

Article

Optimizing Material Usage for Sustainable Shield Tunneling: A Mechanistic Study of Bentonite Slurry Infiltration in Saturated Sands

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Abstract

In slurry shield tunneling, inefficient control of slurry permeability in sandy formations can cause excessive slurry loss, increased material consumption, reduced bentonite reuse, and compromised tunnel-face stability. To address these challenges and enhance material efficiency, this study investigates slurry infiltration and bentonite particle deposition mechanisms in saturated sandy soils. Based on deposited-particle mass conservation and slurry volume conservation coupled with excess pore-water pressure, a mathematical model is established to capture the evolution of slurry rheological properties and soil pore characteristics during infiltration. Through multilayer infiltration column experiments, a multiple regression formula for the filtration coefficient is established, considering the spatiotemporal variability of slurry and soil properties. Furthermore, a dynamic penetration criterion for slurry particles is proposed and verified through single-soil infiltration tests. Results demonstrate that most bentonite particles deposit on the soil surface, with only a minimal fraction migrating into deeper pores until reaching shear stress equilibrium. The maximum infiltration distance is positively correlated with soil particle size but negatively correlated with slurry mass concentration. Increasing the slurry mass concentration or shear strength promotes the development of a well-structured filter cake and infiltration zone. These findings provide a theoretical framework for precisely regulating slurry permeability, thereby minimizing material waste and supporting sustainable shield tunneling operations.

Keywords: sustainable construction materials; slurry shield tunneling; bentonite slurry; infiltration and deposition; material efficiency; mathematical modeling



Academic Editor: Laura Moretti

Received: 15 May 2026

Revised: 20 June 2026

Accepted: 1 July 2026

Published: 8 July 2026

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1. Introduction

Slurry wall protection technology, originally developed in the oil-exploration industry, has become a cornerstone of modern underground construction, particularly in ensuring the stability of excavation trenches and tunnel faces in saturated sandy strata [1]. In slurry shield tunneling, bentonite slurry—typically a suspension comprising bentonite

particles and chemical additives like sodium carboxymethylcellulose (CMC)—serves a dual functional role [2,3]. First, it acts as a critical supporting medium, maintaining tunnel-face stability through controlled infiltration and the formation of an impermeable filter cake. Second, it functions as a transport vehicle, entraining and conveying excavated soil cuttings from the tunnel face to the surface treatment plant [4]. In a sustainable construction paradigm, bentonite slurry is treated not as a single-use consumable but as a recyclable resource. Following the transportation of soil cuttings, the slurry undergoes a separation and filter-pressing process to remove solids, allowing the purified liquid to be reinjected into the tunneling circuit. Consequently, the slurry reuse rate emerges as a decisive factor in determining both the project's techno-economic viability and its environmental footprint. However, in highly porous strata characterized by coarse sand or gravel, swiftly developing an effective filter cake is hindered. Inadequate control of slurry infiltration in these environments leads to excessive slurry loss into the surrounding ground, which is doubly detrimental: it compromises face stability and results in an irreversible loss of recyclable material. From a sustainability perspective, this irrecoverable loss necessitates a higher volume of raw bentonite and increases the environmental burden associated with material supply and waste disposal.

Weiss first summarized the stabilization condition of slurry support in wall protection through a laboratory test: (1) a specific yield strength is required for the slurry to balance the forces on individual soil particles; (2) the slurry pressure must exceed the combined earth and groundwater pressures [5]. Min et al. further investigated the infiltration behaviors of slurry in the laboratory. The experimental results indicate that the retention gradient and the soil particle size distribution significantly influence slurry infiltration [6]. In fine-grained formations like silt, the suspended solids within the slurry typically cannot penetrate deeply and predominantly retain at the soil interface, forming a thin and dense particle layer that presents a highly distinct interface with the underlying soil. In coarser soils such as fine sand, slurry infiltration exhibits a pronounced two-stage characteristic. During the initial stage, the viscous slurry rapidly penetrates the soil pores; subsequently, in the second stage, the infiltration velocity gradually decreases as suspended particles continuously deposit within the pores. Once the shear stress exerted on the suspended particles reaches equilibrium, the slurry undergoes dewatering and consolidation on the surface [7]. This micro-evolutionary process is consistent with the findings of Müller-Kirchenbauer [8] and Thienert et al. [9]. In this case, the infiltration zone exists in soil, and a surface microporous filter cake develops without a well-defined boundary. The stability of the soil interface is primarily maintained by the slurry pressure acting through seepage forces [10]. Several theoretical models have also been developed to describe the mechanism of slurry infiltration. Zhu et al. assumed the slurry to be Bingham fluid and described the slurry infiltration in non-Darcy flow [11]. The dynamic yield point of slurry is considered as a key parameter affecting the transport behavior of slurry within the soil matrix. Nevertheless, from a microstructural perspective, slurry is inherently a solid–liquid two-phase system [12]. Previous single-phase assumptions (e.g., Bingham fluid models) are oversimplifications that often neglect this nature and the critical role of particle deposition, leading to significant discrepancies when predicting slurry consumption and infiltration depth in real-world scenarios.

As slurry permeates through sandy soils, the soil skeleton captures suspended particles, leading to a simultaneous reduction in both the seepage pore size and the slurry mass concentration, which is called filtration effect. Filtration effect is the primary cause of tunnel-face stability in slurry shield tunneling [13]. The migration and deposition of slurry particles within soil pores represent a highly complex process. The filtration effect is affected by multiple factors, including physical interception, gravitational and inertial

forces, hydrodynamic interactions, Brownian motion, and electrostatic repulsion [14–18]. In the macroscopic analysis of slurry infiltration mechanism, filtration coefficient is the critical physical quantity. Grounded in classical filtration theory, previous studies have demonstrated that the adsorption capacity of soil skeleton onto slurry particles is intrinsically linked to the interception, sedimentation and diffusion effect by analogy [19–22]. However, these analytical models require numerous indispensable parameters which are challenging to correspond to engineering parameters in reality. Hence, the determination of the filtration coefficient is still based on empirical approaches up to date [23]. Additionally, these conventional approaches typically fail to account for the dynamic evolution of both slurry rheology and soil pore characteristics during penetration.

The transformation of the infiltration zone to filter cake formation depends on multiple factors, like soil grain composition, the size and rheological properties of solid particles in slurry suspension, and the relative density and uniformity of soil skeleton [24]. In the field of geotechnical grouting, some scholars have proposed soil injectability criteria to quantify the injectability of slurry [25–27]. However, the injectability practically varies with the deposition of suspended particles. The previous theoretical studies neglected the time variation and spatial variation of soil injectability. In addition, most existing studies only focus on the hydromechanical behaviors during slurry infiltration and excavation face stability, while ignoring the resource consumption and circular utilization of bentonite materials from the perspective of sustainable geotechnical construction. This work takes the sustainable construction demand of shield tunneling as the core starting point. The proposed theoretical framework is further applied to quantify the influence of slurry–soil matching parameters on bentonite loss and slurry recyclability.

This study investigates the infiltration and deposition mechanisms of bentonite slurry in saturated sandy soil. The primary objectives as follows: (1) to establish a mathematical model based on mass- and volume-conservation principles to capture the dynamic interplay between particle deposition and slurry rheology; (2) to design multilayer infiltration column experiments to quantify the effects of slurry concentration and soil characteristics (including porosity, grain size, and uniformity) on the filtration coefficient; and (3) to propose a dynamic soil injectability criterion. Finally, a parametric sensitivity analysis is performed to evaluate how suspension viscosity and porosity changes affect maximum infiltration distance and time, providing a theoretical framework for minimizing material waste and maximizing resource efficiency in underground construction.

2. Theoretical Modeling

At the onset of infiltration, most bentonite particles accumulate at the soil–slurry interface under the driving pressure, precipitating a sharp attenuation in local soil porosity. This initial clogging phase is the primary mechanism for establishing a low-permeability barrier, which is essential for minimizing the uncontrolled loss of bentonite resources into the formation. Simultaneously, the displacement of in situ pore-water by the infiltrating slurry results in the development of excessive pore water pressure at the tunnel front [28]. This multi-phase interaction between the suspended solids and the soil skeleton governs the spatial evolution of the infiltration zone and the subsequent stabilization of the excavation face [29]. The schematic representation of slurry infiltration in saturated ground is shown in Figure 1.

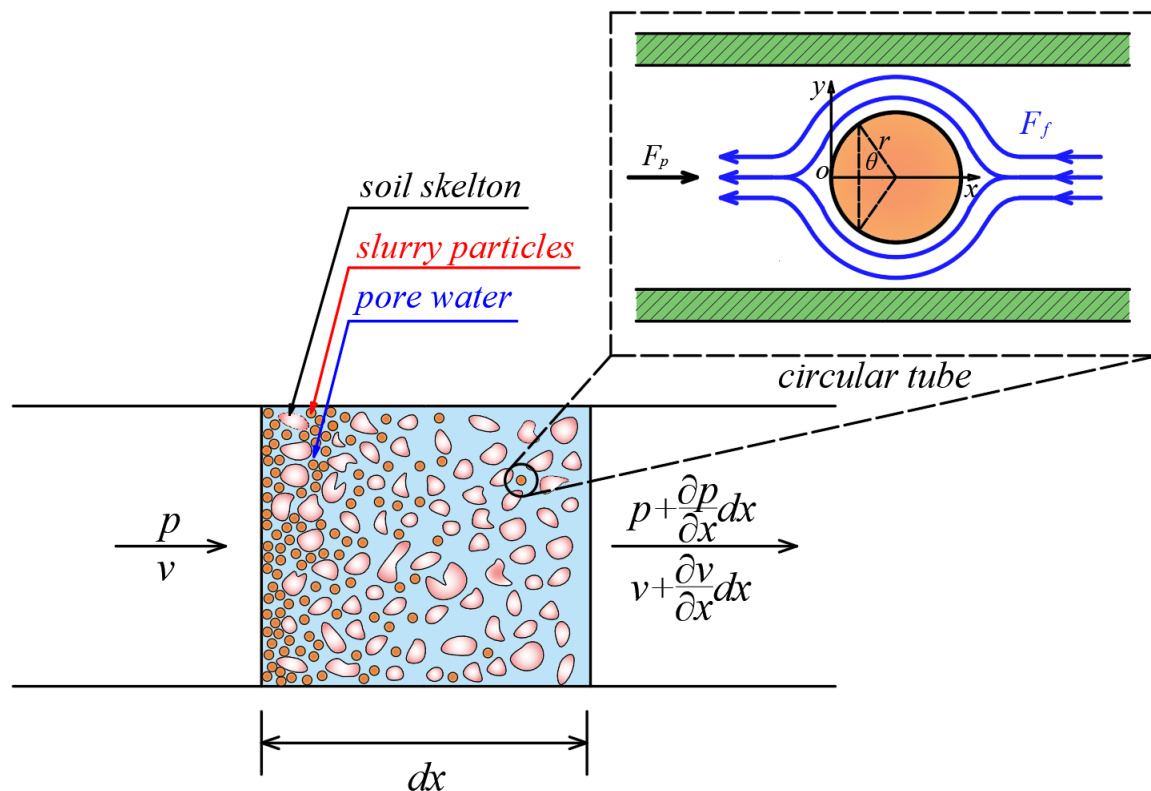


Figure 1. Sketch diagram of slurry particles infiltrating through soil pores modeled as circular tubes.

2.1. Volume and Mass Conservation in Slurry Infiltration

According to the principle of volume conservation, the difference between the influx and efflux of the fluid phase within a soil microelement is given by [30]:

$$\Delta V = vC_w A dt - (v + \frac{\partial v}{\partial x} dx)(C_w + \frac{\partial C_w}{\partial x} dx) A dt = -\frac{\partial v C_w}{\partial x} A dx dt \quad (1)$$

where A denotes the cross-sectional area of the computational unit normal to the direction of infiltration. C_w is the volume fraction of the fluid phase in slurry.

Particle transport within the porous medium exhibits significant nonlinear characteristics. To facilitate theoretical derivation, it is assumed that the slurry follows a non-Darcy flow regime, with its seepage velocity denoted by v [10]. Because bentonite slurry has shear yield strength, seepage can only be initiated when the hydraulic gradient is larger than the hydraulic gradient threshold g_0 :

$$v = -\frac{k}{\gamma_F} \frac{\mu_w}{\mu_f} \left(\frac{\partial p}{\partial x} - g_0 \right) \quad (2)$$

where γ_F denotes the unit weight of the slurry; μ_w is the viscosity of water at ambient temperature; g_0 represents the pressure gradient threshold; and p is the excess pore-water pressure induced by infiltration. The equivalent permeability coefficient k of the soil and the initial permeability coefficient k_0 of the clean soil can be characterized by the following expression [31].

$$k = k_0 \frac{u_w}{u_f} \left(\frac{1 - \varphi}{1 - \varphi_0} \right)^{-4/3} \left(\frac{\varphi}{\varphi_0} \right)^3 \quad (3)$$

φ characterizes the porosity of the soil with deposited slurry particles, while φ_0 represents the initial porosity of the soil.

μ_f characterizes the dynamic viscous properties of the slurry; the bentonite slurry comprises low- and medium-concentration solid–liquid suspensions, which can be expressed by Einstein’s equation [32]:

$$\mu_f = \mu_w \left(1 + \alpha \frac{C_L}{\rho_s} \right) \quad (4)$$

where α is the viscosity coefficient, m^3/kg .

The hydraulic gradient threshold g_0 exhibits a strong dependence on both the slurry’s shear yield limit and the soil pore diameter in Equation (2).

$$g_0 = \frac{2\tau_F}{r_p} \quad (5)$$

where τ_F denotes the slurry’s shear yield strength and r_p represents the equivalent pore radius of infiltration zone, which can be estimated as:

$$r_p = \frac{d_{10}}{2} \left(1 - \beta \frac{C_s}{\rho_s} \right) \quad (6)$$

d_{10} characterizes the effective grain size of the soil skeleton. β denotes the expansion coefficient of the slurry particles, $\beta = 6/\pi$ [15]. C_s quantifies the retained mass of bentonite per unit pore volume. It should be noted that natural soil pores have irregular shapes. In this study, pores are simplified as ideal circular tubes for mechanical analysis, and the calculated pore radius represents the equivalent average pore size of sandy soil.

The volumetric compression, governed by the intrinsic compressibility of the soil, is positively related to excess pore-water pressure.

$$\Delta S_p = \frac{A}{E_S} \frac{\partial p}{\partial t} dx dt \quad (7)$$

where E_S is the elasticity modulus of soil. Given the relatively high permeability of saturated sandy soils, a transient partially drained condition is assumed during the rapid slurry infiltration phase. For computational simplicity, a linear elastic compressibility model is adopted here.

The volume change in the fluid phase caused by slurry particles includes two parts: particles which are deposited versus suspended in soil pores:

$$\Delta S_d = \frac{\varphi_0 A}{\rho_s} \frac{\partial C_s}{\partial t} dx dt \quad (8)$$

$$\Delta S_s = \frac{A}{\rho_s} \frac{\partial \varphi C_L}{\partial t} dx dt \quad (9)$$

ΔS_d characterizes the volumetric retention of particles within the soil voids, whereas ΔS_s quantifies the volumetric variation in the suspended particles.

Based on the volumetric conservation, the volumetric variation in the fluid within the computational soil element strictly balances the net flux across its boundaries. This theoretical premise yields the governing equation for the slurry:

$$\frac{1}{E_S} \frac{\partial p}{\partial t} - \frac{\varphi_0}{\rho_s} \frac{\partial C_s}{\partial t} - \frac{1}{\rho_s} \frac{\partial \varphi C_L}{\partial t} + \frac{\partial v C_w}{\partial x} = 0 \quad (10)$$

The porosity of the soil with bentonite particle deposition is as follows:

$$\varphi = \varphi_0 \left(1 - \frac{\beta C_s}{\rho_s} \right) \quad (11)$$

Slurry mass concentration and particle infiltration velocity are correlated as follows [33]:

$$\frac{\partial C_s}{\partial t} = \lambda v C_L \quad (12)$$

λ represents the filtration coefficient, indicating the soil skeleton's ability to capture bentonite particles in the slurry. The value is determined by the laboratory test shown in Section 3.

The incorporation of Equation (12) into Equation (10) transforms the continuity equation into:

$$\frac{1}{E_s} \frac{\partial p}{\partial t} - \frac{\varphi_0}{\rho_s} \lambda v C_L - \frac{1}{\rho_s} \frac{\partial \varphi C_L}{\partial t} + \frac{\partial v C_w}{\partial x} = 0 \quad (13)$$

Equation (13) is formulated strictly within a fixed spatial control volume (Eulerian framework). The time-derivative term captures the local rate of mass accumulation of the deposited phase by tracking the temporal change in local porosity. This is fundamentally consistent with the continuum-mechanical basis established by [15].

Governed by the mass conservation of bentonite particles during infiltration [30], the net mass influx is equivalent to the total mass of deposited and suspended particles within the soil pores.

$$\lambda v \varphi_0 C_L + \frac{\partial \varphi C_L}{\partial t} + \frac{\partial v C_L}{\partial x} = 0 \quad (14)$$

In combination with Equations (13) and (14), the coupled governing equations are summarized below:

$$\begin{cases} \lambda v \varphi_0 C_L + \frac{\partial \varphi C_L}{\partial t} + \frac{\partial v C_L}{\partial x} = 0 \\ \frac{1}{E_s} \frac{\partial p}{\partial t} - \frac{\varphi_0}{\rho_s} \lambda v C_L - \frac{1}{\rho_s} \frac{\partial \varphi C_L}{\partial t} + \frac{\partial v C_w}{\partial x} = 0 \end{cases} \quad (15)$$

The boundary constraints and initial states are prescribed as follows:

$$\begin{cases} C_L(0, t) = C_{L0} \\ p(0, t) = p_0 \end{cases} \quad (16)$$

where p_0 denotes the pressure differential between the slurry in the chamber and the ambient hydrostatic pressure, and C_{L0} represents the initial mass concentration of slurry. It should be noted that Equation (16) assumes constant slurry concentration at the tunnel face. While bentonite is continuously lost into the formation in practice, the continuous slurry circulation system in modern TBMs dynamically replenishes the chamber, maintaining a relatively stable concentration over the short timeframe of initial infiltration. Thus, this constant boundary simplification is deemed reasonable.

2.2. Dynamic Penetration Criteria of Slurry Particles

Investigating the transition conditions and governing factors between filter cake formation and slurry infiltration is essential for optimizing support efficiency and enhancing resource sustainability in slurry shield tunneling [34]. As the infiltration front advances, the deposition rate of bentonite particles varies spatially. At the leading edge (distal end) of the infiltration zone, most suspended particles are intercepted and retained within the soil matrix. Only a minimal fraction of particles continues to migrate deeper into the pores until they attain a state of shear stress equilibrium against the soil skeleton. This mechanical equilibrium dictates the terminal penetration depth and, consequently, defines the total volume of slurry required to achieve a stable supporting pressure, thereby serving as a critical threshold for minimizing material waste. In the computation element of tube flow, the pressure-gradient force F_p induced by excess pore-water pressure along the infiltration

direction is equal to the viscous resistance F_f acted on the surface of bentonite particles [35]. The pressure distribution caused by the particles' surface pressure gradient is:

$$p = p_0 + r(1 - \cos \theta) \frac{\partial p}{\partial x} \quad (17)$$

r denotes the equivalent radius of bentonite particles.

The lateral area of infinitesimal spherical surface is:

$$dS = 2\pi r^2 \sin \theta d\theta \quad (18)$$

By integral method, the excess pore-water gradient pressure F_p is obtained.

$$F_p = \int_0^\pi \left[p_0 + r(1 - \cos \theta) \frac{\partial p}{\partial x} \right] \cdot 2\pi r^2 \sin \theta \cos \theta d\theta = \frac{4}{3} \pi r^3 \frac{\partial p}{\partial x} \quad (19)$$

The detailed derivation process is provided in Appendix A.

Viscous resistance is denoted by $F_f = 4\pi r^2 \tau$, and τ is the shear stress of slurry:

$$\tau = -\eta \frac{dv}{dr} \quad (20)$$

where η is the coefficient of dynamic viscosity.

Theoretically, the threshold condition for particle immobilization is $F_p = F_f$.

$$\frac{dv}{dr} = \frac{2r}{3\eta} \frac{\partial p}{\partial x} \quad (21)$$

Integrating Equation (21), the bentonite particles stop infiltrating when the equivalent radius of bentonite particles is equal to half of the soil initial mean pore size:

$$v = -\frac{\partial p}{\partial x} \frac{1}{3\eta} \left(\frac{D_0^2}{4} - r^2 \right) \quad (22)$$

where D_0 is the initial soil mean pore size.

As shown in Figure 1, the flow rate Q of slurry is calculated by integrating the circular tube's cross-sectional area:

$$Q = -\frac{\partial p}{\partial x} \frac{\pi D_0^4}{96\eta} \quad (23)$$

The mean flow velocity of slurry in the circular tube $\bar{v}$:

$$\bar{v} = -\frac{\partial p}{\partial x} \frac{\pi D_0^2}{24\eta} \quad (24)$$

The transport of fluid in the porous medium is simplified as pipe flow in a classic circular pipe flow. Consequently, the seepage velocity is equal to the mean flow velocity of slurry.

At the farthest infiltration distance, the rheological parameters of slurry near the bentonite particles are close to pore water, approximately $u_f = u_w$ and $\tau_F = 0$. The permeability coefficient of soil is also close to the initial value k_0 . Combining Equations (3) and (25), the initial average pore diameter of the soil D_0 can be derived.

$$D_0 = \sqrt{\frac{24k_0\eta\mu_w}{\gamma_w\mu_f} \left(1 - g_0 / \frac{\partial p}{\partial x} \right)} \approx \sqrt{\frac{24k_0\eta}{\gamma_w}} \quad (25)$$

The suspended particles are constantly absorbed by the soil skeleton in infiltration. Assuming the cross profile of seepage pores with particles deposited is circular, then the diameter of equivalent soil seepage pores D is:

$$D = D_0(1 - \frac{\beta C_s}{\rho_s}) \quad (26)$$

During the infiltration of suspensions through porous media, the coarser fraction of the particles primarily governs the initial bridging and pore-clogging effects. Therefore, according to the research of Min [36], d_{85} is selected as the equivalent grain size of the bentonite slurry to provide a mechanically sound evaluation for particle interception. The injectability criteria of bentonite particles (BIC) can be expressed as:

$$BIC = \sqrt{\frac{24k_0\eta}{\gamma_w}(1 - \frac{\beta C_s}{\rho_s})} - d_{85} \quad (27)$$

The solution of the coupled governing Equation (15) is performed with COMSOL Multiphysics version 6.0, and Equation (27) is implemented as a threshold criterion to track the penetration front. In addition, the size of each soil computation element is specified as 0.1 mm, alongside the application of Dirichlet boundary conditions.

3. Laboratory Test

3.1. Experimental Setup

To quantitatively analyze the infiltration behavior of bentonite slurry across diverse soil particle distributions, a customized multilayer infiltration column experimental system was developed, as illustrated in Figure 2a. This integrated setup comprises a primary multilayer infiltration column supported by an array of precision auxiliary instruments, including a programmable pressure control system, a high-capacity air compressor, a slurry storage vessel, a filtrate collector, a constant-temperature oven, an analytical electronic balance, and a centralized data acquisition system. The core of the apparatus consists of seven modular organic glass (acrylic) infiltration elements, interconnected via inlet and outlet valves. To ensure airtightness and structural integrity under pressurized conditions, each element is sealed with waterproof rubber gaskets and secured with high-strength bolts. As depicted in Figure 2b, each element features a unique “stepped-diameter” double-column design separated by a partition. The partition contains a 1 cm diameter circular orifice at its center to facilitate controlled flow. Both chambers of the element are 3 cm high, with inner diameters of 10 cm for the upper and 3 cm for the lower chamber. For filtrate sampling, a collection tube (inner diameter 0.5 cm, length 7 cm) is integrated into the sidewall of the lower chamber and controlled by a manual valve. During operation, regulated compressed air is delivered to the slurry storage vessel, driving the slurry into the uppermost column element (Element A) to initiate the infiltration process.

To achieve the controlled injection of slurry, the uppermost column segment (Element A) remains unoccupied to serve as an initial pressurized reservoir, with its manual valve kept closed. Precision filter screens are strategically positioned within the assembly: the bottom of each upper chamber and the interior of each lower chamber are fitted with a single-layer 170-mesh screen and a 40-mesh screen, respectively. These specifications ensure that the bentonite slurry achieves a 100% passage rate through the mesh, whereas the sandy soil particles are completely retained (0% passage). To reduce the flow velocity and facilitate filtrate collection, each lower chamber is filled with 8 mm diameter glass beads. Preliminary calibration shows that the change in slurry concentration after passing through the glass bead layer is below 0.05 kg/m^3 , confirming that its interference with the

concentration measurement is negligible. Furthermore, a 5 mm buffer zone is left between the soil surface and the partition of the preceding column segment. This design ensures that the slurry infiltrates uniformly across the entire cross-sectional area of the soil column, preventing localized flow concentration and ensuring data representativeness.

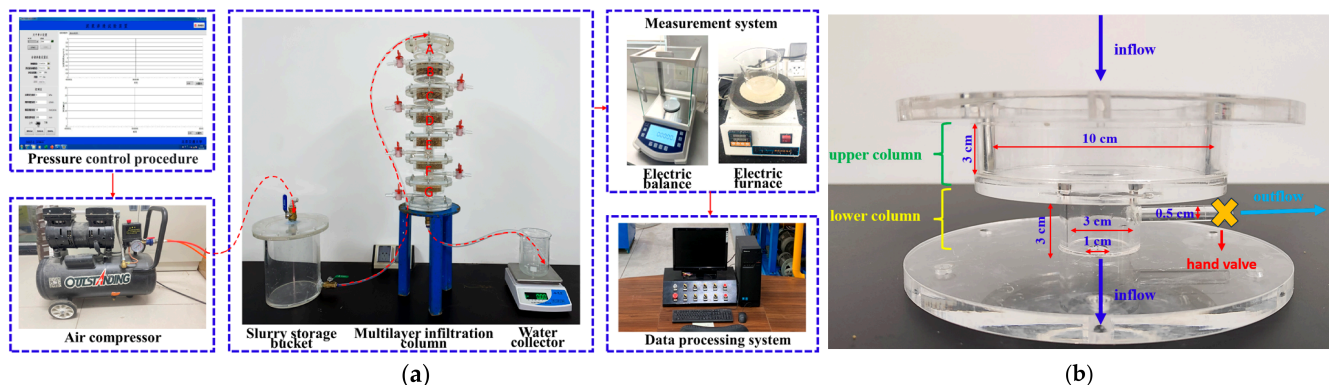


Figure 2. (a) Experimental system for multilayer slurry infiltration. A: unoccupied infiltration elements; B–G: occupied infiltration elements; (b) infiltration column element (all dimensions are in cm).

3.2. Physical Properties of Materials

3.2.1. Bentonite Slurry

The slurry samples were formulated using sodium-bentonite (Na-bentonite), distilled water, and sodium carboxymethylcellulose (CMC). To ensure a stable chemical environment and prevent entrapped air from affecting the infiltration measurements, the test water was maintained in a weakly alkaline state and thoroughly degassed. The preparation procedure involved pre-dissolving the CMC in water to ensure full polymer hydration before blending it with the pre-hydrated bentonite suspension. Given that the rheological properties of bentonite slurry are inherently time-dependent, a standard 24 h expansion (hydration) period was strictly observed prior to the experiments to achieve a stable and consistent viscosity profile [37]. During this period, CMC macromolecules fully interact with and adsorb onto the bentonite surfaces, forming a polymer-clay network that significantly enhances the yield strength and prevents phase separation. For this study, five distinct slurry formulations were prepared by varying the bentonite mass concentration, while the CMC dosage was maintained at a constant 0.05% to isolate and investigate the specific influence of solid phase concentration on the infiltration and deposition mechanisms. Table 1 summarizes the fundamental rheological metrics of the formulated slurries. Dynamic viscosity μ_f and shear strength τ_f are quantified utilizing a 12-speed viscometer under normal atmospheric temperature conditions. Furthermore, the granulometric profile of the suspensions, as depicted in Figure 3, is determined via laser diffraction analysis.

Table 1. Rheological characteristics of different bentonite slurries.

	SL1	SL2	SL3	SL4	SL5
Bentonite content, C_{L0} (kg/m ³)	30	40	50	60	70
Density, ρ (g/cm ³)	1.025	1.027	1.032	1.036	1.041

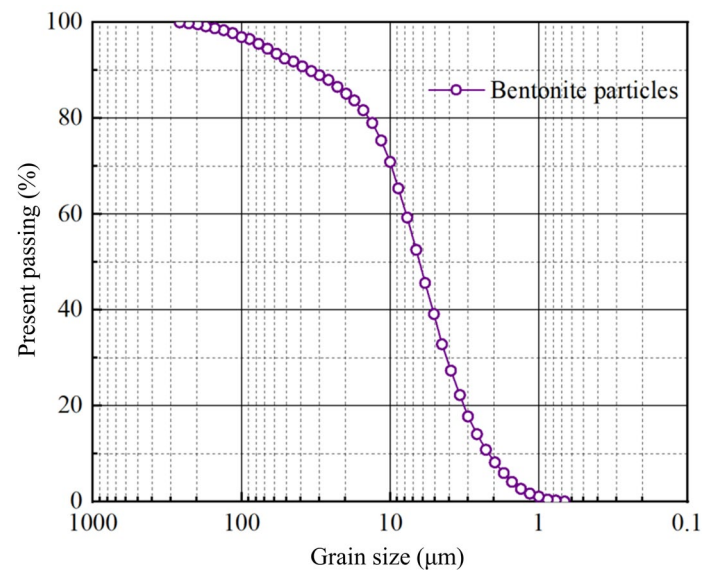


Figure 3. Particle size distribution curves of slurry samples obtained by laser diffraction analysis.

3.2.2. Soil Samples

The particle size distribution of the four sand samples utilized in this research are depicted in Figure 4. Specifically, samples S1 and S2 were prepared by sieving standard sand produced in accordance with Chinese ISO standards, while samples S3–S4 were sourced from natural river sand. To rule out any measurement deviations in the filtrate mass concentration caused by native silt or impurities, every sample was subjected to extensive washing and subsequently baked at 110 °C for 24 h. The fundamental physical properties of these soil samples are summarized in Table 2. The dry density ρ_s of each soil sample is 2.67 kg/m³. The permeability coefficient was determined by constant head test, and the elastic modulus was obtained by laboratory compression test.

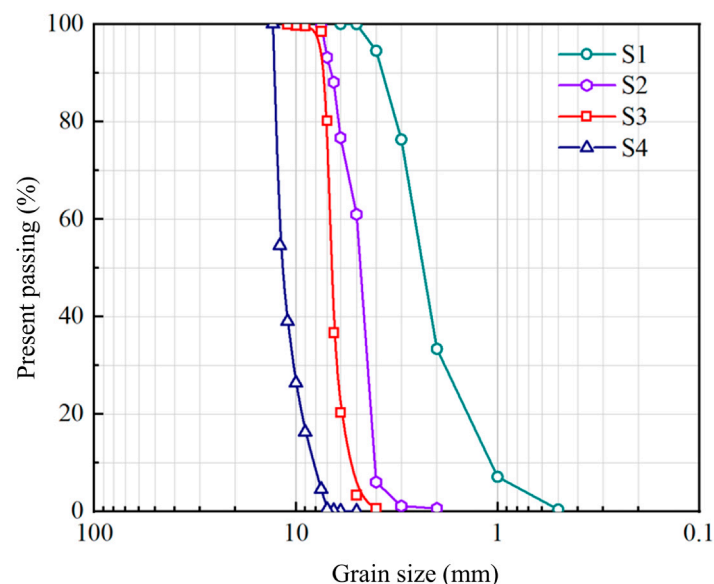


Figure 4. Particle size distribution of soil samples.

Table 2. Physical characteristics of the soil samples.

	S1	S2	S3	S4
Porosity, φ (—)	0.48	0.45	0.41	0.44
Permeability, k_0 (cm/s)	0.17	0.58	2.48	5.22
Effective grain diameter, d_{10} (mm)	0.54	1.03	1.09	4.07

3.3. Experiment Procedure

To accurately reveal the evolution of the filtrate mass concentration during the infiltration process, the following specific experimental procedures were implemented in this study:

(1) Device assembly: Manual valves were installed on each infiltration column element and set to the closed position. The lower valve of the assembly was linked to a water reservoir used for the back-saturation of soil samples. (2) Soil preparation and saturation: To ensure consistent slurry penetration through all stages, the soil samples were packed into the multilayer column such that their particle sizes decreased sequentially from top to bottom. Subsequently, the water reservoir was filled and the bottom valve was opened, allowing water to slowly infiltrate the soil layer from the bottom up, effectively displacing and expelling the air trapped within the pores. Once full saturation was achieved, the outlet valve was closed. (3) Slurry preparation: Bentonite powder and water were blended in the prescribed proportions and stirred for 30 min. The CMC solution was then added to the pre-hydrated slurry, followed by an additional 10 min of stirring. Prior to the commencement of each test, the mixture was stirred for another 5 min to ensure a uniform distribution of bentonite particles. (4) Infiltration experiment: The pressure control system was initiated and set to a constant 50 kPa. Upon opening the bottom manual valve, the slurry initially filled Element A before gradually infiltrating downwards. As the slurry traversed Element B and filled the lower chamber, the manual control valve was promptly opened to collect a 5 s filtrate sample using a graduated cylinder. While physically representing a time-averaged concentration over the collection period, this 5 s interval is sufficiently brief relative to the total infiltration timeframe, allowing the sample to serve as an acceptable quasi-instantaneous snapshot with negligible systematic error. Sampling proceeded sequentially from the top element to the bottom. Pressurization was immediately terminated when slurry began to discharge from the bottom-most valve, signaling the completion of the experimental trial. (5) Filtrate mass concentration measurement: The collected filtrate in the graduated cylinder was transferred into a beaker, and the inner walls of the cylinder were rinsed two to three times to ensure all bentonite particles adhering to the sidewalls were recovered. The samples were then heated at 60 °C until all water had evaporated. This specific temperature was deliberately chosen as it is well below the thermal decomposition threshold of CMC (typically >200 °C), thereby preventing any polymer degradation and ensuring the accuracy of the residual dry mass. The mass of the residual bentonite was measured immediately to mitigate the impact of atmospheric moisture absorption, after which the final filtrate mass concentration was calculated.

3.4. Regression Analysis of Filtration Coefficient

The deposition of bentonite particles within the soil skeleton is commonly characterized by introducing the filtration coefficient λ , which is defined as the ratio of the particle mass retained by the porous medium m_i to the total influent particle mass m_d per unit length along the seepage direction [13]. A higher filtration coefficient facilitates the rapid formation of a low-permeability zone, which is essential for minimizing uncontrolled slurry penetration and maximizing the reuse rate of the bentonite material in sustainable

tunneling operations. In this research, the deposited mass of bentonite particles is derived through the filtrate concentration difference, and λ can be expressed as follows:

$$\lambda = \frac{m_d}{lm_i} \quad (28)$$

l represents the length of the porous medium, which is physically equivalent to the packed height of the soil specimen within the infiltration column.

$$m_d = (C_{Li} - C_{Lo})L \quad (29)$$

$$m_i = v_p t A C_{Lo} \quad (30)$$

C_{Li} and C_{Lo} denote the inflow and outflow concentration of slurry, respectively, which represent the filtrate concentration of the adjacent column elements. L denotes the accumulated volume of the slurry. V_p represents the pore velocity (commonly simplified as $L/\varphi_0 A t$). t stands for the duration of infiltration, and A refers to the cross-sectional area of the upper column section.

By substituting Equations (15) and (16) into Equation (14), the expression for λ is given by:

$$\lambda = \frac{(C_{Li} - C_{Lo})\varphi_0}{C_{Lo}H} \quad (31)$$

In this formulation, l denotes the consistent filling height of the soil sample for each layer, which is strictly controlled at 25 mm across all experiments. Five infiltration experiments were conducted which contain 15 sets of data. C_{Li} , C_{Lo} and λ are detailed in Table 3. The findings reveal that the filtration coefficient is negatively associated with soil particle size. The results indicate that the filtration coefficient is negatively correlated with soil particle size. To statistically validate this, a Spearman rank correlation analysis was performed, yielding a significant negative correlation coefficient between d_{10} and λ ($\rho = -0.680$, $p < 0.01$).

Table 3. Results of slurry infiltration experimentation in multilayer columns.

Test	Slurry	Soil	Mass Concentration of Slurry Filtrate (kg/m ³)		Measurement of Filtration Coefficient λ (m ⁻¹)
			C_{Lin}	C_{Leff}	
1	SL1	S3	27.13	26.15	0.78
2		S2	26.15	24.54	1.29
3		S1	24.54	22.33	1.86
4	SL2	S3	38.03	35.97	1.19
5		S2	35.97	33.15	1.67
6		S1	33.15	29.55	2.29
7	SL3	S3	48.86	45.55	1.51
8		S2	45.55	41.23	2.06
9		S1	41.23	36.10	2.67
10	SL4	S3	58.87	54.10	1.83
11		S2	54.10	48.32	2.34
12		S1	48.32	41.99	2.84
13	SL5	S3	67.97	61.75	2.09
14		S2	61.75	54.30	2.69
15		S1	54.30	46.57	3.12

Utilizing the nonlinear analysis module in SPSS 26, a multiple fitting regression model characterizing the filtration coefficient λ is established. Typical slurry rheological parameter (slurry mass concentration C_L) and pore-characteristic parameters of soil (equivalent grain size d_{15} , porosity φ , nonuniform coefficient C_u) are selected as independent design variables. Before the regression analysis, the correlation forms between each factor and the target function were inferred from the measured scatter plots. Figures 5 and 6 plot the 3D spatial distribution of the observed λ . An analysis of their planar projections reveals a linear relationship between C_L and λ , whereas the pore-characteristic parameters exert an exponential influence on λ .

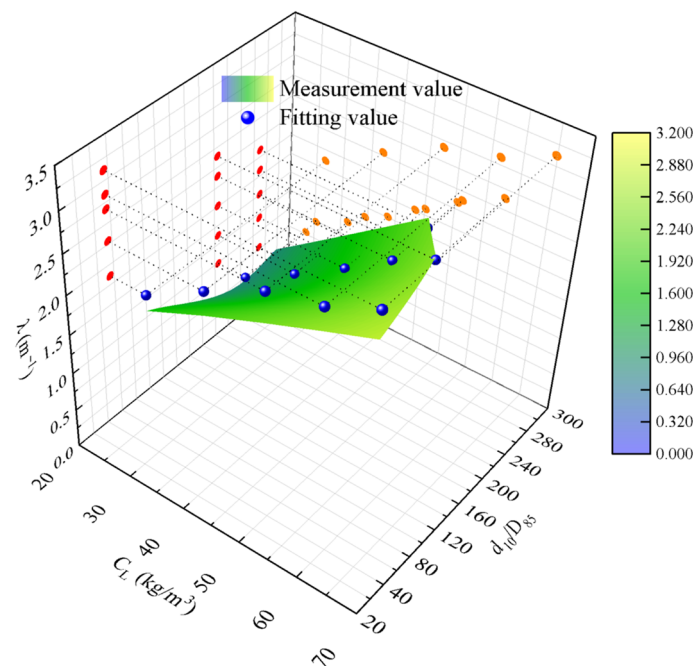


Figure 5. Comparison between the experimental and predicted filtration coefficients under varying C_{L0} and d_{10}/d_{85} .

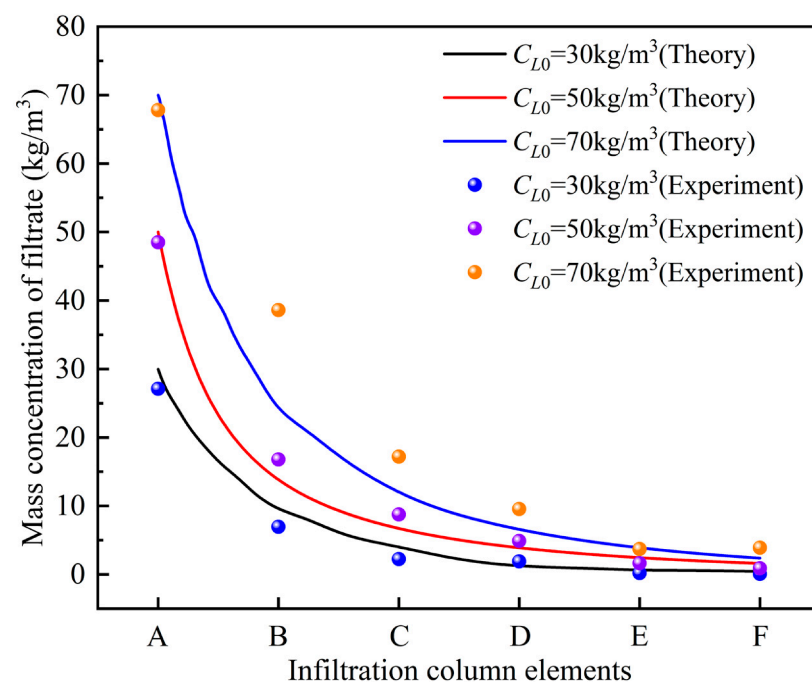


Figure 6. Changes in slurry mass concentration with infiltration distance.

The regression model for filtration coefficient λ_{reg} is formulated as:

$$\lambda_{reg} = 0.052C_L + 1.541e^{\frac{62.02}{d_{10}/d_{85}}} - 0.485 \quad (32)$$

In addition, statistical diagnostics reveal that the determination coefficient $R^2 = 0.96$, with a significance level of $p = 0.002$ (strictly below the 0.05 threshold), confirms a strong regression correlation. To further evaluate the model's robustness and ensure it is not overfitted, the model's RMSE (Root Mean Square Error) is calculated as 0.12. Furthermore, the 95% confidence intervals for all regression coefficients are verified, confirming the statistical stability and power of the established formula. As evidenced by the evolutionary trends in Figures 5 and 6, the theoretical fitted values align highly with the measured data. Significantly, Equation (32) might be only suitable for sandy soil, wherein the particle diameter ratio of effective grain diameter d_{10} and d_{85} of the slurry is less than 270. The filtration coefficient regression equation is fitted based on 15 groups of experimental data from the present test.

4. Model Verification

To evaluate the reliability and robustness of the proposed mathematical model, supplementary infiltration experiments were conducted using a single-layer soil configuration. Subjected to a constant pressure boundary of $p_0 = 50$ kpa, soil sample S3 was tested in conjunction with three slurry formulations (SL1, SL3, and SL5) to characterize the spatial evolution of slurry mass concentration. As illustrated in Figure 6, both the experimental measurements and the theoretical predictions exhibit a consistent trend: the slurry mass concentration undergoes a progressive decline within a limited infiltration depth, with the rate of attenuation negatively correlated with the infiltration distance. This strong agreement confirms the model's capability to capture the particle deposition process. Specifically, the results indicate that the vast majority of bentonite particles are entrapped and deposited adjacent to the soil–slurry interface, while only a minimal fraction of particles continues to migrate into the deeper pore structure under the influence of excess pore-water pressure. This localized deposition mechanism is essential for the rapid formation of a low-permeability barrier, which effectively mitigates the uncontrolled loss of bentonite resources and ensures the material efficiency required for sustainable tunneling operations.

Unlike traditional single-phase models (e.g., Zhu et al. [11]) that assume constant soil properties and yield linear pressure fields, the proposed multi-phase framework successfully captures the non-linear spatiotemporal attenuation of slurry concentration (C_L) validated in Figure 6. This demonstrates that the coupled PDE approach, despite its computational complexity, is essential for accurately replicating dynamic pore-clogging mechanisms.

Furthermore, the influence of slurry mass concentration on the maximum infiltration distance was systematically investigated. To accurately capture this parameter, the experimental configuration was modified from the filling method illustrated in Figure 6. In this setup, the lower chamber was also packed with soil to provide a continuous porous medium. The filter screen previously positioned at the base of the upper chamber was removed, with screens installed exclusively along the sidewalls of the lower chamber. Additionally, the collection tubes remained sealed to prevent the migration and loss of soil particles, thereby ensuring the structural integrity of the soil skeleton (Figure 7). This modification aligns with the findings of Xu and Bezuijen [38], which indicate that the seepage length of slurry is effectively extended when traversing a permeation column with a variable cross-section. Quantifying the infiltration distance is critical for optimizing slurry formulations, as it allows for the precise control of penetration depth to minimize material waste and maximize the reuse of bentonite resources in sandy strata.

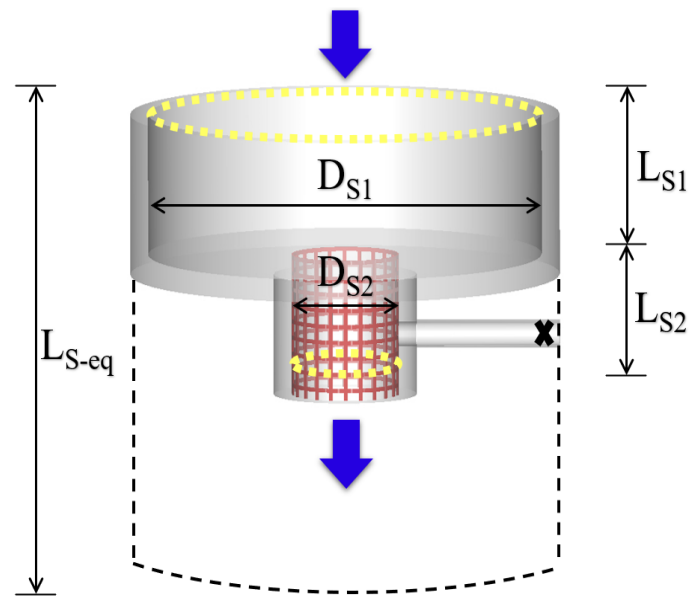


Figure 7. Equivalent infiltration length of column elements.

The equivalent seepage distance of the double-column L_{s-eq} is as follows:

$$L_{s-eq} = L_{S1} + L_{S2} \left(\frac{D_{S1}}{D_{S2}} \right)^2 + \frac{D_{S1}^2}{4D_{S2}} - \frac{D_{S1}}{4} \quad (33)$$

where L_{S1} and D_{S1} are the length and inner diameter of the upper column, respectively. L_{S2} and D_{S2} are the length and inner diameter of the lower column. $D_{S1} > D_{S2}$, so $L_{s-eq} > L_{S1} + L_{S2}$. In this study, $L_{S1} = 25$ mm, which perfectly matches the initial filling depth of the soil medium. L_{S2} is dependent on the practical infiltration distance of slurry in the lower column. The influence of the thickness of the filter screen is ignored. Theoretically, the equivalent infiltration distance of each column element is 417 mm.

Soil sample S3 was selected for the infiltration distance measurement experiment. $p_0 = 50$ kPa and pressure time was set as 180 s. As shown in Figure 8, the mass of retained bentonite per unit pore volume C_S exhibited a distinct decaying trend along the penetration path. In theory, the bentonite particles stop migrating when $C_S = 0$, and BIT will also reach maximum value. Experimental data reveal that the equivalent penetration depth of suspended particles is negatively correlated with the slurry mass concentration, a finding that aligns with the research of Xu and Bezuijen [39]. This inverse relationship demonstrates that a higher bentonite content in the slurry accelerates the clogging of soil pores, thereby facilitating the swift development of a protective filter cake or a concentrated infiltration zone. From a resource-management perspective, increasing the slurry concentration serves as an effective strategy to restrict excessive penetration. By confining the slurry within a narrower infiltration zone, this approach significantly reduces the total volume of bentonite consumed, minimizes material loss in highly permeable sandy strata, and enhances the overall sustainability of shield tunneling.

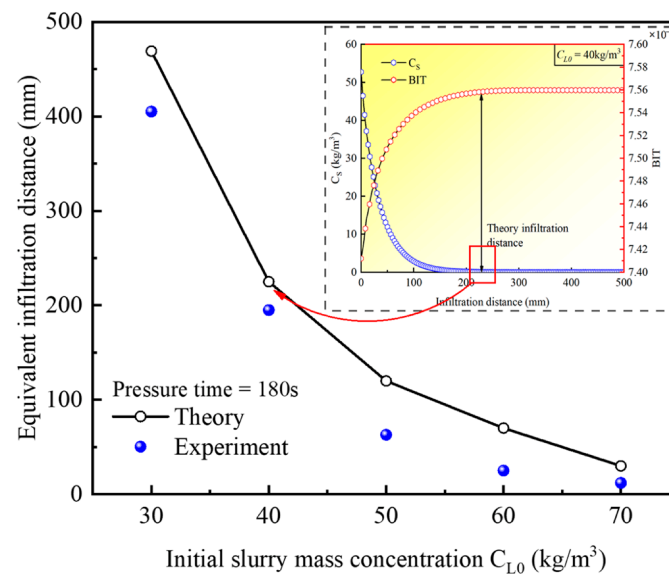


Figure 8. Equivalent infiltration length with different slurry mass concentrations.

5. Parametric Study

Figure 9 shows the influence of the initial soil permeability coefficient on the variation in slurry dynamic viscosity during infiltration. The setting of slurry initial rheological parameters was based on sample SL3, and the magnitude orders of the permeability coefficient ranged from 10^{-3} to 10^{-5} . Pressure time was set as 500 s and p_0 was 50 kPa. The analysis confirms that the dynamic viscosity undergoes a sharp attenuation in the early migration stage, followed by a gradual decline until its rheological behavior completely degenerates to that of water. Under the critical state where μ_f approaches 1.0 cP, the vast majority of the fine bentonite particles carried by the slurry are entrapped and adsorbed within the soil skeleton. In addition, as the initial soil permeability coefficient decreases, the maximum infiltration distance also gradually decreases. From a sustainability perspective, these results highlight that in highly permeable strata, precisely regulating slurry viscosity is paramount to preventing excessive material loss and ensuring dual economic and environmental benefits.

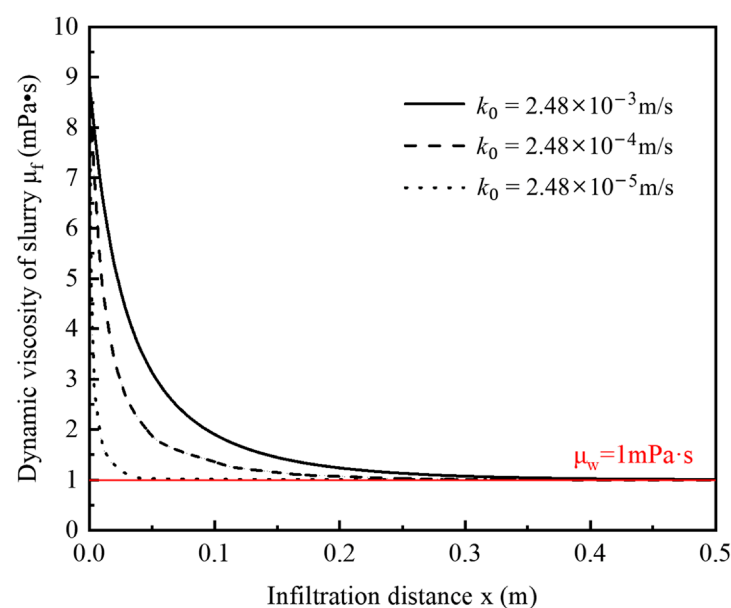


Figure 9. Variation in slurry dynamic viscosity with infiltration distance.

Additionally, the soil porosity is examined with the change in infiltration distance and initial slurry mass concentration C_{L0} . The particle size-characteristic parameters are referenced against soil sample S3. The pressure time lasted for 500 s and p_0 was set as 50 kPa. The pressure boundary refers to the difference between the slurry chamber pressure and the in situ water–earth pressure in actual engineering. As illustrated by the evolution curves in Figure 10, the porosity first rapidly recovers and then slowly approaches the initial value ϕ_0 . This phenomenon fully verifies that the vast majority of bentonite particles struggle to penetrate into the deeper pore spaces. Moreover, the porosity at soil surface ($x = 0$) and the infiltration distance for the porosity to increase to the initial value are significantly negatively correlated with C_{L0} . This suggests that an increased concentration of bentonite particles in the slurry suspension enhances the adsorption capacity of the soil skeleton, leading to a more pronounced and localized clogging effect. From a sustainable construction perspective, optimizing C_{L0} is a key strategy for the rapid establishment of a low-porosity zone, which effectively restricts uncontrolled slurry migration, minimizes material consumption, and promotes the conservation of bentonite resources in sandy formations.

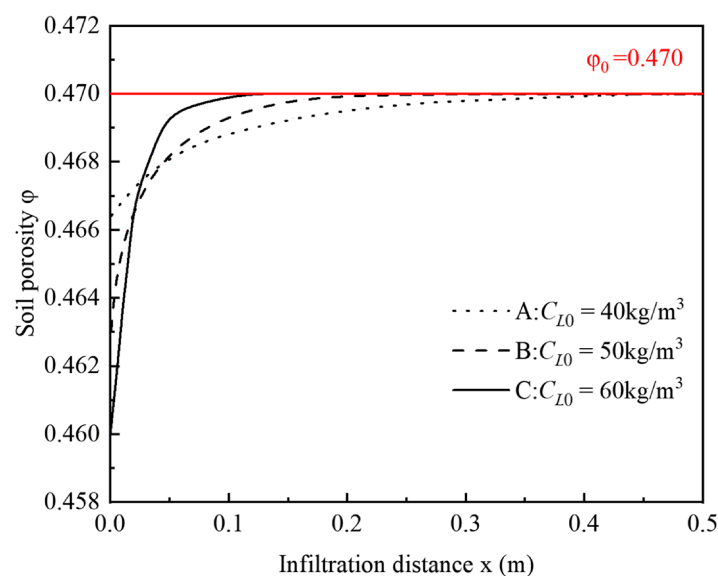


Figure 10. Variation of soil porosity with infiltration distance.

The effect of slurry mass concentration on the seepage velocity v and maximum infiltration distance is also analyzed. The initial parameters were derived from slurry samples SL2, SL3, SL4 and soil sample S1. Pressure time was 30 s. Obviously, v gradually decreases to zero with the increase in infiltration time, as illustrated in Figure 11. The maximum infiltration duration t_d also represents the required time for the bentonite particles to reach an equilibrium state of shear stress. The results show that the mass concentration of the slurry mainly affects t_d but not the quantitative value of v . The maximum infiltration distance and infiltration duration are inversely proportional to C_{L0} . Previous research by Jin and Yuan [40] indicated that the total filtration loss of the slurry is significantly and positively driven by the infiltration distance. Consequently, increasing the slurry mass concentration or its shear strength accelerates the formation rate of the filter cake or infiltration zone. From a sustainable engineering perspective, this accelerated stabilization not only enhances the support efficiency of the tunnel face but also effectively minimizes the cumulative filtration loss. By restricting the penetration depth, this approach significantly reduces the raw-material consumption and enhances the resource-utilization rate in slurry shield tunneling.

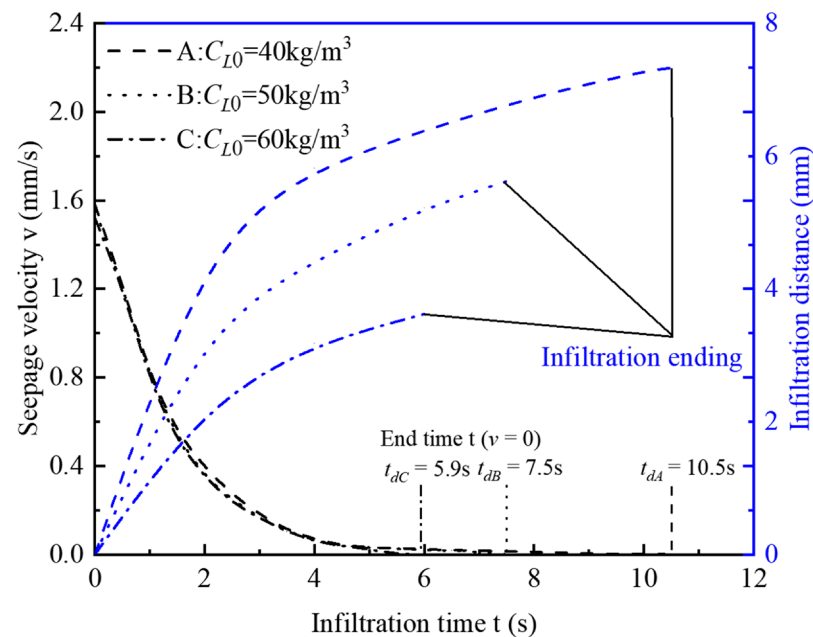


Figure 11. Infiltration ending time and distance of bentonite particles with different slurry mass concentrations.

6. Conclusions

Governed by the principles of mass conservation for bentonite particles and volume conservation for the liquid phase, a multi-phase theoretical model was formulated in this study to analyze the spatiotemporal evolution of slurry rheological properties and soil pore characteristics during infiltration. Relying on multilayer infiltration physical model tests, this study quantitatively revealed the effects of slurry concentration and soil characteristics (including porosity, grain size, and uniformity) on the filtration coefficient. Synthesizing the experimental and theoretical analyses, the principal findings of this study can be summarized as follows:

- (1) Both experimental and theoretical results indicate that slurry mass concentration and dynamic viscosity undergo a rapid initial decline followed by an asymptotic stabilization over distance. Correspondingly, soil porosity exhibits a sharp initial reduction before gradually returning to its initial state. This suggests that the majority of bentonite particles are intercepted and deposited near the soil surface, while only a minimal fraction migrates into deeper pores under excess pore-water pressure until reaching a shear stress equilibrium. This mechanism confirms that the formation of an effective barrier is concentrated at the interface, which is critical for minimizing unnecessary material penetration and loss.
- (2) The filtration coefficient was not only positively driven by the slurry mass concentration but also governed by the negative weakening effects of the soil pore-characteristic parameters (porosity, effective size, and uniformity coefficient). As infiltration progresses, the adsorption capacity of the soil skeleton for bentonite particles diminishes with distance. These findings demonstrate that precisely increasing the bentonite content facilitates the rapid development of a high-quality filter cake or infiltration zone, thereby preventing the excessive escape of slurry into the surrounding environment.
- (3) The reduction in the filtration coefficient effectively shortens the decay distance of the slurry's dynamic viscosity. The maximum infiltration distance is positively related to soil particle size and inversely related to slurry mass concentration, while the maximum infiltration duration is inversely proportional to the bentonite content. From a sustainability perspective, optimizing the slurry formulation—specifically by increas-

ing the concentration of suspending agents or selecting slurries with higher shear strength—can significantly accelerate filter-cake formation. This strategy not only enhances the support efficiency of the tunnel face but also maximizes the reuse rate of bentonite resources and minimizes material waste in shield tunneling operations.

The above research results can serve as a theoretical basis for parameter selection during shield tunneling construction in saturated sandy soils. Considering the differences between indoor test conditions and complex field environments, relevant field data comparison and model revision will be carried out in future studies to promote the on-site application of the research achievements. It should be noted that the quantitative uncertainty evaluations such as stochastic parameter sampling, confidence interval estimation and error propagation analysis are not implemented in the current framework, due to the primary objective of this work being to reveal basic slurry infiltration laws. Comprehensive uncertainty quantification will be carried out in subsequent extended research.

Author Contributions: Conceptualization, B.H., W.W. and C.D.; methodology, B.H. and W.F.; investigation, B.H., W.W. and C.D.; data curation, G.Y.; writing—original draft preparation, B.H.; writing—review and editing, B.H. and W.F.; visualization, G.Y. and J.S.; validation, B.H. and J.S.; project administration, B.H. and W.F.; funding acquisition, B.H. and G.Y. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China (No. 52408422) and the Open Fund Project of the Key Laboratory of Urban Underground Engineering of the Education Ministry, Beijing Jiaotong University (No. TUC2023-01).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available on request from the corresponding author due to privacy restrictions.

Conflicts of Interest: Authors Changyan Du and Jicheng Shu were employed by the company China Railway 14th Corporation Mega Shield Construction Engineering Co., Ltd. Author Gongbiao Yang was employed by the company China Railway 14th Bureau Group Corporation Limited. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as potential conflicts of interest.

Appendix A

The derivation of Equation (19):

$$\begin{aligned}
 F_P &= \int_0^\pi \left[p_0 + r(1 - \cos \theta) \frac{\partial p}{\partial x} \right] \bullet 2\pi r^2 \sin \theta \cos \theta d\theta \\
 &= 2\pi r^2 p_0 \int_0^\pi \sin \theta \cos \theta d\theta + 2\pi r^3 \frac{\partial p}{\partial x} \int_0^\pi (1 - \cos \theta) \sin \theta \cos \theta d\theta \\
 &= \pi r^2 p_0 \int_0^\pi \sin(2\theta) d\theta + 2\pi r^3 \frac{\partial p}{\partial x} \int_0^\pi \sin \theta \cos \theta d\theta - 2\pi r^3 \frac{\partial p}{\partial x} \int_0^\pi \sin \theta \cos^2 \theta d\theta \\
 &= \pi r^2 p_0 \left[-\frac{1}{2} \cos(2\theta) \right]_0^\pi + \pi r^3 \frac{\partial p}{\partial x} \left[-\frac{1}{2} \cos(2\theta) \right]_0^\pi + 2\pi r^3 \frac{\partial p}{\partial x} \int_0^\pi \cos^2 \theta d \cos \theta \\
 &= 0 + 0 + 2\pi r^3 \frac{\partial p}{\partial x} \left[\frac{1}{3} \cos^3 \theta \right]_0^\pi \\
 &= -\frac{4}{3} \pi r^3 \frac{\partial p}{\partial x}
 \end{aligned} \tag{A1}$$

References

1. Xiong, J.; Lin, T.; Li, N. *Geotechnical Chemistry*; Science Press: Beijing, China, 2001.
2. Mao, J.; Yuan, D.; Jin, D.L.; Zeng, J. Optimization and application of backfill grouting material for submarine tunnel. *Constr. Build. Mater.* **2020**, *265*, 120281. [CrossRef]

3. Xu, Z.; Guo, X. Optimization of bentonite parameters for shield tunneling based on response surface method. *Chin. J. Geotech. Eng.* **2021**, *43*, 194–200. [\[CrossRef\]](#)
4. Heinz, A. *Modifizierte Bentonitsuspensionen für Geotechnische Bauverfahren in Böden Hoher Durchlässigkeit*; Eidgenössische Technische Hochschule: Zürich, Switzerland, 2006.
5. Weiss, F. *Die Standfestigkeit Flüssigkeitgestützter Erdwände*; Technische Universität: Berlin, Germany, 1967.
6. Min, F.; Wei, D.; Jiang, T.; Zhang, C. Experimental study of law of slurry infiltration in strata. *Rock Soil Mech.* **2014**, *35*, 2801–2806. [\[CrossRef\]](#)
7. Talmon, A.M.; Mastbergen, D.R.; Huisman, M. Invasion of pressurized clay suspensions into granular soil. *J. Porous Media* **2013**, *16*, 351–365. [\[CrossRef\]](#)
8. Müller-Kirchenbauer, H. Stability of slurry trenches. In *Proceedings of 5th European Conference on Soil Mechanics and Foundation Engineering*; Sociedad Española de Mecánica del Suelo y Cimentaciones (SEMSyC): Madrid, Spain, 1972; pp. 543–553.
9. Thienert, C.; Pulsfort, M. *Neue Erkenntnisse zur Stützdruckübertragung Beim Tunnelvortrieb mit Flüssigkeitgestützter Ortsbrust Tagungs-Bd*; STUVA-Tagung Forschung Praxis; STUVA: Stuttgart, Germany, 2013; pp. 151–157.
10. Jin, D.; Yuan, D.; Mao, J. A novel approach of filter cake formation during the static-dynamic shield tunneling and its support efficiency control method. *China J. Highw. Transp.* **2021**, *35*, 154–167.
11. Zhu, W.; Min, F.; Zhong, X. *Slurry and Filter Cake in Slurry Shield*; Science Press: Beijing, China, 2016.
12. Fritz, P. Additives for slurry shields in highly permeable ground. *Rock Mech. Rock Eng.* **2007**, *40*, 81–95. [\[CrossRef\]](#)
13. Tien, C.; Ramarao, B.V. *Granular Filtration of Aerosols and Hydrosols*; Elsevier Science & Technology Books: Amsterdam, The Netherlands, 2007.
14. Kraemer, H.F.; Johnstone, H.F. Collection of aerosol particles in presence of electrostatic fields. *Ind. Eng. Chem.* **1954**, *47*, 2426–2434.
15. Herzig, J.P.; Leclerc, D.M.; Le, G.P. Flow of suspension through porous media: Application to deep bed filtration. *Ind. Eng. Chem. Res.* **1970**, *62*, 8–35. [\[CrossRef\]](#)
16. Nielsen, K.A.; Hill, J.C. Capture of particles on spheres by inertial and electrical forces. *Ind. Eng. Chem. Fundam.* **1976**, *15*, 157–163. [\[CrossRef\]](#)
17. Zamani, A.; Maini, B. Flow of dispersed particles through porous media—Deep bed filtration. *J. Pet. Sci. Eng.* **2009**, *69*, 71–88. [\[CrossRef\]](#)
18. McCarthy, J.F.; McKay, L.D. Colloid transport in the subsurface: Past, present, and future challenges. *Vadose Zone J.* **2004**, *3*, 326–337. [\[CrossRef\]](#)
19. Yao, K.M.; Habibian, M.T. Water and wastewater filtration: Concepts and applications. *Environ. Sci. Technol.* **1971**, *5*, 1105–1112. [\[CrossRef\]](#)
20. Sharma, P.; Flury, M.; Mattson, E.D. Studying colloid transport in porous media using a geocentrifuge. *Water Resour. Res.* **2008**, *44*, 767–768. [\[CrossRef\]](#)
21. Bedrikovetsky, P.; Zeinijahromi, A.; Siqueira, F.D.; Furtado, C.A.; Antonio, L.S.d.S. Particle Detachment Under Velocity Alternation During Suspension Transport in Porous Media. *Transp. Porous Media* **2012**, *91*, 173–197. [\[CrossRef\]](#)
22. Zhou, S.H.; Zhang, X.; Wu, D.; Di, H. Mathematical modeling of slurry infiltration and particle dispersion in saturated sand. *Transp. Porous Media* **2018**, *124*, 91–116. [\[CrossRef\]](#)
23. Han, W. *Grouting Mechanism Based on Filtration in Porous Media with Cement Grout and Its Engineering Application*. Ph.D. Thesis, ShanDong University, Jinan, China, 2014.
24. Fang, K. *Grout-Soil Interaction During Base Grouting and Its Effects on the Behavior of Grouted Piles*. Ph.D. Thesis, Zhejiang University, Hangzhou, China, 2013.
25. Burwell, E.B. Cement and clay grouting of foundations: Practice of the corps of engineers. *J. Soil Mech. Found. Div.* **1958**, *84*, 1551. [\[CrossRef\]](#)
26. Mitchell, J.K. Soil improvement-state of the art. In *Proceedings of the 10th International Conference on Soil Mechanics and Foundation Engineering*; Balkema A: Stockholm, Sweden, 2006; pp. 509–565.
27. Kim, J.S.; Lee, I.M.; Jang, J.H.; Choi, H. Groutability of cement-based grout with consideration of viscosity and filtration phenomenon. *Int. J. Numer. Anal. Methods Geomech.* **2010**, *33*, 1771–1797. [\[CrossRef\]](#)
28. Broere, W. Influence of excess pore pressures on the stability of the tunnel face. In *Reclaiming the Underground Space*; Routledge: London, UK, 2003; Volume 2, pp. 759–765.
29. Anagnostou, G.; Kov'ari, K. The face stability of slurry-shield-driven tunnels. *Tunn. Undergr. Space Technol.* **1994**, *9*, 165–174. [\[CrossRef\]](#)
30. Yang, Y.; Jin, D.; Yuan, D.; Cheng, P.; Yao, Z.; Cheng, X. A multi-field model for growth and dissipation of excess pore water pressure induced by slurry shield tunneling in semi-confined aquifer. *Tunn. Undergr. Space Technol.* **2023**, *135*, 105068. [\[CrossRef\]](#)
31. Carman, P.C. Fluid flow through granular beds. *Chem. Eng. Res. Des.* **1997**, *75*, S32–S48. [\[CrossRef\]](#)
32. Einstein, A. Eine neue Bestimmung der Moleküldimensionen. *Ann. Phys.* **1906**, *19*, 289–306. [\[CrossRef\]](#)

33. Bai, R.; Chi, T. Effect of deposition in deep-bed filtration: Determination and search of rate parameters. *J. Colloid Interface Sci.* **2000**, *231*, 299–311. [[CrossRef](#)] [[PubMed](#)]
34. Mao, J. Research on Laws of Dynamic Filter Cake Formation and Face Stability During Slurry Shield Tunnelling in Sand Stratum. Ph.D. Thesis, Beijing Jiaotong University, Beijing, China, 2023.
35. Yue, X.A. *Basics of Liquid-Solid Two-Phase Flow*; Petroleum Industry Press: Beijing, China, 1996.
36. Min, F.L. Study on the Law of Slurry Penetration in Stratum and the Filter Cake Formation of Slurry Pressurized Shield Tunnel. Ph.D. Thesis, Hohai University, Nanjing, China, 2012.
37. *API RP 13B-1*; Recommended Practice Standard Procedure for Field Testing Water-Based Drilling Fluids. American Petroleum Institute: Washington, DC, USA, 2026.
38. Xu, T.; Bezuijen, A. Experimental study on the mechanisms of bentonite slurry penetration in front of a slurry TBM. *Tunn. Undergr. Space Technol.* **2019**, *93*, 103052. [[CrossRef](#)]
39. Xu, T.; Bezuijen, A. Pressure infiltration characteristics of bentonite slurry. *Géotechnique* **2018**, *69*, 364–368. [[CrossRef](#)]
40. Jin, D.; Yuan, D. Face stability analysis of the slurry-type shield tunneling considering the dynamic filter cake. *Rock Soil Mech.* **2022**, *43*, 2952–2962. [[CrossRef](#)]

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