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An Approach toward a Q-Neutrosophic Soft Set and Its Application in Decision Making

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Abstract: A neutrosophic set was proposed as an approach to study neutral uncertain information. It is characterized through three memberships, T, I and F, such that these independent functions stand for the truth, indeterminate, and false-membership degrees of an object. The neutrosophic set presents a symmetric form since truth enrolment T is symmetric to its opposite false enrolment F with respect to indeterminacy enrolment I that acts as an axis of symmetry. The neutrosophic set was further extended to a Q-neutrosophic soft set, which is a hybrid model that keeps the features of the neutrosophic soft set in dealing with uncertainty, and the features of a Q-fuzzy soft set that handles two-dimensional information. In this study, we discuss some operations of Q-neutrosophic soft sets, such as subset, equality, complement, intersection, union, AND operation, and OR operation. We also define the necessity and possibility operations of a Q-neutrosophic soft set. Several properties and illustrative examples are discussed. Then, we define the Q-neutrosophic-set aggregation operator and use it to develop an algorithm for using a Q-neutrosophic soft set in decision-making issues that have indeterminate and uncertain data, followed by an illustrative real-life example.

Keywords: decision making; neutrosophic set; Q-neutrosophic set; Q-neutrosophic soft set; soft set

1. Introduction

Fuzzy-set theory was established by Zadeh in 1965 [1]. Since then, fuzzy logic has been utilized in several real-world problems in uncertain environments. Consequently, numerous analysts discussed many results using distinct directions of fuzzy-set theory, for instance, interval valued fuzzy set [2] and intuitionistic fuzzy set [3]. These extensions can deal with uncertain real-world problems. An intuitionistic fuzzy set can only cope with incomplete data through its truth and falsity membership values, but it does not cope with indeterminate data. Thus, Smarandache [4] initiated the neutrosophic idea to overcome this problem. A neutrosophic set (NS) [5] is a mathematical notion serving issues containing inconsistent, indeterminate, and imprecise data. Recent studies on NS include a single-valued neutrosophic set [6] and complex neutrosophic set [7].

Molodtsov [8] proposed the notion of the soft set as an important mathematical notion for handling uncertainties. The main advantage of this notion in data analysis is that it does not need any grade of membership as in fuzzy-set theory. Maji et.al. [9] presented the fuzzy soft set, which is a combination between a soft set and a fuzzy set. Later, many researchers developed several extensions of the soft-set model, such as vague soft set [10], interval-valued vague soft set [11–13], soft expert set [14], and soft multiset theory [15]. Maji [16] extended the notion of the fuzzy soft set to the neutrosophic soft set (NSS) and defined some of its properties.

NS is very appropriate for handling inconsistent, indeterminate, and incomplete information in real applications. Recently, many studies have been done on NSS [17–24], the most recent being on vague soft sets [25], neutrosophic vague soft expert sets [26], n-valued refined neutrosophic soft sets [27], complex neutrosophic soft expert sets [28,29] and time-neutrosophic soft sets [30]. Many researchers [23,31–36] have constructed several aggregation operators, such as simplified neutrosophic prioritized aggregation operators, single-valued neutrosophic Dombi weighted aggregation operators, simplified neutrosophic weighted aggregation operators, interval neutrosophic exponential weighted aggregation operators, and used them in decision-making issues. Aggregation operators perform a vital role in multicriteria decision making (MCDM) issues whose principle target is to aggregate a collection of inputs to a single number. Thus, aggregation operators give us effective tools to handle neutrosophic data in the decision process.

For a two-dimensional universal set, Adam and Hassan [37,38] introduced the Q-fuzzy soft set (Q-FSS) and multi-Q-FSS, which includes a Q-fuzzy soft aggregation operator that allows constructing more efficient decision-making methods. Broumi [39] presented the notion of the Q-intuitionistic fuzzy soft set (Q-IFSS), and defined some basic properties and basic operations. Actually, these notions cannot handle indeterminate data that appear in two universal sets. Inspired by this, Abu Qamar and Hassan [40] initiated the concept of the Q-neutrosophic soft set (Q-NSS) by upgrading the membership functions of the NSS to a two-dimensional entity. As a result, Q-NSS is premium to these models with three two-dimensional independent membership functions. Hence, this concept serves indeterminacy and two-dimensionality at the same time. Moreover, the Q-neutrosophic set (Q-NS) is basically an NS defined over a two-dimensional set. Thus, it has added advantages to NS by treating a two-dimensional universal set, which makes it more valid in modeling real-life problems where two-dimensional sets and indeterminacy majorly appear. Q-NSS is created to keep the advantages of Q-FSS while holding NSS features. Therefore, it incorporates the benefits of these models. Abu Qamar and Hassan studied different aspects of Q-NSS, such as their relations [40], measures of Q-NSS information [41], generalized Q-neutrosophic soft expert set [42], and different decision problems of these concepts.

Motivated by these studies, in this study we discuss the different operations and properties of Q-NSS. To facilitate the discussion, we arranged this article as follows. Section 2 contains a review of basic definitions pertaining to this work. In Section 3, we discuss some operations of Q-NSS, such as subset, equality, complement, intersection, union, AND operation, and OR operation. We discuss necessity and possibility operations in Section 4 along with properties and illustrative examples. In Section 5, we define the aggregation Q-NS in order to utilize it in the algorithm that we constructed to solve decision-making problems using Q-NSS. In Section 6, comparison analysis is presented to validate the proposed approach. Finally, conclusions and future work are in Section 7. Consequently, the concept of Q-NSS will enrich current NSS studies.

2. Preliminaries

In this section, we recall the concepts of soft set, NS, and Q-NS, which are relevant to this paper. Soft-set theory was proposed by Molodtsov [8].

Definition 1 ([8])**.** *A pair (F, E) is a soft set over U if and only if $F : E \to \mathcal{P}(U)$ is mapping. That is, the soft set is a parameterized family of subsets of U.*

In order to handle inconsistent and indeterminate information that exists in some real-world issues, Smarandache [5] initiated the NS concept as follows:

Definition 2 ([5])**.** *An NS Γ on universe U is defined as $\Gamma = \{\langle u, (T_\Gamma(u), I_\Gamma(u), F_\Gamma(u))\rangle : u \in U\}$, where $T, I, F : U \to]^-0, 1^+[$ and $^-0 \leq T_\Gamma(u) + I_\Gamma(u) + F_\Gamma(u) \leq 3^+$.*

Now, we recall some basic NS operations proposed by Smarandache [43].

Definition 3 ([43])**.** *Let Γ and Ψ be two NSs. Then, Γ is a subset of Ψ, written as $\Gamma \subseteq \Psi$, if and only if $T_\Gamma(u) \leq T_\Psi(u)$, $I_\Gamma(u) \geq I_\Psi(u)$ and $F_\Gamma(u) \geq F_\Psi(u) \forall u \in U$.*

Definition 4 ([43])**.** *The union of two NSs* Γ *and* Ψ *in* U*, written as* $\Gamma \cup \Psi = \Lambda$*, where* $\Lambda = \{\langle u, (\max\{T_\Gamma(u), T_\Psi(u)\}, \min\{I_\Gamma(u), I_\Psi(u)\}, \min\{F_\Gamma(u), F_\Psi(u)\})\rangle : u \in U\}$.

Definition 5 ([43])**.** *The intersection of two NSs* Γ *and* Ψ *in* U*, written as* $\Gamma \cap \Psi = \Lambda$*, where* $\Lambda = \{\langle u, (\min\{T_\Gamma(u), T_\Psi(u)\}, \max\{I_\Gamma(u), I_\Psi(u)\}, \max\{F_\Gamma(u), F_\Psi(u)\})\rangle : u \in U\}$.

Definition 6 ([43])**.** *The complement of an NS* Γ *in* U*, denoted by* Γ^c*, where*

$$\Gamma^c = \{\langle u, (1 - T_\Gamma(u), 1 - I_\Gamma(u), 1 - F_\Gamma(u))\rangle : u \in U\}.$$

Abu Qamar and Hassan [40] introduced the idea of Q-NS as follows:

Definition 7 ([40])**.** *A Q-NS* Γ_Q *in* U *is an object of form*

$$\Gamma_Q = \left\{ \left\langle (u,t), T_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), F_{\Gamma_Q}(u,t) \right\rangle : u \in U, t \in Q \right\},$$

where $Q \neq \phi$ *and* $T_{\Gamma_Q}, I_{\Gamma_Q}, F_{\Gamma_Q} : U \times Q \to]^-0, 1^+[$ *are the true, indeterminacy, and false membership functions, respectively, with* $^-0 \leq T_{\Gamma_Q} + I_{\Gamma_Q} + F_{\Gamma_Q} \leq 3^+$.

3. Q-Neutrosophic Soft Sets

In this section, we discuss numerous properties and operations concerning Q-NSSs, namely, union, intersection, and AND and OR operations. The concept of Q-NSS was briefly mentioned in the following definition without any algebraic operations in the discussion on Q-neutrosophic soft relations [40].

Definition 8 ([40])**.** *Let* U *be a universal set,* Q *be a nonempty set and* $A \subseteq E$ *be a set of parameters. Let* $\mu^l QNS(U)$ *be the set of all multi-Q-NSs on* U *with dimension* $l = 1$*. A pair* (Γ_Q, A) *is called a Q-NSS over* U*, where* $\Gamma_Q : A \to \mu^l QNS(U)$ *is a mapping, such that* $\Gamma_Q(e) = \phi$ *if* $e \notin A$*.*

A Q-NSS can be presented as

$$(\Gamma_Q, A) = \{(e, \Gamma_Q(e)) : e \in A, \Gamma_Q \in \mu^l QNS(U)\}.$$

The set of all Q-NSSs in U is denoted by Q-NSS(U).

The following example shows how Q-NSS can represent real-world problems.

Example 1. *Suppose we want to examine the attractiveness of a cell phone that a person is considering buying. Suppose there are two choices in the universe* $U = \{u_1, u_2\}$*,* $Q = \{s = black, t = white\}$ *is the set of colors under consideration and* $E = \{e_1 = price, e_2 = version, e_3 = device\ specification\}$ *is a set of decision parameters. Then, the Q-NSS* (Γ_Q, A) *is given by:*

$$\begin{aligned}(\Gamma_Q, A) = \Big\{ & \Big\langle e_1, [(u_1,s), 0.5, 0.1, 0.2], [(u_1,t), 0.7, 0.5, 0.3], [(u_2,s), 0.7, 0.5, 0.4], [(u_2,t), 0.4, 0.3, 0.1] \Big\rangle, \\ & \Big\langle e_2, [(u_1,s), 0.1, 0.2, 0.6], [(u_1,t), 0.8, 0.1, 0.5], [(u_2,s), 0.4, 0.6, 0.9], [(u_2,t), 0.4, 0.6, 0.7] \Big\rangle, \\ & \Big\langle e_3, [(u_1,s), 0.6, 0.2, 0.2], [(u_1,t), 0.1, 0.8, 0.5], [(u_2,s), 0.6, 0.4, 0.9], [(u_2,t), 0.3, 0.1, 0.5] \Big\rangle \Big\}.\end{aligned}$$

Each element in (Γ_Q, A) *represents the degree of attractiveness of each cell phone with a specific color based on each parameter. For example, element* $[(u_1, s), 0.6, 0.2, 0.2]$ *under parameter* e_3 *represents the degree of true, indeterminacy, and falsity attractiveness of device specification of cell phone* u_1 *with a black color, and they are 0.6, 0.2, and 0.2, respectively.*

Now, we introduce some basic definitions of Q-NSSs.

Definition 9. *Let* $(\Gamma_Q, A) \in$ *Q-NSS(U). If* $\Gamma_Q(e) = \phi$ *for all* $e \in A$*, then* (Γ_Q, A) *is called a null Q-NSS(U) denoted by* (ϕ, A).

Definition 10. *Let* $(\Gamma_Q, A), (\Psi_Q, B) \in$ *Q-NSS(U), then*

1. (Γ_Q, A) *is a Q-neutrosophic soft subset of* (Ψ_Q, B)*, denoted by* $(\Gamma_Q, A) \subseteq (\Psi_Q, B)$*, if* $A \subseteq B$ *and* $\Gamma_Q(e) \subseteq \Psi_Q(e)$ *for all* $e \in A$*, that is* $T_{\Gamma_Q(e)}(u,t) \leq T_{\Psi_Q(e)}(u,t)$, $I_{\Gamma_Q(e)}(u,t) \geq I_{\Psi_Q(e)}(u,t), F_{\Gamma_Q(e)}(u,t) \geq F_{\Psi_Q(e)}(u,t)$*, for all* $(u,t) \in U \times Q$.
2. (Γ_Q, A) *and* (Ψ_Q, B) *are equal, denoted by* $(\Gamma_Q, A) = (\Psi_Q, B)$*, if and only if* $(\Gamma_Q, A) \subseteq (\Psi_Q, B)$ *and* $(\Psi_Q, B) \subseteq (\Gamma_Q, A)$ *for all* $u \in U$.

Now, we define the complement of $Q - NSS(U)$:

Definition 11. *Let* $(\Gamma_Q, A) \in$ *Q-NSS(U). Then, its complement written as* $(\Gamma_Q, A)^c = (\Gamma^c_Q, A)$*, and is defined as*

$$(\Gamma_Q, A)^c = \left\{ \left\langle e, T^c_{\Gamma_Q(e)}(u,t), I^c_{\Gamma_Q(e)}(u,t), F^c_{\Gamma_Q(e)}(u,t) \right\rangle : e \in A, (u,t) \in U \times Q \right\},$$

such that $\forall e \in A, (u,t) \in U \times Q$

$$\begin{aligned} T^c_{\Gamma_Q(e)}(u,t) &= 1 - T_{\Gamma_Q(e)}(u,t), \\ I^c_{\Gamma_Q(e)}(u,t) &= 1 - I_{\Gamma_Q(e)}(u,t), \\ F^c_{\Gamma_Q(e)}(u,t) &= 1 - F_{\Gamma_Q(e)}(u,t). \end{aligned}$$

Example 2. *Let* $U = \{u_1, u_2\}$ *be a universal set,* $A = \{e_1, e_2\}$ *and* $Q = \{s, t\}$.

$$\begin{aligned} (\Gamma_Q, A) = \Big\{ &\Big\langle e_1, [(u_1,s), 0.2, 0.3, 0.5], [(u_1,t), 0.6, 0.4, 0.1], [(u_2,s), 0.1, 0.4, 0.7], [(u_2,t), 0.3, 0.2, 0.8] \Big\rangle, \\ &\Big\langle e_2, [(u_1,s), 0.4, 0.1, 0.9], [(u_1,t), 0.6, 0.2, 0.1], [(u_2,s), 0.5, 0.3, 0.7], [(u_2,t), 0.1, 0.3, 0.2] \Big\rangle \Big\}. \end{aligned}$$

is a Q-NSS and the complement of (Γ_Q, A) *is*

$$\begin{aligned} (\Gamma^c_Q, A) = \Big\{ &\Big\langle e_1, [(u_1,s), 0.8, 0.7, 0.5], [(u_1,t), 0.4, 0.6, 0.9], [(u_2,s), 0.9, 0.6, 0.3], [(u_2,t), 0.7, 0.8, 0.2] \Big\rangle, \\ &\Big\langle e_2, [(u_1,s), 0.6, 0.9, 0.1], [(u_1,t), 0.4, 0.8, 0.9], [(u_2,s), 0.5, 0.7, 0.3], [(u_2,t), 0.9, 0.7, 0.8] \Big\rangle \Big\}. \end{aligned}$$

Proposition 1. *If* $(\Gamma_Q, A) \in$ *Q-NSS(U), then* $((\Gamma_Q, A)^c)^c = (\Gamma_Q, A)$.

Proof. From Definition 11, we have

$$\begin{aligned} (\Gamma_Q, A)^c &= \left\{ \left\langle e, T^c_{\Gamma_Q(e)}(u,t), I^c_{\Gamma_Q(e)}(u,t), F^c_{\Gamma_Q(e)}(u,t) \right\rangle : e \in A, (u,t) \in U \times Q \right\} \\ &= \left\{ \left\langle e, 1 - T_{\Gamma_Q(e)}(u,t), 1 - I_{\Gamma_Q(e)}(u,t), 1 - F_{\Gamma_Q(e)}(u,t) \right\rangle : e \in A, (u,t) \in U \times Q \right\}. \end{aligned}$$

Thus,

$$((\Gamma_Q, A)^c)^c = \Big\{\Big\langle e, 1-(1-T_{\Gamma_{Q(e)}}(u,t)), 1-(1-I_{\Gamma_{Q(e)}}(u,t)), 1-(1-F_{\Gamma_{Q(e)}}(u,t))\Big\rangle : e \in A, (u,t) \in U \times Q\Big\}$$
$$= \Big\{\Big\langle e, T_{\Gamma_{Q(e)}}(u,t), I_{\Gamma_{Q(e)}}(u,t), F_{\Gamma_{Q(e)}}(u,t)\Big\rangle : e \in A, (u,t) \in U \times Q\Big\}$$

This completes the proof. □

Next, we discuss the operations of union, intersection, and AND and OR operations for Q-NSSs, along with some results and examples.

Definition 12. *The union of two Q-NSSs* (Γ_Q, A) *and* (Ψ_Q, B) *is the Q-NSS* (Λ_Q, C) *written as* $(\Gamma_Q, A) \cup (\Psi_Q, B) = (\Lambda_Q, C)$*, where* $C = A \cup B$ *and for all* $c \in C$, $(u,t) \in U \times Q$*, the truth membership, indeterminacy membership, and falsity membership of* (Λ_Q, C) *are as follows:*

$$T_{\Lambda_{Q(c)}}(u,t) = \begin{cases} T_{\Gamma_{Q(c)}}(u,t) & \text{if } c \in A - B, \\ T_{\Psi_{Q(c)}}(u,t) & \text{if } c \in B - A, \\ \max\{T_{\Lambda_{Q(c)}}(u,t), T_{\Psi_{Q(c)}}(u,t)\} & \text{if } c \in A \cap B, \end{cases}$$

$$I_{\Lambda_{Q(c)}}(u,t) = \begin{cases} I_{\Gamma_{Q(c)}}(u,t) & \text{if } c \in A - B, \\ I_{\Psi_{Q(c)}}(u,t) & \text{if } c \in B - A, \\ \min\{I_{\Gamma_{Q(c)}}(u,t), I_{\Psi_{Q(c)}}(u,t)\} & \text{if } c \in A \cap B, \end{cases}$$

$$F_{\Lambda_{Q(c)}}(u,t) = \begin{cases} F_{\Gamma_{Q(c)}}(u,t) & \text{if } c \in A - B, \\ F_{\Psi_{Q(c)}}(u,t) & \text{if } c \in B - A, \\ \min\{F_{\Gamma_{Q(c)}}(u,t), F_{\Psi_{Q(c)}}(u,t)\} & \text{if } c \in A \cap B. \end{cases}$$

Definition 13. *The intersection of two Q-NSSs* (Γ_Q, A) *and* (Ψ_Q, B) *is the Q-NSS* (Λ_Q, C) *written as* $(\Gamma_Q, A) \cap (\Psi_Q, B) = (\Lambda_Q, C)$*, where* $C = A \cap B$ *and for all* $c \in C$ *and* $(u,t) \in U \times Q$ *the truth membership, indeterminacy membership, and falsity membership of* (Λ_Q, C) *are as follows:*

$$T_{\Lambda_{Q(c)}}(u,t) = \min\{T_{\Gamma_{Q(c)}}(u,t), T_{\Psi_{Q(c)}}(u,t)\},$$
$$I_{\Lambda_{Q(c)}}(u,t) = \max\{I_{\Gamma_{Q(c)}}(u,t), I_{\Psi_{Q(c)}}(u,t)\},$$
$$F_{\Lambda_{Q(c)}}(u,t) = \max\{F_{\Gamma_{Q(c)}}(u,t), F_{\Psi_{Q(c)}}(u,t)\}.$$

Example 3. *Let* $U = \{u_1, u_2\}$ *be a universal set,* $E = \{e_1, e_2\}$ *and* $Q = \{s, t\}$*. If* $A = B = \{e_1, e_2\} \subseteq E$,

$$(\Gamma_Q, A) = \Big\{\Big\langle e_1, [(u_1,s), 0.2, 0.3, 0.5], [(u_1,t), 0.6, 0.4, 0.1], [(u_2,s), 0.1, 0.4, 0.7], [(u_2,t), 0.3, 0.2, 0.8]\Big\rangle,$$
$$\Big\langle e_2, [(u_1,s), 0.4, 0.1, 0.9], [(u_1,t), 0.6, 0.2, 0.1], [(u_2,s), 0.5, 0.3, 0.7], [(u_2,t), 0.1, 0.3, 0.2]\Big\rangle\Big\}$$

and

$$(\Psi_Q, B) = \Big\{\Big\langle e_1, [(u_1,s), 0.4, 0.5, 0.2], [(u_1,t), 0.2, 0.3, 0.5], [(u_2,s), 0.3, 0.1, 0.7], [(u_2,t), 0.2, 0.4, 0.6]\Big\rangle,$$
$$\Big\langle e_2, [(u_1,s), 0.9, 0.3, 0.2], [(u_1,t), 0.4, 0.3, 0.1], [(u_2,s), 0.8, 0.6, 0.3], [(u_2,t), 0.2, 0.4, 0.6]\Big\rangle\Big\},$$

then

$$(\Gamma_Q, A) \cup (\Psi_Q, B) = \Big\{ \Big\langle e_1, [(u_1, s), 0.4, 0.3, 0.2], [(u_1, t), 0.6, 0.3, 0.1], [(u_2, s), 0.3, 0.1, 0.7], [(u_2, t), 0.3, 0.2, 0.6] \Big\rangle, \Big\langle e_2, [(u_1, s), 0.9, 0.1, 0.2], [(u_1, t), 0.6, 0.2, 0.1], [(u_2, s), 0.8, 0.3, 0.3], [(u_2, t), 0.2, 0.3, 0.2] \Big\rangle \Big\},$$

and

$$(\Gamma_Q, A) \cap (\Psi_Q, B) = \Big\{ \Big\langle e_1, [(u_1, s), 0.2, 0.5, 0.5], [(u_1, t), 0.2, 0.4, 0.5], [(u_2, s), 0.1, 0.4, 0.7], [(u_2, t), 0.2, 0.4, 0.8] \Big\rangle, \Big\langle e_2, [(u_1, s), 0.4, 0.3, 0.9], [(u_1, t), 0.4, 0.3, 0.1], [(u_2, s), 0.5, 0.6, 0.7], [(u_2, t), 0.1, 0.4, 0.6] \Big\rangle \Big\}.$$

Here, we give a proposition concerning the union and intersection of Q-NSSs.

Proposition 2. *Let* $(\Gamma_Q, A), (\Psi_Q, B)$ *and* $(\Lambda_Q, C) \in$ *Q-NSS(U). Then,*

1. $(\Gamma_Q, A) \cup (\Psi_Q, B) = (\Psi_Q, B) \cup (\Gamma_Q, A)$.
2. $(\Gamma_Q, A) \cap (\Psi_Q, B) = (\Psi_Q, B) \cap (\Gamma_Q, A)$.
3. $(\Gamma_Q, A) \cup ((\Psi_Q, B) \cup (\Lambda_Q, C)) = ((\Gamma_Q, A) \cup (\Psi_Q, B)) \cup (\Lambda_Q, C)$.
4. $(\Gamma_Q, A) \cap ((\Psi_Q, B) \cap (\Lambda_Q, C)) = ((\Gamma_Q, A) \cap (\Psi_Q, B)) \cap (\Lambda_Q, C)$.

Proof. 1. We show that $(\Gamma_Q, A) \cup (\Psi_Q, B) = (\Psi_Q, B) \cup (\Gamma_Q, A)$ by using Definition 12. Consider case $c \in A \cap B$, as other cases are trivial.

$$\begin{aligned}(\Gamma_Q, A) \cup (\Psi_Q, B) &= \Big\{ \Big\langle c, \big(\max\{T_{\Gamma_Q(c)}(u,t), T_{\Psi_Q(c)}(u,t)\}, \min\{I_{\Gamma_Q(c)}(u,t), I_{\Psi_Q(c)}(u,t)\}, \\ &\qquad \min\{F_{\Gamma_Q(c)}(u,t), F_{\Psi_Q(c)}(u,t)\}\big)\} \Big\rangle : (u,t) \in U \times Q \Big\} \\ &= \Big\{ \Big\langle c, \max\{T_{\Psi_Q(c)}(u,t), T_{\Gamma_Q(c)}(u,t)\}, \min\{I_{\Psi_Q(c)}(u,t), I_{\Gamma_Q(c)}(u,t)\}, \\ &\qquad \min\{F_{\Psi_Q(c)}(u,t), F_{\Gamma_Q(c)}(u,t)\} \Big\rangle : (u,t) \in U \times Q \Big\} \\ &= (\Psi_Q, B) \cup (\Gamma_Q, A).\end{aligned}$$

2. The proof is similar to that of Part (1).

3. We show that $((\Gamma_Q, A) \cup (\Psi_Q, B)) \cup (\Upsilon_Q, C) = (\Gamma_Q, A) \cup ((\Psi_Q, B) \cup (\Upsilon_Q, C))$ by using Definition 12. Consider case $c \in A \cap B$, as other cases are trivial.

$$\begin{aligned}(\Gamma_Q, A) \cup (\Psi_Q, B) &= \Big\{ \Big\langle c, \big(\max\{T_{\Gamma_Q(c)}(u,t), T_{\Psi_Q(c)}(u,t)\}, \min\{I_{\Gamma_Q(c)}(u,t), I_{\Psi_Q(c)}(u,t)\}, \\ &\qquad \min\{F_{\Gamma_Q(c)}(u,t), F_{\Psi_Q(c)}(u,t)\}\big) \Big\rangle : (u,t) \in U \times Q \Big\}.\end{aligned}$$

$$\begin{aligned}&((\Gamma_Q, A) \cup (\Psi_Q, B)) \cup (\Upsilon_Q, C) \\ &= \Big\{ \Big\langle c, \Big(\big(\max\big\{ \max\{T_{\Gamma_Q(c)}(u,t), T_{\Psi_Q(c)}(u,t)\}, T_{\Upsilon_Q(c)}(u,t)\big\}, \\ &\qquad \min\big\{ \min\{I_{\Gamma_Q(c)}(u,t), I_{\Psi_Q(c)}(u,t)\}, I_{\Upsilon_Q(c)}(u,t)\big\}, \\ &\qquad \min\big\{ \min\{F_{\Gamma_Q(c)}(u,t), F_{\Psi_Q(c)}(u,t)\}, F_{\Upsilon_Q(c)}(u,t)\big\}\big) \Big) \Big\rangle : (u,t) \in U \times Q \Big\}\end{aligned}$$

$$
\begin{aligned}
&= \Big\{ \Big\langle c, \Big(\big(\max \{ T_{\Gamma_Q(c)}(u,t), T_{\Psi_Q(c)}(u,t), T_{\Upsilon_Q(c)}(u,t) \}, \\
&\qquad \min \{ I_{\Gamma_Q(c)}(u,t), I_{\Psi_Q(c)}(u,t), I_{\Upsilon_Q(c)}(u,t) \}, \\
&\qquad \min \{ F_{\Gamma_Q(c)}(u,t), F_{\Psi_Q(c)}(u,t), F_{\Upsilon_Q(c)}(u,t) \} \big) \Big) \Big\rangle : (u,t) \in U \times Q \Big\} \\
&= \Big\{ \Big\langle c, \Big(\big(\max \{ T_{\Gamma_Q(c)}(u,t), \max\{ T_{\Psi_Q(c)}(u,t), T_{\Upsilon_Q(c)}(u,t) \} \}, \\
&\qquad \min \{ I_{\Gamma_Q(c)}(u,t), \min\{ I_{\Psi_Q(c)}(u,t), I_{\Upsilon_Q(c)}(u,t) \} \}, \\
&\qquad \min \{ F_{\Gamma_Q(c)}(u,t), \min\{ F_{\Psi_Q(c)}(u,t), F_{\Upsilon_Q(c)}(u,t) \} \} \big) \Big) \Big\rangle : (u,t) \in U \times Q \Big\} \\
&= (\Gamma_Q, A) \cup ((\Psi_Q, B) \cup (\Upsilon_Q, C)).
\end{aligned}
$$

4. The proof is similar to that of Part (3). □

Next, we introduce the AND and OR operations of Q-NSSs.

Definition 14. *If (Γ_Q, A) and (Ψ_Q, B) are two Q-NSSs on U, then (Γ_Q, A) AND (Ψ_Q, B) is the Q-NSS denoted by $(\Gamma_Q, A) \wedge (\Psi_Q, B)$ and defined by $(\Gamma_Q, A) \wedge (\Psi_Q, B) = (\Lambda_Q, A \times B)$, where $\Lambda_Q(a,b) = \Gamma_Q(a) \cap \Psi_Q(b)$ for all $(a,b) \in A \times B$ is the operation of intersection of two Q-NSs on U. That is, the truth, indeterminacy, and falsity memberships of (Γ_Q, A) and (Ψ_Q, B) are as follows:*

$$
\begin{aligned}
T_{\Lambda_Q(a,b)}(u,t) &= \min\{ T_{\Gamma_Q(a)}(u,t), T_{\Psi_Q(b)}(u,t) \}, \\
I_{\Lambda_Q(a,b)}(u,t) &= \max\{ I_{\Gamma_Q(a)}(u,t), I_{\Psi_Q(b)}(u,t) \}, \\
F_{\Lambda_Q(a,b)}(u,t) &= \max\{ F_{\Gamma_Q(a)}(u,t), F_{\Psi_Q(b)}(u,t) \},
\end{aligned}
$$

Definition 15. *If (Γ_Q, A) and (Ψ_Q, B) are two Q-NSSs on U, then (Γ_Q, A) OR (Ψ_Q, B) is the Q-NSS denoted by $(\Gamma_Q, A) \vee (\Psi_Q, B)$ and defined by $(\Gamma_Q, A) \vee (\Psi_Q, B) = (\Lambda_Q, A \times B)$, where $\Lambda_Q(a,b) = \Gamma_Q(a) \cup \Psi_Q(b)$ for all $(a,b) \in A \times B$ is the operation of union of two Q-NSs on U. The truth, indeterminacy, and falsity memberships of $(\widehat{\Lambda}^h_Q, A \times B)$ are as follows:*

$$
\begin{aligned}
T_{\Upsilon_Q(a,b)}(u,t) &= \max\{ T_{\Gamma_Q(a)}(u,t), T_{\Psi_Q(b)}(u,t) \}, \\
I_{\Upsilon_Q(a,b)}(u,t) &= \min\{ I_{\Gamma_Q(a)}(u,t), I_{\Psi_Q(b)}(u,t) \}, \\
F_{\Upsilon_Q(a,b)}(u,t) &= \min\{ F_{\Gamma_Q(a)}(u,t), F_{\Psi_Q(b)}(u,t) \},
\end{aligned}
$$

Here, we present an example of AND and OR operations followed by the corresponding propositions.

Example 4. *Reconsider Example 3, then*

$$
\begin{aligned}
&(\Gamma_Q, A) \wedge (\Psi_Q, B) \\
&\quad = \Big\{ \Big\langle (e_1,e_1), [(u_1,s), 0.2, 0.5, 0.5], [(u_1,t), 0.2, 0.4, 0.5], [(u_2,s), 0.1, 0.4, 0.7], [(u_2,t), 0.2, 0.4, 0.8] \Big\rangle, \\
&\qquad \Big\langle (e_1,e_2), [(u_1,s), 0.2, 0.3, 0.5], [(u_1,t), 0.4, 0.4, 0.1], [(u_2,s), 0.1, 0.6, 0.7], [(u_2,t), 0.2, 0.4, 0.8] \Big\rangle, \\
&\qquad \Big\langle (e_2,e_1), [(u_1,s), 0.4, 0.5, 0.9], [(u_1,t), 0.2, 0.3, 0.5], [(u_2,s), 0.3, 0.3, 0.7], [(u_2,t), 0.1, 0.4, 0.6] \Big\rangle, \\
&\qquad \Big\langle (e_2,e_2), [(u_1,s), 0.4, 0.3, 0.9], [(u_1,t), 0.4, 0.3, 0.1], [(u_2,s), 0.5, 0.6, 0.7], [(u_2,t), 0.1, 0.4, 0.6] \Big\rangle \Big\}.
\end{aligned}
$$

$$
\begin{aligned}
&(\Gamma_Q, A) \vee (\Psi_Q, B)\\
&= \Big\{ \Big\langle (e_1, e_1), [(u_1, s), 0.4, 0.3, 0.2], [(u_1, t), 0.6, 0.3, 0.1], [(u_2, s), 0.3, 0.1, 0.7], [(u_2, t), 0.3, 0.2, 0.6] \Big\rangle,\\
&\quad \Big\langle (e_1, e_2), [(u_1, s), 0.9, 0.3, 0.2], [(u_1, t), 0.6, 0.4, 0.1], [(u_2, s), 0.8, 0.4, 0.3], [(u_2, t), 0.3, 0.2, 0.6] \Big\rangle,\\
&\quad \Big\langle (e_2, e_1), [(u_1, s), 0.4, 0.1, 0.2], [(u_1, t), 0.6, 0.2, 0.1], [(u_2, s), 0.5, 0.1, 0.7], [(u_2, t), 0.2, 0.3, 0.2] \Big\rangle,\\
&\quad \Big\langle (e_2, e_2), [(u_1, s), 0.9, 0.1, 0.2], [(u_1, t), 0.6, 0.2, 0.1], [(u_2, s), 0.8, 0.3, 0.3], [(u_2, t), 0.2, 0.3, 0.2] \Big\rangle \Big\}.
\end{aligned}
$$

Proposition 3. *Let $(\Gamma_Q, A), (\Psi_Q, B)$, and (Λ_Q, C) be three Q-NSSs on U. Then, we have the following associative properties:*

1. $(\Gamma_Q, A) \wedge ((\Psi_Q, B) \wedge (\Lambda_Q, C)) = ((\Gamma_Q, A) \wedge (\Psi_Q, B)) \wedge (\Lambda_Q, C)$.
2. $(\Gamma_Q, A) \vee ((\Psi_Q, B) \vee (\Lambda_Q, C)) = ((\Gamma_Q, A) \vee (\Psi_Q, B)) \vee (\Lambda_Q, C)$.

Proof. 1. Let $(\Psi_Q, B) \wedge (\Lambda_Q, C) = (\Upsilon_Q, B \times C)$, where $\Upsilon_Q(b, c) = \Psi_Q(b) \cap \Lambda_Q(c)$.

Now, $(\Gamma_Q, A) \wedge ((\Psi_Q, B) \wedge (\Lambda_Q, C)) = (\Gamma_Q, A) \wedge (\Upsilon_Q, B \times C) = (\Omega_Q, A \times B \times C)$, where $\Omega_Q(a, b, c) = \Gamma_Q(a) \cap \Upsilon_Q(b, c) = \Gamma_Q(a) \cap \Psi_Q(b) \cap \Lambda_Q(c)$.

Also, $(\Gamma_Q, A) \wedge (\Psi_Q, B) = (\Theta_Q, A \times B)$, where $\Theta_Q(a, b) = \Gamma_Q(A) \cap \Psi_Q(B)$. Therefore, $((\Gamma_Q, A) \wedge (\Psi_Q, B)) \wedge (\Lambda_Q, C) = (\Theta_q, A \times B) \wedge (\Lambda_Q, C) = (\Delta_Q, A \times B \times C)$, where $\Delta_Q(a, b, c) = \Theta_Q(a, b) \cap \Lambda_Q(c) = \Gamma_Q(a) \cap \Psi_Q(b) \cap \Lambda_Q(c)$. Hence, $(\Gamma_Q, A) \wedge ((\Psi_Q, B) \wedge (\Lambda_Q, C)) = ((\Gamma_Q, A) \wedge (\Psi_Q, B)) \wedge (\Lambda_Q, C)$.

2. The result can be proved in a similar fashion as in Assertion 1. □

4. Necessity and Possibility Operations on Q-Neutrosophic Soft Sets with Some Properties

In this section, we introduce necessity and possibility operations on Q-NSSs.

Definition 16. *The necessity operation on a Q-NSS on U (Γ_Q, A) is denoted by $\oplus(\Gamma_Q, A)$ and is defined as, for all $e \in A$,*

$$
\oplus(\Gamma_Q, A) = \Big\{ \Big\langle e, [(u, t), T_{\Gamma_Q}(u, t), I_{\Gamma_Q}(u, t), 1 - T_{\Gamma_Q}(u, t)] \Big\rangle : (u, t) \in U \times Q \Big\}.
$$

Example 5. *Reconsider Example 2, then*

$$
\begin{aligned}
\oplus(\Gamma_Q, A) = \Big\{ &\Big\langle e_1, [(u_1, s), 0.2, 0.3, 0.8], [(u_1, t), 0.6, 0.4, 0.4], [(u_2, s), 0.1, 0.4, 0.9], [(u_2, t), 0.3, 0.2, 0.7] \Big\rangle,\\
&\Big\langle e_2, [(u_1, s), 0.4, 0.1, 0.6], [(u_1, t), 0.6, 0.2, 0.4], [(u_2, s), 0.5, 0.3, 0.5], [(u_2, t), 0.1, 0.3, 0.9] \Big\rangle \Big\}
\end{aligned}
$$

Proposition 4. *Let (Γ_Q, A) and (Ψ_Q, B) be two Q-NSSs on U. Then,*

1. $\oplus((\Gamma_Q, A) \cup (\Psi_Q, B)) = \oplus(\Gamma_Q, A) \cup \oplus(\Psi_Q, B)$.
2. $\oplus((\Gamma_Q, A) \cap (\Psi_Q, B)) = \oplus(\Gamma_Q, A) \cap \oplus(\Psi_Q, B)$.
3. $\oplus(\oplus(\Gamma_Q, A)) = \oplus(\Gamma_Q, A)$.

Proof. 1. $(\Gamma_Q, A) \cup (\Psi_Q, B) = (\Lambda_Q, C)$, where $C = A \cup B$
$(\Lambda_Q, C) = \left\{\left\langle e, [(u,t), T_{\Lambda_Q}(u,t), I_{\Lambda_Q}(u,t), F_{\Lambda_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\}$, such that

$$T_{\Lambda_Q}(u,t) = \begin{cases} T_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ T_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \max\{T_{\Gamma_Q}(u,t), T_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$

$$I_{\Lambda_Q}(u,t) = \begin{cases} I_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ I_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \min\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$

and

$$F_{\Lambda_Q}(u,t) = \begin{cases} F_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ F_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \min\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B. \end{cases}$$

Now, by Definition 16, for all $e \in C$
$\oplus(\Lambda_Q, C) = \left\{\left\langle e, [(u,t), \oplus T_{\Lambda_Q}(u,t), \oplus I_{\Lambda_Q}(u,t), \oplus F_{\Lambda_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\}$, where

$$\oplus T_{\Lambda_Q}(u,t) = \begin{cases} T_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ T_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \max\{T_{\Gamma_Q}(u,t), T_{\Psi_{Qi}}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$

$$\oplus I_{\Lambda_Q}(u,t) = \begin{cases} I_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ I_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \min\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$

and

$$\oplus F_{\Lambda_Q}(u,t) = \begin{cases} 1 - T_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ 1 - T_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ 1 - \max\{T_{\Gamma_Q}(u,t), T_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B. \end{cases}$$

Assume for all $e \in A$
$\oplus(\Gamma_Q, A) = \left\{\left\langle e, [(u,t), T_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), 1 - T_{\Gamma_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\}$, and
$\oplus(\Psi_Q, B) = \left\{\left\langle e, [(u,t), T_{\Psi_Q}(u,t), I_{\Psi_Q}(u,t), 1 - T_{\Psi_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\}$. Now, $\oplus(\Gamma_Q, A) \cup \oplus(\Psi_Q, B) = (\Upsilon_Q, C)$, where
$(\Upsilon_Q, C) = \left\{\left\langle e, [(u,t), T_{\Upsilon_Q}(u,t), I_{\Upsilon_Q}(u,t), F_{\Upsilon_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\}$, such that

$$T_{\Upsilon_Q}(u,t) = \begin{cases} T_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ T_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \max\{T_{\Gamma_Q}(u,t), T_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$

$$I_{\Upsilon_Q}(u,t) = \begin{cases} I_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ I_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \min\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$

and

$$\begin{aligned} F_{\Upsilon_Q}(u,t) &= \begin{cases} 1 - T_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ 1 - T_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \min\{1 - T_{\Gamma_Q}(u,t), 1 - T_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases} \\ &= \begin{cases} 1 - T_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ 1 - T_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ 1 - \max\{T_{\Gamma_Q}(u,t), T_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B. \end{cases} \end{aligned}$$

Consequently, $\oplus(\Lambda_Q, C)$ and (Υ_Q, C) are the same. Thus, $\oplus((\Gamma_Q, A) \cup (\Psi_Q, B)) = \oplus(\Gamma_Q, A) \cup \oplus(\Psi_Q, B)$.
2. Can be analogously proven.
3. Assume for all $e \in A$

$$\oplus(\Gamma_Q, A) = \left\{ \left\langle e, [(u,t), T_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), 1 - T_{\Gamma_Q}(u,t)] \right\rangle : (u,t) \in U \times Q \right\}.$$

Now,

$$\begin{aligned} \oplus(\oplus(\Gamma_Q, A)) &= \left\{ \left\langle e, [(u,t), T_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), 1 - T_{\Gamma_Q}(u,t)] \right\rangle : (u,t) \in U \times Q \right\} \\ &= \oplus(\Gamma_Q, A). \end{aligned}$$

□

Definition 17. *The possibility operation on a Q-NSS on U (Γ_Q, A) is denoted by $\otimes(\Gamma_Q, A)$ and is defined as, for all $e \in A$,*

$$\otimes(\Gamma_Q, A) = \left\{ \left\langle e, [(u,t), 1 - F_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), F_{\Gamma_Q}(u,t)] \right\rangle : (u,t) \in U \times Q \right\}.$$

Example 6. *Reconsider Example 2, then*

$$\begin{aligned} (\Gamma_Q, A) = \Big\{ &\Big\langle e_1, [(u_1,s), 0.5, 0.3, 0.5], [(u_1,t), 0.9, 0.4, 0.1], [(u_2,s), 0.3, 0.4, 0.7], [(u_2,t), 0.2, 0.2, 0.8] \Big\rangle, \\ &\Big\langle e_2, [(u_1,s), 0.1, 0.1, 0.9], [(u_1,t), 0.9, 0.2, 0.1], [(u_2,s), 0.3, 0.3, 0.7], [(u_2,t), 0.8, 0.3, 0.2] \Big\rangle \Big\}. \end{aligned}$$

Proposition 5. *Let $(\Gamma_Q, A), (\Psi_Q, B)$ be two Q-NSSs on U. Then,*

1. $\otimes((\Gamma_Q, A) \cup (\Psi_Q, B)) = \otimes(\Gamma_Q, A) \cup \otimes(\Psi_Q, B)$.
2. $\otimes((\Gamma_Q, A) \cap (\Psi_Q, B)) = \otimes(\Gamma_Q, A) \cap \otimes(\Psi_Q, B)$.
3. $\otimes(\otimes(\Gamma_Q, A)) = \otimes(\Gamma_Q, A)$.

Proof. 1. $(\Gamma_Q, A) \cup (\Psi_Q, B) = (\Lambda_Q, C)$, where $C = A \cup B$ and

$$(\Lambda_Q, C) = \left\{ \left\langle e, [(u,t), T_{\Lambda_Q}(u,t), I_{\Lambda_Q}(u,t), F_{\Lambda_Q}(u,t)] \right\rangle : (u,t) \in U \times Q \right\},$$

such that

$$T_{\Lambda_Q}(u,t) = \begin{cases} T_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ T_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \max\{T_{\Gamma_Q}(u,t), T_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$

$$I_{\Lambda_Q}(u,t) = \begin{cases} I_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ I_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \min\{I_{\Gamma_{Qi}}(u,t), I_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$

and

$$F_{\Lambda_Q}(u,t) = \begin{cases} F_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ F_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \min\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B. \end{cases}$$

Now, by Definition 17, for all $e \in A$

$$\otimes(\Lambda_Q, C) = \left\{\left\langle e, [(u,t), \otimes T_{\Lambda_Q}(u,t), \otimes I_{\Lambda_Q}(u,t), \otimes F_{\Lambda_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\}, \text{ where}$$

$$\otimes T_{\Lambda_Q}(u,t) = \begin{cases} 1 - F_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ 1 - F_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ 1 - \min\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$

$$\otimes I_{\Lambda_Q}(u,t) = \begin{cases} I_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ I_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \min\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$

and

$$\otimes F_{\Lambda_Q}(u,t) = \begin{cases} F_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ F_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \min\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B. \end{cases}$$

Assume for all $e \in A$

$$\otimes(\Gamma_Q, A) = \left\{\left\langle e, [(u,t), 1 - F_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), F_{\Gamma_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\},$$

and

$$\otimes(\Psi_Q, B) = \left\{\left\langle e, [(u,t), 1 - F_{\Psi_Q}(u,t), I_{\Psi_Q}(u,t), F_{\Psi_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\}.$$

Now,
$\otimes(\Gamma_Q, A) \cup \otimes(\Psi_Q, B) = (\Upsilon_Q, C)$, where

$$(\Upsilon_Q, C) = \left\{\left\langle e, [(u,t), T_{\Upsilon_Q}(u,t), I_{\Upsilon_Q}(u,t), F_{\Upsilon_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\},$$

such that

$$T_{\Upsilon_Q}(u,t) = \begin{cases} 1 - F_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ 1 - F_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \max\{1 - F_{\Gamma_Q}(u,t), 1 - F_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$
$$= \begin{cases} 1 - F_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ 1 - F_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ 1 - \min\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$

$$I_{\Upsilon_Q}(u,t) = \begin{cases} I_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ I_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \min\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B, \end{cases}$$

and

$$F_{\Upsilon_Q}(u,t) = \begin{cases} F_{\Gamma_Q}(u,t) & \text{if } e \in A - B, \\ F_{\Psi_Q}(u,t) & \text{if } e \in B - A, \\ \min\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\} & \text{if } e \in A \cap B. \end{cases}$$

Consequently, $\otimes(\Lambda_Q, C)$ and (Υ_Q, C) are the same. Thus, $\otimes((\Gamma_Q, A) \cup (\Psi_Q, B)) = \otimes(\Gamma_Q, A) \cup \otimes(\Psi_Q, B)$.
2. Can be analogously proven.
3. Assume for all $e \in A$ $\otimes(\Gamma_Q, A) = \left\{\left\langle e, [(u,t), T_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), 1 - T_{\Gamma_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\}$.
Now,

$$\otimes(\otimes(\Gamma_Q, A)) = \left\{\left\langle e, [(u,t), T_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), 1 - T_{\Gamma_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\}$$
$$= \otimes(\Gamma_Q, A).$$

□

Proposition 6. *Let* (Γ_Q, A) *be a Q-NSS over U and Q; we have the following properties:*

1. $\otimes \oplus (\Gamma_Q, A) = \oplus(\Gamma_Q, A)$.
2. $\oplus \otimes (\Gamma_Q, A) = \otimes(\Gamma_Q, A)$.

Proof. 1. Suppose that, for any $e \in A$,

$$(\Gamma_Q, A) = \left\{\left\langle e, [(u,t), T_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), F_{\Gamma_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\}. \text{ Then,}$$

$$\oplus(\Gamma_Q, A) = \left\{\left\langle e, [(u,t), T_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), 1 - T_{\Gamma_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\} \text{ and}$$

$$\otimes(\Gamma_Q, A) = \left\{\left\langle e, [(u,t), 1 - F_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), F_{\Gamma_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\}.$$

Thus, $\otimes \oplus (\Gamma_Q, A) = \left\{\left\langle e, [(u,t), T_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), 1 - T_{\Gamma_Q}(u,t)]\right\rangle : (u,t) \in U \times Q\right\} = \oplus(\Gamma_Q, A)$.

2. The proof is similar to that of Assertion 1. □

Proposition 7. *Let* (Γ_Q, A) *and* (Ψ_Q, B) *be two Q-NSSs over U and Q, we have the following:*

1. $\oplus((\Gamma_Q, A) \wedge (\Psi_Q, B)) = \oplus(\Gamma_Q, A) \wedge \oplus(\Psi_Q, B)$.
2. $\oplus((\Gamma_Q, A) \vee (\Psi_Q, B)) = \oplus(\Gamma_Q, A) \vee \oplus(\Psi_Q, B)$.
3. $\otimes((\Gamma_Q, A) \wedge (\Psi_Q, B)) = \otimes(\Gamma_Q, A) \wedge \otimes(\Psi_Q, B)$.
4. $\otimes((\Gamma_Q, A) \vee (\Psi_Q, B)) = \otimes(\Gamma_Q, A) \vee \otimes(\Psi_Q, B)$.

Proof. 1. Assume $(\Gamma_Q, A) \wedge (\Psi_Q, B) = (\Lambda_Q, A \times B)$, where for all $e_a \in A, e_b \in B$,

$$\Lambda_Q(e_a, e_b) = \Big\{\langle(e_a, e_b), [(u,t), \min\{T_{\Gamma_Q}(u,t), T_{\Psi_Q}(u,t)\}, \max\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\}, \max\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\}]\Big\rangle : (u,t) \in U \times Q\Big\}.$$

By Definition 16, and for all $e_a \in A$, $e_b \in B$, we have

$$\oplus((\Gamma_Q, A) \wedge (\Psi_Q, B)) = \Big\{\Big\langle(e_a, e_b), [(u,t), \min\{T_{\Gamma_Q}(u,t), T_{\Psi_Q}(u,t)\}, \max\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\}, 1 - \min\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\}]\Big\rangle : (u,t) \in U \times Q\Big\}.$$

Since $\oplus(\Gamma_Q, A) = \Big\{\Big\langle e_a, [(u,t), T_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), 1 - T_{\Gamma_Q}(u,t)]\Big\rangle : e_a \in A, (u,t) \in U \times Q\Big\}$, and $\oplus(\Psi_Q, B) = \Big\{\Big\langle e_b, [(u,t), T_{\Psi_Q}(u,t), I_{\Psi_Q}(u,t), 1 - T_{\Psi_Q}(u,t)]\Big\rangle : e_b \in B, (u,t) \in U \times Q\Big\}$. Then, for all $e_a \in A, e_b \in B$ we have

$$\begin{aligned}
&\oplus(\Gamma_Q, A) \wedge \oplus(\Psi_Q, B) \\
&= \Big\{\Big\langle(e_a, e_b), [(u,t), \min\{T_{\Gamma_Q}(u,t), T_{\Psi_Q}(u,t)\}, \max\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\}, \max\{1 - T_{\Gamma_Q}(u,t), 1 - T_{\Psi_Q}(u,t)\}]\Big\rangle : (u,t) \in U \times Q\Big\}. \\
&= \Big\{\Big\langle(e_a, e_b), [(u,t), \min\{T_{\Gamma_Q}(u,t), T_{\Psi_Q}(u,t)\}, \max\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\}, 1 - \min\{T_{\Gamma_Q}(u,t), T_{\Psi_Q}(u,t)\}]\Big\rangle : (u,t) \in U \times Q\Big\}. \\
&= \oplus((\Gamma_Q, A) \wedge (\Psi_Q, B))
\end{aligned}$$

2. The proof is similar to that of Assertion 1.

3. Since for all $e_a \in A, e_b \in B$

$$(\Gamma_Q, A) \wedge (\Psi_Q, B) = \Big\{\Big\langle(e_a, e_b), [(u,t), \min\{T_{\Gamma_Q}(u,t), T_{\Psi_Q}(u,t)\}, \max\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\}, \max\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\}]\Big\rangle : (u,t) \in U \times Q\Big\}.$$

By Definition 17,

$$(\Gamma_Q, A) \wedge (\Psi_Q, B) = \Big\{\Big\langle(e_a, e_b), [(u,t), 1 - \max\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\}, \max\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\}, \max\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\}]\Big\rangle : (u,t) \in U \times Q\Big\}.$$

Since, $\otimes(\Gamma_Q, A) = \left\{\left\langle e_a, [(u,t), 1 - F_{\Gamma_Q}(u,t), I_{\Gamma_Q}(u,t), F_{\Gamma_Q}(u,t)]\right\rangle : e_a \in A, (u,t) \in U \times Q\right\}$, and $\otimes(\Psi_Q, B) = \left\{\left\langle e_b, [(u,t), 1 - F_{\Psi_Q}(u,t), I_{\Psi_Q}(u,t), F_{\Psi_Q}(u,t)]\right\rangle : e_b \in B, (u,t) \in U \times Q\right\}$. Then, for all $e_a \in A, e_b \in B$ we have

$$\begin{aligned}
\otimes(\Gamma_Q, A) \wedge \otimes(\Psi_Q, B) &= \Big\{\Big\langle (e_a, e_b), [(u,t), \min\{1 - F_{\Gamma_Q}(u,t), 1 - F_{\Psi_Q}(u,t)\}, \max\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\}, \\
&\qquad \max\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\}]\Big\rangle : (u,t) \in U \times Q\Big\} \\
&= \Big\{\Big\langle (e_a, e_b), [(u,t), 1 - \max\{F_{\Gamma_Q}(u,t), F_{B_Q}(u,t)\}, \max\{I_{\Gamma_Q}(u,t), I_{\Psi_Q}(u,t)\}, \\
&\qquad \max\{F_{\Gamma_Q}(u,t), F_{\Psi_Q}(u,t)\}]\Big\rangle : (u,t) \in U \times Q\Big\} \\
&= \otimes((\Gamma_Q, A) \wedge (\Psi_Q, B)).
\end{aligned}$$

4. The proof is similar to that of Assertion 3. □

5. An Application of Q-Neutrosophic Soft Sets

In this section, we present a Q-NS aggregation operator of Q-NSS that produces a Q-NS from a Q-NSS and then reduces it to a Q-fuzzy set in order to use it in a decision-making problem.

Definition 18. *Let (Γ_Q, A) be Q-NSS over U. Then, a Q-NS aggregation operator of (Γ_Q, A), denoted by Γ_Q^{agg}, is defined by $\Gamma_Q^{agg} = \{\langle (u,t), T_Q^{agg}(u,t), I_Q^{agg}(u,t), F_Q^{agg}(u,t)\rangle : (u,t) \in U \times Q\}$, which is a Q-NS over U, where $T_Q^{agg}, I_Q^{agg}, F_Q^{agg} : U \times Q \to [0,1]$*

$$\begin{aligned}
T_Q^{agg} &= \frac{1}{|A|} \sum_{(u,t)\in U\times Q} T_{\Gamma_Q}(u,t), \\
I_Q^{agg} &= \frac{1}{|A|} \sum_{(u,t)\in U\times Q} I_{\Gamma_Q}(u,t) \text{ and} \\
F_Q^{agg} &= \frac{1}{|A|} \sum_{(u,t)\in U\times Q} F_{\Gamma_Q}(u,t).
\end{aligned}$$

Definition 19. *The reduced Q-fuzzy set of a Q-NS Γ_Q is*

$$\widehat{\Gamma}_Q = \{(u,t), \mu_{\Gamma_Q}(u,t) : (u,t) \in U \times Q\},$$

where $\mu_{\Gamma_Q} : U \times Q \to [0,1]$ and given by $\mu_{\Gamma_Q} = \frac{1}{3}[T_Q(u,t) + 2 - I_Q(u,t) - F_Q(u,t)]$.

Now, using the definitions of a Q-NS aggregation operator and a reduced Q- fuzzy set, we construct the following algorithm for a decision method:

Step 1 Construct a Q-NSS over U.
Step 2 Compute the Q-NS aggregation operator.
Step 3 Compute the reduced Q-fuzzy set of the Q-NS aggregation operator.
Step 4 The decision is any element in M, where $M = \max_{(u,t)\in U\times Q}\{\mu_{\Gamma_Q}^{agg}\}$.

Now, we provide an example for Q-NSS decision-making method.

Example 7. *Suppose a university needs to fill a position in the mathematics department to be selected by expert committee. There are three candidates, $U = \{u_1, u_2, u_3\}$, with two types of scientific degree, $Q = \{s = \text{assistant professor}, t = \text{associate professor}\}$, and the hiring committee considers a set of parameters $E = \{e_1, e_2, e_3\}$ representing experience, language fluency, and computer knowledge, respectively.*

Now, we can apply the method to help the committee fill the position with the suitable candidate as follows:

Step 1 *The committee construct the following Q-NSS:*

$$(\Gamma_Q, A) = \Big\{ \Big\langle e_1, [(u_1, s), 0.2, 0.3, 0.5], [(u_1, t), 0.6, 0.4, 0.1], [(u_2, s), 0.1, 0.4, 0.7], [(u_3, t), 0.3, 0.2, 0.8] \Big\rangle, \Big\langle e_2, [(u_1, s), 0.4, 0.1, 0.9], [(u_2, s), 0.6, 0.2, 0.1], [(u_3, s), 0.5, 0.3, 0.7], [(u_3, t), 0.1, 0.3, 0.2] \Big\rangle, \Big\langle e_3, ((u_1, t), 0.2, 0.3, 0.6), ((u_2, t), 0.9, 0.1, 0.2) \Big\rangle \Big\}.$$

Step 2 *The Q-NS aggregation operator is*

$$\Gamma_Q^{agg} = \Big\{ [(u_1, s), 0.2, 0.133, 0.467], [(u_1, t), 0.267, 0.233, 0.233], [(u_2, s), 0.233, 0.2, 0.267], [(u_2, t), 0.3, 0.033, 0.067], [(u_3, s), 0.166, 0.1, 0.233], [(u_3, t), 0.133, 0.167, 0.333] \Big\}.$$

Step 3 *The reduced Q-fuzzy set of the Γ_Q^{agg} is*

$$\widehat{\Gamma}_Q = \Big\{ [(u_1, s), 0.533], [(u_1, t), 0.6], [(u_2, s), 0.589], [(u_2, t), 0.733], [(u_3, s), 0.611], [(u_3, t), 0.544] \Big\}.$$

Step 4 *The largest membership grade is $\mu^{agg}_{\Gamma_Q}(u_2, t) = 0.733$ which implies that the committee is inclined to choose associate-professor candidate u_2 with a scientific degree for the job.*

6. Comparative Analysis

In this section, we compare the concept of the Q-neutrosophic soft method to the neutrosophic soft method [16], Q-FSS [38], and Q-IFSS [39].

In contrast to the neutrosophic soft method that uses the NSS to characterize decision-making data, the novel Q-neutrosophic soft method introduces a new descriptor, that is, Q-NSS, to provide actual decision-making information. From Example 7, it can be seen that the NSS was unable to represent variables in two dimensions. However, the framework of the Q-NSS offers the capacity to simultaneously handle these two dimensions.

On the other hand, Q-NSS is identified through three independent degrees of membership, namely, truth, indeterminacy, and falsity. Hence, it is more accurate than Q-FSS, which is identified by one truth value, and Q-IFSS, which is identified by two dependent memberships for truth and falsity. Thus, the proposed method has certain advantages, that is, this method uses the Q-NSS to represent decision information as an extension of Q-FSS and Q-IFSS. An effective aggregation formula is employed to convert the Q-NSS to Q-NS, which preserves the entirety of the original data without reducing or distorting them. Our method also provides decision making with a simple computational process.

As a result, the Q-NSS has the ability to deal with indeterminate and inconsistent data in two-dimensional sets. Consequently, it is capable to interact with deeper imprecise data. The basic characteristics of Q-NSS were compared to those of NSS, Q-FSS, and Q-IFSS, as shown in Table 1.

Table 1. Characteristic comparison of the Q-neutrosophic soft set (Q-NSS) with other variants.

Method	NSS	Q-FSS	Q-IFSS	Q-NSS
Authors	Maji [16]	Adam and Hassan [38]	Broumi [39]	Proposed Method
Domain	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse
Codomain	$[0,1]^3$	$[0,1]$	$[0,1]^2$	$[0,1]^3$
Q	No	Yes	Yes	Yes
True	Yes	Yes	Yes	Yes
Indeterminacy	Yes	No	No	Yes
Falsity	Yes	No	Yes	Yes

7. Conclusions

Q-NSS is an NSS over a two-dimensional universal set. Thus, a Q-NSS is a tricomponent set that can simultaneously handle two-dimensional and indeterminate data. This study discussed some operations of Q-NSSs, namely, subset, equality, complement, intersection, union, AND operation, and OR operation. It discussed the necessity and possibility operations along with some properties and illustrative examples. Finally, the Q-NS aggregation operator was defined and applied to develop an algorithm for using Q-NSS in decision-making problems that involve uncertainty. This new model provides an important extension to existing studies that can handle indeterminacy, where two-dimensionality appears in the decision process, thus offering the opportunity for further relevant research. Q-NSS encourages the path to various scopes for future research since it deals with indeterminacy and two-dimensionality at the same time. Hence, it can be expanded by utilizing the n-valued refined neutrosophic set [44], possibility neutrosophic set [45], and numerous different structures. Moreover, different algebraic structures, for instance, the field, ring, and group of the Q-NSS and its extensions may be investigated.

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