

Article

Calculation Model for the Scale of Planning Urban Rail Transit Network Based on the Lotka–Volterra Model

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Abstract

There is a typical coopetition relationship between the urban rail transit (URT) network scale and the urban development (UD) level. A reasonable URT network scale is essential for promoting sustainable UD. Currently, the determination of the URT network scale for planning primarily relies on qualitative approaches such as static estimation and analogical methods, which fail to dynamically reflect the coopetition relationship between URT and UD. An improved time-varying parameter Lotka–Volterra (LV) model derived from ecological theory is employed to describe the coopetition relationship between the URT network scale and the UD level. The sliding-window least squares method is applied to estimate parameters of the model. Based on the improved LV model, the lower bound of the URT network scale is obtained by solving for the minimum network scale required to promote sustainable UD under a cooperative relationship; the upper bound of the URT network scale is obtained by solving for the maximum network scale that urban resources can support under competitive conditions. The proposed model is validated using eight Chinese cities with different UD levels. The study offers quantitative theoretical insights for determining the reasonable URT network scale for planning.

Keywords: urban rail transit network scale; urban development; coopetition relationship

1. Introduction

There is a typical coopetition relationship between the urban rail transit (URT) network scale and the urban development (UD) level. URT promotes UD by improving transport efficiency, thereby fostering population concentration and economic growth. In turn, UD provides financial, land, and other resources for URT construction, supporting further URT network scale expansion. Thus, URT and UD exhibit a cooperative relationship. However, under limited urban resources (e.g., fiscal and land resources), an excessively large URT network scale may lead to a financial burden on the city. For example, during the operation of Zhuhai Tram Line 1, the actual passenger transport intensity was only 0.037×10^4 passenger-trips/(km·day), equivalent to approximately 5% of the value forecast in the feasibility study, while the average transport cost was about CNY 67 per passenger-trip. Conversely, insufficient financial resources allocated to URT may result in a network scale that fails to meet urban travel demands, thereby constraining sustainable UD [1–4]. For example, the average daily passenger volume of Beijing’s URT network reached 1.7943 million passenger-trips in 2007. Following the opening of Metro Line 5, network-wide ridership increased by 59% compared with the pre-opening level, with daily passenger volume reaching 2.86 million passenger-trips. Some stations had to implement



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passenger flow control measures, which reduced urban transport operational efficiency and constrained UD. Therefore, determining a reasonable URT network scale is essential for balancing the coopetition relationship between URT and UD and promoting sustainable urban development.

Currently, the determination of the URT network scale in planning relies primarily on static indicators and qualitative analyses, such as static indicator-based estimation, expert judgment, and analogy-based approaches [5]. However, these methods cannot dynamically reflect the coopetition relationship between the URT network scale and the UD level. The dynamic coopetition relationship exhibits nonlinear characteristics that are difficult to describe using traditional linear models. The Lotka–Volterra (LV) theory [6] derived from ecological theory offers a solution to tackle this issue, which can describe relationships, such as cooperation, competition, and mutualism, between species through nonlinear dynamics. The LV theory can determine the scale of species by analyzing the relationships between species. It has been introduced to fields beyond ecological research, such as urban planning and urban rail transit planning [7]. In this study, the LV model is suitable for studying the coopetition relationships between URT and UD.

Therefore, a calculation model is constructed for determining a reasonable URT network scale in planning. An improved time-varying parameter LV model is employed to describe the coopetition relationship between URT and UD. The sliding-window least squares method is applied to estimate parameters of the model. Based on the improved LV model, the lower bound of the URT network scale is obtained by solving for the minimum network scale required to promote sustainable UD under a cooperative relationship; the upper bound of the URT network scale is obtained by solving for the maximum network scale that urban resources can support under competitive conditions. The proposed model is validated using eight Chinese cities with different UD levels. The main contributions of this study are summarized as follows:

- (1) An improved time-varying parameter LV model is proposed to describe the coopetition relationship between URT and UD.
- (2) A calculation model for determining the reasonable URT network scale for planning is constructed, which is constrained by the minimum URT network scale required for UD and the maximum URT network scale that urban resources can support.
- (3) A time-varying parameter estimation approach is developed by integrating the sliding-window least squares method.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the coopetition-based URT network scale model. Section 4 validates the proposed model using empirical data from eight Chinese cities. Section 5 concludes the study and discusses future research directions.

2. Literature Review

In this section, existing studies are reviewed from two main perspectives: the quantitative analysis of the coopetition relationships among system elements and the research on the calculation of URT network scale.

In terms of the quantitative analysis of the coopetition relationships among system elements, existing studies have primarily employed the system dynamics model [8], the coupling coordination model [9], and econometric methods [10]. Additionally, some studies have examined the coopetition relationship through game theory models [11] or complex network approaches [12], thereby exploring system interaction mechanisms from different perspectives. However, the aforementioned methods mainly focus on correlation analysis or the characterization of individual interaction mechanisms. These limitations make it difficult to capture dynamic interactions among system elements. To address this

limitation, the LV model, proposed by Lotka and Volterra, offers an effective solution for describing the coopetition relationship. By constructing nonlinear differential equations, the LV model characterizes the coopetition relationship among system elements and their evolutionary processes.

Currently, the application of the LV model in urban and transportation research remains largely exploratory. Maheshwari et al. [13] applied the LV model to urban system analysis to characterize the coopetition relationship between transportation development and urban economic growth during the UD process. Sun et al. [14] employed the LV model to describe the coopetition relationship among different transportation modes. Liang and Meng [15] further extended the LV model to urban public transportation systems. They examined the coopetition relationship between URT and conventional bus systems. Their results indicate that the degree of competition between subsystems varies with the level of UD. Building on this foundation, some studies have attempted to determine the reasonable scale of system elements and conduct predictions based on the coopetition relationship. Similarly, Sun et al. [16] constructed a coopetition relationship between new energy vehicles and traditional fuel vehicles using the LV model and predicted the development scale of new energy vehicles. These studies indicate that the scale of system elements not only reflects the interaction patterns but also embodies the evolutionary outcomes of the coopetition relationship. In the context of URT construction and UD, a reasonable URT network scale can be regarded as an outcome of the coopetition relationship, providing a new perspective for determining URT network scale.

Regarding the methods for URT network scale calculation, existing studies have mainly adopted two dominant perspectives: static estimation approaches and optimization-based methods. Other approaches, such as simulation-based [17] and data-driven methods [18], have also been explored. Early studies primarily relied on indicators such as population size, travel demand, and service level, and calculated network scale through regression analysis, scale indicator methods, or ridership forecasting approaches [19]. Baum-Snow [20] further examined the relationship between urban transportation and spatial structure, analyzing the demand characteristics of rail transit and their spatial response mechanisms. Some studies have also indicated that there are empirical relationships between URT network scale and indicators such as passenger transport intensity and urban economic development level [21,22], which can be used to indirectly evaluate the rationality of network scale. However, such methods largely depend on existing data and static indicators, and typically determine a single scale value under given demand conditions. As a result, they are unable to capture the dynamic changes during UD processes.

Building on this, some scholars have introduced multi-objective optimization methods to optimize network scale. Relevant studies typically develop optimization models for URT networks by incorporating objectives or constraints such as construction cost, operational efficiency, travel time, and service coverage. Chakroborty [23] applied genetic algorithms and related optimization techniques to rail transit network design problems. Canca et al. [24] further extended these approaches to the integrated optimization of line planning and capacity allocation. In addition, previous studies have employed DEA and its extended models to evaluate the travel efficiency of public transport, the spatial efficiency of URT station areas, and the scale efficiency of passenger railway operators [25–27]. Wang et al. [28] investigated diagnostic and real-time detection methods for freight wagon hunting performance, providing a reference for the evaluation of transportation system performance and efficiency. Overall, optimization-based methods have improved the scientific basis of decision-making in network planning. However, these studies remain primarily focused on engineering optimization under given demand conditions, and do not account for the coopetition relationship between URT and UD.

In summary, existing studies have made important contributions to the competition relationship and to methods for URT network scale calculation. However, several limitations remain in the existing literature:

- (1) Existing studies primarily determine URT network scale using demand forecasting, indicator-based methods, or optimization models, with limited consideration of the competition relationship between URT and UD.
- (2) Most existing approaches mainly rely on static parameters, which restrict their ability to capture the dynamic interactions between transportation and urban systems over the course of UD.
- (3) Existing studies lack a quantitative calculation model for determining the reasonable URT network scale range in planning.

To address these limitations, an improved time-varying parameter LV model derived from ecological theory is employed to describe the competition relationship between URT and UD. The lower bound is calculated based on the minimum URT network scale required to promote sustainable UD under cooperative conditions; the upper bound is calculated based on the maximum URT network scale that urban resources can support under competitive conditions. Therefore, a calculation model is proposed for determining a reasonable URT network scale in planning.

3. Methodology

3.1. LV Model

The LV model is a classical model in ecology used to describe the competition among species. In an ecosystem, the growth of a species is influenced not only by its intrinsic growth rate and the environmental carrying capacity (the resource conditions provided by the ecosystem), but also by its interactions with other species. For example, two populations may form a competitive relationship over limited resources or develop a cooperative relationship through mutualism. These interspecific interactions further affect the population size. For two interacting species, denoted by $x(t)$, $y(t)$, the generalized LV model can be expressed as Equation (1).

$$\begin{cases} \frac{dx(t)}{dt} = r_1 x(t) \left(1 - \frac{x(t)}{K_1} + \frac{\alpha y(t)}{K_2}\right) \\ \frac{dy(t)}{dt} = r_2 y(t) \left(1 - \frac{y(t)}{K_2} + \frac{\beta x(t)}{K_1}\right) \end{cases} \quad (1)$$

where $x(t)$ and $y(t)$ represent the population size of two species at time t , respectively; r_1 and r_2 denote the intrinsic growth rates of the two species; and K_1 , K_2 represent the baseline carrying capacities provided by the limited-resource environment for the growth of the two species when the influence of the other species is not considered. $1 - \frac{x(t)}{K_1}$ and $1 - \frac{y(t)}{K_2}$ describe the self-limiting effects caused by environmental carrying-capacity constraints during the growth processes of the two species. As the species scale gradually approaches its environmental carrying capacity, this term decreases, indicating that the available space for further growth is compressed and the growth rate gradually declines.

α and β are interaction coefficients. Specifically, α represents the effect of species y on species x , while β represents the effect of species x on species y . When $\alpha > 0$, species y has a positive effect on species x ; when $\alpha < 0$, species y has a negative effect on species x . Similarly, when $\beta > 0$, species x has a positive effect on species y ; when $\beta < 0$, species x has a negative effect on species y . In addition, larger absolute values of $|\alpha|$ and $|\beta|$ indicate stronger positive or negative interaction effects. Furthermore, the type of interaction between the two species can be identified according to the sign combination of α , β . When $\alpha > 0$ and $\beta > 0$, the two species exhibit a mutualistic or cooperative relationship. When $\alpha < 0$ and $\beta < 0$, the two species exhibit a competitive relationship. When the signs

of the two coefficients are opposite, the interaction directions between the two species are different.

Therefore, the LV model can describe the interactions between species. More importantly, it can determine the population size of each species by characterizing their interaction relationship. This provides a theoretical basis for further analyzing the interaction between URT network scale and UD level in this study.

3.2. LV Model for the Competition Between URT Network Scale and UD Level

URT promotes UD by improving transport efficiency, thereby fostering population concentration and economic growth. In turn, UD provides financial, land, and other resources for URT construction, supporting further URT network scale expansion. Thus, URT and UD exhibit a cooperative relationship. However, under limited urban resources, an excessively large URT network scale may lead to a financial burden on the city. Conversely, insufficient financial resources allocated to URT may result in a network that fails to meet urban travel demands, thereby constraining sustainable UD. Thus, URT and UD exhibit a competitive relationship.

The competition relationship between URT and UD is analogous to the interaction between species in an ecological system. Based on this analogy, this study abstracts the URT network scale and UD level as two interacting system variables. Specifically, the URT network scale is used to characterize the development level of URT, while the UD level is constructed as a composite index that integrates the major UD dimensions other than URT, including economic growth, population agglomeration, spatial expansion, road infrastructure, and environmental conditions. Within this modeling framework, the analysis is focused on the competition relationship between URT network scale and the UD level.

In the generalized LV model, the intrinsic growth rates, environmental carrying capacities, and interaction coefficients are usually assumed to be constants, which is suitable for describing the interaction relationship between two species under relatively stable environmental conditions. However, as urban economic levels, population sizes, spatial structures, and transport demands continue to change, the competition relationship between URT and UD also evolves over time. The static parameters cannot capture the dynamic characteristics of URT and UD. Therefore, referring to previous studies on time-varying parameter LV models [29,30], the time-varying parameters are introduced into Equation (1) to characterize the dynamic evolution of the competition relationship between URT network scale and UD level, as shown in Equation (2).

$$\begin{cases} \frac{dR(t)}{dt} = r_1(t)R(t)\left(1 - \frac{R(t)}{K_1(t)} + \frac{\alpha(t)U(t)}{K_2(t)}\right) \\ \frac{dU(t)}{dt} = r_2(t)U(t)\left(1 - \frac{U(t)}{K_2(t)} + \frac{\beta(t)R(t)}{K_1(t)}\right) \end{cases} \quad (2)$$

where $R(t)$ represents URT network scale (km); $U(t)$ represents UD level; $r_1(t)$, $r_2(t)$ denote the intrinsic growth rates of URT network scale and UD level, respectively; $\alpha(t)$ and $\beta(t)$ are interaction coefficients that characterize the competition relationship between URT and UD. Specifically, $\alpha(t) > 0$ indicates that UD promotes URT expansion, whereas $\alpha(t) < 0$ indicates UD constrains URT expansion due to limited resources. Similarly, $\beta(t) > 0$ implies that URT promotes UD, reflecting the positive effects of URT on economic growth and population agglomeration, while $\beta(t) < 0$ indicates that URT inhibits UD. $r_1(t)$, $r_2(t)$, $\alpha(t)$ and $\beta(t)$ are obtained through the parameter estimation procedure.

$K_1(t)$ represents the dynamic carrying capacity of the URT network scale in year t under the current UD level $U(t)$. Based on relevant studies [31,32], the Logistic function is used to calculate $K_1(t)$, as shown in Equation (3).

$$K_1(t) = K_{1\min} + \frac{K_{1\max} - K_{1\min}}{1 + \exp[-\lambda_1(U(t) - U(t_c))]} \quad (3)$$

where $K_{1\min}$ denotes the initial supportable URT network scale when the UD level is relatively low. $K_{1\max}$ denotes the long-term supportable URT network scale when the UD level is relatively high, and the conditions of resources, space, environment, and public services are relatively well developed. It can be determined by fitting the actual URT network scale of each city using the Logistic model. λ_1 is the adjustment coefficient, reflecting the influence of changes in the UD level on the dynamically supported URT network scale. t_c denotes the inflection year of the Logistic fitting curve of the UD index, and $U(t_c)$ is the corresponding UD index at the inflection point.

$K_2(t)$ represents the maximum population size that a species can reach without the influence of other species in the classical LV equation. In this study, it can be interpreted as the maximum UD level corresponding to the existing urban support conditions when the promoting effect of URT is not considered. Since the maximum population size dynamically changes with the evolution of UD, $K_2(t)$ is treated as a time-varying parameter in this study. Considering that URT usually has a time-lagged effect on UD, $K_2(t)$ can be approximately represented by the UD level and support conditions formed during the lag period. Accordingly, $U(t - \tau)$ is used to represent $K_2(t)$, where τ denotes the time-lag parameter of the impact of URT on UD.

3.3. Local Stability Analysis

The stable relationship between the URT network scale and the UD level is an important basis for judging whether the calculation results are reasonable. Based on the traditional stability analysis method of the LV model [33–35], the frozen-time method [36–38] is adopted to assess local stability for the proposed LV model. For each year t , the time-varying parameters are fixed at their values for that year. The local stability of each candidate equilibrium point is then analyzed under the corresponding parameter conditions. This analysis determines whether the model has a locally stable equilibrium point with practical significance. Then, the equilibrium equation at year t can be written as Equation (4). The Jacobian matrix corresponding to Equation (4) is constructed, as shown in Equation (5).

$$\begin{cases} f_t(R, U) = r_1(t)R(t)(1 - \frac{R(t)}{K_1(t)} + \frac{\alpha(t)U(t)}{K_2(t)}) = 0 \\ g_t(R, U) = r_2(t)U(t)(1 - \frac{U(t)}{K_2(t)} + \frac{\beta(t)R(t)}{K_1(t)}) = 0 \end{cases} \quad (4)$$

$$J_t = \begin{pmatrix} \frac{\partial f_t}{\partial R} & \frac{\partial f_t}{\partial U} \\ \frac{\partial g_t}{\partial R} & \frac{\partial g_t}{\partial U} \end{pmatrix} = \begin{pmatrix} r_1(t)(1 - \frac{2R(t)}{K_1(t)} + \frac{\alpha(t)U(t)}{K_2(t)}) & r_1(t)R(t)\frac{\alpha(t)}{K_2(t)} \\ r_2(t)U(t)\frac{\beta(t)}{K_1(t)} & r_2(t)(1 - \frac{2U(t)}{K_2(t)} + \frac{\beta(t)R(t)}{K_1(t)}) \end{pmatrix} \quad (5)$$

Equation (5) is evaluated at each candidate equilibrium point, and the local stability of each equilibrium point is determined from the trace and determinant of the corresponding Jacobian matrix, as summarized in Table 1. Specifically, $P_1(t)$ represents an initial boundary state in which both UD and URT are at zero levels. $P_2(t)$ represents a boundary state in which the URT network scale reaches its carrying-capacity level for the corresponding year, whereas the UD level is zero; this state generally lacks practical significance. $P_3(t)$ represents a boundary state in which the URT network scale is zero, whereas the UD level reaches its carrying-capacity level for the corresponding year. These three states cannot

form a coordinated co-evolutionary relationship between URT and UD and are generally unstable in actual urban systems. $P_4(t)$ is the equilibrium point corresponding to year t , representing a nonzero equilibrium state of both the URT network scale and the UD level at year t . Accordingly, when $\alpha(t)\beta(t) < 1$, the equilibrium point is locally stable.

Table 1. Equilibrium points and local stability conditions of the time-varying LV model.

Equilibrium Points	$-\text{tr}[J_t(P_i(t))]$	$\det[J_t(P_i(t))]$	Stability Condition
$P_1(0, 0)$	$-(r_1(t) + r_2(t))$	$r_1(t)r_2(t)$	Unstable
$P_2(K_1(t), 0)$	$r_1(t) - r_2(t)(1 + \beta(t))$	$-r_1(t)r_2(t)(1 + \beta(t))$	Unstable
$P_3(0, K_2(t))$	$r_2(t) - r_1(t)(1 + \alpha(t))$	$-r_1(t)r_2(t)(1 + \alpha(t))$	Unstable
$P_4\left(\frac{(1+\alpha(t))K_1(t)}{1-\alpha(t)\beta(t)}, \frac{(1+\beta(t))K_2(t)}{1-\alpha(t)\beta(t)}\right)$	$\frac{r_1(t)(1+\alpha(t))+r_2(t)(1+\beta(t))}{1-\alpha(t)\beta(t)}$	$\frac{r_1(t)r_2(t)(1+\alpha(t))(1+\beta(t))}{1-\alpha(t)\beta(t)}$	Locally stable $\alpha(t)\beta(t) < 1$

3.4. Parameter Estimation Method for the Model

The sliding-window least squares method is employed to estimate the key parameters $r_1(t), r_2(t), \alpha(t), \beta(t)$. Within each time window, the parameters are assumed to remain locally constant. By sliding the window year by year, a time-varying sequence of the parameters is obtained. For a target year t (from the first year that allows a full window to the last observation year, i.e., $t = t_0, t_0 + 1, \dots, t_n$), the window consists of all observations from year $t-L+1$ to year t . The window slides one year forward each step, yielding a distinct parameter vector for each year t .

Within each sliding window, given the parameter $\theta_t = \{r_1(t), r_2(t), \alpha(t), \beta(t)\}$, the fourth-order Runge–Kutta (RK4) method [39] is employed to obtain approximate solutions. The model is solved iteratively to generate the simulated time series $R_{\text{sim}}(t)$ and $U_{\text{sim}}(t)$ for $t = 1, 2, 3 \dots T$. Then, to minimize the deviation between the simulated values and the observed values, a least squares objective function is constructed for optimal parameter estimation, as shown in Equation (6).

$$\min_{\theta \in \Theta} J(\theta) = \frac{1}{T} \sum_{t=1}^T \left[\frac{(R_{\text{sim}}(t) - R_{\text{obs}}(t))^2}{\sigma_R^2} + \frac{(U_{\text{sim}}(t) - U_{\text{obs}}(t))^2}{\sigma_U^2} \right] + \lambda \|\theta\|^2 \quad (6)$$

where $R_{\text{obs}}(t), U_{\text{obs}}(t)$ represent the observed values; $R_{\text{sim}}(t), U_{\text{sim}}(t)$ represent the simulated values obtained by fourth-order Runge–Kutta integration (time step 0.1 year); σ_R^2, σ_U^2 are the sample variances of the observed series within each sliding-window. λ is the L2 regularization term coefficient; Θ is constrained within predefined bounds.

For each sliding-window, the following optimization is performed independently. The model parameters $\theta_t = \{r_1(t), r_2(t), \alpha(t), \beta(t)\}$ are estimated using a nonlinear least squares approach. Starting from initial parameter values, the parameters are iteratively updated. At each iteration, numerical integration is performed based on the updated parameters to generate the simulated series $R_{\text{sim}}(t)$ and $U_{\text{sim}}(t)$, and the corresponding objective function value is computed. The parameter set is continuously adjusted to minimize the objective function until convergence is achieved. The convergence criteria include that the change in the objective function is smaller than a predefined threshold, the change in parameters falls below a given tolerance, or the maximum number of iterations is reached. The optimal parameter estimates for each sliding-window are finally obtained as $\hat{\theta}_t = \{\hat{r}_1(t), \hat{r}_2(t), \hat{\alpha}(t), \hat{\beta}(t)\}$.

Since future observations are unavailable in the prediction period, the model parameters cannot be directly estimated using the sliding-window least squares method. To ensure the continuity between historical and future parameters, and with reference to existing studies [40,41], this study adopts a stage-based parameter continuation method.

Specifically, the UD index, $U(t)$, of each city is fitted using a Logistic curve [42]. According to the variation characteristics of the fitted curve, the UD process is divided into three stages: the low-development stage (L), the medium-development or rapid-growth stage (M), and the high-development or mature stage (H). The stage division provides a basis for extending historical parameters to the prediction period. It is assumed that the evolution of each city exhibits path dependence, indicating that future parameter values are influenced by the historical trajectory of the city. Meanwhile, cities at similar development stages may have similar evolution patterns; therefore, the parameters of cities in the same stage can be used as a cross-city reference. On this basis, the stage-specific parameters are constructed as shown in Equation (7).

$$\theta_i^{f,s} = \omega_1 \theta_i^{late,s} + \omega_2 \theta_{group}^{late,s}, s \in \{L, M, H\} \quad (7)$$

where $\theta_i^{f,s}$ denotes the future value of parameter θ for the city i at development stage s ; $\theta_i^{late,s}$ denotes the average parameter value of city i during the late historical period of stage s , and $\theta_{group}^{late,s}$ represents the average parameters of cities at the same development stage; the weights ω_1 and ω_2 are the weights assigned to city-specific historical information and cross-city reference information, respectively, satisfying $\omega_1 + \omega_2 = 1$.

3.5. Lower Bound Calculation for the URT Network Scale

The lower bound of the URT network scale, $R_{\min}(t)$, is defined as the minimum URT network scale required to maintain non-negative UD growth under the condition that URT has a positive promoting effect on UD.

According to Equation (2), to ensure that UD can maintain non-negative growth, the following condition should be satisfied: $\frac{dU(t)}{dt} \geq 0$. By setting $\frac{dU(t)}{dt} = 0$, Equation (8) can be obtained. Since $r_2(t)U(t) > 0$, Equation (8) can be simplified as Equation (9). Solving Equation (9) for $R(t)$, the critical URT network scale required to maintain non-negative UD growth can be obtained as Equation (10). Considering the non-negativity of the URT network scale, the lower-bound formula of $R_{\min}(t)$ is expressed as Equation (11).

$$r_2(t)U(t)\left(1 - \frac{U(t)}{K_2(t)} + \frac{\beta(t)R(t)}{K_1(t)}\right) = 0 \quad (8)$$

$$\left(1 - \frac{U(t)}{K_2(t)} + \frac{\beta(t)R(t)}{K_1(t)}\right) = 0 \quad (9)$$

$$R(t) = K_1(t) \cdot \frac{1}{\beta(t)} \cdot \left(\frac{U(t)}{K_2(t)} - 1\right) \quad (10)$$

$$R_{\min}(t) = \max\left\{0, K_1(t) \cdot \frac{1}{\beta(t)} \cdot \left(\frac{U(t)}{K_2(t)} - 1\right)\right\} \quad (11)$$

It should be further noted that the lower-bound calculation in Equation (11) is based on the premise that URT has a positive promoting effect on UD, namely $\beta(t) > 0$. This is consistent with the definition of the lower bound in this study, which aims to identify the minimum URT network scale required to maintain non-negative UD growth when URT can promote UD.

Therefore, $R_{\min}(t)$ can be interpreted as the minimum URT network scale required at time t . This scale is needed to support the increase in the UD level from $K_2(t)$ to the current level $U(t)$. Here, $K_2(t)$ denotes the UD level that can be supported by the urban conditions formed during the lag period. When the actual network scale is lower than $R_{\min}(t)$, the URT network scale may be insufficient to support the current demand for UD level, and the growth of UD may be constrained by transport inefficiency. When the actual network

scale reaches or exceeds $R_{\min}(t)$, the URT network scale can be considered to have the basic supporting capacity required to maintain continuous UD level growth.

3.6. Upper Bound Calculation for the URT Network Scale

URT construction requires substantial financial investment, land space, and other urban resources. If the URT network scale expands excessively, it may lead to insufficient passenger demand, reduced efficiency of resource allocation, and increased local fiscal pressure, thereby adversely affecting sustainable UD. Therefore, the URT network scale should not expand without limit, but should be jointly constrained by the UD level and urban resource-supporting conditions. $R_{\max}(t)$ represents the upper bound of the URT network scale. According to Equation (2), when the URT network scale reaches the critical upper-bound state, its growth rate becomes zero. By setting $\frac{dR(t)}{dt} = 0$, Equation (12) can be obtained. Since $r_1(t)R(t) > 0$, the upper bound is constructed as shown in Equation (13).

$$r_1(t)R(t)\left(1 - \frac{R(t)}{K_1(t)} + \frac{\alpha(t)U(t)}{K_2(t)}\right) = 0 \quad (12)$$

$$R_{\max}(t) = K_1(t)\left(1 + \alpha(t)\frac{U(t)}{K_2(t)}\right) \quad (13)$$

Equation (13) indicates that the upper bound of the URT network scale is jointly determined by the UD level and the dynamically supportable URT network scale. When $\alpha(t) > 0$, an increase in the UD level enhances the demand for URT expansion and improves the resource-supporting capacity of the city, thereby increasing the upper bound of the URT network scale. When $\alpha(t) < 0$, resource, space, or fiscal constraints during UD exert a restrictive effect on URT expansion, thereby reducing the upper bound of the URT network scale.

4. Model Results and Analysis

To validate the effectiveness of the proposed model, eight Chinese cities with different UD levels are selected as case studies. Based on the calculated URT network scale range, a three-part validation strategy is adopted. First, a consistency analysis is conducted to examine whether the actual network scale falls within the model-calculated range. Second, passenger transport intensity is further introduced to validate the rationality of the range from the perspective of supply–demand matching. Third, the calculated results are compared with officially planned network scales.

4.1. Selection of Case Cities

To validate the applicability and effectiveness of the proposed model under different UD levels, eight Chinese cities were selected as case studies, namely Beijing, Shanghai, Guangzhou, Chongqing, Hangzhou, Tianjin, Dalian, and Shijiazhuang. According to the Notice of the State Council on Adjusting the Standards for City-Size Classification, Chinese cities are classified based on the permanent resident population in urban districts. Cities with an urban district population of more than 10 million are classified as megacities; cities with an urban district population of 5–10 million are classified as super-large cities; and cities with an urban district population of 1–5 million are classified as large cities. Based on this classification, the selected cities cover megacities, super-large cities, and large cities, as shown in Table 2. Beijing, Shanghai, and Guangzhou are selected as representatives of megacities; Chongqing, Hangzhou, Tianjin, and Dalian are selected as representatives of super-large cities; and Shijiazhuang is selected as a representative of large cities.

Table 2. Classification of selected case cities.

City-Size Class	Population (Million Persons)	Cities
Megacity	≥ 10	Beijing, Shanghai, Guangzhou
Super-large city	5–10	Chongqing, Hangzhou, Tianjin, Dalian
Large city	1–5	Shijiazhuang

4.2. Input Data

The data and parameter settings required for the model are shown in Table 3. The actual network scale, $R_{\text{obs}}(t)$, is collected from the China Association of Metros and URT operation reports of each city. The study focuses on the issue that the URT network scale is compatible with the UD level from a macro perspective. Therefore, the observed UD index, $U_{\text{obs}}(t)$, is constructed using five key macro indicators, GDP, urban population, built-up area, urban road length, and PM2.5 concentration, with reference to previous studies [43–45]. These indicators are standardized and aggregated using the entropy-weight method [46]. The data are collected from publicly available sources, including China City Statistical Yearbook, China Urban Construction Statistical Yearbook, and the China Environmental Status Bulletin. The dataset covers the period from 1990 to 2024. Given the large volume of annual observations for the eight cities, the complete year-by-year dataset is not presented. The time-varying parameters $r_1(t)$, $r_2(t)$, $\alpha(t)$ and $\beta(t)$ are estimated using the sliding-window least squares method. In addition, other parameters are determined based on model assumptions, Logistic fitting results, and city-specific parameter settings.

Table 3. Model variables, meanings, and sources/settings.

Variable	Meaning	Source/Setting
$R_{\text{obs}}(t)$	the actual URT network scale	Actual operating network length, obtained from URT operation reports
$U_{\text{obs}}(t)$	UD index	Calculated from GDP, population, built-up area, urban road length, and PM2.5 concentration using standardization and the entropy weight method
$r_1(t), r_2(t)$	Intrinsic growth rates of URT and UD	Estimated by the sliding-window least squares method
$\alpha(t), \beta(t)$	Interaction coefficients between URT and UD	Estimated by the sliding-window least squares method
τ	Time-lag parameter	Set to 3 years to reflect the delayed effect of URT on UD [47]
$K_{1\text{min}}$	Initial supportable URT network scale	Determined within the range of 1–10 km according to the UD level of each city in the first year of the study period
$K_{1\text{max}}$	Long-term supportable URT network scale	Obtained by fitting the actual network scale of each city using the Logistic model
$U(t_c)$	UD index at the critical point	Determined according to the inflection point of the Logistic fitting curve of the UD index
λ_1	Adjustment coefficient of $K_1(t)$	0.8

4.3. Model Parameter Calculation

In this section, the parameters $r_1(t)$, $r_2(t)$, $\alpha(t)$ and $\beta(t)$ in Equation (2) are estimated. First, the initial values of the parameters are randomly selected from the preset parameter ranges. These initial values, together with the observed URT network scale and the observed UD index, are then entered into the parameter estimation model. The model is solved to obtain the optimal parameters. With reference to previous studies on the specification of intrinsic growth rates and interaction coefficients in LV models [48,49] and considering the practical context of this study, the ranges of $r_1(t)$ and $r_2(t)$ are set to

(0, 0.5], whereas $\alpha(t)$ and $\beta(t)$ are set to $[-1.5, 1.5]$. In China, national economic and social development plans, comprehensive transportation plans, and major infrastructure construction plans are generally formulated on a five-year cycle. Moreover, previous studies have employed a five-year sliding window to identify local dynamic relationships and estimate time-varying parameters, indicating that this window length can balance local characteristics and sample-size requirements [50]. Therefore, the sliding-window length is set to $L = 5$. Regarding the L2 regularization coefficient, previous studies [51–53] have shown that weak L2 regularization can improve the numerical stability of parameter estimation and suppress excessive parameter fluctuations while avoiding excessive parameter shrinkage. Accordingly, the L2 regularization coefficient is set to $\lambda = 0.0005$.

Since these parameters $r_1(t)$, $r_2(t)$, $\alpha(t)$ and $\beta(t)$ cannot be validated through direct comparison with observed data, the rationality is assessed indirectly based on the goodness-of-fit (R^2) and the root mean square error (RMSE) of the simulated URT network scale and the simulated UD level. The corresponding results are presented in Table 4. As shown in Table 4, the model demonstrates good fitting performance across all cities, with relatively high R^2 values and RMSE maintained within a reasonable range, indicating that the sliding-window least squares parameter estimation method is reliable and robust. In addition, the estimated interaction coefficients satisfy the condition $\alpha(t)\beta(t) < 1$ in all sliding windows, which suggests that the cooperation evolution process between URT and UD satisfies the local stability condition of the LV system. The interaction intensity between URT and UD remains within the local stability range of the LV system. This further supports the rationality of the estimated parameters. Meanwhile, existing studies have indicated that the ability to capture system development trends should also be considered as the capability of capturing the evolutionary trends of system development for complex system models with nonlinear characteristics, rather than merely depending on goodness-of-fit indicators [54].

Table 4. Model goodness-of-fit and error metrics.

City	R^2 for URT	R^2 for UD	RMSE for URT	RMSE for UD
Beijing	0.7851	0.9999	11.8872	0.0002
Shanghai	0.8600	0.9998	19.8399	0.0003
Guangzhou	0.6036	0.9998	20.0517	0.0003
Chongqing	0.6878	0.9999	16.2254	0.0004
Hangzhou	0.8370	0.9998	20.3261	0.0003
Tianjin	0.7524	0.9997	10.9174	0.0003
Dalian	0.6086	0.9996	7.7262	0.0002
Shijiazhuang	0.5524	0.9999	22.3815	0.0001

Figures 1 and 2 present the estimated parameter results for eight case cities over the period from 2025 to 2050. Overall, the parameter trajectories are continuous and show clear evolutionary trends rather than random fluctuations. For Beijing, Shanghai, Guangzhou, and Chongqing, the parameters change smoothly and gradually tend to stabilize, indicating a relatively stable cooperation relationship between URT network scale and UD in cities with higher development levels. For Hangzhou, Tianjin, Dalian, and Shijiazhuang, the parameters show more evident adjustments, reflecting the dynamic evolution during rapid development or stage transition.

These results are consistent with the development stages and city-size characteristics of the selected cities, suggesting that the estimated parameters are reasonable and can be used for the subsequent calculation of reasonable URT network scale ranges.

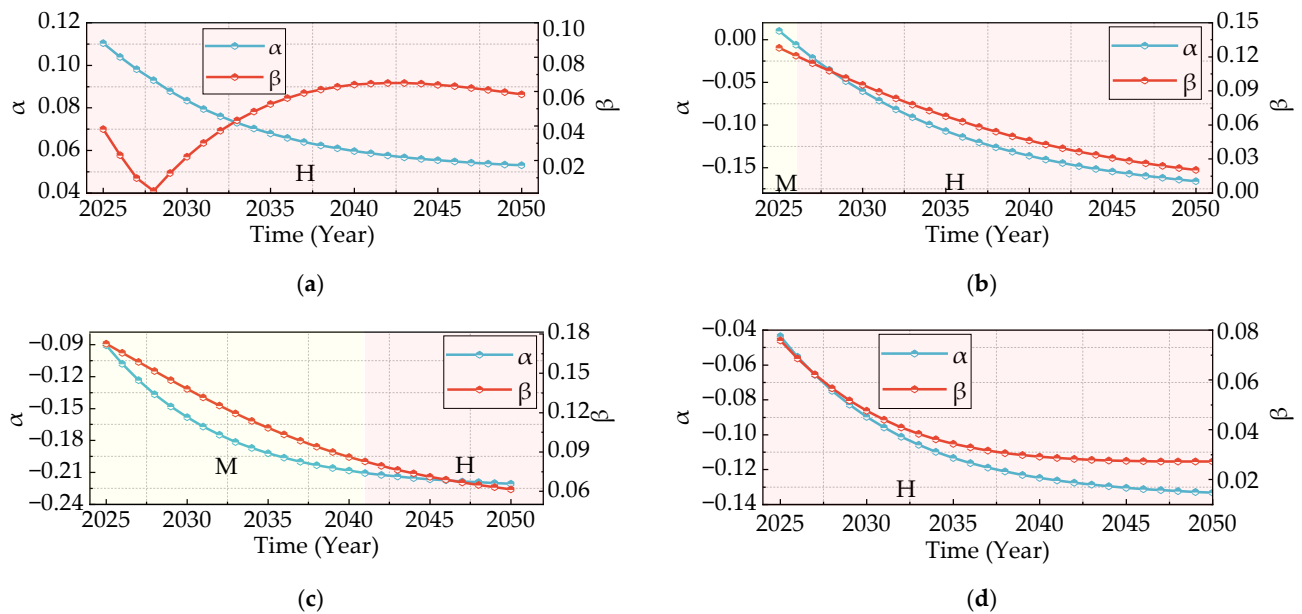


Figure 1. Stage-dependent evolution of LV model parameters (α and β) across cities during 2025–2050. (a) Beijing; (b) Shanghai; (c) Guangzhou; (d) Chongqing. M, and H denote the medium-development or rapid-growth stage, and the high-development or mature stage.

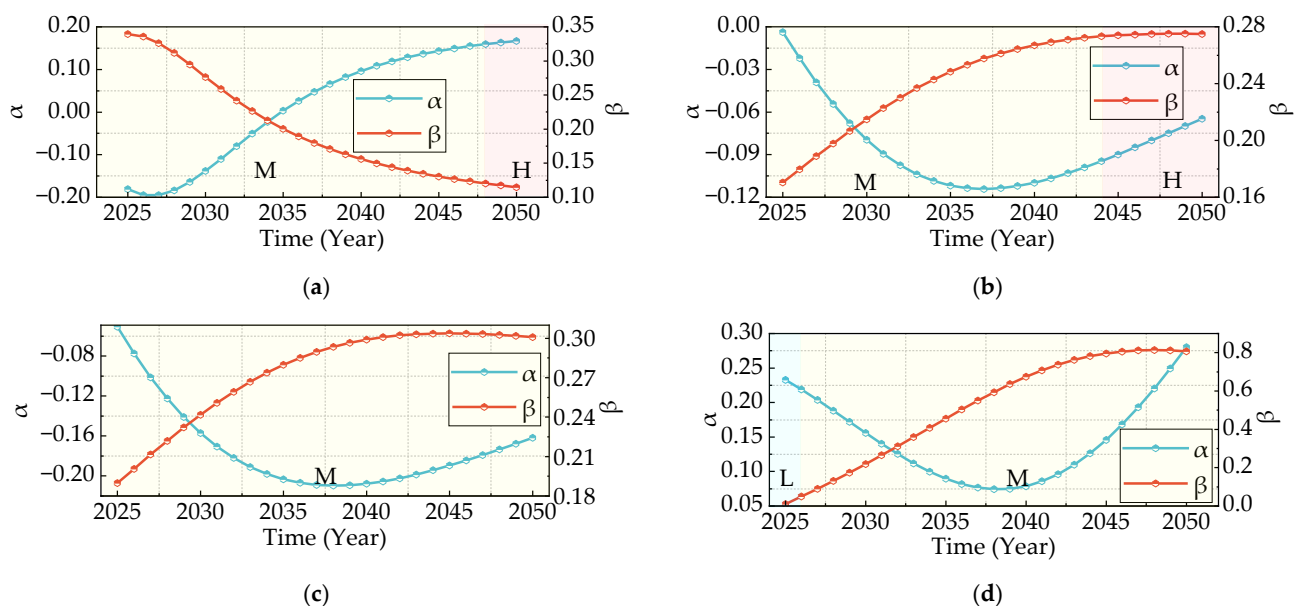


Figure 2. Stage-dependent evolution of LV model parameters (α and β) across cities during 2025–2050. (a) Hangzhou; (b) Tianjin; (c) Dalian; (d) Shijiazhuang. L, M, and H denote the low-development stage, the medium-development or rapid-growth stage, and the high-development or mature stage.

4.4. Results for Reasonable URT Network Scale Ranges

To calculate the reasonable URT network scale, this section first examines the evolution of the dynamic carrying capacity $K_1(t)$ and then presents the resulting reasonable network scale ranges for the eight cities.

Figure 3 presents the dynamic carrying capacity $K_1(t)$ for the eight cities. As shown in the figure, $K_1(t)$ generally increases with the improvement in the UD level and exhibits a staged pattern characterized by slow initial growth, accelerated increase, and gradual stabilization. Among the eight cities, Shanghai has the highest overall dynamic carrying

capacity, whereas Shijiazhuang has the lowest, reflecting differences in UD levels and the corresponding capacity to support URT.

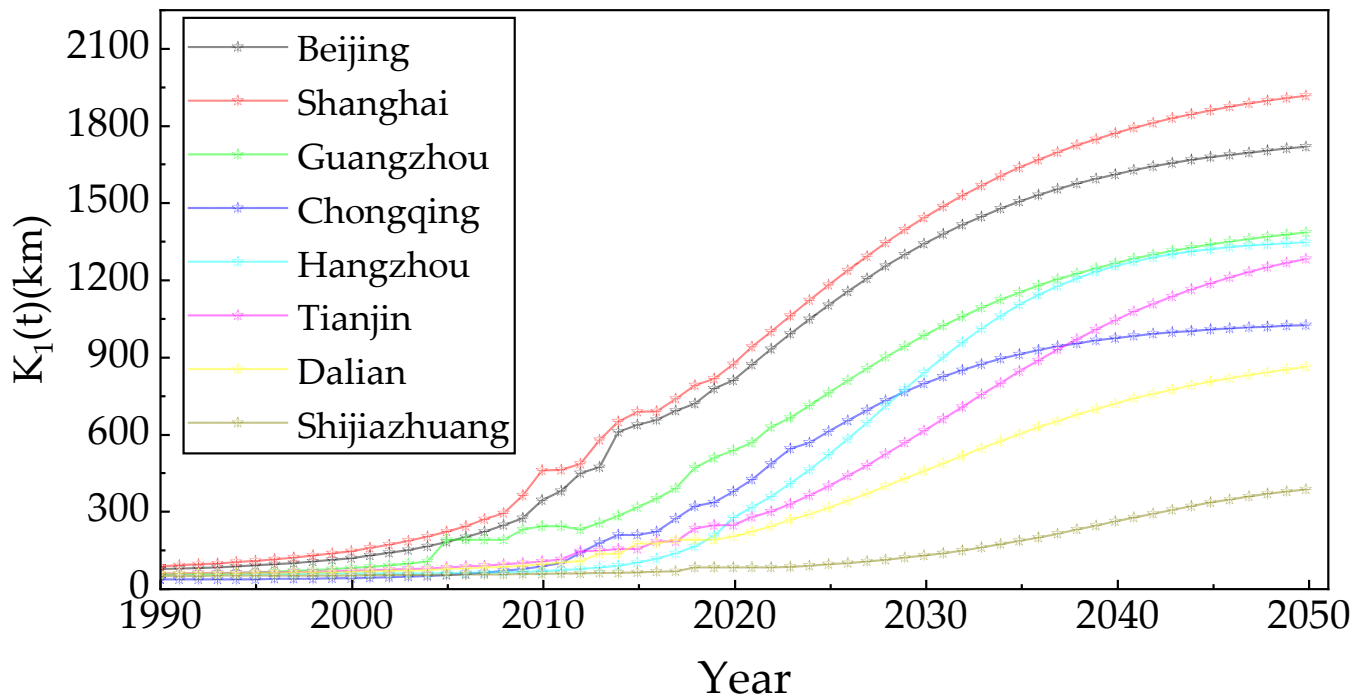


Figure 3. Evolution of the dynamic carrying capacity $K_1(t)$ of the URT network scale in eight cities.

The proposed model is applied to calculate the reasonable URT network scale range for eight cities over the period from 1990 to 2050. Table 5 presents the calculated results for the selected cities in three years: 2030, 2040, and 2050, demonstrating that the model is capable of calculating the network scale ranges for cities.

As shown in Table 5, megacities in the H stage, such as Beijing and Shanghai, maintain relatively large URT network scale ranges throughout the study period, but their growth rates are relatively moderate. For example, the network scale range of Beijing increases from 757.25–1471.18 km in 2030 to 875.66–1768.39 km in 2050, with growth rates of approximately 15.6% and 20.2% for the lower and upper bounds, respectively. The growth rates of Shanghai and Chongqing are both around 20%, indicating relatively stable network scale expansion. For cities transitioning from the M stage to the H stage, including the megacity Guangzhou and the super-large cities Hangzhou and Tianjin, the calculated network scale ranges expand more significantly. Specifically, the calculated URT network scale range of Guangzhou increases from 645.39–821.87 km to 839.67–1069.28 km, with a growth rate of approximately 30.1%. Hangzhou and Tianjin exhibit higher growth rates of about 58.2% and 114.0%, respectively, reflecting rapid expansion. For cities transitioning from the L stage to the M stage (e.g., Dalian and Shijiazhuang), the network scale remains relatively small but shows substantial growth. The growth rates of Dalian and Shijiazhuang are approximately 88.1% and 279.0%, respectively, indicating a transition from low to higher network scale levels.

The results indicate that the proposed model can reasonably calculate differentiated URT network scale ranges for cities with different UD levels, development stages, and city-size classes.

Table 5. Calculated reasonable URT network scale ranges for selected cities in 2030, 2040, and 2050.

City	Year	Lower Bound (km)	Upper Bound (km)
Beijing	2030	757.25	1471.18
	2040	844.60	1666.65
	2050	875.66	1768.39
Shanghai	2030	957.98	1351.28
	2040	1080.01	1523.41
	2050	1127.64	1590.59
Guangzhou	2030	645.39	821.87
	2040	780.26	993.62
	2050	839.67	1069.28
Chongqing	2030	143.71	720.62
	2040	169.22	848.51
	2050	176.19	883.47
Hangzhou	2030	713.12	1001.05
	2040	1047.19	1470.01
	2050	1128.23	1583.77
Tianjin	2030	333.79	553.15
	2040	548.54	909.04
	2050	714.19	1183.55
Dalian	2030	213.33	371.51
	2040	313.68	546.27
	2050	401.27	698.81
Shijiazhuang	2030	57.88	141.30
	2040	114.36	279.18
	2050	219.39	535.58

4.5. Results Analysis

To validate the effectiveness and applicability of the proposed model, the reasonable URT network scale range calculated by the model is compared with the actual network scale, passenger transport intensity, and the officially planned URT network scale of each city.

(1) Comparison with the Actual Network Scale

In the previous section (Section 4.1), the eight case cities were classified into megacities, super-large cities, and large cities according to urban population size. Meanwhile, Figure 4 further illustrates the matching relationship between the actual network scale and the model-calculated range for each city across the L, M, and H stages of UD. Therefore, this section examines whether the actual network scales of different types of cities fall within the model-calculated reasonable range from the perspectives of city-size classification and UD stage evolution.

Figure 4 illustrates the matching relationship between the actual network scale and the model-calculated reasonable range for the eight selected cities. In the figure, the green and red curves represent the lower and upper bounds of the reasonable URT network scale, respectively. An actual network scale below the lower bound indicates insufficient network provision, which may fail to meet the requirements of sustainable UD. In contrast, an actual network scale exceeding the upper bound suggests that URT network scale may surpass the capacity of the current UD level. The blue dots denote the temporal evolution of the actual network scale for each city.

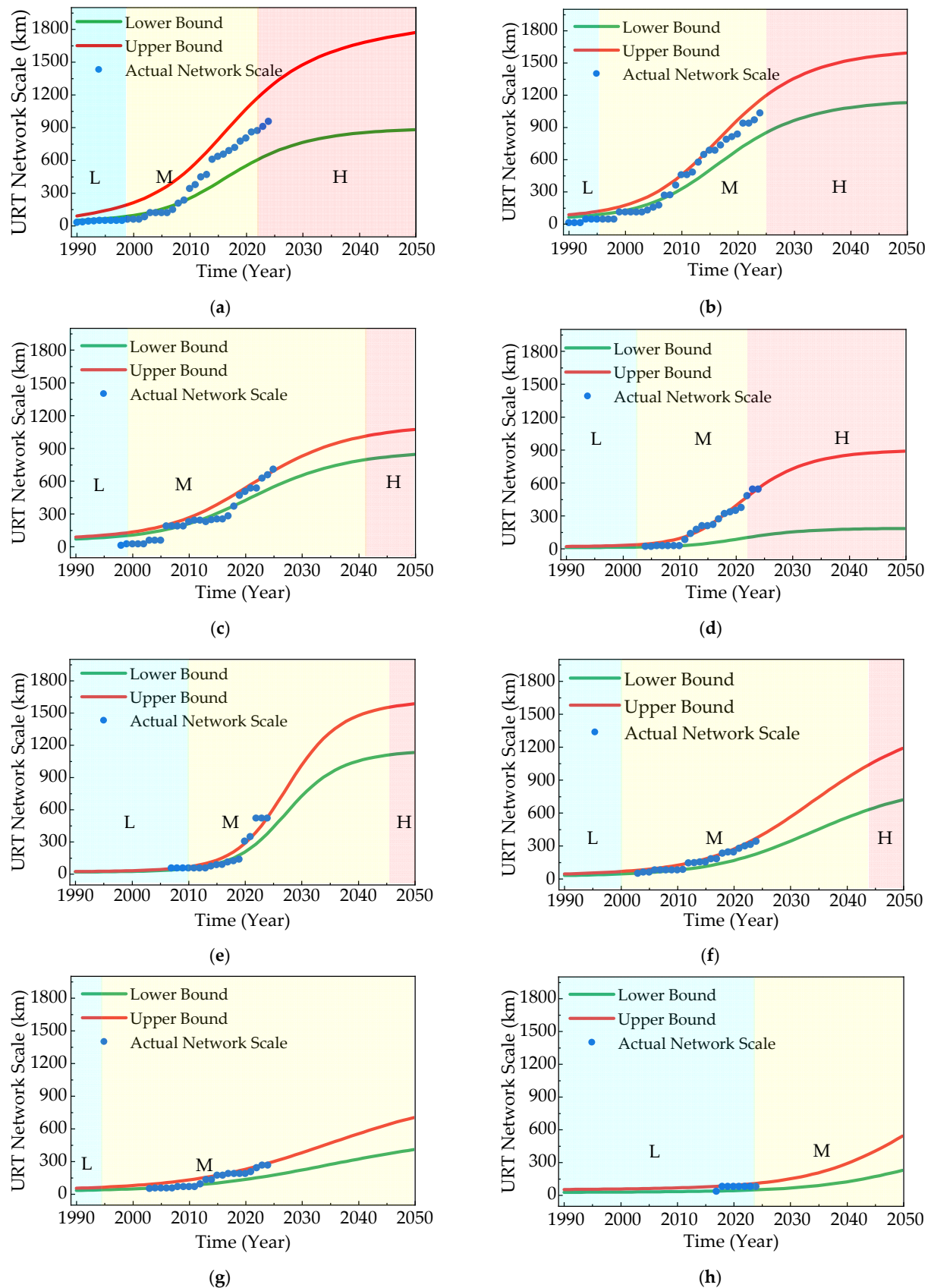


Figure 4. Comparison between actual network scale and calculated scale range in selected cities. (a) Beijing; (b) Shanghai; (c) Guangzhou; (d) Chongqing; (e) Hangzhou; (f) Tianjin; (g) Dalian; (h) Shijiazhuang. L, M, and H denote the low-development stage, the medium-development or rapid-growth stage, and the high-development or mature stage.

For the megacities, Beijing, Shanghai and Guangzhou (Figure 4a–c) had relatively small actual network scales during the L stage. In the early period, their actual network scales were mainly located near or below the lower bound of the model-calculated scale range, indicating that URT construction lagged behind UD to some extent in the early stage of rapid urban growth. After entering the M stage, the actual network scales of the cities increased rapidly and gradually entered the reasonable range calculated by the model. Particularly after 2010, the actual network scales of Beijing and Shanghai were generally located between the lower and upper bounds, suggesting that their URT gradually shifted from insufficient provision in the early period to a state more consistent with the level of UD. This suggests that, for megacities, the proposed model can effectively reflect the transition from insufficient URT provision to relatively coordinated development between URT network scale and UD level.

For the super-large cities, including Chongqing, Hangzhou, Tianjin, and Dalian (Figure 4d–g), the matching relationship between the actual network scale and the model-calculated range shows more differentiated characteristics. Chongqing and Hangzhou exhibit a relatively clear expansion trend. In the early stage, their actual network scales were relatively small and were mainly located near or below the lower bound of the calculated range. With the transition from the L stage to the M stage, the actual network scales increased rapidly and gradually moved into the model-calculated reasonable range. Compared with Chongqing and Hangzhou, Tianjin and Dalian show a more moderate growth pattern. The actual network scale of Tianjin is generally located within the calculated range, indicating that the URT development has maintained a relatively stable matching relationship with UD. For Dalian, the actual network scale also remains largely within the reasonable range, although its growth rate is lower than that of Chongqing and Hangzhou. This suggests that the proposed model can not only identify rapid expansion in super-large cities, but also reflect the relatively steady development process of cities with slower network growth.

For the large city, Shijiazhuang (Figure 4h), the actual network scale is relatively small, but it is generally located within the model-calculated reasonable range. This indicates that the actual network scale of Shijiazhuang is basically consistent with its current UD level.

Therefore, the consistency between the actual network scale and the model-calculated range in case cities supports the effectiveness and applicability of the proposed model.

(2) Validation Using Passenger Transport Intensity

The consistency between passenger transport intensity and the actual network scale is further examined to validate the effectiveness of the proposed model. Passenger transport intensity reflects the degree of supply–demand matching between URT network scale and UD level. When the URT network scale is insufficient, passenger transport intensity per unit length tends to be relatively high; conversely, when the network scale exceeds reasonable demand, passenger transport intensity tends to decrease.

Figure 5 illustrates the relationship between actual network scale and passenger transport intensity for the selected cities. In general, most cities exhibit relatively high passenger transport intensity during the initial stage of URT development, which is consistent with the result shown in Figure 4. In Figure 4, the actual network scale is close to or below the lower bound of the model-calculated range. As the URT network continues to expand and gradually enters the reasonable range calculated by the model, passenger transport intensity decreases from a high level and tends to stabilize. This trend indicates that the calculated range can reflect the changing supply–demand relationship between URT network scale and UD level.

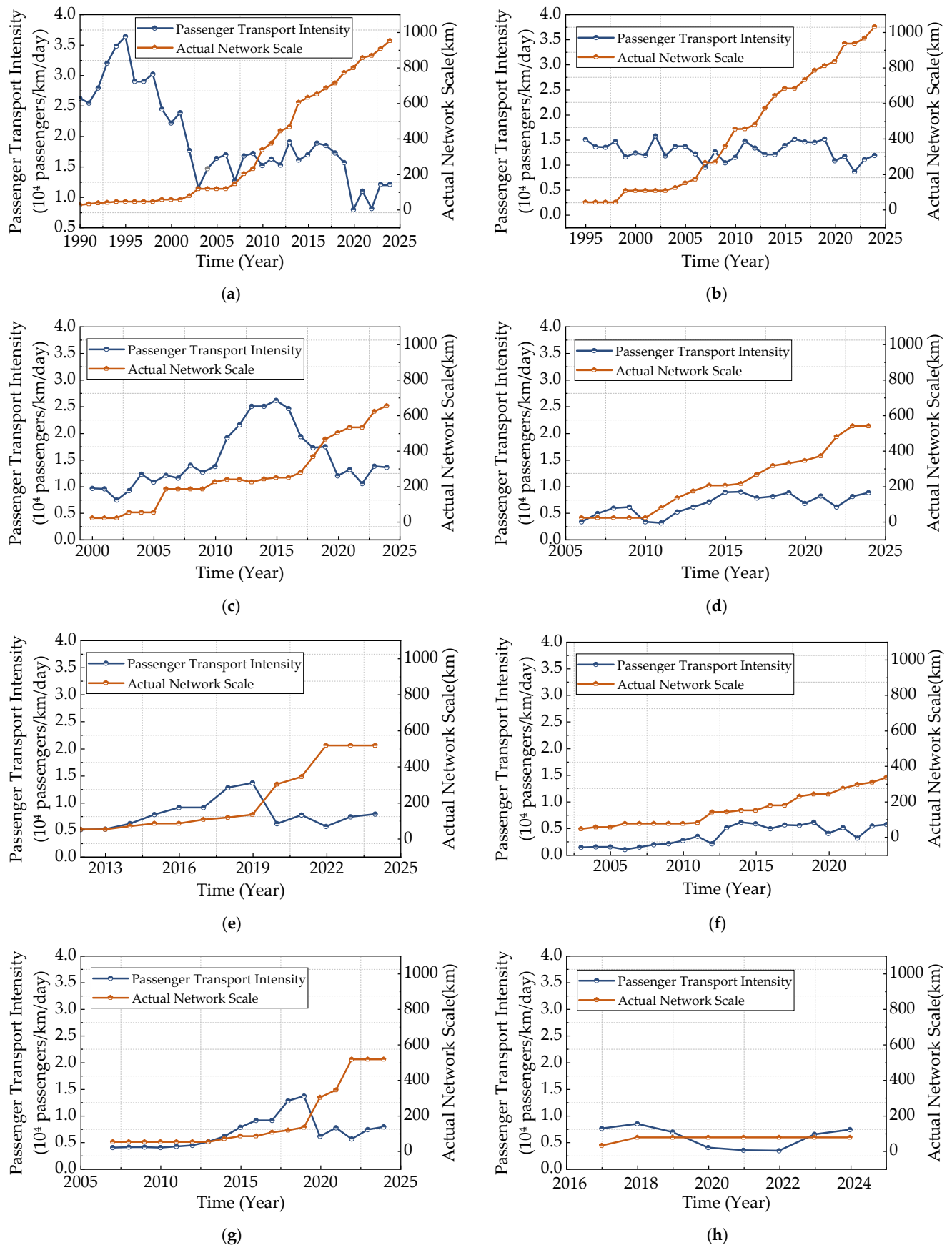


Figure 5. Relationship between passenger transport intensity and the actual network scale in selected cities. (a) Beijing; (b) Shanghai; (c) Guangzhou; (d) Chongqing; (e) Hangzhou; (f) Tianjin; (g) Dalian; (h) Shijiazhuang.

For the megacities, including Beijing, Shanghai, and Guangzhou (Figure 5a–c), passenger transport intensity is relatively high when the URT network scale is small in the early stage. This is consistent with the insufficient network scale identified in Figure 4. With the rapid expansion of the network, the actual network scale gradually enters the model-calculated reasonable range, and passenger transport intensity decreases significantly and becomes more stable.

For the super-large cities, including Chongqing, Hangzhou, Tianjin, and Dalian (Figure 5d–g), the relationship between passenger transport intensity and network scale shows differentiated characteristics. Chongqing and Hangzhou experience rapid network expansion, and their passenger transport intensity decreases as the actual network scale approaches or enters the reasonable range. In contrast, Tianjin and Dalian show relatively low passenger transport intensity, which may indicate that their network scale is relatively sufficient compared with current demand. This is consistent with the results in Figure 4, where the actual network scale of these cities is located within the calculated range or close to the upper bound.

For Shijiazhuang (Figure 5h), the actual network scale is relatively small, but it remains generally within the model-calculated reasonable range. The passenger transport intensity also shows a matching relationship with its relatively moderate network scale, indicating that its URT development is generally consistent with its current UD level. This further suggests that the proposed model can identify reasonable scale ranges not only for megacities and super-large cities, but also for large cities with smaller network scales.

In summary, the consistency between passenger transport intensity and the actual network scale further validates the effectiveness of the proposed model. The model can reflect the differentiated supply–demand characteristics of cities with different city-size classes and UD stages, and provide a reliable basis for determining a reasonable URT network scale in planning.

(3) Comparison with the Officially planned URT Network Scale

To further validate the practical applicability of the model, the model-calculated results are compared with the officially planned URT network scale of the selected cities. Table 6 presents a comparison between the model-calculated results and the officially planned URT network scale. As shown in the table, the officially planned URT network scales of Hangzhou, Dalian, and Shijiazhuang all fall within the model-calculated ranges. Beijing's officially planned URT network scale for 2035 is 1625 km, which is only 17.47 km higher than the model-calculated upper bound of 1607.53 km, corresponding to a relative deviation of approximately 1.09%. Guangzhou's officially planned URT network scale for 2035 is 1004 km, which is 80.54 km higher than the model-calculated upper bound of 923.46 km, with a relative deviation of approximately 8.72%. Shanghai's planning document proposes a rail transit network with a total length of more than 3000 km, comprising more than 1000 km of intercity lines, more than 1000 km of urban lines, and more than 1000 km of local lines. Therefore, the planned urban-line network of more than 1000 km is consistent with the scope and statistical definition of the URT network scale adopted in this study. Accordingly, this value is used for comparison, and it is broadly consistent with the model-calculated range.

Because the officially planned URT network scale data in 2035 for Tianjin are unavailable, the 2025 planning target proposed during Tianjin's 14th Five-Year Plan period is used for comparison. According to the relevant plan, Tianjin's URT network is planned to reach 513 km by 2025, whereas its actual operating network length is 379.3 km. The reasonable network scale range calculated by the proposed model for 2025 is 235.85–390.84 km. The actual operating network scale falls within the model-calculated range, whereas the planning target exceeds the model-calculated upper bound.

Table 6. Comparison of the model-calculated network scale ranges with the officially planned URT network scale.

City	Model-Calculated Network Scale Range (km)	Officially Planned Network Scale (km)	Target Year	Comparison Result
Beijing	[803.31, 1607.53]	1625	2035	Slightly above
Shanghai	[1032.94, 1457.02]	>1000	2035	Within range
Guangzhou	[725.16, 923.46]	1004	2035	Slightly above
Chongqing	[160.31, 803.83]	1312	2035	Above
Hangzhou	[931.83, 1308.07]	1120.9	2035	Within range
Tianjin	[235.85, 390.84]	513	2025	Above
Dalian	[263.60, 459.06]	377.4	2035	Within range
Shijiazhuang	[80.52, 196.56]	137.05	2035	Within range

Overall, the officially planned URT network scales of most cities fall within or close to the model-calculated ranges, supporting the practical applicability of the proposed model.

In the practice of urban rail transit network planning, the network scale should be reasonably determined based on the UD level. For cities where the actual or officially planned URT network scale is close to the upper bound of the model calculation, the additional scale should be determined with caution to avoid the network scale exceeding the development level of the city; for cities where the network scale is close to or below the lower bound of the model calculation, the officially planned URT network scale should be appropriately increased in accordance with the development needs of the city; for cities where the officially planned URT network scale has a large deviation from the model calculation results, the planning goals should be further reviewed.

5. Conclusions

This study develops a time-varying calculation model for identifying a reasonable URT network scale in planning based on the dynamic coopeitition relationship between URT and UD. In this model, an improved time-varying parameter LV model is employed to describe the coopeitition relationship between URT and UD. The sliding-window least squares method is adopted to estimate time-varying parameters of the model. Based on the improved LV model, the lower bound of the URT network scale is obtained by solving for the minimum network scale required to promote sustainable UD under a cooperative relationship; the upper bound of the URT network scale is obtained by solving for the maximum network scale that urban resources can support under competitive conditions. The proposed model is validated using eight cities in China. The findings provide a theoretical basis for URT network planning and decision-making. The main conclusions are summarized as follows:

- (1) The proposed calculation model can determine the reasonable range of URT network scale. The results show that the scale range varies significantly across cities and is consistent with the UD level.
- (2) A time-varying parameter estimation approach is developed by integrating the sliding-window least squares method. The results indicate that model parameters evolve from fluctuations in the early stages to gradual stabilization in later stages. The proposed method can effectively estimate the time-varying parameters of the LV model.
- (3) The proposed model is validated by comparing the model-calculated network scale ranges with the actual URT network scales, passenger transport intensity, and officially planned network scales of the eight case cities. Most actual network scales fall within or close to the model-calculated ranges. Changes in passenger transport intensity are generally consistent with the supply–demand conditions reflected by the calculated

ranges. In addition, the officially planned network scales of most cities fall within or close to the calculated ranges.

Nevertheless, several limitations remain in the model. This study mainly considers the competition relationship between URT network scale and UD level, while external disturbances and policy interventions are not explicitly incorporated. In reality, factors such as economic fluctuations, demographic changes, and land-use policies may significantly influence the evolution of URT. Future studies could extend the proposed model by introducing external control variables or stochastic disturbance mechanisms to improve the adaptability of the model under uncertain conditions.

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