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A Single-Valued Neutrosophic Linguistic Combined Weighted Distance Measure and Its Application in Multiple-Attribute Group Decision-Making

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Abstract: The aim of this paper is to present a multiple-attribute group decision-making (MAGDM) framework based on a new single-valued neutrosophic linguistic (SVNL) distance measure. By unifying the idea of the weighted average and ordered weighted averaging into a single-valued neutrosophic linguistic distance, we first developed a new SVNL weighted distance measure, namely a SVNL combined and weighted distance (SVNLCWD) measure. The focal characteristics of the devised SVNLCWD are its ability to combine both the decision-makers' attitudes toward the importance, as well as the weights, of the arguments. Various desirable properties and families of the developed SVNLCWD were contemplated. Moreover, a MAGDM approach based on the SVNLCWD was formulated. Lastly, a real numerical example concerning a low-carbon supplier selection problem was used to describe the superiority and feasibility of the developed approach.

Keywords: single-valued neutrosophic linguistic set; distance measure; combined weighted average; MAGDM; low-carbon supplier selection

1. Introduction

Multiple-attribute group decision-making (MAGDM) is one of the most commonly used methods to rank and select potential alternatives based on the decision information of multiple decision-makers (or experts). In real MAGDM problems, the increasing uncertainties of objects make it increasingly difficult for people to precisely express judgments about their attributes during the process of decision-making. Indeed, this is related not only to the nature of the objects but also to the ambiguity of the underlying human intervention and cognitive thinking in general. Handling imprecision or vagueness effectively in these complex situations is a matter of great concern in MAGDM problems. Recently, a new tool for solving the uncertainty or inaccuracy of such information was introduced by Ye [1], namely the single-valued neutrosophic linguistic set (SVNLS). By unifying the features of single-valued neutrosophic sets (SVNS) [2,3] and linguistic terms [4], the SVNLS can eliminate both of their shortcomings, and has been proven to be suitable to measure a higher degree of uncertainty for subjective evaluations. As an effective extension of the linguistic terms and SVNS, the basic element of the SVNLS is the single-valued neutrosophic linguistic value (SVNLV), which makes it more effective for handling uncertain and imprecise information when contrasted with the existing fuzzy tools, such as the intuitionistic linguistic set [5] and the Pythagorean fuzzy set [6]. Following the latest research trend, the SVNLS has been widely applied to handle MAGDM problems under indeterminacy and complex environments. Ye [1] investigated the classic technique for order preference by similarity to an ideal solution (TOPSIS) method in SVNLS situation and studied its usefulness for decision-making problems. Ye [7] developed some neutrosophic linguistic operators and investigated their applications in selecting a flexible manufacturing system. Wang et al. [8] extended the Maclaurin symmetric mean operator to aggregate SVNL information. Chen et al. [9] developed a novel distance measure for SVNLS based on the ordered weighted viewpoint. Ji et al. [10] proposed a combined multi-attribute border approximation area comparison (MABAC) and the elimination and choice translating reality (ELECTRE) approach for SVNLS and studied its application in selecting outsourcing providers. Wu et al. [11] investigated the usefulness of the SVNLS in a 2-tuple environment of MAGDM analysis. Kazimieras et al. [12] developed a new SVN decision-making model by applying the weighted aggregated sum product assessment (WASPAS) method. Garg and Nancy [13] proposed several prioritized aggregation operators for SVNLS to handle the priority among the attributes.

Distance measurement is one of the most widely used tools in MAGDM, and can be used to measure the differences between the expected solutions and potential alternatives. Recently, a new distance measurement method based on the ordered weighted viewpoint, i.e., the ordered weighted averaging distance (OWAD) operator proposed by Merigó and Gil-Lafuente [14] has attracted increasing attention from researchers. The essence of this distance operator is that it enables decision-makers to incorporate their attitudinal bias into the decision-making process by imposing some weighting schemes to the individual distances. To date, several OWAD extensions and their subsequent applications in solving MAGDM problems have appeared in recent studies, such as the induced OWAD operator [15], intuitionistic fuzzy OWAD operator [16], hesitant fuzzy OWAD operator [17], probabilistic OWAD operator [18], Pythagorean fuzzy generalized OWAD operator [19], fuzzy linguistic induced Euclidean OWAD operator [20], continuous OWAD operator [21] and the intuitionistic fuzzy weighted induced OWAD operator [22]. More recently, Chen et al. [8] further presented a definition of the single-valued neutrosophic linguistic OWAD (SVNLOWAD) operator, on the basis of which a modified TOPSIS model was then proposed for MAGDM problems in a SVNL situation.

Although the OWAD operator and its numerous extensions, such as the SVNLOWAD operator, have shown their superiority in practical applications, they possess a defect in that they can integrate only the special interests of the experts, while ignoring the importance of the attributes in the outcome of a decision. To overcome this shortcoming, this study develops a combined weighted distance for SVNLSs, called the single-valued neutrosophic linguistic combined weighted distance (SVNLCWD) operator. The proposed combined weighted distance operator is superior in that it involves both subjective information on the importance of the ordered attributes and the importance of specific attributes. We further explored some of the key properties and particular cases of the proposed operator. Finally, we applied the SVNLCWD operator to a MAGDM problem concerning low-carbon supplier selection to verify its effectiveness and superiority.

2. Preliminaries

In this section, we will briefly review some of concepts we need to use in the following sections, including the definition of the SVNLS, the OWAD and the SVNLOWAD operator.

2.1. Linguistic Set

Let $S = \{s_\alpha | \alpha = 1, \ldots, l\}$ be a finitely ordered discrete term set, where s_α indicates a possible value for a linguistic variable (LV) and l is an odd number. For instance, taking $l = 7$, then a linguistic term set S could be specified $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$ = *{extremely poor, very poor, poor, fair, good, very good, extremely good}*. In this case, any two LVs s_i and s_j in S should satisfy rules (1)-(4) [23]:

(1) $Neg(s_i) = s_{-i}$;
(2) $s_i \leq s_j \Leftrightarrow i \leq j$;
(3) $\max(s_i, s_j) = s_j$, if $i \leq j$;
(4) $\min(s_i, s_j) = s_i$, if $i \leq j$.

To minimize information loss in the operational process, the discrete term set S shall be extended to a continuous set $\overline{S} = \{s_\alpha | \alpha \in R\}$. Any two LVs $s_\alpha, s_\beta \in \overline{S}$, satisfy the following operational rules [24]:

(1) $s_{\alpha} \oplus s_{\beta} = s_{\alpha+\beta}$;

(2) $\mu s_{\alpha} = s_{\mu\alpha}, \mu \geq 0$;

(3) $s_{\alpha}/s_{\beta} = s_{\alpha/\beta}$.

2.2. Single-Valued Neutrosophic Set (SVNS)

The neutrosophic set was introduced for the first time by Smarandache in 1998 [2], while Ye introduced the linguistic neutrosophic set in 2015 [1] and Ye developed the single-valued neutrosophic set (SVNS) in 2013 [25].

Definition 1. Let y be an element in a finite set Y. A SVNS P in Y can be defined as in (1):

$$P = \{ \langle y, T_P(y), I_P(y), F_P(y) \rangle | y \in Y \}, \tag{1}$$

where the truth-membership function $T_P(y)$, the indeterminacy-membership function $I_P(y)$, and the falsity-membership function $F_P(y)$ shall satisfy the following conditions:

$$0 \leq T_P(y), I_P(y), F_P(y) \leq 1,\ 0 \leq T_P(y) + I_P(y) + F_P(y) \leq 3. \tag{2}$$

For convenience of calculation, we call the triplet $(T_P(y), I_P(y), F_P(y))$ single-valued neutrosophic value (SVNV) and simply denote it as $y = (T_y, I_y, F_y)$. Let $y = (T_y, I_y, F_y)$ and $z = (T_z, I_z, F_z)$ be two SVNVs, their mathematical operational laws are defined as follows:

(1) $y \oplus z = (T_y + T_z - T_y * T_z, I_y * T_z, F_y * F_z)$;

(2) $\lambda y = (1 - (1 - T_y)^{\lambda}, (I_y)^{\lambda}, (F_y)^{\lambda}),\ \lambda > 0$;

(3) $y^{\lambda} = ((T_y)^{\lambda}, 1 - (1 - I_y)^{\lambda}, 1 - (1 - F_y)^{\lambda}), \lambda > 0$.

2.3. Single-Valued Neutrosophic Linguistic Set (SVNLS)

On the basis of the SVNS, Ye gave the definition and operational laws of the single-valued neutrosophic linguistic set (SVNLS), listed in the definitions 2–5.

Definition 2. Let Y be a finite universe set, a SVNLS Q in Y is defined as in (3):

$$Q = \left\{ \left\langle y, [s_{\theta(y)}, (T_P(y), I_P(y), F_P(y))] \right\rangle \middle| y \in Y \right\}, \tag{3}$$

where $s_{\theta(y)} \in \overline{S}$, the truth-membership function $T_q(y)$, the indeterminacy-membership function $I_q(y)$, and the falsity-membership function $F_q(y)$ satisfy condition (4):

$$0 \leq T_q(y), I_q(y), F_q(y) \leq 1,\ 0 \leq T_q(y) + I_q(y) + F_q(y) \leq 3. \tag{4}$$

For a SVNLS Q in Y, the SVNLV $\left\langle s_{\theta(y)}, (T_P(y), I_P(y), F_P(y)) \right\rangle$ is simply denoted as $y = \left\langle s_{\theta(y)}, (T_y, I_y, F_y) \right\rangle$ for computational convenience.

Definition 3. Let $y_i = \left\langle s_{\theta(y_i)}, (T_{y_i}, I_{y_i}, F_{y_i}) \right\rangle (i = 1, 2)$ be two SVNLVs, then

(1) $y_1 \oplus y_2 = \left\langle s_{\theta(y_1)+\theta(y_2)}, (T_{y_1} + T_{y_2} - T_{y_1} * T_{y_2}, I_{y_1} * T_{y_2}, F_{y_1} * F_{y_2}) \right\rangle$;

(2) $\lambda y_1 = \left\langle s_{\lambda\theta(y_1)}, (1 - (1 - T_{y_1})^{\lambda}, (I_{y_1})^{\lambda}, (F_{y_1})^{\lambda}) \right\rangle, \lambda > 0$;

(3) $y_1^{\lambda} = \left\langle s_{\theta^{\lambda}(y_1)}, ((T_{y_1})^{\lambda}, 1 - (1 - I_{y_1})^{\lambda}, 1 - (1 - F_{y_1})^{\lambda}) \right\rangle, \lambda > 0$.

Definition 4. The distance measure between the SVNLVs $y_i = \left\langle s_{\theta(y_i)}, (T_{y_i}, I_{y_i}, F_{y_i}) \right\rangle (i = 1, 2)$ is defined as in (5):

$$d(y_1, y_2) = \left[\left| \theta(y_1) T_{y_1} - \theta(y_2) T_{y_2} \right|^{\lambda} + \left| \theta(y_1) I_{y_1} - \theta(y_2) I_{y_2} \right|^{\lambda} + \left| \theta(y_1) F_{y_1} - \theta(y_2) F_{y_2} \right|^{\lambda} \right]^{1/\lambda}. \quad (5)$$

If we assign different weights to the individual distances of the SVNLVs, we get the single-valued neutrosophic linguistic weighted distance (SVNLWD) measure [8].

Definition 5. Let y_j, y'_j $(j = 1, \ldots, n)$ be the two collections of SVNLVs, a single-valued neutrosophic linguistic weighted distance measure of dimension n is a mapping SVNLWD: $\Omega^n \times \Omega^n \to R$, which has an associated weighting vector W with $w_j \in [0, 1]$ and $\sum\limits_{j=1}^{n} w_j = 1$, such that:

$$SVNLWD\left((y_1, y'_1), \ldots, (y_n, y'_n) \right) = \sum_{j=1}^{n} w_j d(y_j, y'_j), \quad (6)$$

The OWAD operator developed by Merigó and Gil-Lafuente [14] aims to aggregate individual distances as arguments on the basis of the ordered weighted averaging (OWA) operator [26]. Let $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_n\}$ be two crisp sets, and the OWAD operator can be defined as follows.

Definition 6. An OWAD operator is defined as a mapping OWAD: $R^n \times R^n \to R$ with the weighting vector $W = \{ w_j | \sum\limits_{i=1}^{n} w_j = 1, \;\; 0 \le w_j \le 1 \}$, such that:

$$OWAD(\langle a_1, b_1 \rangle, \ldots, \langle a_n, b_n \rangle) = \sum_{j=1}^{n} w_j d_j, \quad (7)$$

where d_j is the j-th largest number among $|a_i - b_i|$.

On the basis of the OWAD operator, Chen et al. [9] introduced the SVNLOWAD operator to aggregate SVNL information.

Definition 7. Let y_j, y'_j $(j = 1, \ldots, n)$ be the two collections of SVNLVs. If

$$SVNLOWAD\left((y_1, y'_1), \ldots, (y_n, y'_n) \right) = \sum_{j=1}^{n} w_j d(y_j, y'_j), \quad (8)$$

then the SVNLOWAD is called the single-value neutrosophic linguistic OWAD, where $d(y_j, y'_j)$ represents the j-th largest value among the individual distances $d(y_i, y'_i)(i = 1, \ldots, n)$ defined in Equation (5). $w = (w_1, \ldots, w^n)^T$ is a weighting vector related to the SVNLOWAD operator, satisfying $\sum\limits_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$.

The properties of commutativity, monotonicity, boundedness and idempotency can easily be established for the SVNLOWAD operator. Based on the above analysis, we can find that, although the SVNLOWAD and SVNLWD operators have been widely used to solve MAGDM problems in SVNL environments, these two operators exhibit certain deficiencies. Next, we shall propose a combined weighted distance measure to alleviate these shortcomings.

3. SVNL Combined Weighted Distance (SVNLCWD) Operator

The SVNL combined weighted distance (SVNLCWD) operator unifies both the advantages of the SVNLWD and the SVNLOWAD operators in the same framework. Therefore, it is able to integrate the decision-makers' attitudes using ordered weighted arguments as well as embedding the importance of alternatives based on the weighted average method. Moreover, it allows decision-makers to adjust the allocation ratio of the SVNLOWAD and SVNLWD flexibly based on the needs of the particular problem or their interests. The SVNLCWD operator can be defined as follows.

Definition 8. Let y_j, y'_j $(j = 1, \dots, n)$ be the two collections of SVNLVs. If

$$SVNLCWD\big((y_1, y'_1), \dots, (y_n, y'_n)\big) = \sum_{j=1}^{n} \overline{w}_j D_j, \tag{9}$$

then the SVNLCWD is called the single-value neutrosophic linguistic combined weighted distance operator, where D_j represents the j-th largest value among the individual distances $d(y_i, y'_i)(i = 1, 2..., n)$ defined in Equation (5). There are two weights assigned to each distance D_j: ω_j, is the weight for weighted averaging (WA) with $\sum_{j=1}^{n} \omega_j = 1$ and $\omega_j \in [0, 1]$, and w_j, is the weight for the OWA meeting $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$. The integrated weight $\overline{w}_j$ is defined as:

$$\overline{w}_j = \delta\omega_j + (1 - \delta)w_j, \tag{10}$$

where $\delta \in [0, 1]$ and ω_j is indeed ω_i re-ordered to be associated to $d(y_i, y'_i)(i = 1, \dots, n)$.

Based on the basic operational laws (i.e., ordered weighted and weighted average), the SVNLCWD operator can be decomposed linearly into a combination of the SVNLOWAD and SVNLWD:

Definition 9. Let y_j, y'_j $(j = 1, \dots, n)$ be the two collections of SVNLNs. If

$$SVNLCWD\big((y_1, y'_1), \dots, (y_n, y'_n)\big) = \delta \sum_{i=1}^{n} \omega_i d(y_i, y'_i) + (1 - \delta) \sum_{j=1}^{n} w_j D_j, \tag{11}$$

where D_j represents the j-th largest value among the individual distances $d(y_i, y'_i)(i = 1, \dots, n)$ defined in Equation (5), and $\delta \in [0, 1]$. Obviously, the SVNLCWD is reduced to the SVNLOWAD and SVNLWD, when $\delta = 0$ and $\delta = 1$, respectively.

Example 3.1. Let $Y = (y_1, y_2, y_3, y_4, y_5) = (\langle s_2, (0.5, 0.3, 0.4)\rangle, \langle s_5, (0.5, 0.2, 0.2)\rangle, \langle s_5, (0.3, 0.3, 0.6)\rangle, \langle s_2, (0.1, 0.4, 0.6)\rangle, \langle s_7, (0.5, 0.8, 0.2)\rangle)$ and $Y' = (y'_1, y'_2, y'_3, y'_4, y'_5) = (\langle s_5, (0.2, 0.9, 0)\rangle, \langle s_3, (0.5, 0.7, 0.2)\rangle, \langle s_5, (0.4, 0.4, 0.5)\rangle, \langle s_4, (0.5, 0.7, 0.2)\rangle, \langle s_3, (0.4, 0.2, 0.6)\rangle)$ be two SVNLSs defined in set $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$. Let $w = (0.15, 0.3, 0.2, 0.25, 0.1)^T$ be the weighting vector of SVNLCWD measure. Then, the aggregating process by the SVNLCWD can be displayed as follows:

(1) Compute the individual distances $d(y_i, y'_i)(i = 1, 2, \dots, 5)$ (let $\lambda = 1$) according to Equation (5):

$$d(y_1, y'_1) = |2 \times 0.5 - 5 \times 0.2| + |2 \times 0.3 - 5 \times 0.9| + |2 \times 0.4 - 5 \times 0| = 4.7.$$

Similarly, we get

$$d(y_2, y'_2) = 2.4,\ d(y_3, y'_3) = 1.5,$$
$$d(y_4, y'_4) = 3.2,\ d(y_5, y'_5) = 7.7.$$

(2) Sort the $d(y_i, {y'}_i)(i = 1, 2, \ldots, 5)$ in decreasing order:

$$D_1 = d(y_5, {y'}_5) = 7.7,\ D_2 = d(y_1, {y'}_1) = 4.7,\ D_3 = d(y_4, {y'}_4) = 3.2,$$
$$D_4 = d(y_2, {y'}_2) = 2.4,\ D_5 = d(y_3, {y'}_3) = 1.5.$$

(3) Let the weighting vector $\omega = (0.1, 0.15, 0.2, 0.35, 0.2)^T$ and $\delta = 0.4$, calculate the integrated weights $\overline{w}_j$ according to Equation (10):

$$\overline{w}_1 = 0.4 \times 0.2 + (1 - 0.4) \times 0.15 = 0.17,\ \overline{w}_2 = 0.4 \times 0.1 + (1 - 0.4) \times 0.3 = 0.22,$$
$$\overline{w}_3 = 0.4 \times 0.35 + (1 - 0.4) \times 0.2 = 0.26,\ \hat{w}_4 = 0.4 \times 0.15 + (1 - 0.4) \times 0.25 = 0.21,$$
$$\overline{w}_5 = 0.4 \times 0.2 + (1 - 0.4) \times 0.1 = 0.14.$$

(4) Use the SVNLCWD measure defined in Equation (9) to perform the following aggregation:

$$\begin{gathered} SVNLCWD(Y, Y') \\ = 0.17 \times 7.7 + 0.22 \times 4.7 + 0.26 \times 3.2 + 0.21 \times 2.4 + 0.14 \times 1.5 \\ = 3.889 \end{gathered}$$

We can also perform the aggregation process of the SVNLCWD using Equation (11):

$$\begin{gathered} SVNLCWD(Y, Y') \\ = 0.4 \times SVNLWD + (1 - 0.4) \times SVNLOWAD \\ = 0.4 \times 3.79 + 0.6 \times 3.955 \\ = 3.889 \end{gathered}$$

Apparently, we obtain the same results using both methods. However, compared with the SVNLOWAD operator, the proposed SVNLCWD operator can not only incorporate decision-makers' interests and biases according to the ordered weights, but also highlights the importance of the input arguments based on the weighted average tool.

Furthermore, by setting varied weighting schemes on the SVNLCWD operator, we can obtain a series of SVNL weighted distance measures:

- If $w_1 = 1, w_2 = \cdots = w_n = 0$, then max-SVNLWD (SVNLMaxD) is formed.
- If $w_1 = \cdots = w_{n-1} = 0, w_n = 1$, then the min-SVNLWD (SVNLMinD) is obtained.
- The step-SVNLCWD operator is rendered by imposing $w_1 = \cdots = w_{k-1} = 0$, $w_k = 1$ and $w_{k+1} = \cdots = w_n = 0$.
- According to techniques used in the recent literature [27,28], we can create more special cases of the SVNLCWD, such as the Median-SVNLCWD, the Centered-SVNLCWD and the Olympic-SVNLCWD operators.

The SVNLCWD operator has the following desirable properties that all aggregation operators should ideally possess:

Theorem 1. (Commutativity–aggregation operator). Let $((x_1, {x'}_1), \ldots, (x_n, {x'}_n))$ be any permutation of the set of SVNLVs $((y_1, {y'}_1), \ldots, (y_n, {y'}_n))$, then

$$SVNLCWD\big((x_1, {x'}_1), \ldots, (x_n, {x'}_n)\big) = SVNLCWD\big((y_1, {y'}_1), \ldots, (y_n, {y'}_n)\big) \tag{12}$$

The property of commutativity can also be demonstrated from the perspective of distance measure:

$$SVNLCWD\big((y_1,y'_1),\ldots,(y_n,y'_n)\big)=SVNLCWD\big((y'_1,y_1),\ldots,(y'_n,y_n)\big) \tag{13}$$

Theorem 2. (Monotonicity). If $d(y_i,y'_i)\geq d(x_i,x'_i)$ for all i, the following property holds

$$SVNLCWD\big((y_1,y'_1),\ldots,(y_n,y'_n)\big)\geq SVNLCWD\big((x_1,x'_1),\ldots,(x_n,x'_n)\big) \tag{14}$$

Theorem 3. (Boundedness). This feature shows that the aggregation result lies between the minimum and maximum arguments (distances) to be aggregated:

$$\min_i\big(d(y_i,y'_i)\big)\leq SVNLCWD\big((y_1,y'_1),\ldots,(y_n,y'_n)\big)\leq \max_i\big(d(y_i,y'_i)\big) \tag{15}$$

Theorem 4. (Idempotency). If $d(y_i,y'_i)=D$ for all i, then

$$SVNLCWD\big((y_1,y'_1),\ldots,(y_n,y'_n)\big)=D \tag{16}$$

Theorem 5. (Nonnegativity). In case distances are aggregated, the result of aggregation is positive:

$$SVNLCWD\big((y_1,y'_1),\ldots,(y_n,y'_n)\big)\geq 0 \tag{17}$$

Theorem 6. (Reflexivity). In case the two vectors involved in the aggregation coincide, the resulting variable is zero:

$$SVNLCWD((y_1,y_1),\ldots,(y_n,y_n))=0 \tag{18}$$

4. New MAGDM Method Using the SVNLCWD Operator

The SVNLCWD operator can be used in a wide range of environments, such as data analysis, financial investment and engineering applications [29–32]. Subsequently, a new approach was developed for MAGDM problems in SVNL situations. Suppose that $C=\{C_1,C_2,\ldots,C_m\}$ is the set of schemes, and $A=\{A_1,A_2,\ldots,A_n\}$ is a set of finite attributes.

Step 1: Let each decision-maker (DM) $e_k(k=1,2,\ldots,t)$ (whose weight is ε_k, meeting $\varepsilon_k\geq 0$ and $\sum_{k=1}^{t}\varepsilon_k=1$) provide his/her evaluation on the attributes expressed by the SVNLVs, and then form the individual matrix $Y^k=\left(y_{ij}^{(k)}\right)_{m\times n}$.

Step 2: Aggregate all evaluations of the individual DMs into a collective one, and then construct the group matrix:

$$Y=\left(y_{ij}\right)_{m\times n}=\begin{pmatrix} y_{11} & \cdots & y_{1n}\\ \vdots & \ddots & \vdots\\ y_{m1} & \cdots & y_{mn}\end{pmatrix}, \tag{19}$$

where the SVNLN $y_{ij}=\sum_{k=1}^{t}\varepsilon_k y_{ij}^{(k)}$.

Step 3: Construct the ideal levels for each attribute to establish the ideal scheme (see Table 1).

Table 1. Ideal scheme.

	$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\cdots$	$\boldsymbol{A_n}$
I	$\widetilde{y}_1$	$\widetilde{y}_2$	$\ldots$	$\widetilde{y}_n$

Step 4: Utilize the SVNLCWD to compute the distances between the ideal scheme I and the different alternatives $C_i (i = 1, 2, \ldots, m)$.

Step 5: Sort all alternatives and identify the best alternative(s) according to the results derived from Step 4.

5. An Illustrative Example: Low-Carbon Supplier Selection

We will focus on a numerical example of the low-carbon supplier selection problem provided by Chen et al. [9]. Three experts are invited to evaluate and prioritize a suitable low-carbon supplier as a manufacturer, with respect to the four potential suppliers C_i $(i = 1, 2, 3, 4)$ using the attributes: low-carbon technology (A_1), risk factor (A_2), cost (A_3) and capacity (A_4). The preference presented by the experts regarding these four attributes is formed into three individual SVNL decision matrices under the linguistic term set $S = \{s_1$ = extremely poor, s_2 = very poor, s_3 = poor, s_4 = fair, s_5 = good, s_6 = very good, s_7 = extremely good$\}$, as listed in Tables 2–4.

Table 2. SVNL decision matrix Y^1.

	$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\boldsymbol{A_3}$	$\boldsymbol{A_4}$
C_1	$\left\langle s_5^{(1)}, (0.7, 0.0, 0.1) \right\rangle$	$\left\langle s_4^{(1)}, (0.6, 0.1, 0.2) \right\rangle$	$\left\langle s_3^{(1)}, (0.3, 0.1, 0.2) \right\rangle$	$\left\langle s_6^{(1)}, (0.6, 0.1, 0.2) \right\rangle$
C_2	$\left\langle s_6^{(1)}, (0.6, 0.1, 0.2) \right\rangle$	$\left\langle s_5^{(1)}, (0.6, 0.1, 0.2) \right\rangle$	$\left\langle s_4^{(1)}, (0.5, 0.2, 0.2) \right\rangle$	$\left\langle s_3^{(1)}, (0.6, 0.2, 0.4) \right\rangle$
C_3	$\left\langle s_4^{(1)}, (0.3, 0.2, 0.3) \right\rangle$	$\left\langle s_4^{(1)}, (0.5, 0.2, 0.3) \right\rangle$	$\left\langle s_3^{(1)}, (0.5, 0.3, 0.1) \right\rangle$	$\left\langle s_5^{(1)}, (0.3, 0.5, 0.2) \right\rangle$
C_4	$\left\langle s_5^{(1)}, (0.4, 0.2, 0.3) \right\rangle$	$\left\langle s_5^{(1)}, (0.4, 0.2, 0.3) \right\rangle$	$\left\langle s_3^{(1)}, (0.3, 0.2, 0.5) \right\rangle$	$\left\langle s_4^{(1)}, (0.5, 0.3, 0.3) \right\rangle$

Table 3. SVNL decision matrix Y^2.

	$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\boldsymbol{A_3}$	$\boldsymbol{A_4}$
C_1	$\left\langle s_4^{(3)}, (0.6, 0.1, 0.2) \right\rangle$	$\left\langle s_4^{(3)}, (0.5, 0.2, 0.2) \right\rangle$	$\left\langle s_3^{(3)}, (0.4, 0.1, 0.1) \right\rangle$	$\left\langle s_5^{(3)}, (0.7, 0.2, 0.1) \right\rangle$
C_2	$\left\langle s_5^{(3)}, (0.5, 0.2, 0.3) \right\rangle$	$\left\langle s_4^{(3)}, (0.7, 0.2, 0.2) \right\rangle$	$\left\langle s_5^{(3)}, (0.7, 0.2, 0.1) \right\rangle$	$\left\langle s_6^{(3)}, (0.4, 0.6, 0.2) \right\rangle$
C_3	$\left\langle s_6^{(3)}, (0.5, 0.1, 0.3) \right\rangle$	$\left\langle s_5^{(3)}, (0.6, 0.1, 0.3) \right\rangle$	$\left\langle s_4^{(3)}, (0.6, 0.2, 0.1) \right\rangle$	$\left\langle s_4^{(3)}, (0.3, 0.6, 0.2) \right\rangle$
C_4	$\left\langle s_6^{(3)}, (0.5, 0.2, 0.3) \right\rangle$	$\left\langle s_6^{(3)}, (0.6, 0.2, 0.4) \right\rangle$	$\left\langle s_5^{(3)}, (0.2, 0.1, 0.6) \right\rangle$	$\left\langle s_4^{(3)}, (0.5, 0.2, 0.3) \right\rangle$

Table 4. SVNL decision matrix Y^3.

	$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\boldsymbol{A_3}$	$\boldsymbol{A_4}$
C_1	$\left\langle s_4^{(2)}, (0.8, 0.1, 0.2) \right\rangle$	$\left\langle s_5^{(2)}, (0.7, 0.2, 0.3) \right\rangle$	$\left\langle s_4^{(2)}, (0.4, 0.2, 0.2) \right\rangle$	$\left\langle s_6^{(2)}, (0.6, 0.3, 0.3) \right\rangle$
C_2	$\left\langle s_6^{(2)}, (0.7, 0.2, 0.3) \right\rangle$	$\left\langle s_6^{(2)}, (0.7, 0.2, 0.3) \right\rangle$	$\left\langle s_5^{(2)}, (0.6, 0.2, 0.2) \right\rangle$	$\left\langle s_4^{(2)}, (0.5, 0.4, 0.2) \right\rangle$
C_3	$\left\langle s_6^{(2)}, (0.4, 0.2, 0.4) \right\rangle$	$\left\langle s_6^{(2)}, (0.6, 0.3, 0.4) \right\rangle$	$\left\langle s_4^{(2)}, (0.6, 0.1, 0.3) \right\rangle$	$\left\langle s_5^{(2)}, (0.4, 0.4, 0.1) \right\rangle$
C_4	$\left\langle s_5^{(2)}, (0.4, 0.3, 0.4) \right\rangle$	$\left\langle s_6^{(2)}, (0.5, 0.1, 0.2) \right\rangle$	$\left\langle s_5^{(2)}, (0.3, 0.1, 0.6) \right\rangle$	$\left\langle s_3^{(2)}, (0.7, 0.1, 0.1) \right\rangle$

Assume that the weights of the experts are $\varepsilon_1 = 0.37$, $\varepsilon_2 = 0.30$ and $\varepsilon_3 = 0.33$, respectively. Then we can aggregate the individual opinion and form the group SVNL decision matrix, which is listed in Table 5.

Table 5. Group SVNL decision matrix *R*.

	A_1	A_2	A_3	A_4
C_1	$\langle s_{4.37}, (0.714, 0.000, 0.155)\rangle$	$\langle s_{4.33}, (0.611, 0.155, 0.229)\rangle$	$\langle s_{3.67}, (0.365, 0.128, 0.163)\rangle$	$\langle s_{5.70}, (0.633, 0.180, 0.186)\rangle$
C_2	$\langle s_{5.70}, (0.611, 0.155, 0.258)\rangle$	$\langle s_{4.70}, (0.666, 0.155, 0.229)\rangle$	$\langle s_{2.37}, (0.602, 0.200, 0.162)\rangle$	$\langle s_{4.23}, (0.514, 0.350, 0.258)\rangle$
C_3	$\langle s_{5.26}, (0.399, 0.163, 0.330)\rangle$	$\langle s_{4.96}, (0.566, 0.186, 0.330)\rangle$	$\langle s_{3.37}, (0.566, 0.185, 0.144)\rangle$	$\langle s_{4.70}, (0.335, 0.491, 0.159)\rangle$
C_4	$\langle s_{5.30}, (0.432, 0.229, 0.330)\rangle$	$\langle s_{5.63}, (0.450, 0.159, 0.286)\rangle$	$\langle s_{2.37}, (0.271, 0.129, 0.561)\rangle$	$\langle s_{3.67}, (0.578, 0.185, 0.209)\rangle$

According to their objectives, the experts carry out a similar analysis to determine the ideal scheme, which represents the optimal results that a supplier should have. The resulting vector (Table 6) further serves as a reference point.

Table 6. Ideal scheme.

	A_1	A_2	A_3	A_4
I	$\langle s_7, (0.9, 0, 0)\rangle$	$\langle s_7, (1, 0, 0.1)\rangle$	$\langle s_7, (0.9, 0, 0.1)\rangle$	$\langle s_6, (0.9, 0.1, 0)\rangle$

Assume that the weight vectors of the attributes and the SVNLCWD are $\omega = (0.25, 0.40, 0.20, 0.15)^T$ and $w = (0.2, 0.15, 0.3, 0.35)^T$, respectively. Considering the available information, we can employ the developed SVNLCWD (without loss of generality, let $\delta = 0.5$) to compute the distances between the ideal scheme I and the different alternatives $C_i (i = 1, 2, 3, 4)$:

$$SVNLCWD(I, C_1) = 5.176,\ SVNLCWD(I, C_2) = 5.660,$$
$$SVNLCWD(I, C_3) = 6.544,\ SVNLCWD(I, C_4) = 6.641.$$

Note that smaller values of distances show preferable alternatives. Thus, the ranking of the alternatives through the values of $SVNLCWD(I, C_i)(i = 1, 2, 3, 4)$ yields:

$$A_1 \succ A_2 \succ A_3 \succ A_4.$$

The results show that A_1 had the smallest distance from the ideal scheme, which means it was the most desirable alternative.

To better reflect the superiority of the SVNLCWD, we used the SVNLWD and the SVNLOWAD to measure the relative performance of the ideal scheme to all alternatives. For the SVNLWD measure, we obtained:

$$SVNLWD(I, C_1) = 5.249,\ SVNLWD(I, C_2) = 5.669,$$
$$SVNLWD(I, C_3) = 6.621,\ SVNLWD(I, C_4) = 6.789.$$

For the SVNLOWAD operator, we obtained:

$$SVNLOWAD(I, C_1) = 5.103,\ SVNLOWAD(I, C_2) = 5.652,$$
$$SVNLOWAD(I, C_3) = 6.466,\ SVNLOWAD(I, C_4) = 6.492.$$

It is easy to see that the most desirable alternative was A_1 for both the SVNLWD and SVNLOWAD operators, which coincides with the results derived using the proposed SVNLCWD operator. Moreover, the comparison of the SVNLWD and SVNLOWAD operators indicates that the SVNLCWD operator was able to account for the degrees of pessimism or optimism of the attitudes of decision-makers, and the different values of importance assigned to the various criteria during the process of aggregation. Furthermore, this method has more flexibility as it can execute the selection procedure by assigning different parameter values for the operator.

6. Conclusions

In this paper, we proposed a new combined weighted distance measure for SVNLSs, i.e., the SVNL combined weighted distance operator, to overcome the drawbacks of the existing method. Given that the developed combined weighted distance measure for SVNLSs involves both the SVNL weighted average and SVNL ordered weighted models, it takes into account both the attitudes toward separate criteria, as well as toward positions in the ordered array. Moreover, the SVNLCWD operator generalizes different types of SVNL aggregation operators, such as the SVNLMaxD, the SVNLMinD, the SVNLOWAD and the step-SVNLCWD operators. Thus, it provides a further generalization of previous methods by presenting a more general model to deal with the complex environments in a more flexible and efficient manner.

The illustrative example dealt with a selection problem of a low-carbon supplier. We conducted the sensitivity analysis to verify the robustness of the results by means of the changes in the aggregation rules (implemented by switching to different aggregation operators) and the changes in the relative importance of the ordered weights and arithmetic weights. Therefore, the proposed methodology can simulate different degrees of pessimism or optimism displayed by the decision-makers and account for the relative importance imposed on the various criteria in the aggregation process.

In future research, we will propose some methodological extensions and applications of the SVNLCWD with other decision-making approaches, such as induced aggregation and moving averaging.

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