symmetry

Article

An Improved User Association Algorithm for MAP–FAPs Heterogeneous Networks

Fang Ye, Chunxia Su and Yibing Li *

College of Information and Communication Engineering, Harbin Engineering University, Harbin 150001, China; yefang0923@126.com (F.Y.); sue212533@126.com (C.S.)
* Correspondence: liyibing0920@126.com; Tel.: +86-133-0460-5678

Academic Editor: Vladimir Shpilrain
Received: 1 November 2016; Accepted: 7 December 2016; Published: 10 December 2016

Abstract: Heterogeneous networks (HetNets) give users the opportunity to access different access points(APs), which will simultaneously affect user performance and system performance, so user association in HetNets plays a critical role in enhancing the load balancing and the system sum-throughput of networks. Meanwhile, the incremental sum-throughput currently fails to meet the escalating data demands. Besides, ensuring fairness amongst users constitutes another urgent issue in the radio resource management (RRM) of HetNets. What is more, few works consider the maximum service user number constraint in femtocell access points (FAPs). To solve the aforementioned problem, this paper associates users to APs by considering system sum-throughput and fairness at the same time in HetNets under a maximum service user number constraint of FAPs; accordingly, the user association problem is formulated. By releasing constraint, the optimal user association algorithm is obtained by Lagrangian function, and based on this optimal solution, a low complexity suboptimal user association algorithm is proposed. At last, this paper investigates the relationship between system sum-throughput and maximum service user number of FAPs. Numerical simulation results show that the proposed algorithm can improve sum-throughput and fairness at the same time at a specific maximum service user number of FAPs.

Keywords: heterogeneous network; user association; proportional rate constraint; Lagrangian function

1. Introduction

With the development of wireless communication and intelligent terminal technology, service at high rates (such as multimedia business) is attracting people's attention, which exacerbates the demand for high data rate services. According to the latest visual network index (VNI) report from Cisco [1], the global mobile data traffic will increase nearly ten-fold from 2014 to 2019, reaching 24.3 exabytes per month by 2019, wherein three-fourths will be video. Due to the explosion of data traffic and limited spectrum, new wireless communication technology (e.g., heterogeneous networks (HetNets), massive multiple input and multiple output (MIMO)) is emerging to improve utilization efficiency of spectrum resources. In HetNets, additional femtocell access points (FAPs) are deployed within the coverage area of traditional macro-cell access points (MAPs) [2]. Because of the small volume, low cost, and short transmission distances (a few meters to tens of meters in general), these low-power access points can solve "fade area" and "busy area" problems effectively, thereby improving sum-throughput and spectrum efficiency. Though vast improvements in sum-throughput have been made, researchers in the field of communications have reached a consensus that the incremental improvements fail to meet the escalating data demands of the foreseeable future [3–5]. Besides, in most wireless systems of interest, different users require different data rates, which may be accommodated by allowing users to subscribe to different levels of service. So, ensuring system sum-throughput and fairness is still a critical issue in HetNets.

To solve the aforementioned problems, we will investigate the user association-based radio resource management (RRM) problem in this paper. User association—namely associating users with a specific AP—substantially affects the network performance. Four metrics are commonly used for user association in HetNets: spectrum efficiency, energy efficiency, quality of service (QoS), and fairness [1]. Problem formulation is too complicated to implement if all four metrics are considered at the same time, so we mainly consider spectrum efficiency and fairness in this paper.

The problem of user association in HetNets has recently been an area of active research [6]. In existing research, two kinds of user association criteria are prevalent— namely, the bias-based user association criterion and the received power-based user association criterion. For the bias-based user association criterion, the users' power received from FAPs is artificially increased by adding a bias to it to ensure that more users will be associated with FAPs. However, the shortcoming of biased user association is that the users who are forced to access FAPs due to the added bias will experience severe interference from nearby MAPs. As a result, the gain obtained by offloading traffic from MAP to FAPs will be decreased by the severe interference [1]. As for the received power-based user association criterion, users always prefer to access the specific AP which can provide the maximum received signal strength (max-RSS) [6]. However, this criterion does not consider the transmit power difference of MAP and FAPs, which will lead to an overload of MAP, hence resulting the inefficient FAPs deployment in HetNets. Aside from these two common criteria, some literature has also studied other associations; for example, FAP-first criterion and the best AP selection. Song et al. [7] researched user association in the cellular-WLAN heterogeneous network, and users always first try to access WLAN to obtain higher data rate, which is referred to as WLAN-first scheme. References [8,9] focus on the network selection strategy in the WiMAX-WLAN heterogeneous network, and users always try to select access points that can support the highest rate—namely, the best AP selection algorithm.

Game theory and combinatorial optimization are the two prevalent tools to solve the RRM problem in wireless networks. In [10,11], a Stackelberg game is introduced to solve hierarchical resource allocation for HetNets. In [12], a matching game is introduced to solve joint user association and femtocell allocation problems in MAP–FAPs HetNets. Unfortunately, [12] is too complicated for practical systems. Furthermore, it is noted that game theory operates under the assumption that all players are rational individuals acting for their own best interest, which may lead to the "Tragedy of the Commons" [1]. Besides, players (users or APs) in HetNets may not be rational all the time, because multi-parameter optimization is considered in HetNets, and the one maximizing its sum-throughput may be perceived as non-rational by the other ensuring fairness, and vice versa [1]. References [13–18] solve the RRM problem using convex optimization. By formulating a system model and simplifying some conditions, the optimal problem will transfer into a convex optimization problem. Reference [13] focuses on RRM under proportional user rate constraints in LTE-WLAN HetNets, and Karush–Kuhn–Tucker (KKT) conditions are used to obtain the optimal solution. However, the assumptions of multi-homing and resource element sharing are unreasonable; what is more, the average rate ratio-based user association algorithm in [13] uses an exhaustive search to obtain the optimal solution, which is too complicated to implement. References [19,20] use pricing-based user association for the HetNets. Most studies of RRM in HetNets ignore fairness among users, and another inherent nature imposed by FAP—namely, the fact that due to the low power and low cost, a specific FAP can only serve finite users. So, it is necessary to consider this constraint when associating users with FAP. According to the lack of RRM in HetNets, this paper proposes a new user association algorithm in HetNets to maximize system sum-throughput under the constraints of user rate proportion and maximum service user number; meanwhile, the proposed algorithm has a lower complexity than [13].

In this paper, we will develop a user association algorithm in a MAP–FAPs heterogeneous network. To subscribe different kinds of services, various rate requirements are assumed. What is more, a characteristic of FAP makes it so that each FAP can only serve finite users. Therefore, this paper proposes an improved user association algorithm to maximize system sum-throughput, under the constraints of user rate proportion and maximum service user number.

The key contributions of this paper are as follows. First, a user association problem considering the above two constraints is formulated. Second, by relaxing restriction, the optimal solution is obtained by using a Lagrangian function. Based on the optimal solution, a suboptimal user association algorithm that has low complexity is proposed. Third, to provide technical support for people who are dedicated to improving the service number limits of FAPs, this paper investigates the relationship between sum-throughput and the maximum service user number of FAPs.

The remainder of this paper is organized as follows. In Section 2, a MAP–FAPs HetNets system model is introduced. In Section 3, the user association problem considering the both constraints is formulated. An optimal user association algorithm and a suboptimal user association algorithm are obtained by using a Lagrangian function in Section 4. Simulation results are provided in Section 5. Finally, the conclusion is given in Section 6.

2. System Model

We consider a two-tier heterogeneous wireless network, consisting of one MAP and L FAPs, indexed by $i = 0, 1, \ldots, L$. A multiple FAPs heterogeneous network is shown in Figure 1, where FAPs are randomly deployed in the MAP coverage area. For simplicity, we assume that FAPs have non-overlapping coverage areas. We assume that this HetNet owns K users, which can access both MAP and FAPs. In order to distinguish the users served by different access points (APs), K_M and K_F are used to denote the user number served by MAP and FAPs, respectively. K_F is constituted by $K_{F_i}(i = 1, \ldots, L)$, which is used to denote the user number served by different FAPs. Users served by MAP is expressed as set $\mathbb{K}_{\mathbb{M}}$, and users served by the *ith* FAP is expressed as set $\mathbb{K}_{\mathbb{F}_i}$, so the total user set can be denoted by $\mathbb{K} = \mathbb{K}_{\mathbb{M}} \cup \mathbb{K}_{\mathbb{F}_1} \cup \cdots \cup \mathbb{K}_{\mathbb{F}_L}$, where $\mathbb{K}_{\mathbb{M}} = \{1, 2, \cdots, K_M\}$ and $\mathbb{K}_{\mathbb{F}_i} = \{1, 2, \cdots, K_{F_i}\}$.

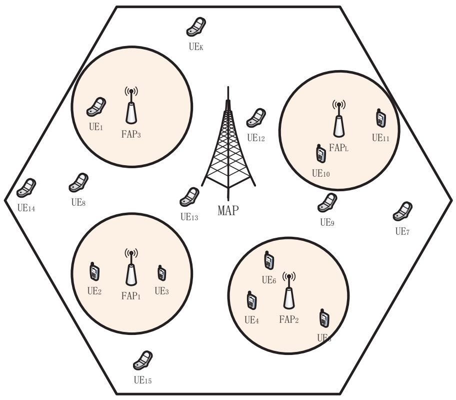

Figure 1. Macro-cell access point–femtocell access points (MAP–FAPs) heterogeneous network model.

In this HetNet, the total spectrum band is divided into N subcarriers, and subcarrier bandwidth is denoted by B. To simplify, MAP and FAPs operate on individual frequency bands without cross-tier interference. In general, this is done by allocating a fixed number of subcarriers to FAPs, or the FAPs leasing some subcarriers from a third party. In addition, there is no interference between FAPs, which can be achieved by assigning non-overlapping spectrum bands.

3. Problem Formulation and Optimal Solution

In this paper, we investigate AP selection which aims to maximize the system sum-throughput under the proportional user rate constraint. Unfortunately, the optimization problem involving AP selection is difficult to solve [13]. To make the problem tractable, restriction is relaxed.

3.1. Single Subcarrier User Rate

The data rate of user k that accesses MAP and FAPs using subcarriers n and m is respectively denoted by

$$r_{0kn} = B\log_2\left(1 + \frac{p_{k,n}G_{k,n}}{\sigma^2}\right) \tag{1}$$

$$r_{ikm} = B\log_2(1 + \frac{p_{k,m}G_{k,m}}{\sum\limits_{k' \in \mathbb{K}_{\mathbb{F}}, k' \neq k} p_{k',m}G_{k',m} + \sigma^2}) \tag{2}$$

where $p_{k,n}$ and $p_{k,m}$ denote the transmission power to user k in subcarrier n and subcarrier m, respectively. $G_{k,n}$ and $G_{k,m}$ are, respectively, the propagation loss of user k in subcarrier n and subcarrier m. σ^2 denotes the noise variance.

3.2. Approximated Problem and Problem Formulation

As we know, user rate is an important parameter in the selection of AP for different users. In this paper, r_{0k} and r_{ik} are chosen as our selection parameter, where $r_{0k} = \frac{\sum_{n=1}^{N} r_{0kn}}{N}$,$r_{ik} = \frac{\sum_{m=1}^{M} r_{ikm}}{M}$. r_{0k} and r_{ik} denote the average single subcarrier user rate, which can imply channel condition. N and M are, respectively, the number of total available subcarriers in MAP and the *ith* FAP.

So the user association problem to maximize sum-throughput with proportional user rate constraint and maximum service user number constraint in the MAP–FAPs heterogeneous network can be formulated as

$$\begin{aligned} &\max \sum_{k=1}^{K} R_k = \max \sum_{k=1}^{K} \left(\alpha_{0k}r_{0k} + \sum_{i=1}^{L} \alpha_{ik}r_{ik}\right) \\ &= \max \sum_{k=1}^{K} \left(\sum_{i=0}^{L} \alpha_{ik}r_{ik}\right) \end{aligned} \tag{3}$$

subject to:

$$\begin{cases} \sum\limits_{i=0}^{L} \alpha_{ik} \leq 1, \forall k; \alpha_{ik} \in \{0,1\}, \forall i,k (3a) \\ \sum\limits_{k=1}^{K} \alpha_{ik} \leq \lambda_{\max}^{i}, i = 0,1,\ldots,L (3b) \\ R_1 : R_2 : \cdots : R_K = \gamma_1 : \gamma_2 : \cdots : \gamma_K (3c) \end{cases}$$

where R_k represents the *ith* user's average single subcarrier user rate. $i(i = 0,\ldots,L)$ represents the index of AP; namely, $i = 0$ implies MAP and $i(i = 1,\ldots,L)$ implies the *ith* FAP. α_{ik} indicates whether or not the *kth* user and the *ith* AP are connected, and $\alpha_{ik} = 1$ implies the *kth* user connected to the *ith* AP. $\lambda_{\max}^{i}$ represents the maximum service user number of the *ith* AP. Condition (3a) guarantees that each user will be connected with only one AP. Condition (3b) states that the *ith* AP can serve a maximum of $\lambda_{\max}^{i}$ users. Condition (3c) is the proportional user rate constraint, in which $\{\gamma_1, \gamma_2, \cdots, \gamma_K\}$ is the set of predetermined values that are used to satisfy different users' demand [14].

It turns out that the above problem is a binary integer programming problem, since it involves binary variables. This kind of problem is generally very difficult to solve. To make this problem tractable, we release the AP selection constraints in (3a),

$$0 \leq \alpha_{ik} \leq 1$$

Assume normalized rate of all users is expressed as y, so

$$y = \frac{R_1}{\gamma_1} = \frac{R_2}{\gamma_2} \cdots = \frac{R_K}{\gamma_K}$$

$$\max \sum_{k=1}^{K} R_k = \max \sum_{k=1}^{K} y\gamma_k$$

As mentioned before, $\{\gamma_1, \gamma_2, \cdots, \gamma_K\}$ is predetermined [15], so the problem (3) can be reformulated as

$$\max y \tag{4}$$

subject to:

$$\begin{cases} y = \frac{R_k}{\gamma_k} = \frac{1}{\gamma_k}\left(\sum_{i=0}^{L} \alpha_{ik} r_{ik}\right), \forall k (4a) \\ \sum_{i=0}^{L} \alpha_{ik} \leq 1, \forall k; 0 \leq \alpha_{ik} \leq 1, \forall i, k (4b) \\ \sum_{k=1}^{K} \alpha_{ik} \leq \lambda_{\max}^{i}, i = 0, 1, \ldots, L (4c) \end{cases}$$

The Lagrangian function of the above problem is given by

$$\begin{array}{l} L = y + \sum_{k=1}^{K} \lambda_k \left[\frac{1}{\gamma_k}\left(\sum_{i=0}^{L} \alpha_{ik} r_{ik}\right) - y\right] + \\ \sum_{k=1}^{K} \mu_k \left(1 - \sum_{i=0}^{L} \alpha_{ik}\right) + \sum_{i=0}^{L} \sum_{k=1}^{K} \theta_{ik} \alpha_{ik} + \\ \sum_{i=0}^{L} \omega_i \left(\lambda_{\max}^{i} - \sum_{k=1}^{K} \alpha_{ik}\right) \end{array} \tag{5}$$

where $\lambda_k \neq 0$, $\mu_k \geq 0$, $\theta_{ik} \geq 0$ and $\omega_i \geq 0$ are the Lagrange multipliers [16]. The Karush–Kuhn–Tucker (KKT) conditions can be expressed as

$$\begin{cases} \frac{\partial L}{\partial y} = 1 - \sum_{k=1}^{K} \lambda_k = 0 (5a) \\ \frac{\partial L}{\partial (\alpha_{ik})} = \frac{\lambda_k r_{ik}}{\gamma_k} - \mu_k + \theta_{ik} - \omega_i = 0, \forall i, k (5b) \\ y = \frac{1}{\gamma_k}\left(\sum_{i=0}^{L} \alpha_{ik} r_{ik}\right), \forall k (5c) \\ \mu_k \left(1 - \sum_{i=0}^{L} \alpha_{ik}\right) = 0, \forall k (5d) \\ \theta_{ik} \alpha_{ik} = 0, \forall i, k (5e) \\ \omega_i \left(\lambda_{\max}^{i} - \sum_{k=1}^{K} \alpha_{ik}\right) = 0, \forall i (5f) \end{cases}$$

Supposing that user k only accesses one network, we assume i; i.e., $\alpha_{ik} > 0$ and $\alpha_{jk} = 0$, for any $j \neq i$. Then, based on (b) and (e), we get

$$\begin{cases} \theta_{ik} = 0, \frac{\lambda_k r_{ik}}{\gamma_k} = \mu_k + \omega_i \\ \theta_{jk} \geq 0, \frac{\lambda_k r_{jk}}{\gamma_k} \leq \mu_k + \omega_j \end{cases}$$

So $\frac{r_{ik}}{r_{jk}} \geq \frac{\mu_k + \omega_i}{\mu_k + \omega_j}$ is obtained.

4. The Proposed Algorithm

As can be seen from the above result, the ratio of a user's achievable rate in different APs is an important selection parameter. In this paper, rate ratio-based user association is proposed, and users prefer to access an AP that can support a relatively higher user rate. Unfortunately, we cannot obtain the ratio threshold using a mathematical method, and the optimal algorithm using exhaustive search is too complicated to implement. Besides, the user association above is an either–or choice:

access the ith AP or access the jth AP, whereas user association in this paper is a multichotomous question. In order to use the above conclusion, we should transfer the multichotomous question into a either–or question.

In summary, a low complexity suboptimal user association based on rate ratio is desirable. First, we divide all users into different sets; namely, each user preselects the AP that can support the highest user rate. Users who preselect MAP are expressed as set $\mathbb{K}_{\mathbb{M}}$, and users who preselect the ith FAP are expressed as set $\mathbb{K}_{\mathbb{F}_i}$. Based on the preselect result, we can execute an either–or choice for users in set $\mathbb{K}_{\mathbb{F}_i}$; that is, users in set $\mathbb{K}_{\mathbb{M}}$ select MAP as the final decision, and users in $\mathbb{K}_{\mathbb{F}_i}$ reselect AP between the ith FAP and MAP. The proposed suboptimal user association algorithm is described as follows.

1. **Initialization**

   L FAPs and K users are randomly generated within the scope of MAP. Initialize $K_M = K_{F_i} = 0(i = 1, \ldots, L)$, $\mathbb{K}_{\mathbb{M}} = \mathbb{K}_{\mathbb{F}_i} = \varnothing(i = 1, \ldots, L)$. Set that the $ith(i = 1, \ldots, L)$ AP can serve a maximum of $\lambda^i_{\max}$ users. To simplify, assume that each FAP has the same maximum service user number, $\lambda_{\max}$, and to make sure that every available user will be served, MAP sets its quota $\lambda^0_{\max}$ to be equal to the maximum number of UEs.

2. **Pre-selection**

   for $k = 1$ to K

   Calculate the kth user rate $r_{ik}(i = 0, 1, \ldots, L)$ when it connects to the ith AP, then the kth user preselects the AP who can support the highest user rate.

   end

   Then K_M, $K_{F_i}(i = 1, \ldots, L)$, $\mathbb{K}_{\mathbb{M}}$ and $\mathbb{K}_{\mathbb{F}_i}(i = 1, \ldots, L)$ are obtained.

3. **Final selection**

   if $i = 0$

   K_M=K_M, $\mathbb{K}_{\mathbb{M}}$=$\mathbb{K}_{\mathbb{M}}$

   end

   for $i = 1$ to L

   Calculate $\frac{r_{ik}}{r_{0k}}(k \in \mathbb{K}_{\mathbb{F}_i})$. Then sort $\frac{r_{ik}}{r_{0k}}(k \in \mathbb{K}_{\mathbb{F}_i})$ in an increasing manner; namely, $\frac{r_{i1}}{r_{01}} \leqslant \frac{r_{i2}}{r_{02}} \leqslant \cdots \leqslant \frac{r_{iK_{F_i}}}{r_{0K_{F_i}}}$. Select the last $min(\lambda_{\max}, K_{F_i})$ users according to the order.

   Accordingly, we obtain

   $\mathbb{K}_{\mathbb{F}_i} = \{K_{F_i} - min(\lambda_{\max}, K_{F_i}) + 1, \ldots, K_{F_i}\}$, $\mathbb{K}_{\mathbb{M}} = \mathbb{K}_{\mathbb{M}} \cup \{1, \ldots, K_{F_i} - min(\lambda_{\max}, K_{F_i})\}$

   $K_{F_i} = min(\lambda_{\max}, K_{F_i})$

   $K_M = K_M + K_{F_i} - min(\lambda_{\max}, K_{F_i})$

   end

In reference [13], Xue et al. calculated $\prod_{i=1}^{L} \left(K_{F_i} + 1\right)$ results, then obtained the final optimal solution by finding the maximum normalized user rate, but it has high complexity. Different from [13], this paper restricts the number of maximum service users for each FAP. Note that the maximum service user number is small compared to the total user number, so to reduce the computational complexity, we only select the last $K_{F_i} = min(\lambda_{\max}, K_{F_i})$ users as our final choice.

5. Simulation Results

5.1. Simulation Configuration

The MAP–FAPs heterogeneous network is shown in Figure 2. To evaluate the performance of the proposed algorithm, we set simulation conditions as follows. We consider one MAP, which is at the center of hexagon cell with the radius of 500 m. Assume the MAP owns four FAPs, which is at the center of a circular network with the radius of 200 m. For simplicity, the FAPs are non-overlapping. FAPs and users are randomly distributed in the MAP. The detailed parameters are listed in Table 1.

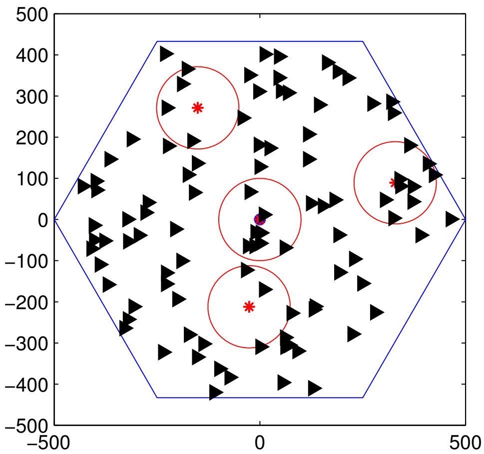

Figure 2. Simulation model of MAP–FAPs heterogeneous network.

Table 1. Simulation parameters.

Parameter	Value
MAP transmission power	43 dBm
FAP transmission power	20 dBm
Bandwidth of subcarrier	15 KHz
Path loss (MAP)	15.3 + 37.6 lg(d)
Path loss (FAP)	38.46 + 20 lg(d)
Shadowing	Log-normal, 8 dB standard deviation
Noise power density	−174 dBm/Hz

To evaluate the advantages of the proposed algorithm, we will compare it with three user association algorithms—namely, MAP-only algorithm, FAP-first algorithm, and the best AP selection algorithm. MAP-only algorithm means that all users can only access MAP. For FAP-first algorithm, users in the FAP coverage area prefer to access FAP, but the maximum number of users that FAP can serve is finite, so only a portion of these users can access FAP. As for the best AP selection algorithm, users always select the AP that can support the highest user rate.

5.2. Simulation Analysis

All curves are generated based on averaging 5000 loops of these algorithms. First, we set $\lambda_{\max} = 6$ for all the FAPs. These four algorithms are first compared in terms of sum-throughput, spectrum efficiency, and fairness, as shown from Figure 3 to Figure 5. The fairness index is defined as $\frac{\left(\sum\limits_{k=1}^{K} R_k\right)^2}{K \sum\limits_{k=1}^{K} R_k^2}$.

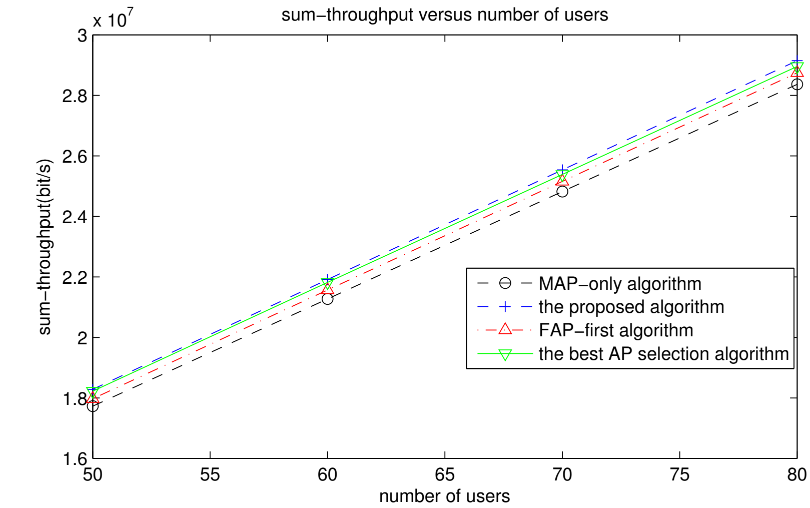

Figure 3. Performance comparison in terms of sum-throughput. AP: access point.

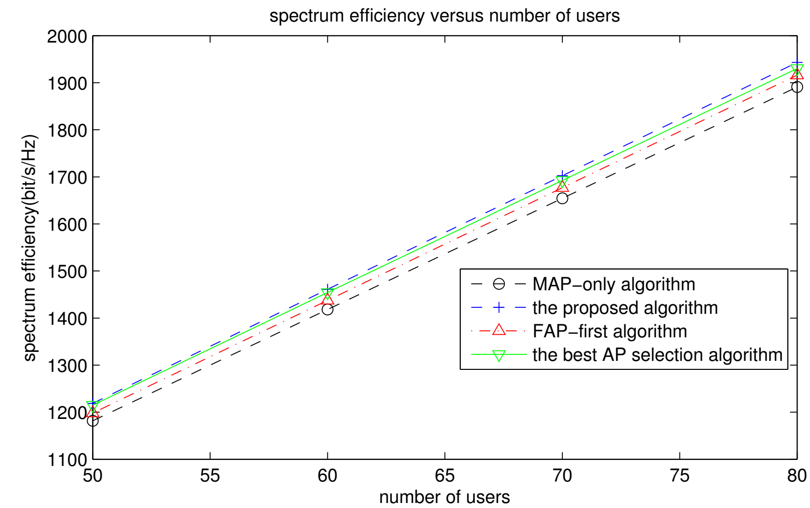

Figure 4. Performance comparison in terms of spectrum efficiency.

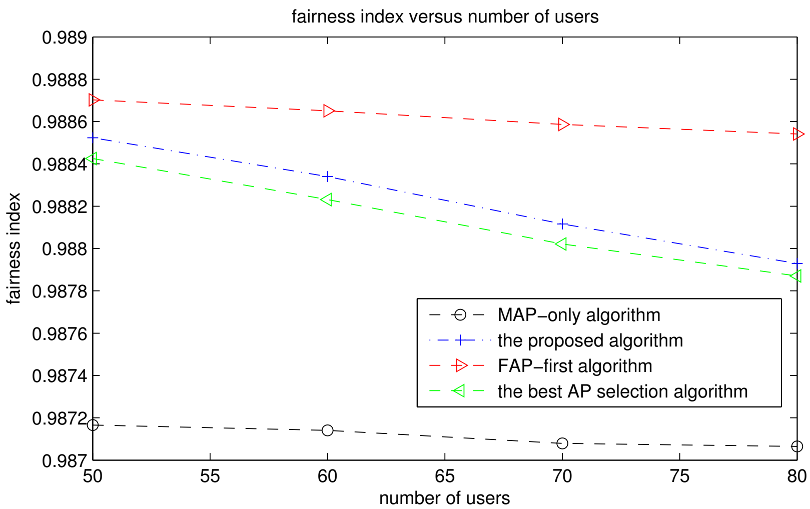

Figure 5. Performance comparison in terms of fairness.

As can be seen from Figures 3–5, the MAP-only algorithm has the worst sum-throughput, the worst spectrum efficiency, and the worst fairness, since no network diversity is utilized. The MAP-only algorithm will make cell-edge users have low user rate and low spectrum efficiency. The FAP-first algorithm has a higher rate and higher spectrum efficiency than MAP-only, and the fairness is the best, because it gives users in the coverage area of FAPs the opportunity to obtain a higher rate. The best AP selection algorithm has slightly higher rate and slightly higher spectrum efficiency than the FAP-first algorithm, but it has a lower fairness index than the FAP-first algorithm, since users in the best AP selection algorithm prefer to select an AP that can support the highest rate. The proposed algorithm can support the highest sum-throughput. Though it has lower fairness index than the FAP-first algorithm, their difference is very small. When total user number is equal to 70, the proposed algorithm's fairness is about 0.051 percent lower than the FAP-first algorithm. The difference is very small, so we can neglect it. When total user number equals 70, the detailed performance comparisons are listed from Tables 2–4.

Table 2. Sum-throughput comparison.

Algorithms	Sum-Throughput (10^7 bit/s)	Improvement over MAP-Only (Percent)
The proposed algorithm	2.5536	2.893
The best AP selection	2.5388	2.297
FAP-first	2.5155	1.358
MAP-only	2.4818	0

Table 3. Spectrum efficiency comparison.

Algorithms	Spectrum Efficiency (10^3 bit/s/Hz)	Improvement over MAP-Only (Percent)
The proposed algorithm	1.7024	2.895
The best AP selection	1.6925	2.297
FAP-first	1.6770	1.360
MAP-only	1.6545	0

Table 4. Fairness comparison.

Algorithms	Fairness Index	Decrease from FAP-First (Percent)
FAP-first	0.9886	0
The proposed algorithm	0.9881	0.051
The best AP selection	0.9880	0.061
MAP-only	0.9871	0.152

Figures 6 and 7 depict that both sum-throughput and spectrum efficiency of the proposed algorithm are dominated by maximum service user number of FAP. With the increase of maximum service user number, we can see a slower increase of system sum-throughput and spectrum efficiency from Figures 6 and 7, respectively.

To illustrate the relationship between sum-throughput and maximum service user number, we set that the total user number equals to 50. Figure 8 compares sum-throughput of different users with different maximum service user numbers. A stable sum-throughput is obtained in MAP-only algorithm, since no users in MAP-only can access FAP. Except for MAP-only algorithm, all the other algorithms have the same tendency; namely, sum-throughput increases with the increase of maximum service user number in the early stage, and becomes a stable sum-throughput when maximum service user number reaches a certain value $\lambda_{threshold}$. As depicted in Figure 8, different algorithms have different $\lambda_{threshold}$. Similarly, the relationship between spectrum efficiency and maximum service user number is the same as the relationship between spectrum efficiency and maximum service user number, as shown in Figure 9.

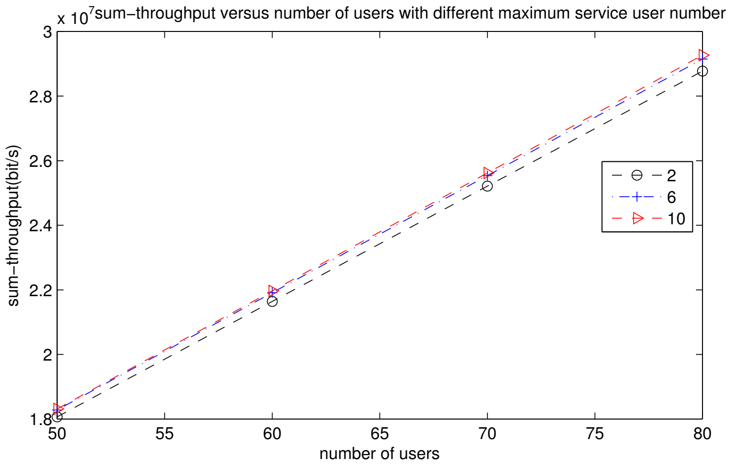

Figure 6. Performance comparison in terms of sum-throughput with different maximum service user numbers.

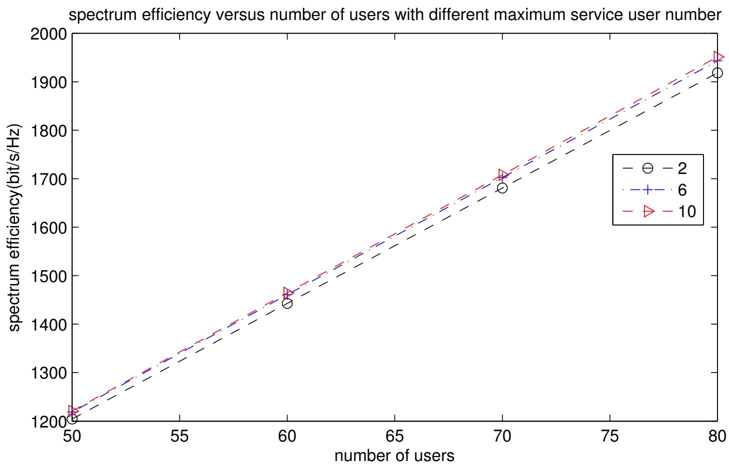

Figure 7. Performance comparison in terms of spectrum efficiency with different maximum service user numbers.

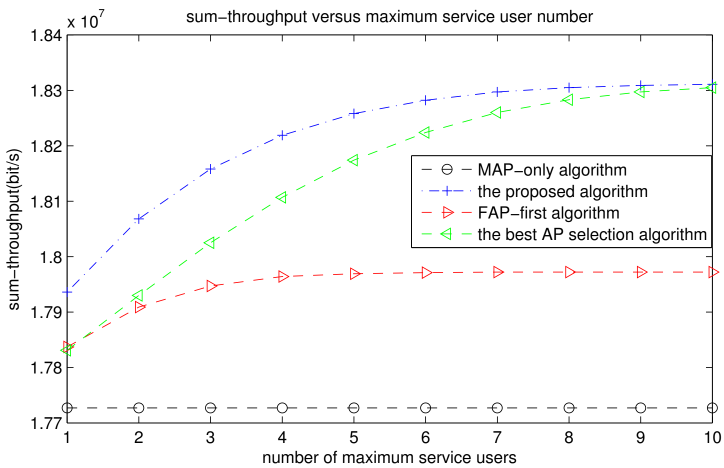

Figure 8. Performance comparison in terms of sum-throughput with different maximum service user numbers.

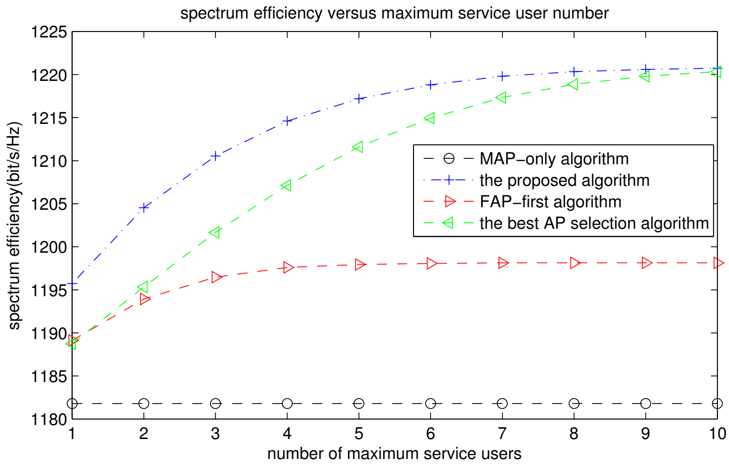

Figure 9. Performance comparison in terms of spectrum with different maximum service user numbers.

6. Conclusions

This paper investigated the user association problem with proportional rate constraint and maximum service user number constraint a in MAP–FAPs heterogeneous network. By using Lagrangian function, the optimal solution is obtained on the basis of problem formulation. However, the actual rate ratio threshold cannot be obtained by using a mathematical method. Therefore, an approximate optimal algorithm is proposed. Simulation results show that the proposed algorithm has the highest user rate while sacrificing minor fairness. Additionally, this paper investigates the influence of maximum service user number on system sum-throughput and spectrum efficiency, and the conclusion will provide technical support for people who are dedicated to improving the service number limits of FAPs.

Acknowledgments: This paper is funded by the National Natural Science Foundation of China (Grant No. 51509049), the Natural Science Foundation of Heilongjiang Province, China (Grant No. F201345), the Fundamental Research Funds for the Central Universities of China (Grant No. GK2080260140), the National Key Research and Development Program of China (Grant No. 2016YFF0102806).

Author Contributions: Fang Ye conceived the concept and performed the research, Chunxia Su conducted experiments to evaluate the performance of the proposed algorithm and wrote the manuscript, Yibing Li reviewed the manuscript. All authors have read and approved the final manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Liu, D.; Wang, L.; Chen, Y.; Elkashlan, M.; Wong, K.; Schober, R.; Hanzo, L. User association in 5G networks: A survey and an outlook. *IEEE Commun. Surv. Tutor.* **2016**, *18*, 1018–1044.
2. Andrews, J.G.; Claussen, H.; Dohler, M.; Rangan, S.; Reed, M.C. Femtocells: Past, present, and future. *IEEE J. Sel. Areas Commun.* **2012**, *30*, 497–508.
3. Peng, M.; Wang, C.; Li, J.; Xiang, H.; Lau, V. Recent advances in underlay heterogeneous networks: Interference control, resource allocation, and self-organization. *IEEE Commun. Surv. Tutor.* **2015**, *17*, 700–729.
4. Li, Y.B.; Yang, R.; Ye, F.; Gao, Z. Improved spectrum sharing algorithm based on feedback control information in cognitive radio networks. *J. Syst. Eng. Electron.* **2013**, *24*, 564–570.
5. Lee, Y.L.; Chuah, T.C.; Loo, J.; Vinel, A. Recent advances in radio resource management for heterogeneous LTE/LTE-A networks. *IEEE Commun. Surv. Tutor.* **2014**, *16*, 2142–2180.
6. Luan, Z.; Qu, H.; Zhao, J.; Chen, B. Low complexity distributed max-throughput algorithm for user association in heterogeneous network. *Wirel. Pers. Commun.* **2016**, *87*, 1147–1156.
7. Song, W.; Jiang, H.; Zhuang, W. Performance analysis of the WLAN-first scheme in cellular/WLAN interworking. *IEEE Trans. Wirel. Commun.* **2007**, *6*, 1932–1952.

8. Nie, J.; Wen, J.C.; Dong, Q.; Zhou, Z. A seamless handoff in IEEE 802.16 a and IEEE 802.11 n hybrid networks. In Proceedings of the IEEE 2005 International Conference on Communications, Circuits and Systems, Hong Kong, China, 27–30 May 2005; Volume 1, pp. 383–387.
9. Roy, J.J.J.; Vaidehi, V.; Srikanth, S. Always best-connected QoS integration model for the WLAN, WiMAX heterogeneous network. In Proceedings of the IEEE First International Conference on Industrial and Information Systems, Moratuwa, Sri Lanka, 8–11 August 2006; pp. 361–366.
10. Lasaulce, S.; Tembine, H. *Game Theory and Learning for Wireless Networks: Fundamentals and Applications*; Academic Press: New York, NY, USA, 2011.
11. Liang, L.; Feng, G.; Jia, Y. Game-theoretic hierarchical resource allocation for heterogeneous relay networks. *IEEE Trans. Veh. Technol.* **2015**, *64*, 1480–1492.
12. Bayat, S.; Louie, R.H.Y.; Han, Z.; Vucetic, B.; Li, Y. Distributed user association and femtocell allocation in heterogeneous wireless networks. *IEEE Trans. Commun.* **2014**, *62*, 3027–3043.
13. Xue, P.; Gong, P.; Park, J.H.; Park, D.; Kim, D.K. Radio resource management with proportional rate constraint in the heterogeneous networks. *IEEE Trans. Wirel. Commun.* **2012**, *11*, 1066–1075.
14. Wong, I.C.; Shen, Z.; Evans, B.L.; Andrews, J.G. A low complexity algorithm for proportional resource allocation in OFDMA systems. In Proceedings of the SIPS IEEE Workshop on Signal Processing Systems, Austin, TX, USA, 13–15 October 2004; pp. 1–6.
15. Shen, Z.; Andrews, J.-G.; Evans, B.-L. Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints. *IEEE Trans. Wirel. Commun.* **2005**, *4*, 2726–2737.
16. Zheng, J.; Li, J.D.; Shi, H.; Liu, Q.; Yang, X.N. Joint subcarrier, code, and power allocation for parallel multi-radio access in heterogeneous wireless networks. *Sci. China Inf. Sci.* **2014**, *57*, 1–5.
17. Li, Y.; Nie, W.; Ye, F. A complex mixing matrix estimation algorithm based on single source points. *Circuits Syst. Signal Process.* **2015**, *34*, 3709–3723.
18. Jia, C.; Lim, T.J. Resource partitioning and user association with sleep-mode base stations in heterogeneous cellular networks. *IEEE Trans. Wirel. Commun.* **2015**, *14*, 3780–3793.
19. Ye, Q.; Rong, B.; Chen, Y.; Al-Shalash, M.; Caramanis, C.; Andrews, J.G. User association for load balancing in heterogeneous cellular networks. *IEEE Trans. Wirel. Commun.* **2013**, *12*, 2706–2716.
20. Shen, K.; Yu, W. Distributed pricing-based user association for downlink heterogeneous cellular networks. *IEEE J. Sel. Areas Commun.* **2014**, *32*, 1100–1113.