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A Consistent Markov Chain-Based Framework for Life-Cycle Optimization and Cost–Benefit Evaluation of Infrastructure Maintenance Policies

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Abstract

A consistent computational framework is presented that integrates Markov chain modeling, decision optimization, and cost–benefit analysis for the life-cycle management of engineering assets. The approach combines deterioration modeling with a complete economic evaluation and optimization of maintenance decisions. Each condition state of the system is associated with possible actions such as do-nothing, preventive maintenance, major repair, and replacement, each defined by its own transition matrix or generator describing state changes. The expected one-step reward is formulated as the difference between benefits and all relevant costs including operating, action, and failure costs. The optimization problem is expressed as a discounted Markov decision process and solved by linear programming. The resulting stationary policy specifies the optimal decision rule for every state. Both discrete-time and continuous-time variants are implemented. Transition matrices and generator matrices are linked using a matrix exponential mapping for the selected step length. Under state-dependent policies, the discrete step model and the continuous-time feedback model may lead to different long-run state shares. This is caused by different decision timing. The continuous-time variant also provides reliability indicators such as survival and hazard. It can also provide mean time to absorption under an absorbing failure interpretation. Simulation under the optimal policy yields state trajectories, present values of benefits and costs, and key economic indicators such as net present value, benefit–cost ratio, equivalent annual cost, and equivalent annual net benefit. The framework forms a unified and practical tool that connects reliability analysis, Markov optimization, and life-cycle cost–benefit evaluation for long-term infrastructure management.

Keywords: Markov decision process; cost–benefit analysis; life-cycle optimization; infrastructure maintenance policy; reliability analysis; engineering asset management



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1. Introduction

Life-cycle decisions in infrastructure management depend on both technical reliability and economic performance [1]. Maintenance and replacement actions dominate total life-cycle costs, and the optimal timing of these actions determines long-term savings. Decision makers require tools that connect probabilistic forecasts of deterioration with cost and benefit information expressed in monetary units. Such tools allow consistent evaluation of maintenance strategies and support transparent asset management practice that links engineering reliability with financial indicators [2–4]. For public infrastructure, cost–benefit

analysis is a policy requirement, and agencies report measurable savings when probabilistic optimization replaces deterministic age-based rules [5].

From a sustainability perspective, maintenance optimization is important because it can extend the useful life of existing assets and reduce premature replacement. This may limit unnecessary material use, waste generation, service disruption, and avoidable capital expenditure. A life-cycle framework also supports more transparent long-term planning, because technical reliability and economic consequences are evaluated together. In this sense, the proposed method contributes to sustainable infrastructure management by helping decision makers select maintenance policies that preserve asset functionality while using resources more efficiently.

In practice, the rate of deterioration depends not only on age but also on how the building is used. High intensity use, frequent tenant rotation or heavy commercial activity can speed up wear and create different maintenance needs than in buildings with stable and predictable use. This is similar to service recommendations in other technical areas where the correct maintenance interval depends on the real usage profile. A Markov-based decision model can include such differences in a transparent way.

Markov chains are widely used to describe deterioration as transitions between discrete condition states [6,7]. They provide transition probabilities that can be estimated from inspection data and directly integrated into life-cycle cost models. Continuous-time Markov chains extend this concept by modeling degradation as a stochastic process on a real-time axis. This supports reliability quantities such as survival and hazard. It can also provide mean time to absorption under an absorbing failure interpretation [7–9]. However, traditional Markov analysis stops at prediction. It does not indicate which maintenance action should be performed or when. The present framework extends the probabilistic model [10] into a decision environment by coupling the Markov chain with an optimization step and an explicit cost–benefit formulation.

Recent studies show that infrastructure maintenance management is moving toward predictive and data-driven approaches. Current research includes prognostic and health management-driven maintenance strategies, adaptive inspection and maintenance planning, and digital twins for intelligent infrastructure maintenance [11–13]. These approaches improve condition forecasting and support earlier maintenance decisions. However, they still require clear decision models that translate technical condition data into maintenance policies and economic indicators. The framework proposed in this paper addresses this need by combining Markov deterioration modeling, decision optimization, and cost–benefit evaluation in one transparent procedure.

The methodology treats maintenance planning as a Markov Decision Process (MDP) [14,15]. Each state of the system is associated with a set of available actions such as do-nothing, preventive maintenance, major repair, and replacement. Each action has its own transition matrix or generator and its own economic profile including direct costs, operational costs, and expected benefits. The reward function combines these components into a single monetary balance. The optimization is formulated as a discounted dynamic programming problem and is solved by linear programming [16,17]. The resulting stationary policy gives the economically optimal decision rule for each condition state under the assumed rates, costs, and benefits.

The model supports discrete-time and continuous-time variants. The transition model is linked across time domains at the action level. This link is exact for a fixed step length. Under a state-dependent policy, the discrete model holds the chosen action over the whole step. In the continuous-time feedback model, the action can change right after a state change. For this reason, long-run stationary distributions can differ between the two variants. Continuous-time calculations also provide reliability functions, including survival

and hazard, which can be combined with the economic results. Simulation under the optimal policy produces state trajectories and discounted economic indicators such as net present value, benefit–cost ratio, equivalent annual cost, and equivalent annual net benefit.

The contribution is methodological and practical. The framework integrates deterioration modeling, reliability analysis, and cost–benefit optimization into one consistent process. It produces results that can be directly used for maintenance scheduling, budgeting, and asset management at strategic level. The numerical steps are reproducible and auditable, which makes the approach suitable for practical implementation in engineering organizations and for future extensions toward fuzzy [18,19] and time-dependent formulations. All computations were performed in MATLAB R2024b using the CVX 2.2 (Build 1148) modeling framework and the MOSEK 9.1.9 optimization engine, ensuring reproducible and numerically stable solutions [20,21].

This study continues the authors' previous research on the application of Markov processes to building service-life prediction. Earlier works established the probabilistic and reliability background for discrete- and continuous-time formulations [22,23]. The present paper extends that methodology toward full integration with decision optimization and cost–benefit analysis, thus providing a unified framework for the maintenance planning and life-cycle economic evaluation of building and infrastructure systems.

The objective of this study is to develop and test a consistent Markov-based framework for optimizing maintenance policies and evaluating their economic effects. The study addresses three research questions. First, can deterioration modeling, decision optimization, and cost–benefit analysis be integrated into one transparent computational procedure? This question is addressed in Section 2, where the theoretical formulation is presented. Second, can the same framework be applied to realistic building-management data and economic assumptions? This question is addressed in Section 3, where the input data and model assumptions are defined. Third, do discrete-time and continuous-time formulations lead to consistent maintenance policies and comparable economic indicators? This question is addressed in Section 4, where the numerical results, reliability indicators, and economic outcomes are compared. The answers to these questions are summarized in Section 5. The main contribution of the paper is a reproducible framework that connects Markov deterioration modeling, Markov decision optimization, reliability analysis, and cost–benefit evaluation for long-term infrastructure maintenance planning.

2. Theoretical Background

This section introduces the theoretical basis of the proposed framework. It first defines the Markov state-transition model used to describe deterioration. It then presents the reward structure, the decision optimization problem, the economic evaluation procedure, and the continuous-time reliability measures. These elements form the mathematical basis for the numerical case study presented in the following sections.

2.1. Markov-State Transition Model

The condition of an engineering asset is represented by a finite set of discrete states that describe different levels of performance. The last state represents failure or replacement. The probability of moving from state i to state j under a selected maintenance action a is given by the element $P_{ij}(a)$ of the transition matrix $P(a)$

$$P(a) = [P_{ij}(a)], \quad i, j = 1, 2, \dots, n \quad (1)$$

Each row of the transition matrix must satisfy the following stochastic conditions:

$$\sum_{j=1}^n P_{ij}(a) = 1, \quad P_{ij}(a) \geq 0 \quad (2)$$

The Markov property ensures that future transitions depend only on the current state and on the chosen action. In the continuous-time formulation, the process is defined by a generator matrix $Q(a)$. The corresponding discrete matrix for a given time step Δt is obtained from the matrix exponential [24,25].

$$P(a) = \exp[Q(a)\Delta t] \quad (3)$$

The generator matrix satisfies the following conditions which guarantee positivity of all probabilities and a zero sum in each row:

$$Q_{ij}(a) \geq 0 \quad (i \neq j), \quad \sum_{j=1}^n Q_{ij}(a) = 0 \quad (4)$$

This mapping is exact for each action and for a fixed step length. It ensures that the discrete transition over one step matches the continuous generator for that action. Under a state-dependent policy, the discrete model applies one action over the full step. The continuous-time feedback model can switch actions immediately after a state change. Therefore, the long-run stationary probabilities may differ between the two decision semantics.

2.2. Reward and Discount Structure

Each action executed in state produces an expected one-step reward that expresses the balance between benefits and costs during one analysis period. The reward function is defined as [16,26]:

$$R(i, a) = \mathbb{E}[B] - \mathbb{E}[C_{op}] - \mathbb{E}[C_{fail}] - C_{act}(i, a) \quad (5)$$

where $\mathbb{E}[B]$ is the expected benefit obtained from the system after applying action a in state i . It represents the expected service value, income, or utility generated by the asset and is computed as the probability-weighted average of the benefit levels $B(j)$ in the next states j .

$$\mathbb{E}[B] = \sum_{j=1}^n P_{ij}(a) B(j) \quad (6)$$

Moreover $\mathbb{E}[C_{fail}]$ is the expected failure cost, which accounts for the probability of entering the failure state multiplied by the economic consequence of failure C_{fail} . For example, if the failure state corresponds to index n , this term is evaluated as:

$$\mathbb{E}[C_{fail}] = P_{in}(a) C_{fail} \quad (7)$$

Finally, $C_{act}(i, a)$ is the direct action cost, which includes the immediate expenditure required to perform the selected maintenance action a while the asset is in state i . It may include materials, equipment, and labor specific to that intervention.

The three cost categories used in this study form a basic cost structure for the optimization problem. Operating costs describe the regular use and management of the asset. Action costs describe direct expenditure related to maintenance, repair or replacement. Failure costs describe the economic consequence of entering the failure state. These categories were selected because they are common to many maintenance-planning problems and can be directly included in the reward function. Other costs may also be important in specific applications. These may include user disruption, environmental costs, social costs, indirect business losses, administrative costs, or residual value effects. They were not included in the numerical example in order to keep the case study transparent. However, the framework allows such costs to be added if reliable data are available.

All future rewards are discounted to their present value using the discount factor γ defined as [3,27]:

$$\gamma = (1 + r)^{-\Delta t} \quad (8)$$

where r is the annual discount rate and Δt is the time step in years. This ensures that future monetary effects have lower weight than current ones, which is consistent with life-cycle cost principles.

2.3. Decision Optimization Problem

The maintenance planning problem is formulated as a Markov Decision Process (MDP). The objective is to find an optimal stationary policy that maximizes the expected total discounted reward over an infinite horizon. The optimal value function satisfies the Bellman equation [16,26]:

$$V(i) = \max_a \left[R(i, a) + \gamma \sum_{j=1}^n P_{ij}(a) V(j) \right] \quad (9)$$

The value function $V(i)$ expresses the expected total discounted benefit of being in state i and acting optimally thereafter. To compute $V(i)$ and the optimal policy $\pi^*(i)$, the Bellman equation is written as a system of inequalities and solved as a linear program. The linear programming formulation is as follows [28].

Let $\mu(i) > 0$ denote the state-weighting coefficient used in the objective function of the linear program:

$$\begin{aligned} & \text{minimize} \quad \sum_{i=1}^n \mu(i) V(i) \\ & \text{subject to} \quad V(i) \geq R(i, a) + \gamma \sum_{j=1}^n P_{ij}(a) V(j), \quad \forall i, a \end{aligned} \quad (10)$$

In this study, $\mu(i)$ is chosen as the initial state distribution. These coefficients are not weights of different decision criteria. The optimization is not formulated as a multi-criteria problem. It maximizes one scalar objective, namely the expected total discounted reward expressed in monetary units. Solving this linear program yields the optimal value function and the stationary policy [29]:

$$\pi^*(i) = \operatorname{argmax}_a \left[R(i, a) + \gamma \sum_{j=1}^n P_{ij}(a) V^*(j) \right] \quad (11)$$

The optimization is solved numerically in MATLAB using the CVX modeling framework with the MOSEK solver, which ensures stable and reproducible results for both dense and sparse transition matrices [20,21]. The complete computational procedure is summarized in Appendix A.

The computational workflow can be summarized as follows:

- Inspection data and economic assumptions are defined;
- Condition states and available maintenance actions are introduced;
- Transition matrices or generator matrices are assigned to each action;
- Expected rewards are computed as benefits minus operating, action, and failure costs;
- The Markov decision problem is solved to obtain the optimal stationary policy;
- The optimal policy is used to simulate state trajectories;
- Reliability indicators are computed in the continuous-time variant;
- Benefits and costs are converted into economic indicators, including net present value, benefit–cost ratio, equivalent annual cost, and equivalent annual net benefit.

The complete algorithm is presented in Appendix A.

2.4. Discrete-Time Simulation and Economic Evaluation

Once the optimal policy is found, a single transition matrix P_π is constructed by selecting, for each state, the row corresponding to the optimal action. The time evolution of the system is then simulated through the recurrence relation [8,30].

$$p_{t+1} = p_t P_\pi \quad (12)$$

The probability vector p_t describes the distribution of assets among condition states at time step t . At each step, the expected benefits and costs are calculated as averages weighted by p_t , and all values are discounted by the factor γ^t . The cumulative present values of benefits and costs are obtained as [3,27].

$$PV_B = \sum_{t=0}^{T-1} \gamma^t \mathbb{E}[B_t] \quad (13a)$$

$$PV_C = \sum_{t=0}^{T-1} \gamma^t \mathbb{E}[C_t] \quad (13b)$$

The resulting values are used to compute the net present value and the benefit–cost ratio by applying the following equations:

$$NPV = PV_B - PV_C \quad (14a)$$

$$BCR = \frac{PV_B}{PV_C} \quad (14b)$$

Equivalent annual indicators are derived using the annuity factor f_{ann} as follows:

$$f_{ann} = r \left[1 - (1 + r)^{-T_{years}} \right]^{-1} \quad (15a)$$

$$EAC = PV_C \cdot f_{ann} \quad (15b)$$

$$EANB = NPV \cdot f_{ann} \quad (15c)$$

These indicators translate the probabilistic model into standard economic terms used in cost–benefit analysis and infrastructure planning [31].

Throughout the manuscript, NPV is reported in MEUR, BCR is dimensionless, and EAC and EANB are reported in MEUR/year. Calculated economic indicators are rounded to two decimal places.

2.5. Continuous-Time Reliability Analysis

In the continuous-time formulation, the system evolution is governed by the Kolmogorov equation [7]

$$\frac{d\mathbf{p}(t)}{dt} = \mathbf{p}(t)\mathbf{Q}_\pi, \quad \mathbf{p}(0) = \mathbf{p}_0 \quad (16)$$

The analytical solution of this differential system is given by the matrix exponential

$$\mathbf{p}(t) = \mathbf{p}_0 \exp(\mathbf{Q}_\pi t) \quad (17)$$

Reliability indicators are derived from the continuous-time state probabilities. We define $S(t)$ as the probability that the system is not in the failure state at time t . If the failure state is repairable, this is an availability-type measure. We also define the failure entry intensity $\lambda_{in}(t)$ as the inflow rate into the failure state from all non-failure states [8,32]. The hazard is defined as the conditional failure entry intensity, $\lambda_{in}(t)/S(t)$.

$$S(t) = 1 - p_{failure}(t) \quad (18)$$

$$\lambda_{in}(t) = \sum_{i \neq failure} p_i(t) q_{i,failure} \quad (19)$$

$$h(t) = \frac{\lambda_{in}(t)}{S(t)} \quad (20)$$

If the failure state is absorbing, MTTA can be computed by integrating $S(t)$ over time.

2.6. Model Assumptions

The approach assumes that the process is Markovian, meaning that the future state depends only on the current state and the selected action. This assumption means that

the current condition state is treated as a sufficient summary of the previous deterioration history; it is appropriate when inspection data describe the actual technical condition with enough accuracy. However, it may be less accurate for systems with strong cumulative damage, fatigue history, aging effects or hidden degradation mechanisms that are not fully captured by the observed state. In such cases, semi-Markov models could be used to include state residence times, while hidden Markov models could be used when the true deterioration state is only indirectly observed. These extensions are outside the scope of the present paper but are relevant directions for future research. Transition probabilities or rates are constant within each time step, and all economic parameters are deterministic. The discount rate and the analysis time step are fixed throughout the study. The condition states are mutually exclusive and collectively exhaustive. The last state represents failure. In the do-nothing deterioration model it can be treated as absorbing. Under maintenance actions it can be repairable, because interventions may restore the asset from the failed condition. The optimal policy is stationary and does not change with time. All cash flows are expressed in constant monetary units, and no stochastic interaction between technical and financial parameters is introduced. These assumptions keep the model mathematically consistent, transparent, and easy to calibrate while maintaining compatibility with standard reliability and cost–benefit frameworks used in engineering asset management.

The economic parameters used in the numerical example are scenario-based values. They represent a high-value hotel asset and are used to demonstrate the proposed computational framework. They should not be treated as audited financial data from a specific building operator. Therefore, the numerical values of NPV, BCR, EAC, and EANB depend on the assumed discount rate, capitalization rate, action costs, downtime factor, failure cost, and transition probabilities. The main value of the example is comparative. It shows how different maintenance policies can be evaluated within one consistent Markov–CBA framework. Future work should include broader inspection datasets, real maintenance costs, downtime records, and wider uncertainty analysis of transition and economic parameters.

The framework is not intended to be universal without recalibration. In this paper, it is applied to a building asset represented by a hotel case study. The approach is suitable for assets that can be described by discrete condition states, inspection-based transition data, defined maintenance actions, and monetary cost–benefit parameters. Other infrastructure systems, such as transport networks, utilities, or industrial facilities, may require different state definitions, deterioration models, operational constraints, and cost structures. For this reason, each new application should be calibrated according to the specific asset type and management context.

In practical building management the frequency and quality of inspections vary, and this affects how deterioration is detected and corrected. These differences influence the empirical transition data and the timing of maintenance actions. Including information on inspection practice and typical repair delays in future work could improve the calibration of transition rates and lead to more realistic forecasts of long-term economic performance.

3. Materials and Methods: Input Data

The numerical study is based on a scenario-based hotel asset case. The case is used to demonstrate and verify the proposed computational workflow for a building-management problem. It should not be interpreted as a full empirical validation for all infrastructure systems. The assumed initial market value of the building is 50 million EUR, and the capitalization rate is 7% per year. The annual net operating income is therefore 3.5 million EUR, which corresponds to a 5-year gross benefit of 17.5 million EUR when the asset is in the best condition. The analysis uses a constant time step of 5 years and a discount rate of 4% per year. The total horizon consists of 12 steps, which represent 60 years of operation.

The technical condition of the hotel is represented by four discrete states. State S1 corresponds to a very good condition, S2 to a moderate condition, S3 to a poor condition, and S4 to failure. Each state is assigned a service value multiplier that scales the expected benefit over one 5-year step. The multipliers equal 1.00, 0.93, 0.75, and 0.00 respectively, meaning that the expected benefit gradually decreases with deterioration and vanishes in the failure state. The resulting 5-year benefit vector B is equal to [17.50, 16.28, 13.13, 0.00] million EUR.

Routine operating costs depend on the expected condition reached after the transition. They are defined as [2.00, 2.50, 3.20, 0.00] million EUR per 5-year step for states S1 to S4 respectively. The monetary consequence of failure is represented by a single scalar value $C_{fail} = 4.0$ MEUR, which is weighted by the probability of entering the failure state in the next step. All cost and benefit flows are expressed in constant prices. Four maintenance actions are considered in the analysis: do-nothing (DN), preventive maintenance (PM), major repair (MR), and replacement (RE). Each action is characterized by a direct cost and a specific transition model that reflects the technical effect of the intervention. The direct costs per 5-year step are given in Table 1.

Table 1. Direct action costs per 5-year step (MEUR).

State	DN	PM	MR	RE
S1	0.00	1.20	5.00	15.00
S2	0.00	1.20	5.00	15.00
S3	0.00	1.20	5.00	15.00
S4	0.00	0.00	0.00	15.00

In this study the maintenance actions—do-nothing, preventive maintenance, major repair and replacement—are defined at the level of the whole hotel asset. The building components used to construct the inspection count matrix in the authors' previous studies [22,23], such as foundations and structural walls, contribute only to the global condition rating. They are not treated as separately replaceable items in the present hotel-level model. The replacement action is therefore interpreted as a complete renewal or redevelopment of the building rather than a physical replacement of a single component.

A downtime factor of 0.20 is applied to the benefit during the period of replacement, which means that only 80% of the 5-year income is credited in that step. Logical constraints are imposed to exclude non-physical decisions. The do-nothing and preventive maintenance actions are not permitted in the failure state, reflecting the practical rule that a failed asset must be either repaired or replaced.

The initial condition of the asset is estimated from inspection data collected over a reference population of similar hotels. The inspection count matrix (see Table 2) contains the number of observed transitions between states during the previous service period. Each row corresponds to the current state and each column to the next state [22].

Table 2. Inspection count matrix C used for the do-nothing (DN) transition model.

From/To	S1	S2	S3	S4
S1	1	1	0	0
S2	0	1	6	0
S3	0	0	5	5
S4	0	0	0	6

The matrix C provides a realistic empirical basis for the estimation of transition probabilities under the do-nothing scenario. Each row is normalized by its total count,

and when a row contains zero observations, a uniform distribution is substituted. This standard correction guarantees a valid row-stochastic structure of the empirical matrix P_{DN}^{emp} . The initial state distribution p_0 is defined from the row totals of C , which results in $[0.08, 0.28, 0.40, 0.24]$. This distribution is used both as the starting condition for forward simulation and as the weighting vector in the linear program that computes the optimal value function.

The inspection dataset is small, and this limits the statistical reliability of the empirical transition probabilities. Therefore, in this paper the count matrix is treated as a scenario-based input for demonstrating the proposed framework. In practical applications, transition matrices should be calibrated using larger inspection datasets and statistically regularized estimation procedures, especially when some transition counts are low or zero [8,15,16]. In practical applications, transition matrices should be calibrated using larger inspection datasets [15,22,23]. As a practical screening rule, at least 20 observed departures from each origin state and at least five observations for each admissible transition estimated without regularization are recommended. If these thresholds are not met, the estimates should be treated as exploratory and supported by pooled data or regularized estimation. These values are heuristic and do not provide a universal guarantee of statistical validity.

The transition data used in this study follow the inspection-based procedure developed in the authors' previous works on the Markov modeling of building service life [22,23]. These studies describe the definition of condition states, the use of inspection records, and the construction of empirical count matrices in more detail. In the present paper, the same data structure is used as input for the Markov decision and CBA framework. The count matrix should therefore be understood as an inspection-based transition dataset for a reference group of similar hotel assets. It is not intended to represent a complete statistical database for all building types.

Transition models for the actions PM, MR, and RE are calibrated to represent typical effects of maintenance on the deterioration process. Preventive maintenance slows down deterioration relative to the do-nothing case and reduces the probability of downgrades. Major repair shifts probability mass toward better states and almost eliminates the chance of direct failure in the next step. Replacement resets the system to the best condition with a dominant probability of entering S1 and a smaller probability of entering S2. These assumptions are implemented in both discrete-time and continuous-time forms, using either transition matrices $P(a)$ or generator matrices $Q(a)$ that satisfy Equation (3).

The parameters described above define the economic and technical environment of the case study. The annual discount rate of 4% results in a 5-year discount factor $\gamma = (1.04)^{-5} = 0.8219$. The total simulation horizon of 60 years is sufficient to reach a steady distribution of states and to evaluate the long-term performance of alternative maintenance policies. A terminal value is included as a perpetuity of the last step flows scaled by $\frac{\gamma^T}{(1-\gamma)}$, which provides realistic cumulative economic indicators without the need to extend the time horizon further. In summary, the data set represents a high-value hotel property with predictable deterioration behavior and economically meaningful maintenance actions. The chosen parameters are consistent with practical experience in building management and provide a clear numerical base for comparing discrete and continuous Markov formulations and for evaluating the optimal cost-benefit strategy.

4. Results and Discussion

Figures 1–5 were prepared from one computational workflow. First, the optimal stationary policy was obtained from the Markov decision problem. This policy was then used in the discrete-time and continuous-time simulations. Figures 1 and 2 present the discrete-time results, including state probabilities and cumulative discounted economic

values. Figures 3–5 present the continuous-time results, including state probabilities, reliability indicators, and cost–benefit values obtained by time integration. The horizontal axes show time, either in 5-year steps or in years. The vertical axes show probabilities, reliability measures, or monetary values. The numerical checks included stochastic consistency of transition matrices, generator consistency in the continuous-time model, non-negative probabilities, conservation of total probability, stationary distribution checks, and consistency between the discrete and continuous representations where applicable.

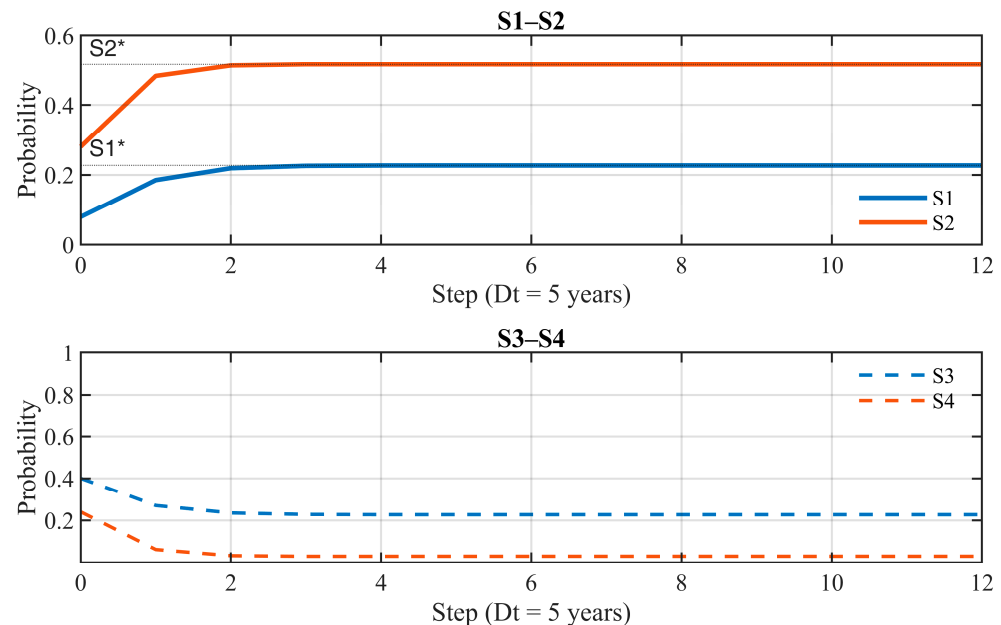


Figure 1. Discrete-time state probabilities under the optimal policy. The asterisk (*) denotes the stationary probability of the corresponding state.

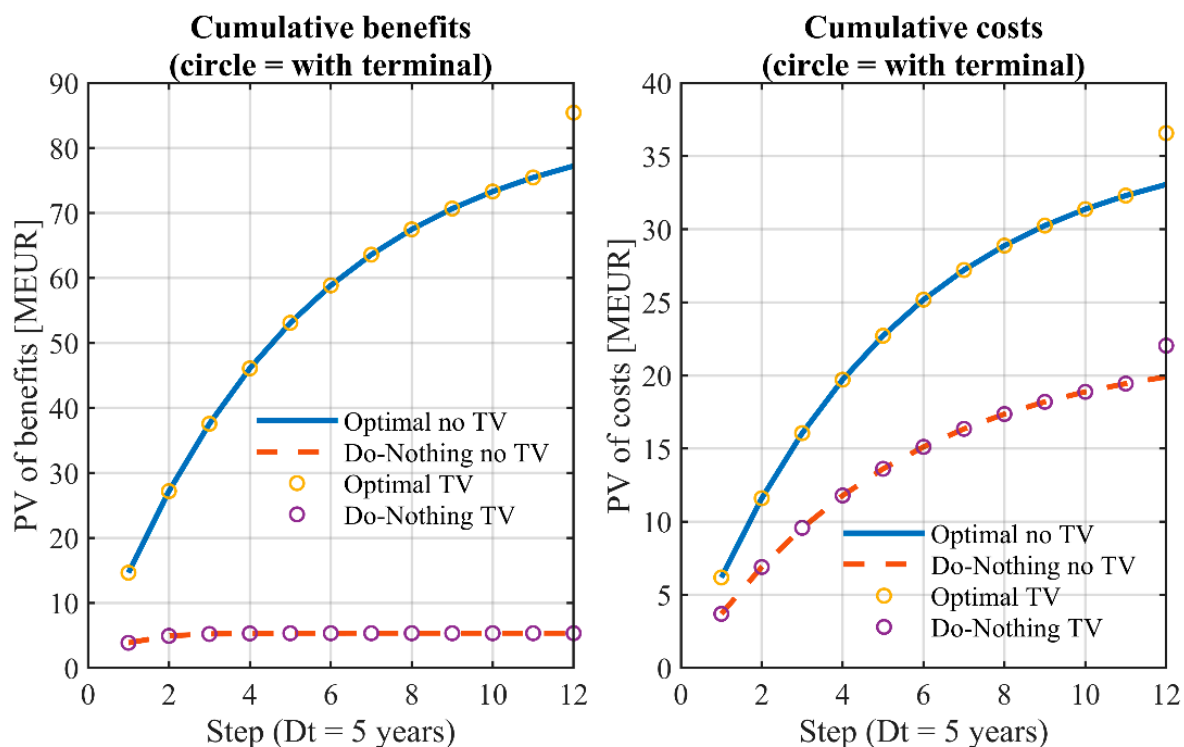


Figure 2. Cumulative discounted benefits and costs in the discrete-time model.

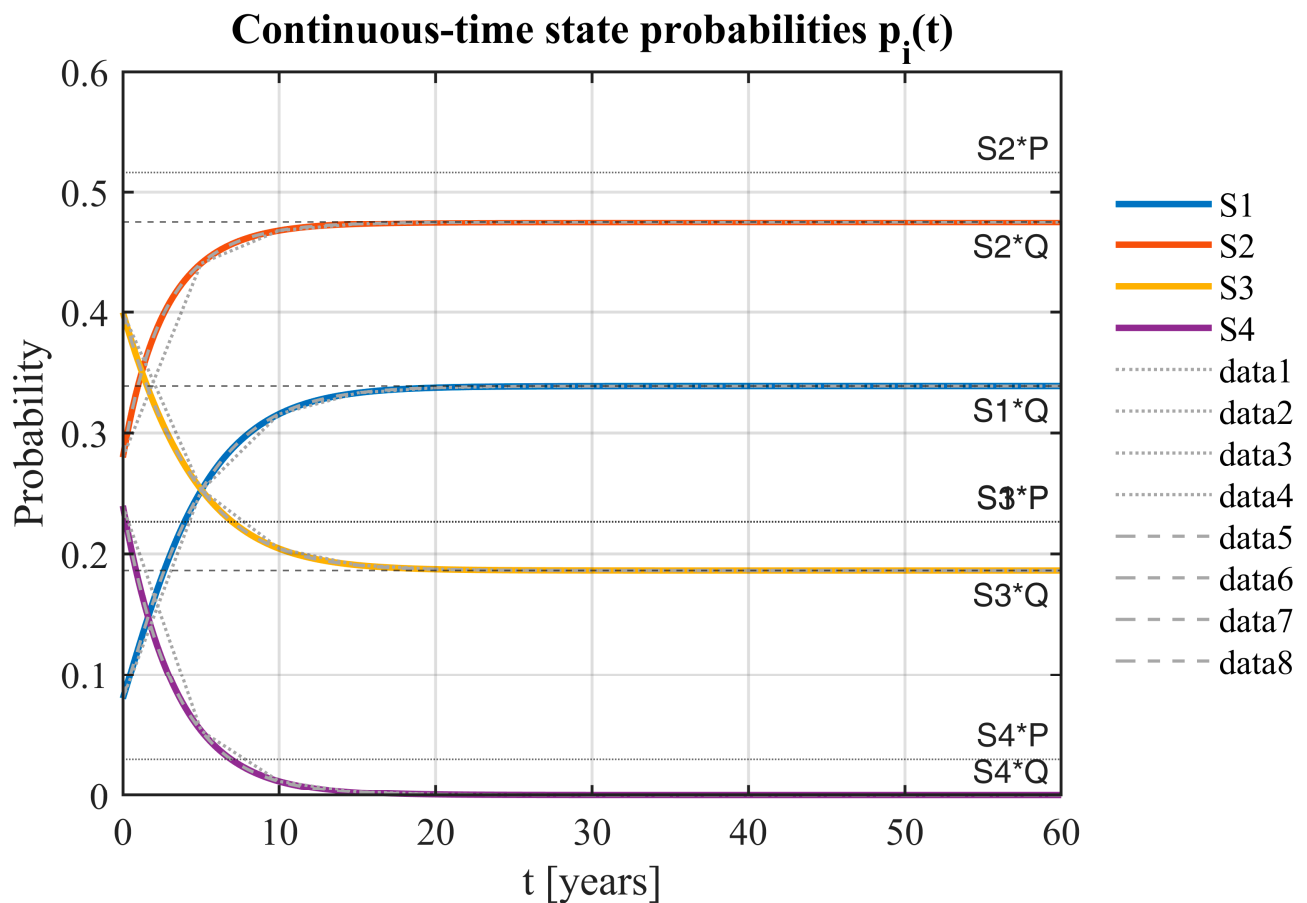


Figure 3. Discrete- and continuous-time state probabilities under the optimal policy. The asterisk denotes a stationary value, while P and Q refer to the discrete transition matrix and the continuous-time generator, respectively.

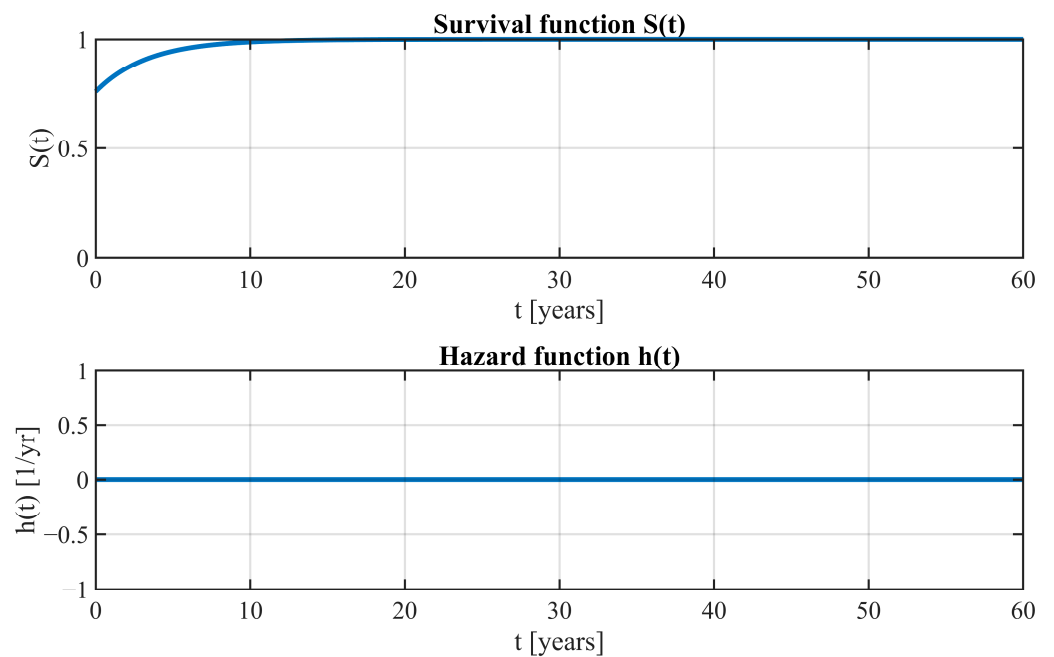


Figure 4. Survival and hazard indicators in the continuous-time model.

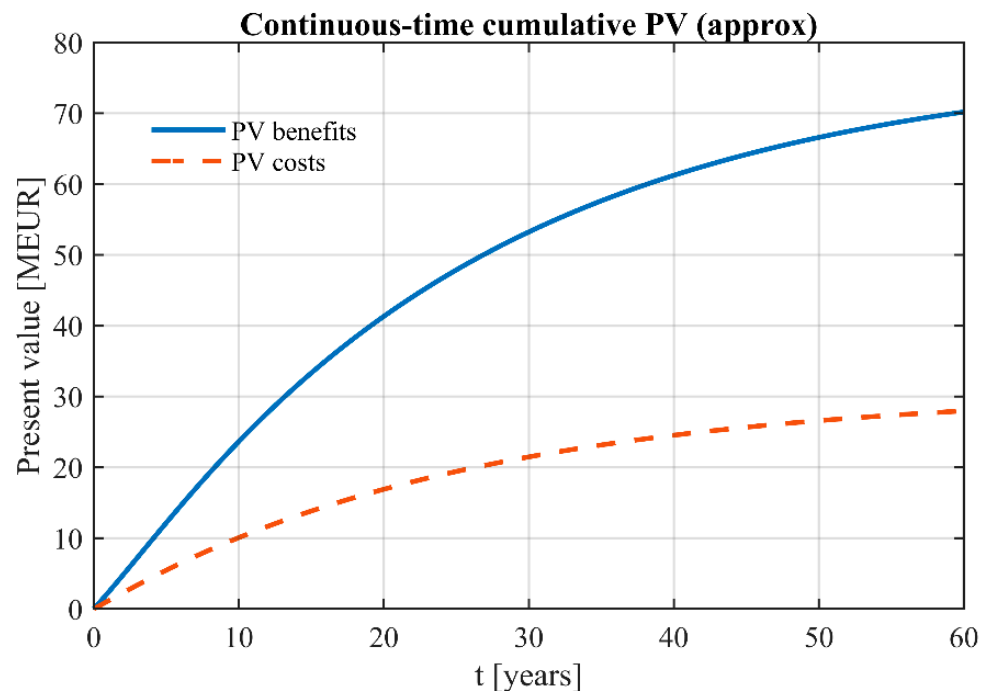


Figure 5. Continuous-time cost–benefit analysis under the optimal policy.

The continuous-time variant of the model was used to evaluate the optimal maintenance policy for the hotel asset. The optimization returned a stationary policy prescribing preventive maintenance in the best condition and major repair in all other states ($S1 \rightarrow PM$, $S2 \rightarrow MR$, $S3 \rightarrow MR$, $S4 \rightarrow MR$). The corresponding trajectories of the discrete-time model, shown in Figure 1, demonstrate a fast redistribution of state probabilities during the first three steps and stabilization after approximately 15 years. The share of the failure state $S4$ decreases from 0.24 to below 0.03, while the combined share of the two best states exceeds 0.70. This confirms that the optimal strategy suppresses degradation and concentrates the system in the upper condition levels.

The cumulative economic outcomes are summarized in Figure 2, which shows the present value of benefits and costs over the 60-year analysis period. Dots correspond to 5-year time steps representing the cumulative discounted total value (TV) under the optimal and do-nothing strategies. The left panel indicates that the total discounted benefits under the optimal policy reach about 76.00 MEUR, whereas the do-nothing case remains below 10.00 MEUR. The right panel illustrates that the optimal strategy requires higher expenditures in the early decades but still yields a substantially larger benefit–cost margin at the end of the period. The resulting economic indicators, $NPV = 48.86$ MEUR and $BCR = 2.34$, clearly outperform the do-nothing baseline with $NPV = -16.70$ MEUR and $BCR = 0.24$. The equivalent annual cost and net benefit are $EAC = 1.62$ MEUR/year and $EANB = 2.16$ MEUR/year, confirming that the optimized policy provides stable and positive long-term returns.

The comparison between the discrete and continuous representations is shown in Figure 3. In this figure, data1 to data4 denote the discrete-time state probabilities for states $S1$ to $S4$. Data5 to data8 denote the continuous-time trajectories computed from the generator. The curves overlap closely, especially at the inspection instants. This confirms the action-level link between the discrete and continuous transition models. The embedded discrete model based on the generator reproduces the CTMC stationary distribution to numerical precision. The check gives $L_1(\pi_Q, \pi_{P[\exp(Q\pi\Delta t)]}) \approx 5 \times 10^{-16}$. The discrete stationary distribution π_P differs from π_Q , with $L_1(\pi_P, \pi_Q) = 0.224$.

This difference is not a numerical artifact. It is caused by decision timing. In the discrete model the selected action is held over the full step Δt . In the continuous-time feedback model the action can change immediately after a state change.

The reliability indicators under the optimal policy are presented in Figure 4. The function $S(t) = 1 - p_4(t)$ is the probability that the system is not in the failure state S4 at time t . Since S4 is repairable under the selected actions, $S(t)$ should be interpreted as an availability-type measure. The hazard $h(t)$ is computed as the conditional intensity of entering S4, based on the generator. For the present optimal policy and the adopted generator, the transition rates into S4 from states S1 to S3 are zero. Therefore $\lambda_{in}(t) = 0$ and $h(t) = 0$ for all t . The probability mass in S4 decays quickly because transitions from S4 back to a functioning state are allowed.

The zero hazard obtained here is a property of the illustrative generator and should not be interpreted as zero real-world risk. Practical applications should include a small non-zero baseline transition rate from each operating state to S4. This rate can represent sudden failures, hidden defects, accidents, or force majeure events. It should be calibrated using failure records or expert risk assessment. Its inclusion would produce a positive hazard and may lead to a more conservative maintenance policy.

The continuous-time cost–benefit evolution shown in Figure 5 confirms the same qualitative trends observed in the discrete model. The discounted benefits grow smoothly to about 70.00 MEUR over the 60-year horizon. The discounted costs grow to about 28.00 MEUR. The continuous-time evaluation gives NPV = 42.22 MEUR and BCR = 2.51. These values are close to the discrete-time results, where NPV = 48.86 MEUR and BCR = 2.34. Differences are expected because the discrete plots include a perpetuity terminal value, while the continuous-time plot is a finite horizon integral. The continuous-time cost curve includes operating costs, action costs, and the expected failure entry cost based on the failure entry intensity.

This comparison also quantifies the practical effect of the different decision-timing assumptions. The difference between the discrete and continuous stationary distributions is reflected in the economic indicators. In the present case, the discrete-time NPV is higher by 6.64 MEUR, while the continuous-time BCR is higher by 0.17. These differences are noticeable, but they do not change the main economic conclusion. In both formulations, the optimized policy remains clearly beneficial.

The sensitivity analysis for reference study periods of 50, 75, and 100 years yields identical NPV and BCR values in the discrete-time model. This confirms that the perpetuity terminal captures the long tail of discounted flows. The consistency check between the CTMC stationary distribution and the embedded discrete model gives $L_1(\pi_Q, \pi_{P[\exp(Q\pi\Delta t)]}) \approx 5 \times 10^{-16}$. This confirms steady state equivalence for the embedded matrix $\exp(Q\pi\Delta t)$. The residual difference $L_1(\pi_P, \pi_Q) = 0.224$ is not a numerical artifact. It reflects the different decision timing between the discrete step model and the continuous-time feedback model.

A sensitivity analysis was also performed for two key economic parameters: the annual discount rate and the failure cost. The discount rate varied from 2% to 6%, and the failure cost varied from 2 to 8 MEUR. These parameters were selected because they directly affect the present value of future cash flows and the economic consequence of entering the failure state.

The results confirmed the robustness of the main economic conclusion. In the discrete model, the optimized policy remained almost unchanged over the tested range. The NPV varied from 43.56 to 129.19 MEUR, while BCR varied from 3.12 to 3.82. In the continuous-time model with discrete cost–benefit evaluation, NPV varied from 33.71 to 93.66 MEUR, while BCR varied from 2.28 to 2.99. In the continuous-time model with continuous cost–

benefit integration, NPV varied from 29.45 to 74.34 MEUR, while BCR varied from 2.48 to 3.39. In all variants, the optimized policy remained economically superior to the do-nothing strategy. The sensitivity analysis shows that the discount rate has the strongest effect on NPV, whereas the failure cost mainly affects cases in which the policy allows a higher probability of entering the failure state.

An additional stress test was performed at the base discount rate of 4%, with failure costs of 10.00, 15.00, and 20.00 MEUR. At 10.00 and 15.00 MEUR, the optimal policies remained unchanged. At 20.00 MEUR, the CTMC-based policy changed from preventive maintenance in S1 and major repair in S2–S4 to major repair in all states. The discrete synchronized policy remained unchanged. This result shows that very high failure consequences can shift the strategy toward earlier and more conservative intervention.

Overall, the results demonstrate that preventive maintenance in the best condition and systematic major repair in lower states form an economically optimal and technically stable policy for the assumed inputs. The discrete simulation shows a fast reduction in the failure share and rapid stabilization of state probabilities. The continuous-time trajectories provide a smooth interpolation and confirm the same short-term trend. The action-level link between discrete and continuous transitions is exact. Policy level steady states can differ when decision timing differs between discrete and continuous formulations. This should be considered when interpreting long-run probabilities and reliability indicators.

To verify the robustness of the conclusions, the analysis was repeated using the discrete-time variant with enforced $P \leftrightarrow Q$ consistency. The optimal policy in this case prescribes preventive maintenance in states S1 and S2, major repair in S3, and replacement in S4, reflecting a more preventive strategy than in the continuous-time formulation.

The system spends more than 90% of the time in the two best condition states $\pi_P \approx [0.60, 0.33, 0.07, 0.00]$, and the probability of being in the failure state falls below 0.2%.

The corresponding value function increases slightly due to lower maintenance costs, and the total net present value rises to 64.80 MEUR with a benefit–cost ratio of 3.41. This confirms that the decision outcome is insensitive to the modeling framework and remains economically sound when discrete transitions are synchronized with their continuous-time counterparts. The consistency metrics $L_1(\pi_P, \pi_Q) = 0.045$ and $L_1(\pi_Q, \pi_{P[expmQ]}) \approx 10^{-15}$ demonstrate excellent numerical agreement between the two formulations.

In summary, the discrete synchronized model reproduces the same qualitative behavior observed in Figures 1–5, strengthens the economic indicators, and validates the stability of the integrated Markov–CBA approach under alternative parameterizations.

5. Conclusions

This study answered the research questions stated in the Introduction. The first question concerned the possibility of combining deterioration modeling, decision optimization, and cost–benefit analysis in one consistent framework. The results show that such integration is possible. The Markov model describes the evolution of condition states. The Markov decision process identifies the optimal maintenance policy. The cost–benefit module converts the technical results into economic indicators useful for asset management.

The second question concerned the practical use of the framework for a realistic building-management case. The numerical example confirmed that the method can be applied to a high-value hotel asset with inspection-based transition data and scenario-based economic parameters. The model produced an interpretable maintenance policy and allowed comparison with the do-nothing strategy. The optimized policy reduced the probability of failure and improved the long-term economic performance of the asset.

The third question concerned the relation between discrete-time and continuous-time formulations. The study showed that both formulations are consistent at the action level when the transition matrix and generator are linked for the selected time step. At the same time, they may lead to different stationary distributions and slightly different economic indicators. This difference is not a numerical error. It results from different decision timing. In the discrete-time model, the selected action is kept over the full time step. In the continuous-time feedback model, the action can change immediately after a state transition.

The main added value of the paper is the unified computational structure. It connects Markov deterioration modeling, reliability analysis, maintenance policy optimization, and cost–benefit evaluation. The framework is transparent and reproducible. It can support strategic maintenance planning, budgeting, and comparison of alternative intervention policies. It also shows how technical reliability indicators and economic indicators can be evaluated within the same model.

Several difficulties were identified during the study. The first difficulty concerns the calibration of transition probabilities from limited inspection data. The second concerns the interpretation of economic parameters, because real repair costs, downtime, and failure consequences may vary between assets and operators. The third difficulty concerns the comparison of discrete-time and continuous-time results, since both models use different decision-timing assumptions. These issues do not invalidate the method, but they should be considered when applying it to real asset portfolios.

The study has some limitations. The transition probabilities, maintenance costs, discount rate, and operating conditions were assumed to be fixed within the analyzed scenario. The economic parameters were used as scenario-based values and should not be interpreted as audited financial data from a specific operator. The numerical results therefore illustrate the behavior of the proposed framework rather than provide a universal economic benchmark for all building assets.

Future research should include broader empirical datasets from building managers and infrastructure operators. Further work should also include real maintenance costs, repair delays, downtime records, and observed failure consequences. Further sensitivity analysis should cover transition probabilities, action costs, benefit assumptions, and other sources of economic uncertainty. The framework can also be extended to include uncertainty, fuzzy parameters, time-dependent deterioration rates, Bayesian updating, and non-stationary maintenance policies. These extensions would improve empirical validation and increase the practical value of the method for long-term infrastructure management.

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Abbreviations

The following abbreviations are used in this manuscript:

MDP	Markov Decision Process
CBA	Cost–Benefit Analysis
CTMC	Continuous-Time Markov Chain
NPV	Net Present Value
BCR	Benefit–Cost Ratio
EAC	Equivalent Annual Cost
EANB	Equivalent Annual Net Benefit

Appendix A

Markov Chain Cost–Benefit Optimization Framework

The algorithm presents the computational framework used to couple the Markov chain deterioration model with the cost–benefit optimization. The algorithm integrates a Bellman-based linear programming formulation for optimal policy selection with the subsequent simulation of system evolution and discounted economic evaluation.

Input:

- Transition matrices $P(a)$ or generators $Q(a)$ for each action a in {DN, PM, MR, RE}
- Economic parameters: $B(i)$, $Cop(i)$, C_{fail} , $C_{act}(i,a)$
- Time parameters: step Dt , annual rate r , discount factor $\gamma = (1+r)^{-Dt}$
- Simulation horizon T and terminal value option
- Initial state distribution p_0
- Model options: `mode_trans` in {ctmc, discrete}, `enforce_consistency` in {true, false}
- Optimization mode: `mode_mdp` in {valueLP, occupancyLP}

Output:

- Optimal policy $\pi^*(i)$
- State trajectories p_t under π^*
- Stationary distributions π_P and π_Q (if CTMC)
- Economic indicators: NPV, BCR, EAC, EANB
- Continuous-time economic indicators (if CTMC)
- Reliability metrics (if CTMC): $S(t)$, hazard $h(t)$, MTTA (for absorbing interpretation)

Procedure:

1. Initialize model
 - Read input data and normalize transition counts.
 - Compute initial distribution p_0 and discount factor γ .
2. Generate consistent transition model
 - if `mode_trans` == 'ctmc' then
 - For each action a :
 - Clean generator $Q(a)$
 - Compute discrete matrix $P(a) = \expm(Q(a) \cdot Dt)$
 - Project $P(a)$ to a valid row-stochastic matrix
 - else if `mode_trans` == 'discrete' then
 - For each action a :
 - Read empirical $P(a)$
 - Project $P(a)$ to a valid row-stochastic matrix
 - if `enforce_consistency` == true then
 - Regularize $P(a)$ to ensure positive eigenvalues
 - Compute $Q(a) = \logm(P(a)) / Dt$

- Clean $Q(a)$ to a valid generator
 - Overwrite $P(a) = \expm(Q(a)*Dt)$
 - Project $P(a)$ to a valid row-stochastic matrix
 - end if
- end if
- 3. Define economic environment
 - For each state i and action a :
 - Compute one-step reward:
 - $R(i,a) = E[B] - E[Cop] - E[Cfail] - Cact(i,a)$
 - where expectations are evaluated over $P(i, :, a)$
 - Apply downtime reduction for RE actions if required.
- 4. Solve Markov Decision Process (MDP)
 - if mode_mdp == 'valueLP' then
 - Solve Bellman LP:
 - minimize $\sum_i \mu(i)*V(i)$
 - subject to $V(i) \geq R(i,a) + \gamma*(P(i, :, a)*V)$
 - for all states i and all actions a
 - Obtain optimal values $V^*(i)$ and policy:
 - $\pi^*(i) = \operatorname{argmax}_a [R(i,a) + \gamma*(P(i, :, a)*V^*)]$
 - else if mode_mdp == 'occupancyLP' then
 - Solve discounted occupancy LP:
 - maximize $\sum_{i,a} x(i,a)*R(i,a)$
 - subject to flow-balance constraints
 - Obtain optimal occupancy x^* and policy $\pi^*(i)$
 - end if
- 5. Build stationary transition matrix under π^*
 - For each state i :
 - $P_{\pi}(i, :) = P(i, :, a = \pi^*(i))$
 - Compute stationary distribution π_P from P_{π}
 - If $Q(a)$ available:
 - Construct Q_{π} from selected rows of $Q(a)$
 - Compute π_Q
 - Compute embedded matrix $P_{emb} = \expm(Q_{\pi}*Dt)$
 - Check consistency between π_Q and the stationary distribution of P_{emb}
 - Report also the difference between π_P and π_Q
- 6. Simulate discrete-time trajectory
 - Set $p(0) = p_0$
 - For $t = 0$ to $T-1$:
 - $p(t+1) = p(t)*P_{\pi}$
 - Compute expected step benefits and costs:
 - $EB_t = \sum_i p(t,i)*E[B \mid i, \pi^*(i)]$
 - $EC_t = \sum_i p(t,i)*(E[Cop \mid i, \pi^*(i)] + E[Cfail \mid i, \pi^*(i)] + Cact(i, \pi^*(i)))$
 - where expectations are evaluated over $P(i, :, a = \pi^*(i))$
 - Discounted present values:
 - $PV_B(t) = \gamma^t * EB_t$
 - $PV_C(t) = \gamma^t * EC_t$
 - End loop
 - Compute cumulative sums:
 - $PV_B = \sum_t PV_B(t)$
 - $PV_C = \sum_t PV_C(t)$

$$NPV = PV_B - PV_C$$

$$BCR = PV_B / PV_C$$

EAC, EANB from standard annuity factor.

7. Continuous-time analysis (optional, if CTMC)
 - Compute state probabilities $p(t) = p_0 \cdot \expm(Q_{pi} \cdot t)$
 - Derive survival function $S(t) = 1 - p(t, \text{failure})$
 - Compute failure-entry intensity $\lambda_{in}(t) = \sum_{i \neq \text{failure}} p_i(t) \cdot q_{i, \text{failure}}$
 - Compute hazard $h(t) = \lambda_{in}(t) / S(t)$
 - If absorbing interpretation is used:
 - Integrate mean time to absorption (MTTA) = integral of $S(t)$
 - Approximate continuous-time NPV by integration over discounted benefit and cost rates.
8. Reporting and visualization
 - Print optimal policy $\pi^*(i)$ and value function $V^*(i)$
 - Report stationary distributions π_P and π_Q
 - Display cost-benefit indicators: NPV, BCR, EAC, EANB
 - Plot:
 - Discrete state trajectories p_t
 - Cumulative PV of benefits and costs
 - Continuous-time $p_i(t)$, $S(t)$, $h(t)$ (if CTMC)
 - Sensitivity of CBA over RSP horizons (50, 75, 100 years)

End Procedure

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