

Article

Allocation Efficiency and Shadow Prices of Energy Inputs in China: A Multi-Energy Approach

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Abstract

Enhancing energy efficiency is a critical pathway for China's achievement of carbon peaking and carbon neutrality goals while maintaining economic growth. The deviation between actual factor prices and shadow prices is a major source of low total factor energy efficiency. Therefore, evaluating the allocative efficiency of production factors, including energy, should be addressed. In consideration of China's energy endowment, energy transition, and unbalanced sustainability of various energy sources, this study adopts a stochastic frontier model with a generalized cost function to measure the allocative inefficiency of coal, oil, natural gas, and electricity and heat in China. The results indicate the following: (1) From 2005 to 2022, both technical efficiency and energy factor allocative efficiency remained below 1 at the provincial level, while technical efficiency declined gradually and allocative efficiency exhibited an upward spiral trend. (2) Labor, coal, oil, natural gas, and electricity and heat are found to be overutilized relative to capital; moreover, except for capital, all other factors are excessively employed relative to electricity and heat. (3) Setting capital factors as standardization factors, the shadow prices of coal, oil, natural gas, and electricity and heat are all higher than the actual prices. This indicates that China can reasonably allocate factors through full market competition and government regulation, thereby improving energy utilization efficiency.

Keywords: energy allocative efficiency; factor price distortion; shadow price; generalized cost function



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1. Introduction

Since China surpassed the United States as the world's largest energy consumer in 2009, the former's primary energy consumption has continued to grow rapidly. A pronounced structural contradiction exists between the rigid constraints of China's energy resource endowment and the dynamic demands of economic and social development. Moreover, empirical studies indicate that China's energy efficiency remains below the

global average [1,2]. To achieve high-quality economic development, it is necessary to improve energy efficiency and alleviate the energy supply-and-demand imbalance.

Farrell [3] argued that the divergence between actual and shadow prices in imperfect factor markets is a critical determinant of total factor productivity (TFP). The inefficiency caused by such price distortions is called allocative inefficiency, also known as price inefficiency. From the perspective of production function theory, allocative inefficiency is present when input costs cannot be minimized at a given level of technology and factor prices. In other words, allocative inefficiency emerges when the marginal rates of substitution among factors are inconsistent with their respective price ratios. Moreover, allocative inefficiency implies that actual input usage exceeds the optimal level, indicating that the market fails to minimize costs.

Ouyang [4] noted that price distortions in China's energy markets are closely associated with policy interventions, including subsidies and energy quotas. In China, enhancing energy efficiency is widely regarded as a low-hanging fruit for reducing pollution and mitigating carbon emissions and as a form of non-regret mitigation to achieve the country's carbon peaking and carbon neutrality goals without compromising its economic growth.

China is currently at a critical period for economic development, low-carbon energy transition, and energy security [5,6], especially as the demand for green energy continues to increase [7]. Breaking down energy into different types of energy factors is necessary when measuring the efficiency of factor allocation for three main reasons: First, China's carbon peaking and carbon neutrality goals, along with its long-term socioeconomic development, require energy restructuring; for example, the 14th Five-Year Plan for People's Republic of China (PRC)'s National Economic and Social Development and the Outline of the Long-term Target in 2035 [8] aim for a 20% share of non-fossil energy by 2025. Second, China's energy endowment is highly unbalanced: the country is rich in coal, poor in oil, and scarce in gas. Coal remains dominant in primary energy, while oil holds strategic importance. Natural gas offers greater environmental advantages than coal and oil, and non-fossil energy has become central to pollution and carbon mitigation. Third, energy types are only partially substitutable. Renewables cannot fully replace coal due to intermittency and stability issues, while the electrification of end-use energy faces technological, behavioral, and cost-related challenges. For example, Wang [9] showed that heavy rain disasters can cause power outages, leading to unstable electricity and heat supply and requiring the government to have a consistent supply of fossil fuels, such as coal and oil, in case of emergencies. Therefore, subdividing energy types is essential when assessing allocative efficiency.

This study focuses on the structural contradictions between China's resource endowment of—rich in coal, poor in oil, and scarce in gas—and its energy demand while also addressing key sustainable development challenges, such as rational energy allocation and CO₂ emission reduction, in the context of global warming. Energy elements are subdivided into coal, oil, natural gas, as well as heat and electricity. The allocation inefficiency among China's energy types is measured on this basis. Specifically, the study addresses the following questions: (1) Do allocative distortions exist in China's energy factor markets? (2) How efficiently are different energy sources allocated within the energy market? (3) What are the shadow prices of the four energy types under the assumption of no allocative distortions?

This study's primary contributions are as follows:

- i. It optimizes the framework for estimating energy input. Existing studies generally treat energy as a single factor and neglect the differences among energy types. This paper divides energy into coal, oil, and natural gas, as well as electricity and heat, effectively avoiding the problems of offsetting deviations across different energy

prices, flattening heterogeneity and significantly improving the accuracy of measuring factor allocation distortions and shadow prices.

- ii. It enriches the literature perspective from the three dimensions of shadow prices, factor allocation, and input distortions. The adoption of a detailed perspective can precisely capture structural distortions that broader studies cannot identify, revealing specific allocation contradictions relating to China's resource endowment and providing new empirical evidence for understanding the misallocation of energy factors. The research results can then be combined with the measurement of shadow prices from the perspective of emission-reduction costs, thereby serving as a research basis for the construction of a carbon pricing policy system and offering directional suggestions for strategies to adjust the energy structure.

This paper is organized as follows. Section 2 provides a relevant literature review. Section 3 introduces the stochastic frontier analysis model and the generalized cost function model and describes the data sources and data processing. Section 4 presents the regression results and the analytical discussion. Section 5 summarizes the findings and discusses the limitations.

2. Literature Review

2.1. Allocative Efficiency

Farrell [3] was the first to distinguish between technical efficiency and allocative efficiency from a microeconomic perspective, defining the latter as the extent to which producers employ inputs in optimal proportions given their prices. Christopoulos and Tsionas [10] argued that allocative efficiency arises when a firm minimizes costs by equating the marginal value product of each input with its marginal cost. At the macroeconomic level, allocative inefficiency is indicative of the failure to achieve potential profit maximization or cost minimization. In practice, firms typically base their cost-minimization decisions on observed market prices. However, deviations between actual and shadow prices may generate allocative inefficiency and result in unwarranted costs. Christopoulos and Tsionas [10], Burki and Khan [11], and Khiabani and Hasani [12] quantified allocative inefficiency in the manufacturing sectors of Greece, Pakistan, and Iran, respectively, and examined its implications for resource allocation and factor substitutability.

2.2. Factor Price Distortion

Factor price distortion is a key indicator of allocative inefficiency. Scholars have developed various approaches to quantifying the extent of such distortions. Kim [13] argued that under perfect competition, the cost share of a factor should equal its marginal output share in total output. The discrepancy between these shares is used to measure distortion: the larger the gap, the lower the efficiency. Wang and Wu [14] assessed distortion using the ratio of marginal revenue product (MRP) to factor price. Khademvatani and Gordon [15] introduced a profit function that incorporates the shadow price of energy to evaluate distortions. Schmidt and Knox Lovell [16] used a mathematical model to demonstrate how incorporating errors into first-order conditions can eliminate distortions and achieve cost minimization. Han and Hu [17] analyzed panel data for 30 Chinese provinces from 2002 to 2014, concluding that substantial distortions that reduce capacity utilization and exacerbate overcapacity exist.

2.3. Shadow Price

The concept of shadow price was first introduced by Dutch economist Jan Tinbergen in the late 1930s. Shadow price denotes the theoretical price that ensures the optimal allocation of production factors, thereby placing the economy in a socially optimal state [18]. It reflects

the value of a resource when it is allocated as efficiently as possible (i.e., the marginal product value or marginal cost of that factor of production). When resources are optimally allocated, the marginal value of a resource is its shadow price. As the shadow price is a theoretical construct, it cannot be directly observed and must instead be estimated through statistical methods before being inferred.

Methods for measuring shadow prices are generally divided into two categories: parametric and non-parametric approaches [19]. Parametric approaches include the stochastic frontier analysis (SFA) model and the parametric linear programming (PLP) model. For example, Färe et al. [20] applied an SFA model combined with a directional input distance function to estimate pollutant shadow prices from US coal-fired power plants during 1985–1988, while Peng et al. [21] employed a quadratic directional distance function to estimate industrial pollutant shadow prices in over 200 Chinese prefecture-level and above cities from 2003 to 2008. Du and Mao [22] applied a PLP model to evaluate environmental technical efficiency, emission-reduction potential, and the shadow price of CO₂ from China’s coal-fired power plants. Non-parametric approaches are primarily represented by the data envelopment analysis (DEA) model developed by Charnes and Cooper [23], which derives shadow prices from productivity measures. For example, Zhang et al. [24] evaluated pollutant shadow prices in China’s Poyang Lake ecological–economic zone using a pairwise slack-based measure (SBM) model.

Existing research on shadow prices has primarily focused on energy and carbon dioxide. Studies on energy shadow prices have concentrated on the energy sector as a whole, breaking energy down into non-fossil fuels and other energy sources. Research on carbon dioxide shadow prices has prioritized analyzing these prices across sectors and regions. Relevant studies are listed in Table 1.

Table 1. Research on shadow prices.

Related Study	Research Object	Research Aim	Methods	Comments
Xie [25]	Energy	Estimating the extent to which non-fossil fuels can replace fossil fuels	Quadratic input distance function	In most years, the shadow price of non-fossil energy was significantly higher than that of fossil energy. The average values were 995.11 yuan per ton of standard coal (CNY/tce) and 19.41 CNY/tce, respectively.
Ru [26]	Energy	Estimating the impact of distorted energy prices on economic growth and carbon emissions reduction	Marginal opportunity cost pricing approach	Price distortions exist in all four types of energy products. Fossil fuel prices are subject to negative price distortion, meaning that their shadow prices are higher than their actual prices. Non-fossil fuel prices are subject to positive price distortion, which indicates that their shadow prices are lower than their actual prices.
Ouyang [4]	Energy	Measuring factor price distortions using a shadow price model for China’s industrial sector	Stochastic frontier analysis	The Chinese industrial sector uses too much energy relative to its capital.

Table 1. *Cont.*

Related Study	Research Object	Research Aim	Methods	Comments
Molinos Senante [27]	CO ₂	Estimating the CO ₂ shadow price associated with energy consumption at 25 wastewater treatment plants in Spain	Quadratic directional distance function	The average shadow price of carbon dioxide is €88 per ton.
Deng and Du [28]	CO ₂	Estimating the shadow price of carbon dioxide in countries along the Belt and Road Initiative	Endogenous directional distance function	In 2016, the average CO ₂ shadow price in countries along the Belt and Road route was \$3.26 per ton.
Lee and Zhang [29]	CO ₂	Shadow prices of CO ₂ across 30 manufacturing sectors in China	Non-logarithmic input distance functions	The average shadow price for CO ₂ is \$3.13 per ton.
Du [30]	CO ₂	Shadow prices of CO ₂ in China's eastern, central, and western regions and nationwide	Quadratic directional distance functions	China's CO ₂ shadow price has continued to rise, increasing from 1000 yuan per ton in 2001 to 2100 yuan per ton in 2010.
Chen and Xiang [31]	CO ₂	Estimation of shadow prices of CO ₂ for 20 power stations in Shanghai	Non-radial directional distance functions	The average shadow price for CO ₂ emissions from power stations in Shanghai was 20.58 yuan per ton in 2007, rising to 39.67 yuan per ton by 2010. This is significantly lower than the average international shadow price for CO ₂ emissions.
Wei [32]	CO ₂	Estimation of shadow prices of CO ₂ across China's provinces	Super-efficient SBM	Between 1995 and 2007, Beijing had the highest average marginal abatement cost (266.5 yuan per ton), while China's main coal-producing region, Shanxi, had the lowest (31.1 yuan per ton).

The deviation between China's energy shadow prices and actual prices primarily stems from externalities and is specifically reflected in environmental costs [25,33]. For example, although China has implemented policies such as ad valorem resource taxes and pollutant emission controls for coal, most provinces have resource tax rates concentrated only in the 2~10% range. At the same time, while ultra-low emission retrofit policies have gradually been promoted from the power industry to key industrial sectors, such as cement and coking, prompting energy production enterprises to internalize part of the environmental costs, companies have not borne the full costs of environmental damage. Substantial uninternalized externalities, including negative impacts on residents' health [34,35] and intergenerational equity losses [36,37], remain. From an economic perspective, incompletely internalized environmental costs cause coal market prices to be lower than their shadow prices, leading to the overconsumption of coal resources. Similar price distortions and overuse issues exist for fossil energies such as oil and natural gas. In contrast, non-fossil energy alternatives to fossil energy provide significant environmental and economic double dividends, reducing pollutant and greenhouse gas emissions while supporting high-quality economic development. Therefore, China can continue to internalize the environmental externalities of traditional fossil energy consumption through optimizing market-based policy tools such as environmental taxes, thereby steadily increasing the share of non-fossil energy consumption.

2.4. General Summary

Existing studies have yet to conduct an in-depth discussion of allocative efficiency across subdivided energy types. One category of the literature on subdivided energy examines the contribution of various energy types to overall energy inefficiency, primarily focusing on technical efficiency without involving the dimension of allocative efficiency. Another category of studies focuses on exploring the substitution relationship between energy types [38] and their impact on carbon emissions [25], with few articles paying attention to allocative efficiency among subdivided energy types. In addition, energy is usually regarded as a single production factor for consideration in studies related to factor allocation, without further accounting for the mismatch within energy structure [4,39]. From the perspective of shadow prices, there are significant differences in actual prices, development characteristics, and allocation states across various energy types. Conducting detailed research on energy can expand the analytical dimensions of shadow prices and fully exploit micro-level data information. From the perspective of factor misallocation, the approach of aggregating energy as a whole tends to obscure structural misallocation issues. In this study, quantitative measurements are performed separately for four types of energy from the perspective of energy input distortions. These measurements accurately determine the input distortion coefficients for each type of energy. This study further enriches and expands research in the field of allocative efficiency by assessing the differentiated manifestations of allocation distortions across different energy types.

Most current studies measure the shadow price by the expected output forgone to reduce one unit of undesired output. However, the inefficient allocation of input factors prevents the market from achieving cost minimization and generates additional costs. In this study, we evaluate the shadow prices of various energy types using a detailed classification. Our findings can enrich research on the comprehensive social cost assessment of greenhouse gas reduction and energy transition.

3. Methodology and Data

3.1. Methodology

We develop a two-stage method to estimate the shadow prices of different energy sources across various provinces. The methods related to the first stage are discussed in Sections 3.1.1–3.1.3. Sections 3.1.1 and 3.1.2 detail how the technical efficiency of each province for each year was measured using stochastic frontier analysis. In Section 3.1.3, outputs are projected onto the technically efficient production frontier to obtain the effective output. At this point, allocative efficiency and cost efficiency are numerically equivalent. The methods related to the second stage are discussed in Section 3.1.4. The effective output obtained in the first stage is substituted into the generalized cost function to measure allocative efficiency. At the same time, the generalized cost function is based on shadow prices and incorporates the price distortion factors for each input. After estimating the model to obtain the distortion factors, the actual shadow prices for each factor are derived in reverse. See Figure 1 for details.

Specifically, we carry out our analysis in four steps. First, we measure the technical efficiency of each province using SFA; second, we adjust the actual output based on the calculated efficiency to obtain the effective output; third, we substitute the effective output into the generalized cost function for analysis; and fourth, we measure the degree of factor distortion, derive shadow prices, and assess the efficiency of resource allocation.

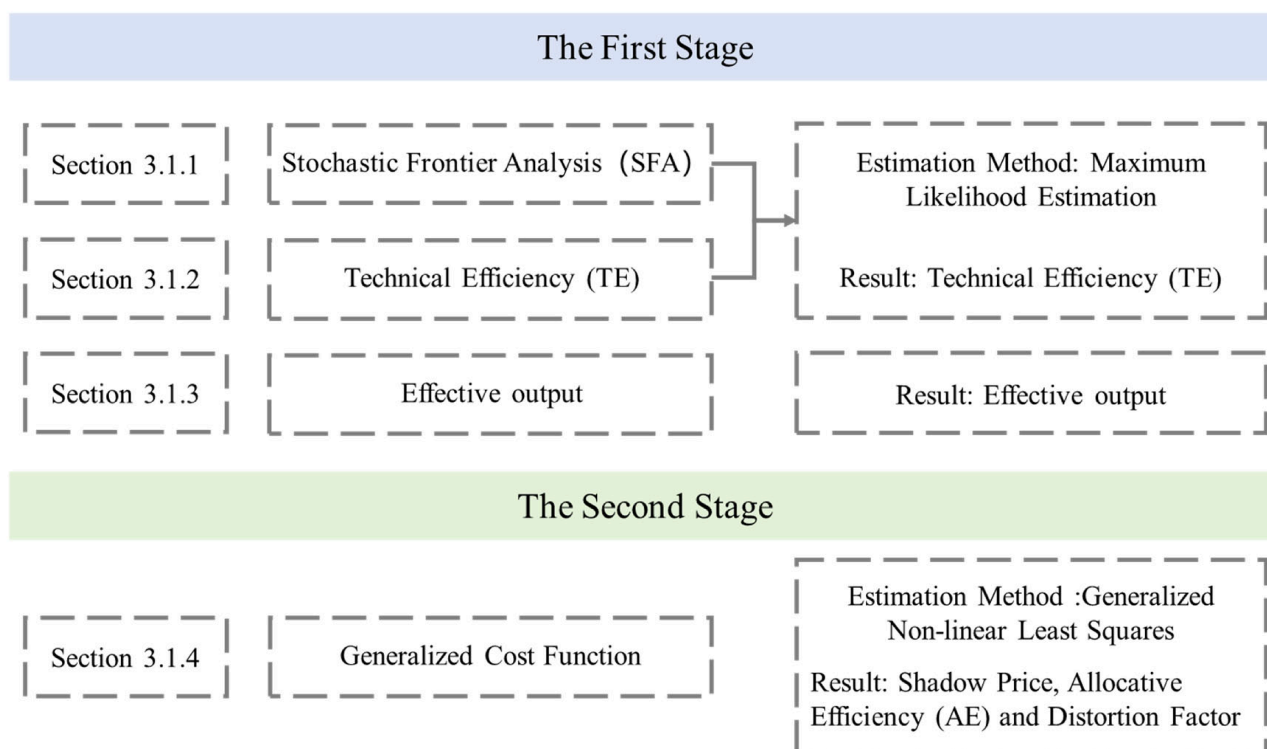


Figure 1. Research technology roadmap.

3.1.1. Stochastic Frontier Production Function

The frontier production function represents the relationship between input combinations and the maximum output attainable, given technological conditions and factor combinations. For the case of a single input and a single output, considering the effects of technical efficiency and stochastic disturbances, the actual output can be expressed by Equation (1), whose expression is as follows:

$$y_i = f(x_i) \times TE_i \times \exp(v_i) \quad (1)$$

where y_i is the output vector, and x_i is the input vector of the i -th production unit; $f(x_i)$ denotes the potential output of the i -th production unit; TE_i denotes the technical efficiency of the i -th production unit; and v_i denotes the stochastic disturbance term for the i -th production unit, which follows a normal distribution, that is, $v_i \sim N(0, \sigma_v^2)$.

The maximum output is represented by the production frontier, which is divided into a deterministic frontier and a stochastic frontier. The latter introduces stochastic disturbances based on the former. Equation (1) can be further transformed into the following equation:

$$y_i = [f(x_i) \times \exp(v_i)] \times TE_i \quad (2)$$

where $f(x_i)$ is the deterministic production frontier, which is the same for all production units; $\exp(v_i)$ is the stochastic disturbance, which varies for different production units. The stochastic frontier may be above or below the deterministic frontier; $[f(x_i) \times \exp(v_i)]$ represents the stochastic production frontier. As illustrated in Figure 2, points AT and BT are actual production points. In the absence of technical inefficiency, the frontier surfaces corresponding to these two points are stochastic frontiers, which are denoted as points AS and BS, respectively. The output at AS lies above the deterministic frontier, whereas the output at BS lies below it. This occurs because the disturbance component at AS is positive,

while that at BS is negative. Taking the logarithm of both sides of Equation (2) yields the following:

$$\ln y_i = \ln[f(x_i)] + v_i - u_i \quad (3)$$

where $u_i = -\ln TE_i$. Moreover, because $TE_i \in [0, 1]$, $u_i = -\ln TE_i \geq 0$. Since the actual output $\ln y_i$ is always below the stochastic frontier, u_i is defined as technical inefficiency. We assume that u_i follows a half-normal distribution: $u_i \sim N^+(0, \sigma_u^2)$.

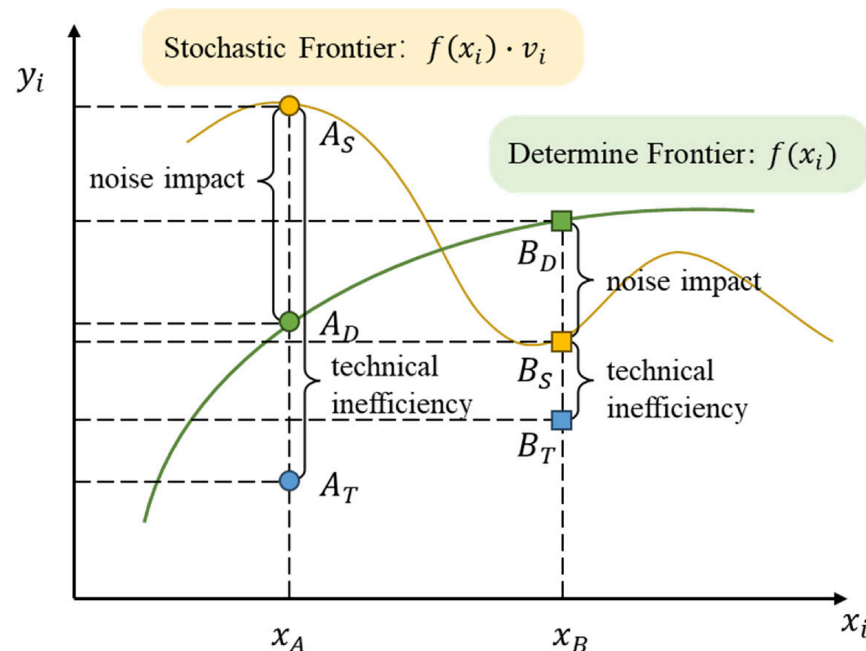


Figure 2. Schematic diagram of the stochastic frontier approach.

Furthermore, following the work of Tu and Xiao [40], technical efficiency can be expressed as follows:

$$TE_i = \exp(-u_i) \quad (4)$$

As panel data were used in this study, our analysis must account for the effects of time on technical inefficiency and stochastic effects. Equation (3) is thus rewritten as follows to measure the technical efficiency of g decision-making units over T periods:

$$\begin{aligned} \ln y_{it} &= \ln[f(x_{it})] + v_{it} - u_{it}, (t = 1, 2, \dots, T) \\ u_{it} &= \delta(t)u_i, \delta(t) = \exp(\eta(T - t)) \\ \gamma &= \frac{\sigma_u^2}{\sigma_v^2} \\ \sigma^2 &= \sigma_v^2 + \sigma_u^2 \end{aligned} \quad (5)$$

where u_{it} is the time-varying technical inefficiency term composed of $\delta(t)$ and u_i and where $\delta(t)$ represents the trend of technical efficiency over time. When $\eta > 0$, $\delta(t)$ is a monotonically decreasing function of η , indicating that technical inefficiency decreases and technical efficiency increases over time. When $\eta < 0$, $\delta(t)$ is a monotonically increasing function of η , indicating that technical inefficiency increases and technical efficiency decreases over time. $\eta = 0$ implies that technical inefficiency does not change over time. γ is used to measure whether inefficiency is dominated by stochastic error or technical inefficiency. σ_u^2 is the variance of the technical inefficiency term, and σ_v^2 is the variance of the stochastic error term. u and v are independent. The range of γ is $[0, 1]$. As γ approaches 0, the composed error term is primarily driven by the stochastic disturbance term v , meaning that fluctuations in output are dominated by external stochastic factors. As γ approaches 1, the composed

error term is primarily driven by the technical inefficiency term u , meaning that deviations in output from the frontier are dominated by technical shortcomings or efficiency losses in the production process.

Common forms of the production function in SFA include the Cobb–Douglas production function (CD function) and the translog production function. While the CD function is simple and has clear economic implications, its assumption that the elasticity of substitution between factors is always 1 makes it difficult for the function to reflect the complex relationships in actual production. In contrast, the transcendental logarithmic production function introduces quadratic and cross terms, thus enabling a more accurate description of nonlinear relationships between factors and the complex characteristics of production technology. Given the flexibility of the translog function and its superior ability to fit actual production relationships, we adopt the translog form for the production function in SFA. It considers capital, labor, coal, oil, natural gas, and electricity and heat as its input factors and gross domestic product (GDP) as its output factor. The function's specific form is shown below:

$$\begin{aligned} \ln y_{it} = & \alpha_0 + \sum_i \alpha_i \ln x_{it} + \alpha_t t + \sum_i \sum_j \alpha_{ij} \ln x_{it} \ln x_{jt} \\ & + \alpha_{tt}(t)^2 + \sum_i \alpha_{it} \ln x_{it} t + v_{it} - u_{it} \end{aligned} \quad (6)$$

$$t = 1, 2, 3 \dots T$$

where the factor inputs for the i -th production unit in period t include capital (K), labor (L), coal (E1), oil (E2), natural gas (E3), and electricity and heat (E4).

We use the maximum likelihood estimation (MLE) method in the Frontier 4.1 software to estimate the stochastic frontier analysis model and conduct robustness checks by further changing the distributional assumptions of the technical inefficiency term. In the robustness checks, we assume that the technical inefficiency term follows a truncated normal distribution; thus, we conduct a likelihood ratio (LR) test to assess the reasonableness of this assumption.

3.1.2. Technical and Allocative Efficiency

From an input-oriented perspective, technical efficiency is defined as the ratio of the minimum feasible inputs to the actual inputs for a given level of output. In contrast, allocative efficiency reflects the extent to which input proportions are optimally adjusted across factors, under given technology, factor prices, and output levels, to minimize total input cost.

Suppose there are two input factors: x_1 and x_2 . In Figure 3, $L(y)$ denotes the production frontier, B is the actual production point, and A is a technically efficient point with the same input ratio as B. The technical efficiency at point B is defined as $TE = OA/OB$. Since points A and B share the same input ratio, they have identical allocative efficiency. We then examine the allocative efficiency at point A. As illustrated in Figure 3, both A and A' lie on the production frontier $L(y)$; thus, they are technically efficient. However, they differ in their input combinations. If A'C is an iso-cost line tangent to the production frontier $L(y)$, then A' represents the cost-minimizing input combination. Given fixed output and factor prices, the input mix at point A can be adjusted to A' by altering the input ratio. This adjustment reduces the cost at A to the level represented by point C, thereby achieving cost minimization. The allocative efficiency at A is therefore provided by allocative efficiency $(AE) = OC/OA$. As noted above, the allocative efficiency of B can likewise be expressed as AE.

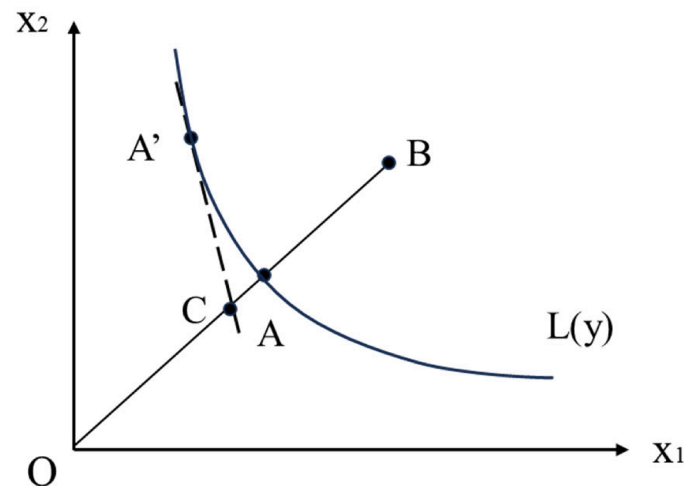


Figure 3. Schematic diagram of technical and allocative efficiency under two inputs and one output.

3.1.3. Effective Output Level

In this study, technical efficiency is measured by assuming the proportions of factor inputs remain constant. Allocative efficiency is evaluated based on full technical efficiency, that is, when $TE = 1$. Accordingly, before calculating allocative efficiency, we find it necessary to determine the effective total output under full technical efficiency ($TE = 1$).

Three approaches have commonly been employed in the literature to estimate effective total output: i. All enterprises are assumed to be fully technically efficient; therefore, technical inefficiency is disregarded when evaluating allocative efficiency [10]. ii. Technical efficiency is estimated using corrected ordinary least squares (COLS) based on the residuals of the total cost function, and this estimate is then used to derive the effective output. This approach typically assumes a deterministic production frontier [41]. iii. Actual output is used to infer the output at the technological frontier from which the effective output is calculated [42]. This approach typically employs a stochastic frontier production function. Given that technical inefficiency is generally present in production and that the stochastic frontier framework accounts for inefficiency through stochastic disturbances, we adopt the third approach to estimating effective output. The effective actual output (Y^*) is defined as the ratio of the actual output (Y) to technical efficiency.

3.1.4. Generalized Cost Function

Cost functions have been widely applied in productivity analysis, as shown in research by Berndt and David [43], Gollop and Roberts [44], and Diewert and Kevin [45]. When factor allocative inefficiencies are present in a market, the cost-minimization assumption is violated. Many studies have introduced generalized cost functions to analyze production efficiency and address this issue, such as those by Zhang [46], Guan [47], and Lee [48].

As illustrated in Figure 4, allocative inefficiency causes actual costs to deviate from the minimum attainable cost. The cost-minimizing factor combination is denoted as point M under the assumptions of a perfectly competitive market, given factor prices and an output level of $L(y)$. In the presence of allocative inefficiency, the optimal choice consistent with the same output level as point M deviates from M. This alternative cost-minimizing choice is denoted as point M^* . If the optimal choice shifts from M to M^* along the isoquant $L(y)$, allocative inefficiency results in an additional cost of ΔC . The difference in slopes between the iso-cost lines at points M and M^* indicates that the actual factor prices deviate from their corresponding shadow prices. Following relevant studies [11,48,49], the shadow price of factors is defined as follows:

$$w_m^s = d_m w_m \quad (7)$$

where w_m^s denotes the shadow price of the m -th input; w_m denotes the actual price of the m -th input; and d_m denotes the distortion factor for the m -th input, representing the degree of deviation between the actual market price and the shadow price. When $d_m - 1 = 0$, the actual price and the shadow price are identical. The greater the absolute value of $(d_m - 1)$, the greater the deviation between the actual price and the shadow price.

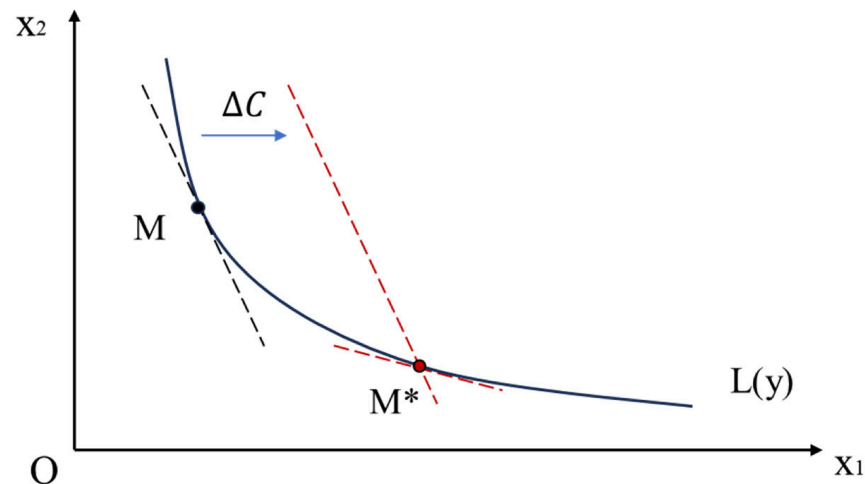


Figure 4. Schematic diagram of factor market allocation distortion.

The cost-minimization problem under allocative inefficiency is expressed as follows in accordance with the research of Christopoulos and Tsionas, Atkinson and Halvorsen, and Eakin and Kniesner [10,50,51]:

$$\begin{aligned} \text{MinGC} &= \sum_i^n w_i^s x_i \\ \text{s.t. } f(x_1 \dots x_n) &= Y^* \end{aligned} \quad (8)$$

where generalized cost (GC) denotes the shadow cost, w_m^s denotes the shadow price, x_m denotes the m -th input, $f(x_1 \dots x_n)$ denotes the production function, and Y^* denotes the effective gross output.

The ratio of the shadow prices of the two input factors must equal their marginal technical rate of substitution to minimize costs:

$$\frac{w_m^s}{w_n^s} = \frac{d_m w_m}{d_n w_n} = \frac{MP_m}{MP_n} \quad (9)$$

where MP_m denotes the marginal output of the m -th input.

When $\frac{d_m}{d_n} \neq 1$, the marginal rate of technical substitution between inputs m and n does not equal their actual price ratio, indicating allocative inefficiency. If $\frac{d_m}{d_n} > 1$, the actual price of input m is lower relative to its shadow price compared with input n , indicating that input m is overutilized. Conversely, if $\frac{d_m}{d_n} < 1$, the actual price of input m exceeds its shadow price relative to input n , indicating that input m is underutilized.

Since time may influence factors such as the technological level of the production process, the time variable t can be incorporated into the generalized cost function alongside output and cost. The function can be expressed as follows:

$$GC = GC(Y^*, d_m w_m, t) \quad (10)$$

According to Shephard's lemma [52] and combining Equations (8) and (9), the shadow cost-share equations (M_m^s) for each factor can be derived as follows:

$$M_m^s = \frac{\partial \ln GC}{\partial \ln(d_m w_m)} = \frac{\partial GC}{\partial(d_m w_m)} \cdot \frac{d_m w_m}{GC} = \frac{(d_m w_m) \cdot x_m}{GC} \quad (11)$$

We solve the system of simultaneous equations by setting up a system of generalized cost functions and cost-share equations using the feasible generalized nonlinear least squares (FGNLS) method.

Since the GC and shadow cost share (M_m^s) cannot be directly determined, we refer to Ouyang's study [4] and transform the generalized cost function and the shadow cost-share equation (the related derivation process is shown in Appendix A) to derive the actual cost-share equation and the actual cost function. We then assign a transcendental logarithmic functional form to the actual cost function, thereby obtaining the regression equation of the actual cost function as follows:

$$\begin{aligned} \ln TC &= \ln GC + \ln \sum_i \frac{M_i^s}{d_i} \\ &= \beta_0 + \beta_Y \ln Y^* + \sum_m \beta_m \ln(d_m w_m) + \beta_t t + \frac{1}{2} \beta_{YY} (\ln Y^*)^2 + \frac{1}{2} \beta_{tt} (t)^2 \\ &\quad + \frac{1}{2} \sum_m \sum_n \beta_{mn} \ln(d_m w_m) \ln(d_n w_n) + \sum_m \beta_{Ym} \ln Y^* \ln(d_m w_m) + \beta_{Yt} \ln Y^* t \\ &\quad + \sum_m \beta_{mt} \ln(d_m w_m) t + \ln \left(\sum_m \left(\frac{\beta_m + \sum_n \beta_{mn} \ln(d_n w_n) + \beta_{Ym} \ln Y^* + \beta_{mt} t}{d_m} \right) \right) \end{aligned} \quad (12)$$

$$M_m = \frac{(\beta_m + \sum_n \beta_{mn} \ln(d_n w_n) + \beta_{Ym} \ln Y^* + \beta_{mt} t) / d_m}{\sum_m [(\beta_m + \sum_n \beta_{mn} \ln(d_n w_n) + \beta_{Ym} \ln Y^* + \beta_{mt} t) / d_m]} \quad (13)$$

Since the generalized cost function must satisfy the assumption of linear homogeneity in input factor prices, the following restriction is imposed in the joint regression of Equations (12) and (13):

$$\begin{aligned} \sum_m \beta_m &= 1 \\ \beta_{mn} &= \beta_{nm}, \sum_m \sum_n \beta_{mn} = 0 \\ \sum_m \beta_{Ym} &= 0 \\ \sum_m \beta_{mt} &= 0 \end{aligned} \quad (14)$$

The following three adjustments were made during the parameter estimation: First, since the actual cost function and actual cost-share equations are homogeneous of degree zero with respect to d_i , it is necessary to select the distortion factor of one input factor for normalization. Capital has a relatively high degree of marketization among the various production factors and the lowest degree of price distortion. Therefore, selecting capital as the normalization factor can better reflect the market's distortion structure; that is, let $dk = 1$. Based on this, we rank the degree of price distortion for each energy subcategory.

Second, considering that the sum of the actual cost shares is 1 and that the actual cost function Equation (12) and all the actual cost-share equations are estimated directly, the resulting error covariance matrix from linear correlation becomes singular. Removing a cost-share equation overcomes the singularity of the error covariance matrix without altering the model's estimation results. Therefore, this paper omits the actual cost-share equation for capital to facilitate subsequent parameter estimation.

Third, given that the parameters to be estimated in Equations (12) and (13) are nested within the exponential function, we employ nonlinear estimation methods for parameter estimation. Primary nonlinear estimation methods include nonlinear least squares (NLS),

generalized nonlinear least squares (GNLS), and feasible generalized nonlinear least squares (FGNLS). Due to differences in the degree of energy dependence, cost fluctuations in response to price changes vary across provinces. Therefore, considering the existence of heteroscedasticity and the unknown covariance matrix of errors, we further select the FGNLS method for parameter estimation based on Ouyang [4].

Therefore, we used the GNLS method with the `nlshr` command in Stata 17 to fit the cost function, labor share equation, coal share equation, oil share equation, natural gas share equation, and electricity and heat share equation simultaneously.

In summary, we estimate parameters using Stata after normalizing the distortion factors and removing the capital cost component from the share equation. Following model identification, the estimation is conducted using the `nlshr` command. i. Initial residuals are calculated using NLS, and the error covariance matrix is constructed using heteroskedasticity. ii. The estimated error covariance matrix is incorporated into GNLS for iterative estimation, updating the residuals and the error covariance matrix until the parameters converge.

Following the work of Zhang Ning et al. [46], we employed the Wald test for hypothesis testing to examine the relevant characteristics of the generalized cost function. The specific hypotheses are as follows:

- i. Test whether the production function exhibits linear homogeneity.

$$\beta_{YY} = \beta_{YK} = \beta_{YL} = \beta_{YE1} = \beta_{YE2} = \beta_{YE3} = \beta_{YE4} = \beta_{YT} = 0 \quad (15)$$

- ii. Test whether the input factors are allocatively efficient.

$$d_L = d_{E1} = d_{E2} = d_{E3} = d_{E4} = 1 \quad (16)$$

Should the hypothesis hold, it would imply that the shadow prices of the input factors are equal to their actual prices. In other words, relative to the capital market, the markets for labor, coal, oil, natural gas, and electricity and heat have no allocative efficiency, and the mix of factors in the factor input markets is rational. The hypothesis not holding would suggest that the shadow prices of input factors differ from their actual prices. In other words, allocative inefficiencies relative to the capital market in the markets for labor, coal, oil, natural gas, and electricity and heat will exist, and cost minimization will have to be achieved through optimal allocation.

Based on the definition of shadow prices in Equation (7), the shadow prices for the four energy sources are computed after estimating the total cost function and the actual cost-share equations using feasible generalized nonlinear least squares (FGNLS). At a given technological level, the minimum cost ($\hat{C}_{min}$) is estimated by evaluating the actual cost function when $d_i w_i = w_i$. Following Khiabani and Hasani [12], AE is defined as the ratio of the minimum cost to the fitted cost of the actual cost function ($\hat{C}$):

$$AE = \frac{\hat{C}_{min}}{\hat{C}} \times 100\% \quad (17)$$

where the minimum cost ($\hat{C}_{min}$) of the actual cost function is provided by the following equation:

$$\begin{aligned} \ln C_{min} = & \beta_0 + \beta_Y \ln Y^* + \sum_m \beta_m \ln w_m + \beta_t t + \frac{1}{2} \beta_{YY} (\ln Y^*)^2 \\ & + \frac{1}{2} \sum_m \sum_n \beta_{mn} \ln w_m \ln w_n + \frac{1}{2} \beta_{tt} (t)^2 + \sum_{Ym} \beta_{Ym} \ln Y^* \ln w_m \\ & + \sum_m \beta_{mt} \ln w_m t + \ln \left\{ \sum_m (\beta_m + \sum_n \beta_{mn} \ln w_n + \beta_{Ym} \ln Y^* + \beta_{mt} t) \right\} \end{aligned} \quad (18)$$

The fitted value of the total cost function is as follows:

$$\begin{aligned} \ln C &= \beta_0 + \beta_Y \ln Y^* + \sum_m \beta_m \ln(d_m w_m) + \beta_t t + \frac{1}{2} \beta_{YY} (\ln Y^*)^2 + \frac{1}{2} \beta_{tt} (t)^2 \\ &+ \frac{1}{2} \sum_m \sum_n \beta_{mn} \ln(d_m w_m) \ln(d_n w_n) + \sum_m \beta_{Ym} \ln Y^* \ln(d_m w_m) + \beta_{Yt} \ln Y^* t \\ &+ \sum_m \beta_{mt} \ln(d_m w_m) t + \ln \left(\sum_m \left(\frac{\beta_m + \sum_n \beta_{mn} \ln(d_n w_n) + \beta_{Ym} \ln Y^* + \beta_{mt} t}{d_m} \right) \right) \end{aligned} \quad (19)$$

3.2. Data

Due to data availability constraints, panel data for 30 Chinese provinces (excluding Tibet, Taiwan, Hong Kong, and Macau) from 2005 to 2022 are analyzed. The variables include input quantities, price data, and economic output. The data sources are as follows:

3.2.1. Capital

Capital input is measured by capital stock. The perpetual inventory method is employed to estimate capital stock, with adjustments following the approach of Shan Haojie [53] to fit Chinese statistical data. Data are obtained from the *China Statistical Yearbook (2005–2022)* [54], and the calculation is performed with the following equation:

$$K_t = I_t + K_{t-1} \times (1 - \gamma_t) \quad (20)$$

where K represents the capital stock in period t , I represents the amount of new investment in period t , and γ represents the depreciation rate in period t ; following the work of Shan Haojie, we have chosen 10.96% [53].

Following Yang Mian et al. [55], the price of capital is measured as the sum of the depreciation rate and the interest rate minus the inflation rate. The interest rate is proxied by the People's Bank of China's benchmark lending rate, while inflation is proxied by each province's consumer price index (CPI). Capital data are obtained from the Almanac of *China's Finance and Banking (2005–2022)* [56] and the *China Statistical Yearbook (2005–2022)* [54].

3.2.2. Labor

Labor input is measured by the total number of employees at the end of the year in each province (including municipalities and autonomous regions). To smooth short-term fluctuations in labor input, the average of the current and previous year-end employee numbers is used as a more stable measure.

The price of labor is determined using the method of Wang Ning and Shi Jinchuan [57] and is proxied by labor wages. Labor data are obtained from the *China Statistical Yearbook (2005–2022)* [54], provincial statistical yearbooks, and the *China Rural Statistical Yearbook (2005–2022)* [58].

3.2.3. Energy

Energy inputs are classified into four categories based on end-use consumption in the energy balance: (A) coal, which includes raw coal, washed coal, other washed coal, coal products (coal briquettes), coal gangue, coke, coke-oven gas, blast-furnace gas, converter gas, other gas, and other coke-chemical products; (B) petroleum, which includes crude oil, gasoline, kerosene, diesel fuel, fuel oil, refinery dry gas, liquefied petroleum gas, and other petroleum products; (C) natural gas, which includes natural gas and liquefied natural gas; and (D) electricity and heat. All energy inputs are converted into standard coal equivalents using coal conversion factors. Data are obtained from the *China Energy Statistical Yearbook (2005–2022)* [59].

Electricity and heat prices are proxied by provincial and municipal electricity sales tariffs, with data obtained from the *National Electricity Price Situation Regulatory Circular* [60].

Since direct price data for coal, oil, and natural gas are unavailable, we employ the method proposed by Tao Xiaoma [61] to estimate the prices of these three energy sources for each province. The calculation equation we employ is detailed as follows:

$$\begin{aligned} \text{Energy price} &= \text{Energy price of 2005} \times \text{Energy price index} \\ \text{Energy price index} &= \frac{\sum \frac{\text{Major energy price of 2005}}{\text{Major energy consumption of 2005}}}{\text{Major energy consumption of 2005}} \end{aligned} \quad (21)$$

(1) Energy prices of 2005

The price of coal is represented by the prices of three types of energy: raw coal, coke, and coal gas. Conversely, the price of oil is denoted by the prices of four energy types: crude oil, gasoline, kerosene, and diesel. Lastly, the price of natural gas is represented by the prices of two types of energy: industrial gas and residential gas. The specific types of energy, sources, and methods for converting to constant prices are shown in the Table 2.

Table 2. Data sources and conversion to constant prices for major energy sources.

Energy Category	Subcategory	Source	Converted to Constant Prices
Coal	Raw coal	<i>Third National Industrial Census of the People's Republic of China (1995)</i> [62]	We use the producer price index (PPI) to adjust the 1995 raw coal prices to 2005 constant prices.
	Coke	<i>Price Yearbook Of China (2004)</i> [63]	We use the producer price index (PPI) to adjust the 2003 coke prices to 2005 constant prices.
	Coal gas	<i>Price Yearbook Of China (2006)</i> [63]	/
Oil	Crude oil	<i>Third National Industrial Census of the People's Republic of China (1995)</i> [62]	We use the producer price index (PPI) to adjust the 1995 crude oil prices to 2005 constant prices.
	Gasoline	<i>Third National Industrial Census of the People's Republic of China (1995)</i> [62]	We use the producer price index (PPI) to adjust the 1995 gasoline prices to 2005 constant prices.
	Kerosene	<i>Price Yearbook Of China (2006)</i> [63]	/
	Diesel	<i>Price Yearbook Of China (2006)</i> [63]	/
Natural gas	Industrial gas	<i>Price Yearbook Of China (2006)</i> [63]	/
	Residential gas	<i>Price Yearbook Of China (2006)</i> [63]	/

Note: Data from the *Third National Industrial Census of the People's Republic of China (1995)* are at the provincial level, while data from the *Price Yearbook of China (2004–2006)* are at the city level. Provincial energy price data are represented by the prices in each province's respective provincial capital.

(2) Energy Price Index from 2006 to 2022 (with 2005 energy prices as 100)

We use the Bohai-Rim Steam-Coal Price to convert the coal price index. The Daqing Oilfield price is employed to convert the oil price index. We set the fuel and power price index as the natural gas price index.

3.2.4. Economic Output

Economic output is measured by each province's GDP, which is deflated to constant 2005 prices. The data are obtained from the *China Statistical Yearbook* [54].

3.2.5. Geographic Location Division

In reference to the regional division of China's economy provided by the National Bureau of Statistics, we divide China into four major regions (northeastern, eastern, central, and western) in our subsequent regional distribution analysis. China's specific regions are shown in Table 3.

Table 3. Regional division of China's economy.

Region	Provinces
Northeastern region	Liaoning, Jilin, Heilongjiang
Eastern region	Beijing, Tianjin, Hebei, Shanghai, Jiangsu, Zhejiang, Fujian, Shandong, Guangdong, Hainan
Central region	Shanxi, Anhui, Jiangxi, Henan, Hubei, Hunan
Western region	Inner Mongolia, Guangxi, Chongqing, Sichuan, Guizhou, Yunnan, Shaanxi, Gansu, Qinghai, Ningxia, Xinjiang

4. Results

4.1. Assessment of Technical Efficiency

A parameter analysis of Equation (6) was performed using Frontier 4.1. The relevant results are shown in Table 4. The estimated value of γ in the parameter fit is 0.9993, which indicates that the technical inefficiency component accounts for a significant proportion of the composite error term, and deviations of decision-making units from the frontier can be attributed to technical inefficiencies in production. The negative value of η suggests that overall technical efficiency is decreasing on a yearly basis.

Table 4. Results of stochastic frontier parameter estimation.

Variable	Coefficient	Parameter Estimate	Standard Deviation	T-Statistic
Constant	β_0	4.2245 ***	1.1518	3.6678
lnK	β_K	0.0948	0.1392	0.6809
lnL	β_L	0.6923 ***	0.2958	2.3406
lnE1	β_{E1}	0.0962	0.0989	0.9728
lnE2	β_{E2}	0.4703 ***	0.1035	4.5457
lnE3	β_{E3}	0.0068	0.0498	0.1361
lnE4	β_{E4}	−0.6350 ***	0.1650	−3.8496
t	β_t	0.1472 ***	0.0238	6.1921
$0.5(\ln K)^2$	β_{KK}	0.0382	0.0348	1.0983
$0.5(\ln L)^2$	β_{LL}	−0.0241	0.0462	−0.5206
$0.5(\ln E1)^2$	β_{E1E1}	0.0137 *	0.0085	1.6030
$0.5(\ln E2)^2$	β_{E2E2}	0.0192	0.0237	0.8111
$0.5(\ln E3)^2$	β_{E3E3}	−0.0087 **	0.0038	−2.3240
$0.5(\ln E4)^2$	β_{E4E4}	0.2814 ***	0.0458	6.1415
$0.5(t)^2$	β_{tt}	−0.0005	0.0008	−0.6021
lnK × lnL	β_{KL}	0.0205	0.0244	0.8392
lnK × lnE1	β_{KE1}	0.0084	0.0211	0.3995
lnK × lnE2	β_{KE2}	−0.0290 **	0.0174	−1.6626
lnK × lnE3	β_{KE3}	0.0340 ***	0.0138	2.4559
lnK × lnE4	β_{KE4}	−0.0719 **	0.0349	−2.0614
lnK × t	β_{Kt}	0.0026	0.0044	0.5841

Table 4. Cont.

Variable	Coefficient	Parameter Estimate	Standard Deviation	T-Statistic
$\ln L \times \ln E1$	β_{LE1}	−0.0704 ***	0.0214	−3.2950
$\ln L \times \ln E2$	β_{LE2}	0.0375 **	0.0204	1.8404
$\ln L \times \ln E3$	β_{LE3}	−0.0243 ***	0.0094	−2.5864
$\ln L \times \ln E4$	β_{LE4}	−0.0161	0.0292	−0.5516
$\ln L \times t$	β_{Lt}	0.0034	0.0040	0.8527
$\ln E1 \times \ln E2$	β_{E1E2}	0.0205 *	0.0157	1.3119
$\ln E1 \times \ln E3$	β_{E1E3}	−0.0235 ***	0.0062	−3.7635
$\ln E1 \times \ln E4$	β_{E1E4}	0.0339 *	0.0255	1.3330
$\ln E1 \times t$	β_{E1t}	−0.0009	0.0035	−0.2617
$\ln E2 \times \ln E3$	β_{E2E3}	0.0121 *	0.0089	1.3699
$\ln E2 \times \ln E4$	β_{E2E4}	−0.1000 ***	0.0248	−4.0405
$\ln E2 \times t$	β_{E2t}	0.0029	0.0028	1.0305
$\ln E3 \times \ln E4$	β_{E3E4}	0.0015	0.0120	0.1289
$\ln E3 \times t$	β_{E3t}	−0.0061 ***	0.0017	−3.5656
$\ln E4 \times t$	β_{E4t}	−0.0149 ***	0.0054	−2.7712
σ^2			1.3864	
γ			0.9993	
η		−0.0084	0.0012	−6.9679
Log-likelihood			975.56962	
LR statistic			1398.6965	

Note: *, **, and *** denote significance levels at the 10%, 5%, and 1% levels, respectively. K, L, E1, E2, E3, and E4 denote capital, labor, coal, oil, natural gas, as well as electricity and heat, respectively.

The results of the technical efficiency calculations are shown in Table 5. Although the results cover 30 provinces over 18 years, only the results for 2005, 2010, 2015, 2020, and 2022 are presented in the main text due to space limitations. From a temporal perspective, China's technical efficiency exhibits a downward trend over time, which is consistent with the direction indicated by the parameter η in the table. This result aligns with the findings of Li [64] and Zhou et al. [65] regarding the fact that technical efficiency has declined both nationwide and in the energy sector. The reasons for this decline in technical efficiency may include the following: the actual market prices of production factors deviate from shadow prices, and production entities are unable to allocate factors according to resource scarcity, resulting in a decline in technical efficiency [66].

Table 5. Technical efficiency of 30 provinces, 2005–2022 (five-year intervals).

Region	2005	2010	2015	2020	2022
Beijing	0.6610	0.6494	0.6375	0.6254	0.6205
Tianjin	0.3910	0.3756	0.3603	0.3449	0.3387
Hebei	0.4425	0.4274	0.4121	0.3968	0.3907
Shanxi	0.3151	0.3000	0.2849	0.2700	0.2641
Inner Mongolia	0.3445	0.3292	0.3139	0.2988	0.2927
Liaoning	0.4239	0.4087	0.3934	0.3780	0.3718
Jilin	0.2537	0.2392	0.2250	0.2112	0.2057
Heilongjiang	0.3673	0.3519	0.3366	0.3213	0.3152
Shanghai	0.7495	0.7403	0.7309	0.7211	0.7172
Jiangsu	0.8781	0.8732	0.8682	0.8629	0.8608
Zhejiang	0.7086	0.6983	0.6877	0.6767	0.6723
Anhui	0.4505	0.4354	0.4202	0.4049	0.3988
Fujian	0.5536	0.5398	0.5258	0.5115	0.5058
Jiangxi	0.3594	0.3440	0.3287	0.3134	0.3073

Table 5. Cont.

Region	2005	2010	2015	2020	2022
Shandong	0.6414	0.6294	0.6171	0.6045	0.5993
Henan	0.5988	0.5858	0.5726	0.5591	0.5536
Hubei	0.4696	0.4547	0.4396	0.4245	0.4184
Hunan	0.4692	0.4543	0.4392	0.4241	0.4180
Guangdong	0.9894	0.9889	0.9884	0.9880	0.9877
Guangxi	0.2987	0.2837	0.2688	0.2542	0.2484
Hainan	0.1571	0.1452	0.1337	0.1227	0.1184
Chongqing	0.4149	0.3996	0.3843	0.3689	0.3627
Sichuan	0.5138	0.4994	0.4848	0.4700	0.4641
Guizhou	0.2277	0.2138	0.2001	0.1869	0.1816
Yunnan	0.2981	0.2831	0.2683	0.2536	0.2478
Shaanxi	0.3664	0.3510	0.3357	0.3204	0.3143
Gansu	0.2248	0.2109	0.1973	0.1841	0.1789
Qinghai	0.1398	0.1286	0.1178	0.1075	0.1035
Ningxia	0.1320	0.1210	0.1106	0.1006	0.0968
Xinjiang	0.2664	0.2518	0.2374	0.2232	0.2177

The spatial distribution pattern of technical efficiency in 2022 is shown in Figure 5. From the perspective of economic regional distribution, technical efficiency is proportional to economic development, presenting a spatial distribution pattern of high in the east and low in the west, that is, the eastern region > the central region > the northeastern region > the western region. The eastern region has the highest technical efficiency. Guangdong's technical efficiency is 0.98, which is very close to the production frontier. The technical efficiency of the central region is second only to the eastern region, while the technical efficiency of the northeastern region ranks third. The western region has the lowest technical efficiency, with all areas except Sichuan and Chongqing having efficiency below 0.36.

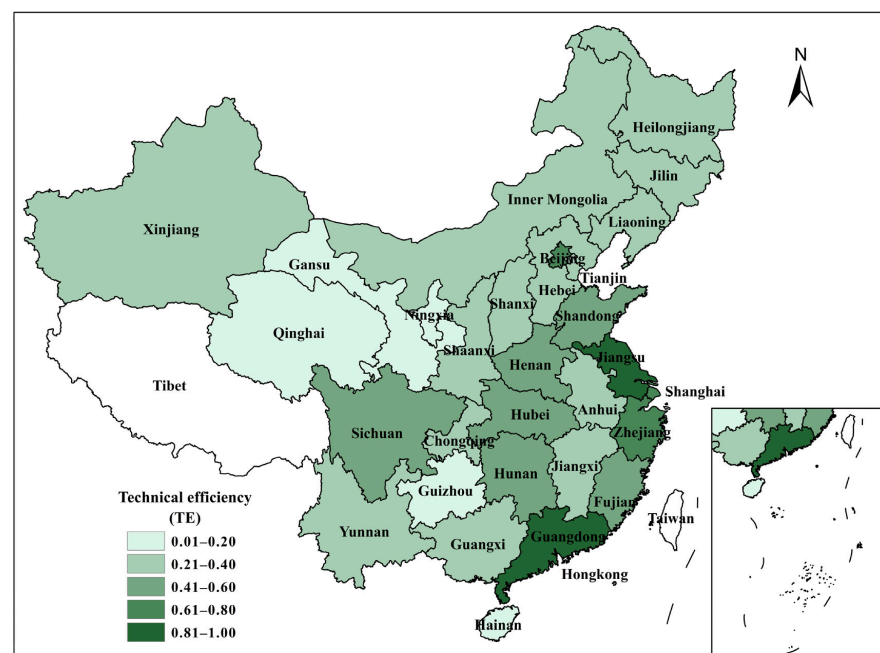


Figure 5. Distribution of technical efficiency in 2022. Notes: This map is created based on the standard map with the review number GS (2024)0650 downloaded from the Standard Map Service website of the National Administration of Surveying, Mapping, and Geoinformation. The base map has not been modified.

Taking 2022 as an example, the provinces in the eastern region and their technical efficiency are as follows: Beijing (0.62), Tianjin (0.33), Hebei (0.39), Shanghai (0.72), Jiangsu (0.86), Zhejiang (0.67), Fujian (0.50), Shandong (0.60), Guangdong (0.99), and Hainan (0.12). With the exception of Hainan, Tianjin, and Hebei, the technical efficiency of all eastern provinces is above 0.5. The provinces in central China and their technical efficiency are as follows: Shanxi (0.26), Anhui (0.39), Jiangxi (0.30), Henan (0.55), Hubei (0.42), and Hunan (0.42). The technical efficiency of provinces in the central region ranges from 0.26 to 0.55. The provinces in northeast China and their technical efficiency are as follows: Liaoning (0.37), Jilin (0.20), and Heilongjiang (0.31). The technical efficiency in the provinces of northeast China ranges from 0.20 to 0.37. The western regions and their technical efficiency are as follows: Inner Mongolia (0.29), Guangxi (0.25), Chongqing (0.36), Sichuan (0.46), Guizhou (0.18), Yunnan (0.24), Shaanxi (0.31), Gansu (0.17), Qinghai (0.10), Ningxia (0.10), and Xinjiang (0.21). Except for Sichuan, Chongqing, and Shaanxi, the technical efficiency in all western provinces is below 0.30. In summary, the technical efficiency of each region is as follows: the technical efficiency of each province in the eastern region > the provinces in the central region > the provinces in the northeast > and the provinces in the western region.

The distribution assumption for the stochastic error term v is generally a normal distribution, while the distribution assumptions for the technical inefficiency term u include a normal-half distribution, a normal-exponential distribution, a normal-truncated distribution, and a Gamma distribution [67]. For ease of operation and control, we adopt the more common distribution assumptions in SFA, namely, that the stochastic error term v follows a normal distribution, and that the technical inefficiency term u is non-negative and follows a half-normal distribution.

We assume that the technical inefficiency term follows a truncated normal distribution to assess robustness. The parameter estimation results of the truncated normal distribution are shown in Table 6.

Table 6. Parameter estimation results of total factor energy efficiency of overall energy (inefficiency term is truncated distribution).

Variable	Coefficient	Parameter Estimate	Standard Deviation	T-Statistic
Constant	α_0	2.1786 *	0.9736	2.2376
lnK	α_K	0.0279	0.1887	0.1476
lnL	α_L	1.3769 ***	0.2604	5.2881
lnE1	α_{E1}	−0.0402	0.1326	−0.3035
lnE2	α_{E2}	0.6040 ***	0.1287	4.6928
lnE3	α_{E3}	0.0302	0.0590	0.5116
lnE4	α_{E4}	−0.6426 **	0.2031	−3.1639
t	α_t	0.1048 ***	0.0284	3.6893
$0.5(\ln K)^2$	α_{KK}	0.0440	0.0544	0.8088
$0.5(\ln L)^2$	α_{LL}	−0.1337 *	0.0531	−2.5195
$0.5(\ln E1)^2$	α_{E1E1}	0.0088	0.0133	0.6622
$0.5(\ln E2)^2$	α_{E2E2}	−0.0087	0.0296	−0.2951
$0.5(\ln E3)^2$	α_{E3E3}	−0.0088	0.0062	−1.4020
$0.5(\ln E4)^2$	α_{E4E4}	0.2336 ***	0.0617	3.7870
$0.5(t)^2$	α_{tt}	−0.0012	0.0010	−1.2048
$\ln K \times \ln L$	α_{KL}	0.0107	0.0353	0.3020
$\ln K \times \ln E1$	α_{KE1}	0.0010	0.0280	0.0374
$\ln K \times \ln E2$	α_{KE2}	−0.0220	0.0276	−0.7969
$\ln K \times \ln E3$	α_{KE3}	0.0387 *	0.0197	1.9651
$\ln K \times \ln E4$	α_{KE4}	−0.0638	0.0437	−1.4607
$\ln K \times t$	α_{Kt}	0.0018	0.0059	0.3047

Table 6. Cont.

Variable	Coefficient	Parameter Estimate	Standard Deviation	T-Statistic
$\ln L \times \ln E1$	α_{LE1}	−0.0552	0.0301	−1.8302
$\ln L \times \ln E2$	α_{LE2}	0.0449	0.0268	1.6718
$\ln L \times \ln E3$	α_{LE3}	−0.0316 *	0.0141	−2.2507
$\ln L \times \ln E4$	α_{LE4}	0.0005	0.0440	0.0107
$\ln L \times t$	α_{Lt}	0.0066	0.0058	1.1403
$\ln E1 \times \ln E2$	α_{E1E2}	0.0149	0.0246	0.6066
$\ln E1 \times \ln E3$	α_{E1E3}	−0.0236 **	0.0090	−2.6093
$\ln E1 \times \ln E4$	α_{E1E4}	0.0598	0.0351	1.7024
$\ln E1 \times t$	α_{E1t}	−0.0033	0.0043	−0.7679
$\ln E2 \times \ln E3$	α_{E2E3}	0.0209	0.0139	1.5010
$\ln E2 \times \ln E4$	α_{E2E4}	−0.1077 **	0.0349	−3.0860
$\ln E2 \times t$	α_{E2t}	0.0032	0.0041	0.7920
$\ln E3 \times \ln E4$	α_{E3E4}	−0.0080	0.0184	−0.4345
$\ln E3 \times t$	α_{E3t}	−0.0067 **	0.0023	−2.8532
$\ln E4 \times t$	α_{E4t}	−0.0089	0.0072	−1.2306
σ^2			0.0820	
γ			0.9878	
η		−0.0007	0.0008	−0.7816
Log-likelihood			906.6445	
LR statistic			1260.8462	

Note: *, **, and *** denote significance levels at the 10%, 5%, and 1% levels, respectively. K, L, E1, E2, E3, and E4 denote capital, labor, coal, oil, natural gas, as well as electricity and heat, respectively.

The test finds that, assuming technical inefficiency follows a truncated normal distribution, only four parameters are significant at the 1% level, four at the 5% level, and four at the 10% level; the remaining parameters are not significant. In comparison, the parameters in Table 4 are more significant overall, and the log-likelihood estimates are larger, indicating that the assumption that the technical inefficiency term u_i follows a half-normal distribution is more appropriate.

To further test the parsimony and robustness of the model specification, we replace the production function with a Cobb–Douglas (CD) function for re-estimation, with the results shown in Table 7. Compared to the translog model in Table 4, the overall significance of the CD model is similar to that of the original model. Moreover, the log-likelihood of the original model is higher, indicating that the quadratic and interaction terms have a limited impact on the core estimation results, and the model specification is robust and reliable.

Table 7. Parameter estimation results of total factor energy efficiency of overall energy (CD function).

Variable	Coefficient	Parameter Estimate	Standard Deviation	T-Statistic
Constant	α_0	4.0027 ***	0.2414	16.5833
$\ln K$	α_K	0.2916 ***	0.0102	28.4534
$\ln L$	α_L	0.3246 ***	0.0254	12.8011
$\ln E1$	α_{E1}	0.0040	0.0067	0.5941
$\ln E2$	α_{E2}	0.0257 ***	0.0059	4.3767
$\ln E3$	α_{E3}	0.0003	0.0040	0.0836
$\ln E4$	α_{E4}	0.0004	0.0037	0.1170
t	α_t	0.0423 ***	0.0016	26.1978
σ^2			1.1193	
γ			0.9978	
η		−0.0069	0.0011	−6.3723
Log-likelihood			738.6783	
LR statistic			1521.6669	

Note: *** denotes significance level at the 1% level. K, L, E1, E2, E3, and E4 denote capital, labor, coal, oil, natural gas, as well as electricity and heat, respectively.

4.2. Assessment of Factor Allocation Distortions

A regression on the system of equations composed of Equations (12) and (13) using FGNLS is performed, with the fitting results shown in Table 8. The parameter estimation results are shown in Table 9. As seen in Table 8, the results indicate that four of the six equations yield R^2 values above 0.9, with the R^2 of the actual cost function equation reaching 0.9908. Table 9 shows that 38 of the 45 parameters of the generalized cost function reached the 1% significance level. This suggests that the generalized cost function effectively captures the distortion patterns of the various input factors.

Table 8. Fitting results of the system of equations.

Equation	Sample Size	Parameter	Mean Square Error	R^2
Actual cost function	540	41	0.0932	0.9908
Labor cost share equation	540	35	0.0721	0.9580 *
Coal cost share equation	540	35	0.0049	0.7904 *
Oil cost share equation	540	35	0.0110	0.9090 *
Natural gas cost share equation	540	35	0.0064	0.6788 *
Electricity and heat cost share equation	540	35	0.0443	0.9144 *

Note: * denotes the non-centralized R^2 in model fitting, which arises from the absence of a constant term in the equation.

Table 9. Parameter estimates of the system of generalized cost functions.

Variable	Coefficient	Parameter Estimate	Standard Deviation	p -Value
Constant	β_0	−13.1513 ***	1.7001	0.0000
$\ln Y$	β_Y	3.1399 ***	0.3640	0.0000
$\ln(d_K w_K)$	β_K	0.0471	0.0479	0.3250
$\ln(d_L w_L)$	β_L	−0.2037 **	0.1039	0.0500
$\ln(d_{E1} w_{E1})$	β_{E1}	−0.0168	0.1164	0.8850
$\ln(d_{E2} w_{E2})$	β_{E2}	0.2710 ***	0.0749	0.0000
$\ln(d_{E3} w_{E3})$	β_{E3}	0.5049 ***	0.1271	0.0000
$\ln(d_{E4} w_{E4})$	β_{E4}	0.3974 ***	0.0709	0.0000
t	β_t	−0.3886 ***	0.0406	0.0000
$0.5 \times \ln Y^2$	β_{YY}	−0.1509 ***	0.0390	0.0000
$0.5 \times \ln(d_K w_K)^2$	β_{KK}	0.1195 ***	0.0150	0.0000
$0.5 \times \ln(d_L w_L)^2$	β_{LL}	0.1084 ***	0.0082	0.0000
$0.5 \times \ln(d_{E1} w_{E1})^2$	β_{E1E1}	0.0492 **	0.0187	0.0080
$0.5 \times \ln(d_{E2} w_{E2})^2$	β_{E2E2}	0.1093 ***	0.0086	0.0000
$0.5 \times \ln(d_{E3} w_{E3})^2$	β_{E3E3}	0.0110	0.0083	0.1880
$0.5 \times \ln(d_{E4} w_{E4})^2$	β_{E4E4}	0.0945 ***	0.0186	0.0000
$0.5 \times t^2$	β_{tt}	0.0004	0.0007	0.5230
$\ln Y \times \ln(d_K w_K)$	β_{YK}	0.0364 ***	0.0067	0.0000
$\ln Y \times \ln(d_L w_L)$	β_{YL}	0.0413 ***	0.0110	0.0000
$\ln Y \times \ln(d_{E1} w_{E1})$	β_{YE1}	0.0211 *	0.0124	0.0890
$\ln Y \times \ln(d_{E2} w_{E2})$	β_{YE2}	−0.0178 **	0.0082	0.0300
$\ln Y \times \ln(d_{E3} w_{E3})$	β_{YE3}	−0.0555 ***	0.0135	0.0000
$\ln Y \times \ln(d_{E4} w_{E4})$	β_{YE4}	−0.0255 ***	0.0071	0.0000
$\ln Y \times t$	β_{Yt}	0.0294 ***	0.0044	0.0000
$\ln(d_K w_K) \times \ln(d_L w_L)$	β_{KL}	−0.0327 ***	0.0058	0.0000
$\ln(d_K w_K) \times \ln(d_{E1} w_{E1})$	β_{KE1}	−0.0171 **	0.0060	0.0040
$\ln(d_K w_K) \times \ln(d_{E2} w_{E2})$	β_{KE2}	−0.0353 ***	0.0059	0.0000
$\ln(d_K w_K) \times \ln(d_{E3} w_{E3})$	β_{KE3}	−0.0236 ***	0.0056	0.0000
$\ln(d_K w_K) \times \ln(d_{E4} w_{E4})$	β_{KE4}	−0.0108 **	0.0058	0.0650
$\ln(d_K w_K) \times t$	β_{Kt}	0.0098 ***	0.0014	0.0000

Table 9. Cont.

Variable	Coefficient	Parameter Estimate	Standard Deviation	p-Value
$\ln(d_L w_L) \times \ln(d_{E1} w_{E1})$	β_{LE1}	−0.0518 ***	0.0110	0.0000
$\ln(d_L w_L) \times \ln(d_{E2} w_{E2})$	β_{LE2}	−0.0160 **	0.0060	0.0080
$\ln(d_L w_L) \times \ln(d_{E3} w_{E3})$	β_{LE3}	0.0144 **	0.0051	0.0040
$\ln(d_L w_L) \times \ln(d_{E4} w_{E4})$	β_{LE4}	−0.0223 **	0.0073	0.0020
$\ln(d_L w_L) \times t$	β_{Lt}	−0.0069 ***	0.0017	0.0000
$\ln(d_{E1} w_{E1}) \times \ln(d_{E2} w_{E2})$	β_{E1E2}	−0.0141 **	0.0061	0.0200
$\ln(d_{E1} w_{E1}) \times \ln(d_{E3} w_{E3})$	β_{E1E3}	0.0264 ***	0.0072	0.0000
$\ln(d_{E1} w_{E1}) \times \ln(d_{E4} w_{E4})$	β_{E1E4}	0.0075	0.0065	0.2470
$\ln(d_{E1} w_{E1}) \times t$	β_{E1t}	−0.0026	0.0018	0.1570
$\ln(d_{E2} w_{E2}) \times \ln(d_{E3} w_{E3})$	β_{E2E3}	−0.0016	0.0034	0.6350
$\ln(d_{E2} w_{E2}) \times \ln(d_{E4} w_{E4})$	β_{E2E4}	−0.0424 ***	0.0081	0.0000
$\ln(d_{E2} w_{E2}) \times t$	β_{E2t}	−0.0062 ***	0.0011	0.0000
$\ln(d_{E3} w_{E3}) \times \ln(d_{E4} w_{E4})$	β_{E3E4}	−0.0265 ***	0.0073	0.0000
$\ln(d_{E3} w_{E3}) \times t$	β_{E3t}	0.0037 **	0.0015	0.0140
$\ln(d_{E4} w_{E4}) \times t$	β_{E4t}	0.0022 **	0.0011	0.0340
d_L/d_K	d_L	3.5148 ***	0.5958	0.0000
d_{E1}/d_K	d_{E1}	44.9554 ***	12.4344	0.0000
d_{E2}/d_K	d_{E2}	16.0930 ***	2.7914	0.0000
d_{E3}/d_K	d_{E3}	15.2063 ***	4.4325	0.0010
d_{E4}/d_K	d_{E4}	2.8266 ***	0.8258	0.0010

Note: *, **, and *** denote significance levels at the 10%, 5%, and 1% levels, respectively. K, L, E1, E2, E3, and E4 denote capital, labor, coal, oil, natural gas, as well as electricity and heat, respectively.

We employ the Wald test to conduct a series of hypothesis tests. The results are shown in Table 10. The Wald statistic of Equation (15) is 249.92, which is significantly greater than the critical value, leading to the rejection of the null hypothesis that the production function does not exhibit homogeneity characteristics. This means that the economies of scale at the provincial level in China will change over time; thus, the time variable must be incorporated. The Wald statistic in Equation (16) is 32.71, which is greater than the critical value, resulting in the rejection of the null hypothesis; that is, there is a significant gap between the shadow price and the actual price of input factors, which, in turn, leads to distortions in the allocation of factor markets. This also validates the findings of Hsieh and Klenow [68].

Table 10. Results of hypothesis tests.

Hypothesis Testing	Wald Statistic	Critical Value ($\alpha = 0.01$)	Degrees of Freedom	Result
Equation (15)	249.92	13.28	7	Refuse
Equation (16)	32.71	16.81	5	Refuse

The results of the distortion factor calculation show that the distortion factors of labor, coal, oil, natural gas, and electricity and heat relative to capital are 3.5148, 44.9554, 16.0930, 15.2063, and 2.8266, respectively (see Table 11). This indicates that the input of other factors is excessive relative to capital input. The extremely high distortion of coal reflects that its actual price deviates significantly from the shadow price, which may be associated with the low level of internalization of its external costs. Electricity and heat show the lowest distortion; moreover, their actual price is closer to the shadow price relative to other factors, properties that may be driven by industrial policy support for renewable energy. This discrepancy indicates that China's current energy price system still suffers from structural

distortions that tend to mislead investment and production decisions, reduce the overall allocative efficiency of energy resources, and further exacerbate environmental pollution.

Table 11. Distortions of input factors relative to capital and to electricity and heat.

Distortion	Relative to Capital ($m = K$)	Relative to Electricity and Heat ($m = E4$)
d_K/d_m	1.0000	0.3538
d_L/d_m	3.5148	1.2435
d_{E1}/d_m	44.9554	15.0944
d_{E2}/d_m	16.0930	5.6934
d_{E3}/d_m	15.2063	5.3798
d_{E4}/d_m	2.8266	1.0000

Since the measurement reflects relative distortions, we can further analyze the distortion factors with respect to electricity and heat sources. This enables us to assess the relative distortion of electricity and heat compared with other input factors.

Where $d_K/d_{E4} < 1$, electricity and heat is overutilized relative to capital; where $d_L/d_{E4} > 1$, labor is overutilized relative to electricity and heat; where $d_{E1}/d_{E4} > 1$, coal is overutilized relative to electricity and heat; where $d_{E2}/d_{E4} > 1$, oil is overutilized relative to electricity and heat; and where $d_{E3}/d_{E4} > 1$, natural gas is overutilized relative to electricity and heat.

Based on Equation (17), we estimate the allocative efficiency of various provinces in China. Overall, the allocative efficiency across all 30 provinces is less than 1, indicating inefficiency in factor allocation in each province of China. From the perspective of time, except for 2022, the allocative efficiency of each province shows an increasing trend year by year. This is consistent with the findings of Lu et al. [69] (see Figure 6). Compared with Ouyang's study [4], which treated energy as a whole, our measured allocative efficiency is lower after subdividing energy. The decline in allocative efficiency in 2022 may be related to the ongoing impact of the global pandemic and the shock of geopolitical conflicts on the global energy supply chain. Research indicates that disruptions in production and logistics caused by the pandemic, as well as fluctuations in global energy prices, have had short-term negative impacts on China's allocative efficiency [70].

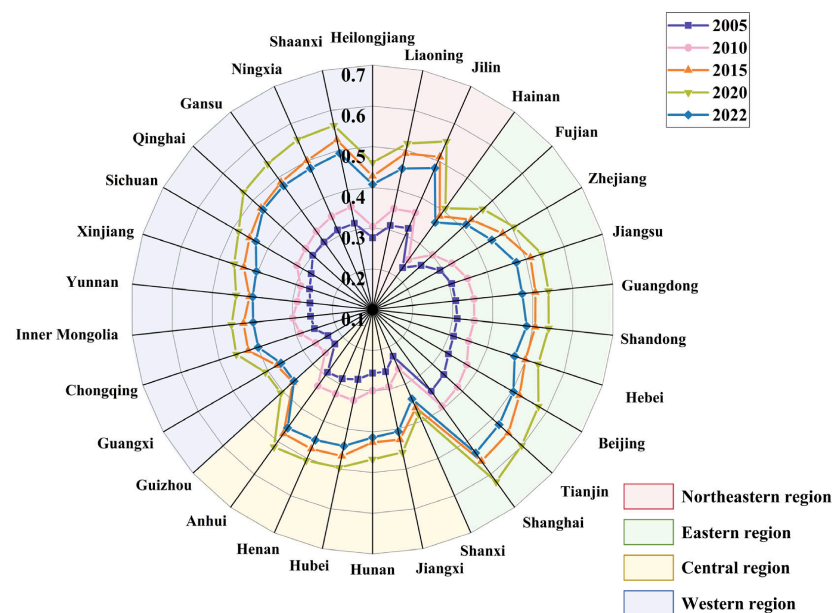


Figure 6. Allocative efficiency across provincial-level administrative regions in China.

In terms of spatial distribution, the large provinces in the eastern region have relatively high resource allocation efficiency, and overall, regional resource allocation tends to be reasonable. The central region has the lowest overall allocation efficiency, with obvious shortcomings in resource utilization. In the northeastern and western regions, the efficiency among provinces varies significantly, and some provinces still need to continuously optimize the structure of their resource allocation (see Figure 7).

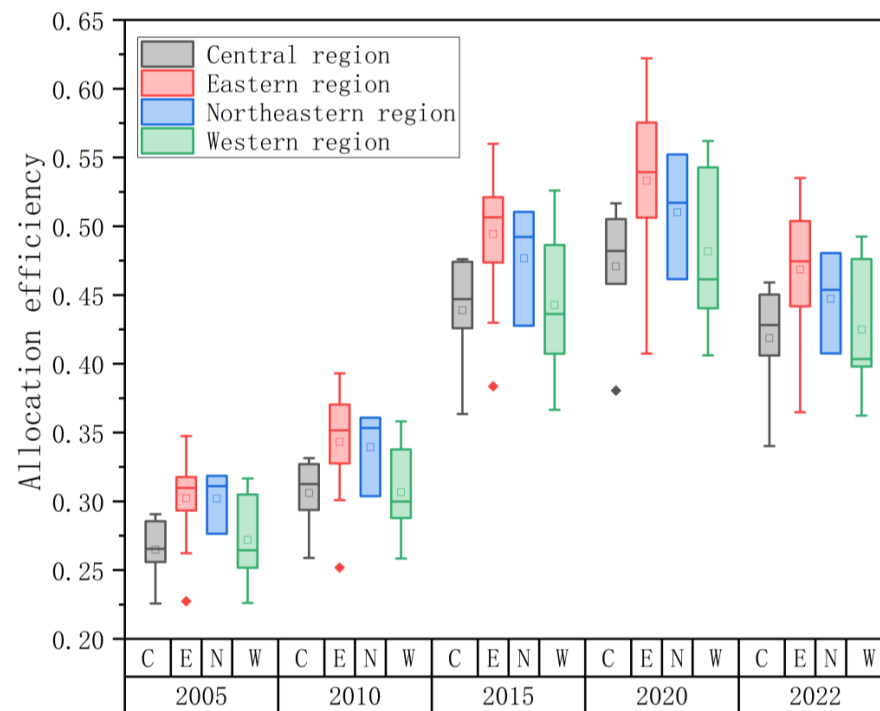


Figure 7. Box plot of China's allocation efficiency from a regional perspective.

4.3. Shadow Price Measurement

The results in Section 4.2 reveal the presence of allocative inefficiency in all provinces of China. To better understand the impact of input factors on allocation efficiency across provinces, the shadow prices of coal, oil, natural gas, electricity, and heat for each province are calculated using Equation (7). Figure 8 compares the actual prices and shadow prices of coal, oil, natural gas, and electricity and heat, and analyzes the current utilization of various factors and energy. Overall, the shadow prices for these four energy types are generally higher than their actual prices, indicating a common pattern of overuse. From a dimensional perspective, Figure 8a shows the trend of the two types of coal prices, with the difference between them remaining at a relatively high level, thus indicating a prominent coal overuse problem; Figure 8b presents the price trend of oil, with significant overall fluctuations, indicating that its price is easily affected by external factors; Figure 8c shows the price changes of natural gas and its obvious fluctuations, although the overall trend remains stable; Figure 8d corresponds to electricity and heat, whose prices indicate a relatively smooth overall trend with a noticeable decline occurring only in 2013.

The calculation results show that the shadow prices for coal, oil, natural gas, and electricity and heat are 7000–11,000, 17,000–30,000, 18,000–27,000, and 12,000–20,000 yuan per ton of standard coal equivalent, respectively. In terms of capital, the distortion factors for all four energy sources are greater than 1, indicating that all factors are overused relative to capital. This is consistent with the findings of Ouyang [4]. The shadow prices for all four energy sources are higher than their actual prices, with the order of price distortion being coal > oil > natural gas > electricity and heat. This aligns with the measurement

results of Ru et al. [26] regarding the degree of distortion of the four types of energy. In addition, our measurement results for electricity and heat differ significantly from those of Xie et al. [25]. This may be due to the difference in measurement methods between the studies: Xie et al. [25] calculated shadow prices using the substitution effect between energies, while we infer shadow prices through distortion factors.

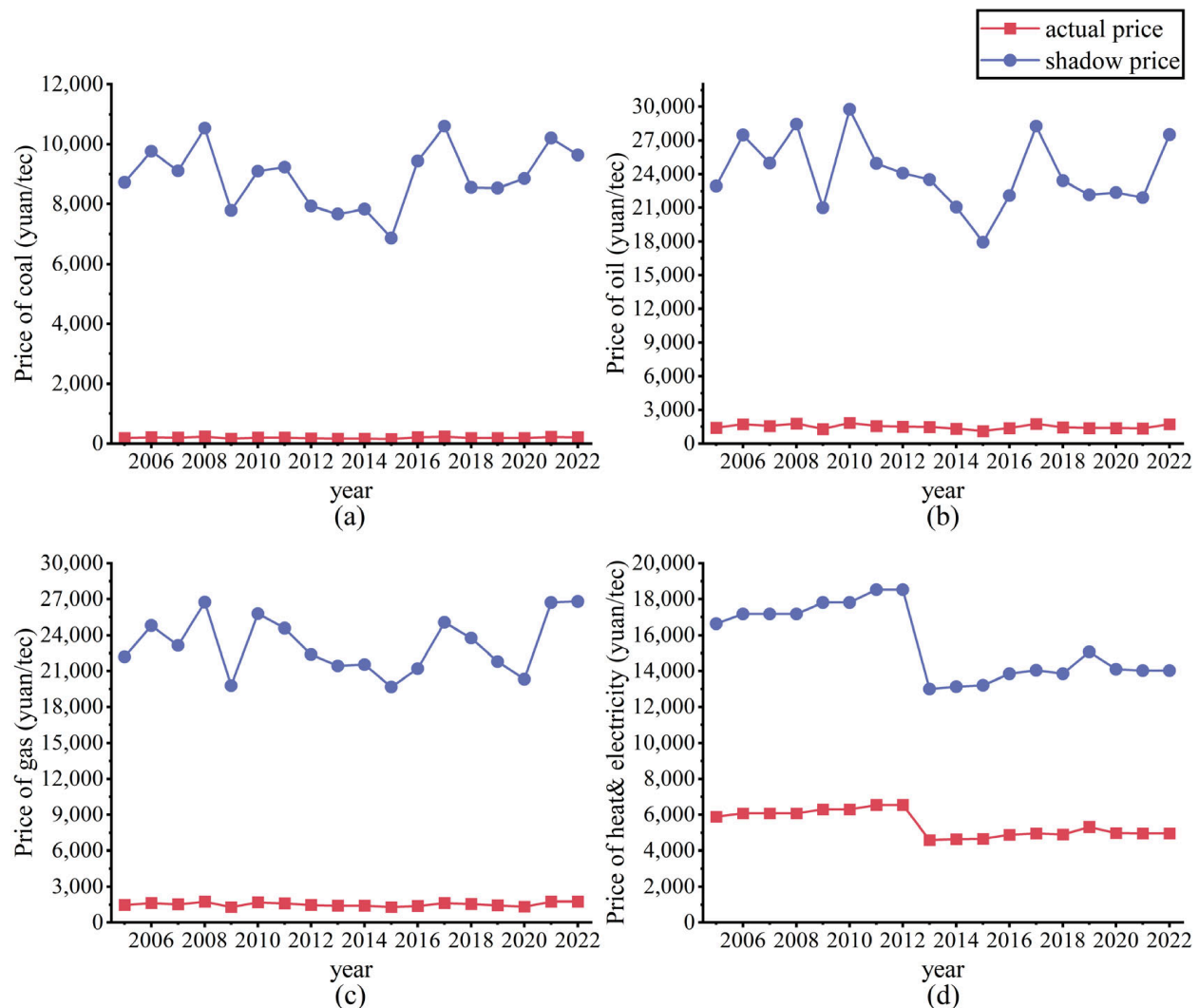


Figure 8. Comparison of actual prices and shadow prices for coal (a), comparison of actual prices and shadow prices for oil (b), comparison of actual prices and shadow prices for gas (c), comparison of actual prices and shadow prices for electricity and heat (d).

5. Conclusions

Using provincial panel data from 30 administrative regions in China between 2005 and 2022, we employ methods such as SFA and the generalized cost function approach to examine total factor energy efficiency and the optimization of factor allocation at the provincial level across three dimensions: technical efficiency, factor allocation distortions, and shadow prices. Our main conclusions are as follows:

- i. From 2005 to 2022, both technical efficiency and energy factor allocation efficiency were less than 1 at the provincial level in China, indicating room for further optimization in both technical efficiency and allocation efficiency. The slight decline in technical efficiency from 2005 to 2022 reflects the fact that certain challenges in terms of the speed of technological progress, innovation capability, and industrial upgrading remain. Allocation efficiency showed a spiral upward trend, implying that China

has gradually achieved some results in resource allocation. However, the decline in allocation efficiency from 2020 to 2022 may be related to the continued impact of the global pandemic and geopolitical conflicts on the global energy supply chain. Factors such as production and logistics disruptions caused by the pandemic and global energy price fluctuations may have had short-term negative effects on provincial-level allocation efficiency in China [70].

- ii. Research has found relatively obvious distortions in the allocation of factors in China, with factor price distortions generally present among different production factors. This is mainly reflected as follows: (1) Using capital as the standardized factor, the distortion factors for coal, oil, natural gas, electricity, and heat are all greater than 1, indicating that these four types of energy are overused relative to capital. The excessive use of traditional energy sources such as coal, oil, and natural gas reflects China's current high dependence on high-pollution, high-energy-consumption energy. (2) Using electricity and heat as the standardized factors, the distortion factors for coal, oil, and natural gas are all greater than 1, indicating that coal, oil, and natural gas are also overused relative to electricity and heat. Electricity and heat may be constrained by various factors such as policy support, infrastructure construction, technological maturity, and market mechanisms; thus, their use remains relatively limited.
- iii. The actual prices of coal and oil are lower than their shadow prices, which means that coal and oil are overused in China. This price distortion may encourage enterprises to overuse fossil energy, hinder the development and widespread adoption of electricity and heat, and cause environmental pollution. The actual prices of electricity and heat are also lower than their shadow prices, indicating that the market prices of electricity and heat do not fully reflect their potential value and lack sufficient policy support and market incentives.

The empirical findings suggest that data and decision-making references should be provided to the government for areas such as environmental tax formulation, macro-level control of energy prices, and the optimization of the energy structure. Optimizing the energy structure and improving energy utilization efficiency would help facilitate the low-carbon transformation of the energy system and effectively support the achievement of sustainable energy development goals. The study's relevant policy recommendations include the following:

- i. In response to the issue identified in this study of low and continuously declining technical efficiency, it is recommended that policy incentives be strengthened to guide enterprises to increase investment in technological research and development. This can be achieved by implementing tax incentives and establishing dedicated R&D funds to support the transformation and upgrading of traditional industries and optimize the industrial chain structure. At the same time, the Chinese government should incentivize enterprises engaged in energy transition and technological renovation with special subsidies to reduce transformation costs and steadily advance the green transition of energy.
- ii. The issues identified in this study, namely, the high degree of distortion in coal factors and China's excessively inefficient use of energy resources, indicate that enterprises are still accustomed to using traditional fossil energy and existing equipment and that they have insufficient motivation to shift toward the greater use of clean, low-carbon energy through technological transformation and innovation. The government can further optimize measures such as environmental taxes and carbon quotas to promote the return of coal prices to their shadow prices, thereby compelling enterprises to optimize their energy structure and reduce reliance on fossil energy.

- iii. This study's finding that electricity and thermal energy are less distorted compared to other forms of energy indicates that the current consumption of electricity and thermal energy is relatively low and that there is an urgent need to further expand the coverage of electricity and thermal energy and increase their consumption share. Tiered pricing for electricity and thermal energy can be implemented in the residential sector to encourage residents to increase their proportion of energy consumption from electricity and thermal sources. Additionally, the construction of electricity and thermal infrastructure should be accelerated, and the deployment of distributed photovoltaic systems, clean heating equipment, and similar technologies should be promoted to expand their application.
- iv. In response to the current situation identified in this study, characterized by inefficient allocation and significant mismatches in energy factors, it is recommended that macro-level regulation be strengthened to optimize the spatial layout of energy and prevent supply risks. Cross-regional energy dispatch projects should be relied on to coordinate resource allocation and achieve differentiated and reasonable allocation that accounts for regional differences. At the same time, a multidimensional energy early warning and dispatch model that integrates meteorological, grid, and dispatch data to optimize power dispatch plans and improve the emergency support system, thereby ensuring safe and stable electricity supply in each implementing region [71,72], should be established.

This study has the following limitations: (1) The distortion factors calculated in this study reflect the overall distortions of the national factor market and fail to capture the dynamic changes in allocative efficiency across different provinces and years. (2) This study employs province-level data that cannot reflect the situation at the micro level (e.g., enterprises). In addition, this study standardizes the distortion factor of capital and uses the standardized distortion factor to measure the shadow prices of other factors. In this case, shadow prices represent the relative prices of other factors relative to capital and cannot be directly used as a pricing standard.

Future research in this area can be built on existing research through the following paths: (1) We can construct the distortion factor function $d_i = g(\theta_1, \theta_2, \dots, \theta_k; \theta)$ by introducing k influencing factors θ of distortion factors, such as investment behavior, technological progress, and provincial resource endowment, in order to show how these dynamically changing variables affect factor allocation. (2) We can enrich data at the micro level and collect and use data from different types of enterprises (such as state-owned and private enterprises and high-energy-consuming and low-energy-consuming enterprises) from the perspective of enterprises to carry out more micro-level analyses. (3) We should conduct more spatial econometrics tests. Future research could comprehensively consider the differences in energy structure, industrial base, resource endowment, and policy implementation across provinces and conduct regional divisions through cluster analysis. Corresponding spatial econometric tests will be conducted.

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Appendix A

Since the generalized cost (GC) and shadow cost share (M_m^s) cannot be directly observed, we convert Equation (11) and reformulate it into demand functions for each input factor, as shown in Equation (A1). The demand for various input factors can be directly observed in these functions.

$$x_m = \frac{M_m^s \cdot GC}{d_m w_m}, x_m = K, L, E1, E2, E3, E4 \quad (A1)$$

In contrast to the generalized cost (GC), the actual cost (TC) is defined as the sum of each factor input multiplied by its actual price as follows:

$$TC = \sum_m w_m x_m = \sum_m w_m \frac{M_m^s \cdot GC}{d_m w_m} = GC \sum_m \frac{M_m^s}{d_m} \quad (A2)$$

The actual share equation (M_m) can be expressed as follows:

$$M_m = \frac{w_m x_m}{TC} = \frac{w_m \frac{M_m^s \cdot GC}{d_m w_m}}{GC \sum_m \frac{M_m^s}{d_m}} = \frac{\frac{M_m^s}{d_m}}{\sum_m \frac{M_m^s}{d_m}} \quad (A3)$$

The following baseline regression equation is derived by specifying a translog functional form for the generalized cost function:

$$\begin{aligned} \ln GC &= \beta_0 + \beta_Y \ln Y^* + \sum_m \beta_m \ln(d_m w_m) + \beta_t t + \frac{1}{2} \beta_{YY} (\ln Y^*)^2 \\ &+ \frac{1}{2} \sum_m \sum_n \beta_{mn} \ln(d_m w_m) \ln(d_n w_n) + \frac{1}{2} \beta_{tt} (t)^2 \\ &+ \sum_m \beta_{Ym} \ln Y^* \ln(d_m w_m) + \beta_{Yt} \ln Y^* t + \sum_m \beta_{mt} \ln(d_m w_m) t \end{aligned} \quad (A4)$$

Based on Equations (11) and (A4), the shadow cost-share equation can be expressed as follows:

$$M_m^s = \beta_m + \sum_n \beta_{mn} \ln(d_n w_n) + \beta_{Ym} \ln Y^* + \beta_{mt} t \quad (A5)$$

Substituting Equation (A4) into Equation (A2) yields the actual cost function:

$$\begin{aligned} \ln TC &= \ln GC + \ln \sum_i \frac{M_i^s}{d_i} \\ &= \beta_0 + \beta_Y \ln Y^* + \sum_m \beta_m \ln(d_m w_m) + \beta_t t + \frac{1}{2} \beta_{YY} (\ln Y^*)^2 + \frac{1}{2} \beta_{tt} (t)^2 \\ &+ \frac{1}{2} \sum_m \sum_n \beta_{mn} \ln(d_m w_m) \ln(d_n w_n) + \sum_m \beta_{Ym} \ln Y^* \ln(d_m w_m) + \beta_{Yt} \ln Y^* t \\ &+ \sum_m \beta_{mt} \ln(d_m w_m) t + \ln \left(\sum_m \left(\frac{\beta_m + \sum_n \beta_{mn} \ln(d_n w_n) + \beta_{Ym} \ln Y^* + \beta_{mt} t}{d_m} \right) \right) \end{aligned} \quad (A6)$$

Substituting Equation (A5) into Equation (A3) yields the actual cost-share equation:

$$M_m = \frac{(\beta_m + \sum_n \beta_{mn} \ln(d_n w_n) + \beta_{Ym} \ln Y^* + \beta_{mt} t) / d_m}{\sum_m [(\beta_m + \sum_n \beta_{mn} \ln(d_n w_n) + \beta_{Ym} \ln Y^* + \beta_{mt} t) / d_m]} \quad (A7)$$

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