

## Article

# Resource Allocation Using the Balanced Scorecard and Cooperative Game Theory: An Axiomatic Framework for Sustainable Strategic Decision-Making

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## Abstract

Allocating scarce budgetary resources across the strategic objectives of the balanced scorecard (BSC) and the sustainability balanced scorecard (SBSC) is a central problem in sustainability-oriented strategic management. Existing approaches typically operate at the level of aggregated perspectives, rely on subjective weighting schemes, or ignore the causal structure of the strategy map. This article formalizes the problem as a cooperative game in which each strategic objective is a player and the value of every coalition combines the individual performance of its members with the synergies arising from the causal relationships of the strategy map, estimated through the decision-making trial and evaluation laboratory (DEMATEL). The Shapley value is adopted as the allocation rule. The resulting game is shown to be superadditive and convex, the Shapley value belongs to the Core, and the rule satisfies exhaustiveness, fairness, sensitivity to the strategy map, monotonicity, and convergence to the proportional rule. Computational experiments and sensitivity analyses support the robustness of the model. In an illustrative case—a sustainability balanced scorecard with ten objectives organized in five perspectives, whose dedicated sustainability perspective hosts a carbon-emission and an employee-safety objective, and a budget of USD 1M—the framework shifts 2.7% of the total budget relative to a proportional allocation (individual objectives change by up to 12.3%), funds employee safety above financial cost reduction for any synergy weight  $\gamma \geq 0.15$ , keeps the combined share of the sustainability objectives stable within 0.6 percentage points across the entire synergy range, and grants seed funding to sustainability objectives that enter the map without a measured baseline, providing a transparent and auditable basis for sustainable budget allocation.



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**Keywords:** resource allocation; sustainable strategic management; balanced scorecard; sustainability balanced scorecard; Shapley value; DEMATEL; employee safety; carbon emissions

## 1. Introduction

The efficient allocation of resources is one of the fundamental problems of sustainability-oriented strategic management. Under strict budget constraints, both in private firms and in public organizations pursuing sustainable development goals, decision-makers must distribute scarce resources among interdependent strategic objectives that compete with one another [1,2]. An inadequate allocation produces underutilization of capabilities and a

deterioration of the organization's competitive and socio-environmental position [3]. In sustainability contexts, the same problem extends to the simultaneous management of economic, environmental, and social objectives, where a poor allocation directly compromises long-term sustainability outcomes.

Performance measurement systems are the main instrument for translating strategy into quantifiable objectives and guiding resource allocation [4–7]. However, most existing systems exhibit documented limitations: subjective weighting frameworks, the absence of formal mechanisms to quantify interdependencies, and a lack of equitable rules for resource distribution [8–10]. These limitations are more restrictive in implementations of the sustainability balanced scorecard (SBSC), where the causal links between sustainability drivers (learning and growth, internal processes) and outcomes (customer, financial, and stakeholder) must be explicitly considered to avoid funding decisions that erode triple bottom line performance [11,12].

The balanced scorecard (BSC) [13–15] structures objectives along four perspectives (Financial, Customer, Internal Processes, and Learning and Growth) and represents causal hypotheses through the strategy map [16]. This causal architecture encodes the extent to which the performance of one objective is enabled by others, but the BSC lacks a formal mechanism that translates this causal information into a mathematically grounded allocation rule [8–10].

The SBSC extends this architecture to sustainability-oriented management, and two design choices distinguish it from the classical BSC. The first is architectural: environmental and social objectives can be integrated into the four classical perspectives, added as a fifth non-market perspective, or organized in a derived sustainability scorecard [17,18]. The choice is not neutral: it determines which causal links the strategy map can express, and it remains the subject of an active debate on whether architecture shapes—or merely reflects—the firm's posture toward sustainability [19,20]. The second difference concerns the causal texture of the map. Environmental and social objectives typically influence several perspectives simultaneously (an emission-reduction objective lowers operating costs and protects revenue from sustainability-conscious customers), operate with longer and more uncertain time lags, and create value partly through non-market channels—regulators, communities, and employees—rather than through the market transactions assumed in the classical  $LG \rightarrow IP \rightarrow C \rightarrow F$  chain [17,19,21]. A budget allocation mechanism for sustainable strategic management must therefore account for the *position* of each objective within this denser and more heterogeneous causal network, in addition to its stand-alone performance.

The central question is: *how should a budget be distributed across BSC objectives so that the allocation reflects not only the individual performance of each objective, but also its position and contribution within the causal network of the strategy map?* The question is relevant for sustainability management because cumulative allocation distortions widen the gap between the strategic intent of the SBSC and realized sustainability outcomes.

Cooperative game theory [22] provides an appropriate framework for this problem. The analogy is direct: the strategic objectives of the BSC can be interpreted as *players* that, by working together through the strategy map, generate a collective value greater than the sum of their individual contributions. The budget is the *resource to be distributed* among these players, and the causal map defines how they reinforce one another, that is, the *synergies*. The problem then consists in finding an allocation rule that is equitable, efficient, and aligned with the sustainability strategy.

The Shapley value [23] is the suitable tool for this purpose for three reasons. First, it assigns each objective a payoff proportional to its average marginal contribution to all possible coalitions, capturing both its own performance and the value generated through interaction with other objectives. Second, it is the unique solution concept that simultane-

ously satisfies efficiency (full distribution of the budget), symmetry (equivalent objectives receive the same amount), null player (an objective that contributes no value receives no resources), and additivity [23,24]. Third, under the conditions of the proposed model, the Shapley value belongs to the Core of the game, ensuring that no subset of objectives is underfunded relative to the value it generates.

One assumption delimits the domain of the model from the outset. Formalizing budget allocation as a game with transferable utility (TU) presumes that monetary resources can move across strategic objectives—from a customer-experience program to an information-systems upgrade—within a planning cycle. This is realistic in organizations with high budgetary flexibility and cross-functional integration: firms that fund strategy from a single pool of strategic initiatives, operate rolling or zero-based budgeting, and govern allocation through a committee whose authority spans all perspectives [25,26]. It is restrictive where line items are earmarked by statute or by multi-year capital plans, or where sustainability funds are ring-fenced for compliance reasons. Section 2.8 makes these conditions explicit, and Section 2.11 discusses non-transferable extensions for the restrictive case.

An axiomatic budget-allocation rule for sustainability-oriented strategy maps must satisfy four conditions simultaneously: (i) it must operate at the level of *individual objectives*, because sustainability objectives coexist with economic ones inside each perspective and aggregation removes objective-level trade-offs; (ii) it must take the *causal structure* of the strategy map as the input that generates coalition value; (iii) it must offer *axiomatic guarantees*—efficiency, symmetry, treatment of null players, and coalitional stability—so that the resulting allocation is defensible before boards, auditors, and stakeholders; and (iv) it must output an *explicit budget allocation* in monetary units, not merely a ranking or a vector of weights. The literature satisfies these conditions only in partial combinations. Game-theoretic BSC studies [8–10,27,28] provide (iii) in part, but model aggregated perspectives as players and stop at rankings, failing (i), (ii), and (iv). Hybrids of the BSC with multi-criteria decision-making (MCDM) methods—the analytic hierarchy process (AHP), the analytic network process (ANP), the decision-making trial and evaluation laboratory (DEMATEL), and their combinations [11,29–36]—quantify interdependencies, satisfying (ii), but their weighting schemes carry no axiomatic guarantees and rarely reach monetary allocation. Shapley-based allocation in logistics, energy, and production [37–42] satisfies (iii) and (iv), but its players are autonomous entities and the characteristic function ignores strategy maps. No published model satisfies the four conditions jointly.

The gap has persisted for a structural reason: converting a strategy map into a characteristic function—deciding how causal influence becomes coalition value—is a modeling step that belongs to neither parent literature, and it is a step required for sustainability applications. When sustainability drivers (training, safety, emission-reducing process capabilities) precede sustainability outcomes through long causal chains, allocation rules that ignore the map systematically underfund the drivers and overfund the outcomes, eroding triple-bottom-line performance over successive planning cycles [11,18,43]. This article closes that gap: it formalizes the (S)BSC as a cooperative game whose characteristic function encodes the strategy map, proves that the resulting game is superadditive and convex, and derives a Shapley-based budget rule with the four properties above.

This article formalizes the resource allocation problem as a cooperative game  $(N, v)$ . The players are the individual strategic objectives of the BSC; the function  $v(S)$  integrates autonomous performance with causal synergies. Six contributions are made. First, a characteristic function is proposed that encodes the structure of the strategy map, with proofs of superadditivity, convexity, and membership of the Shapley value in the Core. Second, five formal properties of the allocation rule are established, complemented by a characterization of the null player. Third, a theoretical analysis of the synergy coefficient is developed, in-

cluding managerial criteria for its calibration and a discussion of the transferable utility (TU) assumption versus its non-transferable utility (NTU) extensions. Fourth, a reproducible methodological framework for computing the exact allocation of each objective is described in detail. Fifth, computational experiments are reported for games ranging from eight to twenty strategic objectives, together with a sensitivity analysis showing that the allocation is robust to moderate estimation errors in the causal relationships and that it responds in a predictable and controllable manner to the weight assigned to synergies. Sixth, the framework is applied to a complete illustrative case of a manufacturing firm operating a sustainability balanced scorecard whose dedicated sustainability perspective hosts a carbon-emission and an employee-safety objective, showing how the resulting allocation departs from a purely proportional distribution by channeling additional resources toward the objectives that drive the rest of the strategy map; at the organizational level, this allows management to fund the causes of performance—including its environmental and social enablers—rather than only its outcomes.

### 1.1. *The Balanced Scorecard, the Sustainability Balanced Scorecard, and Strategy Maps*

The BSC [13] integrates financial and non-financial indicators across four perspectives: Financial (F), Customer (C), Internal Processes (IP), and Learning and Growth (LG) [14]. The strategy map [16] represents causal relationships between objectives through a vertical chain: improvements in LG lead to better IP, which enhance C and drive F performance [44]. Recent work on the SBSC extends this architecture by incorporating environmental and social dimensions alongside the four classical perspectives [11,12], providing the strategic backbone for sustainability-oriented management control.

Three decades of research document both the diffusion of the BSC and the contested nature of its performance effects [45,46]. Recent evidence from the small and medium-sized enterprise (SME) segment is directly relevant to the allocation problem addressed here. Gallo et al. [43] surveyed industrial SMEs and found that although firms with more than twenty employees do implement BSC-type systems with linked key performance indicators (KPIs), BSC adopters exhibit no statistically significant advantage in return on equity (ROE) or turnover, and the authors diagnose a systematic lack of consistency between strategic objectives and the indicators that operationalize them. Malagueño et al. [47] reach a complementary conclusion in a sample of Spanish SMEs: performance gains materialize only when the scorecard is used for strategy implementation rather than as a measurement inventory. These findings indicate that the main deficiency of BSC practice lies not in measurement but in the mechanism connecting measured objectives to funding decisions. The proposed model formalizes this mechanism.

For sustainability-oriented organizations, the SBSC literature provides the architectural foundation. Figge et al. [17] established the seminal formulation, linking sustainability management to business strategy by embedding environmental and social objectives in the value-creation logic of the scorecard; Epstein and Wisner [21] documented early implementation practice. Hansen and Schaltegger [18] systematized two decades of designs into a typology of architectures (integrated, additive, and derived), while Hahn and Figge [19] argued that architecture alone cannot resolve the tension between instrumental and integrative views of corporate sustainability. Hristov et al. [20] showed empirically that environmental and social objectives are often disconnected from economic success precisely because the causal conditions under which the SBSC creates value remain unmodeled. The model proposed here is architecture-agnostic by construction: the perspective label of an objective plays no role in the characteristic function, so sustainability value is represented by the causal network, not by the architectural label. The illustrative case adopts

the additive architecture, hosting the environmental and social objectives in a dedicated sustainability perspective.

Despite its widespread adoption, the BSC presents recognized limitations: a static nature, unidirectional causality without feedback, the absence of time lags, and a lack of formal mechanisms to validate causal hypotheses or select indicators [8–10]. These limitations have motivated its complementation with quantitative methods, which is also necessary in SBSC implementations, where allocation decisions must reconcile economic, environmental, and social trade-offs.

### 1.2. Traditional Resource-Allocation Methods in BSC Settings

In BSC practice, budgets are typically allocated through proportional, incremental, or negotiation-based rules administered by management control systems [25,26]. Formal support, where it exists, takes the form of project prioritization: Asosheh et al. [48] integrate DEA with the BSC to select information-technology projects, and Quezada et al. [44] prioritize strategic projects with DEMATEL using the BSC perspectives as criteria. These approaches allocate resources *to projects*, taking the relative importance of the strategic objectives themselves as given.

*Limitations and gaps.* This stream leaves the upstream question unanswered: how much funding should each strategic objective receive, given that objectives are causally interdependent? Proportional and incremental rules ignore the strategy map altogether; project-selection methods inherit whatever objective weights the organization already uses. None of these methods offers formal guarantees of efficiency, fairness, or stability, and none prices the causal contribution of one objective to the rest of the map.

### 1.3. Game Theory in Performance Management

Cooperative game theory has been explored as a complement to the BSC. Abedian et al. [9] treated each BSC perspective as a player in a four-player cooperative game in a manufacturing firm; the Shapley value revealed that the Customer perspective was the highest ranked. Seyedi and Yousefi Hanoomarvar [10] combined system dynamics with game theory to build a dynamic BSC for strategic performance management in a municipality. Naini et al. [8] combined evolutionary game theory with the BSC for environmental supply chains.

More recently, Abedian et al. [28] proposed an integrated BSC, DEA, and game theory model. Their proposal defines an efficiency change ratio as a superadditive characteristic function and uses the Shapley value to rank BSC indicators. Li and Liang [49] had previously proposed a Shapley-value-based index to measure the importance of variables in DEA models. Although these works advance the integration with efficiency techniques, their purpose is to rank indicators rather than allocate budget. Moreover, the characteristic function is constructed from DEA efficiencies without incorporating the causal structure of the strategy map. In a similar vein, Abedian et al. [27] developed a bi-objective model based on Nash equilibrium to determine the optimal combination of BSC indicators, treating the perspectives as players.

Huang and Chen [42] combined AHP/ANP with Shapley values to allocate budget among transportation alternatives, incorporating cooperative dynamics into the multi-criteria ranking. These works confirm the usefulness of the Shapley value as an allocation mechanism, but operate without the structure of the BSC strategy map.

Outside the BSC domain, the Shapley value has become a well-established tool for equitable allocation in horizontal cooperation problems. Lozano et al. [38] formulated a linear programming model to quantify the economies of scale arising from merging transport demands across firms. The authors compared the Shapley value with the nucleolus and

Core approximations, and reported that the Shapley value produces stable allocations in most scenarios. Frisk et al. [39] studied cost allocation in collaborative forest transport, and proposed an Equal Profit Method based on cooperative game concepts.

An et al. [40] integrated network data envelopment analysis (DEA) with cooperative game theory in a three-stage production system, treating each stage as a player and distributing the gains from resource sharing through the Shapley value. Alonso Pedrero et al. [41] addressed the scalability of the Shapley value in large energy communities, proposing a scalable approximation scheme for the fair allocation of investments. Both contributions share an interest in stability and fairness in the allocation; however, their players are autonomous entities (firms, productive stages, or prosumers) rather than strategic objectives connected by causal relationships.

On the theoretical side, Slime et al. [50] extended the classical notions of rationality, Core, and Shapley value to games defined on the Cartesian product of two sets of players. The authors confirm the uniqueness of the Shapley value as a point-valued solution, in contrast to the set-valued nature of the Core, which reinforces its relevance as an allocation criterion in complex cooperation structures.

In machine learning, Lundberg and Lee [51] introduced SHapley Additive exPlanations. Each predictor feature is interpreted as a cooperative player, and its contribution to the prediction is measured through the Shapley value. Rozemberczki et al. [52] systematize this line of work in a recent survey that catalogs applications of the Shapley value to explainability, feature selection, outlier detection, and federated learning. This application illustrates the axiomatic versatility of the concept. The link to the present model is methodological rather than analogical: as in exact SHAP variants, which exploit the structure of the value function to avoid exhaustive enumeration, the allocation rule proposed here obtains exact Shapley values through the additive decomposition of the characteristic function (Remark 1 and Appendix A). A comprehensive view of its theoretical and applied extensions is found in the volume edited by Algaba et al. [53], which documents its use in cost-sharing, voting, networks, and resource allocation.

*Limitations and gaps.* Two patterns emerge. Game-theoretic BSC studies treat entire perspectives as players, which erases objective-level information and produces rankings rather than allocations; their characteristic functions are built from DEA efficiencies or expert scores, not from the causal structure of the strategy map. Conversely, Shapley-based allocation outside the BSC domain achieves axiomatic rigor and monetary output, but its players are autonomous entities (firms, prosumers, production stages) whose cooperation is technological or contractual, not strategic-causal. No existing model combines objective-level players with a characteristic function derived from the strategy map.

#### 1.4. MCDM–BSC Integration

Another line of research has addressed the prioritization of BSC objectives through MCDM methods. Yüksel and Dağdeviren [29] applied ANP with fuzzy logic to weight BSC perspectives and indicators in a manufacturing firm, exploiting ANP's ability to capture interdependencies between criteria. Janeš et al. [30] empirically compared AHP and ANP for prioritizing BSC strategic objectives, concluding that ANP is more compatible with the BSC because it models influences between objectives via network dependencies.

A second generation of works has combined multiple MCDM techniques in hybrid architectures to mitigate the limitations of individual methods. Wu, Tzeng, and Chen [31] developed a banking performance evaluation framework in which fuzzy AHP determines the weights of BSC indicators. SAW, TOPSIS, and VIKOR are then applied comparatively to rank three Taiwanese banks and identify specific gaps relative to aspired performance. Chen, Hsu, and Tzeng [32] proposed a hybrid model for the hotel industry that combines

DEMATEL with ANP. DEMATEL captures the causal relationships between BSC criteria and constructs a network relations map; ANP, fed by the total relation matrix, computes the influence weights. This approach addresses dependence and feedback between criteria, an aspect that conventional AHP ignores.

Tavana, Khalili-Damghani, and Rahmatian [33] extended this hybrid architecture to the pharmaceutical sector through DEMATEL, ANP, and DEA models in a fuzzy environment. Efficiency scores are aggregated using Shannon entropy. The study identifies the most relevant BSC indices for 21 firms listed on the Swiss stock exchange and produces a robust ranking against multiple DEA models. Varmazyar, Dehghanbaghi, and Afkhami [34] advanced in the same direction with a hybrid framework for evaluating research and technology centers, in which DEMATEL reflects the interdependencies between the four BSC perspectives and ANP determines the weights of the indicators. A correlation-based mechanism combines the rankings produced by ARAS, COPRAS, MOORA, and TOPSIS to obtain a consensus ranking.

More recently, Nazari-Shirkouhi et al. [35] developed an integrated approach for higher education institutions based on fuzzy DEMATEL and *Importance-Performance Analysis* on BSC indicators, simultaneously quantifying the relative importance and the performance level of each indicator.

López-Ospina et al. [36] proposed a quantitative method based on fuzzy DEMATEL and an optimization model for the simultaneous selection of strategic objectives and the causal relationships to be included in the strategy map, explicitly addressing the choice of objectives themselves.

*Limitations and gaps.* Hybrid MCDM architectures capture interdependencies between criteria with increasing sophistication, and DEMATEL in particular yields exactly the kind of influence matrix our model requires as input. However, the outputs of this stream are weights and rankings whose properties depend on the aggregation heuristic chosen; they offer no counterpart to efficiency, the null-player axiom, or Core stability, and they stop short of monetary allocation. In sustainability settings this is consequential: a weight vector cannot certify that no subset of objectives—for example, the environmental ones—is collectively underfunded relative to the value it contributes.

### 1.5. Allocation Problems in Sustainability Management

Sustainability management modifies the allocation problem in three ways. First, SBSC implementations multiply the number and heterogeneity of objectives: environmental and social objectives coexist with economic ones inside each perspective or in additional perspectives [17,18,20]. Second, the empirical SBSC literature reports persistent difficulty in connecting sustainability objectives to economic outcomes [11,20], so outcome-based funding rules assign low priority to objectives whose value is mainly enabling (safety culture, emission-reducing capabilities). Third, implementation studies in SMEs show that sustainability scorecards fail when they add measurement burden without changing decision routines [43,54]. Deng et al. [11] evaluate sustainable performance with a hybrid DEMATEL–ANP–VIKOR model over an SBSC extended with environmental and social dimensions, and Mayo-Alvarez et al. [12] embed industrial safety in a dynamic BSC for the mining-metallurgical sector; both quantify sustainability performance, neither allocates budget.

*Limitations and gaps.* The sustainability stream has produced evaluation frameworks but no allocation rule: the step from “how sustainable is our performance” to “how much should each sustainability objective receive, given its causal role” is absent. Because sustainability drivers sit early in long causal chains, this absence systematically biases budgets toward outcome objectives. An axiomatic rule that funds objectives by their

marginal contribution to the whole map, with stability guarantees for every subset of objectives, addresses precisely this bias.

### 1.6. Research Gap

The four conditions stated at the end of the Introduction organize the comparison. Table 1 classifies the reviewed studies along five dimensions: the unit of analysis, the treatment of interdependencies, the game-theoretic foundation, the methodological level of the output, and the sustainability scope.

**Table 1.** Positioning of the reviewed literature. Unit: unit of analysis (P = perspectives, O = objectives, I = indicators, E = autonomous entities). Interdep.: treatment of interdependencies (— = none, Q = qualitative, D = quantified via DEMATEL/ANP or network models). Game found.: game-theoretic foundation (— = none, NC = non-cooperative, CH = cooperative-heuristic, CA = cooperative-axiomatic). Method. level: IR = indicator ranking, WA = weight allocation, BA = budget allocation, EE = effect evaluation, Fr. = conceptual framework. Sust.: sustainability scope (— = absent, ○ = partial, ● = explicit).

| Study                                                       | Unit     | Interdep. | Game Found. | Method. Level | Sust.    |
|-------------------------------------------------------------|----------|-----------|-------------|---------------|----------|
| <i>(a) Resource allocation in BSC settings</i>              |          |           |             |               |          |
| Asosheh et al. [48]                                         | I        | —         | —           | BA            | —        |
| Quezada et al. [44]                                         | O        | D         | —           | WA            | —        |
| Gallo et al. [43]                                           | I        | —         | —           | EE            | —        |
| Malagueño et al. [47]                                       | I        | —         | —           | EE            | —        |
| <i>(b) Game theory in performance management</i>            |          |           |             |               |          |
| Naini et al. [8]                                            | P        | Q         | NC          | EE            | ●        |
| Abedian et al. [9]                                          | P        | Q         | CH          | IR            | —        |
| Abedian et al. [27]                                         | P        | —         | NC          | IR            | —        |
| Abedian et al. [28]                                         | I        | —         | CH          | IR            | —        |
| Seyedi, Yousefi H. [10]                                     | P        | Q         | NC          | EE            | —        |
| Yang, Zhang [37]                                            | E        | —         | CA          | BA            | —        |
| Huang, Chen [42]                                            | E        | D         | CA          | BA            | —        |
| Lozano et al. [38]                                          | E        | —         | CA          | BA            | —        |
| Frisk et al. [39]                                           | E        | —         | CA          | BA            | —        |
| An et al. [40]                                              | E        | —         | CA          | BA            | —        |
| Alonso Pedrero et al. [41]                                  | E        | —         | CA          | BA            | ○        |
| <i>(c) MCDM–BSC integration</i>                             |          |           |             |               |          |
| Yüksel, Dağdeviren [29]                                     | I        | D         | —           | WA            | —        |
| Janeš et al. [30]                                           | O        | D         | —           | WA            | —        |
| Wu et al. [31]                                              | I        | D         | —           | WA            | —        |
| Chen et al. [32]                                            | I        | D         | —           | WA            | —        |
| Tavana et al. [33]                                          | I        | D         | —           | EE            | —        |
| Varmazyar et al. [34]                                       | I        | D         | —           | EE            | —        |
| Nazari-Shirkouhi et al. [35]                                | I        | D         | —           | WA            | —        |
| López-Ospina et al. [36]                                    | O        | D         | —           | WA            | —        |
| <i>(d) Allocation problems in sustainability management</i> |          |           |             |               |          |
| Figge et al. [17]                                           | O        | Q         | —           | Fr.           | ●        |
| Hansen, Schaltegger [18]                                    | O        | Q         | —           | Fr.           | ●        |
| Hristov et al. [20]                                         | O        | Q         | —           | EE            | ●        |
| Deng et al. [11]                                            | I        | D         | —           | EE            | ●        |
| Mayo-Alvarez et al. [12]                                    | O        | Q         | —           | EE            | ○        |
| Falle et al. [54]                                           | O        | Q         | —           | EE            | ●        |
| <b>This study</b>                                           | <b>O</b> | <b>D</b>  | <b>CA</b>   | <b>BA</b>     | <b>●</b> |

Table 1 shows that the four conditions are never satisfied jointly. Reading by columns: cooperative-axiomatic foundations (CA) appear only in studies whose unit of analysis is the autonomous entity (E), never the strategic objective (O); quantified interdependencies (D) appear only in streams without axiomatic foundations; budget allocation (BA) as methodological level never coincides with an explicit sustainability scope (●). This

study occupies the empty intersection: objective-level unit, DEMATEL-quantified interdependencies, cooperative-axiomatic foundation, budget allocation as output, and explicit sustainability scope.

## 2. Materials and Methods

This section presents the model that connects the structure of the BSC with cooperative game theory. The strategy map of the BSC describes how objectives influence one another, but does not specify how to translate those relationships into a concrete resource allocation rule. Cooperative game theory provides this mechanism: it allows one to compute how much each group of objectives “is worth” when working together and, on that basis, determine the share of the budget that corresponds to each objective according to its actual contribution to the system.

### 2.1. Foundations of Cooperative Games

A cooperative game with transferable utility is defined as a pair  $(N, v)$ , where  $N = \{1, \dots, n\}$  is the set of players and  $v : 2^N \rightarrow \mathbb{R}$  is the *characteristic function*, with  $v(\emptyset) = 0$  [22]. Each subset  $S \subseteq N$  is called a *coalition*, and  $v(S)$  represents the value that the players in  $S$  can generate by cooperating.

In the context of strategic management, the players are the objectives of the BSC, the coalitions represent subsets of objectives that work together, and  $v(S)$  measures the strategic value generated by that subset, considering both the individual performance of its members and the synergies arising from their causal relationships.

The game is *superadditive* when the value of the union of two disjoint coalitions is at least the sum of their separate values; in management terms, joining two groups of objectives never destroys value, so cooperating is always at least as good as working separately. This notion is stated formally for the proposed game in Proposition 1.

**Definition 1** (Shapley value). *Let  $(N, v)$  be a cooperative game with  $n$  players. The Shapley value of player  $i \in N$  is*

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} \Delta_i(S), \quad (1)$$

where  $\Delta_i(S) = v(S \cup \{i\}) - v(S)$  is the marginal contribution of  $i$  to coalition  $S$ .

Intuitively, the Shapley value answers the question: *how much, on average, does the inclusion of a strategic objective add across all possible work groups?* To compute it, one considers all possible orders in which objectives could be added one by one. Each time an objective joins, the resulting increase in the value of the group is measured. The Shapley value is the average of those increments across all orders.

The Shapley value is the unique solution concept that simultaneously satisfies four axioms [23]: *efficiency* ( $\sum_{i \in N} \phi_i = v(N)$ , the budget is allocated in full), *symmetry* (objectives with equal contribution receive the same), *null player* (an objective that contributes no value receives no resources), and *additivity*. These properties make it suitable for budget allocation: they guarantee a complete, equitable distribution based on marginal contribution.

### 2.2. Representation of the BSC as a Cooperative Game

**Players.** Let  $N = \{1, \dots, n\}$  be the set of strategic objectives, where each objective  $i$  belongs to a perspective  $\pi(i) \in \{F, C, IP, LG\}$  in the classical BSC [14], augmented with a dedicated sustainability perspective (SUS) in additive SBSC architectures [17,18]. The label  $\pi(i)$  plays no role in the characteristic function—only the causal links collected

in  $A$  do—so the model accommodates any SBSC architecture without modification. In contrast to previous works that group entire perspectives as a single player [9,10,27], or that prioritize objectives through MCDM without axiomatic properties [29,30], here each individual objective is a player. This granularity allows for a specific allocation for each objective and provides formal guarantees of fairness and stability.

**Performance score.** A score  $p_i \in [0, 1]$  is associated with each objective  $i$ :

$$p_i = \max\left(0, \min\left(\frac{r_i}{r_i^*}, 1\right)\right), \quad (2)$$

where  $r_i$  is the actual value of the KPI of objective  $i$  and  $r_i^*$  its target value. If the indicator is “smaller is better”,  $p_i = \max(0, \min(r_i^*/r_i, 1))$  is used. It is assumed that  $r_i^* > 0$ . The lower truncation guarantees  $p_i \geq 0$  and the upper one ensures  $p_i \leq 1$ : an objective that exceeds its target is considered fully achieved. In management terms,  $p_i$  represents the degree of progress of objective  $i$  toward its target, where  $p_i = 1$  indicates full attainment and  $p_i = 0$  indicates no progress.

**Influence matrix.** The causal relationships of the strategy map [16] are represented by a matrix  $A = (a_{ij})$ , where  $a_{ij} \in [0, 1]$  is positive when objective  $j$  has a causal effect on objective  $i$ , and  $a_{ii} = 0$ . The weight  $a_{ij}$  quantifies the intensity with which objective  $j$  enables objective  $i$  within the strategy map. The weights can be estimated using DEMATEL [44,55] or expert judgment. Row normalization is required:

$$\sum_{j=1}^n a_{ij} \leq 1 \quad \forall i \in N. \quad (3)$$

This normalization ensures that the total influence received by each objective is bounded, allowing  $a_{ij}$  to be interpreted as the proportion of the external influence on  $i$  attributable to  $j$ .

### 2.3. Reliability of the Influence Matrix

Three documented limitations should be made explicit. First, the influence values originate in expert judgment, so the quality of  $A$  depends on the size and consistency of the panel; the literature recommends panels of five to ten experts and verification of the internal consistency of the individual matrices ( $\alpha \geq 0.7$ ) before averaging [55–61]. Second, small estimation errors in the entries of  $A$  propagate nonlinearly to the final allocation [62,63]; for this reason, the model incorporates a systematic sensitivity analysis (Section 3.2), following recommended practice [56,58]. Third, DEMATEL produces a static picture of a strategy map that evolves with the planning cycle [12,16,64]; fuzzy extensions of the method [35,36,65] can incorporate the linguistic imprecision of expert judgments and are compatible with the proposed model without altering the structure of the cooperative game.

### 2.4. Characteristic Function Based on the Strategy Map

The central step of the model is the definition of the characteristic function  $v(S)$ , which translates the causal structure of the strategy map into a numerical value for each coalition of objectives.

**Definition 2** (Characteristic function). For every  $S \subseteq N$ ,

$$v(S) = \underbrace{\sum_{i \in S} p_i}_{\text{autonomous value}} + \gamma \underbrace{\sum_{i \in S} \sum_{\substack{j \in S \\ j \neq i}} a_{ij} p_j}_{\text{causal synergy}}, \quad (4)$$

where  $\gamma \in [0, 1]$  is the synergy coefficient.

The characteristic function has two components with a direct interpretation in strategic management. The *autonomous value* is the sum of the performance scores of the objectives in  $S$ : what they contribute on their own. The *causal synergy*, weighted by  $\gamma$ , captures the additional value generated when causally connected objectives cooperate within the same coalition. For example, if “Personnel training” has a causal effect on “Production efficiency”, having both objectives in the same coalition yields a value bonus proportional to the intensity of that causal relation ( $a_{ij}$ ) and to the performance of the influencing objective ( $p_j$ ). This approach extends the work of Yang and Zhang [37] to the domain of strategy maps and differs from Huang and Chen [42] in that the characteristic function is constructed directly from the causal structure of the BSC, rather than deriving coalition values from external AHP/ANP weightings.

The parameter  $\gamma$  controls the balance between individual performance and synergies. With  $\gamma = 0$ , the strategy map has no effect and the allocation is purely proportional to performance. As  $\gamma$  approaches 1, causal synergies gain greater importance in the distribution of the budget. Under row normalization (3), the synergy received by any objective remains bounded, ensuring that the strategy map enhances objectives without distorting the structure of the game.

A direct calculation from (4) gives the marginal contribution of objective  $i$  to any coalition  $S \subseteq N \setminus \{i\}$ :

$$\Delta_i(S) = p_i + \gamma \sum_{k \in S} (a_{ik} p_k + a_{ki} p_i), \quad (5)$$

an expression used repeatedly in what follows.

**Remark 1** (Closed form of the Shapley value). *The characteristic function (4) is a sum of single-player terms  $p_i$  and pairwise terms  $\gamma a_{ij} p_j$  that activate exactly when both  $i$  and  $j$  belong to the coalition. Each pairwise term is proportional to a two-player unanimity game, whose Shapley value splits the term equally between  $i$  and  $j$ ; additivity of the Shapley value then yields, for every  $i \in N$ ,*

$$\phi_i(v) = p_i + \frac{\gamma}{2} \sum_{j \neq i} (a_{ij} p_j + a_{ji} p_i). \quad (6)$$

Equation (6) shows that the rule rewards the product of connectivity and performance symmetrically on both endpoints of every causal link, and reduces the computation of the allocation from  $O(2^n)$  coalition evaluations to  $O(n^2)$  arithmetic operations (Appendix A). In all experiments of Section 3.2, (6) coincides with the brute-force computation of Definition 1 to machine precision.

## 2.5. Game Properties: Superadditivity and Convexity

The game defined by (4) satisfies two structural properties that underpin the fairness and stability of the allocation; we state them below in management terms and defer all proofs to Appendix A. *Superadditivity* means that merging two disjoint groups of objectives never destroys value: joining coalitions can only activate additional causal synergies, never cancel existing ones. *Convexity* means that an objective contributes more the larger the coalition it joins, because larger coalitions contain more of its causal partners. Convexity is the stronger property and has the following managerial consequence: by Shapley’s theorem on convex games [66], the Shapley value lies in the Core, so no subset of objectives—in particular, no subset of environmental or social objectives—receives less funding than the value it generates by cooperating.

**Proposition 1** (Superadditivity). *The game  $(N, v)$  defined by (4) with  $\gamma \geq 0$  is superadditive. For every pair of disjoint coalitions  $S, T \subseteq N$ ,*

$$v(S \cup T) \geq v(S) + v(T), \quad (7)$$

*with equality if and only if there is no causal relationship between  $S$  and  $T$ , or  $\gamma = 0$ .*

**Proposition 2** (Convexity). *The game  $(N, v)$  defined by (4) with  $\gamma \geq 0$  and  $A \geq 0$  is convex. For every  $i \in N$  and every pair  $S \subseteq T \subseteq N \setminus \{i\}$ ,*

$$\Delta_i(T) \geq \Delta_i(S). \quad (8)$$

**Corollary 1** (Membership in the Core). *For convex games, the Shapley value always belongs to the Core [66]. Therefore,*

$$\sum_{i \in S} \phi_i(v) \geq v(S) \quad \forall S \subseteq N. \quad (9)$$

## 2.6. Budget Allocation Rule

Once the Shapley values have been computed, the budget allocation is obtained directly.

**Definition 3** (Shapley-Based Allocation Rule). *Let  $R > 0$  be the total budget. The amount allocated to objective  $i$  is*

$$R_i = \omega_i \cdot R, \quad \omega_i = \frac{\phi_i(v)}{\sum_{j \in N} \phi_j(v)}, \quad (10)$$

*with  $\sum_{i \in N} \omega_i = 1$ .*

The normalization ensures that the entire budget is fully allocated. Each objective receives a fraction of the budget proportional to its Shapley value, which already incorporates both its individual performance and its synergistic contribution to the system.

## 2.7. Properties of the Allocation Rule

The proposed rule satisfies five formal properties that distinguish it from subjective weighting methods. The properties are stated below and follow either from the Shapley axioms [23] or from the structure of the characteristic function (4).

**Property 1** (Budget Exhaustiveness).  $\sum_{i \in N} R_i = R$ . *Consequence of the efficiency axiom [23].*

**Property 2** (Fairness Based on Marginal Contribution). *If  $i, j \in N$  have identical marginal contributions to every coalition, then  $R_i = R_j$ . Consequence of the symmetry axiom [23,67].*

**Property 3** (Sensitivity to the Strategy Map). *For  $\gamma > 0$ , let  $i, j \in N$  with  $p_i = p_j$ . If  $i$  has causal connections in  $A$  and  $j$  is isolated, then  $\phi_i(v) > \phi_j(v)$ . Follows directly from (5), which strictly exceeds  $\Delta_j(S) = p_j$  for every coalition containing some  $k$  causally connected to  $i$ .*

**Property 4** (Monotonicity of the Shapley Value). *If  $p_i$  increases by  $\varepsilon > 0$  while everything else remains unchanged, then  $\phi_i(v)$  increases strictly, since for every  $S \subseteq N \setminus \{i\}$ ,  $\Delta'_i(S) - \Delta_i(S) = \varepsilon(1 + \gamma \sum_{k \in S} a_{ki}) > 0$ .*

**Property 5** (Convergence to the Base Case). *For  $\gamma = 0$ ,  $v(S) = \sum_{i \in S} p_i$  and  $\phi_i = p_i$ . The allocation is proportional to individual performance, with no effect from the strategy map.*

The monotonicity of  $\phi_i$  does not directly imply the monotonicity of  $R_i = \omega_i R$ , since increasing  $p_i$  may also raise  $\phi_j(v)$  for every  $j \neq i$  with  $a_{ji} > 0$ . A sufficient condition for  $R_i$  to increase strictly is that  $i$  be a sink node of the strategy map ( $a_{ki} = 0$  for every  $k \neq i$ ).

Under Property 3, an objective with  $p_i = 0$  may receive a positive budget when it receives causal influence from objectives with positive performance. The following proposition characterizes the conditions under which an objective is effectively null in  $(N, v)$ , reconciling the model with the classical null player axiom.

**Proposition 3** (Null Player Characterization). *Objective  $i \in N$  is a null player of  $(N, v)$ , and therefore  $\phi_i(v) = 0$ , if and only if (i)  $p_i = 0$  and (ii)  $\sum_{j \in N \setminus \{i\}} a_{ij} p_j = 0$ .*

The five properties hold under the assumptions of the model and together provide a complete, fair, and strategically informed budget allocation rule.

## 2.8. Conditions for Applicability

The allocation rule assumes that funds withdrawn from one objective can be reallocated to any other. Four conditions must hold; together they define the settings in which the model applies directly.

**(C1) Budget fungibility.** The budget  $R$  is a single strategic fund without statutory earmarks. Transferring resources from objective C1 (customer satisfaction) to LG2 (information systems) is feasible when both draw on one pool of strategic initiatives—as in rolling or zero-based budgeting—and infeasible when systems investments are locked into multi-year capital plans or when sustainability funds are ring-fenced for regulatory compliance [25,26].

**(C2) Cross-functional governance.** A single decision body holds allocation authority across all perspectives. If marketing and IT budgets are approved by separate hierarchies, the C1→LG2 transfer requires a negotiation the model does not represent. Organizations with integrated planning routines and a strategy office satisfy this condition; federated structures satisfy it only within divisions [6].

**(C3) Commensurability through normalized scores.** Heterogeneous objectives enter the game only through their normalized progress  $p_i \in [0, 1]$  and their causal weights. This deliberate abstraction makes a training objective and an emissions objective comparable, at the cost of compressing qualitative differences; it is acceptable when KPIs and targets are set through a disciplined process, and unreliable when targets are negotiable (Section 6).

**(C4) Within-cycle stationarity.** The matrix  $A$  and the scores  $p_i$  remain valid for the planning cycle over which  $R$  is allocated. If the map is redesigned within the cycle, the framework is re-executed (Figure 1).

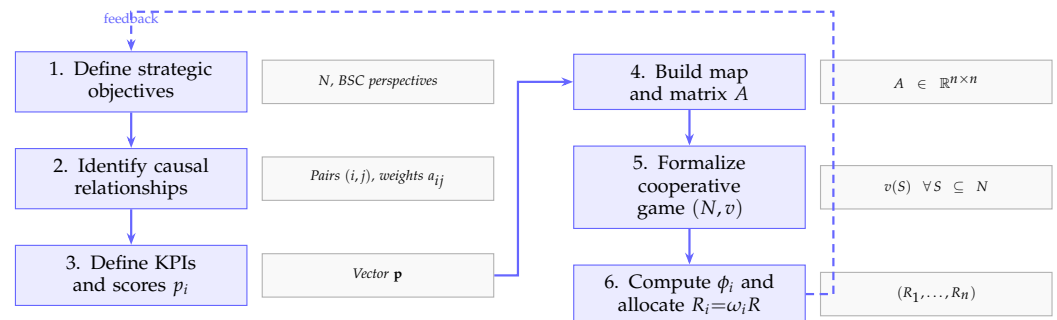
These conditions are typically met by single-business firms and SMEs that fund strategy from one pool under a management committee—the population studied by Gallo et al. [43]—and by divisions of larger firms within their delegated budgets. They are typically violated by regulated utilities with earmarked programs, grant-funded organizations, and groups whose ESG funds are contractually ring-fenced; for those settings, Section 2.11 outlines non-transferable extensions.

## 2.9. Theoretical Analysis of the Synergy Coefficient $\gamma$

The coefficient  $\gamma \in [0, 1]$  is a structural parameter that controls the degree of superadditivity of the game. Its calibration is not purely empirical: it admits a theoretical characterization that informs its a priori choice.

**Proposition 4** (Limiting Cases of  $\gamma$ ). *Let  $(N, v_\gamma)$  be the family of games defined by (4) with  $\gamma \in [0, 1]$ . Then: (i) for  $\gamma = 0$ , the causal synergy is null and  $\phi_i(v_0) = p_i$  (Property 5); (ii) for*

$\gamma = 1$ , the synergy is fully transferred to  $v(S)$ , so that  $v_1(N) = \sum_i p_i + \sum_i \sum_{j \neq i} a_{ij} p_j$ ; (iii)  $\phi_i(v_\gamma)$  is linear in  $\gamma$ .



**Figure 1.** Integrative methodological framework. The flow articulates the six steps described in Section 2.12: (1) definition of strategic objectives across the four BSC perspectives; (2) identification of causal relationships through DEMATEL; (3) definition of KPIs and scores  $p_i$ ; (4) construction of the strategy map and the influence matrix  $A$ ; (5) formalization of the cooperative game  $(N, v)$  with the synergy coefficient  $\gamma$ ; and (6) computation of the Shapley value  $\phi_i$  and allocation  $R_i = \omega_i R$ . Solid arrows indicate the sequential flow between steps; the dashed arrow represents the feedback loop that allows the procedure to be re-executed when the strategy map changes.

#### 2.10. Organizational Decision-Making Process for the Synergy Coefficient

Calibrating  $\gamma$  is part of using the model, not a preliminary statistical exercise. The coefficient expresses how much confidence the organization places in the causal hypotheses of its strategy map—and, therefore, how much budget it is willing to shift from outcome indicators toward performance drivers [15,16,25,44,45]. In sustainability-oriented management, a low  $\gamma$  favors control of observable individual performance, whereas a high  $\gamma$  channels resources toward enabling capabilities with high causal centrality, consistent with a long-term investment logic [3,5,7,68]. Because this confidence changes as the map matures, we recommend that a standing strategy committee—typically the CFO or planning office, the owners of each perspective, and the sustainability officer where an SBSC is in use—set  $\gamma$  at the start of every planning cycle through a four-step protocol.

**Step G1 (maturity audit).** Review the age of the strategy map, the number of planning cycles it has survived, and the internal consistency of the DEMATEL panel that produced  $A$  (Cronbach's  $\alpha$ ; Section 2.3).

**Step G2 (band selection).** Select the calibration band using the criteria (a)–(d) below, which map audit outcomes to intervals of  $\gamma$ .

**Step G3 (robustness inspection).** Run the  $\gamma$ -sweep and the perturbation analysis of Section 3.2 and inspect the robustness interval around the candidate value: if small variations of  $\gamma$  would reorder the top-funded objectives, the committee should either gather more evidence on the contested causal links or choose a  $\gamma$  inside the invariant interval.

**Step G4 (documented approval).** Record the chosen  $\gamma$ , the supporting evidence, and the dissenting views. This documentation makes the allocation auditable before boards and external stakeholders. The committee should recalibrate extraordinarily whenever the map is redesigned, a merger or divestiture changes the objective set, or a new sustainability objective enters the map with an immature KPI baseline.

The following bands summarize the accumulated evidence and serve as the committee's decision table in Step G2. The choice should reflect the maturity of the map and the reliability of matrix  $A$  [46,56,58]: (a)  $\gamma \in [0.00, 0.25]$  for preliminary maps or weakly consensual estimates, typical of recent BSC implementations, where amplifying causal specification errors should be avoided [45,46,62]; (b)  $\gamma \in [0.25, 0.50]$  for maps consolidated over two or more planning cycles with partial operational feedback [12,32,44,45];

(c)  $\gamma \in [0.50, 0.75]$  for empirically validated maps with highly consistent expert panels ( $\alpha \geq 0.8$ ), where the allocation can lean on documented synergies [10,11,35,36,59,61]; and (d)  $\gamma = 1$  only as a theoretical benchmark, since managerial practice cannot assume that the map fully accounts for value creation [45,46]. The value  $\gamma = 0.5$  used in the numerical illustration corresponds to an operational map of intermediate maturity, representative of firms with a fully deployed BSC but without longitudinally audited causal metrics [12,44].

### 2.11. The TU Assumption and Non-Transferable Extensions

The proposed model assumes transferable utility: the budget  $R$  can be freely redistributed across objectives and perspectives [22,69]. This assumption is standard in cooperative game theory applied to cost and resource allocation [23,38–40] and allows direct application of the Shapley value as an axiomatically grounded distribution rule [23,67,70]. In organizational practice, however, budgets are often compartmentalized into heterogeneous categories (capex versus opex, training versus maintenance) subject to differentiated controls, which limits the liquidity with which resources can be transferred between them [6,7,25,26]. As long as such constraints are loose relative to the Shapley optimum,  $\phi_i$  continues to provide an interpretable signal of strategic value [23,41,53]; when they are strict, the TU rule loses direct implementability and must be adapted using NTU solution concepts, such as the Harsanyi value [71] or the NTU Shapley value [69,72], or cooperative games with restricted communication structure that model limited transfers between budget compartments [50,53,73]. The NTU extension of the model is discussed as future work in Section 6.

### 2.12. Methodological Framework

The methodology articulates, in six sequential steps, the construction of the BSC, the quantification of the causal relationships, and the computation of the Shapley value that yields the budget allocation  $R_i$ . Each step produces a verifiable intermediate output, which facilitates managerial auditing and replicability [16,44], and a feedback loop allows the procedure to be re-executed whenever the strategy map changes (Figure 1).

**Step 1: Strategic objectives.** The set  $N = \{1, \dots, n\}$  is established, distributed across the four BSC perspectives [14]—augmented, in SBSC implementations, with a dedicated sustainability perspective or with environmental and social objectives embedded in the classical perspectives [18]—with at least two objectives per perspective.

**Step 2: Causal relationships.** Causal influence is quantified using DEMATEL [44]: experts assign  $a_{ij} \in \{0, 1, 2, 3, 4\}$ , the values are averaged and normalized to  $[0, 1]$ , and each row is normalized so as to satisfy condition (3).

**Step 3: KPIs and scores.** For each objective  $i$ , a KPI  $r_i$ , its target  $r_i^*$ , and  $p_i$  are defined according to Equation (2).

**Step 4: Strategy map and matrix  $A$ .** The objectives are organized into the four perspectives (LG, IP, C, F). The matrix  $A = (a_{ij})$  takes the values from Step 2.

**Step 5: Cooperative game.** A value  $\gamma \in [0, 1]$  is selected and  $v(S)$  is evaluated for every  $S \subseteq N$ .

**Step 6: Shapley value and allocation.** The value  $\phi_i(v)$  is computed,  $\omega_i$  is normalized, and  $R_i = \omega_i \cdot R$  is assigned (Definition 3, Equation (10)).

The allocation itself follows from the closed form of Remark 1 in  $O(n^2)$  operations; the exhaustive evaluation of the  $2^n$  coalitions is used to verify the Core condition numerically and remains tractable up to  $n \approx 20$  on a personal computer.

### 3. Results

This section reports the results of applying the framework. Section 3.1 presents an illustrative case: a sustainability balanced scorecard with  $n = 10$  objectives organized in five perspectives—including a dedicated sustainability perspective that hosts a carbon-emission and an employee-safety objective—and a budget of US\$1,000,000. Section 3.2 reports computational experiments with 8, 15, and 20 objectives, the sweep over  $\gamma$  with its ranking-invariance analysis, and the sensitivity analysis of matrix  $A$ .

#### 3.1. Illustrative Case: An SBSC with a Dedicated Sustainability Perspective

The framework is applied to an electronics components manufacturer with an annual strategic budget  $R = \text{US\$ } 1,000,000$ . The firm operates an *additive* SBSC architecture [17,18]: it extends the four classical perspectives with a fifth, dedicated sustainability perspective (SUS) that hosts an environmental objective (EN1, carbon emissions) and a social objective (SO1, employee safety). The firm chose this architecture to give both objectives board-level visibility and dedicated stakeholder reporting; its context—industrial B2B customers increasingly require low-carbon suppliers, and safety incidents directly interrupt production—ties the new perspective causally to operations, customers, and finance. The case demonstrates that the model values non-financial objectives under the same axioms as financial ones. Because the perspective label plays no role in the characteristic function, the allocation depends only on the causal map: the same objectives with the same links would receive identical budgets under an integrated architecture.

##### 3.1.1. Step 1: Strategic Objectives

The board defined  $n = 10$  objectives (Table 2).

**Table 2.** Strategic objectives, perspective, and KPI target. EN1 and SO1 are members of the dedicated sustainability perspective (SUS).

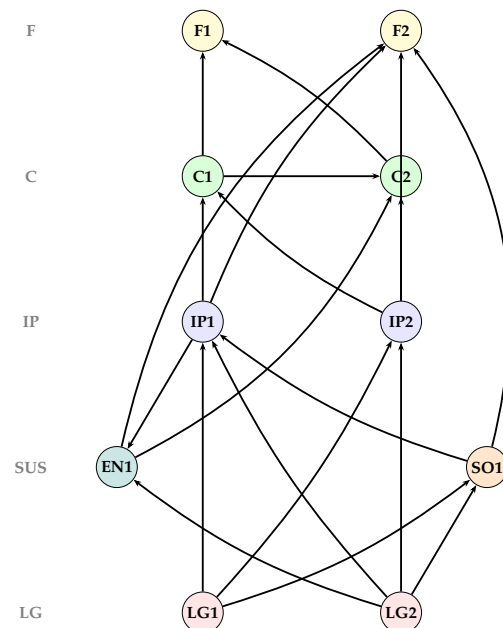
| Id  | Persp. | Objective                      | KPI/Target                               |
|-----|--------|--------------------------------|------------------------------------------|
| F1  | F      | Maximize profitability         | ROE $\geq 15\%$                          |
| F2  | F      | Reduce operating costs         | Cost $\leq \text{US\$}2.8\text{M}$       |
| C1  | C      | Improve customer satisfaction  | NPS $\geq 75$                            |
| C2  | C      | Increase customer retention    | Rate $\geq 90\%$                         |
| IP1 | IP     | Optimize production efficiency | OEE $\geq 85\%$                          |
| IP2 | IP     | Reduce delivery time           | Lead time $\leq 5 \text{ d}$             |
| EN1 | SUS    | Reduce carbon emissions        | $\leq 32.0 \text{ t CO}_2\text{e/US\$M}$ |
| SO1 | SUS    | Improve employee safety        | LTIFR $\leq 1.75$                        |
| LG1 | LG     | Train personnel                | $\geq 40 \text{ h/year}$                 |
| LG2 | LG     | Improve information systems    | Avail. $\geq 99\%$                       |

Table 2 lists the ten objectives. EN1 measures Scope 1+2 emissions intensity per million dollars of output; SO1 measures the lost-time injury frequency rate (LTIFR) per million hours worked. Both are “smaller is better” indicators and enter the model through Equation (2) exactly as the financial and customer KPIs do: the model requires no special treatment for non-financial objectives.

##### 3.1.2. Step 2: Causal Relationships and Matrix $A$

A DEMATEL session [44,55] with five executives (operations, finance, commercial, HSE, and IT) quantified twenty causal relationships, including the sustainability-specific channels: emission reductions cut energy costs (EN1→F2) and protect retention among industrial customers with low-carbon procurement policies (EN1→C2); safety improvements reduce accident-driven downtime (SO1→IP1) and insurance and disruption costs (SO1→F2); training builds the safety culture (LG1→SO1), while monitoring systems enable

both incident reporting ( $LG2 \rightarrow SO1$ ) and emissions measurement ( $LG2 \rightarrow EN1$ ); finally, process efficiency lowers energy waste per unit ( $IP1 \rightarrow EN1$ ). The raw weights were row-normalized to satisfy condition (3). Figure 2 shows the strategy map; Table 3 lists the weights by receiving objective, and Table 4 reports the normalized matrix  $A$ .



**Figure 2.** Strategy map of the SBSC example. The dedicated sustainability perspective (SUS) hosts EN1 (carbon emissions) and SO1 (employee safety); the weights  $a_{ij}$  are listed in Table 3 and collected in the matrix of Table 4.

Figure 2 shows the causal structure. The sustainability objectives are neither pure drivers nor pure outcomes: EN1 receives influence from LG2 and IP1 and sends influence to F2 and C2; SO1 receives from LG1 and LG2 and sends to IP1 and F2. This intermediate causal position—typical of environmental and social objectives in SBSC architectures, wherever they are displayed [18,20]—cannot be represented by perspective-level models and is quantified by the characteristic function (4).

**Table 3.** Weights  $a_{ij}$  of the strategy map, grouped by receiving objective.

| Receiver | Sender → Weight                                                                          | Channel                       |
|----------|------------------------------------------------------------------------------------------|-------------------------------|
| F1       | $C1 \rightarrow 0.57; C2 \rightarrow 0.43$                                               | $F \leftarrow C$              |
| F2       | $IP1 \rightarrow 0.30; IP2 \rightarrow 0.20; EN1 \rightarrow 0.30; SO1 \rightarrow 0.20$ | $F \leftarrow \{IP, SUS\}$    |
| C1       | $IP1 \rightarrow 0.46; IP2 \rightarrow 0.54$                                             | $C \leftarrow IP$             |
| C2       | $C1 \rightarrow 0.40; IP2 \rightarrow 0.30; EN1 \rightarrow 0.30$                        | $C \leftarrow \{C, IP, SUS\}$ |
| IP1      | $LG1 \rightarrow 0.35; LG2 \rightarrow 0.35; SO1 \rightarrow 0.30$                       | $IP \leftarrow \{LG, SUS\}$   |
| IP2      | $LG1 \rightarrow 0.45; LG2 \rightarrow 0.55$                                             | $IP \leftarrow LG$            |
| EN1      | $LG2 \rightarrow 0.40; IP1 \rightarrow 0.35$                                             | $SUS \leftarrow \{LG, IP\}$   |
| SO1      | $LG1 \rightarrow 0.60; LG2 \rightarrow 0.20$                                             | $SUS \leftarrow LG$           |

**Table 4.** Normalized influence matrix  $A$ .  $a_{ij} > 0$ :  $j$  (column) influences  $i$  (row).

|     | F1 | F2 | C1   | C2   | IP1  | IP2  | EN1  | SO1  | LG1  | LG2  |
|-----|----|----|------|------|------|------|------|------|------|------|
| F1  | 0  | 0  | 0.57 | 0.43 | 0    | 0    | 0    | 0    | 0    | 0    |
| F2  | 0  | 0  | 0    | 0    | 0.30 | 0.20 | 0.30 | 0.20 | 0    | 0    |
| C1  | 0  | 0  | 0    | 0    | 0.46 | 0.54 | 0    | 0    | 0    | 0    |
| C2  | 0  | 0  | 0.40 | 0    | 0    | 0.30 | 0.30 | 0    | 0    | 0    |
| IP1 | 0  | 0  | 0    | 0    | 0    | 0    | 0    | 0.30 | 0.35 | 0.35 |
| IP2 | 0  | 0  | 0    | 0    | 0    | 0    | 0    | 0    | 0.45 | 0.55 |
| EN1 | 0  | 0  | 0    | 0    | 0.35 | 0    | 0    | 0    | 0    | 0.40 |
| SO1 | 0  | 0  | 0    | 0    | 0    | 0    | 0    | 0    | 0.60 | 0.20 |
| LG1 | 0  | 0  | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| LG2 | 0  | 0  | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |

### 3.1.3. Step 3: KPIs and Scores $p_i$

Scores follow Equation (2); for “smaller is better” indicators (F2, IP2, EN1, SO1), the ratio is inverted (Table 5).

**Table 5.** Actual values, targets, and scores  $p_i$ .

| Id  | KPI                                    | $r_i$ | $r_i^*$ | $p_i$ |
|-----|----------------------------------------|-------|---------|-------|
| F1  | ROE (%)                                | 12.75 | 15.00   | 0.850 |
| F2  | Cost (M US\$) <sup>1</sup>             | 3.89  | 2.80    | 0.720 |
| C1  | NPS                                    | 67.50 | 75.00   | 0.900 |
| C2  | Retention (%)                          | 70.20 | 90.00   | 0.780 |
| IP1 | OEE (%)                                | 57.80 | 85.00   | 0.680 |
| IP2 | Lead time (d) <sup>1</sup>             | 6.67  | 5.00    | 0.750 |
| EN1 | t CO <sub>2</sub> e/US\$M <sup>1</sup> | 50.00 | 32.00   | 0.640 |
| SO1 | LTIFR <sup>1</sup>                     | 2.50  | 1.75    | 0.700 |
| LG1 | Training (h/year)                      | 32.00 | 40.00   | 0.800 |
| LG2 | Availability (%)                       | 64.35 | 99.00   | 0.650 |

<sup>1</sup> “Smaller is better”:  $p_i = \max(0, \min(r_i^*/r_i, 1))$ .

Table 5 reports the scores. The two sustainability objectives carry the lowest progress values of the map ( $p_{EN1} = 0.640$ ,  $p_{SO1} = 0.700$ ), a realistic pattern: environmental and social indicators are typically the most recent additions to the measurement system and the farthest from target [20,43]. Under a purely proportional rule, this immaturity would translate directly into the lowest funding priorities; Section 3.1.6 shows how the causal structure modifies this outcome.

### 3.1.4. Step 4: Characteristic Function

The strategy committee sets  $\gamma = 0.5$  following the protocol of Section 2.10 (operational map of intermediate maturity). The characteristic function (Definition 2) was evaluated for the  $2^{10} = 1024$  coalitions. Table 6 reports representative values.

**Table 6.** Values  $v(S)$  for selected coalitions ( $\gamma = 0.5$ ).

| Coalition $S$           | $v(S)$  |
|-------------------------|---------|
| {LG1}                   | 0.8000  |
| {SO1}                   | 0.7000  |
| {LG1, SO1} <sup>1</sup> | 1.7400  |
| {EN1, LG2}              | 1.4200  |
| {IP1, SO1}              | 1.4850  |
| {C1, C2}                | 1.8600  |
| {F1, F2}                | 1.5700  |
| {LG1, LG2, SO1, EN1}    | 3.2250  |
| $N$ (grand coalition)   | 10.2561 |

<sup>1</sup> Example:  $v(\{LG1, SO1\}) = (0.80 + 0.70) + 0.5 \times 0.60 \times 0.80 = 1.740$ : training and safety generate joint value beyond the sum of their scores.

Table 6 illustrates how the strategy map generates coalition value. The pair {LG1, SO1} is worth 0.240 more than the sum of its members' scores because training causally enables safety ( $a_{SO1, LG1} = 0.60$ ); the pair {F1, F2} generates no synergy because no causal link connects the two financial objectives. The grand coalition is worth  $v(N) = 10.2561$ , of which 7.510 is autonomous performance and 2.746 is causal synergy activated at  $\gamma = 0.5$ .

### 3.1.5. Steps 5–6: Shapley Value and Allocation

Applying Remark 1—verified against the brute-force computation of Definition 1 over all 1,024 coalitions—yields Table 7. Efficiency holds exactly:  $\sum_i \phi_i = 10.2561 = v(N)$ . The Core condition (Corollary 1) was verified computationally over the 1,022 non-trivial coalitions with zero violations (minimum slack 0.1715), confirming the coalitional stability of the allocation.

**Table 7.** Shapley value, weight  $\omega_i$ , and budget  $R_i$  ( $R = \text{US\$1,000,000}$ ,  $\gamma = 0.5$ ).

| Id           | Persp. | $\phi_i$       | $\omega_i$    | $R_i$ (US\$)     |
|--------------|--------|----------------|---------------|------------------|
| C1           | C      | 1.2977         | 0.1265        | 126,530          |
| IP2          | IP     | 1.1244         | 0.1096        | 109,630          |
| LG1          | LG     | 1.0800         | 0.1053        | 105,303          |
| F1           | F      | 1.0621         | 0.1036        | 103,558          |
| C2           | C      | 1.0581         | 0.1032        | 103,168          |
| IP1          | IP     | 1.0481         | 0.1022        | 102,190          |
| SO1          | SUS    | 0.9400         | 0.0917        | 91,653           |
| LG2          | LG     | 0.8938         | 0.0871        | 87,143           |
| F2           | F      | 0.8915         | 0.0869        | 86,924           |
| EN1          | SUS    | 0.8605         | 0.0839        | 83,901           |
| <b>Total</b> |        | <b>10.2561</b> | <b>1.0000</b> | <b>1,000,000</b> |

Table 7 reports the allocation. Comparison with the proportional benchmark ( $\gamma = 0$ ; see Section 3.2.2) quantifies the effect of the causal structure: the rule moves US\$26,564 (2.7% of the budget) across objectives, funding the process drivers IP1 (+12.3%) and IP2 (+9.2%) and the high-connectivity customer objective C1 (+5.0%) at the expense of the financial sinks F1 (−9.0%) and F2 (−9.8%). The sustainability objectives hold their shares almost unchanged (EN1 −2.1%, SO1 −2.2%): the model neither subsidizes nor penalizes them as a class; the allocation reflects their causal role (Section 3.1.6).

### 3.1.6. Interpretation of the Allocation

Four results distinguish the allocation from a proportional rule.

**C1 receives the largest allocation; the financial objectives lose share.** C1 combines the highest score ( $p = 0.90$ ) with strong outgoing links to F1 and C2, so it earns the largest

allocation (US\$126,530). Both financial objectives, in contrast, are pure sinks: they receive influence but send none, and the rule accordingly shifts part of their proportional share toward the objectives that produce their results (Property 3). The rule shifts funding from outcome objectives to their drivers.

**The safety objective outranks cost reduction.** SO1 receives US\$91,653 versus US\$86,924 for F2, although their scores are similar (0.700 vs. 0.720). The difference is positional: SO1 sends influence to IP1 and F2, so every coalition containing production or cost objectives values its presence. For any  $\gamma \geq 0.146$  the model funds employee safety above financial cost reduction—an ordering no performance-only rule produces with these scores.

**EN1 approximately retains its proportional share.** EN1 carries the lowest score of the map ( $p = 0.640$ ) and closes the ranking (US\$83,901), yet its share falls only 2.1% below its proportional benchmark, because its links to F2 and C2 compensate for its immature KPI. Moreover, had EN1 entered the map with no measured progress at all ( $p_{\text{EN1}} = 0$ ), the rule would still assign it US\$13,211 (1.3% of the budget) by virtue of the synergies it receives (Proposition 3): newly added sustainability objectives obtain seed funding from their causal position while their data baseline is being built.

**LG1's allocation reflects its role as a safety enabler.** LG1 ranks third (US\$105,303). Its synergy volume decomposes by outgoing link: the safety channel LG1→SO1 contributes 0.480 of its total 1.120—that is, 42.9% of LG1's synergy-based funding is earned by enabling employee safety, a channel not identifiable under a proportional rule or a perspective-level game.

### 3.2. Computational Experiments and Sensitivity Analysis

We examine three empirical questions: (i) the scalability of the computation; (ii) the sensitivity of the allocation  $R_i$  to the coefficient  $\gamma$ , including the ranking-invariance intervals; and (iii) the sensitivity to perturbations of matrix  $A$ . The experiments were run in Python 3.11 on a 2.6 GHz Intel i7 processor (Intel Corporation, Santa Clara, CA, USA) with 16 GB of RAM.

#### 3.2.1. Scalability

The allocation itself is computed through the closed form of Remark 1 in  $O(n^2)$  operations for any practical map size. Exponential cost arises only in the numerical verification of the axioms and of the Core condition, which traverses the  $2^n$  coalitions. Table 8 reports the time required for this brute-force verification for games with 8, 15, and 20 objectives, using sparse random  $A$  matrices (density 20%) and a uniform  $\mathbf{p}$  vector.

**Table 8.** Time for the brute-force evaluation of all  $2^n$  coalitions (verification of the Core condition) as a function of game size  $n$ .

| $n$ | Method     | Time (s) |
|-----|------------|----------|
| 8   | Exhaustive | 0.04     |
| 15  | Exhaustive | 1.80     |
| 20  | Exhaustive | 63.5     |

Exhaustive verification is tractable up to  $n \approx 20$  on a personal computer, a range that covers most real-world strategy maps used in management practice; beyond it, convexity (Proposition 2) still guarantees Core membership analytically.

#### 3.2.2. Sensitivity to the Synergy Coefficient

Table 9 shows the allocation  $R_i$  from the illustrative case for five values of  $\gamma$ .

**Table 9.** Allocation  $R_i$  (thousands of US\$) for five values of  $\gamma$  ( $n = 10$ ,  $R = \text{US\$1,000,000}$ ).

| Id  | $\gamma = 0$ | $\gamma = 0.25$ | $\gamma = 0.5$ | $\gamma = 0.75$ | $\gamma = 1$ |
|-----|--------------|-----------------|----------------|-----------------|--------------|
| F1  | 113.8        | 107.9           | 103.6          | 100.3           | 97.7         |
| F2  | 96.4         | 90.9            | 86.9           | 83.9            | 81.5         |
| C1  | 120.5        | 124.0           | 126.5          | 128.5           | 130.0        |
| C2  | 104.4        | 103.7           | 103.2          | 102.8           | 102.5        |
| IP1 | 91.0         | 97.5            | 102.2          | 105.8           | 108.6        |
| IP2 | 100.4        | 105.7           | 109.6          | 112.6           | 114.9        |
| EN1 | 85.7         | 84.6            | 83.9           | 83.3            | 82.9         |
| SO1 | 93.7         | 92.5            | 91.7           | 91.0            | 90.5         |
| LG1 | 107.1        | 106.1           | 105.3          | 104.7           | 104.3        |
| LG2 | 87.0         | 87.1            | 87.1           | 87.2            | 87.2         |

Table 9 shows the allocation across the full range of  $\gamma$ ; Figure 3 plots seven representative trajectories. Because  $\phi_i$  is linear in  $\gamma$  (Proposition 4), the funding *ranking* can change only at exactly computable crossing points, and the choice of  $\gamma$  therefore determines a *robustness interval*: the largest interval around the operating point within which the allocation ordering is invariant. Thirteen crossings occur in  $[0, 1]$ . At the operating point  $\gamma = 0.5$ , the complete ranking is invariant on  $(0.484, 0.530)$ —the nearest reorderings are LG2 overtaking F2 at  $\gamma^* = 0.484$  and C2 overtaking F1 at  $\gamma^* = 0.530$ —while the composition and internal order of the top three funded objectives  $\{C1, IP2, LG1\}$  is invariant on the much wider interval  $(0.368, 0.681)$ . Choices of  $\gamma$  below 0.368 would return F1 to the top three (at these values the synergy term does not displace the financial outcome objective), whereas choices above 0.681 would promote IP1 above LG1. Two crossings involve the sustainability objectives: SO1 overtakes F2 at  $\gamma^* = 0.146$ , so employee safety outranks cost reduction under any non-trivial synergy weight, and EN1 overtakes F2 only at  $\gamma^* = 0.816$ , indicating that the ranking of EN1 depends primarily on KPI maturity rather than on the synergy weight. Under Step G3 of Section 2.10, the committee can select any  $\gamma$  inside  $(0.368, 0.681)$ ; within this interval, the identity and order of the top-funded objectives do not depend on the value chosen. Finally, the combined share of the two sustainability objectives varies only from 17.9% ( $\gamma = 0$ ) to 17.3% ( $\gamma = 1$ ): the funding of EN1 and SO1 is largely insensitive to  $\gamma$ .

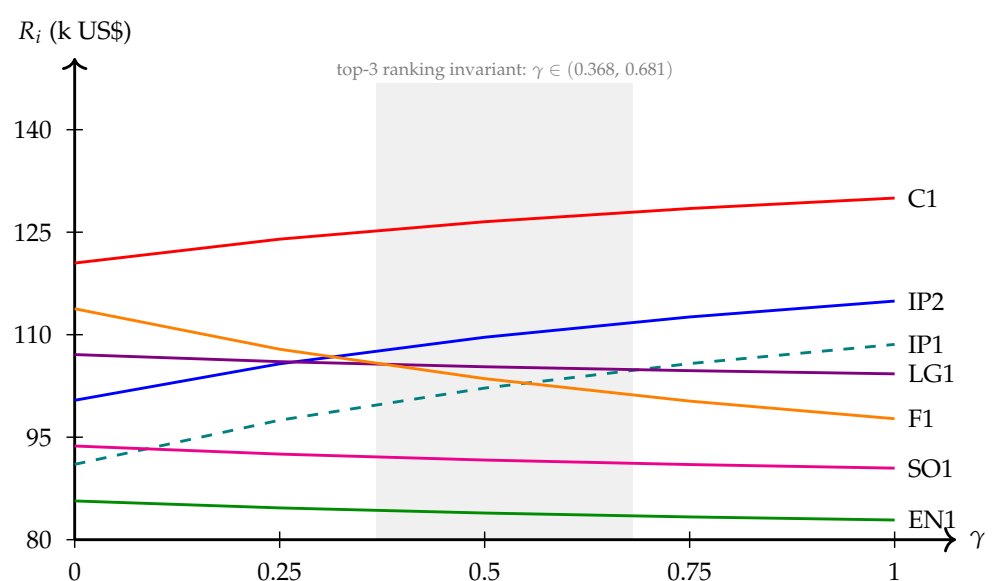
**Figure 3.** Allocation  $R_i$  as a function of  $\gamma$  for seven representative objectives, including the environmental (EN1) and social (SO1) objectives. The shaded band marks the interval  $(0.368, 0.681)$  over which the composition and order of the top three funded objectives are invariant.

Figure 3 shows the structural roles. Senders with high performance and connectivity (C1, IP2, IP1) gain budget as  $\gamma$  grows; the financial sink F1 declines steadily; and the two sustainability objectives trace nearly flat trajectories (EN1 from US\$85.7k to US\$82.9k; SO1 from US\$93.7k to US\$90.5k), because their incoming and outgoing synergies are of comparable size and offset each other in the closed form (6). This insensitivity is relevant for governance: the funding of the sustainability objectives does not depend on the parameter that is hardest to calibrate.

### 3.2.3. Perturbation of Matrix $A$

To assess robustness against DEMATEL estimation errors, we generated  $K = 500$  perturbed matrices  $\tilde{A}^{(k)} = A + E^{(k)}$ , with  $E_{ij}^{(k)} \sim \mathcal{U}[-\delta, +\delta] \cdot a_{ij}$ , renormalizing any row whose perturbed sum exceeded one, with a fixed random seed for reproducibility. We tested  $\delta \in \{0.10, 0.20\}$  with  $\gamma = 0.5$  fixed.

Table 10 reports the results. Mean deviations remain below 0.6% for  $\delta = 10\%$  and below 1.2% for  $\delta = 20\%$ ; worst cases stay under 2.4% and 4.9%, respectively. The sustainability objectives exhibit above-average relative deviations (SO1: mean 0.6%, max 2.4% at  $\delta = 10\%$ ) because a larger share of their value flows through the map rather than through their own scores; even so, the top-three ranking  $\{C1, IP2, LG1\}$  is preserved in 96.4% of the  $\delta = 10\%$  replications and 76.8% of the  $\delta = 20\%$  replications. Estimation error in the DEMATEL panel therefore perturbs allocation amounts marginally and rarely alters the funding ranking.

**Table 10.** Mean and maximum deviation of  $R_i$  (%) under perturbations  $\pm\delta$  on  $A$  ( $K = 500$  replications,  $\gamma = 0.5$ ).

| Id  | $\delta = 10\%$ |     | $\delta = 20\%$ |     |
|-----|-----------------|-----|-----------------|-----|
|     | Mean            | max | Mean            | max |
| F1  | 0.3             | 1.5 | 0.7             | 3.0 |
| F2  | 0.3             | 1.3 | 0.6             | 2.6 |
| C1  | 0.5             | 2.0 | 1.0             | 4.0 |
| C2  | 0.4             | 1.8 | 0.9             | 3.5 |
| IP1 | 0.5             | 1.8 | 1.0             | 3.5 |
| IP2 | 0.5             | 1.6 | 1.0             | 3.1 |
| EN1 | 0.5             | 2.0 | 1.1             | 4.0 |
| SO1 | 0.6             | 2.4 | 1.2             | 4.9 |
| LG1 | 0.6             | 1.8 | 1.2             | 3.6 |
| LG2 | 0.6             | 2.2 | 1.1             | 4.3 |

### 3.2.4. Comparison: $\gamma$ Sweep vs. DEMATEL Sensitivity

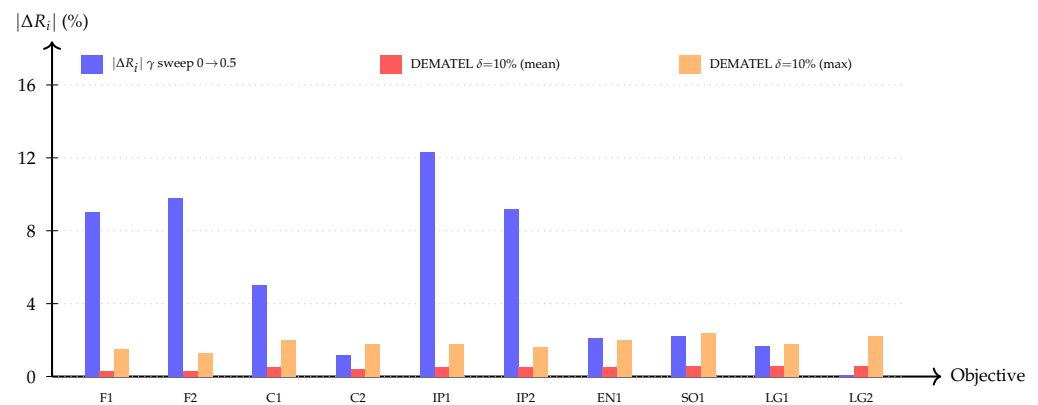
Both analyses evaluate the stability of the allocation  $R_i$  but answer different questions. The  $\gamma$  sweep (Section 3.2.2) keeps  $A$  fixed and measures how the allocation responds to the weight assigned to causal synergy: it quantifies *design* uncertainty. The perturbation of  $A$  (Section 3.2.3) keeps  $\gamma$  fixed and quantifies *estimation* uncertainty. Table 11 summarizes both effects for each objective.

**Table 11.** Design effect versus estimation effect at  $\gamma = 0.5$ . Columns 2–3: deviation of  $R_i$  when moving from  $\gamma = 0$  to  $\gamma = 0.5$ . Columns 4–5: mean and maximum deviation under DEMATEL perturbation ( $\delta = 10\%$ ). Column 6: amplification factor  $\rho_i$  (design over mean estimation effect).

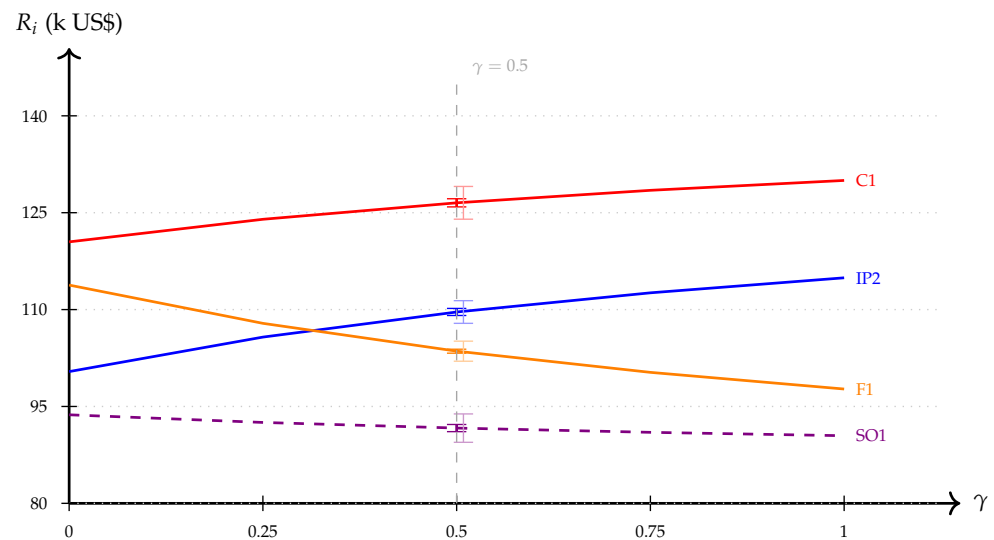
| Id          | $\gamma$ Sweep: 0 $\rightarrow$ 0.5 |                  | DEMATEL $\delta=10\%$ |            | Ratio       |
|-------------|-------------------------------------|------------------|-----------------------|------------|-------------|
|             | $\Delta R_i$ (k\$)                  | $\Delta R_i$ (%) | Mean (%)              | max (%)    | $\rho_i$    |
| F1          | −10.2                               | −9.0             | 0.3                   | 1.5        | 26.5        |
| F2          | −9.5                                | −9.8             | 0.3                   | 1.3        | 31.2        |
| C1          | +6.0                                | +5.0             | 0.5                   | 2.0        | 10.4        |
| C2          | −1.2                                | −1.2             | 0.4                   | 1.8        | 2.7         |
| IP1         | +11.2                               | +12.3            | 0.5                   | 1.8        | 25.2        |
| IP2         | +9.2                                | +9.2             | 0.5                   | 1.6        | 18.8        |
| EN1         | −1.8                                | −2.1             | 0.5                   | 2.0        | 3.9         |
| SO1         | −2.1                                | −2.2             | 0.6                   | 2.4        | 3.6         |
| LG1         | −1.8                                | −1.7             | 0.6                   | 1.8        | 2.9         |
| LG2         | +0.1                                | +0.1             | 0.6                   | 2.2        | 0.3         |
| <b>Avg.</b> | —                                   | <b>5.3</b>       | <b>0.5</b>            | <b>1.8</b> | <b>12.5</b> |

Table 11 separates the two sources of variation, and Figure 4 compares them objective by objective (see Figure 4). Activating the causal synergy redistributes up to 12.3% of an objective's budget, whereas estimation noise in  $A$  moves allocations by 0.5% on average: the design effect dominates the estimation effect by an average factor of  $\bar{\rho} = 12.5$ . The exception is LG2, a nearly  $\gamma$ -neutral node with  $\rho = 0.3$ . The moderate ratios of the sustainability objectives ( $\rho_{EN1} = 3.9$ ,  $\rho_{SO1} = 3.6$ ) indicate that their allocations are stable under both analyses.

Figure 4 disaggregates the comparison by objective: for the high-connectivity senders (IP1, IP2, C1) and the financial sink objectives (F1, F2), the design effect exceeds both Monte Carlo statistics; for LG2 the two effects are of the same order. Figure 5 overlays the DEMATEL uncertainty bands on the  $\gamma$  trajectories at the operating point (see Figure 5).



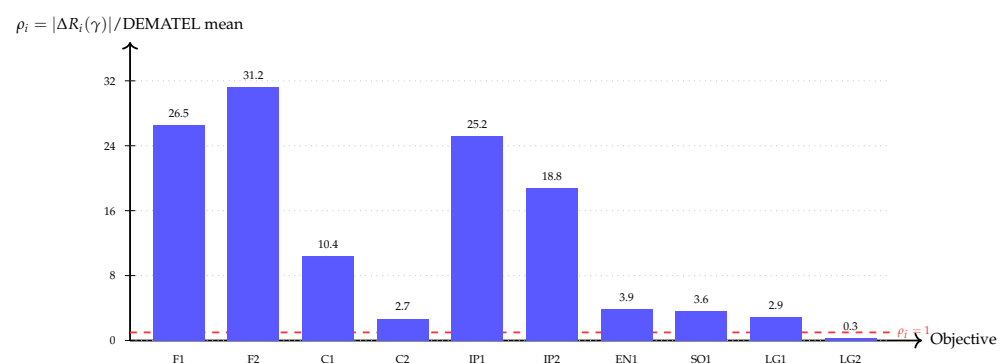
**Figure 4.** Objective-by-objective comparison between the design effect ( $\gamma$  sweep from 0 to 0.5) and the DEMATEL estimation effect ( $\delta = 10\%$ , mean and maximum over  $K = 500$  replications). All quantities are expressed as a percentage of  $R_i$ .



**Figure 5.** Overlay of the  $\gamma$  sweep with DEMATEL uncertainty bands ( $\delta = 10\%$ ) at the operating point  $\gamma = 0.5$ , for four representative objectives (C1, IP2, F1, and the social objective SO1). The thick bar marks the mean deviation; the light bar marks the maximum deviation observed in 500 replications.

Figure 5 shows that the DEMATEL uncertainty bands at  $\gamma = 0.5$  are narrow relative to the slopes of the trajectories: none of the four plotted bands overlaps an adjacent plotted trajectory, so estimation error alone cannot reorder the objectives shown. Finally, Figure 6 reports the amplification factors.

Three conclusions follow. First, the allocation is stable: perturbations of  $\pm 10\%$  in  $A$  preserve the top-three ranking  $\{C1, IP2, LG1\}$  in 96.4% of replications (76.8% under  $\pm 20\%$ ). Second, the choice of  $\gamma$  affects the allocation more than residual estimation error: the design effect exceeds the mean estimation effect for every objective except LG2, with an average amplification of  $\bar{\rho} = 12.5$ . Third, the sustainability objectives EN1 and SO1 display low sensitivity on both axes, so their funding is defensible under any reasonable specification of the map and of  $\gamma$ .



**Figure 6.** Amplification factor  $\rho_i$  at  $\gamma = 0.5$ . The dashed line indicates parity ( $\rho_i = 1$ ); every objective except the nearly  $\gamma$ -neutral LG2 lies above it, and the average across objectives is  $\bar{\rho} = 12.5$ .

## 4. Discussion

### 4.1. Comparison with Prior Approaches

The results confirm and extend three streams of literature. Methodologically, the model differs from the game-theoretic BSC stream in unit and output. Abedian et al. [9] and Seyedi and Yousefi Hanoomarvar [10] treat four aggregated perspectives as players and obtain importance rankings; our game has ten objective-level players and returns dollars. The distinction is substantive: in our example, a five-player perspective game

would assign the dedicated sustainability perspective a single share, erasing the 9% funding gap between its two members (SO1, US\$91,653; EN1, US\$83,901) and the fact that the map's least-funded objective coexists there with a mid-ranked one. A perspective-level game averages away precisely the heterogeneity that an SBSC exists to manage. Relative to Abedian et al. [28], who build a superadditive characteristic function from DEA efficiency ratios to rank indicators, our characteristic function is built from the strategy map itself, which allows causal position, in addition to measured efficiency, to determine the allocation.

Against the Shapley-allocation stream outside the BSC [37–41], the contribution is symmetric: those models allocate among autonomous entities whose cooperation is technological or contractual; ours allocates among strategic objectives whose “cooperation” is the causal texture of the strategy map. The closed form of Remark 1 has no counterpart in that stream because it derives from the bilinear structure of the map-based characteristic function.

Empirically, our robustness results compare favorably with the MCDM–BSC stream, where rankings are known to be sensitive to the weighting method: Janeš et al. [30] report method-dependent priority reversals between AHP and ANP on the same scorecard, and hybrid frameworks [31,32,34] require consensus mechanisms across multiple rankers. In our model the source of variation is explicit and governable: the allocation ordering is invariant to the synergy weight on (0.368, 0.681) around the operating point and survives  $\pm 10\%$  DEMATEL noise in 96.4% of replications. Finally, the finding that outcome objectives cede budget to drivers (F1 and F2 lose 9–10% of their proportional share) formalizes the claim of the strategy-map literature that organizations should fund performance drivers [16,44]; to our knowledge, no prior model quantifies how much funding should move, or certifies with Core membership that the resulting redistribution leaves no coalition of objectives underfunded.

#### 4.2. The Functional Form of Sustainability Synergies

The characteristic function (4) encodes every synergy—economic or sustainability-related—as a bilinear term  $\gamma a_{ij}p_j$ : proportional to the strength of the causal link and to the current performance of the sender, and additive across pairs. For classical economic linkages this is a reasonable first-order approximation: process–cost and quality–revenue relations are approximately proportional within a planning cycle, and their interaction effects beyond pairs are second-order. For sustainability linkages, three departures require examination. First, *thresholds and regime shifts*: the value of an emissions objective may jump discontinuously at a regulatory limit or a customer's procurement cutoff, so the marginal value of progress is not constant in  $p_j$ . Second, *temporal displacement*: safety culture and decarbonization investments generate value with lags that exceed the planning cycle, so a within-cycle  $p_j$  understates the sender's true contribution. Third, *option-like risk value*: part of what safety and compliance objectives produce is the *avoidance* of low-probability, high-severity losses—an insurance value invisible to expected-progress scores. In our example these features surface empirically: EN1's priority depends on KPI maturity rather than on  $\gamma$  (it overtakes F2 only at  $\gamma^* = 0.816$ ), consistent with a sender whose transmitted synergy  $a_{ij}p_j$  is attenuated by a low  $p_j$ .

The model accommodates these departures. The proofs of superadditivity and convexity (Appendix A) use only the non-negativity and pairwise-additive structure of the synergy terms, not their bilinearity: replacing  $a_{ij}p_j$  with any non-negative pair function  $g_{ij}(p_j)$ —for instance, a logistic form for threshold effects, or a lag-discounted score  $\tilde{p}_j$ —preserves superadditivity, convexity, Core membership, and the equal-split closed form with  $g_{ij}$  in place of  $a_{ij}p_j$ . What changes is estimation, not axiomatics: the DEMATEL protocol must then elicit a curve rather than a weight. In summary, sustainability synergies do not always take the same functional form as economic ones, but they take the same

game-theoretic form, and the allocation framework is preserved under the generalization. Genuinely higher-order effects—value created only when three or more objectives progress jointly, as in system-level transitions—would require coalition-dependent terms and are the one case in which the closed form is lost; we identify this as future work.

#### 4.3. Robustness and Boundary Conditions

The two sensitivity analyses answer different governance questions. The  $\gamma$ -sweep quantifies *design* uncertainty—how much the allocation depends on a parameter the organization chooses—and yields the invariance intervals reported in Section 3.2. The perturbation of  $A$  quantifies *estimation* uncertainty—how much the allocation depends on expert error—and remains an order of magnitude smaller ( $\bar{\rho} = 12.5$ ). Both results, however, hold within the boundary conditions of Section 2.8: a fungible budget, a single governance body, commensurable scores, and a stationary map. Outside those conditions the model's outputs remain informative as *shadow allocations*—signals of where value is created—but their direct implementation requires the NTU adaptations discussed in Section 2.11.

## 5. Implications

### 5.1. Theoretical Implications

The article makes three theoretical contributions. First, it extends cooperative game theory to a new class of players: strategic objectives connected by a causal map, rather than autonomous economic agents. The strategy map acts as the technology of cooperation, and the resulting game is always superadditive and convex—a structural result that does not depend on the example. Second, it provides the axiomatic link between performance measurement and resource allocation: efficiency, symmetry, null-player treatment, and Core stability translate, in management terms, into full budget use, equal pay for equal contribution, principled seed funding, and the guarantee that no subset of objectives subsidizes the rest. Third, the closed form of Remark 1 shows that map-based Shapley allocation is computable in  $O(n^2)$  operations and transparent term by term, which removes the computational objection to axiomatic methods in strategy settings and provides the SBSC literature with a tractable formal core [17,18].

### 5.2. Practical Implications

For planning practice, the framework converts artifacts most organizations already possess—a strategy map, KPIs with targets, and a DEMATEL or expert-judgment session—into a defensible budget without new data infrastructure. The allocation is auditable: every deviation from a proportional budget traces to identified causal links, which supports the documentation requirements of ESG assurance and sustainability reporting. The two sensitivity analyses run in seconds and give budget committees confidence bands rather than point estimates. For SMEs—where scorecards are frequently used for measurement but not for decision-making [43,47,54]—the model links the scorecard directly to funding decisions; the ten-objective example is computable in a spreadsheet.

### 5.3. Managerial Implications

Five recommendations follow directly from the results. First, fund drivers, not only outcomes: the model moved 9–10% of the financial objectives' proportional share toward process and enabling objectives; managers should expect and communicate this direction of redistribution. Second, treat  $\gamma$  as a governed decision parameter: set it by committee at each planning cycle (Section 2.10) and choose it inside the reported invariance interval so that the funding ranking is robust. Third, maintain funding for recently introduced sustainability objectives: immature environmental and social indicators reduce  $p_i$ , but

causal position sustains funding—and the null-player characterization (Proposition 3) justifies seed funding for objectives whose baseline is still being built (US\$13,211 for EN1 at  $p = 0$  in the example). Fourth, improvements in KPI quality have a double effect: raising the quality and maturity of the emissions KPI raises both EN1's autonomous value and every synergy it transmits. Fifth, audit the KPIs of sink objectives with particular attention: their allocation responds approximately linearly to their own scores, so measurement error propagates directly into misallocation.

## 6. Limitations and Future Research

Three groups of limitations bound the contribution.

**Model-theoretic limitations.** The model assumes transferable utility within the conditions of Section 2.8; outside them, NTU solution concepts or restricted-cooperation structures are required (Section 2.11). The synergy terms are bilinear and pairwise: threshold effects, lags, and higher-order complementarities are captured only approximately (Section 4.2). The matrix  $A$  is static within the planning cycle, whereas strategy maps evolve; dynamic extensions could couple the game to system-dynamics formulations [10,12,64]. Although the allocation itself is computable in  $O(n^2)$  (Remark 1), the numerical verification of the Core condition still enumerates  $2^n$  coalitions and is practical only up to  $n \approx 20$ ; for larger maps, convexity guarantees Core membership analytically, but independent numerical verification becomes sampling-based.

**Sustainability-measurement limitations.** The model inherits the quality of environmental and social indicators through  $p_i$ , and Property 4 makes this inheritance monotone: understated emissions boundaries (e.g., excluded Scope 3 categories) or underreported safety incidents translate directly into allocation error, and target-based scores create incentives to negotiate lenient targets—a gaming channel that requires audit controls outside the model. Commensurating heterogeneous horizons is a second limitation: a decarbonization objective whose value accrues over decades and a quarterly cost objective are compared through same-cycle progress scores, which structurally favors the short cycle. Third, the model prices sustainability objectives by their causal contribution to the mapped strategy; stakeholder value that the organization has not mapped is, by construction, not represented—a formal analogue of the instrumental-versus-integrative tension in the SBSC debate [19,20]. Governance safeguards (minimum-funding floors, materiality reviews of the map) should complement the rule where integrative ambitions exceed the map.

**SBSC implementation limitations.** The allocation depends on the SBSC architecture only through the causal links the architecture leads the organization to elicit: integrating sustainability objectives into existing perspectives, adding a dedicated perspective, or deriving a separate scorecard changes which links are expressed, and hence  $A$  and the allocation, even though the perspective labels themselves play no role [18]. The DEMATEL panel must include sustainability expertise; a panel drawn only from finance and operations will systematically under-specify environmental and social links. Finally, implementation evidence in SMEs shows that scorecard-based systems fail when they add measurement burden without decision consequences [43,54]; the model mitigates this risk by making the scorecard directly consequential for funding, but it cannot substitute for the managerial capability its inputs require.

**Future research.** Two extensions follow. First, we propose formulating the problem as a cooperative game with non-transferable utility, in which each perspective  $\pi \in \{F, C, IP, LG, SUS\}$  has a partial budget  $R_\pi$  with  $\sum_\pi R_\pi = R$ ; the Shapley value is then replaced by NTU solution concepts such as the Harsanyi value or the NTU Shapley value [53,71,72], or by a restricted cooperative game [53,73] in which the communication structure  $\mathcal{L} \subseteq 2^N$  represents the feasible transfers between budget compartments. Second,

we propose empirical validation: replicating the model in real organizations across sectors, including SBSC implementations in mining, energy, and education, evaluating its performance against heuristic rules, and eliciting nonlinear synergy functions  $g_{ij}$  for sustainability linkages (Section 4.2), for which fuzzy DEMATEL [35] is a suitable elicitation method.

## 7. Conclusions

This article presents a resource allocation model that integrates the causal structure of the BSC strategy map with cooperative game theory, with the explicit goal of supporting sustainable strategic management. The central contribution is a characteristic function (Definition 2) that simultaneously encodes the autonomous performance of each objective and the synergies derived from the causal relationships between them. On this basis, we prove superadditivity, convexity, and membership of the Shapley value in the Core, which guarantees that no subset of objectives is underfunded relative to the value it generates by cooperating.

The derived allocation rule satisfies formal properties of exhaustiveness, fairness based on marginal contribution, sensitivity to the strategy map, monotonicity, and convergence to the proportional rule, complemented by a characterization of the null player and by a closed form that makes the allocation computable in  $O(n^2)$  operations (Remark 1). The SBSC example with  $n = 10$  objectives and a dedicated sustainability perspective illustrates the managerial content of these properties: the customer objective C1 obtains the largest allocation through its causal position rather than its score; the employee-safety objective outranks financial cost reduction for every  $\gamma \geq 0.15$ ; and a sustainability objective entering the map without a measured baseline still receives seed funding through the synergies it channels. The computational experiments confirm tractability (exhaustive verification of the Core up to  $n \approx 20$ ) and robustness: the top-three funding ranking is invariant for  $\gamma \in (0.368, 0.681)$ , mean deviations remain below 0.6% under  $\pm 10\%$  perturbations of  $A$ , and the top-three ranking survives in 96.4% of replications.

The framework provides sustainability-oriented organizations with an allocation that is efficient, equitable, coalitionally stable, and auditable—properties that subjective weighting schemes do not guarantee. Section 6 delimits the contribution and outlines the extensions—non-transferable budgets, nonlinear sustainability synergies, dynamic strategy maps, and empirical validation—that follow.

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## Abbreviations

The following abbreviations are used in this manuscript:

|                   |                                                 |
|-------------------|-------------------------------------------------|
| AHP               | Analytic Hierarchy Process                      |
| ANP               | Analytic Network Process                        |
| BSC               | Balanced Scorecard                              |
| C                 | Customer (perspective)                          |
| CO <sub>2</sub> e | Carbon Dioxide Equivalent                       |
| DEA               | Data Envelopment Analysis                       |
| DEMATEL           | Decision-Making Trial and Evaluation Laboratory |
| ESG               | Environmental, Social, and Governance           |
| F                 | Financial (perspective)                         |
| IP                | Internal Processes (perspective)                |
| KPI               | Key Performance Indicator                       |
| LG                | Learning and Growth (perspective)               |
| LTIFR             | Lost-Time Injury Frequency Rate                 |
| MCDM              | Multi-Criteria Decision-Making                  |
| NPS               | Net Promoter Score                              |
| NTU               | Non-Transferable Utility                        |
| OEE               | Overall Equipment Effectiveness                 |
| ROE               | Return on Equity                                |
| SBSC              | Sustainability Balanced Scorecard               |
| SUS               | Sustainability (perspective)                    |
| TU                | Transferable Utility                            |

## Appendix A. Formal Statements and Proofs

### Appendix A.1. Superadditivity, Convexity, and Core Membership

**Proof of Proposition 1.** For disjoint  $S$  and  $T$ ,

$$v(S \cup T) - v(S) - v(T) = \gamma \left( \sum_{i \in S} \sum_{j \in T} a_{ij} p_j + \sum_{i \in T} \sum_{j \in S} a_{ij} p_j \right) \geq 0,$$

since  $\gamma \geq 0$ ,  $a_{ij} \geq 0$ , and  $p_j \geq 0$ . Equality holds if and only if every cross term vanishes, that is, when no causal relationship connects  $S$  and  $T$  or  $\gamma = 0$ .  $\square$

**Proof of Proposition 2.** By (5),  $\Delta_i(T) - \Delta_i(S) = \gamma \sum_{j \in T \setminus S} (a_{ij} p_j + a_{ji} p_i) \geq 0$  for  $S \subseteq T$ .  $\square$

Corollary 1 follows directly from convexity through the theorem of Shapley [66] on convex games. Note that both proofs use only the non-negativity and pairwise-additive structure of the synergy terms, not their bilinearity; this observation supports the generalization discussed in Section 4.2.

### Appendix A.2. Null-Player Characterization

**Proof of Proposition 3.** By (5), conditions (i)–(ii) imply  $\Delta_i(S) = 0$  for every  $S$ , hence  $\phi_i(v) = 0$ . Conversely, if  $\phi_i(v) = 0$  and  $\gamma > 0$ , choosing  $S = \{j\}$  with  $a_{ij} p_j > 0$  would yield  $\Delta_i(S) > 0$ , a contradiction.  $\square$

### Appendix A.3. Limiting Cases and Linearity in $\gamma$

**Proof of Proposition 4.** The characteristic function (4) decomposes as  $v_\gamma = v_0 + \gamma(v_1 - v_0)$ , where  $v_0(S) = \sum_{i \in S} p_i$  is the game without synergy and  $v_1$  the game with full synergy. Since the Shapley value is linear in the characteristic function [23],  $\phi_i(v_\gamma) = \phi_i(v_0) + \gamma[\phi_i(v_1) - \phi_i(v_0)]$ , which proves (iii). For (i),  $v_0$  is additive, so  $\phi_i(v_0) = p_i$ ; (ii) follows by substituting  $S = N$  and  $\gamma = 1$  into (4).  $\square$

### Appendix A.4. Derivation of the Closed Form (Remark 1)

For each ordered pair  $(i, j)$  with  $i \neq j$ , define the unanimity-type game  $u_{ij}(S) = 1$  if  $\{i, j\} \subseteq S$  and  $u_{ij}(S) = 0$  otherwise, and let  $e_i(S) = 1$  if  $i \in S$  and 0 otherwise. The characteristic function (4) decomposes as

$$v = \sum_{i \in N} p_i e_i + \gamma \sum_{i \neq j} a_{ij} p_j u_{ij}.$$

The Shapley value of  $e_i$  assigns 1 to player  $i$  and zero to the rest; the Shapley value of  $u_{ij}$  assigns 1/2 to each of  $i$  and  $j$  and zero to the rest [23]. Linearity of the Shapley value in the characteristic function then yields (6). Computing (6) requires one pass over the nonzero entries of  $A$ , that is,  $O(n^2)$  operations.

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