

Article

Magnetohydrodynamics Stagnation-Point Flow of a Nanofluid Past a Stretching/Shrinking Sheet with Induced Magnetic Field: A Revised Model

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Received: 1 August 2019; Accepted: 19 August 2019; Published: date



Abstract: The revised Buongiorno's nanofluid model with the effect of induced magnetic field on steady magnetohydrodynamics (MHD) stagnation-point flow of nanofluid over a stretching or shrinking sheet is investigated. The effects of zero mass flux and suction are taken into account. A similarity transformation with symmetry variables are introduced in order to alter from the governing nonlinear partial differential equations into a nonlinear ordinary differential equations. These governing equations are numerically solved using the *bvp4c* function in Matlab solver, a very adequate finite difference method. The influences of considered parameters (Pr , M , χ , Le , Nb , Nt , S , and λ) on velocity, induced magnetic, temperature, and concentration profiles together with the reduced skin friction and heat transfer rate are discussed. Results from these criterion exposed the existence of dual solutions when magnetic field and suction are applied for a specific range of λ . The stability of the solutions obtained is carried out by performing a stability analysis.

Keywords: boundary layer; dual solutions; stagnation-point flow; nanoparticles; stability analysis

1. Introduction

In preceding decades, a very active topic of studies continues to be the study of fluid flow and heat transfer past a stretching/shrinking surface. Although a large sum of works have made remarkable contributions to the development of the theory, a proportionally good amount of efforts have also been dedicated to daily engineering applications that include electronic equipment, thermal energy storage systems, glass-fiber manufacturing, wire drawing, paper milling, and extraction of polymer sheets (see Fisher[1]). As a matter of fact, stretching confers a unidirectional orientation to the extrudate, thereby improving its mechanical properties as the features of the end product are substantially influenced by the rate of cooling. Correspondingly, it is crucial to meticulously control the fluid and heat transfer mechanism as the desired quality of the final product relies heavily on the cooling procedure. Following the famous works of Tsou et al. [2], Sakiadis [3], and that of Crane [4] in spearheading the stretching sheet problem with boundary layer approximation, several similar studies have arose, emphasizing on the shrinking sheets aspect (see Miklavčič and Wang [5] and Wang [6]). The general deduction in previously reported manuscripts suggested that the shrinking/stretching is linearly proportional to the axial distance. The solution approach somehow leaves behind uncertainties pertaining the uniqueness of the solution.

The concept of magnetohydrodynamics (MHD), which analyses the performance of magnetically induced nanofluids in various boundary-layer flow control systems can be integral in the theoretical

and experimental studies of fluid flow and heat transfer. Several instances where MHD concept is fundamentally applicable encompass nuclear reactor engineering, plasma studies, geothermal energy harvesting, polymer development, metallurgy, and MHD heat/mass transfer systems of which the details can be found in the books by [7–9], and in the classical papers such as [10–13]. Gul et al. [14] has stressed that the polarization is very powerful and the process persists even when the magnetic flux was eliminated entirely. Stimulus in the research of MHD stems contribute to diverse profitable applications in oil and gas sectors, solving geophysical problems, and agricultural engineering. In addition, the effects of magnetic flux on electrically conducting liquid consequently influences the efficiency of industrial equipment like bearings, MHD generators, and pumps. The versatility of MHD concept also means that it can be implemented in medical sector such as in tumors ablation, wound treatment, gastric medications, and sanitization of apparatus [15–17]. Tashtoush and Magableh [18] studied the blood flow in multi-stenosis arteries and discovered that the introduction of magnetic flux could alter the flow patterns. Mukhopadhyay [19] investigated MHD boundary layer flow in the presence of thermal stratification and concluded that the higher magnetic parameter suppressed the velocity field and can enhance the skin friction coefficient over some period of time. A similar output was acquired by Tian et al. [20], who explored the effects of radiation optical properties and Lorentz force on MHD boundary layer flow over a stretching plate. They explained that the magnetic force repressed the fluid flow to a significant magnitude, hence, the convective heat transfer was eliminated effectively. An equally notable paper by Ali et al. [21] presented a study of dual solutions in MHD flow on a nonlinear porous shrinking sheet in a viscous fluid, while another distinct paper by Jusoh et al. [22] emphasized the MHD rotating flow and permeable stretching/shrinking sheet. On the other hand, it should be mentioned that in these papers, Ali et al. [23,24] has studied the effect of induced magnetic field on the MHD flow of an incompressible, viscous, and electrically conducting fluid over horizontal and vertical flat plates.

Nanotechnology has been broadly applied in varying industries since nanomaterials possess distinctive physicochemical properties. It is regarded by many as one of the prominent forces that catalyze the forth industrial revolution, especially in this century. It appears that Choi and Eastman [25] are probably the first authors to describe the term nanofluid, referring to the colloid-containing suspended nanoparticles with a length scale on the order of 1–100 nm which specializes at manipulating the structure of the matter at a molecular level. For applications in thermal engineering, heat conductivity can be powered up by mixing a minute quantity of nanoparticles to a fluid. Several experimental studies [26–30] have shown that the enhancement is greater than that computed from the conventional models for composites, such as the Maxwell-Garnett (MG) model. However, the mechanism that contributes to the improvement is not thoroughly perceived, henceforth, a large number of analytical and numerical investigations have been put into motion to better comprehend the phenomena. These next-generation nanofluids can be implemented in various heat transfer applications such as automotive cooling systems, solar collector panels, and heat exchangers. Titanium oxide, aluminum oxide, and copper are among the archetypal nanoparticles used in nanofluid-centered research. Additionally, ethylene glycol, water, and mineral oil are often used as a base fluid in nanofluids formulation. For example, Bondareva et al. [31] have considered alumina-water nanofluid in their study. A detailed review on the application of nanofluids can be found in Saidur et al. [32]. References on nanofluids can be found in the book by [33–36] and in the review papers by [37–43]. These reviews discuss in detail the development of nanofluids, theoretical and experimental investigations of rheological properties (thermal conductivity and viscosity) of nanofluids, and the work done on convective transport in nanofluids. It would also be interesting to mention the study of hybrid nanofluid, which is one of the new classifications of nanofluid by Sheikholeslami et al. [44].

Over the past few decades, an impressive number of studies have demonstrated the presence of dual solutions in the boundary layer flow problems. There are few considerations that contribute to dual solutions such as a moving, stretching, or shrinking plate, mixed convection, permeable surface,

where suction is an existent and unsteady case. For those interested in engineering analysis, the idea of discovering dual solutions and defining stability is of practical significance, as it offers a way to determine if a steady state solution is physically relevant [45]. This kind of research is therefore very useful in determining the dual solutions of fluid flow problems. Here, the work of Hussaini and Lakin [46] is worth mentioning among the pioneering researchers who discovered the uniqueness of solutions in the boundary layer problems. Magyari et al. [47] has presented dual solutions in homogenous boundary layer flows caused by continuous surface stretching with rapidly decreasing power-law and exponential velocities. Subsequently, in unsteady cases, Merrill et al. [48] studied the mixed stagnation-point convection flow on a vertical surface in a fluid-saturated porous medium and discovered that there exist dual solutions for certain buoyancy parameter values. By conducting a stability analysis by Merkin [49], Merrill et al. [48] demonstrated that the solutions of the upper branch are stable and therefore asymptotically available solutions. Very recently, the effect of induced magnetic field on MHD stagnation-point flow of nanofluids towards stretching/shrinking sheet was analyzed by Junoh et al. [50]. Since the existence of dual solutions for a certain range of shrinking rate, a stability analysis has been performed and the result showed that only the upper branch solution is stable and physically acceptable.

The purpose of this present paper is to extend the work by Junoh et al. [50] by considering the presence of suction on MHD stagnation-point flow of heat and mass transfer over a permeable stretching/shrinking sheet in a nanofluid. The effect of induced magnetic field is also taken into account. In this study, the new boundary condition proposed by Kuznetsov and Nield [51] will be implemented. This work also highlights the stability of the solutions obtained. Therefore, we presume the findings are new and original, which all those interested in stretching/shrinking sheet problems in nanofluids can use with strong confidence.

2. Problem Formulation

We consider a steady two-dimensional MHD stagnation-point flow of heat and mass transfer over a permeable stretching/shrinking sheet in a nanofluid with induced magnetic field. The coordinate system is selected, in which the x -axis is in horizontal direction and the y -axis is in vertical direction. u and v represents the velocity and H_1 and H_2 are the magnetic components along the x - and y -axes, respectively. u_e is the velocity at the edge of the boundary layer while u_w and v_w are, respectively, the velocity and mass flux velocity at the surface. At the plate, $y = 0$, and the temperature T and concentration C take constant values T_w and C_w . The ambient values as $y \rightarrow \infty$, temperature and concentration, respectively, are T_∞ and C_∞ . The physical flow model and coordinate system are shown in Figure 1.

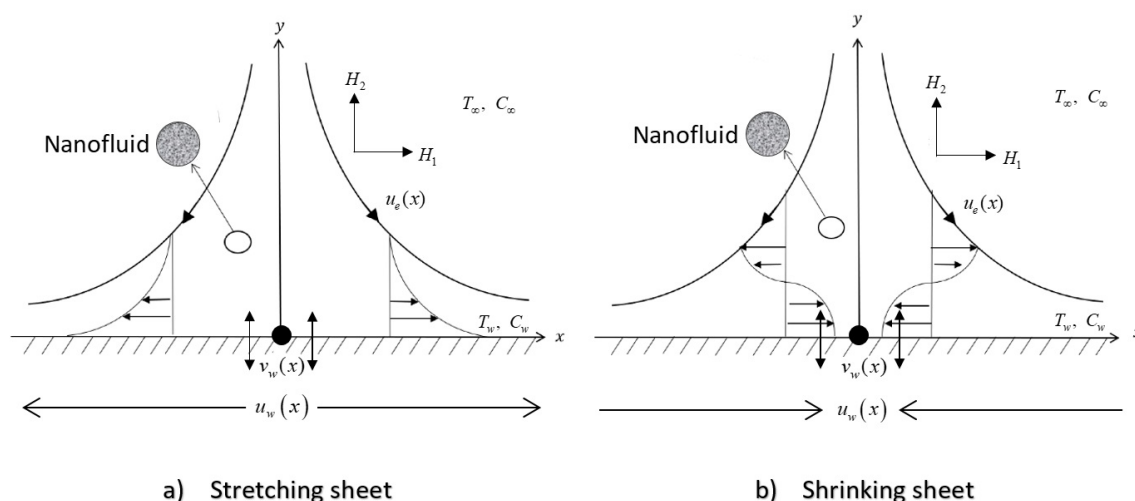


Figure 1. A sketch of physical model and coordinate system.

Following the mathematical nanofluid model proposed by Davies [52] and Kuznetsov and Nield [53], the governing equations of this problem can be derived as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0, \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + u_e \frac{du_e}{dx} + \frac{\mu}{4\pi\rho} \left(H_1 \frac{\partial H_1}{\partial x} + H_2 \frac{\partial H_1}{\partial y} \right) - \frac{\mu H_e}{4\pi\rho} \frac{dH_e}{dx}, \quad (3)$$

$$u \frac{\partial H_1}{\partial x} + v \frac{\partial H_1}{\partial y} - H_1 \frac{\partial u}{\partial x} - H_2 \frac{\partial u}{\partial y} = \mu_e \frac{\partial^2 H_1}{\partial y^2}, \quad (4)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \epsilon \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right], \quad (5)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}, \quad (6)$$

along with their boundary conditions:

$$\begin{aligned} u = \lambda u_w(x), \quad v = v_w(x), \quad \frac{\partial H_1}{\partial y} = 0, \quad H_2 = 0, \quad T = T_w, \quad D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0, \\ u = u_e \rightarrow ax, \quad H_1 = H_e(x) \rightarrow H_0(x), \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty, \end{aligned} \quad (7)$$

where $\epsilon = \frac{(\rho C_p)_p}{(\rho C_p)_f}$ is defined as the ratio of nanoparticle heat capacity to the base fluid heat capacity.

The boundary conditions $D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0$ at $y = 0$ in Equation (7) states that the normal flux of nanoparticles is zero at the boundary when the thermophoresis is considered (Kuznetsov and Nield [51]).

Following Kuznetsov and Nield [53], we introduce symmetry variables

$$\begin{aligned} \eta = \left(\frac{a}{\alpha} \right)^{1/2} y, \quad u = ax f'(\eta), \quad v = -(a\alpha)^{1/2} f(\eta), \quad H_1 = H_0 x h'(\eta), \\ H_2 = - \left(\frac{\alpha}{a} \right)^{1/2} H_0 h(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_\infty}, \end{aligned} \quad (8)$$

where primes denote differentiation with respect to η .

After substitution Equation (8) into Equations (1)–(6), we get the following reduced form:

$$Pr f''' + f f'' - f'^2 + 1 + M(h'^2 - h h'' - 1) = 0, \quad (9)$$

$$\chi h''' + f h'' - f'' h = 0, \quad (10)$$

$$\theta'' + f \theta' + Nb \phi' \theta' + Nt \theta'^2 = 0, \quad (11)$$

$$\phi'' + Le f \phi' + \frac{Nt}{Nb} \theta'' = 0, \quad (12)$$

subjected to the new boundary conditions:

$$\begin{aligned} f(0) = S, \quad f'(0) = \lambda, \quad h(0) = 0, \quad h''(0) = 0, \quad \theta(0) = 1, \quad Nb \phi' + Nt \theta' = 0, \\ f'(\infty) \rightarrow 1, \quad h'(\infty) \rightarrow 1, \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0. \end{aligned} \quad (13)$$

In the above equations, Pr is the Prandtl number, M is the magnetic parameter, χ is the reciprocal magnetic parameter, Nb is the Brownian motion parameter, Nt is the thermophoresis parameter, Le is the Lewis number, S is the suction/injection parameter which is defined as

$$\begin{aligned} Pr &= \frac{\nu}{\alpha}, \quad M = \frac{\mu H_0^2}{4\pi\rho a^2}, \quad \chi = \frac{\mu_e}{\nu}, \quad Nb = \frac{\epsilon D_B C_w}{\alpha}, \\ Nt &= \frac{\epsilon D_T (T - T_\infty)}{\alpha T_\infty}, \quad Le = \frac{\alpha}{D_B}, \quad S = -\frac{v_0}{(a\alpha)^{1/2}}, \end{aligned} \quad (14)$$

and λ stands for stretching/shrinking parameter. Here, $\lambda > 0$ indicates that the plate is stretched; while when $\lambda < 0$ the plate is shrunk. It should be highlighted that the use of the variables (8) will change the position of the Pr from the energy Equation (11) to the momentum Equation (9), which is a new fact for stretching/shrinking sheet problems.

The physical quantities of interest in this study are the skin friction or shear stress coefficient C_f and the local Nusselt number Nu_x are

$$C_f = \frac{\tau_w}{\rho u_e^2}, \quad Nu_x = \frac{x q_w}{k(T_w - T_\infty)}. \quad (15)$$

Here τ_w is the surface shear stress and q_w is the surface heat flux given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}. \quad (16)$$

Then, applying the symmetry variables (8), we obtain

$$C_f Re_x^{1/2} Pr^{-1/2} = f''(0), \quad Nu_x Re_x^{-1/2} Pr^{-1/2} = -\theta'(0), \quad (17)$$

where $Re_x = u_e x / \nu$ is the local Reynolds number. However, the local Sherwood number Sh_x can be derived from the boundary conditions (13) as $\phi'(0) = -\frac{Nt}{Nb} \theta'(0)$.

3. Stability Analysis

To conduct the stability analysis of the solutions, we consider this problem in unsteady form. The continuity in Equations (1) and (2) hold, in the time Equations (3)–(6) are redressed by

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + u_e \frac{du_e}{dx} + \frac{\mu}{4\pi\rho} \left(H_1 \frac{\partial H_1}{\partial x} + H_1 \frac{\partial H_2}{\partial y} \right) - \frac{\mu H_e}{4\pi\rho} \frac{dH_e}{dx}, \quad (18)$$

$$\frac{\partial H_1}{\partial t} + u \frac{\partial H_1}{\partial x} + v \frac{\partial H_1}{\partial y} - H_1 \frac{\partial u}{\partial x} - H_2 \frac{\partial u}{\partial y} = \mu_e \frac{\partial^2 H_1}{\partial y^2}, \quad (19)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \epsilon \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right], \quad (20)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}, \quad (21)$$

where t denotes the time. The following new dimensionless variables are introduced:

$$\begin{aligned} \eta &= \left(\frac{a}{\alpha} \right)^{1/2} y, \quad \tau = at, \quad u = ax f'(\eta, \tau), \quad v = -(a\alpha)^{1/2} f(\eta, \tau), \quad H_1 = H_0 x h'(\eta, \tau), \\ H_2 &= -\left(\frac{\alpha}{a} \right)^{1/2} H_0 h(\eta, \tau), \quad \theta(\eta, \tau) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta, \tau) = \frac{C - C_\infty}{C_\infty}, \end{aligned} \quad (22)$$

thus, Equations (18)–(21) can be composed as

$$Pr \frac{\partial^3 f}{\partial \eta^3} - \left(\frac{\partial f}{\partial \eta} \right)^2 + f \frac{\partial^2 f}{\partial \eta^2} + 1 + M \left[\left(\frac{\partial h}{\partial \eta} \right)^2 - h \left(\frac{\partial^2 h}{\partial \eta^2} \right) - 1 \right] - \frac{\partial^2 f}{\partial \eta \partial \tau} = 0, \quad (23)$$

$$\chi \frac{\partial^3 h}{\partial \eta^3} + f \frac{\partial^2 h}{\partial \eta^2} - h \frac{\partial^2 f}{\partial \eta^2} - \frac{\partial^2 h}{\partial \eta \partial \tau} = 0, \quad (24)$$

$$\frac{\partial^2 \theta}{\partial \eta^2} + f \frac{\partial \theta}{\partial \eta} + Nb \frac{\partial \phi}{\partial \eta} \frac{\partial \theta}{\partial \eta} + Nt \left(\frac{\partial \theta}{\partial \eta} \right)^2 - \frac{\partial \theta}{\partial \tau} = 0, \quad (25)$$

$$\frac{\partial^2 \phi}{\partial \eta^2} + Le f \frac{\partial \theta}{\partial \eta} + \frac{Nt}{Nb} \frac{\partial^2 \phi}{\partial \eta^2} - \frac{\partial \phi}{\partial \tau} = 0, \quad (26)$$

along with the boundary conditions:

$$\begin{aligned} f(0, \tau) = S, \quad \frac{\partial f}{\partial \eta}(0, \tau) = \lambda, \quad h(0, \tau) = 0, \quad \frac{\partial^2 h}{\partial \eta^2}(0, \tau) = 0, \quad \theta(0, \tau) = 1, \\ Nb \frac{\partial \phi}{\partial \eta}(0, \tau) + Nt \frac{\partial \theta}{\partial \eta}(0, \tau) = 0, \\ \frac{\partial f}{\partial \eta}(\eta, \tau) \rightarrow 1, \quad \frac{\partial h}{\partial \eta}(\eta, \tau) \rightarrow 0, \quad \theta(\eta, \tau) \rightarrow 0, \quad \phi(\eta, \tau) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty. \end{aligned} \quad (27)$$

To check the stability of the solution $f(\eta) = f_0(\eta)$, $h(\eta) = h_0(\eta)$, $\theta(\eta) = \theta_0(\eta)$, and $\phi(\eta) = \phi_0(\eta)$ attaining the boundary value problem (9)–(12), we write

$$\begin{aligned} f(\eta, \tau) = f_0(\eta) + e^{-\gamma \tau} F(\eta, \tau), \quad h(\eta, \tau) = h_0(\eta) + e^{-\gamma \tau} H(\eta, \tau), \\ \theta(\eta, \tau) = \theta_0(\eta) + e^{-\gamma \tau} T(\eta, \tau), \quad \phi(\eta, \tau) = \phi_0(\eta) + e^{-\gamma \tau} P(\eta, \tau), \end{aligned} \quad (28)$$

where $F(\eta, \tau)$, $H(\eta, \tau)$, $T(\eta, \tau)$, and $P(\eta, \tau)$ are small relative to $f_0(\eta)$, $h_0(\eta)$, $\theta_0(\eta)$, $\phi_0(\eta)$, respectively, and γ is an unknown eigenvalue. Introducing (28) into Equations (23)–(27), we get the following linearized problem:

$$Pr \frac{\partial^3 F}{\partial \eta^3} + f_0 \frac{\partial^2 F}{\partial \eta^2} - (2f'_0 - \gamma) \frac{\partial F}{\partial \eta} + f_0'' F + M \left[2h'_0 \frac{\partial H}{\partial \eta} - h_0 \frac{\partial^2 H}{\partial \eta^2} - h_0'' H \right] - \frac{\partial^2 F}{\partial \eta \partial \tau} = 0, \quad (29)$$

$$\chi \frac{\partial^3 H}{\partial \eta^3} + f_0 \frac{\partial^2 H}{\partial \eta^2} - h_0 \frac{\partial^2 F}{\partial \eta^2} + h_0'' F - f_0'' H + \gamma \frac{\partial H}{\partial \eta} - \frac{\partial^2 H}{\partial \eta \partial \tau} = 0, \quad (30)$$

$$\frac{\partial^2 T}{\partial \eta^2} + f_0 \frac{\partial T}{\partial \eta} + \theta'_0 F + Nt \left(2\theta'_0 \frac{\partial T}{\partial \eta} \right) + Nb \left(\phi'_0 \frac{\partial T}{\partial \eta} + \theta'_0 \frac{\partial P}{\partial \eta} \right) + \gamma T - \frac{\partial T}{\partial \tau} = 0, \quad (31)$$

$$\frac{\partial^2 P}{\partial \eta^2} + Le \left(f_0 \frac{\partial P}{\partial \eta} + \phi'_0 F \right) + \frac{Nt}{Nb} \frac{\partial^2 T}{\partial \eta^2} + \gamma P - \frac{\partial P}{\partial \tau} = 0, \quad (32)$$

the boundary conditions in Equation (27) are now reduced to

$$\begin{aligned} F(0, \tau) = 0, \quad \frac{\partial F}{\partial \eta}(0, \tau) = 0, \quad H(0, \tau) = 0, \quad \frac{\partial^2 H}{\partial \eta^2}(0, \tau) = 0, \quad T(0, \tau) = 0, \\ Nb \frac{\partial P}{\partial \eta}(0, \tau) + Nt \frac{\partial T}{\partial \eta}(0, \tau) = 0, \\ \frac{\partial F}{\partial \eta}(\eta, \tau) \rightarrow 0, \quad H(\eta, \tau) \rightarrow 0, \quad T(\eta, \tau) \rightarrow 0, \quad P(\eta, \tau) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty. \end{aligned} \quad (33)$$

The solutions $f(\eta) = f_0(\eta)$, $h(\eta) = h_0(\eta)$, and $\theta(\eta) = \theta_0(\eta)$ of the steady flow Equations (9)–(13) are obtained by setting $\tau = 0$. Accordingly, $F(\eta) = F_0(\eta)$, $H(\eta) = H_0(\eta)$, and $T(\eta) = T_0(\eta)$ in Equations (29)–(32) pinpoint the initial growth or decay of perturbation of the solution (28). In this regard, we need to work out the following linear eigenvalue problem:

$$PrF_0''' + f_0''F_0 + f_0F_0'' - (2f_0' - \gamma)F_0' + M(2h_0'H_0' - h_0H_0'' - h_0''H) = 0, \quad (34)$$

$$\chi H_0''' + f_0H_0'' + h_0''F_0 - f_0''H_0 - h_0F_0'' + \gamma H_0' = 0, \quad (35)$$

$$T_0'' + f_0T_0' + \theta_0'F_0 + Nt(2\theta_0'T_0') + Nb(\phi_0'T_0' + \theta_0'P_0') + \gamma T_0 = 0, \quad (36)$$

$$P_0'' + Le(f_0T_0' + \phi_0'F_0) + \frac{Nt}{Nb}T_0'' + \gamma P_0 = 0, \quad (37)$$

along with the boundary conditions:

$$F_0(0) = 0, \quad F_0'(0) = 0, \quad H_0(0) = 0, \quad H_0''(0) = 0, \quad T_0(0) = 0, \quad NbP_0'(0) + NtT_0'(0) = 0, \quad (38)$$

$$F_0'(\eta) \rightarrow 0, \quad H_0'(\eta) \rightarrow 0, \quad T_0(\eta) \rightarrow 0, \quad P_0(\eta) \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty.$$

The smallest eigenvalue γ_1 obtained delineates the stability of this dual solution of the steady-state flow. Consequently, as recommended by Harris et al. [54], the boundary condition $F_0'(\eta)$, $H_0'(\eta)$, $T_0(\eta)$, and $P_0(\eta)$ can be put at rest in order to demonstrate the range of possible eigenvalue. In this problem, the condition $F_0'(\eta)$ as $\eta \rightarrow \infty$ is chosen to be relaxed and, for a fixed value of γ , the following system of Equations (34)–(38) will be solved by introducing a new boundary condition which is $F_0''(0) = 1$.

4. Results and Discussion

The governing Equations (9)–(12) together with the boundary conditions (13) have been numerically-solved adopting bvp4c function in Matlab solver. It should be emphasized that the parameters used in this study are the $Pr(0.72-3)$, $M(0.0-0.5)$, $\chi(1-3)$, $Le(1-5)$, $Nb(0.01-0.05)$, $Nt(0.01-0.05)$, $S(0-3)$, and $\lambda(\leq 2)$. We fix $Pr = 0.72$, $M = 0.5$, $\chi = 1$, $Le = 1$, $Nb = 0.01$, $Nt = 0.01$, and $S = 3$ into our computation procedure, modifying one parameter at a time. We take $\eta_\infty = 10$ as our far-field boundary condition. All the profiles presented in the form of figures (Figures 2–10) satisfy the boundary condition (13) and produce asymptotic graph. A comparison of the results for some values of $f''(0)$ has been carried out with the results in the existing literature [55,56] in order to validate this study. This comparison data is shown in Table 1 and shows a positive agreement. Consequently, we are assured that the numerical values obtained in this study are veritable.

Table 1. Comparison values of the skin friction coefficient $f''(0)$ for some values of λ when $M = 0$.

λ	Present Result		Aman et al. [55]		Bhattacharyya et. al [56]	
	Upper Branch	Lower Branch	Upper Branch	Lower Branch	Upper Branch	Lower Branch
−0.25	1.40224		1.4022		1.40224051	
−0.3	1.42758		1.4276			
−0.4	1.46861		1.4686			
−0.5	1.49567		1.4957		1.49566948	
−0.615	1.50724		1.5072		1.50724089	
−0.75	1.48929		1.4893		1.48929834	
−1.0	1.32881	0	1.3288	0	1.32881689	0
−1.15	1.08223	0.11670	1.0822	0.1167	1.08223164	0.11667340
−1.18	1.00045	0.17836	1.0004	0.1784		
−1.2465	0.55430	0.55430	0.5543	0.5543	0.55428565	0.55428565

Figures 2 and 3 show the velocity profiles with the effects of M and Pr , respectively. It is seen that as the value of M and Pr increases, the boundary layer thickness of these profiles also increase. Figures 4–6 portray the impact of M , χ , and Pr on the induced magnetic profiles. As the value of these parameters increase, the boundary layer thickness become thicker. Figure 7 illustrates the influence of the Pr on temperature profile. It is noticed that the thermal boundary layer thickness increases as the Pr increase.

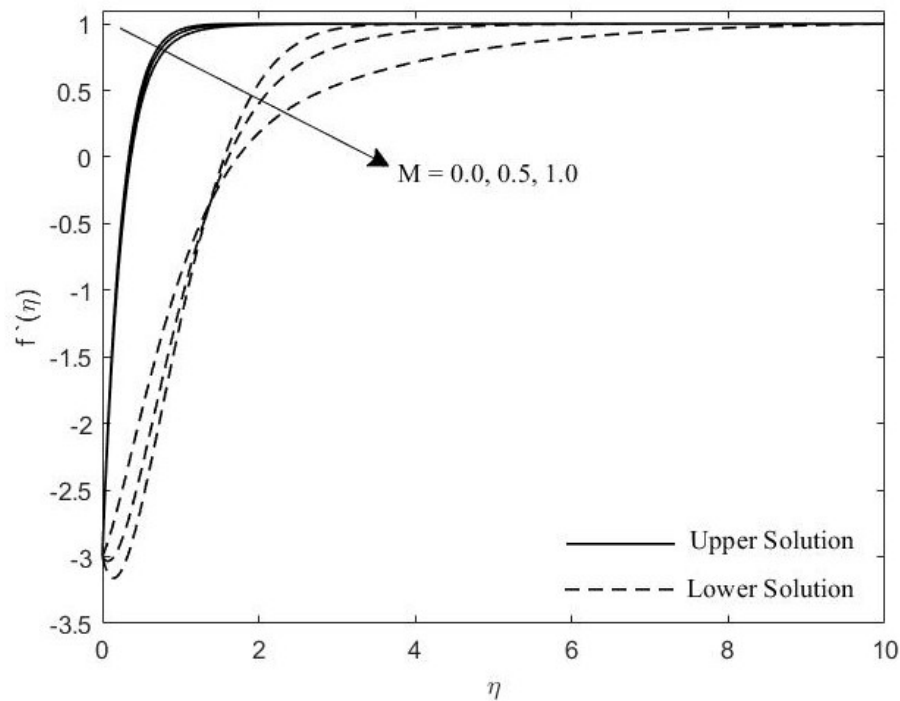


Figure 2. Velocity profile for various values of M .

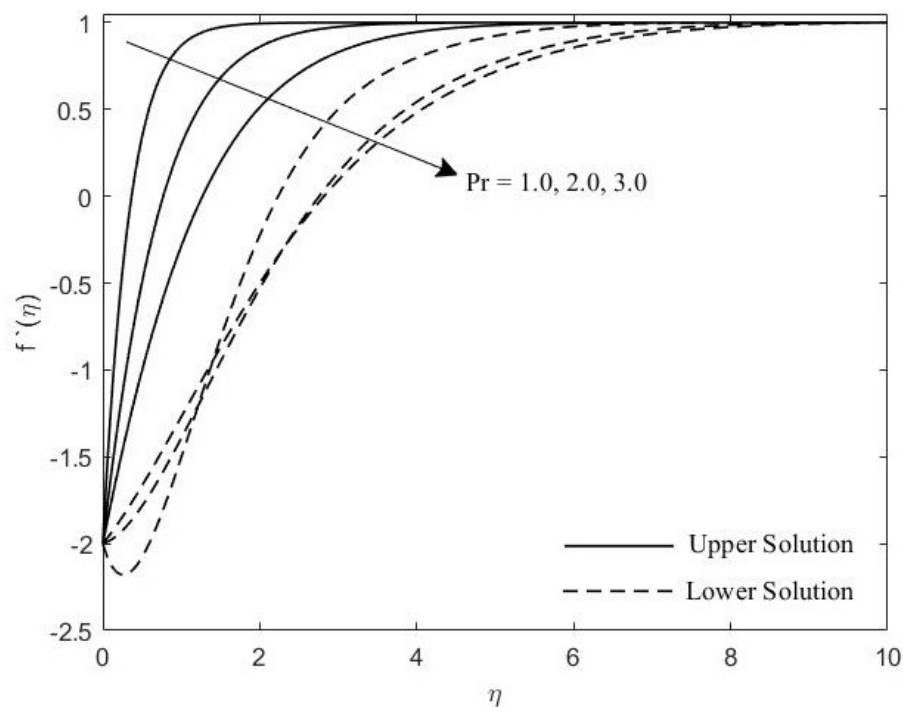


Figure 3. Velocity profile for various values of Pr .

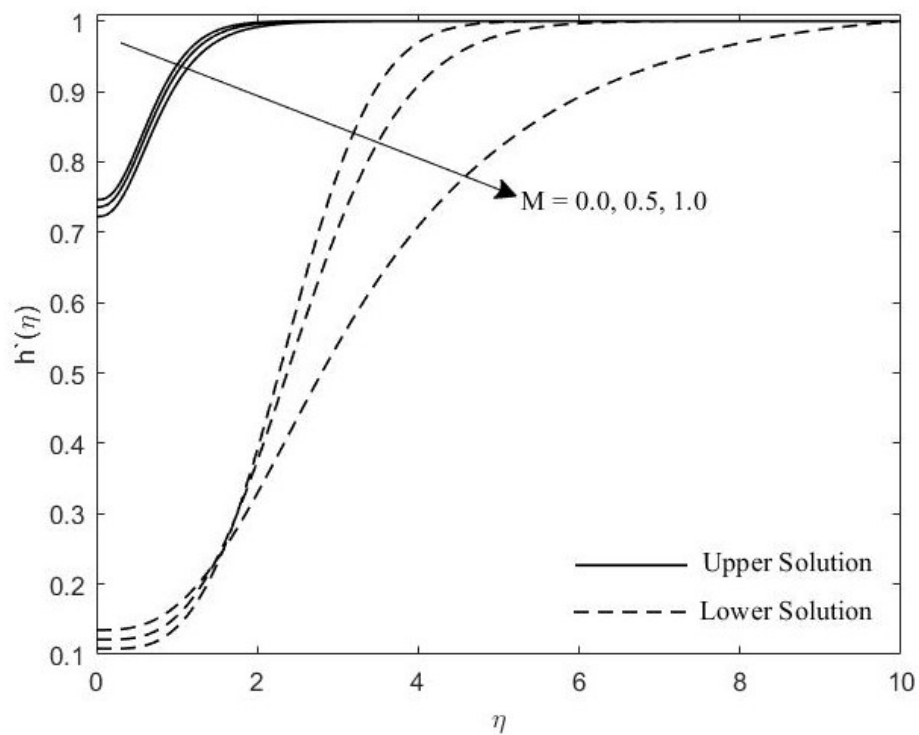


Figure 4. Induced magnetic profile for various values of M .

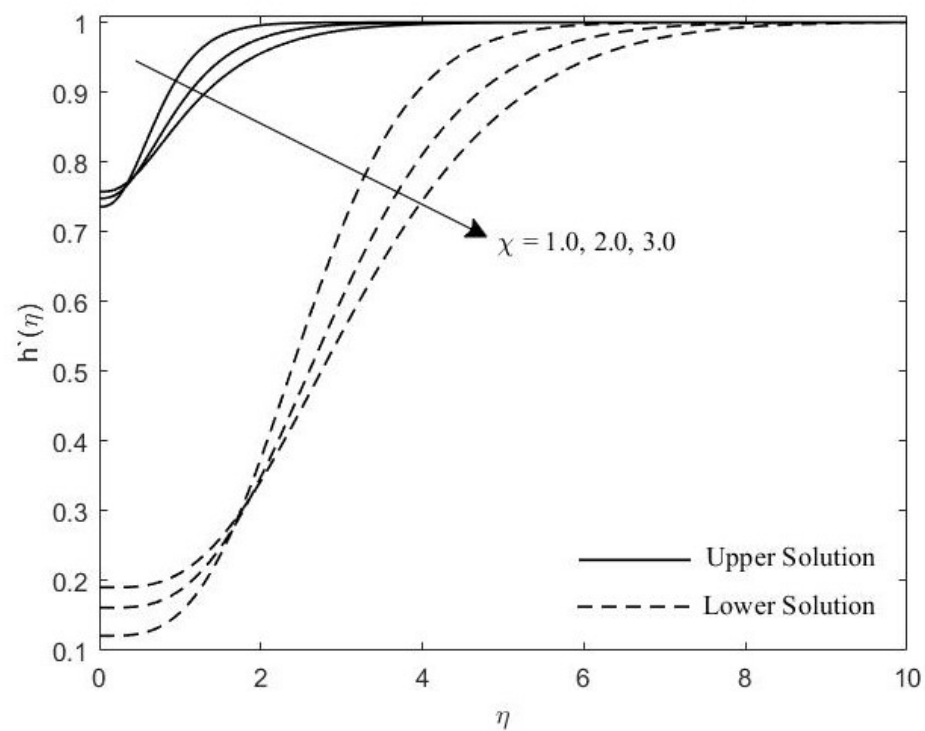


Figure 5. Induced magnetic profile for various values of χ .

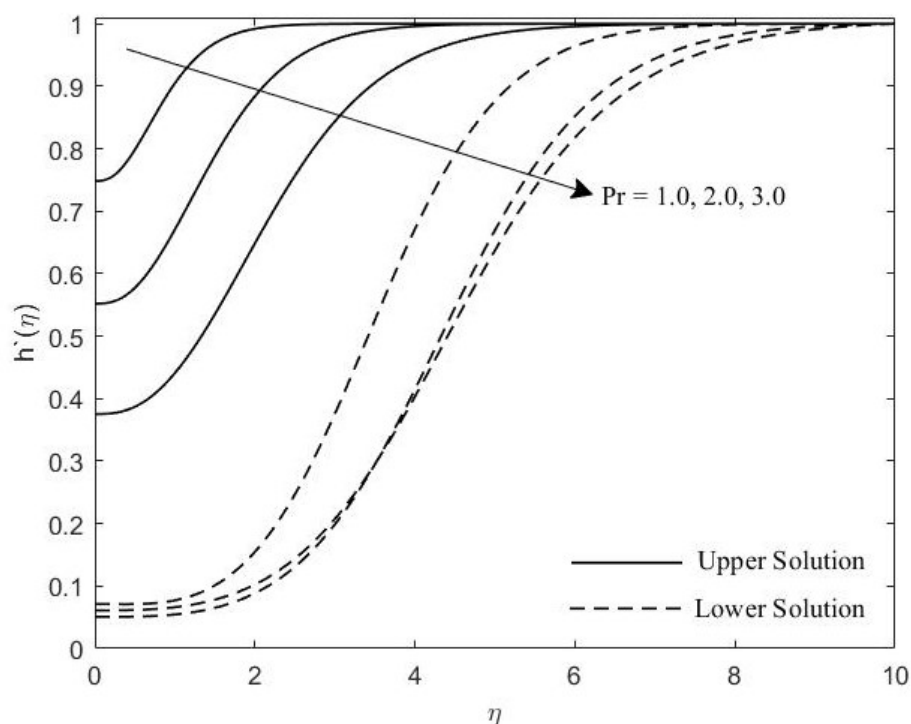


Figure 6. Induced magnetic profile for various values of Pr .

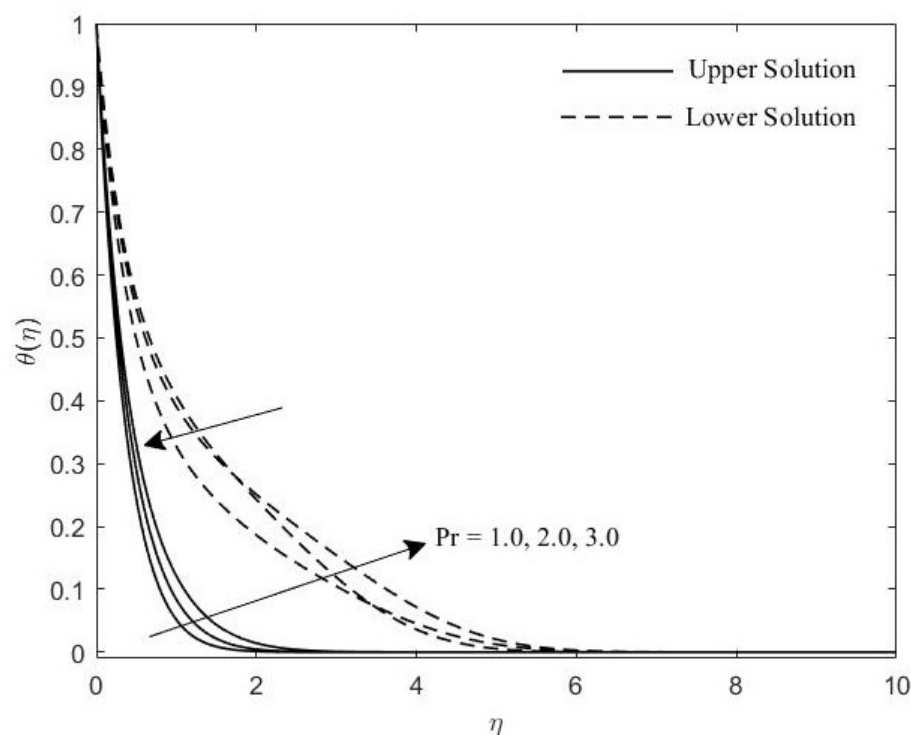


Figure 7. Temperature profile for various values of Pr .

Figures 8–10 demonstrate the concentration profile with respect to the changes of Le , Nt , and Nb , respectively. Figure 8 exhibits the influence of Le on concentration graph profile. The concentration boundary layer thickness decreases as Lewis number increases. This is likely due to the fact that mass transfer rate increases parallel to Le . Likewise divulges that the concentration gradient at the plate

surface is also increasing. As can be seen in Figure 9, increasing values of Nt leads to an increase in the thickness of concentration graph profile. A different behavior is shown in Figure 10, the concentration graph profile is decreasing when the value of Nb increasing. Thus, the concentration boundary layer also decreases. From these figures it may very well be seen that the boundary layer thickness for the upper solution is slightly slimmer contrasted with that of the lower branch solution.

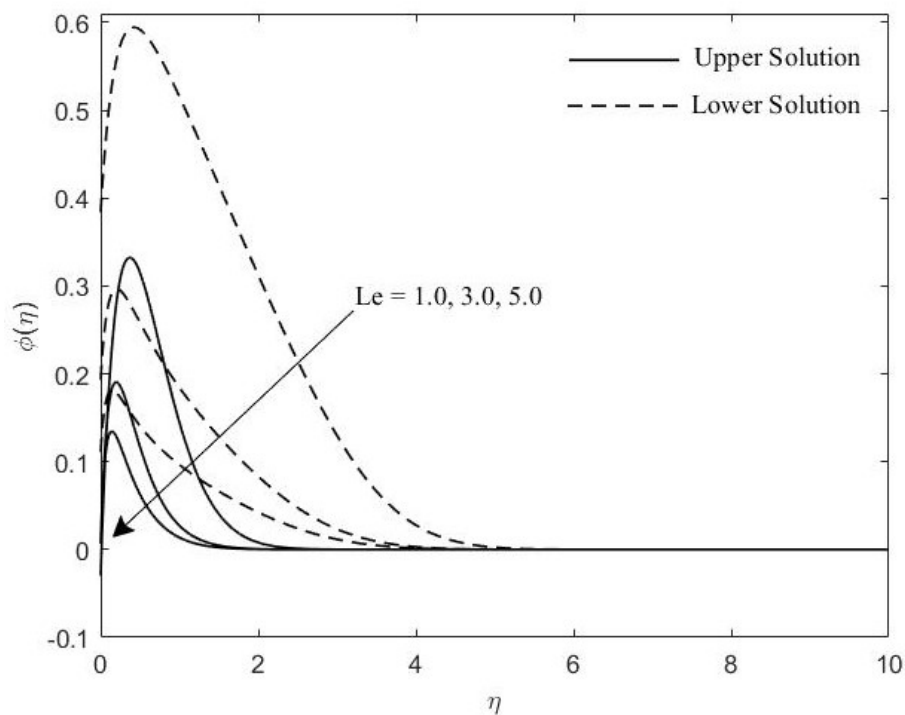


Figure 8. Concentration profile for various values of Le .

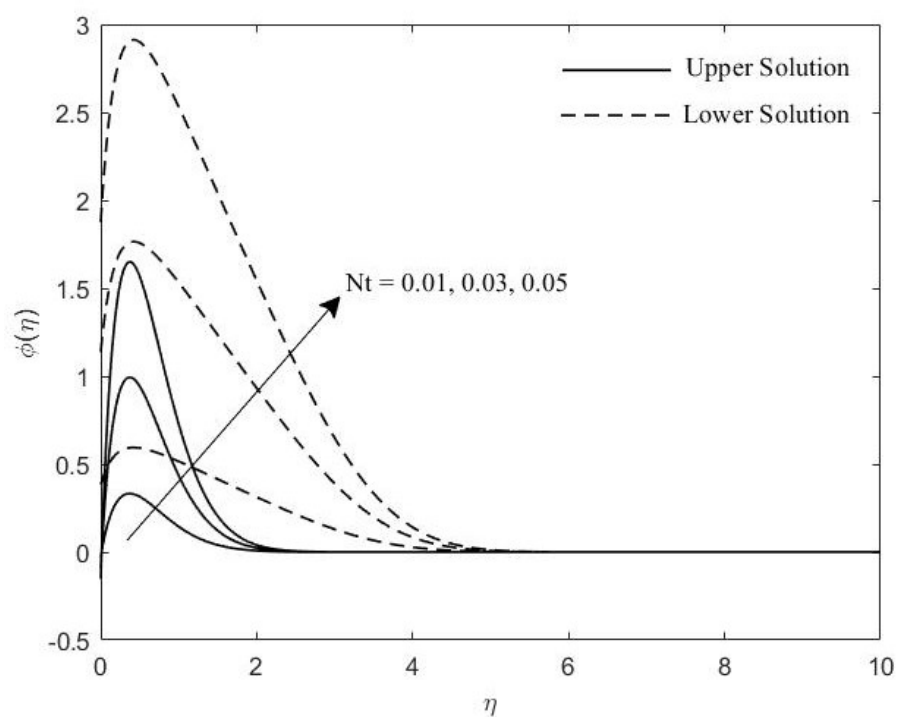


Figure 9. Concentration profile for various values of Nt .

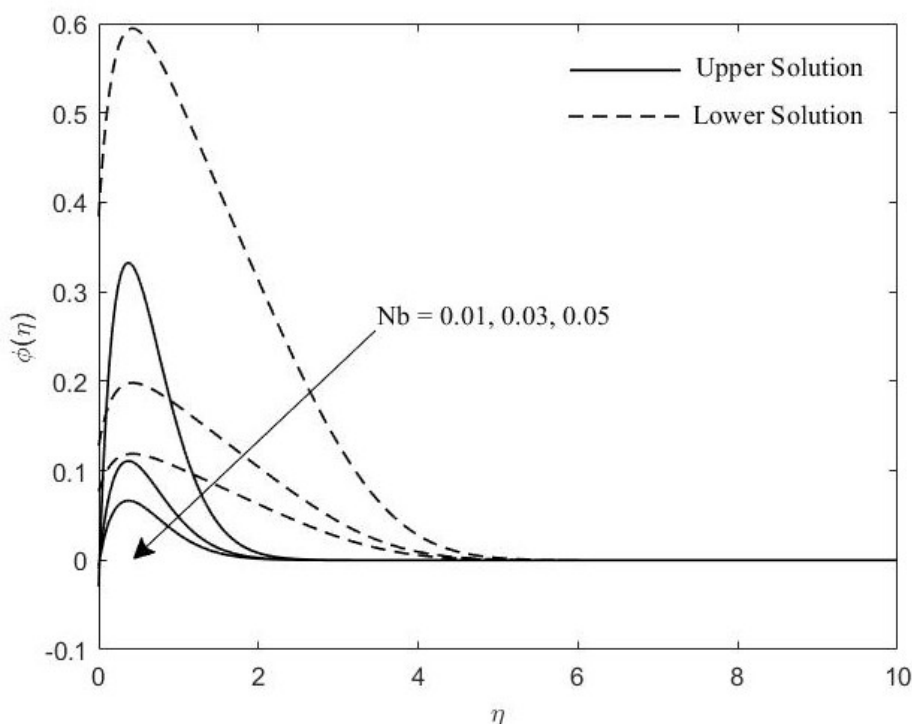


Figure 10. Concentration profile for various values of Nb .

Figures 11–14 illustrate the dual solutions for the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$. Based on the observation, dual solutions of Equations (9)–(12) can be obtained both for shrinking ($\lambda < 0$) and stretching ($\lambda > 0$) cases. Figure 11 exhibits the variation of the $f''(0)$ increasing as the value of S increases. The presence of suction will escalate the skin friction at the surface of sheet. Thereupon, the flow speed will decrease and as a result increase the velocity gradient at the surface of the sheet. The variation of the $-\theta'(0)$, as shown in Figure 12, indicates that when S increases, the heat transfer rate at the surface also increases. The heat flux becomes larger when suction is applied in the flow. Hence, it will also increase the magnitude of the temperature gradient at the surface of the sheet. Based on the critical value λ_c in these figures, the influence of suction S also causes a delaying of the boundary layer separation. From Figures 13 and 14, we can observe that as the values of M increase, the $f''(0)$ and $-\theta'(0)$ decrease. In magnetohydrodynamics problems, it is a fact that the presence of transverse magnetic field sets in Lorentz force, which exudes a retarding force on the velocity field. The existing of this force has the urgency to decelerate the fluid motion in the boundary layer. As a result, the boundary layer separation occurs faster when the magnetic field is adapted.

Based on all these figures, the existence of nonunique solutions (dual solutions) are clearly shown. The stability of these solutions are tested to analyze which solutions are stable and physically realizable in practice. It is also intended to find out which solutions are not suitable as well. Therefore, the eigenvalue problem (34)–(38) is solved for the smallest eigenvalues γ_1 regarding the upper and lower solution branches. These calculations are presented in Table 2 for some values of λ when $S = 1, 2, 3$. From the table, it can be noticed that the smallest eigenvalue γ_1 gives a positive value for the first solution and a negative value for the second solution. Hence, a conclusion is drawn that only the upper branch solutions are physically significant; whilst the lower branch solutions are not. Furthermore, as the value of λ approaches the critical point λ_c , the smallest eigenvalue γ_1 converges to 0 for both upper and lower branches, as discovered by Merkin [49].

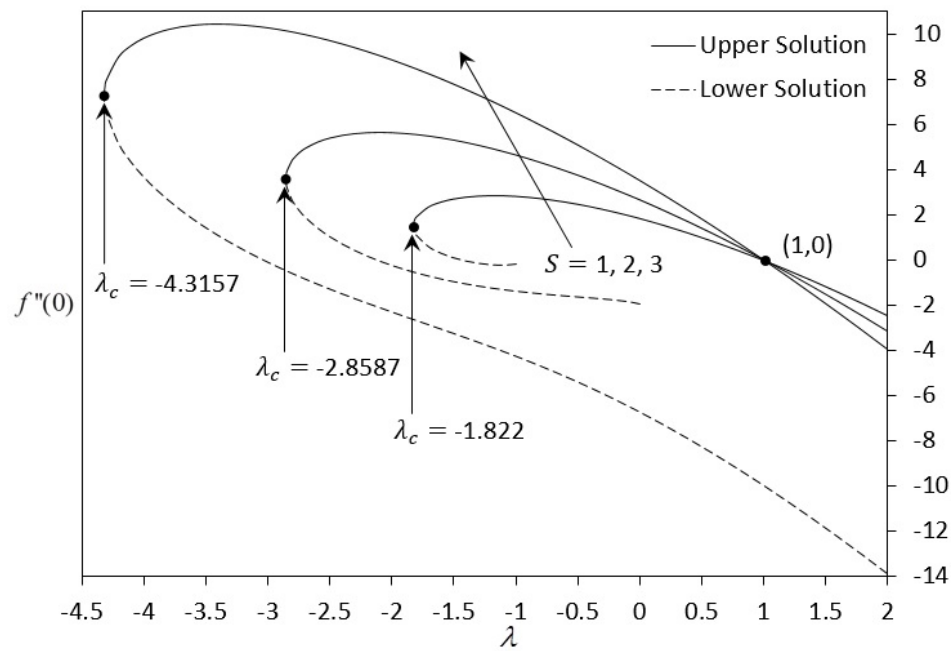


Figure 11. Variation of skin friction for various values of S .

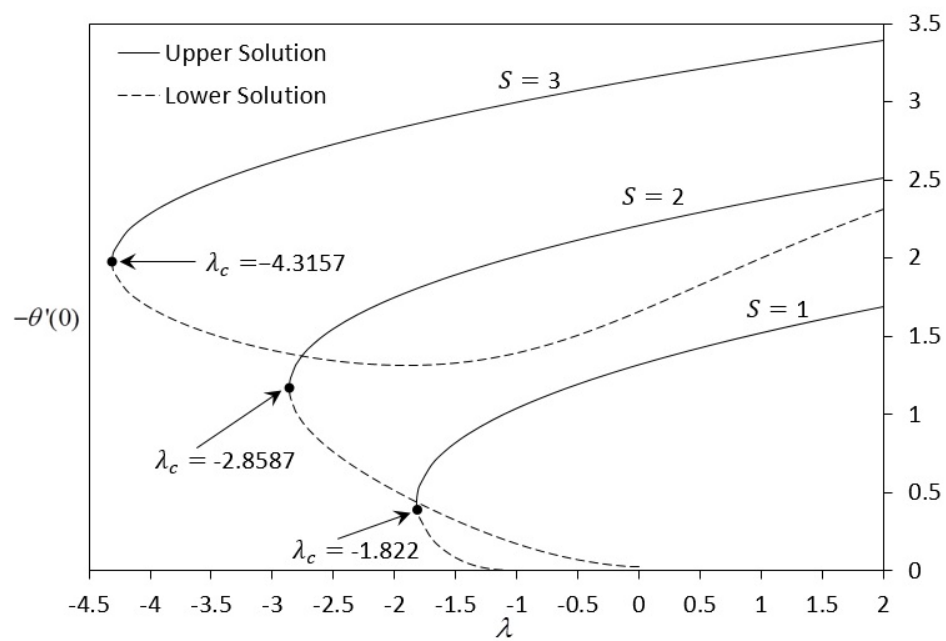


Figure 12. Variation of local Nusselt number for various values of S .

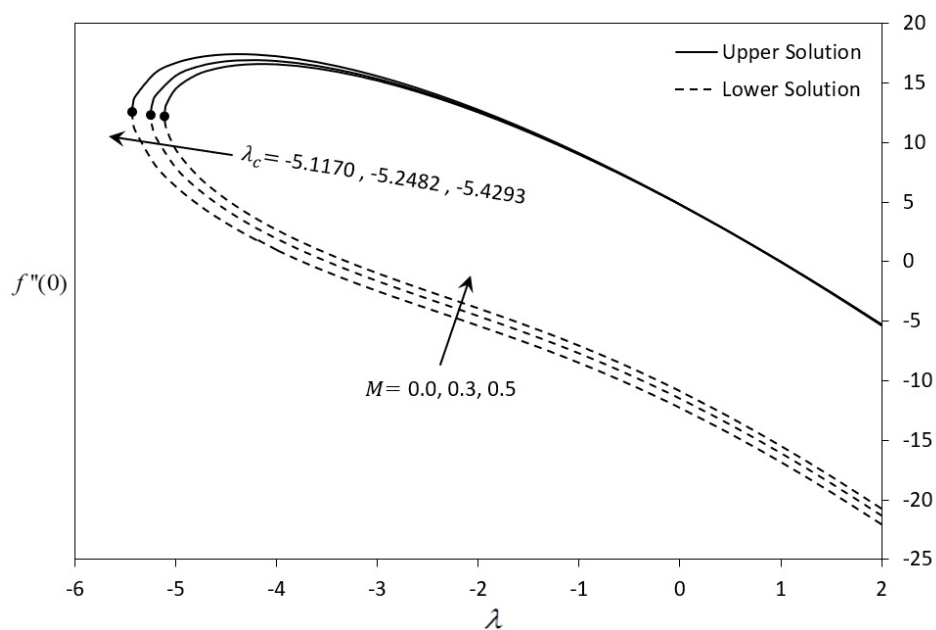


Figure 13. Variation of skin friction for various values of M .

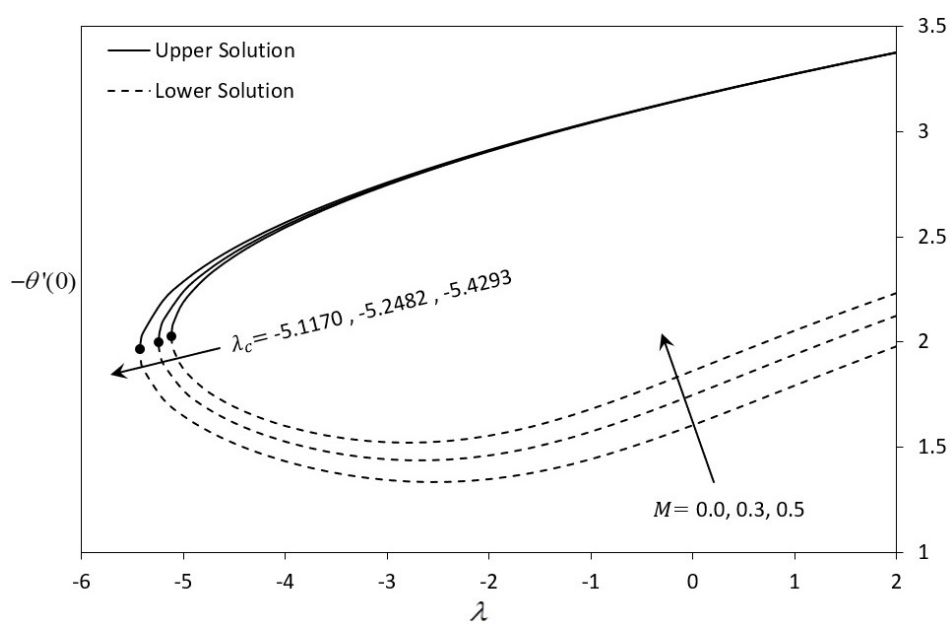


Figure 14. Variation of local Nusselt number for various values of M .

Table 2. Smallest eigenvalue γ_1 for some values of λ with different S .

S	λ	First Solution	Second Solution
1	-1.8	0.40484	-0.39610
	-1.82	0.12550	-0.12458
	-1.822	0.03052	-0.03049
2	-2.8	0.73421	-0.71159
	-2.85	0.28333	-0.27970
	-2.858	0.08134	-0.08104
3	-4.3	0.42854	-0.42058
	-4.31	0.25712	-0.25423
	-4.315	0.08807	-0.08773

5. Conclusions

A numerical study is carried out for the revised model of the steady MHD stagnation-point flow of nanofluid over a stretching or shrinking sheet with induced magnetic field. The effect of all parameters are studied numerically and graphically. The suction detains the boundary layer separation, however, the magnetic parameter enhances the boundary layer separation. Non-unique (dual) solutions are found to exist for both stretching and shrinking cases. Therefore, a stability analysis is done via *bvp4c* function in MATLAB software, and their results found that the first solution (upper branch) is stable and valid physically; while the second solution (lower branch) is not stable.

Author Contributions: M.M.J., F.M.A., and I.P. designed the research; M.M.J. constructed the mathematical model and generate the numerical results; M.M.J. and F.M.A. analyzed the results; M.M.J. and I.P. wrote the manuscript; M.M.J., F.M.A., and I.P. have read and approved this manuscript.

Acknowledgments: The authors gratefully acknowledge the financial support received in the form of a Putra Grant (Project number: GP-IPS/2018/9570000) from Universiti Putra Malaysia. The work of Ioan Pop was supported by the grant PN-III-P4-ID-PCE-2016-0036, UEFISCDI, Romania. We thank the respected reviewers for their constructive comments that clearly enhanced the quality of the manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

Roman Letters

a	constant variable
C	nanoparticle volume fraction
C_f	skin friction coefficient
C_p	specific heat capacity
D_B	Brownian diffusion coefficient
D_T	thermophoresis diffusion coefficient
$f(\eta)$	dimensionless stream function
$h(\eta)$	dimensionless induced magnetic field
$H_0(x)$	applied magnetic field
$H_e(x)$	magnetic field at the edge
H_1, H_2	induced magnetic field components along the x and y directions, respectively
k	thermal conductivity
Le	Lewis number
M	magnetic parameter
Nb	Brownian motion parameter
Nt	thermophoresis parameter
Nu_x	local Nusselt number
Pr	Prandtl number
q_w	surface heat flux
Re_x	local Reynolds number
S	suction/injection parameter
Sh_x	local Sherwood number
t	time
T	temperature of the nanofluid
$u_e(x)$	velocity at the edge of the boundary layer
u, v	velocity components along the x and y directions, respectively
v_0	constant mass velocity
x, y	Cartesian coordinates

Greek Symbols

α	thermal diffusivity of the nanofluid
γ	eigenvalue
γ_1	smallest eigenvalue
ϵ	ratio of nanoparticle heat capacity to the base fluid heat capacity
η	similarity variable
$\theta(\eta)$	dimensionless temperature
λ	stretching/shrinking parameter
μ	magnetic permeability
μ_e	magnetic diffusivity
ν	kinematic viscosity
ρ	density
τ	dimensionless time
τ_w	surface shear stress
$\phi(\eta)$	dimensionless nanoparticle volume fraction
χ	reciprocal of the magnetic Prandtl number

Subscripts

w	condition at the surface
∞	condition outside of boundary layer
c	critical value
f	base fluid
p	nanoparticle

Superscripts

$'$	differentiation with respect to η
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