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Bayesian Tree–Spline Modeling of Circular Bioeconomy Potential in Regional Agricultural and Livestock Production Systems

Dariusz Sala ^{1,*} , Olena Pavlova ^{1,2} , Kostiantyn Pavlov ² , Olexander Korniietskyi ³ , Volodymyr Orel ³, Anna Orel ⁴, Mohammad Jammal ⁵  and Michał Pyzalski ^{1,6,*} 

¹ Faculty of Management, AGH University of Krakow, Gramatyka 10, 30-067 Kraków, Poland; strishenez@gmail.com

² Faculty of Economics and Management, Lesya Ukrainka Volyn National University, Voli Ave 13, 43025 Lutsk, Ukraine; pavlovkostya@gmail.com

³ Livestock Farming Institute, National Academy of Agrarian Sciences of Ukraine, Tvarynnykyv 1A, 61026 Kharkiv, Ukraine; ksmank@gmail.com (O.K.); vova7003@gmail.com (V.O.)

⁴ Faculty of Economics and Management, Kyiv National University of Technology and Design, Mala Shihanovska 2, 01011 Kyiv, Ukraine; anna.orel1904@gmail.com

⁵ Department of Management, Faculty of Business and Law, The British University in Dubai, Gate Village, Building 3-2, Level-1, 2 and 3—Zaa'beel Second, DIFC, Dubai P.O. Box 345015, United Arab Emirates; muhammad.jammal@buid.ac.ae

⁶ Faculty of Materials Science and Ceramics, AGH University of Krakow, al. Adama Mickiewicza 30, 30-059 Kraków, Poland

* Correspondence: sala@agh.edu.pl (D.S.); michal.pyzalski@agh.edu.pl (M.P.)

Abstract

This study investigates the structural transformation of regional livestock production systems toward an integrated Circular Bioeconomy (CBE) framework. Using an integrated Bayesian Tree–Spline approach, spatially disaggregated regional data were analyzed to evaluate the environmental, technological, and logistical determinants of sustainable livestock production. The empirical model incorporated key indicators, including infrastructure readiness, market accessibility, manure intensity, biogas conversion rates, methane emissions, alternative feed utilization, and transport-related emissions. The probabilistic classification results revealed substantial spatial heterogeneity among the analysed regions, enabling the identification of areas with high circular bioeconomy implementation potential, transitional regions, and areas requiring further technological and infrastructural development. Shannon entropy measures were applied to quantify classification uncertainty and identify transitional zones, while penalized B-splines captured non-linear saturation effects associated with alternative feed utilization and the integration of bioenergy technologies. The findings provide a data-driven basis for supporting the implementation of circular bioeconomy principles, enhancing resource-use efficiency, reducing greenhouse gas emissions, and facilitating the transition toward climate-neutral livestock production systems. Compared with the deterministic benchmark model, the proposed Bayesian Tree–Spline framework additionally quantifies classification uncertainty, captures nonlinear threshold and saturation effects, and provides probabilistic regional typologies that support more robust and evidence-based policy recommendations.

Keywords: circular bioeconomy; sustainable livestock production; Bayesian tree–spline modelling; spatial heterogeneity; soft clustering; resource efficiency; biogas conversion; sustainable supply chains; decarbonization



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1. Introduction

The contemporary agricultural sector faces extensive challenges that necessitate a fundamental shift in development and modernization strategies. Enhancing the efficiency and resilience of agricultural production systems requires the formulation of new strategies for the utilization of regional agricultural resource potential [1] and the establishment of specialized agricultural market clusters capable of strengthening long-term competitiveness [2]. In the context of rapidly changing economic and environmental conditions, disruptions in supply chains significantly affect the resilience of logistics systems [3], making last-mile delivery optimization a critical structural challenge [4]. Addressing these issues requires advanced logistics planning, including the integration of automated transport networks [5] and the development of collaborative delivery systems [6].

At the same time, environmental innovation and the low-carbon transformation of the agri-food sector have become increasingly important [7]. Livestock production, often identified as a significant source of greenhouse gas emissions, requires the implementation of solutions aligned with the principles of the circular bioeconomy. This approach enables the efficient utilization of biomass as a renewable energy resource, supporting the development of decentralized energy systems in rural areas [8] and enhancing energy independence through the application of biogas technologies [9]. However, this transition is constrained by substantial spatial disparities in regional production capacities and infrastructure development [10], as well as by persistent logistical limitations associated with the transportation and distribution of agricultural products [11].

From a methodological perspective, traditional deterministic models are insufficient for analysing complex and spatially heterogeneous agricultural systems. The assessment of latent regional typologies requires advanced data analysis methods based on multilevel hierarchical frameworks [12] and machine learning pattern recognition techniques [13]. Furthermore, identifying non-linear threshold effects associated with the adoption of biological and environmental innovations necessitates the application of generalized additive models [14] and advanced statistical learning approaches [15].

The theoretical foundations of sustainable supply chain management emphasize the necessity of developing decentralized and localized infrastructure, which enhances the resilience of production systems to disruptions while promoting more efficient resource utilization [16]. International institutional guidelines indicate that the transition of livestock production toward a circular bioeconomy constitutes one of the key mechanisms for achieving sustainable development objectives [17]. In the European context, the acceleration of climate-neutral farming systems is primarily based on optimizing regional nutrient cycles, improving biomass utilization efficiency, and integrating stringent carbon footprint certification standards [18,19].

However, the contemporary agricultural sector operates under conditions of increasing economic, social, and structural uncertainty, necessitating the adaptation of global development concepts to specific regional circumstances [20]. Research indicates that effective agricultural modernization should be based on the implementation of innovative technological and organizational solutions that enhance resource-use efficiency, reduce environmental pressures, and strengthen systemic resilience [21,22]. In this context, the livestock sector plays a particularly important role. On the one hand, it represents a significant source of biomass and by-products that can be utilized within circular economy frameworks; on the other hand, it is responsible for a substantial share of greenhouse gas emissions generated by agriculture.

The assessment of processes occurring within complex livestock production systems requires the application of advanced analytical methods [23]. To quantitatively describe the relationships among environmental, technological, and economic factors, researchers

commonly employ vector autoregression models for panel data (VAR) [24], structural VAR models [25], and spatial theories related to economic geography and trade flows [26]. At the same time, the analysis of non-linear relationships and threshold effects characteristic of circular economy systems necessitates the use of more advanced statistical approaches, such as Monte Carlo simulations [27]. Increasing attention has also been devoted to soft clustering techniques based on entropy theory [28], which enable the identification of transitional zones and provide a quantitative description of spatial ambiguity without the need to assign analysed units to rigid categories.

Despite these important methodological advances, existing studies still face several limitations when assessing regional circular bioeconomy potential. Most approaches rely either on deterministic clustering methods or on conventional regression models that assume fixed regional boundaries and predominantly linear relationships among explanatory variables. Such methods often provide limited insight into gradual transitions between regional development profiles, nonlinear saturation effects associated with technology adoption, and uncertainty in regional classification. Consequently, there remains a need for analytical frameworks capable of simultaneously integrating hierarchical decision structures, nonlinear functional relationships, and probabilistic assessment of regional heterogeneity within a unified modelling environment.

To bridge theoretical spatial models with the real-world conditions of regional livestock production systems, it is essential to utilize reliable and validated data sources. To operationalize the adopted theoretical framework, a spatial dataset covering the analysed regions was developed. Based on available institutional databases, seven key indicators were identified to characterize both the environmental potential and the logistical constraints associated with the implementation of circular bioeconomy principles.

The first of these is the infrastructure readiness index, which reflects the level of development of the technical and logistical infrastructure required for the implementation of modern environmental technologies. This indicator enables the identification of regions that require substantial infrastructure investments, as well as those that can effectively utilize existing resources to implement circular economy solutions. The second indicator is the market accessibility index, which reflects the spatial accessibility of domestic and international agricultural product markets and, consequently, influences logistics costs and the competitiveness of regional production systems.

Another important parameter is manure intensity, which measures the number of livestock units per unit area of agricultural land. This indicator serves as a fundamental measure of biomass availability for biogas production processes. Complementing this characteristic is the biogas conversion rate, which describes the extent to which agricultural waste is utilized for the production of biomethane and renewable energy, thereby reflecting the technological maturity of a region and its level of advancement in implementing circular bioeconomy solutions [29].

An important component of the analysis is methane intensity, which serves as an indicator of the environmental pressure associated with livestock production. It enables the identification of regions requiring the implementation of greenhouse gas emission mitigation measures and supports the assessment of the sector's decarbonization potential. Another indicator, the alternative feed ratio, reflects the extent to which conventional feed is replaced by by-products originating from the agri-food industry. This parameter serves as a measure of supply chain resilience and resource-use efficiency within the framework of the circular bioeconomy. The final parameter considered in the analysis is the transport emissions proxy, which quantifies the carbon footprint associated with the operation of regional supply chains and enables the assessment of the actual environmental effects of implementing decarbonization strategies in the agricultural sector [30].

The aim of this study was to assess the potential for implementing circular bioeconomy principles within regional livestock production systems by identifying the key environmental, technological, and logistical factors influencing the development of sustainable production models. Furthermore, the study sought to identify patterns of regional heterogeneity through advanced statistical modelling and probabilistic classification methods.

The principal methodological contribution of this study lies in integrating Classification and Regression Trees (CART) with penalized cubic B-splines within a Bayesian inference framework. Unlike conventional deterministic classification approaches, the proposed methodology simultaneously captures nonlinear relationships, quantifies posterior classification uncertainty, and identifies transitional regional development typologies, thereby providing a more flexible basis for evaluating circular bioeconomy implementation potential.

2. Methods

Assessing the potential for implementing circular bioeconomy principles in regional agricultural and livestock production systems requires analytical methods capable of simultaneously accounting for environmental, technological, and logistical factors. Due to the complexity of the relationships under investigation and the presence of substantial regional heterogeneity, this study employs an approach that combines conventional data exploration techniques with advanced probabilistic modelling. The proposed methodological framework enables the identification of regional development typologies, the assessment of classification uncertainty, and the analysis of non-linear relationships associated with the implementation of circular bioeconomy solutions.

Figure 1 presents the integrated analytical architecture developed to model the potential for circular bioeconomy implementation in regional agricultural and livestock production systems. The framework illustrates the process of integrating data derived from seven spatial indicators, the methodological division into a deterministic K-means benchmark model and a probabilistic Bayesian Tree–Spline model, and the final soft-clustering outputs used to support regional development recommendations. The analytical framework is structured into four distinct phases.

The analytical workflow was designed to improve methodological transparency by combining deterministic exploratory analysis with probabilistic classification within a unified modelling framework. The deterministic branch serves exclusively as a benchmark for comparison, whereas the Bayesian Tree–Spline model constitutes the principal analytical framework used for regional classification, uncertainty quantification, and nonlinear relationship estimation.

The process begins with the construction of a multidimensional dataset comprising seven environmental, technological, and logistical indicators. At this stage, agricultural production data are integrated with key spatial variables, particularly the Infrastructure Readiness Index and the Market Accessibility Index, which reflect differences in regional development conditions. All variables are standardized using the *z-score* transformation to ensure statistical stability and comparability among variables expressed in different measurement units, such as emission intensities, production indicators, and logistical parameters.

Phase 2. Comparative Modelling Pathways

The methodological framework is divided into two distinct analytical streams.

The first stream represents the deterministic benchmark model (red pathway), which employs Principal Component Analysis (PCA) for dimensionality reduction, followed by K-means clustering ($K = 3$). This approach serves as a hard-clustering baseline for identifying fundamental regional typologies. However, its main limitation lies in its

inability to adequately capture high levels of spatial uncertainty and gradual transitions between regional categories.

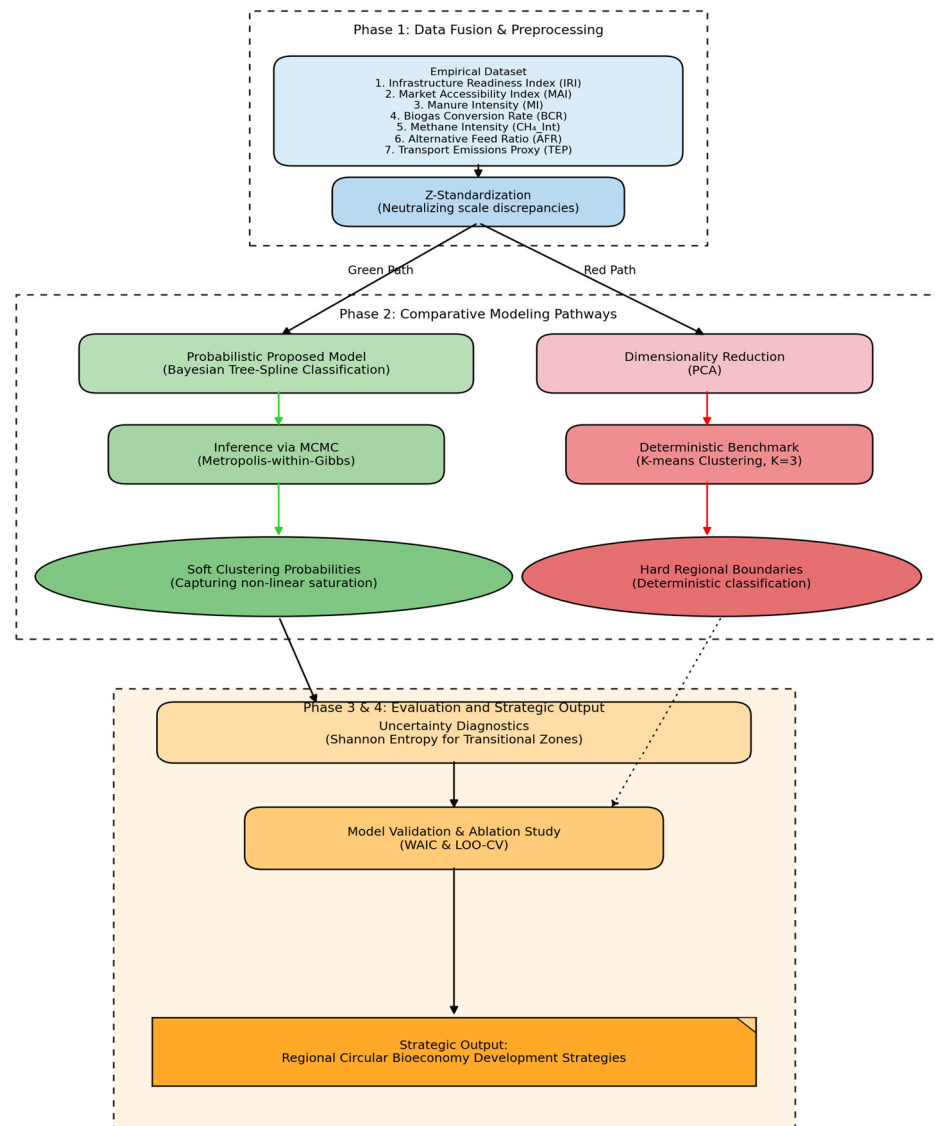


Figure 1. Data Integration and Preprocessing.

The second stream constitutes the proposed probabilistic model (green pathway), which represents the central methodological component of the study. The model is based on a Bayesian Tree–Spline Classification framework that combines the non-linear partitioning logic of Classification and Regression Trees (CART) with penalized cubic B–splines. Within this framework, the CART component recursively partitions the multidimensional predictor space into relatively homogeneous regional groups, while the penalized cubic B–spline component models smooth nonlinear relationships between the explanatory variables and posterior class probabilities within individual terminal nodes. Consequently, the proposed hybrid architecture combines interpretable hierarchical decision rules with flexible nonlinear estimation, allowing both robust regional classification and probabilistic uncertainty assessment within a single Bayesian modelling framework. This hybrid approach enables the identification of saturation effects associated with the implementation of circular bioeconomy technologies, such as diminishing returns resulting from increasing levels of biogas utilization or alternative feed adoption. Model parameters are estimated using Markov Chain Monte Carlo (MCMC) methods implemented through the

Metropolis-within-Gibbs algorithm [27,31]. Model convergence was evaluated using standard MCMC diagnostic procedures, including inspection of trace plots and convergence statistics presented in Appendix A. These procedures ensured numerical stability and reliable estimation of posterior class probabilities.

Phase 3. Soft Clustering and Uncertainty Analysis

The Bayesian framework generates soft-clustering results rather than rigid class boundaries. Instead of assigning a region to a single category, the model estimates posterior probabilities of membership in each developmental typology. This approach captures the uncertainty associated with classification outcomes and provides a more realistic representation of regional heterogeneity across the analysed regions. To quantify classification uncertainty, Shannon entropy is calculated according to $H = -\sum p \cdot \log_2 p$. All entropy values reported in the revised manuscript were recalculated directly from the posterior class-membership probabilities to ensure numerical consistency. High entropy values indicate the presence of transitional zones characterized by ambiguous development profiles and greater uncertainty regarding regional classification. Such regions may represent transitional development profiles that warrant further investigation, rather than directly indicating higher adaptive capacity or transformation potential.

Phase 4. Model Evaluation and Strategic Outputs

The final phase involves a comprehensive assessment of the model's predictive performance using the Watanabe–Akaike Information Criterion (WAIC) and Leave-One-Out Cross-Validation (LOO-CV). These validation metrics provide complementary measures of model adequacy, balancing goodness-of-fit, predictive performance, and model complexity, while reducing the risk of overfitting in Bayesian inference. In addition, an ablation analysis is conducted by systematically removing selected spatial variables to evaluate their contribution to classification performance and predictive accuracy. The final output consists of data-driven recommendations supported by the comparative validation results obtained from the WAIC, LOO-CV, and ablation analyses, providing a transparent basis for evaluating the robustness and predictive reliability of the proposed Bayesian Tree–Spline framework.

2.1. Data Collection and Selection of Variables

The empirical approach is based on a multidimensional dataset aggregating indicators characterising regional agricultural and livestock production systems. The data integration was carried out with the use of verified institutional sources, which ensured high reliability and comparability of the analysed information. Prior to the start of the analysis, all continuous variables were standardized with *z-score* in order to eliminate differences resulting from different measurement scales. Table 1 presents the multidimensional variable space and data sources used to assess the potential for circular economy implementation in regional livestock production systems.

Seven key indicators have been selected to reflect the core pillars of regenerative agriculture, including waste management, resource efficiency, and decarbonization of supply chains.

- Environmental and technological indicators: methane emission intensity, conversion to biogas and production intensity of natural fertilizers are the basic measures of environmental performance, biomass availability and the level of technological advancement of biomass into renewable energy conversion processes.
- Economic and logistics indicators: the share of alternative feed reflects the degree of development of regional resource cycles and the resilience of supply chains, while the transport emissions index, developed on the basis of the GLEC methodology, allows

the assessment of the level of decarbonisation of logistics processes, in particular at the *last-mile logistics* stage.

- Spatial variables: the model takes into account the *Infrastructure Readiness Index* and the *Market Accessibility Index*, which reflect regional variation in development conditions, the availability of technical and logistics infrastructure, and the possibilities of integration with national and international agricultural markets.

All data points were standardized with *z-score* to ensure convergence and stability of the calculation during the further modelling process using the Bayesian Tree–Spline model.

Prior to model estimation, all continuous variables were transformed using *z-score* standardization to ensure numerical comparability across heterogeneous measurement scales. The standardized variables served as input covariates for both the exploratory analysis and the Bayesian Tree–Spline model, thereby improving numerical stability and facilitating convergence of the Bayesian estimation procedure.

2.2. Exploratory Dimensionality Reduction

Prior to the use of probabilistic modelling, a preliminary analysis of the spatial data structure was performed. For this purpose, *Principal Component Analysis (PCA)* was used, enabling orthogonal transformation of a multidimensional variable space into the form of hidden components explaining the largest part of the data variance. Next, a deterministic K-means grouping algorithm ($K = 3$) was used [13,15,32], which served as a reference model based on *hard clustering*.

The parameter $K = 3$ was not chosen arbitrarily. Its value was determined a priori on the basis of the observed regional diversity of the analysed agricultural systems and livestock production. The number of clusters adopted reflects three basic development typologies of regions: areas with a high potential for the implementation of the circular economy, transition regions with a moderate level of development, and regions requiring further technological and infrastructure investments. To evaluate the robustness of this choice, exploratory analyses with alternative cluster numbers ($K = 2$ and $K = 4$) were also examined during the preliminary stage. These alternatives either merged developmentally distinct regional profiles or produced clusters with limited interpretability and reduced practical relevance. Consequently, $K = 3$ was retained as the most parsimonious and policy-relevant representation of regional circular bioeconomy development typologies.

However, the classical K-means algorithm has a limited ability to account for classification uncertainty, gradual transitions between regional categories, and the complex heterogeneity observed across regional agricultural systems [3,10,33]. Therefore, a more flexible probabilistic approach was needed, allowing simultaneous modelling of classification uncertainty and nonlinear relationships between the variables analysed. In the proposed analytical framework, the deterministic K-means solution serves exclusively as a benchmark for comparison with the probabilistic Bayesian Tree–Spline classification. Rather than replacing the exploratory clustering stage, the Bayesian framework extends it by quantifying posterior membership probabilities and allowing regions to exhibit transitional characteristics between development typologies. This complementary strategy improves the interpretation of regional classification under heterogeneous development conditions and provides a more realistic representation of uncertainty than deterministic partitioning alone. Consequently, the sequential application of deterministic and probabilistic methods enables evaluation of the additional explanatory value obtained through Bayesian inference while preserving the interpretability of the initial exploratory classification.

To facilitate a systematic comparison of the selected regional typologies, the principal characteristics of each cluster were summarized using a common analytical framework. The resulting classification provides a concise overview of the distinguishing features asso-

ciated with different levels of circular bioeconomy readiness and serves as a reference for the subsequent probabilistic modelling. In addition, the identified cluster profiles establish the baseline against which the Bayesian Tree–Spline model evaluates posterior membership probabilities and nonlinear transitions between regional development typologies. This sequential interpretation improves the transparency of the analytical framework and facilitates the comparison between deterministic and probabilistic classification outcomes. The main characteristics of the identified regional clusters are presented in Table 1.

Table 1. Characterization of variables, indicators and institutional data sources used in the study (baseline: 2025–2026).

Variable Code	Indicator Name	Unit	Primary Institutional Source	Reference
IRI	Infrastructure Readiness Index	Score (0–10)	World Bank Group & KSE Institute	[31,32]
MAI	Market Accessibility Index	Kilometers (km)	State Border Guard Service of Ukraine	[33]
MI	Manure Intensity	Heads/ha	USDA FAS & KSE Agrocenter	[34,35]
BCR	Biogas Conversion Rate	Percentage (%)	Bioenergy Association of Ukraine (UABIO)	[36]
CH ₄ _Int	Methane Intensity	Score (0–10)	Ministry of Environmental Protection	[37]
AFR	Alternative Feed Ratio	Percentage (%)	Green Transition Office (Ukraine)	[38]
TEP	Transport Emissions Proxy	Kilometers (km)	Smart Freight Centre (GLEC Framework)	[39–41]

2.3. Bayesian Tree–Spline Classification Model

The main analytical tool used in this study is a hybrid Bayesian model combining Classification and Regression Trees (CART) with penalized cubic B-splines. This approach allows for simultaneous modelling of linear and nonlinear relationships, identification of threshold effects, and consideration of uncertainty in the classification of regional agricultural systems and livestock production. In contrast to classical grouping methods, the proposed model allows probabilistic assignment of regions to specific developmental typologies, thereby providing a more realistic representation of regional heterogeneity across the analysed regions. Unlike conventional Bayesian additive models, the proposed framework explicitly combines recursive partitioning and smooth nonlinear estimation within a single hierarchical structure. The CART component identifies interpretable decision boundaries separating relatively homogeneous regional groups, whereas the penalized B-spline component captures gradual nonlinear responses within each partition. This integration allows the model to represent both abrupt structural changes and continuous saturation effects that characterize circular bioeconomy implementation across heterogeneous regional production systems.

$$y_i \in \{1, 2, \dots, C\}$$

denotes a hidden developmental typology (cluster) for regional observation. The model relies on a hidden additive predictor for each class, which takes the following form: η_{ic}

$$\eta_{ic} = \alpha_c + \sum_{j=1}^P f_j(X_{ij}) + \sum_{m=1}^M \beta_{mc} Z_{imc} + T(R_i) \quad (1)$$

where:

η_{ic} indicates a hidden additive predictor for a region belonging to the cluster ic ;

α_c is a class-specific free expression reflecting the basic probability of assigning a region to a specific developmental typology;

$\sum_{j=1}^P f_j(X_{ij})$ represents *penalized cubic B-splines*, describing nonlinear relationships between the analyzed variables and saturation effects associated with the implementation of circular economy solutions;

$\sum_{m=1}^M \beta_{mc} Z_{imc}$ represents the linear component of the model, where β_{mc} denotes the class-specific regression coefficients, and Z_{imc} denotes the vector of linear predictor variables, including, among others, the Market Accessibility Index (MAI) and the Infrastructure Readiness Index (IRI).

$T(R_i)$ means a component of the Bayesian Classification Tree (CART), responsible for the division of regions based on the Readiness Index (IRI). This indicator reflects the ability of regions to adapt to economic, environmental and technological changes and their level of preparation for the implementation of circular economy solutions.

In the proposed model, B-spline functions are responsible for identifying nonlinear relationships between the analyzed indicators, while the CART component allows for the detection of hidden data structures and decision thresholds that cannot be captured with classical linear models. Such a combination allows for simultaneous modelling of continuous quantitative relationships and complex regional variation among geographically distributed observations.

In order to reduce the risk of *overfitting* non-linear functions, a *random-walk penalty* was applied to the coefficients of the B-spline function. This solution allows you to maintain the flexibility of the model while reducing over-fitting to empirical data. Due to the lack of an analytical solution for posterior distribution for such a complex model architecture, parameter estimation was carried out using *Markov Chain Monte Carlo simulations (MCMC)* implemented using the Metropolis-within-Gibbs algorithm [26]. The obtained posterior distributions then form the basis for a probabilistic classification of regions and an assessment of the uncertainty associated with their assignment to specific developmental typologies. The proposed Bayesian Tree–Spline architecture integrates the complementary strengths of recursive partitioning and spline-based estimation within a unified probabilistic framework. The CART component identifies hierarchical decision rules and locally homogeneous regional subsets, whereas the penalized B-spline component captures smooth nonlinear responses that cannot be adequately represented by conventional linear specifications. Consequently, the model simultaneously accounts for threshold effects, nonlinear saturation patterns, and posterior classification uncertainty, providing a transparent analytical framework for evaluating regional circular bioeconomy potential.

2.4. Soft Classification and Quantification of Uncertainty

In contrast to deterministic approaches, the Bayesian framework used transforms the results generated by the Tree–Spline model into probabilistic assignments to individual classes using a *multinomial logistic regression function with a softmax binding function*:

$$P(y_i = c \mid X_i, Z_i) = \frac{\exp(\eta_{ic})}{\sum_{k=1}^C \exp(\eta_{ik})} \quad (2)$$

where:

$P(y_i = c \mid X_i, Z_i)$ means the probability a posteriori that the region will belong to the cluster c , conditioned by observed indicators and spatial variables, $X_i Z_i$;

C means the total number of hidden developmental typologies (clusters) identified in the model;

k is an index that includes all possible clusters in the denominator of the equation, ensuring that the sum of the probabilities of belonging to all typologies is exactly 1;

$\exp(\eta_{ic})$ means an exponential function applied to a hidden additive predictor generated by the Tree–Spline architecture;

$\exp(\eta_{ik})$ denotes the exponential transformation of the linear predictor for cluster k . The sum of these values for all clusters C acts as a normalization constant, ensuring the correct interpretation of the results obtained as a probability distribution.

The resulting posterior probability distribution provides the basis for probabilistic regional classification and allows uncertainty to be quantified directly from the estimated class membership probabilities. A key feature of the proposed Bayesian Tree–Spline framework is its ability to quantify classification uncertainty, which is not directly represented by conventional deterministic clustering methods.

The applied soft clustering approach generates for each region a full distribution of the probability of belonging to individual developmental typologies. This allows not only to identify the dominant class, but also to assess the degree of uncertainty associated with the classification process. In order to quantify this uncertainty, Shannon’s entropy was calculated for each region [28]:

$$H(i) = - \sum_{c=1}^C P(y_i = c) \log_2 P(y_i = c) \quad (3)$$

where:

$H(i)$ denotes the Shannon entropy value for a region, which is a quantitative measure of classification uncertainty;

$\log_2$ stands for base 2 logarithm, commonly used in information theory.

High entropy values indicate the presence of transitional regions characterized by ambiguous developmental profiles and comparable posterior probabilities of belonging to multiple regional typologies. It should be emphasized that Shannon entropy measures classification uncertainty rather than regional adaptive capacity or development potential. Consequently, regions exhibiting higher entropy values should be interpreted as transitional cases requiring further investigation rather than as regions inherently possessing greater transformation capacity. Accordingly, Shannon entropy should be interpreted as a complementary diagnostic measure supporting probabilistic classification rather than as an independent indicator of regional performance.

The last stage of the analysis was the assessment of the predictive quality and robustness of the model. For this purpose, the Leave-One-Out Cross-Validation (LOO-CV) procedure and the Watanabe–Akaike Information Criterion (WAIC) information criterion was used [12]. The results showed a higher predictive efficiency of the hybrid Bayesian Tree–Spline architecture compared with the baseline polynomial logistic regression model without nonlinear spline components. The corresponding quantitative validation statistics obtained from the WAIC, LOO-CV, and ablation analyses are reported in Appendix A to facilitate independent evaluation of model robustness and predictive performance. These results confirm the importance of accounting for nonlinear relationships and hierarchical partitioning introduced by the classification tree component when analysing complex and spatially heterogeneous agricultural systems.

This study integrates Bayesian Classification and Regression Trees (CART), penalized cubic B-splines, probabilistic soft clustering, and entropy-based uncertainty quantification within a unified analytical framework for assessing regional circular bioeconomy implementation potential.

3. Results

3.1. Exploratory Dimensionality Reduction and Deterministic Reference Models

The exploratory stage of the analysis was designed to establish a deterministic reference classification that served as a benchmark for evaluating the subsequent probabilistic Bayesian Tree–Spline model. To illustrate the distribution of regions within the principal component space, a geographic visualization of the identified regional typologies was constructed. The application of the K-means grouping algorithm ($K = 3$) made it possible to determine unambiguous boundaries between clusters, which allowed for the identification of basic types of regional agricultural systems and livestock production. The results of deterministic classification are shown in Figure 2. The deterministic classification provides an initial exploratory representation of regional development patterns and serves as a benchmark against which the probabilistic classification results presented in the following sections can be compared.

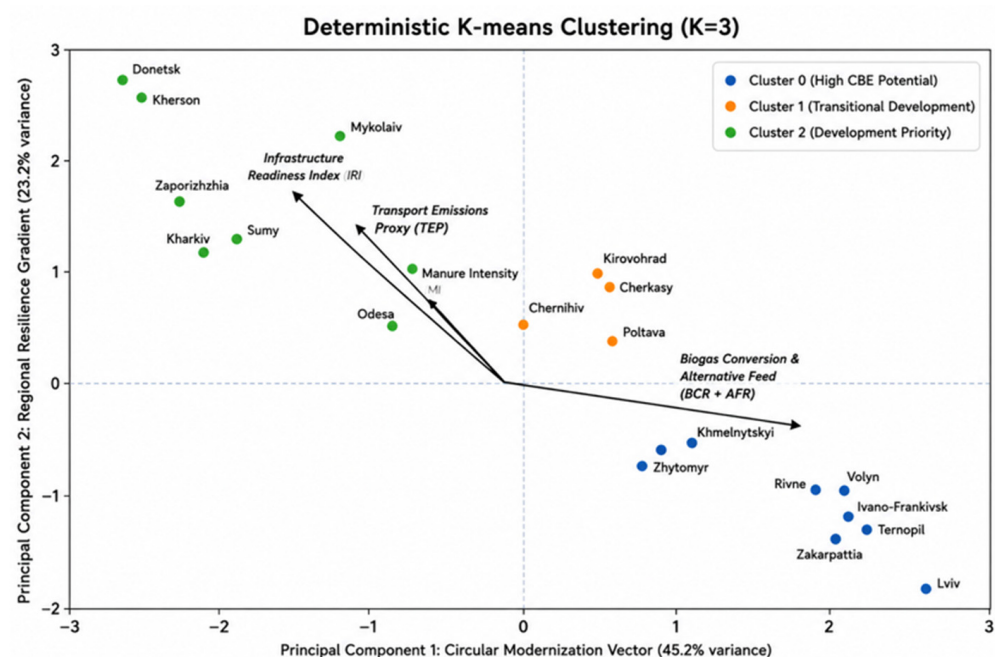


Figure 2. Principal Component Analysis and Deterministic K-Means Clustering of Regional Circular Bioeconomy Typologies.

Directional vectors represent *variable loadings* for key environmental, technological and logistical indicators. The colored dots indicate the assignment of regions to individual clusters on the basis of deterministic classification using the K-means method ($K = 3$), indicating a clear differentiation between regions with a high potential for the implementation of the circular economy and regions requiring further development activities. The orientation and magnitude of the loading vectors illustrate the relative contribution of the analysed indicators to the first two principal components. Variables pointing in similar directions indicate positive associations, whereas opposing vectors suggest contrasting patterns among the regional development characteristics.

Figure 2 illustrates the regional heterogeneity of the study area, allowing the identification of three basic developmental typologies. The first group consists of regions characterized by a high level of integration of circular economy solutions, high availability of biomass and favorable logistics conditions. The second group consists of transition regions, showing a moderate level of development and the potential for further transformation. The third group consists of regions with a lower level of development resilience,

requiring additional infrastructural, technological and organizational investments in order to increase the efficiency of resource use and improve competitiveness.

Although the presented deterministic regional classification is an important reference point for the interpretation of the results, the boundaries set by the K-means algorithm are deterministic in nature and do not reflect the uncertainty related to the belonging of regions to individual typologies. As a consequence, this classification may simplify the actual relationships between regions with similar developmental characteristics. In order to more accurately identify the environmental, technological, logistical and economic factors shaping individual clusters, further analysis focuses on the distributions of variables and the characteristics of the isolated groups presented in Figure 3.

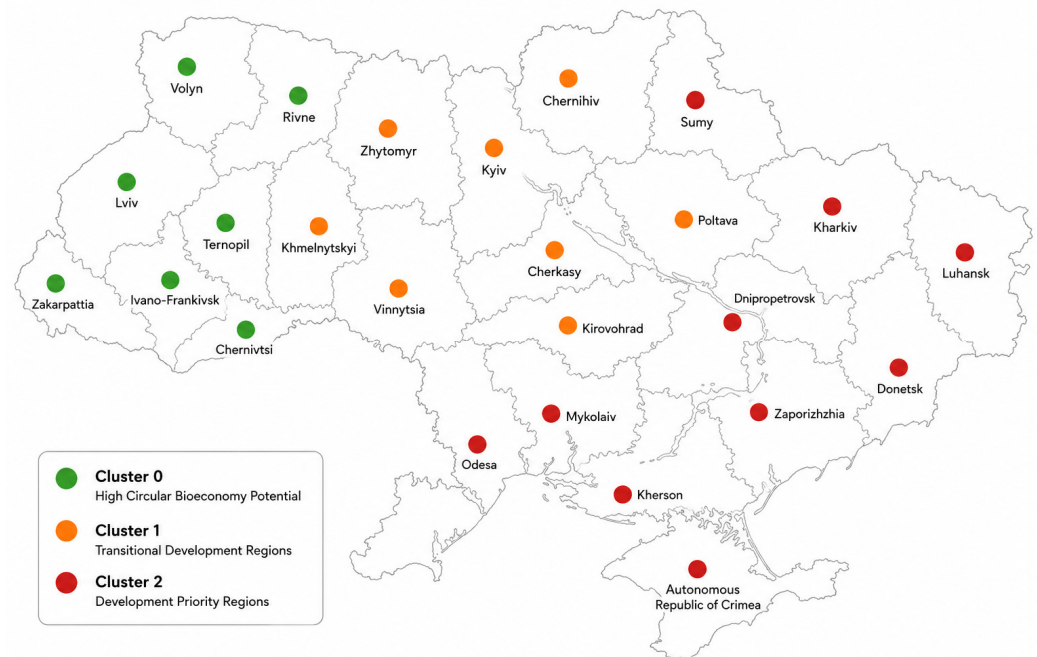


Figure 3. Geographic distribution of regional circular bioeconomy development clusters identified by deterministic K-means clustering ($K = 3$). Green regions indicate high circular bioeconomy potential, orange regions represent transitional development areas, and red regions denote development-priority regions. Ukraine is presented as an empirical case study.

Although the regional patterns exhibit a clear geographic structure, the presented map illustrates the spatial distribution of classification outcomes rather than formal spatial dependence. Consequently, the identified clusters should be interpreted as regional development typologies derived from multivariate similarity rather than results of spatial econometric modelling. As shown in Figure 3, the classification reveals a distinct geographical pattern in the distribution of circular bioeconomy potential across the analysed regions. Cluster 0 (High Circular Bioeconomy Potential) includes primarily regions characterized by a high level of infrastructure readiness, efficient resource utilization, and advanced implementation of circular bioeconomy technologies. Cluster 1 (Transitional Development Regions) consists of regions with an intermediate level of development, demonstrating the potential for further transformation and increasing the degree of integration of circular economy solutions. Cluster 2 (Development Priority Regions), on the other hand, includes regions that require further infrastructural, technological and organisational investments to improve the environmental performance and competitiveness of the agricultural sector and livestock production. These deterministic clusters provide a simplified representation of regional development patterns. However, regions located close to cluster boundaries

may exhibit characteristics shared by multiple typologies, which cannot be adequately represented using hard classification alone.

In order to quantify the infrastructural, environmental and economic factors responsible for the observed spatial differentiation, a comparative analysis of the median values for all analysed structural indicators was carried out.

The analysis of the heat map presented in Figure 4 confirms the significant structural differences between the identified developmental typologies. Cluster 0 is characterized by high median values of the Infrastructure Readiness Index (IRI), Market Accessibility Index (MAI) and Biogas Conversion Rate (BCR), as well as a high share of alternative feed (AFR) and low transport emissions (TEP). These results indicate a high potential for implementing circular economy assumptions, efficient use of resources and a high level of integration of solutions supporting the sustainable development of regional agricultural systems and livestock production.

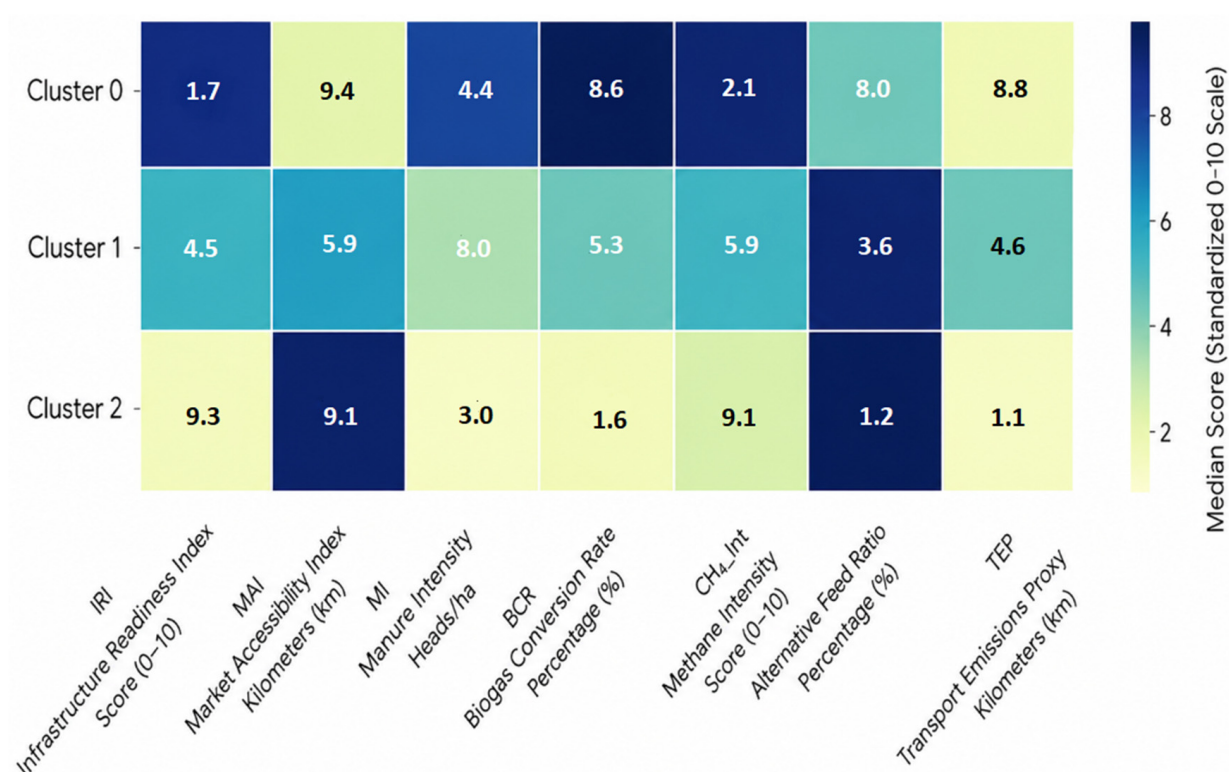


Figure 4. Heatmap of median standardized scores for the seven structural indicators across the three regional circular bioeconomy development clusters identified by deterministic K-means clustering ($K = 3$). Higher scores indicate stronger development potential, except for Methane Intensity and Transport Emissions Proxy, where higher values represent greater environmental and logistical pressure.

Cluster 2, on the other hand, is characterized by low values of regional resilience indicators, infrastructure readiness and the use of circular economy technologies, with high methane emission intensity and a high level of transport emissions. This indicates that there are significant infrastructure, environmental and logistical constraints that may hinder the transition towards more sustainable production models. Cluster 1 achieves intermediate values for most of the analysed indicators, which confirms its transitional nature and the potential for further development towards a circular economy.

Although the K-means classification provides a useful description of the spatial differentiation of regions, it does not allow for an unambiguous determination of the hierarchical importance of individual variables responsible for the observed differences between clus-

ters. In order to identify probabilistic decision rules and determine the most important structural factors responsible for regional differentiation, a Bayesian model of the classification tree was developed. This model made it possible to assess the relative importance of variables such as the Infrastructure Readiness Index (IRI), Market Accessibility Index (MAI) and Manure Intensity (MI) in the process of shaping regional development typologies.

3.2. Tree–Spline Architecture and Division Dynamics

The Bayesian Classification and Regression Trees (CART) module made it possible to identify the hierarchical decision-making rules responsible for shaping regional developmental typologies. Unlike conventional decision trees used solely for deterministic partitioning, the Bayesian CART component estimates posterior probabilities for each terminal node, thereby explicitly accounting for uncertainty in regional classification while preserving the interpretability of the hierarchical decision structure.

Unlike conventional deterministic decision trees, the Bayesian Tree component not only identifies the hierarchical sequence of decision rules but also quantifies the confidence associated with each terminal classification through posterior probabilities. This additional probabilistic information enables the identification of robust regional typologies while simultaneously distinguishing observations located close to decision boundaries. Consequently, the proposed framework provides a more informative and transparent basis for regional policy design than deterministic classification alone.

Node sample sizes and posterior class probabilities are reported to facilitate interpretation of the hierarchical decision process and to illustrate the statistical confidence associated with each terminal regional typology. Together, these characteristics improve the transparency and interpretability of the Bayesian classification procedure. As shown in Figure 5, the resulting hierarchical classification tree identifies the principal decision rules responsible for assigning regions to individual development clusters based on the analysed structural indicators. The model indicates that the Readiness Index (IRI) is the main factor in the split of the main node of the decision tree. Regions with a lower level of regional resilience are directly classified in Cluster 2 (Development Priority), which indicates the need for further actions to support the infrastructural, technological and organizational development necessary for the effective implementation of circular economy assumptions. The dominant role of the Infrastructure Readiness Index indicates that institutional and technical preparedness constitutes the primary prerequisite for successful circular bioeconomy implementation, preceding the influence of production-related characteristics.

For regions with higher regional resilience index, Manure Intensity (MI) is the second key differentiator. This variable makes it possible to distinguish between regions characterized by a high potential for the implementation of circular economy solutions (Cluster 0—High CBE Potential) and transition regions (Cluster 1—Transitional Development), which show a moderate level of biomass use and integration of environmental technologies. This hierarchical structure also indicates the presence of threshold behaviour, whereby the influence of biomass availability becomes relevant only after a sufficient level of infrastructure readiness has been achieved.

Further interpretation of the model structure indicates that the probability of belonging to clusters with a higher level of development of the circular economy is additionally related to factors such as Biogas Conversion Rate (BCR), Alternative Feed Ratio (AFR) and Transport Emissions Proxy (TEP), which reflect the level of technological advancement, resource efficiency and environmental impact of logistics processes.

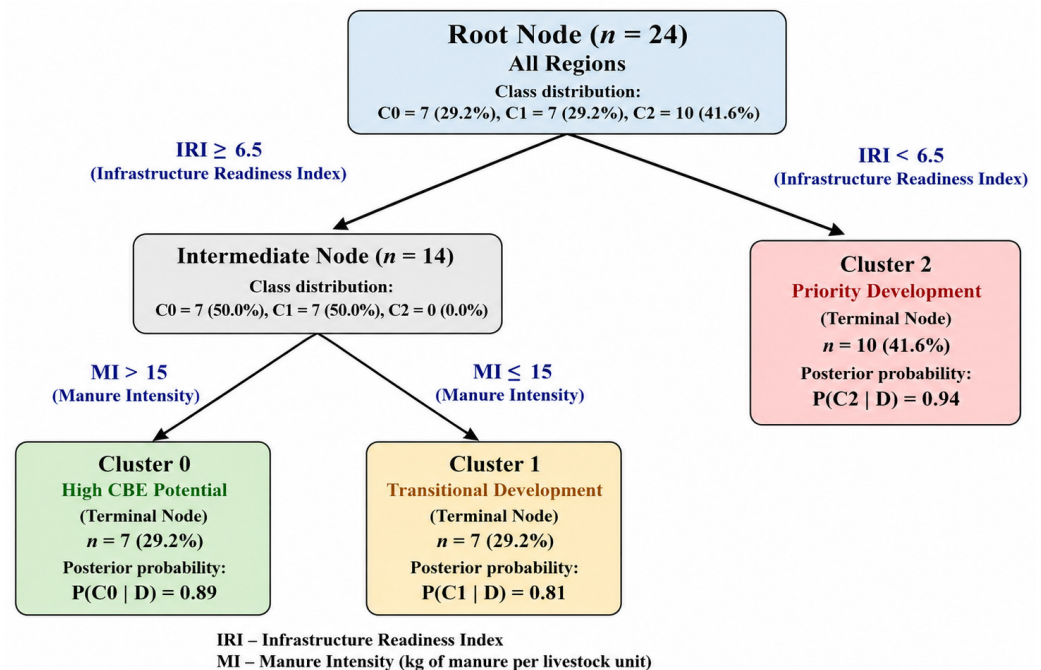


Figure 5. Bayesian hierarchical classification tree illustrating the principal decision rules for assigning regions to the identified circular bioeconomy development typologies. Note: $P(C_x | D)$ denotes the Bayesian posterior probability of cluster membership conditional on the observed data. Splitting rules are based on the standardized Infrastructure Readiness Index (IRI, 0–10) and Manure Intensity (MI, $t\ ha^{-1}$). n denotes the number of regions assigned to each node.

The end nodes of the tree represent the Bayesian posterior probabilities of belonging to individual clusters, providing a measure of classification certainty. The results obtained confirm that regional resilience, biomass availability, and the level of implementation of circular economy solutions are among the most important factors shaping the regional heterogeneity of agricultural systems and livestock production. These findings demonstrate that the Bayesian CART component provides an interpretable representation of the principal decision rules governing regional classification, while the subsequent spline component captures the continuous nonlinear relationships that cannot be represented by tree partitioning alone.

3.3. Probabilistic Classification, Simplex Projections and Transition Regions

The use of the multinomial logistic link function made it possible to map regions in a continuous probability space, represented graphically by means of a simplex projection. In contrast to the deterministic classification of K-means, the simplex projection presented in Figure 6 reveals the complex structure of inter-region relationships and allows for the identification of areas with ambiguous affiliation to individual developmental typologies. The concentration of transition regions in the central part of the space of probability is particularly visible, indicating the gradual nature of the transition towards a circular economy. Unlike deterministic clustering, the simplex representation characterises each region by its complete posterior probability distribution across all development typologies rather than assigning it exclusively to a single class. Consequently, transitional regions are identified through probabilistic proximity to multiple clusters instead of fixed class boundaries, providing a more informative representation of regional development heterogeneity.

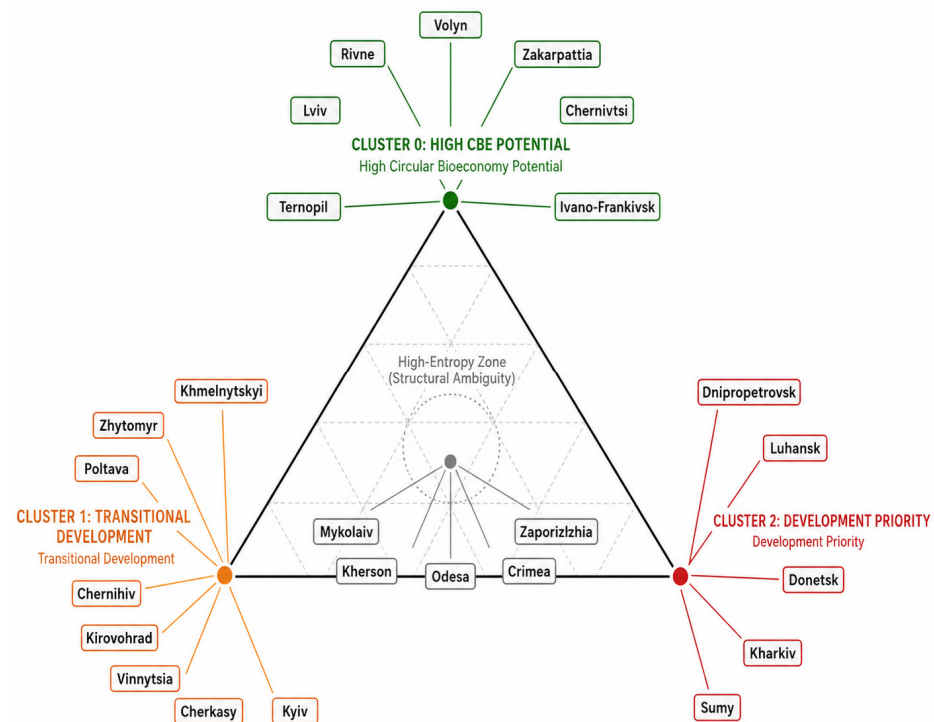


Figure 6. Simplex projection of posterior cluster membership probabilities for regional circular bioeconomy development typologies. Regions located near the center exhibit higher classification uncertainty, while those positioned closer to the vertices show stronger affiliation with a specific development cluster.

This visualization presents the distribution of regional systems of animal production and agriculture in relation to the level of implementation of circular economy solutions, resource efficiency and logistical conditions. A ternary plot allows you to divide regions into three basic functional domains:

- Cluster 0 (High Circular Bioeconomy Potential) includes regions characterized by high biogas production potential, efficient use of biomass and a high level of integration of circular economy solutions.
- Cluster 1 (Transitional Development) includes regions in transition, using mixed production and logistics models and gradually implementing technologies supporting the circular economy.
- Cluster 2 (Development Priority) includes regions that require further action in the field of infrastructure development, increasing resource efficiency, and improving environmental and logistical parameters in order to accelerate the transition towards sustainable production systems.

The Daisy-type structure shown in Figure 6 allows for the visualization of the degree of uncertainty in the classification of regions, the identification of transition areas and the assessment of the potential for the implementation of circular economy solutions in regional agricultural and livestock production systems.

“Ambiguous cases” located in the central part of the probability space, whose level of classification uncertainty has been quantified by Shannon entropy (H).

As shown in Table 2, Shannon entropy values provide a quantitative measure of classification uncertainty derived directly from the posterior probability distributions estimated by the Bayesian Tree-Spline model. Regions exhibiting low entropy values are characterized by dominant posterior probabilities for a single development typology, indicating stable and well-defined classification profiles.

Table 2. Regions with the highest classification uncertainty based on Shannon entropy and posterior probabilities (the complete results are provided in Table A1 (expanded to include all seven indicators, including Methane Intensity (CH₄_Int) and Alternative Feed Ratio (AFR)), Appendix A).

Country/Region	P (Cluster 0)	P (Cluster 1)	P (Cluster 2)	Shannon Entropy (H)
Kharkiv oblast	0.42	0.55	0.03	1.15
Poltava oblast	0.38	0.41	0.21	1.53
Sumy oblast	0.51	0.39	0.10	1.36

Note: Cluster probabilities may not sum exactly to 1.00 due to MCMC rounding constraints.

In contrast, transitional regions are associated with substantially higher entropy values, reflecting more balanced posterior probabilities across multiple clusters and therefore greater uncertainty in regional assignment. For example, Poltava Oblast (H = 1.53), Kirovohrad Oblast (H = 1.51), Chernihiv Oblast (H = 1.46), Odesa Oblast (H = 1.49), Dnipropetrovsk Oblast (H = 1.45), and Sumy Oblast (H = 1.36) exhibit the highest entropy values, indicating ambiguous classification profiles located between the principal development typologies.

These findings should be interpreted as evidence of probabilistic classification ambiguity rather than as direct indicators of regional development potential or adaptive capacity. Consequently, Shannon entropy serves as a complementary diagnostic measure that enhances the interpretation of posterior probabilities by explicitly identifying regions with transitional characteristics requiring more detailed assessment before region-specific policy interventions are formulated.

3.4. Saturation Effects and Nonlinear Relationships

The use of *penalized B-splines* made it possible to identify complex nonlinear relationships related to the implementation of circular economy technologies, revealing the occurrence of saturation effects for selected technological and logistics processes. The non-linear nature of these relationships was confirmed by the probability distributions presented in Figure 7, which illustrate the constraints associated with resource efficiency and the implementation of bioenergy technologies.

As shown in Panel A, the use of alternative feeds is subject to the principle of diminishing marginal returns. The initial increase in the share of alternative feed leads to a clear improvement in environmental performance and reduced pressures related to the functioning of supply chains. However, the analysis indicates the presence of a clear saturation point. Once a certain level of alternative feed is exceeded, further increasing their use brings less and less environmental benefits. This is due, among other things, to an increase in logistics expenditure related to the transport, storage and processing of larger quantities of feed materials from various sources.

Panel B, on the other hand, reveals a non-linear relationship between the Biogas Conversion Rate (BCR) and the overall environmental performance of the system. The initial increase in the level of conversion of biomass to biogas is associated with improved environmental performance and more efficient use of available resources. However, once the optimal production range has been reached, a saturation effect occurs, in which further increasing the process intensity no longer leads to proportional benefits. This means that there is an optimal range of system operation in which the balance between biomass use, energy efficiency and logistical requirements ensures the highest level of environmental sustainability.

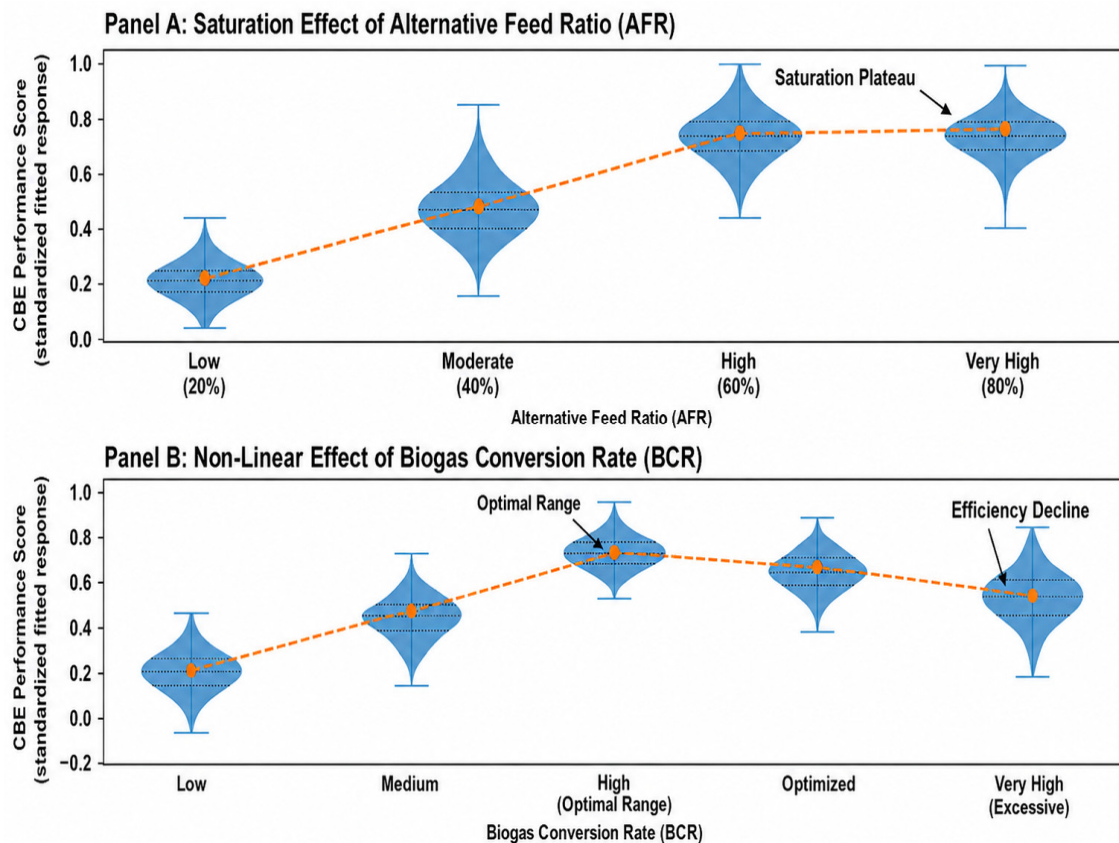


Figure 7. Nonlinear partial effects of key technological and managerial variables on the fitted CBE Performance Score estimated by the Bayesian Tree–Spline model. Panel (A) shows the saturation effect associated with increasing Alternative Feed Ratio (AFR), whereas Panel (B) illustrates the nonlinear relationship between Biogas Conversion Rate (BCR) and the fitted CBE Performance Score. Note: Panel (A) illustrates the saturation effect associated with increasing Alternative Feed Ratio (AFR), where marginal improvements in the CBE Performance Score diminish beyond a critical threshold. Panel (B) demonstrates the nonlinear relationship between Biogas Conversion Rate (BCR) and the CBE Performance Score, indicating the existence of an optimal operating range beyond which further increases are associated with diminishing improvements in the fitted response. The CBE Performance Score represents the standardized fitted response estimated by the Bayesian Tree–Spline model and reflects the combined influence of the analysed explanatory variables rather than a directly observed empirical indicator.

The resulting decrease in efficiency indicates the occurrence of a significant limiting effect related to the functioning of the system. As biomass utilization increases, additional requirements for transport, storage, and processing also increase, leading to higher logistical costs and resource consumption. Consequently, further increases in biomass conversion to biogas generate progressively smaller improvements in system efficiency, indicating diminishing marginal returns. The vertical axis (CBE Performance Score) represents the standardized fitted response of the Bayesian Tree–Spline model and reflects the relative contribution of each explanatory variable to the estimated circular bioeconomy potential rather than a directly observed empirical indicator.

The results indicate that simple linear intensification models may not adequately capture the complex relationships observed in regional circular bioeconomy systems. The robustness analyses, including Leave-One-Out Cross-Validation (LOO-CV), WAIC, and ablation experiments based on the sequential removal of selected explanatory variables, confirmed the stability of the fitted Bayesian Tree–Spline model and the identified nonlinear response patterns. These findings suggest that the analysed relationships are characterized

by threshold and saturation effects, indicating that improvements in circular bioeconomy performance are not necessarily proportional to increases in individual technological indicators. Consequently, balanced development of biomass availability, conversion efficiency, and supporting infrastructure appears to be more consistent with the estimated model than strategies focused solely on maximizing individual production parameters.

4. Discussion

The results obtained in this study challenge traditional, linear approaches to the development of regional agricultural systems and livestock production. The use of the Bayesian Tree–Spline architecture instead of classical deterministic methods has shown that the effective integration of the assumptions of the circular bioeconomy (CBE) is not only a function of capital expenditure or availability of resources. It is a much more complex process, shaped by regional heterogeneity, classification uncertainty, and nonlinear relationships between environmental, technological and logistical factors.

4.1. Regional Resilience as a Major Differentiator

The Bayesian hierarchical classification tree (Figure 5) confirms that the Readiness Index (IRI) is the most influential factor associated with differences among regional agricultural systems and livestock production. This indicator has been identified as the dominant effect of the division of the main node of the classification tree, exceeding in importance many traditional economic and logistical conditions. The results suggest that higher levels of regional resilience are associated with a greater capacity to implement circular economy solutions.

Regions classified in Cluster 2 (Development Priority) are characterized by lower values of resilience and infrastructure preparedness indicators, which limits their ability to effectively implement innovative solutions related to the use of biomass, biogas production and closing material cycles. On the other hand, regions belonging to Cluster 0 (High CBE Potential) show more favorable conditions for the development of circular technologies and greater readiness to implement modern resource management models.

These results are in line with current research showing that regional resilience, the quality of infrastructure and the availability of local resources are key factors for the successful implementation of circular economy strategies in the agricultural and livestock sectors. The results suggest that development activities may therefore benefit from being adapted to the specificities of individual regions, with special attention being paid to areas requiring improvement in infrastructure, technology availability and biodiversity efficiency.

4.2. Probabilistic Classification Uncertainty in Transitional Regions

An important contribution of this study is a new look at regions classified in Cluster 1 (Transitional Development). In classical grouping methods such as K-means, these regions are treated as homogeneous intermediate units between extreme developmental typologies. However, the use of probabilistic classification (*soft clustering*) and Shannon entropy analysis (Table 2) allowed for the quantitative determination of the level of uncertainty related to their cluster affiliation.

Elevated entropy values observed for transitional regions indicate that they have characteristics of more than one developmental typology. These results indicate that these regions occupy intermediate positions between the identified development typologies and therefore exhibit greater uncertainty in probabilistic cluster assignment. From a strategic planning perspective, this ambiguity should not be seen as a weakness of the system, but as a potential opportunity for policy intervention for the effective implementation of

measures aimed at improving resource efficiency, developing bioenergy technologies and strengthening local value chains.

4.3. Spatial Ambiguity as a Developmental Potential (Continued)

Rather than treating elevated entropy values as a sign of weakness or instability in the system, the results of this study indicate that they may represent an important source of development flexibility. Transition regions with greater classification uncertainty may possess greater development flexibility than regions that are already highly specialised. In contrast to regions belonging to cluster 0 (High Circular Bioeconomy Potential), which have achieved a high level of development of specific technological and organizational solutions, transition regions retain a greater ability to adapt to new directions of development.

For this reason, these areas can be particularly attractive places for the implementation of innovative solutions related to the use of biomass, biogas production, the development of local energy systems and the closure of material cycles. Their relatively high structural flexibility allows for the implementation of new organizational and technological models with less risk of limitations resulting from previous investment decisions. As a consequence, transition regions may play an important role in accelerating the transition towards a circular economy in line with international sustainability objectives.

4.4. Saturation Effects and Scaling Limits of Bioenergy Systems

One of the most important conclusions from the analyses is the identification of non-linear saturation effects detected by *penalized B-splines* shown in Figure 7. The results indicate that scaling up the implementation of bioenergy solutions is not necessarily associated with an unlimited increase in the environmental or economic efficiency of the system.

In particular, the analysis showed that as the Biogas Conversion Rate (BCR) and the Alternative Feed Ratio (AFR) share increase, the benefits of further process intensification gradually decrease. Once a certain level of biomass utilization is reached, additional inputs related to its acquisition, transport, storage and processing begin to limit the efficiency of the entire system. As a result, there are decreasing marginal benefits, and further intensification may lead to a decline in overall environmental and resource efficiency.

The non-linear nature of the relationship observed for the regions belonging to Cluster 0 and Cluster 1 confirms the existence of an Optimized Operating Range in which the most favorable balance between biomass use, technological process efficiency and logistical requirements is achieved. Once this range is exceeded, the system shows decreasing efficiency, indicating the need for careful planning to further scale up bioenergy production.

The results suggest that circular economy development strategies should not focus solely on maximising the volume of biogas production or biomass use. The results suggest that greater attention should be paid to the optimization of the entire system, including the availability of resources, infrastructure, organization of logistics processes and local spatial conditions. Measures to support the development of the circular economy should therefore be adapted to the real potential of individual regions and their ability to make efficient use of available resources.

In practice, this means that development policies and support mechanisms should be designed in such a way that they promote the functioning of bioenergy systems within the identified areas of optimal efficiency. This reduces the risk of excessive intensification of processes and increases the long-term environmental and economic performance of regional agricultural systems and livestock production.

4.5. Practical Implications and Recommendations for Development Policy

The clear regional differentiation identified using probabilistic modelling indicates the need to implement development policies tailored to the specificities of individual

regions. The planned actions should not be based on national averages, but should take into account the characteristics and development potential of the regions representing individual clusters.

The differentiated policy recommendations presented below are derived directly from the probabilistic regional typologies identified by the Bayesian Tree–Spline model. They also reflect the nonlinear threshold and saturation effects observed for infrastructure readiness and biomass utilisation. Consequently, policy interventions should be adapted to the estimated characteristics of each regional typology rather than applying uniform development measures.

Regions requiring priority actions (Cluster 2—Development Priority). In the regions belonging to this group, the primary objective should be to increase regional resilience through the development of technical infrastructure, the improvement of the availability of environmental technologies and the creation of local biomass use systems. The development of decentralised biogas installations enabling the local use of natural fertilisers and other agricultural by-products may be of particular importance. Such solutions contribute to increasing resource efficiency, reducing emissions and strengthening local energy security.

Transition regions (Cluster 1—Transitional Development). These regions are characterized by the highest transformation potential resulting from their relatively high development flexibility. High entropy values indicate greater uncertainty in probabilistic classification rather than inherently higher adaptive capacity. Nevertheless, these regions may benefit most from well-targeted investment aimed at improving infrastructure and resource efficiency. In particular, it is recommended to support *Short Food Supply Chains* (SFSCs), the development of local biomass processing systems, and investments that improve logistics and infrastructure efficiency.

Regions with High Circular Bioeconomy Potential (Cluster 0).

These regions should act as leaders in the implementation of innovative environmental technologies and demonstrators of good practices related to the circular economy. Development activities should focus on further increasing resource efficiency, developing advanced bioenergy technologies and implementing certification schemes confirming high environmental standards of agricultural and livestock production. This will increase the competitiveness of regional production systems on both domestic and international markets.

Compliance with European sustainable development strategies. Effective implementation of the presented strategies requires their close connection with the objectives of the European *Green Deal*, the Common Agricultural Policy (CAP) and the principles of the circular economy. Financial support schemes should take greater account of environmental and resource efficiency indicators, such as the level of conversion of biomass to biogas, emission reduction or the efficiency of the use of by-products, rather than relying solely on the volume of production or farm area. Such an approach can contribute to increasing the competitiveness, resilience and long-term sustainability of regional agricultural systems and livestock production.

4.6. Limitations of the Study and Directions for Further Research

Despite the high efficiency and flexibility of the Bayesian Tree–Spline model used, this study has some limitations that set potential directions for further research work. Firstly, the analysis is based on *cross-sectional data*, which provides a picture of regional agricultural systems and livestock production at a specific point in time. Due to the dynamic nature of transformation processes related to the implementation of the circular economy, future research should use *longitudinal panel data* [12]. This would make it possible to track changes

in regional development typologies and identify delayed effects of the implementation of specific technological and organizational solutions.

Secondly, although the developed model incorporates geographically referenced explanatory variables, there is a need to further develop the analyses by integrating *spatial network models*. Such an approach would allow for a more accurate mapping of the flows of biomass, energy and by-products between the different components of circular economy systems. This would allow for better consideration of logistical dependencies and the spatial organisation of regional value chains.

Another promising direction is the investigation of causal relationships. In this context, the use of Causal Bayesian Networks (CBNs) and advanced methods of non-linear modeling seems particularly promising [14]. These tools could identify the real drivers of circular economy technology deployment, bioenergy systems and resource efficiency improvements.

A better understanding of cause-and-effect mechanisms would allow for the development of more precise instruments to support the transformation of regional farming and livestock production systems towards circular economy models. At the same time, it would make it possible to adapt development strategies to the specific environmental, infrastructural and socio-economic conditions of individual regions.

From a methodological perspective, the proposed Bayesian Tree–Spline framework combines the interpretability of hierarchical decision trees with the flexibility of spline-based nonlinear modelling and Bayesian probabilistic inference. Unlike deterministic clustering methods such as K-means, the proposed framework explicitly quantifies classification uncertainty. Compared with Generalized Additive Models (GAM), it additionally identifies hierarchical decision structures, while in contrast to machine-learning approaches such as Random Forests and Gradient Boosting Machines (GBM), it provides directly interpretable decision rules together with posterior probabilities supporting transparent uncertainty assessment. Consequently, the proposed methodology offers a flexible and interpretable analytical framework for investigating complex regional sustainability systems characterized by nonlinear relationships and heterogeneous development trajectories.

5. Conclusions

The transformation of regional livestock production systems towards a circular economy represents an important opportunity to increase resource efficiency, improve environmental performance and strengthen the long-term resilience of agricultural systems. The results of this study suggest that traditional linear development models may not be sufficient to describe the regional heterogeneity and nonlinear relationships associated with the implementation of circular economy strategies.

The use of the Bayesian Tree–Spline model enabled the application of an integrated analytical framework to identify probabilistic regional typologies and quantify the interactions between environmental, technological, and logistical factors. The proposed methodology allows for the consideration of classification uncertainty, the level of infrastructure readiness and the availability of resources, providing a more comprehensive representation of the functioning of regional agricultural systems than classical deterministic approaches.

The results suggest that development strategies may benefit from being adapted to the characteristics of individual regional typologies. Regions classified in the Development Priority group require investments in infrastructure, efficient resource management and the development of decentralised bioenergy systems. On the other hand, regions with a high potential for the circular economy should focus on further technological development, efficient use of biomass and the implementation of advanced circular solutions. Transition regions deserve particular attention because their probabilistic classification indicates

greater development flexibility and therefore a greater opportunity for targeted policy interventions supporting the transition towards a circular economy.

The integration of circular economy principles can be regarded not only as an environmental measure, but also as an effective mechanism to increase resource efficiency, strengthen regional resilience and improve the competitiveness of agricultural and livestock production systems in the long term. Ultimately, the proposed approach provides a practical analytical tool to support data-driven decision-making and the design of development strategies tailored to the specific circumstances of each region, thereby supporting evidence-based regional planning and the implementation of sustainable agricultural development strategies.

Beyond the present case study, the proposed Bayesian Tree–Spline framework provides a transferable analytical architecture that can support evidence-based decision-making, regional planning, resource allocation, and the development of future digital agriculture systems. Its ability to integrate hierarchical decision rules, nonlinear relationships, and probabilistic uncertainty assessment offers a flexible methodological foundation for analysing complex regional sustainability challenges across diverse geographical contexts.

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Appendix A

Appendix A.1

This section provides a comprehensive empirical framework for research covering all the regions analysed. In order to increase the transparency of the presentation of the data, the statement was divided into two parts. Table A1 (expanded to include all seven indicators, including Methane Intensity (CH₄_Int) and Alternative Feed Ratio (AFR)) shows the standardized macrostructural indicators used as input covariates in the Bayesian Tree–Spline model.

Table A1 (expanded to include all seven indicators, including Methane Intensity (CH₄_Int) and Alternative Feed Ratio (AFR)). Standardised macrostructural input variables describing the potential of the circular economy and the logistical conditions of regional agricultural systems and livestock production.

Table A1. Regional values of the standardized indicators used in the Bayesian Tree–Spline model.

Oblast	IRI	MAI	MI	BCR	CH ₄ _Int	AFR	TEP
Lviv	9.8	9.4	0.65	4.2	1.8	8.2	0.15
Ivano-Frankivsk	9.9	8.9	0.58	3.8	2.0	7.8	0.18
Ternopil	9.8	8.3	0.72	4.5	2.2	8.5	0.22
Volyn	9.7	9.6	0.68	5.1	1.9	8.4	0.14
Rivne	9.6	8.8	0.54	3.5	2.4	7.6	0.19
Zakarpattia	9.9	10.0	0.42	2.1	1.5	8.0	0.11
Chernivtsi	9.9	9.7	0.48	2.8	2.1	7.9	0.13
Khmelnyskyi	9.7	7.5	0.81	5.8	2.5	8.8	0.28
Zhytomyr	8.2	7.1	0.49	3.2	5.4	4.2	0.31
Kyiv	7.5	5.9	0.52	4.1	5.8	4.5	0.42
Chernihiv	5.8	4.4	0.44	4.9	6.1	3.8	0.55
Sumy	3.2	3.1	0.38	3.1	8.8	1.5	0.68
Kharkiv	1.8	2.5	0.22	1.8	9.1	1.2	0.79
Poltava	8.5	4.8	0.78	5.6	5.6	4.8	0.51
Cherkasy	9.2	6.0	0.69	4.3	5.9	4.6	0.41
Vinnytsia	9.5	7.2	0.75	5.5	5.2	4.9	0.32
Kirovohrad	8.9	5.3	0.41	2.9	6.0	3.6	0.48
Dnipropetrovsk	4.6	3.2	0.31	2.4	8.9	1.4	0.72
Odesa	6.2	6.8	0.28	1.5	8.5	1.6	0.46
Mykolaiv	3.9	5.2	0.19	0.8	9.0	1.3	0.61
Kherson	1.1	4.0	0.08	0.4	9.4	1.1	0.78
Zaporizhzhia	1.5	2.7	0.11	0.5	9.2	1.2	0.84
Donetsk	0.5	1.2	0.05	0.3	9.5	1.0	0.95
Luhansk	0.2	0.0	0.02	0.1	9.8	0.9	0.99

Notes: IRI—Region Readiness Index; MAI—Market Accessibility Index; MI—Manure Intensity (heads ha^{−1}); BCR—Biogas Conversion Rate (%); CH₄_Int—Methane Intensity (score 0–10); AFR—Alternative Feed Ratio (%); TEP—Transport Emissions Proxy. All regional indicators were compiled and standardized from official statistical registers and databases on regional development, agriculture, and sustainable resource management [30,34], supplemented by infrastructure, environmental, and logistics datasets for the period 2024–2026 [28,29,32].

Table A2 presents the obtained probabilities of belonging to clusters (*posterior soft clustering probabilities*) together with the calculated Shannon entropy values, which are a measure of the uncertainty of the classification of regions and confirm regional differences in circular bioeconomy potential across agricultural systems and livestock production.

Table A2. Posterior Probability of Cluster Membership and Structural Ambiguity Estimation.

Region	P (Cluster 1)	P (Cluster 2)	P (Cluster 3)	Shannon Entropy (H)
Lviv	0.91	0.07	0.02	0.51
Ivano-Frankivsk	0.89	0.09	0.02	0.58
Ternopil	0.86	0.11	0.03	0.69
Volyn	0.88	0.10	0.02	0.61
Rivne	0.85	0.12	0.03	0.72
Zakarpattia	0.83	0.14	0.03	0.77
Chernivtsi	0.81	0.16	0.03	0.82
Khmelnyskyi	0.87	0.11	0.02	0.64
Zhytomyr	0.62	0.33	0.05	1.17
Kyiv	0.56	0.39	0.05	1.21
Chernihiv	0.32	0.51	0.17	1.46
Sumy	0.51	0.39	0.10	1.36
Kharkiv	0.42	0.55	0.03	1.15
Poltava	0.38	0.41	0.21	1.53

Table A2. Cont.

Region	P (Cluster 1)	P (Cluster 2)	P (Cluster 3)	Shannon Entropy (H)
Cherkasy	0.41	0.49	0.10	1.36
Vinnytsia	0.46	0.47	0.07	1.30
Kirovohrad	0.34	0.46	0.20	1.51
Dnipropetrovsk	0.16	0.34	0.50	1.45
Odesa	0.21	0.29	0.50	1.49
Mykolaiv	0.09	0.24	0.67	1.19
Kherson	0.04	0.11	0.85	0.74
Zaporizhzhia	0.05	0.14	0.81	0.86
Donetsk	0.02	0.07	0.91	0.51
Luhansk	0.01	0.04	0.95	0.32

Note: Due to rounding of posterior estimates obtained from MCMC sampling, the reported cluster membership probabilities may not sum exactly to 1.00.

Appendix A.2. Computational Configuration and Bayesian Model Convergence Diagnostics

To ensure full reproducibility of the analyses, the proposed model was implemented in a dedicated Python 3.11 environment. The computational workflow comprised the following stages:

Preparation and standardization of variables.

The input data matrix, including spatial, logistic, environmental and technological variables describing the analysed regions, was standardized using the StandardScaler module available in the scikit-learn library. This preprocessing step removed scale-related differences among variables and improved numerical stability during subsequent modelling in a multidimensional feature space.

Bayesian modelling of probabilistic mixtures.

Latent regional development typologies were identified using Bayesian probabilistic modelling implemented in the PyMC (v5.x) framework [13,15]. The proportions of individual clusters were modelled using a symmetric Dirichlet prior ($\alpha = [1.0, 1.0, 1.0]$), while the parameters describing the position and variability of the clusters were estimated using weakly informative normal (*Gaussian*) and semi-normal (*Half-Normal*) distributions [12].

MCMC sampling.

Estimation of a posteriori parameters was carried out using the No-U-Turn Sampler (NUTS) algorithm, which is an adaptive variation of the Hamiltonian Monte Carlo algorithm (HMC) [26]. The sampling process involved four independent Markov chains running in parallel. For each chain, 2000 *posterior draws* and 1500 *tuning iterations* were generated, resulting in a total of 8000 a posterior sample after the *warm-up* phase. The target acceptance rate was set to 0.95 to minimise *divergent transitions*.

Evaluation of model convergence.

All estimated parameters were subjected to detailed diagnostics using the ArviZ package. The stability and mixing properties of Markov chains were confirmed on the basis of Gelman–Rubin statistics, for which the following values were obtained:

$$\hat{R} < 1.05$$

for all analysed model parameters [12]. In addition, high values of the effective sample number (*Effective Sample Size*, $ESS > 4000$) were obtained for each parameter, confirming the correct exploration of a posteriori distributions. On the basis of the obtained posterior distributions, the individual probabilities of the regions belonging to individual clusters were then determined, which were used to calculate the Shannon entropy value (H), which is a measure of the uncertainty of the classification of regional development typologies [27].

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