	0.0335	0.0530	0.0834	0.1361	0.2450

6.2. Analysis of Pricing and Cold Chain Investment Strategies

This section examines pricing and cold-chain investment strategies under different government subsidy schemes. Based on the benchmark parameter settings and incorporating Propositions 4 and 5, we obtain $\bar{g}_0 = 0.3361$. The effects of the two subsidy policies are analyzed by examining how changes in the subsidy intensity s_1 and s_2 , influence pricing decisions under two scenarios: (i) a low initial level of product freshness, where $g_0 \leq \bar{g}_0$ ($g_0 = 0$), and (ii) a high initial level of freshness, where $g_0 > \bar{g}_0$ ($g_0 = 0.8$). The corresponding price dynamics under the two subsidy schemes are illustrated in Figures 1–4.

Figures 1 and 2 show that when the initial level of product freshness is low, take $g_0 = 0$ as an example, firms consistently benefit from adopting a penetration pricing strategy combined with higher levels of cold-chain investment, regardless of the government subsidy intensity. In this case, improving product freshness and stimulating market demand at an early stage of the selling period are crucial for enhancing supply chain profitability. In contrast, when the initial level of freshness is high, take $g_0 = 0.8$ as an example, we obtain $\hat{s}_1 = 0.575$ and $\hat{s}_2 = 0.138$, which indicates that the intensity of government subsidies becomes a key determinant of pricing strategy. Figures 3 and 4 show that when

$s_1 < 0.575$ and $s_2 < 0.138$, the retail price of agricultural products decreases over time until reaching a steady-state, implying that supply chain members should adopt a price-skimming strategy. Conversely, when $s_1 > 0.575$ and $s_2 > 0.138$, the retail price increases over time and converges to a steady-state, suggesting that all supply chain members should implement a penetration pricing strategy.

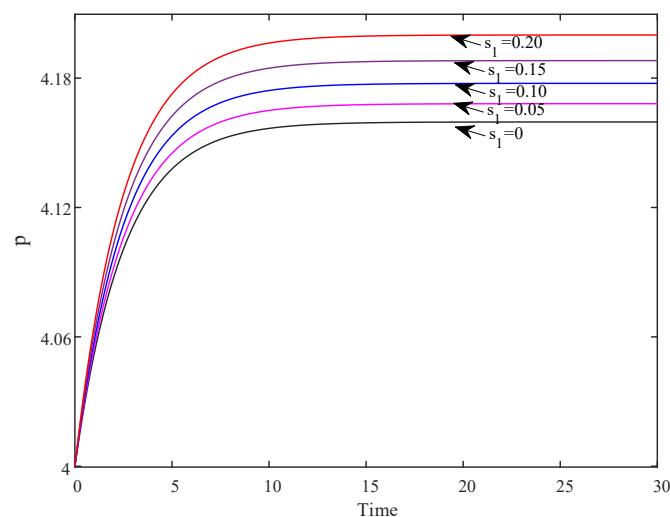


Figure 1. The impact of s_1 on pricing ($g_0 < \bar{g}_0$).

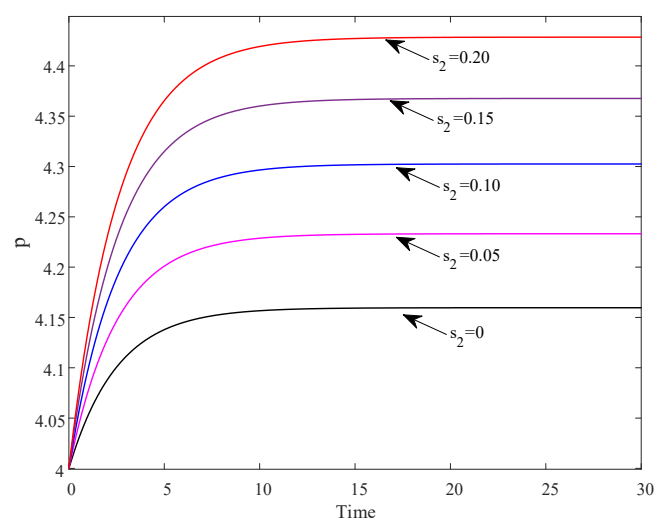


Figure 2. The impact of s_2 on pricing ($g_0 < \bar{g}_0$).

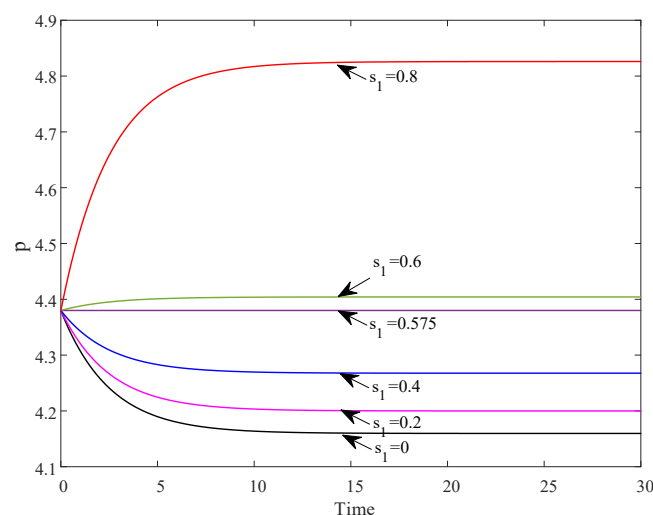


Figure 3. The impact of s_1 on pricing ($\mathcal{G}_0 > \bar{\mathcal{G}}_0$).

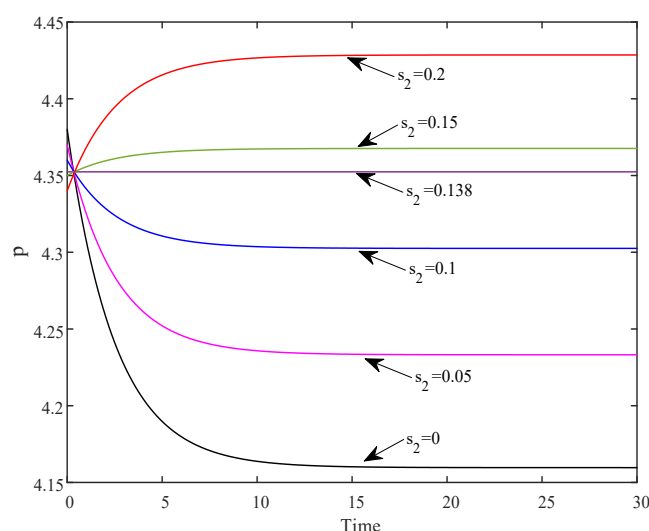


Figure 4. The impact of s_2 on pricing ($\mathcal{G}_0 > \bar{\mathcal{G}}_0$).

Overall, these results imply that when the initial freshness of agricultural products is low, firms should adopt a penetration pricing strategy to maximize economic profits, regardless of the intensity of government subsidies. However, when the initial freshness is high, firms' pricing decisions are contingent on subsidy intensity. In particular, a threshold value of the subsidy coefficient exists, beyond which enterprises optimally shift from a skimming pricing strategy to a penetration pricing strategy.

Additionally, when firms in the fresh product supply chain adopt a penetration pricing strategy, they should appropriately increase their level of cold-chain investment. Conversely, when firms opt for a skimming pricing strategy, a corresponding reduction in cold-chain investment is optimal. This finding indicates that the adjustment and evolutionary trajectories of cold-chain investment and retail prices in the fresh supply chain are closely aligned. Moreover, variations in subsidy intensity do not alter the qualitative conclusions of this study. Therefore, for the sake of clarity, this paper presents only the time evolution of cold-chain investment for the two representative cases of $s_1 = 0.1$ and $s_2 = 0.0047$, as illustrated in Figure 5. As shown in Figure 5, compared with price dynamics, the evolution of cold-chain investment over time is more gradual. This suggests that time exerts a weaker influence on cold-chain investment decisions than on retail prices,

implying that cold-chain investment strategies in the fresh supply chain are relatively stable over time. Figures 1–5 illustrate Propositions 4 and 5.

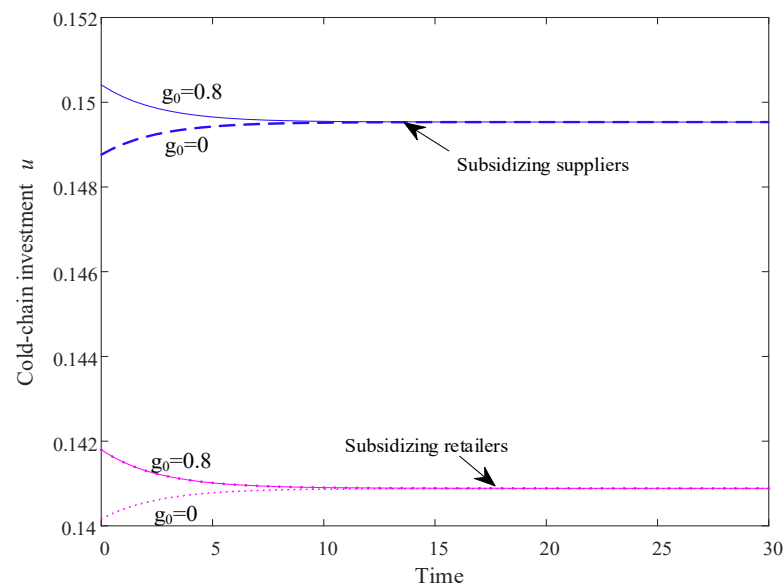


Figure 5. Change in cold-chain investment over time T.

6.3. Analysis of the Steady-State Subsidy Effects

Under the assumption that the government's steady-state expenditures are equal, the effects of subsidy intensity on supply chain freshness, retail prices, cold-chain preservation investment, supplier profit, retailer profit, consumer surplus, and social welfare are reported in Table 3. Several observations can be drawn from Table 3.

(1) Under both subsidy schemes, supply chain freshness, retail prices, cold-chain investment, supplier profits, retailer profits, consumer surplus, and social welfare all increase with government subsidy intensity. This indicates that regardless of whether the government subsidizes suppliers or retailers, government intervention is beneficial to suppliers, retailers, consumers, and overall social welfare.

(2) Under equal government expenditure and from a steady-state perspective, subsidizing suppliers is more conducive than subsidizing retailers in promoting cold chain investment in the supply chain and enhancing the freshness of agricultural products. Consequently, under supplier subsidies, suppliers raise wholesale prices to offset the increased cold-chain investment costs, which illustrates Proposition 6. From the consumer perspective, although subsidizing suppliers implies higher prices, the greater improvement in agricultural product freshness results in a more favorable freshness-adjusted price-performance ratio than under retailer subsidies.

(3) The calculations show that when $s_1 < 0.25$, we obtain $V_{s\infty}^{S*} > V_{s\infty}^{R*}$, $V_{r\infty}^{S*} > V_{r\infty}^{R*}$, $CS_{\infty}^{S*} > CS_{\infty}^{R*}$, and $SW_{\infty}^{S*} > SW_{\infty}^{R*}$; that is, when government subsidy intensity is relatively low, subsidizing suppliers is more effective than subsidizing retailers in enhancing supplier and retailer profits, consumer surplus, and social welfare. When $s_1 > 0.25$, we obtain $V_{s\infty}^{S*} < V_{s\infty}^{R*}$, $V_{r\infty}^{S*} < V_{r\infty}^{R*}$, $CS_{\infty}^{S*} < CS_{\infty}^{R*}$, and $SW_{\infty}^{S*} < SW_{\infty}^{R*}$, which indicates that when government subsidy intensity is relatively high, subsidizing retailers outperforms subsidizing suppliers in terms of enhancing supplier and retailer profits, consumer surplus, and social welfare. This demonstrates that the effectiveness of subsidy schemes differs across subsidy intensity levels, which illustrates Proposition 7.

Table 3. The equilibrium values of two subsidy scenarios under equal expenditures.

s_1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$g_s^{S^*}$	0.3738	0.4211	0.4819	0.5634	0.6780	0.8511	1.1428	1.7391
$g_s^{R^*}$	0.3522	0.3749	0.4062	0.4517	0.5201	0.6290	0.8253	1.2766
$p_s^{S^*}$	4.1776	4.2000	4.2289	4.2676	4.3221	4.4043	4.5428	4.8261
$p_s^{R^*}$	4.1669	4.1770	4.1909	4.2108	4.2402	4.2856	4.3640	4.5282
$u_s^{R^*}$	0.1495	0.1684	0.1928	0.2253	0.2712	0.3404	0.4571	0.6957
$u_s^{R^*}$	0.1409	0.1499	0.1625	0.1807	0.2081	0.2516	0.3301	0.5106
$V_{s\infty}^{S^*}$	40.4493	40.5061	40.5795	40.6776	40.8159	41.0249	41.3779	42.1020
$V_{s\infty}^{R^*}$	40.4432	40.5015	40.5881	40.7254	40.9582	41.3909	42.3573	45.3822
$V_{r\infty}^{S^*}$	20.3756	20.4233	20.4848	20.5673	20.6837	20.8601	21.1592	21.7769
$V_{r\infty}^{R^*}$	20.3705	20.4194	20.4921	20.6076	20.8037	21.1701	21.9961	24.6468
$CS_{s\infty}^{S^*}$	1.0188	1.0212	1.0242	1.0284	1.0342	1.0430	1.0580	1.0888
$CS_{s\infty}^{R^*}$	1.0185	1.0209	1.0246	1.0304	1.0402	1.0585	1.0998	1.2323
$SW_{s\infty}^{S^*}$	61.8403	61.9421	62.0718	62.2429	62.4785	62.8237	63.3758	64.3871
$SW_{s\infty}^{R^*}$	61.8288	61.9334	62.0881	62.3329	62.7469	63.5152	65.2338	70.6807

6.4. Analysis of the Dynamic Effects of Government Subsidy

To analyze the differences in the dynamic effects between subsidizing suppliers and subsidizing retailers, we consider the case where $g_0 = 0$, and $c_1 = 0.4$, with relatively high subsidy intensity, i.e., $s_1 = 0.3$, and $s_2 = 0.0204$. Figures 6–9 sequentially illustrate the time-evolution trajectories of supply chain performance indicators, including agricultural product freshness and social welfare. Observations from Figures 6–9 are summarized as follows.

- (1) Supplier profit, retailer profit, consumer surplus, and social welfare all increase over time and eventually reach a steady state. This indicates that, within a certain time horizon, increasing government subsidy expenditures can enhance the economic profits of supply chain members, as well as consumer surplus and social welfare. However, once the supply chain reaches its steady state, continued government subsidy expenditures at the same subsidy intensity impose a greater fiscal burden on the government without further improving economic profits, consumer surplus, or social welfare. At this point, the marginal efficiency of government subsidies becomes zero, suggesting that subsidy phase-out policies should be considered.
- (2) Over the life cycle of fresh agricultural products $[0, T]$, $V_s^{S^*}(t) < V_s^{R^*}(t)$, $V_r^{S^*}(t) < V_r^{R^*}(t)$, $CS^{S^*}(t) < CS^{R^*}(t)$, $SW^{S^*}(t) < SW^{R^*}(t)$, subsidizing retailers yields higher supplier profits, retailer profit, consumer surplus, and social welfare than subsidizing suppliers. This implies that, from the perspectives of economic profits, consumer surplus, and social welfare, subsidizing retailers is more effective than subsidizing suppliers.
- (3) Throughout the entire operational period, the performance gap between the two subsidy schemes gradually widens over time and reaches its maximum at the steady state. From a long-term perspective, the advantages of subsidizing retailers become more pronounced.

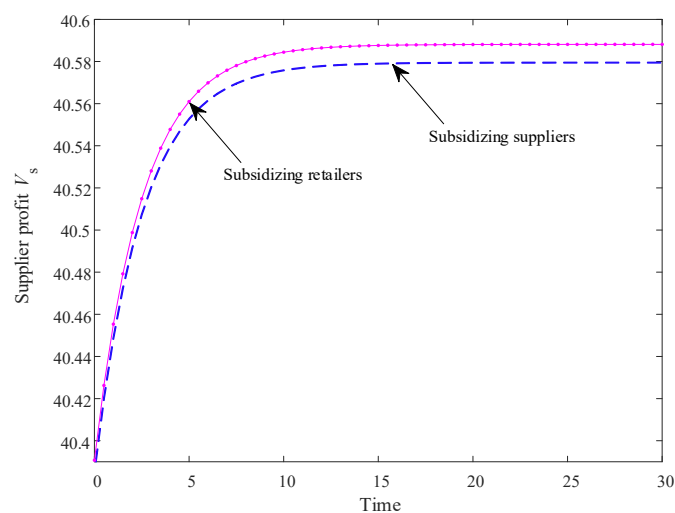


Figure 6. Change in supplier profit over time T .

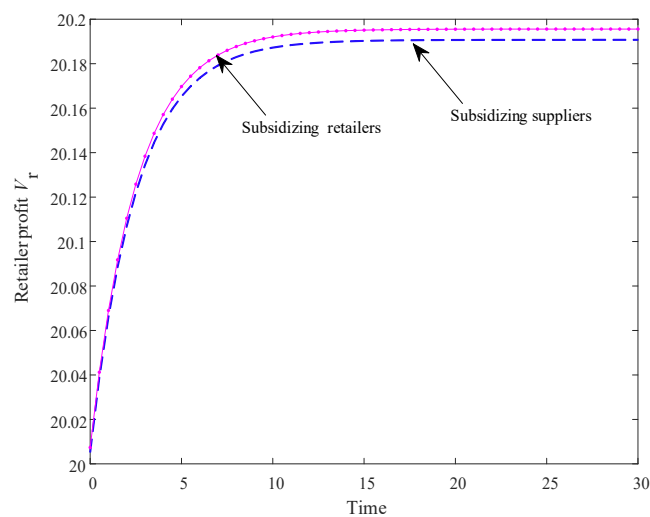


Figure 7. Change in retailer profit over time T .

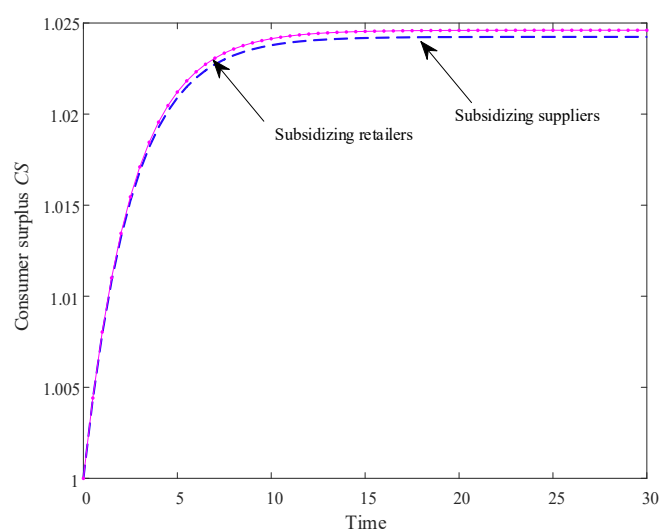


Figure 8. Change in consumer surplus over time T .

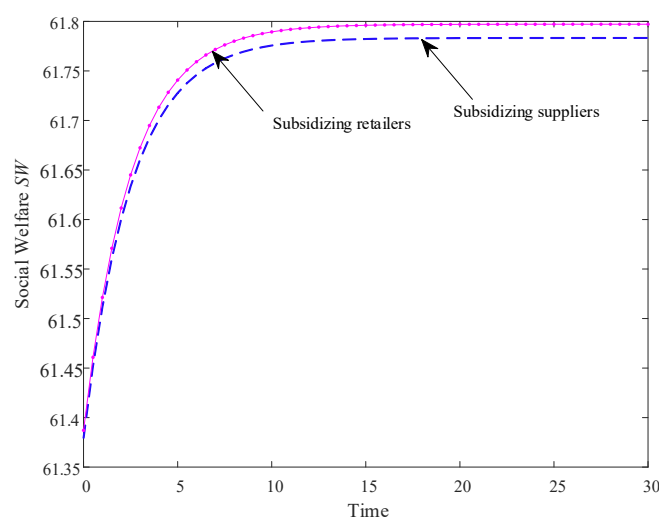


Figure 9. Change in social welfare over time T .

7. Conclusions

This paper examines a fresh agricultural product supply chain consisting of one supplier and one retailer. Using a differential game framework, it analyzes dynamic pricing and cold chain investment decisions under two government subsidy schemes: subsidizing the supplier or subsidizing the retailer. The study explicitly incorporates consumer freshness preference, operational inefficiency coefficient, and government subsidy intensity, and compares the impacts of these factors on the supply chain pricing, cold chain investment, economic profits, consumer surplus, and social welfare. Under the assumption of equal government expenditures across schemes, both steady-state and dynamic perspectives are adopted to comprehensively evaluate the effectiveness of alternative subsidy schemes.

7.1. Main Findings

The main conclusions are as follows:

(1) Under different government subsidy scenarios, when the initial freshness of agricultural products is low, fresh supply chain members should adopt a penetration pricing strategy and increase cold-chain investment, regardless of subsidy intensity. In contrast, when the initial freshness of agricultural products is high, a critical subsidy intensity exists that induces a strategic shift: firms transition from a skimming pricing strategy to a penetration pricing strategy, while cold-chain investment changes from decreasing to increasing. Moreover, in contrast to the pronounced temporal sensitivity of pricing strategies, cold-chain investment decisions exhibit greater stability over the planning horizon.

(2) While consumer freshness preference and government subsidy intensity stimulate higher prices and cold-chain investment, operational inefficiency exerts a negative impact, reducing both pricing levels and cold-chain investment. This finding highlights that effective fresh supply chain management requires firms not only to respond to external drivers such as consumer freshness preference and government subsidies, but to improve internal operational efficiency to reduce marginal costs.

(3) Under equal government expenditure and from a steady-state perspective, subsidizing the fresh product suppliers, relative to subsidizing retailers, more effectively promotes cold chain investment and improves agricultural product freshness, but also leads to higher retail prices. As a result, the economic profits of fresh product supply chain members, consumer surplus, and social welfare are jointly influenced by a subsidy intensity threshold. Specifically, when the government subsidy intensity is relatively low

($s_1 < 0.25$), subsidizing suppliers yields higher profits, consumer surplus, and social welfare; when the government subsidy intensity is relatively high ($s_1 > 0.25$), subsidizing retailers becomes the superior policy option.

(4) Under equal government expenditure and from a dynamic perspective, profits of supply chain members, consumer surplus, and social welfare all increase over time and eventually reach a steady state under both subsidy schemes. Once the steady state is reached, the marginal effectiveness of government subsidies becomes zero, indicating that continued subsidies only increase a fiscal burden without generating additional social benefits. Therefore, a subsidy phase-out policy should be considered. Over the entire operational horizon, the difference in effectiveness between the two subsidy schemes widens over time and reaches its maximum at the steady state.

7.2. Managerial and Policy Implications

This study yields several important implications for both policymakers and supply chain managers.

For policymakers: First, governments should promote cold-chain development through targeted subsidies. They should implement clear, accessible subsidy programs to stimulate investment in cold-chain technology and infrastructure. Such foundational support—targeting both upstream facilities (e.g., refrigerated storage, processing) and downstream market access (e.g., cold transportation, retail refrigeration)—reduces capital barriers, accelerates technology adoption, and enhances the quality and value of agricultural output. This is crucial for reducing post-harvest losses, ensuring food security, and improving overall supply chain sustainability. Second, governments should tailor subsidy allocation to budget intensity to maximize impact. A one-size-fits-all subsidy strategy is suboptimal. Policymakers should adopt a nuanced, evidence-based approach to fund allocation. The findings indicate that at the early stages or with limited budgets (low subsidy intensity), directing subsidies to suppliers is more effective for building foundational cold-chain capacity. Conversely, when greater public funds are available (high subsidy intensity), subsidizing retailers generates stronger market pull, incentivizing the entire chain to align with freshness-driven demand. This strategic allocation maximizes the long-term return on public expenditure. Third, governments should adopt a dynamic and long-term perspective in subsidy design. The effectiveness of subsidies evolves as the supply chain matures. Policies should be framed as dynamic interventions with phased objectives. Initial programs may focus on infrastructure building, while subsequent stages can shift towards performance-based incentives (e.g., tied to verified freshness levels). Recognizing that the full benefits of an optimal subsidy strategy compound over time underscores the need for policy consistency and a commitment to multi-year planning, allowing the supply chain to stabilize into a more efficient and resilient steady-state operation.

For supply chain managers: First, supply chain managers must dynamically adjust their strategies in response to evolving subsidy schemes and market conditions. Operational decisions must be agile. They should continuously monitor the level and target (supplier vs. retailer) of government subsidies, using them as strategic signals. Concurrently, real-time data on product freshness and shelf-life should inform pricing mark-downs or premiums. This requires integrated decision-making models where cold-chain investment levels are calibrated to anticipated subsidy benefits and the resulting inventory freshness state, not just immediate costs. Second, firms should quantitatively incorporate consumer freshness preferences into core operations. Freshness is a primary value driver. Firms should leverage market research and data analytics to precisely quantify how freshness attributes influence willingness-to-pay. This demand sensitivity should be directly embedded into financial models for pricing (e.g., setting freshness-based price

tiers) and capital budgeting (e.g., justifying investments in better preservation technology), ensuring a direct link between customer value creation and operational actions. Third, firms should balance market-driven freshness with operational efficiency. Long-term profitability requires synthesizing external market signals with internal capability constraints. While pursuing higher freshness can capture market share, unbridled investment without regard to operational efficiency (e.g., energy costs of cooling, technology maintenance, logistics coordination) erodes margins. Firms should develop integrated performance metrics that weigh freshness achievement against cost efficiency. This holistic approach ensures that the pursuit of quality is economically sustainable, fostering a competitive advantage that is both market-relevant and operationally resilient.

7.3. Limitations and Future Research

This study has several limitations that suggest directions for future research. First, the model assumes deterministic conditions and does not incorporate stochastic risks in fresh agricultural product supply chains, such as production uncertainty, demand volatility, transportation delays, temperature fluctuations, or supply disruptions. Future work would integrate stochastic elements—for example, through stochastic differential games or robust control frameworks—to enable a more realistic examination of how uncertainty influences dynamic pricing, cold-chain investment decisions, and the effectiveness of alternative subsidy policies. Second, the analysis focuses on supplier-led cold-chain investments and does not account for retailer-side investments in preservation technology or handling costs. Extending the model to incorporate endogenous retailer investment decisions would allow researchers to examine the interplay between upstream and downstream preservation efforts under different subsidy schemes. This extension would provide a more comprehensive understanding of vertical coordination, cost-sharing mechanisms, and responsibility allocation in sustainable fresh produce supply chains. Third, the modeling framework adopts a simplified one-supplier–one-retailer structure. While this parsimonious setting facilitates a tractable analysis of the core dynamic interactions among government subsidy policies, pricing strategies, and preservation investments, it abstracts from important real-world complexities such as competition among multiple suppliers or retailers and potential horizontal interactions. Future research could extend the framework to oligopolistic or network settings to investigate how market competition and multi-agent strategic interactions shape subsidy effectiveness, investment incentives, and overall sustainability performance. Such extensions would enhance the model’s external validity and applicability across a broader range of industrial contexts.

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Data Availability Statement: The data used in this study are publicly available and have been correctly cited. Data sets used or analyzed in the current study are available from the corresponding authors upon reasonable request.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Proof of Theorem 1. This paper employs the Hamilton–Jacobi–Bellman (HJB) equation to address differential game scenarios. The objective functions of fresh produce suppliers and retailers are: $J_s^S = e^{-\rho t} V_s^S$, $J_r^S = e^{-\rho t} V_r^S$. Among them, V_s^S and V_r^S represent the optimal value functions of fresh agricultural product suppliers and retailers, respectively, and the HJB equation can be obtained as:

$$\rho V_s^S = \max_{w,u} \left((w - c_0 - c_1 g)(\alpha - \beta p + \delta g) - \frac{1}{2}(1 - s_1)\theta u^2 + \frac{\partial V_s^S}{\partial g}(\varepsilon u - \sigma g) \right), \quad (A1)$$

$$\rho V_r^S = \max_p \left((p - w)(\alpha - \beta p + \delta g) + \frac{\partial V_r^S}{\partial g}(\varepsilon u - \sigma g) \right). \quad (A2)$$

The problem is solved by using inverse induction method. First, the calculation of the first-order partial derivatives of p in Equation (A2) is as follows.

$$p^S = \frac{\alpha + \delta g + \beta w}{2\beta}. \quad (A3)$$

Based on the retailer's reaction function, the HJB equation for the fresh produce supplier is:

$$\rho V_s^S = \max_{w,u} \left((w - c_0 - c_1 g) \frac{\alpha + \delta g - \beta w}{2} - \frac{1}{2}(1 - s_1)\theta u^2 + \frac{\partial V_s^S}{\partial g}(\varepsilon u - \sigma g) \right). \quad (A4)$$

To maximize the supplier's profit, the first-order conditions with respect to w and u are solved, yielding:

$$w^S = \frac{\alpha + \beta c_0 + (\delta + \beta c_1)g}{2\beta}, \quad (A5)$$

$$u^S = \frac{\varepsilon}{\theta(1 - s_1)} \frac{\partial V_s^S}{\partial g}. \quad (A6)$$

Substituting Equation (A5) into Equation (A3), we get:

$$p^S = \frac{3\alpha + \beta c_0 + (3\delta + \beta c_1)g}{4\beta}. \quad (A7)$$

Substituting the above Equations (A5)–(A7) into the HJB Equations (A2) and (A4) of the supplier and retailer, and assuming V_s^S and V_r^S are quadratics, we obtain the optimal value function as:

$$\begin{aligned} V_s^S &= A_1 g^2 + A_2 g + A_3 \\ V_r^S &= B_1 g^2 + B_2 g + B_3 \end{aligned} \quad (A8)$$

The first partial derivatives of Equation (A8) for g are as follows:

$$\begin{aligned}\frac{\partial V_s^S}{\partial g} &= 2A_1g + A_2 \\ \frac{\partial V_r^S}{\partial g} &= 2B_1g + B_2\end{aligned}\quad (A9)$$

The six Riccati equations are solved using the undetermined coefficient method, and the corresponding six parameters are obtained. Let $A_i, B_i, i = 1, 2, 3$. The six parameters are as follows:

$$\begin{aligned}A_1 &= \frac{\beta\theta(1-s_1)(\rho+2\sigma)-\Delta_1}{4\beta\varepsilon^2}, & A_2 &= \frac{\theta(\alpha-\beta c_0)(\delta-\beta c_1)(1-s_1)}{2[\rho\beta\theta(1-s_1)+\Delta_1]}, \\ A_3 &= \frac{(\alpha-\beta c_0)^2}{8\rho} \left[\frac{1}{\beta} + \frac{\theta\varepsilon^2(\delta-\beta c_1)^2(1-s_1)}{[\rho\beta\theta(1-s_1)+\Delta_1]^2} \right], & B_1 &= \frac{\theta(\delta-\beta c_1)^2(1-s_1)}{16\Delta_1}, \\ B_2 &= \frac{(\alpha-\beta c_0)(\delta-\beta c_1)\{\beta\theta^2\varepsilon^2(\delta-\beta c_1)^2(1-s_1)^2+2\Delta_1\theta(1-s_1)[\rho\beta\theta(1-s_1)+\Delta_1]\}}{8\Delta_1[\rho\beta\theta(1-s_1)+\Delta_1]^2}, \\ B_3 &= \frac{(\alpha-\beta c_0)^2}{\rho} \left\{ \frac{1}{16\beta} + \frac{\theta\varepsilon^2(1-s_1)(\delta-\beta c_1)^2}{8[\rho\beta\theta(1-s_1)+\Delta_1]^2} + \frac{\beta\theta^2\varepsilon^4(1-s_1)^2(\delta-\beta c_1)^4}{16\Delta_1[\rho\beta\theta(1-s_1)+\Delta_1]^3} \right\},\end{aligned}\quad \text{where}$$

$$\Delta_1 = \sqrt{\beta\theta(1-s_1)[\beta\theta(1-s_1)(\rho+2\sigma)^2-\varepsilon^2(\delta-\beta c_1)^2]}.$$

After all the coefficients are obtained, substitute Equation (A9) into Equation (A6) to obtain Equation (12), namely, $g^{S*} = g_\infty^{S*} + e^{-v^S t}(g_0 - g_\infty^{S*})$. Substituting Equation (A8) into $J_s^S = e^{-\rho t}V_s^S, J_r^S = e^{-\rho t}V_r^S$ gives Equation (13). Substituting Equation (12) into Equation (1), and then solving the differential equation, we get

$$v^R = \frac{1}{2} \left[\sqrt{(\rho+2\sigma)^2 - \frac{\varepsilon^2(\delta-\beta c_1+\beta s_2)^2}{\beta\theta}} - \rho \right], \quad \text{and} \quad g_\infty^{R*} = \frac{\beta\theta\varepsilon^2(\alpha-\beta c_0)(\delta-\beta c_1+\beta s_2)}{\Delta_2 - \beta^2\theta^2\rho^2}.$$

Within the life cycle of fresh agricultural products, the steady-state freshness value g_∞^{S*} is globally stable if and only if $v^S > 0$, then we get: $\beta\theta(\rho+2\sigma)^2(1-s_1) - \varepsilon^2(\delta-\beta c_1)^2 - \beta\theta\rho^2(1-s_1) > 0$. This completes the proof of Theorem 1. $\square$

Proof of Theorem 2. Based on Theorem 1, substituting Equation (12) into Equations (9)–(11) directly yields Equations (14)–(16). The derivation is straightforward and thus omitted. Theorem 2 is proved accordingly. $\square$

Proof of Theorem 3. The objective functions of fresh produce suppliers and retailers are: $J_s^R = e^{-\rho t}V_s^R, J_r^R = e^{-\rho t}V_r^R$. Among them, V_s^R and V_r^R represent the optimal value functions of fresh agricultural product suppliers and retailers, and the HJB equation can be obtained as:

$$\rho V_s^R = \max_{w,u} \left((w - c_0 - c_1g)(\alpha - \beta p + \delta g) - \frac{1}{2}\theta u^2 + \frac{\partial V_s^R}{\partial g}(\varepsilon u - \sigma g) \right), \quad (A10)$$

$$\rho V_r^R = \max_p \left((p - w + s_2g)(\alpha - \beta p + \delta g) + \frac{\partial V_r^R}{\partial g}(\varepsilon u - \sigma g) \right). \quad (A11)$$

The inverse induction method is used to solve the problem. First, the calculation of the first-order partial derivatives of p in Equation (A11) is shown below.

$$p^R = \frac{\alpha + \delta g + \beta w - \beta s_2 g}{2\beta}. \quad (\text{A12})$$

Based on the retailer's reaction function, the HJB equation for the fresh produce supplier is:

$$\rho V_s^R = \max_{w,u} \left((w - c_0 - c_1 g) \frac{\alpha + \delta g - \beta w + \beta s_2 g}{2} - \frac{1}{2} \theta u^2 + \frac{\partial V_s^R}{\partial g} (\varepsilon u - \sigma g) \right). \quad (\text{A13})$$

The supplier's goal is to maximize profits; thus, the right-hand side of Equation (A13) w and u is solved, yielding:

$$w^R = \frac{\alpha + \beta c_0 + (\delta + \beta c_1 + \beta s_1)g}{2\beta}, \quad (\text{A14})$$

$$u^R = \frac{\varepsilon}{\theta} \frac{\partial V_s^R}{\partial g}. \quad (\text{A15})$$

Substituting Equation (A14) into Equation (A12) yields:

$$p^R = \frac{3\alpha + \beta c_0 + (3\delta + \beta c_1 - \beta s_2)g}{4\beta}. \quad (\text{A16})$$

Substituting Equations (A14)–(A16) into the HJB Equation (A10) and Equation (A12) of the supplier and retailer, and assuming that V_s^R and V_r^R are quadratic, the optimal value function is obtained as:

$$\begin{aligned} V_s^R &= X_1 g^2 + X_2 g + X_3 \\ V_r^R &= Y_1 g^2 + Y_2 g + Y_3 \end{aligned} \quad (\text{A17})$$

The first partial derivatives of Equation (A17) for g are as follows:

$$\begin{aligned} \frac{\partial V_s^R}{\partial g} &= 2X_1 g + X_2 \\ \frac{\partial V_r^R}{\partial g} &= 2Y_1 g + Y_2 \end{aligned} \quad (\text{A18})$$

The six Riccati equations are solved using the undetermined coefficient method, and the corresponding six parameters are obtained. $X_i, Y_i, i = 1, 2, 3$, and the six parameters are as follows:

$$\begin{aligned} X_1 &= \frac{\beta \theta (\rho + 2\sigma) - \Delta_2}{4\beta \varepsilon^2}, & X_2 &= \frac{\theta (\alpha - \beta c_0) (\delta - \beta c_1 + \beta s_2)}{2(\rho \beta \theta + \Delta_2)}, \\ X_3 &= \frac{(\alpha - \beta c_0)^2}{8\rho} \left[\frac{1}{\beta} + \frac{\theta \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2}{(\rho \beta \theta + \Delta_2)^2} \right], & Y_1 &= \frac{\theta (\delta - \beta c_1 + \beta s_2)^2}{16\Delta_2}, \\ Y_2 &= \frac{(\alpha - \beta c_0) (\delta - \beta c_1 + \beta s_2) \{ \beta \theta^2 \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2 + 2\Delta_2 \theta (\rho \beta \theta + \Delta_2) \}}{8\Delta_2 (\rho \beta \theta + \Delta_2)^2}, \\ Y_3 &= \frac{(\alpha - \beta c_0)^2}{\rho} \left\{ \frac{1}{16\beta} + \frac{\theta \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2}{8(\rho \beta \theta + \Delta_2)^2} + \frac{\beta \theta^2 \varepsilon^4 (\delta - \beta c_1 + \beta s_2)^4}{16\Delta_2 (\rho \beta \theta + \Delta_2)^3} \right\}, & \text{where} \\ \Delta_2 &= \sqrt{\beta \theta [\beta \theta (\rho + 2\sigma)^2 - \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]}. \end{aligned}$$

After all the coefficients are obtained, substituting Equation (A18) into Equation (A15) yields Equation (22), namely, $g^{R*} = g_{\infty}^{R*} + e^{-v^R t} (g_0 - g_{\infty}^{R*})$, and substituting Equation (A8) into $J_s^R = e^{-\rho t} V_s^R$ and $J_r^R = e^{-\rho t} V_r^R$ yields Equation (23). Substituting Equation (22) into Equation (1), and then solving the differential equation, we get

$$v^R = \frac{1}{2} \left[\sqrt{(\rho + 2\sigma)^2 - \frac{\varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2}{\beta \theta}} - \rho \right], \quad \text{and} \quad g_{\infty}^{R*} = \frac{\beta \theta \varepsilon^2 (\alpha - \beta c_0) (\delta - \beta c_1 + \beta s_2)}{\Delta_2^2 - \beta^2 \theta^2 \rho^2}.$$

Within the life cycle of fresh agricultural products, the freshness steady-state value g_{∞}^{R*} is globally stable if and only if $v^R > 0$; that is: $\beta \theta (\rho + 2\sigma)^2 - \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2 - \beta \theta \rho^2 > 0$. Theorem 3 is thus proved. $\square$

Proof of Theorem 4. Based on Theorem 3, Substitute Equation (22) into Equations (19)–(21), we can obtain Equations (24)–(26). Theorem 4 is proved. $\square$

Proof of Proposition 1. According to $g_{\infty}^{S*} = \frac{\beta \theta \varepsilon^2 (\alpha - \beta c_0) (\delta - \beta c_1) (1 - s_1)}{\Delta_1^2 - \beta^2 \theta^2 \rho^2 (1 - s_1)^2}$, we have

$$\frac{\partial g_{\infty}^{S*}}{\partial \delta} = \frac{\varepsilon^2 (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma) (1 - s_1) + \varepsilon^2 (\delta - \beta c_1)^2]}{[4\beta \theta \sigma (\rho + \sigma) (1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2]^2}.$$

Because of $\beta > 0$, $\theta > 0$, $\delta > 0$, $\alpha - \beta c_0 > 0$, $\beta \theta (\rho + 2\sigma)^2 (1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2 - \beta \theta \rho^2 (1 - s_1) > 0$, $0 < \varepsilon < 1$, $0 < \sigma < 1$, $0 < \rho < 1$, $0 < s_1 < 1$, $\delta - \beta c_1 > 0$, we obtain $\frac{\partial g_{\infty}^{S*}}{\partial \delta} > 0$. Similarly, we obtain

$$\frac{\partial g_{\infty}^{S*}}{\partial s_1} = \frac{4\varepsilon^2 \beta \theta \sigma (\rho + \sigma) (\alpha - \beta c_0) (\delta - \beta c_1)}{[4\beta \theta \sigma (\rho + \sigma) (1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2]^2} > 0,$$

$$\frac{\partial g_{\infty}^{S*}}{\partial c_1} = -\frac{\beta \varepsilon^2 (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma) (1 - s_1) + \varepsilon^2 (\delta - \beta c_1)^2]}{[4\beta \theta \sigma (\rho + \sigma) (1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2]^2} < 0,$$

$$\frac{\partial g_{\infty}^{R*}}{\partial \delta} = \frac{\varepsilon^2 (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma) + \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]}{[4\beta \theta \sigma (\rho + \sigma) - \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]^2} > 0,$$

$$\frac{\partial g_{\infty}^{R*}}{\partial s_2} = \frac{\beta \varepsilon^2 (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma) + \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]}{[4\beta \theta \sigma (\rho + \sigma) - \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]^2} > 0,$$

$$\frac{\partial g_{\infty}^{R*}}{\partial c_1} = -\frac{\beta \varepsilon^2 (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma) + \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]}{[4\beta \theta \sigma (\rho + \sigma) - \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]^2} < 0.$$

Proposition 1 is proved. $\square$

Proof of Proposition 2. According to $u_{\infty}^{S*} = \frac{\varepsilon \sigma (\alpha - \beta c_0) (\delta - \beta c_1)}{4\beta \theta \sigma (\rho + \sigma) (1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2}$, we have

$$\frac{\partial u_{\infty}^{S*}}{\partial \delta} = \frac{\sigma \varepsilon (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma) (1 - s_1) + \varepsilon^2 (\delta - \beta c_1)^2]}{[4\beta \theta \sigma (\rho + \sigma) (1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2]^2}.$$

Because of $\beta > 0$, $\theta > 0$, $\delta > 0$, $\alpha - \beta c_0 > 0$, $\beta \theta (\rho + 2\sigma)^2 (1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2 - \beta \theta \rho^2 (1 - s_1) > 0$, $0 < \varepsilon < 1$, $0 < \sigma < 1$, $0 < \rho < 1$, $0 < s_1 < 1$, $\delta - \beta c_1 > 0$, we obtain $\frac{\partial u_{\infty}^{S*}}{\partial \delta} > 0$. Similarly, we obtain

$$\frac{\partial u_{\infty}^{S*}}{\partial s_1} = \frac{4\varepsilon \beta \theta \sigma^2 (\rho + \sigma) (\alpha - \beta c_0) (\delta - \beta c_1)}{[4\beta \theta \sigma (\rho + \sigma) (1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2]^2} > 0,$$

$$\frac{\partial u_{\infty}^{S*}}{\partial c_1} = -\frac{\beta \sigma \varepsilon (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma) (1 - s_1) + \varepsilon^2 (\delta - \beta c_1)^2]}{[4\beta \theta \sigma (\rho + \sigma) (1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2]^2} < 0,$$

$$\frac{\partial u_{\infty}^{R*}}{\partial \delta} = \frac{\sigma \varepsilon (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma) + \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]}{[4\beta \theta \sigma (\rho + \sigma) - \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]^2} > 0,$$

$$\frac{\partial u_{\infty}^{R^*}}{\partial \delta} = \frac{\sigma \varepsilon (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma) + \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]}{[4\beta \theta \sigma (\rho + \sigma) - \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]^2} > 0 ,$$

$$\frac{\partial u_{\infty}^{R^*}}{\partial c_1} = - \frac{\beta \sigma \varepsilon (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma) + \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]}{[4\beta \theta \sigma (\rho + \sigma) - \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]^2} < 0 . \text{ Proposition 2 is proved. } \square$$

Proof of Proposition 3. According to

$$p_{\infty}^{S^*} = \frac{3\alpha + \beta c_0}{4\beta} + \frac{\varepsilon^2 (3\delta + \beta c_1)(\alpha - \beta c_0)(\delta - \beta c_1)}{4\beta [4\beta \theta \sigma (\rho + \sigma)(1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2]} , \quad \text{we have}$$

$$\frac{\partial p_{\infty}^{S^*}}{\partial \delta} = \frac{\varepsilon^2 (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma)(1 - s_1)(3\delta - \beta c_1) + 2\varepsilon^2 \beta c_1 (\delta - \beta c_1)^2]}{2\beta [4\beta \theta \sigma (\rho + \sigma)(1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2]^2} . \text{ Because of } \beta > 0 ,$$

$$\theta > 0 , \quad \delta > 0 , \quad \alpha - \beta c_0 > 0 , \quad \beta \theta (\rho + 2\sigma)^2 (1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2 - \beta \theta \rho^2 (1 - s_1) > 0 , \quad 0 < \varepsilon < 1 ,$$

$$0 < \sigma < 1 , \quad 0 < \rho < 1 , \quad 0 < s_1 < 1 , \quad \delta - \beta c_1 > 0 , \text{ we obtain } \frac{\partial p_{\infty}^{S^*}}{\partial \delta} > 0 . \text{ Similarly, we obtain}$$

$$\frac{\partial p_{\infty}^{S^*}}{\partial s_1} = \frac{\varepsilon^2 \theta \sigma (\rho + \sigma)(\alpha - \beta c_0)(\delta - \beta c_1)(3\delta + \beta c_1)}{[4\beta \theta \sigma (\rho + \sigma)(1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2]^2} > 0 ,$$

$$\frac{\partial p_{\infty}^{S^*}}{\partial c_1} = - \frac{\varepsilon^2 (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma)(1 - s_1)(\delta + \beta c_1) + 2\delta \varepsilon^2 (\delta - \beta c_1)^2]}{2\beta [4\beta \theta \sigma (\rho + \sigma)(1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2]^2} < 0 ,$$

$$\frac{\partial p_{\infty}^{R^*}}{\partial \delta} = \frac{\varepsilon^2 (\alpha - \beta c_0) [4\beta \theta \sigma (\rho + \sigma)(3\delta - \beta c_1 + \beta s_2) + 2\varepsilon^2 (\beta c_1 - \beta s_2)(\delta - \beta c_1 + \beta s_2)^2]}{2\beta [4\beta \theta \sigma (\rho + \sigma) - \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]^2} > 0 ,$$

$$\frac{\partial p_{\infty}^{R^*}}{\partial s_2} = \frac{2\varepsilon^2 (\alpha - \beta c_0) [\beta \theta \sigma (\rho + \sigma)(\delta + \beta c_1 - \beta s_2) + \varepsilon^2 \delta (\delta - \beta c_1 + \beta s_2)^2]}{\beta [4\beta \theta \sigma (\rho + \sigma) - \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]^2} > 0 ,$$

$$\frac{\partial p_{\infty}^{R^*}}{\partial c_1} = - \frac{2\varepsilon^2 (\alpha - \beta c_0) [\beta \theta \sigma (\rho + \sigma)(\delta + \beta c_1 - \beta s_2) + \varepsilon^2 \delta (\delta - \beta c_1 + \beta s_2)^2]}{\beta [4\beta \theta \sigma (\rho + \sigma) - \varepsilon^2 (\delta - \beta c_1 + \beta s_2)^2]^2} < 0 . \text{ Proposition 3 is}$$

thus proved. $\square$

Proof of Proposition 4. Following Theorem 2, under the scenario where the supplier receives subsidies, the monotonicity of the retail price and cold-chain investment level depends on the relationship between g_0 and $g_0^{S^*}$, as well as the magnitude of $s_1 \in (0, \bar{s}_1)$,

where according to $v^S = \frac{1}{2} \left[\sqrt{(\rho + 2\sigma)^2 - \frac{\varepsilon^2 (\delta - \beta c_1)^2}{\beta \theta (1 - s_1)}} - \rho \right] > 0$, we have

$4\beta \theta \sigma (\rho + \sigma)(1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2 > 0$. Let $4\beta \theta \sigma (\rho + \sigma)(1 - s_1) - \varepsilon^2 (\delta - \beta c_1)^2 = 0$, we obtain $\bar{s}_1 = \frac{4\beta \theta \sigma (\rho + \sigma) - \varepsilon^2 (\delta - \beta c_1)^2}{4\beta \theta \sigma (\rho + \sigma)}$. When $g_0 \leq \bar{g}_0$, for any $s_1 \in (0, \bar{s}_1)$, both the supplier

and retailer adopt a penetration pricing strategy and increase cold chain investment. When $g_0 > \bar{g}_0$, there exists a threshold $\hat{s}_1$ that triggers a shift in pricing strategy. Specifically, if $s_1 \in (0, \hat{s}_1)$ is bound to hold, supply chain members should adopt a skimming pricing strategy and reduce cold chain investment; whereas if $s_1 \in (\hat{s}_1, \bar{s}_1)$ is bound to hold, supply chain members should adopt a penetration pricing strategy and increase cold chain investment. First, according to $g_{\infty}^{S^*} = \frac{\beta \theta \varepsilon^2 (\alpha - \beta c_0)(\delta - \beta c_1)(1 - s_1)}{\Delta_1^2 - \beta^2 \theta^2 \rho^2 (1 - s_1)^2}$, let $g_0 = g_{\infty}^{S^*}$, we

can obtain

$$\hat{s}_1 = \frac{2\{g_0 [4\beta \theta \sigma (\rho + \sigma) - \varepsilon^2 (\delta - \beta c_1)^2] - \varepsilon^2 (\alpha - \beta c_0)(\delta - \beta c_1)\}}{g_0 [8\beta \theta \sigma (\rho + \sigma) - \varepsilon^2 (\delta - \beta c_1)^2] + \varepsilon \sqrt{g_0 (\delta - \beta c_1) [g_0 \varepsilon^2 (\delta - \beta c_1)^3 + 16\beta \theta \sigma (\rho + \sigma)(\alpha - \beta c_0)_1]}}$$

. Moreover, because of $\frac{\partial \hat{s}_1}{\partial g_0} > 0$, $\hat{s}_1$ is a monotonically increasing function of g_0 . Next,

let $\hat{s}_1 = 0$, we have $\bar{g}_0 = \frac{\varepsilon^2(\alpha - \beta c_0)(\delta - \beta c_1)}{4\beta\theta\sigma(\rho + \sigma) - \varepsilon^2(\delta - \beta c_1)^2}$. Then, when $g_0 \leq \bar{g}_0$, we have

$\hat{s}_1 < 0$. Since $g_\infty^{S^*}$ is a monotonically increasing function of s_1 , for any $s_1 \in (0, \bar{s}_1)$, we have $g_\infty^{S^*} > g_0$, meaning firms would adopt a penetration pricing strategy and increase cold-chain investment. When $g_0 > \bar{g}_0$, we have $\hat{s}_1 > 0$. Because $g_\infty^{S^*}$ is a monotonically increasing function with s_1 , when $s_1 \in (0, \hat{s}_1)$, we have $g_\infty^{S^*} < g_0$, meaning firms would adopt a skimming pricing strategy and reduce cold-chain investment. Similarly, when $s_1 \in (\hat{s}_1, \bar{s}_1)$, we have $g_\infty^{S^*} > g_0$, firms would adopt a penetration pricing strategy and increase cold-chain investment. Proposition 4 is thus proved. $\square$

Proof of Proposition 5. Following Theorem 4, under the scenario where the retailer receives subsidies, the monotonicity of the retail price and cold-chain investment level depends on the relationship between g_0 and $g_\infty^{R^*}$, as well as the magnitude of $s_2 \in (0, \bar{s}_2)$,

where according to $v^R = \frac{1}{2} \left[\sqrt{(\rho + 2\sigma)^2 - \frac{\varepsilon^2(\delta - \beta c_1 + \beta s_2)^2}{\beta\theta}} - \rho \right] > 0$, we have

$4\beta\theta\sigma(\rho + \sigma) - \varepsilon^2(\delta - \beta c_1 + \beta s_2)^2 > 0$. Let $4\beta\theta\sigma(\rho + \sigma) - \varepsilon^2(\delta - \beta c_1 + \beta s_2)^2 = 0$, we obtain

$\bar{s}_2 = \frac{2\sqrt{\beta\theta\sigma(\rho + \sigma)} - \varepsilon(\delta - \beta c_1)}{\beta\varepsilon}$. Specifically, when $g_0 \leq \bar{g}_0$, for any $s_2 \in (0, \bar{s}_2)$, both suppliers and retailers adopt a penetration pricing strategy and increase their cold chain investment levels.

When $g_0 < \bar{g}_0$, there exists a threshold $\hat{s}_2$ that triggers a shift in pricing strategy. If $s_2 \in (0, \hat{s}_2)$ is bound to hold, supply chain members should adopt a skimming pricing strategy and reduce cold chain investment levels. If $s_2 \in (\hat{s}_2, \bar{s}_2)$ is bound to hold, supply chain members should adopt a penetration pricing strategy and increase cold chain investment levels. First, according to $g_\infty^{R^*} = \frac{\beta\theta\varepsilon^2(\alpha - \beta c_0)(\delta - \beta c_1 + \beta s_2)}{\Delta_2^2 - \beta^2\theta^2\rho^2}$, let

$g_0 = g_\infty^{R^*}$, we have $\hat{s}_2 = \frac{4\{g_0[4\beta\theta\sigma(\rho + \sigma) - \varepsilon^2(\delta - \beta c_1)^2] - \varepsilon^2(\alpha - \beta c_0)(\delta - \beta c_1)\}}{2\beta\varepsilon[\varepsilon(\alpha - \beta c_0) + 2g_0\varepsilon(\delta - \beta c_1) + \sqrt{16g_0^2\beta\theta\sigma(\rho + \sigma) + \varepsilon^2(\alpha - \beta c_0)^2}]}$.

Moreover, because $\frac{\partial \hat{s}_2}{\partial g_0} > 0$, $\hat{s}_2$ is a monotonically increasing function of g_0 . Next, let

$\hat{s}_2 = 0$, and we can obtain $\bar{g}_0 = \frac{\varepsilon^2(\alpha - \beta c_0)(\delta - \beta c_1)}{4\beta\theta\sigma(\rho + \sigma) - \varepsilon^2(\delta - \beta c_1)^2}$. Then, when $g_0 \leq \bar{g}_0$, we

have $\hat{s}_2 < 0$. Because $g_\infty^{R^*}$ is a monotonically increasing function of s_2 , for any $s_2 \in (0, \bar{s}_2)$, we have $g_\infty^{R^*} > g_0$, meaning that the firms would adopt a penetration pricing strategy and increase cold-chain investment. When $g_0 > \bar{g}_0$, we have $\hat{s}_2 > 0$. Because $g_\infty^{R^*}$ is a monotonically increasing function with s_2 , when $s_2 \in (0, \hat{s}_2)$, we have $g_\infty^{R^*} < g_0$, meaning that the firm would adopt a skimming pricing strategy and reduce cold-chain investment. Similarly, when $s_2 \in (\hat{s}_2, \bar{s}_2)$, we have $g_\infty^{R^*} > g_0$, the firm adopts a penetration pricing strategy and increase cold-chain investment. Proposition 5 is thus proved. $\square$

Proof of Propositions 6 and 7. This paper employs the Numerical Simulation to compare the equilibrium strategies, supplier and retailer profits, consumer surplus, and social welfare across different subsidy schemes under equal government expenditure. The results are summarized in Tables A1–A4.

Table A1. The equilibrium values of two subsidy scenarios under equal expenditures ($\delta = 1.5$, and $\delta = 1.25$).

δ_1	0.1	0.2	0.3	0.4	0.5	0.6	0.1	0.2	0.3	0.4	0.5	0.6
$g_e^{S^*}$	1.4621	1.6716	1.9512	2.3430	2.9319	3.9160	0.8743	0.9896	1.1401	1.3445	1.6382	2.0960
$g_e^{R^*}$	1.3765	1.4916	1.6671	1.9482	2.4401	3.4402	0.8234	0.8814	0.9654	1.0911	1.2906	1.6423
$p_\infty^{S^*}$	4.9686	5.1074	5.2926	5.5523	5.9424	6.5944	4.4972	4.5628	4.6484	4.7647	4.9317	5.1921
$p_\infty^{R^*}$	4.9062	4.9730	5.0732	5.2299	5.4955	6.0136	4.4661	4.4956	4.5377	4.5994	4.6943	4.8539
$u_\infty^{S^*}$	0.5848	0.6686	0.7805	0.9372	1.1727	1.5664	0.3497	0.3958	0.4560	0.5378	0.6552	0.8384
$u_\infty^{R^*}$	0.5506	0.5966	0.6668	0.7793	0.9760	1.3761	0.3293	0.3525	0.3861	0.4364	0.5162	0.6569
$V_{\infty}^{S^*}$	46.2719	47.1920	48.4283	50.1773	52.8406	57.3868	42.3800	42.6969	43.1112	43.6760	44.4911	45.7705
$V_{\infty}^{R^*}$	46.1849	47.1613	48.7514	51.5252	56.9563	69.7659	42.3467	42.6751	43.1832	44.0140	45.4922	48.5193
$V_{\infty}^{S^*}$	25.4449	26.2786	27.4123	29.0416	31.5781	36.0545	22.0155	22.2887	22.6475	23.1396	23.8558	24.9941
$V_{\infty}^{R^*}$	25.3665	26.2507	27.7109	30.3170	35.6229	49.0856	21.9870	22.2699	22.7101	23.4357	24.7451	27.4963
$CS_\infty^{S^*}$	1.2722	1.3139	1.3706	1.4521	1.5789	1.8027	1.1007	1.1144	1.1323	1.1569	1.1927	1.2497
$CS_\infty^{R^*}$	1.2683	1.3125	1.3855	1.5158	1.7811	2.4542	1.0993	1.1134	1.1355	1.1717	1.2372	1.3748
$SW_\infty^{S^*}$	72.9377	74.6504	76.9370	80.1438	84.9661	93.0358	65.4780	66.0531	66.7976	67.7990	69.2177	71.3818
$SW_\infty^{R^*}$	72.7684	74.5905	77.5738	82.8309	93.3289	119.0975	65.4148	66.0115	66.9353	68.4479	71.1525	76.7578
δ	$\delta = 1.5$						$\delta = 1.25$					

Table A2. The equilibrium values of two subsidy scenarios under equal expenditures ($c_1 = 0.3$, and $c_1 = 0.2$).

δ_1	0.1	0.2	0.3	0.4	0.5	0.6	0.1	0.2	0.3	0.4	0.5	0.6
$g_e^{S^*}$	0.7692	0.8696	1.0000	1.1764	1.4285	3.0769	1.2121	1.3793	1.6000	1.9047	2.3529	3.0769
$g_e^{R^*}$	0.7670	0.7743	0.8457	0.9518	1.1173	2.7263	1.1008	1.2296	1.3611	1.5652	1.9083	2.7263
$p_\infty^{S^*}$	4.3462	4.3913	4.4500	4.5294	4.6428	5.3076	4.5151	4.5862	4.6800	4.8095	5.0000	5.3076
$p_\infty^{R^*}$	4.3173	4.3440	4.3718	4.4120	4.4724	4.9645	4.4666	4.5119	4.5569	4.6240	4.7308	4.9645
$u_\infty^{S^*}$	0.3077	0.3478	0.4000	0.4706	0.5714	0.7273	0.4848	0.5517	0.6400	0.7619	0.9412	1.2307
$u_\infty^{R^*}$	0.2828	0.3097	0.3383	0.3807	0.4469	0.5613	0.4403	0.4918	0.5444	0.6261	0.7633	1.0905
$V_{\infty}^{S^*}$	41.8580	42.1021	42.4200	42.8512	43.4694	44.4298	44.4298	45.0511	45.8752	47.0204	48.7197	51.5029
$V_{\infty}^{R^*}$	41.8000	42.0728	42.6277	43.3558	44.6171	47.1106	44.0814	45.0204	46.0579	47.8141	51.1158	60.4031
$V_{\infty}^{S^*}$	21.5680	21.7769	22.0500	22.4221	22.9592	23.8017	23.8017	24.3519	25.0880	26.1224	27.6816	30.2958
$V_{\infty}^{R^*}$	21.5002	21.7067	22.1739	22.7686	23.8145	25.9372	23.4948	24.3246	25.2522	26.8471	29.9278	39.1209
$CS_\infty^{S^*}$	1.0784	1.0888	1.1025	1.1211	1.1479	1.1901	1.1900	1.2175	1.2544	1.3061	1.3840	1.5147
$CS_\infty^{R^*}$	1.0736	1.0881	1.1047	1.1318	1.1798	1.2782	1.1747	1.2162	1.2626	1.3423	1.4963	1.9560
$SW_\infty^{S^*}$	64.4902	64.9316	65.5005	66.2616	67.3316	68.9456	69.3862	70.5293	72.0333	74.1006	77.1211	81.9502
$SW_\infty^{R^*}$	64.3597	64.8687	65.8343	67.1234	69.3665	73.8498	68.7157	70.4700	72.3883	75.6552	81.8757	100.1167
δ	$c_1 = 0.3$						$c_1 = 0.2$					

Table A3. The equilibrium values of two subsidy scenarios under equal expenditures ($\sigma = 0.405$, and $\sigma = 0.395$).

s_1	0.1	0.2	0.3	0.4	0.5	0.6	0.1	0.2	0.3	0.4	0.5	0.6
g_{∞}^{S*}	0.3654	0.4116	0.4711	0.5507	0.6627	0.8318	0.3824	0.4307	0.4930	0.5764	0.6937	0.8709
g_{∞}^{R*}	0.3446	0.3664	0.3974	0.4419	0.5087	0.6150	0.3603	0.3831	0.4153	0.4618	0.5315	0.6434
p_{∞}^{S*}	4.1736	4.1955	4.2237	4.2616	4.3147	4.3951	4.1816	4.2046	4.2342	4.2738	4.3295	4.4137
p_{∞}^{R*}	4.1633	4.1730	4.1867	4.2062	4.2349	4.2792	4.1707	4.1809	4.1951	4.2155	4.2454	4.2922
u_{∞}^{S*}	0.1480	0.1667	0.1908	0.2231	0.2684	0.3368	0.1510	0.1701	0.1947	0.2277	0.2740	0.3440
u_{∞}^{R*}	0.1396	0.1484	0.1609	0.1789	0.2060	0.2490	0.1423	0.1513	0.1640	0.1824	0.2099	0.2541
V_{∞}^{S*}	40.4385	40.4939	40.5655	40.6614	40.7961	40.999	40.4604	40.5187	40.5939	40.6945	40.8363	41.0507
V_{∞}^{R*}	40.4334	40.4895	40.5747	40.7090	40.9361	41.3583	40.4542	40.5131	40.6017	40.7424	40.9796	41.4246
V_{∞}^{S*}	20.3671	20.4137	20.4739	20.5545	20.6682	20.8404	20.3842	20.4331	20.4961	20.5806	20.6997	20.8804
V_{∞}^{R*}	20.3628	20.4099	20.4816	20.5947	20.7864	21.1444	20.3790	20.4283	20.5027	20.6208	20.8204	21.1967
CS_{∞}^{S*}	1.0186	1.0207	1.0237	1.0277	1.0334	1.0420	1.0192	1.0216	1.0248	1.0290	1.0349	1.0440
CS_{∞}^{R*}	1.0181	1.0205	1.0241	1.0297	1.0393	1.0572	1.0189	1.0214	1.0251	1.0310	1.0410	1.0598
SW_{∞}^{S*}	61.8207	61.9201	62.0467	62.2137	62.4437	62.7803	61.8605	61.9648	62.0978	62.2731	62.5147	62.8686
SW_{∞}^{R*}	61.8111	61.9116	62.0641	62.3036	62.7079	63.4578	61.8488	61.9542	62.1125	62.3632	62.7848	63.5747
δ	$\sigma = 0.405$						$\sigma = 0.395$					

Table A4. The equilibrium values of two subsidy scenarios under equal expenditures ($\theta = 3.25$, and $\theta = 2.75$).

s_1	0.1	0.2	0.3	0.4	0.5	0.6	0.1	0.2	0.3	0.4	0.5	0.6
g_{∞}^{S*}	0.3448	0.3883	0.4444	0.5194	0.6250	0.7843	0.4081	0.4597	0.5263	0.6154	0.7407	0.9302
g_{∞}^{R*}	0.3248	0.3457	0.3749	0.4167	0.4797	0.5796	0.3845	0.4093	0.4440	0.4938	0.5684	0.6883
p_{∞}^{S*}	4.1637	4.1844	4.2111	4.2467	4.2968	4.3725	4.1938	4.2184	4.2500	4.2923	4.3518	4.4418
p_{∞}^{R*}	4.1539	4.1632	4.1761	4.1944	4.2215	4.2632	4.1822	4.1932	4.2086	4.2304	4.2624	4.3125
u_{∞}^{S*}	0.1379	0.1410	0.1777	0.2077	0.2500	0.3137	0.1632	0.1839	0.2105	0.2461	0.2962	0.3721
u_{∞}^{R*}	0.1299	0.1383	0.1499	0.1667	0.1918	0.2318	0.1538	0.1637	0.1776	0.1975	0.2273	0.2753
V_{∞}^{S*}	40.4143	40.4667	40.5343	40.6247	40.7519	40.9442	40.4906	40.5527	40.6329	40.7403	40.8916	41.1206
V_{∞}^{R*}	40.4087	40.4625	40.5432	40.6697	40.8840	41.2819	40.4891	40.5504	40.6837	40.8599	41.1553	41.7074
V_{∞}^{S*}	20.3463	20.3902	20.4469	20.5228	20.6298	20.7920	20.4102	20.4624	20.5297	20.6201	20.7475	20.9410
V_{∞}^{R*}	20.3416	20.3866	20.4543	20.5607	20.7412	21.0777	20.4096	20.4621	20.5588	20.6984	20.9336	21.3761
CS_{∞}^{S*}	1.0173	1.0195	1.0223	1.0261	1.0314	1.0396	1.0205	1.0231	1.0264	1.0310	1.0373	1.0470
CS_{∞}^{R*}	1.0170	1.0193	1.0227	1.0280	1.0370	1.0538	1.0202	1.0229	1.0269	1.0332	1.0439	1.0641
SW_{∞}^{S*}	61.7749	61.8686	61.9881	62.1456	62.3625	62.6798	61.9177	62.0290	62.1709	62.3581	62.6162	62.9944
SW_{∞}^{R*}	61.7643	61.8607	62.0048	62.2304	62.6115	63.3176	61.9152	62.0217	62.2512	62.5584	63.0726	64.0335
δ	$\theta = 3.25$						$\theta = 2.75$					

Tables A1–A4 reveal several key observations: (1) Under both subsidy schemes, an increase in government subsidy intensity raises supply chain freshness, retail prices, cold-chain investment, supplier profits, retailer profits, consumer surplus, and social welfare.

(2) $u_{\infty}^{S*} > u_{\infty}^{R*}$, $g_{\infty}^{S*} > g_{\infty}^{R*}$, $p_{\infty}^{S*} > p_{\infty}^{R*}$. (3) When $s_1 < 0.25$, $V_{\infty}^{S*} > V_{\infty}^{R*}$, $V_{\infty}^{S*} > V_{\infty}^{R*}$, $CS_{\infty}^{S*} > CS_{\infty}^{R*}$, and $SW_{\infty}^{S*} > SW_{\infty}^{R*}$; when $s_1 > 0.25$, $V_{\infty}^{S*} < V_{\infty}^{R*}$, $V_{\infty}^{S*} < V_{\infty}^{R*}$, $CS_{\infty}^{S*} < CS_{\infty}^{R*}$,

and $SW_{\infty}^{S*} < SW_{\infty}^{R*}$. These observations demonstrate that the relative effectiveness of each subsidy scheme depends on the intensity of government support. Propositions 6 and 7 are thus proved. $\square$

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