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Heat Induction by Viscous Dissipation Subjected to Symmetric and Asymmetric Boundary Conditions on a Small Oscillating Flow in a Microchannel

Chih Ping Tso , Chee Hao Hor , Gooi Mee Chen * and Chee Kuang Kok

Multimedia University, Faculty of Engineering and Technology, Jalan Ayer Keroh Lama, Melaka 75450, Malaysia; cptso@mmu.edu.my (C.P.T.); andyhorcheehao@yahoo.com (C.H.H.); ckkok@mmu.edu.my (C.K.K.)

* Correspondence: gmchen@mmu.edu.my; Tel.: +60-252-3342

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Abstract: The heat induced by viscous dissipation in a microchannel fluid, due to a small oscillating motion of the lower plate, is investigated for the first time. The methodology is by applying the momentum and energy equations and solving them for three cases of standard thermal boundary conditions. The first two cases involve symmetric boundary conditions of constant surface temperature on both plates and both plates insulated, respectively. The third case has the asymmetric conditions that the lower plate is insulated while the upper plate is maintained at constant temperature. Results reveal that, although the fluid velocity is only depending on the oscillation rate of the plate, the temperature field for all three cases show that the induced heating is dependent on the oscillation rate of the plate, but strongly dependent on the parameters Brinkman number and Prandtl number. All three cases prove that the increasing oscillation rate or Brinkman number and decreasing Prandtl number, when it is less than unity, will significantly increase the temperature field. The present model is applied to the synovial fluid motion in artificial hip implant and results in heat induced by viscous dissipation for the second case shows remarkably close agreement with the experimental literature.

Keywords: Microchannel; oscillating fluid flow; viscous dissipation; Brinkman number; Prandtl number; synovial fluid

1. Introduction

Heat transfer in parallel plates flow plays an essential role in processes such as in heat exchangers, extrusion, glass fiber drawing, and metal forming, where the heat is exchanged continuously with the surrounding fluid and plate surfaces [1]. The insight of fluid rheology behaviour is vital and can affect the quality efficiency during the heat transfer process. Viscous dissipation, a development of heat induced due to the work done by viscous friction acting within a fluid may cause significant rise in fluid temperature led by the presence of large velocity gradients [2]. The effect of viscous dissipation can contribute to a significant amount of heat generation under certain situations such as flow in microchannels and microtubes [3–5]. Ignoring the viscous dissipation in such circumstances could markedly affect fluid flow accuracy [6]. Often, the viscous dissipation effect in the conservation of energy is in the form of a dimensionless term, known as the Brinkman number, Br , with a zero value of Br implying no viscous dissipation.

Previous studies have been reported in the literature regarding the viscous dissipation effect on forced convective heat transfer [1–11]. For example, Aydin and Avci [1] revealed the effect of viscous dissipation on the fully-developed convection heat transfer in pipes, subjected to constant wall temperature and constant wall heat flux, respectively, and concluded that the temperature effect

is evident when Br is large. Sheela and Tso [3] addressed the Newtonian fluid flow study with asymmetric thermal boundary conditions and taking into account the viscous dissipation. The fluid flow was subjected to either a fixed or moving boundary and results were expressed in temperature profiles together with the Nusselt number.

While the case of one of the parallel plates moving in constant velocity and the other plate stationary is well-known as Couette flow or Couette-Poiseuille flow, the case when the plate motion is oscillating had only been alluded to and consequences of the full solution has less commonly been discussed [12–15]. It is even rare when viscous dissipation effect is considered in a finite oscillating flow. Most of the literature focus on unidirectional flows when studying the effect of viscous dissipation. It is noteworthy that the classic case of a single plate moving in an infinite fluid medium has been studied as Stoke's first problem, when the plate is suddenly accelerated on a stationary fluid [15]. Stoke's second problem involves a single oscillating plate in the infinite medium [15].

Therefore, the novelty of the present study is to provide a vital insight into the effect of viscous dissipation within a microchannel, where the fluid is hydrodynamically driven by an oscillating lower plate. This study will include the symmetric and asymmetric thermal boundary conditions that shows the wide coverage of viscous dissipation impact on the flow. The authors notice that some relevant experimental results had been reported on the related problem of synovial fluid motion in an artificial hip joint, which will be discussed in Section 3.3. The thin layer of synovial fluid is encapsulated within the moving femoral head and the stationary acetabular cup. Thus, the developed model is used to simulate the thermal hydraulics characteristics in the synovial fluid.

2. Problem Description and Analysis

Figure 1 considers an unsteady laminar Newtonian fluid flow within a microchannel with two infinite parallel plates distanced W apart from each other, where the upper plate is fixed while the lower one is oscillating with time dependent velocity $u = U \sin(\omega t)$. The upper and lower plates are subjected to arbitrary temperatures denoted in general as T_a and T_b , respectively.

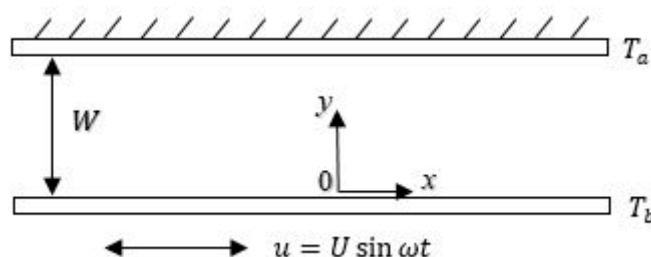


Figure 1. Schematic diagram of the microchannel.

The general conservation of momentum and energy are represented in typical symbols (see Nomenclature) in Equation (1) and Equation (2), respectively. These are for Newtonian, laminar, and incompressible flow with constant properties [16,17]:

$$\rho \left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = -\nabla p + \mu \nabla^2 \vec{u} + \rho \vec{F}. \quad (1)$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T \right) = k \nabla^2 T + \mu \Phi + q''' . \quad (2)$$

For unsteady infinite one-dimensional fluid flow with no pressure gradient and body force, the conservation of momentum in Equation (1) reduces to Equation (3), comprising the unsteady term and the viscous term. In addition, in the absence of heat source, the conservation of energy in Equation (2) reduces to Equation (4). The first term on the right side of Equation (4) is the conduction term. The second term on the right side is the remnant viscous dissipation term, and this is the cause of a temperature rise in the flow. The convective term in this case is negligible as the changes of

temperature in x-direction is insignificant in an oscillating flow (see Appendix A), hence leaving on the left side of the heat storage term.

$$\frac{\partial u}{\partial t} - \nu \frac{\partial^2 u}{\partial y^2} = 0. \quad (3)$$

$$\rho c_p \frac{\partial T}{\partial t} = k \left(\frac{\partial^2 T}{\partial y^2} \right) + \mu \left(\frac{\partial u}{\partial y} \right)^2. \quad (4)$$

In the above equations, u is the fluid velocity driven by the bottom plate, ν is the kinematic viscosity, ρ is the fluid density, c_p is the specific heat, T is the fluid temperature, k is the thermal conductivity of fluid, and μ is the dynamic viscosity. Solutions to Equations (3) and (4) can only be obtained by specifying the appropriate boundary conditions. As far as the momentum equation is concerned, the upper plate is stationary and the lower plate oscillates according to $u = U \sin(\omega t)$.

For the energy equation, three cases of boundary conditions will be considered:

Case A. Symmetric boundary conditions, where both plates are kept at the same constant temperature T_1 . ($T_a = T_b = T_1$)

Case B. Symmetric boundary conditions, where both plates are insulated. (Temperature gradients zero)

Case C. Asymmetric boundary conditions, where the upper plate is insulated and the bottom plate is kept at constant temperature T_1 . ($T_b = T_1$)

As this is an unsteady problem, in addition to the above, the fluid is assumed to be initially at rest, and the fluid temperature is equal to an arbitrary initial temperature T_1 within the parallel plates.

Before solving, it is more efficient to re-cast the equations into dimensionless forms, using the following definitions:

$$u^* = \frac{u}{U}, \quad y^* = \frac{y}{W}, \quad Pr = \frac{\nu}{\alpha}, \quad T^* = \frac{T - T_1}{T_m - T_1}, \quad Br = \frac{\mu U^2}{k(T_m - T_1)}, \quad t^* = \frac{\nu t}{W^2}, \quad \text{and } \omega^* = \frac{\omega W^2}{\nu}, \quad (5)$$

where u^* is the dimensionless fluid velocity, y^* is the dimensionless y-axis position, Pr is the Prandtl number, which is a measure of momentum diffusion over heat diffusion in a fluid, T^* is the dimensionless fluid temperature, Br is the Brinkman number consisting of viscous dissipation effect, T_m is the mean fluid temperature, t^* is the dimensionless time dependent term, ω^* is the dimensionless angular frequency, and α is the thermal diffusivity.

The dimensionless forms of Equations (3) and (4) are:

$$\frac{\partial u^*}{\partial t^*} - \frac{\partial^2 u^*}{\partial y^{*2}} = 0. \quad (6)$$

$$Pr \left(\frac{\partial T^*}{\partial t^*} \right) - \frac{\partial^2 T^*}{\partial y^{*2}} = Br \left(\frac{\partial u^*}{\partial y^*} \right)^2. \quad (7)$$

$$\begin{aligned} \text{Case A : } u^*(0, t^*) &= \sin(\omega^* t^*), \quad u^*(1, t^*) = 0 \text{ and } u^*(y^*, 0) = 0. \\ T^*(0, t^*) &= 0, \quad T^*(1, t^*) = 0 \text{ and } T^*(y^*, 0) = 0. \end{aligned} \quad (8)$$

The corresponding dimensionless boundary conditions for both equations for the three cases are as follows:

$$\begin{aligned} \text{Case B : } u^*(0, t^*) &= \sin(\omega^* t^*), \quad u^*(1, t^*) = 0 \text{ and } u^*(y^*, 0) = 0. \\ \frac{dT^*}{dy^*}(0, t^*) &= 0, \quad \frac{dT^*}{dy^*}(1, t^*) = 0 \text{ and } T^*(y^*, 0) = 0. \end{aligned} \quad (9)$$

$$\begin{aligned} \text{Case C : } u^*(0, t^*) &= \sin(\omega^* t^*), \quad u^*(1, t^*) = 0 \text{ and } u^*(y^*, 0) = 0. \\ \frac{dT^*}{dy^*}(0, t^*) &= 0, \quad T^*(1, t^*) = 0 \text{ and } T^*(y^*, 0) = 0. \end{aligned} \quad (10)$$

Hence, the aim of this study is to analyse the viscous dissipation effect (the effect of the Br term in Equation (7)) towards the fluid within the parallel plates when it is subjected to the three different

thermal boundary conditions. The methodology is to solve the Equations (6) and (7) simultaneously and numerically.

3. Results and Discussion

The model is solved numerically by the finite difference method and all computations are solved using the Matlab software. Typically, the problem is first solved by the hydrodynamic and thermal initial conditions stated, say those in Equation (8), which yield a set of velocity and temperature profiles for the fluid at a specified time step. After that, the set of data obtained will then be used as the next conditions for the next time step, and so on. An optimum time discretization is desirable in a numerical solution because a large time step gives an unpredictable numerical error while too small a time step requires more time.

In the present problem, the variable parameters in the model are Br , t^* , Pr and ω^* , which consist of the geometry as well as the fluid properties, such as the gap between the parallel plates, the fluid thermal diffusivity, and the time duration of fluid oscillation. The Br contains information on the fluid properties and the overall temperature change, which is always positive. The t^* , Pr and ω^* are always positive too. The range for Br , t^* , Pr and ω^* in this study are $1 \leq Br \leq 10$, $1 \leq t^* \leq 5$, $0.5 \leq Pr \leq 5$, and $1 \leq \omega^* \leq 10$. From trial computations, the optimum time step for these suggested ranges is found to be about $\Delta t^* = 0.02$.

3.1. Velocity Profiles

Since Cases A to C have the same velocity boundary conditions, the fluid velocity profiles shown in Figure 2 are common for all three cases. Figure 2 shows the velocity profiles within the selected time range $1 \leq t^* \leq 5$. The Br and Pr does not affect the velocity profile as it does not appear in Equation (6).

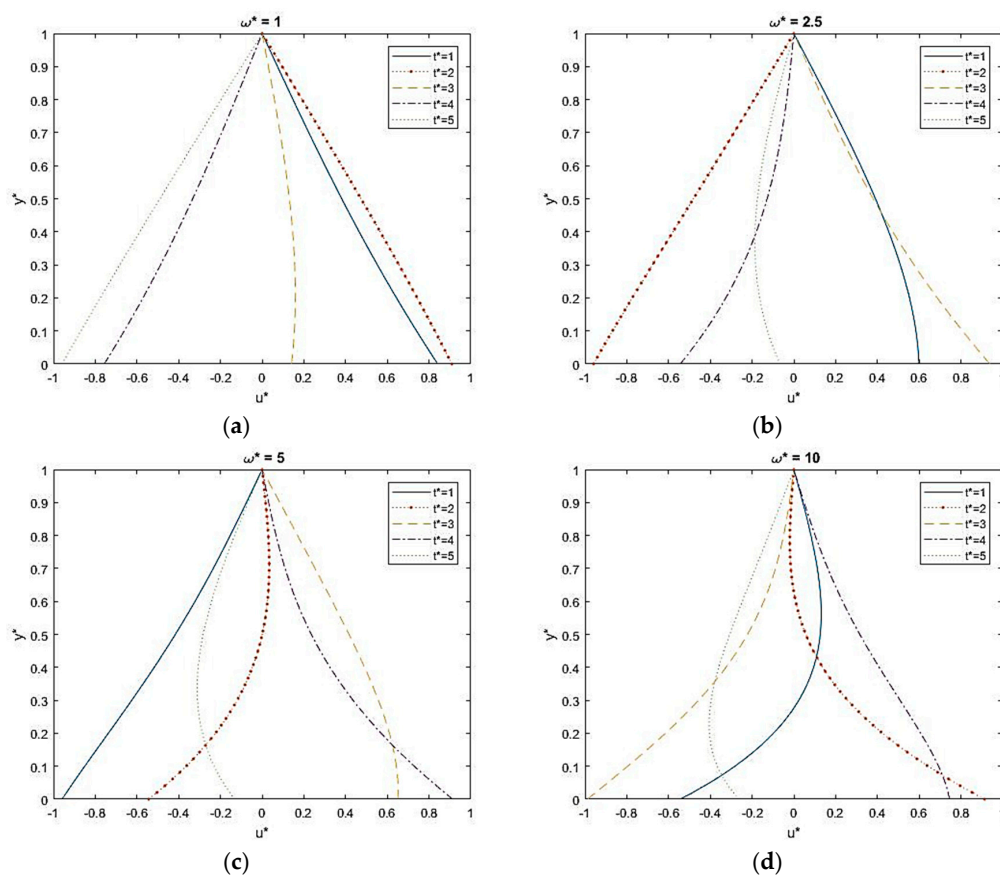


Figure 2. Various velocity profiles of y^* versus u^* at (a) $\omega^* = 1$, (b) $\omega^* = 2.5$, (c) $\omega^* = 5$, and (d) $\omega^* = 10$.

Comparing Figure 2a–d, the time dependent velocity profiles seem to vary widely as ω^* increases and bounded within the velocity range $-1 \leq u^* \leq 1$. These profiles satisfy the boundary conditions, where the fluid velocity at the upper plate is always zero and lower plate follows the time dependent oscillation. The fluid inertia influences the acceleration and give rise to a velocity phase lag with time to the fluid layers above the oscillating plate. This explains the wide variation in the profiles in Figure 2a–d, which reflect the variation in the position of the moving plate at $\omega = 1, 2.5, 5$, and 10 , influencing fluid layers above it in various degrees.

3.2. Temperature Profiles

3.2.1. Case A

Figure 3 displays the various fluid temperature profiles when the fluid is driven by the oscillating lower plate subjected to constant surface temperature as Equation (8). The temperature profiles are selected to show the effects of Br , ω^* , and Pr .

Figure 3a–b illustrate the effect of Brinkman number Br , at the selected time range from $1 \leq t^* \leq 5$. It is clear that Br has a significant effect on temperature rise and the maximum temperature increases with the Br . For instance, the maximum fluid temperature in Figure 3a is $T^* \cong 0.6$ at $t^* = 5$, $\omega^* = 1$, while in Figure 3b, $T^* \cong 1.2$ at $t^* = 5$, $\omega^* = 1$. It is worth mentioning that the peak temperature occurs at about 12% of the assigned Br at a constant ω^* . For example, the highest temperature at $t^* = 5$, $\omega^* = 1$ in Figure 3a,b, is at $T^* \cong 0.6$, when $Br = 5$ and $T^* \cong 1.2$, when $Br = 10$, respectively.

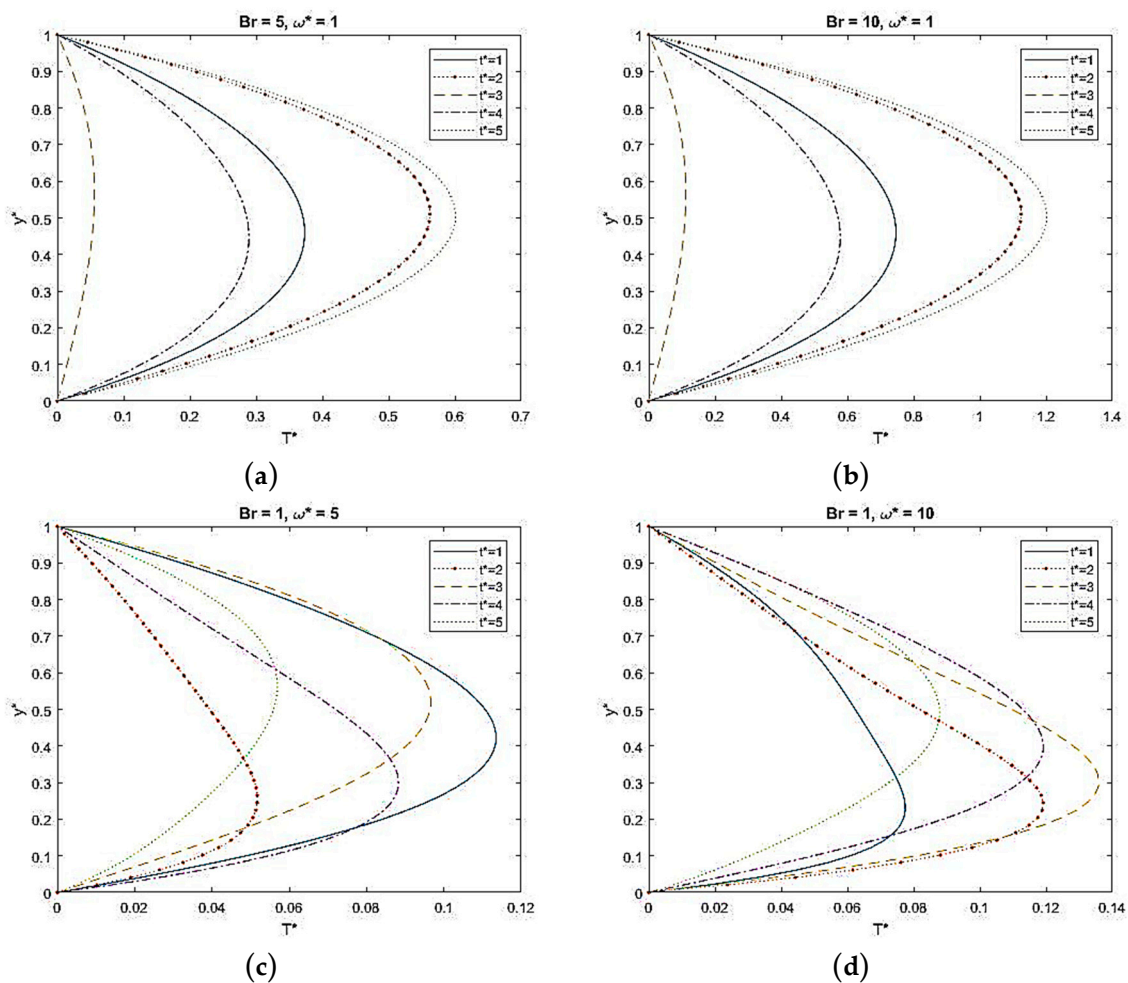


Figure 3. Cont.

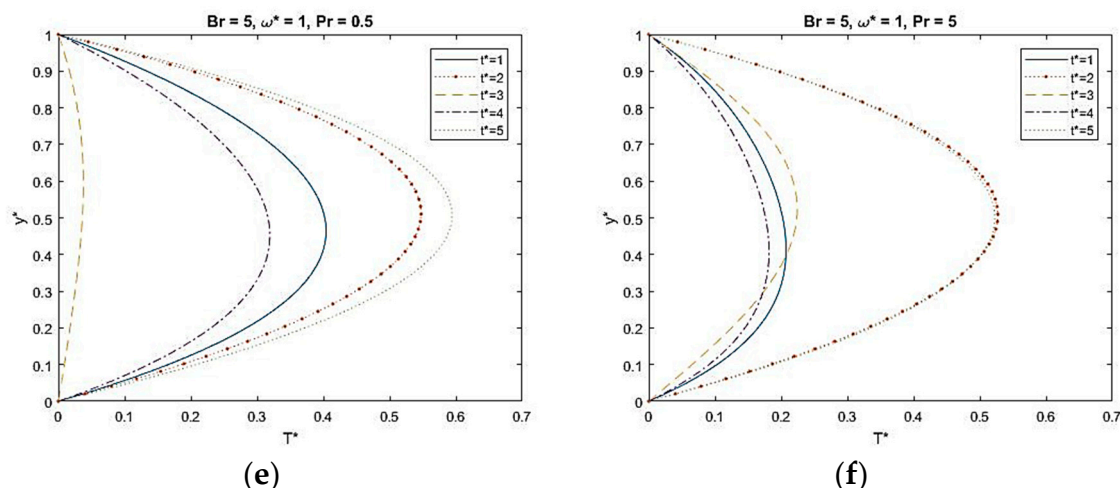


Figure 3. Fluid temperature profiles for both plates with constant surface temperature at (a) $Br = 5$, $\omega^* = 1$, $Pr = 1$, (b) $Br = 10$, $\omega^* = 1$, $Pr = 1$, (c) $Br = 1$, $\omega^* = 5$, $Pr = 1$, (d) $Br = 1$, $\omega^* = 10$, $Pr = 1$, (e) $Br = 5$, $\omega^* = 1$, $Pr = 0.5$ and (f) $Br = 5$, $\omega^* = 1$, $Pr = 5$.

As observed in Figure 3c–d, the peak temperature happens to be nearer to the lower plate when the dimensionless angular frequency ω^* increases. Additionally, the maximum temperature seems to increase with the ω^* as well. For instance, the temperature peak is approximately $T^* \cong 1.2$ at $t^* = 1, \omega^* = 5$ in Figure 3c and $T^* \cong 1.4$ at $t^* = 1, \omega^* = 10$ in Figure 3d, respectively, demonstrating that the heat induced due to viscous dissipation increases with the fluid motion.

The effect of Pr is illustrated in Figure 3a,e,f, all with the same values of Br and ω^* . As Pr indicates the relative momentum diffusion versus heat diffusion, it is seen that Figure 3f with more momentum diffusion, the temperature are lower compared to Figure 3a which has equal diffusion rates. However, Figure 3e with greater heat diffusion does increase the temperature profiles significantly.

The pattern of temperature profiles varies with time because the heat induced due to the viscous dissipation is allowed to transfer out from both plates at constant temperature conditions. At a lower rate of ω^* , the peak temperature is more likely to occur around the central location, having a symmetric temperature profile about the mid-plane of the parallel plates.

3.2.2. Case B

Figure 4 reveals the fluid temperature profiles when the thermal boundary conditions for both upper and lower plates are insulated, following Equation (9). This simply means that no heat is allowed to diffuse out of the system. The assigned values for the parameters Br , ω^* , Pr , and t^* are the same as in the previous case.

At low dimensionless angular frequency, $\omega^* = 1$, the induced heat is diffused evenly throughout the fluid, as shown in Figure 4a–b. Consequently, the temperature increases in a form that is closer to a vertical line as time moves on. Increase in heat induction by Br will only yield a higher rate of temperature increase, provided at a constant and low ω^* .

As observed on the ω^* effect alone in Figure 4c–d at constant Br , the heat induced seems to be significant with ω^* , which amplify the fluid momentum. The fluid temperature difference between both plates is more obvious when ω^* increases, as in Figure 4c–d, respectively.

As in Case A, the effect of Pr is explained in Figure 4a,e,f, respectively for $Pr = 0.5, 1, 5$, while keeping $Br = 5$, $\omega^* = 1$. As before, Figure 4e, with $\alpha > \nu$, shows a greater increase in temperature, whereas Figure 4f, with $\alpha < \nu$, a lower increase in temperature.

Since the induced heat is not able to diffuse out from the parallel plates, the fluid temperature within the parallel plates will only increase with time, more significantly than the other two cases.

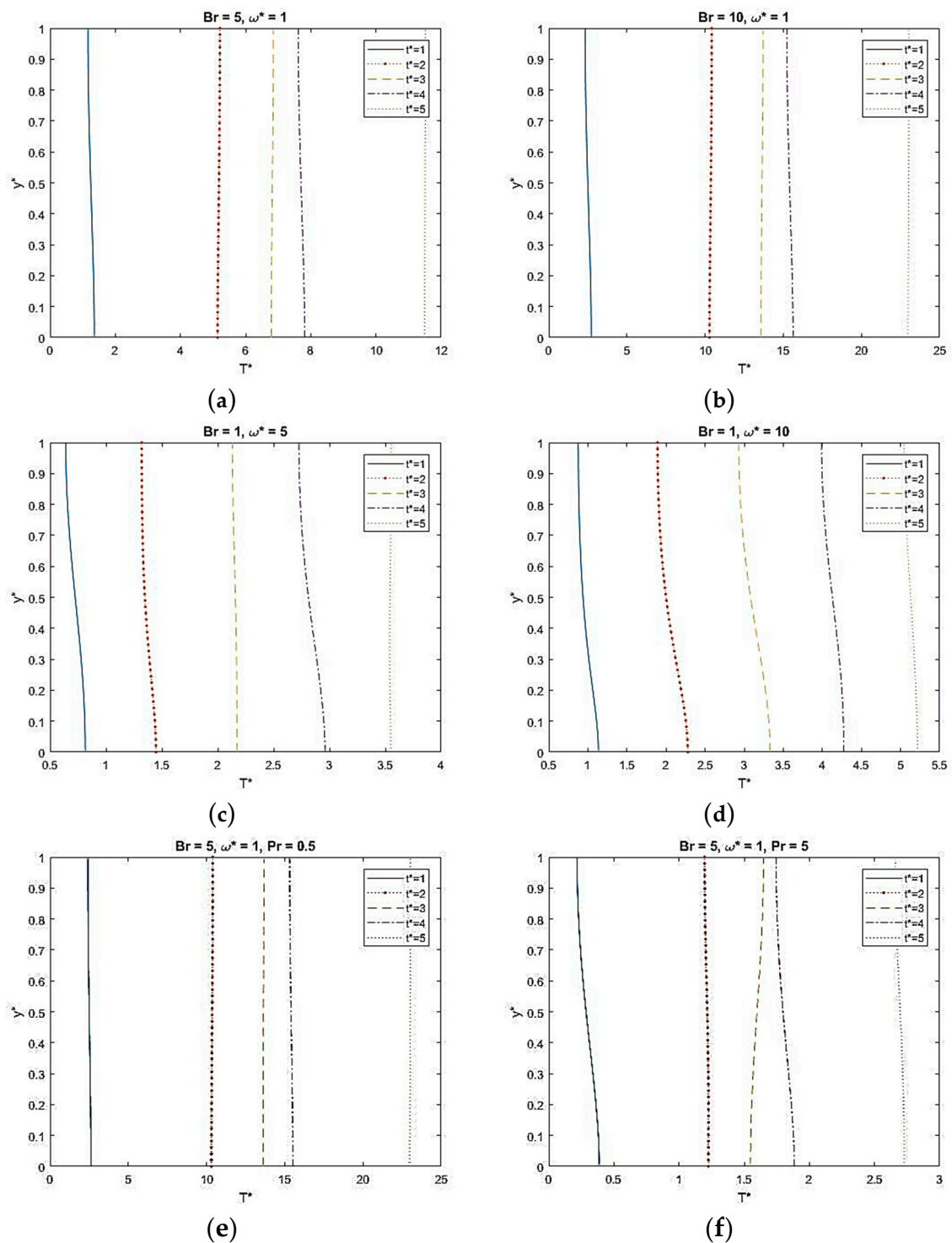


Figure 4. Fluid temperature profiles for both plates insulated at (a) $Br = 5, \omega^* = 1, Pr = 1$, (b) $Br = 10, \omega^* = 1, Pr = 1$, (c) $Br = 1, \omega^* = 5, Pr = 1$, (d) $Br = 1, \omega^* = 10, Pr = 1$, (e) $Br = 5, \omega^* = 1, Pr = 0.5$, and (f) $Br = 5, \omega^* = 1, Pr = 5$.

3.2.3. Case C

Figure 5 shows six fluid temperature profile variations with the parameters Br , ω^* and Pr . The selected time range is again $1 \leq t^* \leq 5$. The thermal boundary conditions for this model is asymmetric, where the upper plate is subjected to constant surface temperature while the lower plate is insulated, as per Equation (10).

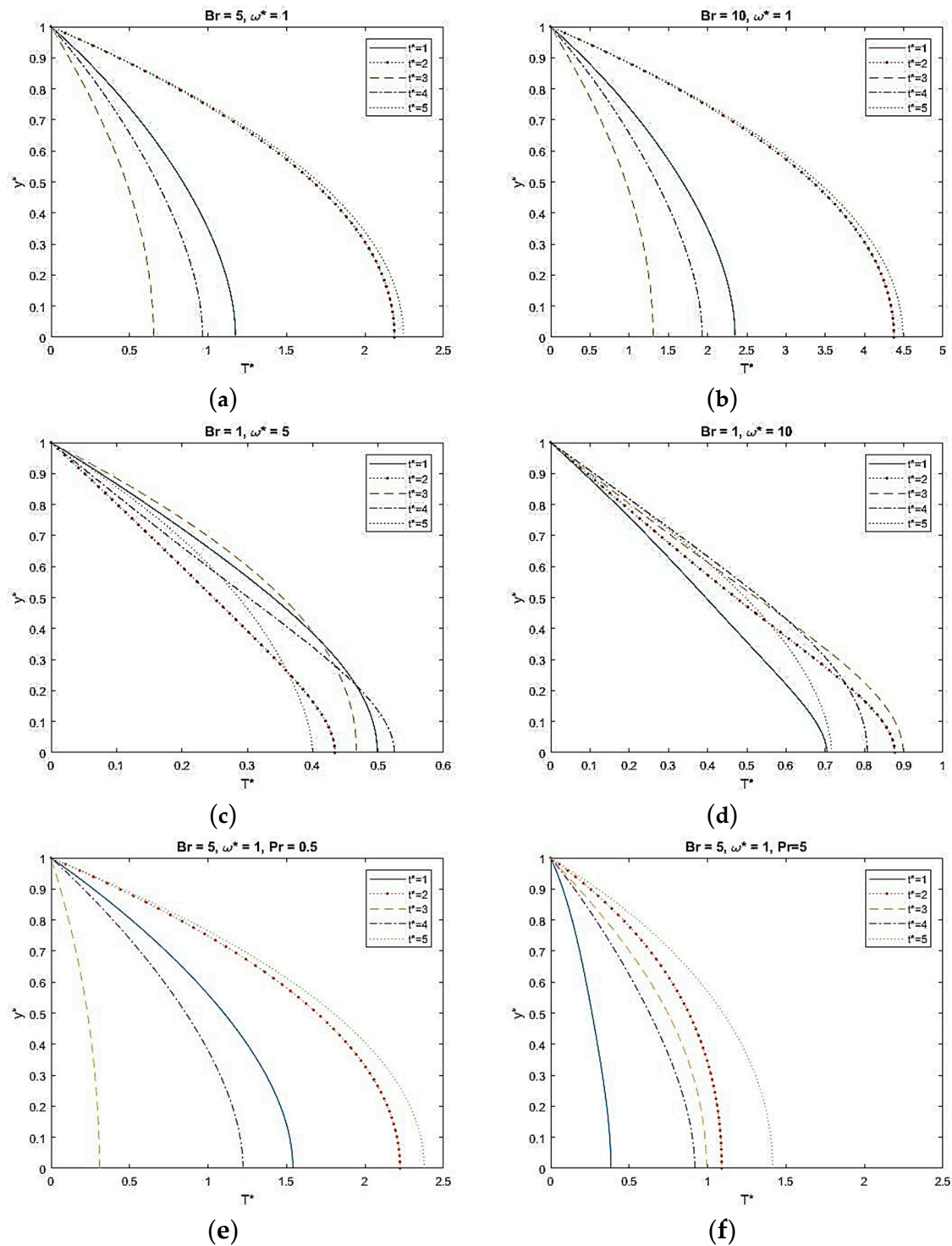


Figure 5. Fluid temperature profiles for upper plate with constant surface temperature and lower plate insulated when at (a) $Br = 5, \omega^* = 1, Pr = 1$, (b) $Br = 10, \omega^* = 1, Pr = 1$, (c) $Br = 1, \omega^* = 5, Pr = 1$, (d) $Br = 1, \omega^* = 10, Pr = 1$, (e) $Br = 5, \omega^* = 1, Pr = 0.5$, and (f) $Br = 5, \omega^* = 1, Pr = 5$.

Figure 5a–b reveal the temperature pattern at the same dimensionless angular frequency, $\omega^* = 1$. By analysing the Br variation, it is worth noting that the overall temperature profiles have similar curvatures to each other but increases in T^* for all t^* . Figure 5a–b reflect that the peak temperature occurs at $t^* = 5$ and $\omega^* = 1$, where $T^* \cong 2.25$ when $Br = 5$ and $T^* \cong 4.5$ when $Br = 10$, respectively. Thus, the peak temperature of T^* is about 45% of the arbitrary Br value at low ω^* .

It is understandable that the fluid temperature near the lower plate is much higher compared to Case A, as a result of the insulated thermal boundary condition. The heat induced is accumulated closer to the lower plate and the only way is to diffuse upwards through the upper boundary, where heat is allowed to leave by heat conduction through the upper plate.

Figure 5c–d illustrate the variation of ω^* at a constant Br . It shows the increase in ω^* consequently increase the fluid temperature due to the heat induced by the fluid motion. From these two figures, it is noteworthy that the range of temperature variation is narrower and the temperature curvature pattern seems to become more linear as ω^* increases. When the Br and ω^* are fixed as a constant, as per Figure 5a,e,f, the Pr are effecting on the rate of heat diffused out from the plates just like previous cases.

3.3. Comparison with Synovial Fluid Motion

Since no experimental result is found, this section intends to apply these developed models to predict and compare with reported experimental rise in temperature in the synovial fluid motion. The rise in hip joint temperature due to motion is crucial since the critical temperature before bone necrosis happens is at 47 °C [18].

The geometry and properties for the synovial fluid in artificial hip joint are taken as in the following: The joint gap is 50 μm [19], the dynamic viscosity is 10 $\text{mPa}\cdot\text{s}$ [19], the average sliding velocity is 0.06 m/s [20], the average angular frequency is 1 rad/s [21], the thermal conductivity is 0.62 $\text{W/m}\cdot\text{K}$ [22], the specific heat is 3900 $\text{J/kg}\cdot\text{K}$ [22], the fluid density is 1650 kg/m^3 [23], the body temperature is 37 °C [24], and the net mean fluid temperature is 5.5 °C [24], respectively. Utilising these gathered available information, the calculated values for Brinkman number and Prandtl number are 1.0557×10^{-5} and 62.9, respectively. The simulated outcome is set at a 60 minutes duration, as to be same with the duration recorded by the reported experiment, for comparison purpose.

The temperature field induced in Case A and Case C are insignificant, with a temperature rise much lower than 1 °C, and thus will not be discussed further.

However, in Case B, the temperature rise is approximately $T = 4.02$ °C at 60 min mark, as shown in Figure 6. It is noteworthy that the literatures reported experimental temperature rise, after the same period, in the ranges 2 °C–8 °C and 3 °C–14 °C, depending on in-vivo and in-vitro conditions, respectively [24–26]. The simulation results are somewhat able to match remarkably well with the literature.

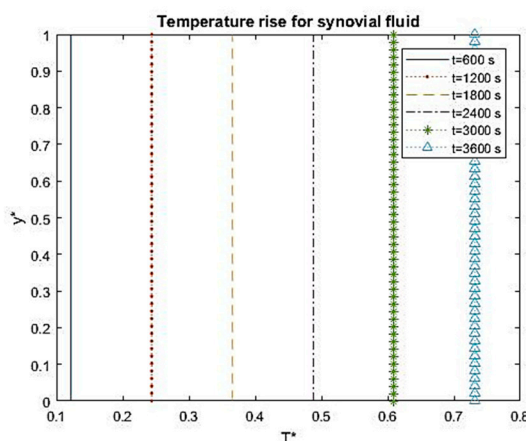


Figure 6. The temperature rise for synovial fluid due to the viscous dissipation at 6 selected times.

Thus, the developed model in Case B seems capable to predict the temperature field induced within the joint at a small oscillatory flow, as compared to the other two cases.

4. Conclusions

In this study on heat induced by viscous dissipation caused by an oscillating microchannel fluid flow that is laminar, unsteady, constant properties, and Newtonian, the governing equations are solved numerically for the same unsteady hydrodynamic conditions, but three different cases of thermal boundary conditions. They are parallel plates with both sides subjected to constant surface temperature, both sides insulated and the third case with upper side subject to constant surface temperature and lower oscillating side insulated.

Concern has been focused on the internal heat induced by the viscous dissipation term in the energy equation. Parametric studies are done on the effects of ω^* , Br , and Pr . The fluid momentum is shown to be strongly dependent on ω^* and not affected by the Br and Pr . However, the heat induced is strongly depending on the Br and Pr , while depending on ω^* as well. The temperature field increases with increasing Br and decreasing Pr , when $Pr < 1$.

Table 1 summarises these effects in the present study. The Br plays a central role in the induced heat. The Pr is also important, especially when its value is less than unity, more so in Case B, where both plates are insulated.

Table 1. The effects of the parameters ω^* , Br and Pr on fluid momentum and heat induction on the present three cases.

Transport Quantity	Parameters	Effects
Fluid momentum (velocity)	ω^*	Strongly dependent
	Br	Independent
	Pr	Independent
Induced heat (temperature)	ω^*	Dependent
	Br	Strongly dependent
	Pr	Strongly dependent

Since the synovial fluid motion within the artificial hip implants is subjected to a small oscillation motion and constant human body temperature, it is applied to these 3 present cases and compared to the experimental literature. It is noteworthy that the heat induced in Case B concurs remarkably with the literature. The present results will also be useful for verification when analytical solutions are available, since the three cases represent standard thermal boundary conditions in the discipline.

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Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

Br	Brinkman number defined in Equation (5)
c_p	Specific heat (J/kg·K)
k	Thermal conductivity of fluid (W/m·K)
Re	Reynold number defined as $Re = \frac{UW}{\nu}$
Pe	Peclet number defined in Equation (A1)
Pr	Prandtl number defined in Equation (5)
q'''	Heat source (W/m ³)
T	Fluid temperature (°C)
T^*	Dimensionless fluid temperature defined in Equation (5)
T_1	Specified plate temperature (°C)
T_a	Top plate temperature (°C)
T_b	Bottom plate temperature (°C)
T_m	Mean fluid temperature (°C)
t	Time (s)
t^*	Dimensionless time defined in Equation (5)
U	Maximum velocity magnitude at lower plate (m/s)
u	Velocity component in the x-direction (m/s)
u^*	Dimensionless velocity component in the x-direction defined in Equation (5)
W	Distance between parallel surfaces (m)
x	Distance along x-axis (m)
x^*	Dimensionless distance along x-axis defined in Equation (A1)
y	Distance along y-axis (m)
y^*	Dimensionless distance along y-axis defined in Equation (5)
Greek Letters	
α	Thermal diffusivity (m ² /s)
Φ	Viscous dissipation function (1/s ²)
μ	Dynamic viscosity (Pa·s)
ρ	Density (kg/m ³)
ν	Kinematic viscosity (m ² /s)
ω	Angular frequency (rad/s)
ω^*	Dimensionless parameter defined in Equation (5)

Appendix A. Justification of Neglecting The Convective Term

The relevant convection term missing on the left side of Equation (4) is $\rho c_p u \frac{\partial T}{\partial x}$. After re-casting into the dimensionless form as in Equation (7), the term becomes $Pe u^* \left(\frac{\partial T^*}{\partial x^*} \right)$, where Pe is the Peclet number and x^* is the dimensionless distance along x-axis, defined respectively, as

$$Pe = Re \cdot Pr, \quad x^* = \frac{x}{W}. \quad (A1)$$

A minute characteristic length W in the microchannel implies a low Re , and consequently, a low Pe , justifying neglecting the convective term in the governing energy equation in this study. Therefore, any temperature increase can be regarded as an increase at a point, depending only on the y-position.

Appendix B. Numerical Method of Solution

This problem is solved numerically. The conservation of momentum and energy in Equations (6) and (7), assuming constant properties for this model, is reformulated as a system of first order equations as in Equations (A2) to (A5), respectively. Since it is a boundary value problem, initial guesses for the slopes of the dependent variables are required. Numerical integration is carried out

from $y^* = 0$ to $y^* = 1$ by employing Runge-Kutta fourth order scheme until the solutions satisfy the transformed boundary conditions.

$$\frac{\partial u^*(i)}{\partial y^*} = u^*(i+1). \quad (\text{A2})$$

$$\frac{\partial u^*(i+1)}{\partial y^*} = \frac{u^*(i) - u^*(ini)}{t}. \quad (\text{A3})$$

$$\left(\frac{\partial u^*(i+2)}{\partial y^*} \right) = u^*(i+3). \quad (\text{A4})$$

$$\left(\frac{\partial u^*(i+3)}{\partial y^*} \right) = Pr \frac{u^*(i+2) - u^*(ini)}{t} - Br \left(\frac{\partial u^*(i+1)}{\partial y^*} \right)^2. \quad (\text{A5})$$

The boundary conditions and the initial condition for the first order equations followed accordingly to the cases stated in Equations (8), (9) and (10), respectively.

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