Violation of the Dominant Energy Condition in Geometrodynamics

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Abstract: It is shown that in Einstein’s theory and in the theory of gravity with Logunov constraints, there is a field-theoretical model of dark energy that is consistent with the observational data indicating that the Hubble value increases over time. In the developed model of dark energy, the isotropic energy dominant condition is violated. It solves the problem of the cosmological singularity and the singularity of “black holes”. The compact configuration of the scalar field can generate a flux of particles by the pairs of particles production mechanism from the vacuum by a field of barrier and in the process of transformation of thermal energy (Hawking radiation) and acceleration energy into radiation. The scalars can play the role of the so-called “black holes” with no singularity inside themselves.

Keywords: condition of energy dominance; problem of singularity; scalar

1. Introduction

According to Wheeler, the problem of cosmological singularity is similar to the well-known atom collapse problem, which was solved by quantizing the atom. Therefore, the task of developing the quantum theory of gravity is a relevant issue [1–30]. In Reference [26], an atomic model of the quantum theory of gravity and the Big Bang was developed. It is based on quantum geometrodynamics with a non-zero Hamiltonian and on the Logunov concept of gravity asymptotically combined with the idea of a vacuum-like Gliner medium. The role of the Big Bang model is played by the Lemaître–Friedmann primordial atom in the superspace-time, where the “spatial coordinate” is the so-called scale factor of the Logunov metric of effective Riemannian space. The Lemaître–Friedmann primordial atom in the superspace-time corresponds to the local space-time structures filled with the scalar field, with a constant negative density of the potential energy.

Isotropic dominant energy condition takes the form $T_{\mu\nu}n^\mu n^\nu > 0$, where $n^\mu$ is a lightlike vector $(g_{\mu\nu}n^\mu n^\nu = 0)$, $T_{\mu\nu}$ is the energy-momentum tensor of matter [10]. This condition is the condition of the applicability of the Penrose singularity theorem in General Relativity (GR) [31]. According to the Penrose theorem, due to the dominant energy condition, the expansion of the universe started from the singular state with the infinite density and the infinite rate of expansion ($a' \rightarrow \infty$).

In this regard, within the framework of GR and the relativistic theory of gravity with Logunov constraints, we developed a theoretic field model where the isotropic dominant energy condition is violated. It automatically solves the problem of singularity that inevitably arises in the classical theory of gravity if the conditions of applicability of the Penrose theorem are satisfied.

2. Violation of the Isotropic Dominant Energy Condition in Einstein’s Geometrodynamics

Firstly, within the framework of the general theory of relativity (GTR), we examine the evolution of an isotropic and homogeneous Universe filled with a scalar field.
For the Friedman–Lemaître–Robertson–Walker metric $ds^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j$, Einstein equations for the scale factor-gravitational field $a(t)$ and the scalar field $\varphi(x, t)$ give

$$\frac{a''}{a} = -\frac{4\pi G}{3} (\varepsilon + 3p) \tag{1}$$

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi G}{3} \varepsilon - \frac{\kappa}{a^2} \tag{2}$$

$$\varphi'' + 3\frac{a''}{a} \varphi' - \frac{1}{a^2} \Delta \varphi = -\frac{dU(\varphi)}{d\varphi} \tag{3}$$

For the modern Universe, which has a high accuracy spatial flat ($\kappa = 0$), from the Friedman Equations (1) and (2), there follows the equation of the Hubble parameter evolution $H = \frac{\dot{a}}{a}$

$$H' = -4\pi G (\varepsilon + p) \tag{4}$$

Here, the nonzero components of the energy-momentum tensor of the homogeneous and isotropic matter of such a Universe are equal to $T_{00} = \varepsilon$, $T_{ij} = a^2 \gamma_{ij} p$. Then if $n^\mu = \left(1, \frac{\varphi}{\sqrt{2}}\right)$, $\gamma_{ij} n^i n^j > 0$ takes the form $\varepsilon + p > 0$. Moreover, from Equation (4), it follows that this condition is not consistent with the observational data indicating that the Hubble value increases over time. From Equations (1) and (2), the conservation law follows $d\varepsilon = -3(\varepsilon + p) \frac{\dot{a}^2}{a}$. It suggests that considering the dominant energy condition, energy density in the expanding Universe decreases over time so that the expansion of the Universe started from singularity ($\varepsilon \to \infty$, $a' \to \infty$).

In this regard, within the framework of GR, we find the solutions to the Friedman equations and the scalar field equation, where the dominant energy condition is violated. We find a solution to the system of Equations (1)–(3) for the potential energy density of a homogenous scalar field $U(\varphi(t)) = \frac{W_0}{2} \left[1 - \text{sh}^2 \left(\frac{\varphi(t)}{\sqrt{W_0}}\right)\right]$ which at $\frac{\varphi}{\sqrt{W_0}} < 1$, takes a standard form $U(\varphi) \approx \frac{m^2 \varphi^2}{2} + \frac{W_0}{2}$.

Using direct substitution, it is easy to verify that Equations (1)–(3) always have $(t \in (-\infty, \infty))$ a smooth solution

$$a = a_0 \cosh \left(\frac{\sqrt{2}}{2} H_0 t\right), \quad H_0 = \sqrt{\frac{8\pi G |W_0|}{3}} \tag{5}$$

$$\varepsilon_a = \varepsilon_\varphi = |W_0| \text{th}^2 \left(\frac{\sqrt{2} H_0 t}{2}\right), \quad \rho_a = \rho_\varphi = -|W_0|,$n

$$\varphi = i \varphi_0 \text{arcctg} \left[\text{sh} \left(\frac{\sqrt{2} H_0 t}{2}\right)\right] \tag{6}$$

where $\varphi_0 = \frac{1}{\sqrt{6m}}$, $m = \frac{3}{2} H_0$ is the effective mass of the scalar field and $W_0$ is the constant density of the potential energy of the scalar field. From Equation (6), it is obvious that the scalar field is always finite $|\varphi| \to \frac{\pi \varphi_0}{2}$ if $t \to \infty$ and $\frac{\varphi}{\sqrt{W_0}} < \frac{\pi}{2} \approx 1.57$, so that almost always $U(\varphi) \approx \frac{m^2 \varphi^2}{2} + \frac{W_0}{2}$. For Solution (6), the density of potential energy of the scalar field

$$U(\varphi(t)) = \frac{|W_0|}{2} \left[1 - \text{sh}^2 \left(\frac{\varphi(t)}{\sqrt{W_0}}\right)\right] = \frac{|W_0|}{2} \left[1 + \text{th}^2 \left(\frac{3}{2} H_0 t\right)\right]$$

so that the scalar field density and pressure

$$\varepsilon_\varphi = \frac{\varphi^2}{2} + U(\varphi) = |W_0| \text{th}^2 \left(\frac{3}{2} H_0 t\right) \tag{7}$$

$$p_\varphi = \frac{\varphi^2}{2} - U(\varphi) = -|W_0|, \quad \varepsilon_a = \varepsilon_\varphi, \quad \rho_a = \rho_\varphi \tag{8}$$

According to Relations (5)–(7) $\varepsilon_\varphi = \varepsilon_a, \quad p_\varphi = p_a$, it follows that the scalar field itself provides fluid density and the pressure of the cosmological fluid $\varepsilon_a, \quad p_a$. 
For such a solution

$$\epsilon_a + p_a = \epsilon_q + p_q = -\frac{|W_0|}{\cosh^2\left(\frac{H_0}{t}\right)} < 0, \quad H' > 0$$

This solution demonstrates that “dark energy” with positive density $\epsilon_a = \epsilon_q = |W_0|\sin^2\left(\frac{H_0}{t}\right) > 0$ and with negative constant pressure $p_a = p_q = -|W_0|$ changes the energy-dominant condition $\epsilon_a + p_a = \epsilon_q + p_q > 0$ and eliminates the cosmological singularity, since for such a cosmological medium, the Penrose theorem does not work.

The solution $\epsilon_a = |W_0|\sin^2\left(\frac{H_0}{t}\right)$ can be represented in the form of density deviation $\epsilon_a$ from its asymptotic value $W_0$:

$$\delta \epsilon = \left(\frac{a_0}{a}\right)^3 = \frac{1}{\cosh^2\left(\frac{H_0}{t}\right)}$$

where $\delta \epsilon = \frac{|W_0| - \epsilon}{\epsilon_0}$. In the modern era $|W_0| \approx 10^{72} \frac{\text{GeV}}{\text{cm}}$, when $H_0t = 1$ the relative deviation $\delta \epsilon \approx 0.2$, so that the state of dark energy in the modern era is close to a vacuum-like state $\epsilon + p \approx 0$ imitating a cosmological constant $\Lambda = 8\pi G|W_0|$. While $H_0t = 2$, the deviation $\delta \epsilon = 0.01$ is less than the error in measuring the density of dark energy $|W_0|$.

It should be noted that applying different methods provides different values of the Hubble value: $67 \frac{\text{km}}{\text{Mpc}} \leq H(t) \leq 74 \frac{\text{km}}{\text{Mpc}}$, where the left value is obtained by measuring the parameters of the radiation and the right value is found from the luminosity of Cepheids. For Solution (5), the Hubble value $H(t) = H_0\sin^2\left(\frac{H_0}{t} + a_0\right)$, so that if $H_0 = 74 \frac{\text{km}}{\text{Mpc}}$, $a_0 = 1.5$, $0 \leq H_0t \leq 1$, where theoretical values of the Hubble value correspond to the measurement data. Therefore, the reason for the discrepancy in the measurement data can be a temporary increase in the Hubble value. For all the other known modes of the scale factor evolution, such interpretation is impossible.

3. Violation of the Energy-Dominant Condition in the Geometrodynamics with Logunov Constraints

It will be shown below that the scalar field of the compact configuration (scalar) can be located in the center of a star or contain a star, a galaxy, a galaxy cluster, and the entire Universe following the value of the potential energy density of the scalar field. In the framework of the relativistic theory of gravity (RTG) with Logunov’s connections [32–35], we consider the evolution of the scalar of a universal size, and then we examine the evolution of the scalar of a stellar size.

A similar solution also exists in the relativistic theory of gravity (RTG) with Logunov constraints [32]. In the Logunov relativistic theory of gravity, constant $k$ of the homogeneous and isotropic Logunov metric [32]

$$dS^2 = N^2(d\psi)^2 - \left(\frac{a(\psi)}{a_0}\right)^2 d\tau^2, \quad d\tau^2 = \frac{dr^2}{1 - kr^2} + r^2\sin^2(\psi)dy^2 + d\delta^2$$

is determined uniquely and equal to zero, which follows from the field equations of Logunov’s relativistic theory of gravity $D_\mu \tilde{g}^{\mu\nu} = \partial_\mu \tilde{g}^{\mu\nu} + \gamma_\mu^{\alpha\beta} \tilde{g}^{\alpha\beta} = 0$, where $\tilde{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}$ $g^{\mu\nu}$ is the metric tensor of the effective Riemannian space, $g$ is the determinant of the metric tensor of the effective Riemannian space, $\gamma_\mu^{\alpha\beta}$ are the Christoffel symbols of Minkowski space where the nonzero components in the spherical coordinates $r, \psi, \phi$ are $\gamma_1^{12} = -r$, $\gamma_1^{13} = -r \sin^2(\psi)$, $\gamma_2^{13} = -\sin(\psi) \cos(\psi)$, $\gamma_2^{3} = \gamma_3^{3} = 1$, $\gamma_2^{3} = \gamma_3^{3} = \frac{a_0}{a}$, $\gamma_3^{3} = \frac{a_0}{a}$, where it follows that $N^2 = \left(\frac{a_0}{a}\right)^6 k = 0$, i.e., $g^{\mu\nu} = (g^{\alpha\beta})^2$. In the theory of gravity with Logunov’s connections, two components $g^{00}$ and $g^{\phi\phi}$ of the metric tensor are dependent. That is why, with Logunov’s connections, variation of the action leads to Equation (9).
The action of the considered model of the Universe taking into account the Logunov constraint has the form $S = \int L(a^\alpha, a, a^\alpha, \varphi, \varphi') dt$, where the Lagrange function without considering the graviton mass is determined by the expression

$$L = N \sqrt{-\frac{3M_\text{p}^2}{8\pi N^2} \left(\frac{a'}{a}\right)^2 + \frac{\phi'^2}{2N^2} - U(\varphi) + \lambda \chi(a, N)}$$

$$a^\alpha = \frac{da}{dx^\alpha}, \quad \varphi^\alpha = \frac{d\varphi}{dx^\alpha}, \quad \lambda$$ is the Lagrangian multiplier, \(\chi(a, N) = \left(\frac{a_0}{a}\right)^6 - \frac{1}{N^2}\) is the function of the Logunov constraint, \(G = M_\text{p}^2\) is the gravitational constant, and \(M_\text{p}\) is the Planck mass, \(V = \frac{4\pi a^3}{3}\).

From the action by varying metric coefficients \(a, a^\alpha, N\) and \(\varphi, \varphi', \lambda\), one can obtain Lagrange equations that determine the stationary trajectory

$$\left(\frac{a''}{a}\right) + 2 \left(\frac{a'}{a}\right)^2 - 8\pi G U(\varphi) = 0$$  \(\text{(9)}\)

$$\varepsilon_\varphi - \varepsilon_a = \frac{2\lambda}{N^2}, \quad 2\lambda = |W_0|, \quad N^2 = \left(\frac{a}{a_0}\right)^6$$  \(\text{(10)}\)

$$\phi'' + 3 \left(\frac{a'}{a}\right) \phi' = -\frac{dU(\varphi)}{d\phi}$$  \(\text{(11)}\)

In Equations (9)–(11), the Logunov constraint \(\chi(a, N) = 0\) and physical time \(dt = \left(\frac{a}{a_0}\right)^3 dx^0\) are used after variation so that in Equations (1)–(3) \(a' = \frac{da}{dt}, \varphi' = \frac{d\varphi}{dt}, \varepsilon_a = \frac{3}{8\pi G} \left(\frac{a'}{a}\right)^2, \varepsilon_\varphi = \frac{\varphi'^2}{2} + U(\varphi), p_\varphi = \frac{\varphi'^2}{2} - U(\varphi)\). However, the constraint \(\frac{1}{N^2} = \left(\frac{a_0}{a}\right)^6\) does not influence Equation (11).

It should be noted, that for the Logunov metric \(ds^2 = \left(\frac{a(t)}{a_0}\right)^6 \left(dt^0\right)^2 - \left(\frac{a(t)}{a_0}\right)^2 g_{ij} dx^i dx^j\), the absolute value of the scalar factor does not have physical meaning since the components of the metric tensor are determined by a relative dimensionless quantity \(\frac{a(t)}{a_0}\), where \(a(t)\) and \(a_0\) can have a dimension of spatial extension that is essential while developing quantum geometrodynamics. While varying the action from the Logunov equation \(D_\mu \widetilde{g}^{\mu\nu} = 0\), the conservation law follows \(D_\mu \rho^{\mu\nu} = 0\). The converse is also true. The necessary and sufficient nature of the relation of these equations implies that any of them can be used as the initial postulate of the theory. In the original approach, equations \(D_\mu \widetilde{g}^{\mu\nu} = \partial_\mu \widetilde{g}^{\mu\nu} + \gamma^{\nu}_{\mu\lambda} \widetilde{g}^{\mu\lambda} = 0\) were postulated, where \(\gamma^{\nu}_{\mu\lambda}\) are the Christoffel symbols of Minkowski space.

Logunov’s connections lead to several significant conclusions:

1. In this approach, the field equations of the theory are derived from the conditional extremum of the action with constraints \(D_\mu \widetilde{g}^{\mu\nu} = 0\) due to which in «RTG with Logunov constraints» instead of two Friedmann Equations (1) and (2), one Equation (9) arises

   $$\left(\frac{a''}{a}\right) + 2 \left(\frac{a'}{a}\right)^2 = 4\pi G (\varepsilon_\varphi - p_\varphi)$$

2. This equation is more general as any solution of Friedman equations is the solution of Equation (9). However, not every solution of Equation (9) is the solution of Equations (1) and (2). Formally, Equation (9) is a linear combination of Equations (1) and (2). For example, for a vacuum-like medium with the state equation \(\varepsilon_\varphi + p_\varphi = 0\), \(\varepsilon_\varphi = U_0\), \(p_\varphi = -U_0\), \(U_0 > 0\), the solution of Equation (9) has the form

   $$a(t) = a_0 \sqrt{3 \cosh(3H_0 t)}$$  \(\text{(12)}\)
which is not the solution of the Friedmann equations. The solution of the Friedmann equations
\(a = a_0 e^{H t}\) satisfies Equation (9).

3. Since \(\varepsilon_p - p_\varphi = U(\varphi)\), Equation (9) shows that evolution of the scaling factor is determined only
by the potential energy density, a scalar field, and does not depend on both the kinetic energy
density and the rest energy density of the quantum vacuum. Thus, the values of the kinetic energy
density and the rest energy density of the quantum vacuum \(\varepsilon_p = \frac{1}{4\pi} \int_0^M k^2 \sqrt{k^2 + m^2} dk = \frac{M^4}{16\pi}\)
do not influence Universe evolution. We estimate the vacuum energy using Einstein’s formula
\(k_0 = \sqrt{k^2 + m^2}\) that does not involve the vacuum energy in the potential form. This explains the
fact that astronomical observations and theoretical calculations demonstrate significant difference
in the value of the cosmological constant.

4. In the case of inhomogeneity of the Newtonian type \((\Delta \varphi = 0)\), the metric remains homogeneous.

System of Equations (9) and (11) when \(U(\varphi) = \frac{m^2 \varphi^2}{2} + U_0\) has a homogeneous solution [30]

\[\varphi(t) = \frac{iq_0}{\cosh\left(\frac{3}{2}H_0 t\right)}, \quad \varphi_0 = \frac{1}{\sqrt{8\pi G}}, \quad m = \frac{3}{2} H_0, \quad a = a_0 \cosh\left(\frac{3}{2}H_0 t\right),\]

for which

\[\varepsilon_a = \frac{3}{8\pi G} \left(\frac{a'}{a}\right)^2 = U_0 \text{th}^2\left(\frac{3}{2}H_0 t\right),\]

\[\varepsilon_\varphi = \frac{q'^2}{2} + U(\varphi) = U_0 - \frac{U_0}{2 \cosh^2\left(\frac{3}{2}H_0 t\right)} \left[1 + \text{th}^2\left(\frac{3}{2}H_0 t\right)\right] > 0\]

\[p_\varphi = \frac{q'^2}{2} - U(\varphi) = -U_0 + \frac{U_0}{2 \cosh^2\left(\frac{3}{2}H_0 t\right)} \left[1 - \text{th}^2\left(\frac{3}{2}H_0 t\right)\right] < 0\]

\(p_\varphi\)—pressure of the scalar field, \(\varepsilon_\varphi\)—its density. The undertaken research has shown that the
solutions \(\varepsilon_a \geq 0, \varepsilon_\varphi > 0, p_\varphi < 0, \varphi\) are always finite and \(\varepsilon_a \neq \varepsilon_\varphi\). Furthermore, \(\frac{U_0}{2} \leq \varepsilon_\varphi < U_0, \quad \frac{U_0}{2} \leq p_\varphi < -U_0\), that is if \(t = 0\) and when \(t \rightarrow \pm \infty\), the scalar field is vacuum-like \((\varepsilon_\varphi + p_\varphi = 0)\).

Next, we consider the evolution of the scalar field of the compact configuration of the stellar size.

In the case of the inhomogeneous scalar field in Equation (9), instead of the potential function
\(U(\varphi)\), there arises function \(\bar{U}(\varphi) = U(\varphi) - \frac{q'^2}{2} \Delta \varphi\). Therefore, in RTG, in case of inhomogeneity of the
Newtonian type \((\Delta \varphi = 0)\), the metric remains homogeneous as such inhomogeneity does not affect
Equation (9). On the contrary, in GR for an inhomogeneous scalar field from the second Friedmann
equation, it is clear that the metric must be inhomogeneous. It is because in GR, instead of one Equation
(9), two Friedmann Equations (1) and (2) appear so the relation must be satisfied

\[\varepsilon_a = \varepsilon_\varphi(\bar{x}, t) = \frac{q'^2}{2} + \frac{(\partial \varphi)^2}{2} + U(\varphi)\]

For Solution (12), Equation (11) has solution \(\varphi(r, \vartheta, \psi, t) = \varphi(r, \vartheta, \psi)\varphi(t)\), where \(\varphi(t) = C_1 \text{arctg} (e^{-3H_0 t}) + C_2\) and the spatial function satisfies the Laplace equation \(\Delta \varphi = 0\). We find the
integration constant \(C_1\) from the initial condition \(\varphi^2(0) = U_0\), so that \(C_1 = \frac{2 \sqrt{\pi}}{3} H_0\). The second
integration constant is expressed through the obvious final condition \(\varphi(\infty) = C_2\).

The Laplace equation \(\Delta \varphi = 0\) has a spatial inhomogeneous Newtonian-type solution \(\varphi(r) = \frac{\Phi_0}{\rho}, \quad \rho = kr\) and a homogeneous solution \(\varphi = \Phi_0 = \text{const}\). The solution combining these two particular
solutions takes the form

\[\varphi(\rho) = \frac{\Phi_0}{\rho} \int_{-\infty}^{\rho} \frac{\eta(\bar{\rho})}{|\rho - \bar{\rho}|} d\bar{\rho} = \frac{\Phi_0}{\rho} F(\rho)\]
where \( F(\rho) = \int_{-\infty}^{\rho} \frac{\rho(p)}{\sqrt{1-p^2}} dp = \begin{cases} 0, & \rho < 0 \\ 1, & 0 \leq \rho \leq 1 \\ 0, & \rho > 1 \end{cases} \). It can be seen from Formula (13), that on the interval \( 0 \leq r \leq r_0, \ r_0 = \frac{1}{k} \), the solution is asymptotically vacuum-like \( (\epsilon_0 \rightarrow U_0, p_0 \rightarrow -U_0) \). This means, in case of a collapse \( (r \rightarrow 0) \), a spatial inhomogeneous time-variable scalar field \( \Phi_0(\rho) \frac{\partial^\prime}{\partial t} \) can prevent further collapse of the scalar-sphere filled with a homogeneous scalar field. The collapse without singularity is determined by the transformation of the scalar state of such field into the asymptotically vacuum-like state \( C_2 \Phi_0 \). It leads to antigravity because \( p_\rightarrow < 0 \). In this respect, the so-called active mass of such a field is also negative

\[
\epsilon_\rightarrow + 3p_\rightarrow = -2U_0 t^2 (3H_0) < 0, \quad \epsilon_\rightarrow + 3p_\rightarrow \to -2U_0
\]

while the passive mass rapidly tends to zero \( \epsilon_\rightarrow + p_\rightarrow = \frac{U_0}{\cosh^2(3H_0 t)} \to 0 \). Besides, to solve equation \( a(t) = a_0 \cosh(3H_0 t) \), the effective density determined by the relative rate of changing the scale factor is \( \epsilon_a = \frac{3}{8 \pi} \cosh^2(3H_0 t) = U_0 t^2 (3H_0) \), \( \epsilon_a = \frac{U_0}{\cosh^2(3H_0 t)} + U_0 \), and the relative acceleration is positive \( a^\prime = 3H_0^2 [1 - \frac{2}{3} t^2 (3H_0 t)] > 0 \), such that the metric of the effective Riemannian space evolves with positive acceleration. The effective density \( \epsilon_a \) is connected with the scalar field \( \epsilon_0 \) by the untraditional relation \( \epsilon_0 = \frac{1}{3} (\epsilon_a - \epsilon_\rightarrow), \) which only asymptotically \( \epsilon_a \to \epsilon_\rightarrow \) coincides with the traditional identical relation \( \epsilon_0 = \epsilon_\rightarrow \). Thus, if \( 3H_0 t > 1 \), densities \( \epsilon_a \) and \( \epsilon_\rightarrow \) do not coincide identically but are only approximately equal \( \epsilon_\rightarrow \approx \epsilon_a \).

The nonstationarity of the scalar field allows introducing its temperature. It is easy to verify that solutions \( a(t) = a_0 \cosh(3H_0 t) \), \( \phi(t) = C_1 \arctg(e^{-3H_0 t}) + C_2 \) satisfy the equation similar to the first law of thermodynamics \( dE = -p_0 dV \), where \( E_0 = \epsilon_0 V, \quad V = 4\pi a^3, \quad d_f = \frac{d}{dt} \) being the intermediate differential for variable \( t \). Value \( \epsilon_0 \) contains the kinetic component that allows introducing the temperature of the nonstationary scalar field based on the Kelvin relation \( \frac{\partial \epsilon_0}{\partial T} = T \frac{\partial \epsilon_0}{\partial T} \), where the solution is equal to \( T(t) = \frac{T_0}{\cosh(3H_0 t)} \) and where the initial temperature value is \( T_0 = 1.4 \cdot 10^9K \).

The radius of a scalar sphere \( r_0 = \frac{1}{k} \) can be expressed in terms of a particle horizon

\[
r_0 = a_0 \int_{0}^{\tau} \frac{d\tau}{a(\tau)} = \frac{\sqrt{2}}{H_0} \left( 2F_1 \left( \frac{1}{3}, \frac{1}{6}; \frac{7}{6}; 1 \right) - \exp(-6H_0 \tau) \right) _{2F_1 \left( \frac{1}{3}, \frac{1}{6}; \frac{7}{6}; 1 \right) - \exp(-6H_0 \tau)}
\]

(13)

Here, \( 2F_1(a, b; c; x) \) is a hypergeometric function. Then if \( \tau \to \infty \), from Equation (13), we obtain

\[
r_0 = \frac{\sqrt{2}}{H_0} \left( \frac{1}{2F_1 \left( \frac{1}{3}, \frac{1}{6}; \frac{7}{6}; -1 \right) - \exp(6H_0 \tau)} \right)_0 = \frac{1}{H_0}
\]

At nuclear density \( U_0 \approx 10^{14} \frac{g}{cm^3} \), the radius of the sphere \( r_0 \approx 10 \), km, and its effective mass \( M \approx 10^{34}g \) are comparable to the parameters of “black holes" of stellar scale, although without singularity. Cosmic vacuum densities \( U_0 \approx 10^{-29} \frac{g}{cm^3} \) correspond to values \( r_0 \approx 10^{27}cm, M \approx 10^{56}g \), coinciding with the values of the observable parameters of the Universe. A vacuum-like scalar field can also exist inside the ordinary space objects (stars, galaxies, clusters of galaxies) determining their final state. For example, with the density of the scalar potential \( U_0 \approx 10^{21} \frac{g}{cm^3} \), the mass of the scalar \( M \approx 10^{31}g \), and its radius \( r_0 \approx 10^3 \) cm, so that such a Scalar may quite exist in the center of the Sun. The obtained results showed that depending on the value of the scalar potential \( U_0 \) scalars can be located both in the center of stars, as well as include stars, galaxies, clusters of galaxies, and the Universe as a whole.
As soon as the temperature of the scalar is estimated, we determine its entropy. For this purpose, we find the volume density of the quanta of the scalar field \( \tilde{c}_q \) with the continuity equation

\[
\frac{d\tilde{c}_q}{dt} = \frac{4\pi}{r_0^2} \left( \frac{1}{r_0} \right)
\]

while the total entropy of the scalar is constant and defined by the expression

\[
S_{\tilde{c}_q} = s_{\tilde{c}_q} a_0^3 = 4\pi^2 GM^2 = S_{BH}
\]

here \( a_0 = r_0 = \frac{1}{\hbar} = GM, M = \frac{1}{2\sqrt{\hbar G}}, d\tilde{t} = 4\pi \Delta t d\tilde{c}_q = 3H_0 d\tilde{c}_q, \Delta t = \left( \frac{a_0}{r_0} \right)^3 \Delta x^0, \Delta x_0 = \frac{\hbar}{2}, \quad t_0^{-1} = |U_0| V_p, \quad V_p = \frac{4\pi}{3} l_p^{3} - \text{Planck volume}, l_p = \frac{\hbar}{M c}. \quad \text{Value} \ S_{BH} = 4\pi^2 GM^2 \text{is the Beckenstein-Hawking entropy} [36–39]. For example, at nuclear density \( U_0 \approx 10^{44} \frac{g}{cm^3} \), the Scalar entropy is equal to \( S_{\tilde{c}_q} \approx 10^{80} \) and at a space vacuum density \( U_0 \approx 10^{-29} \frac{g}{cm^3} \), the scalar entropy of a cosmic scale is \( S_{\tilde{c}_q} \approx 10^{123} \).

4. Radiation of a Compact Scalar Field Configuration

In works [6,40,41], a new direction based on the existence of quantum solutions of the equations of classical physics is developed. On this basis, theoretical models of exotic atoms of Newton–Hook, Maxwell–Bagrov, Navier–Stokes, Kolmogorov–Burgers are constructed. The existence of quantum solutions of classical physics is determined by the nonstationarity of the potential and the Ehrenfest theorem. In addition, the corresponding solutions are independent of the Planck constant, instead there automatically arises its diffusion analog \( \tilde{h} = 2mD >> \hbar \). Quantum solutions of the equations of classical physics have all the attributes of quantum mechanics: quantization of the energy value, wave-particle duality, uncertainty principle, the principle of superposition, quantum interference, the causality principle both with respect to the wave function and to the rates, radiation with a discrete spectrum, tunneling, spin effects, violation of Bell’s inequalities. The synthesis of the mathematical principles of Newton’s natural philosophy and quantum physics may become the basic formalism for the second quantum revolution. The developed theoretical foundations of a new scientific direction are of interest to a wide range of researchers and can be applied in various spheres of science and technology: quantum biology, synthetic biology, medicine, quantum theory of consciousness, biological electronics, quantum computer, nature-like technologies, financial mathematics, and geometrodynamics.

In the framework of the developed approach, we investigated various processes of generating particles by the scalar field of the compact configuration of the stellar size (the scalar). Such a structure can be interpreted as a gravitational atom, in which the scalar is the nucleus, and charged test particles from the accretion disk are electrons.

For this purpose, we applied the one-dimensional differential equation of classical mechanics which describes the displacement \( \tilde{R} \equiv R(t) - r_0 \) of a test charged particle from the equilibrium position \( r_0 \) that determines the scalar radius,

\[
m \frac{d^2\tilde{R}}{dt^2} = -\frac{\partial U(\tilde{R}, t)}{\partial \tilde{R}}
\]

where \( U(\tilde{R}, t) = mH_0^2[\lambda - V(\tau)], \tau = H_0 t, H_0 = \sqrt{\frac{8\pi G m_0}{3}}, G \) is Newton’s gravitational constant, \( m_0 \) is the volume density of potential energy, \( \lambda = \frac{2G}{\sqrt{3}} \). It should be noted that the potential function can be represented in the Newtonian form \( U(\tilde{R}, t) = \frac{GM(\tilde{R}, t)}{\tilde{R}} \), where the effective mass \( M(\tilde{R}, t) = \frac{4\pi \tilde{R}^3}{3} \rho(t) \), volume energy density \( \rho(t) = 2e + 3\tilde{p}(t), \tilde{p}(t) = -\frac{U_0}{\tilde{R}^2} \). The function \( \rho(t) \) has a pressure dimension. Therefore, if function \( \tilde{p}(t) \) is interpreted as the pressure of a continuous medium, it can be considered as an analog of the active mass of Einstein’s relativistic theory of gravity. We supplement Equation (13) with the continuity equation \( \frac{dp}{dt} + 3\frac{\tilde{p}}{\tilde{R}} = 0 \). From the continuity equation, it follows that \( \rho = \frac{C_0}{\tilde{R}^3} \).

Then from Equation (13), considering \( \rho = \frac{C_0}{\tilde{R}^3} \), we can easily obtain equation

\[
\left( \frac{\tilde{R}}{\tilde{R}_0} \right)^2 = \frac{8\pi G}{3} \rho.
\]
It is easy to verify that Equation (14) can be represented in the form of a dimensionless eigenvalue equation
\[ \hat{H}_t \tilde{R}(\tau) = \lambda \tilde{R}(\tau) \]  
(15)
\[ \hat{H}_t = [\hat{\pi}_t^2 + V(\tau)] \], \( \hat{\pi}_t = -i \frac{d}{d\tau}, \lambda \) is a dimensionless value which depending on the physical problem can be related to the surface density \( \sigma \), volume energy density \( \varepsilon \), or energy \( \lambda = \frac{2\varepsilon}{m_o^2}, \quad \lambda = \frac{2\sigma}{\varepsilon_0}, \quad \lambda = \frac{2\varepsilon}{h\omega} \),
where \( m_o^2 \) is the quantum of the surface energy density, \( \varepsilon_0 \) is the quantum of the volume energy density, \( h\omega \) is the energy quantum, \( h = 2mD \) is the diffusion analog of the Planck constant, and \( D \) is the diffusion coefficient. Equation (15) has many quantum solutions since for each corresponding function \( V(\tau) \), there is its own quantum solution of Equation (15). For example, for a time-harmonious dimensionless “potential” \( V(\tau) = \tau^2 \), the solution of Equation (14) and the quantization condition have a known form
\[ \tilde{R}_n(\tau) = N \psi_n(\tau), \quad \psi_n(\tau) = e^{-\frac{\tau^2}{2}} \hat{P}n(\tau) \]
\[ \lambda_n = 2n + 1, \quad \varepsilon_n = |U_0|(n + \frac{1}{2}) \]
where \( \hat{P}n(\tau) \) is a Hermite polynomial, \( N = \frac{1}{\sqrt{2^n n! \sqrt{n}}} \) is a dimensionless integration constant which

is found from the normalization condition \[ \int_{-\infty}^{\infty} |\tilde{R}_n(\tau)|^2 d\tau = 1 \], \( \lambda = \frac{2\varepsilon_n}{|U_0|} \), such that the dimensionless function \( \tilde{R}_n(t) \) is a wave function. At the same time, the function that has a dimension of length

is its own quantum solution of Equation (15). For example, for a time-harmonious

\[ \int \tilde{R}_n(\tau)d\tau = (n + \frac{1}{2}) \int d\tilde{R}^2 \]
\[ \text{where} \quad a' = \frac{d}{d\tau} = \sqrt{2} a_{n-1} - \frac{n+1}{2} a_{n+1}, \quad \int a_n d\tau = \delta_{nk}. \]
\[ \text{In the nonrelativistic case, the average pressure} \quad \langle \tilde{p} \rangle = \int \tilde{p}(\tau) a_n^2 d\tau = -\frac{\varepsilon_n}{2} \quad \text{while in the relativistic} \]
\[ \text{case} \quad \langle \tilde{p} \rangle = \int \tilde{p}(\tau) a_n^2 d\tau = \int (\varepsilon - \frac{2K_0}{\varepsilon_0} \tau^2) a_n^2 d\tau = \frac{\varepsilon_n}{2} \int \tau^2 a_n^2 d\tau = n + \frac{1}{2}, \quad \text{such that in the nonrelativistic} \]
\[ \text{approximation, the medium being close to the scalar surface is the quintessence, and in the relativistic} \]
\[ \text{case the medium is a relativistic plasma.} \]
\[ \text{Let us consider the scalar field radiation with a compact configuration. From Equation (15), it is} \]
\[ \text{obvious that the acceleration of a charged test particle} \quad \frac{d^2\tilde{R}}{d\tau^2} = H_0^2(\tau^2 - \lambda_n) \tilde{R}_n \quad \text{is nonzero. Therefore, its} \]
\[ \text{acceleration motion should be followed by the quadrupole radiation of a basic} \quad (\Delta n = n - n' = 0), \]
\[ \text{where the quantum intensity is determined by the relation} \]
\[ I_n = \frac{2q^2}{3c^3} \langle \tilde{R}'' \rangle = \frac{q^2H_0^2}{c} \left( \left(n + \frac{1}{2} \right)^2 + \frac{1}{4} \right) \]
While considering the Sterling formula, we find that the energy of quanta of all the cells-oscillators of this group is the same while the approach. Let there be \( N \) energy cells-oscillators near the scalar surface consisting of a large set of exotic atoms. We solve this problem using the Planck study the thermal radiation (Hawking radiation) of an individual exotic atom and an exotic medium if \( q = \alpha, \xi = 1, n = 1 \), the intensity of electromagnetic radiation of the test particle has a Planck scale \( I \approx 10^{58} \frac{\text{erg}}{\text{cm}^2} \), and if \( \xi = 10^{-8} \), it coincides with the luminosity of the gamma-ray burst GRB 990123 [42]. While \( \xi = 10^{-8} \), the energy of quanta \( hI_0 \approx 10^{2} \text{eV} \).

The generation of a particle flux on the scalar surface is also possible by the pairs of particle production mechanisms from the vacuum by a barrier field. In the case of an “inverted” oscillator \( V(\tau) = -\tau^2 \), the solutions of Equation (15) are the functions of the parabolic cylinder

\[
D_r(z), D_\nu(z), \quad \nu = -\frac{1}{2} \pm i \frac{\lambda}{2}, \quad z = \pm (1 \pm i)\tau
\]

Quantum phenomena also occur in case of a potential barrier. For example, the barrier equation arises in the theory of particle production from a vacuum by an intense homogenous electric field. The functions of the parabolic cylinder conceptually coincide with the solutions leading to the Klein paradox that implies that the current of transmitted particles exceeds the current of incident particles by a potential barrier with height > \( 2m_0c^2 \). The excess current mentioned by Klein is due to the increase in the total number of particles as a result of pairs production by the barrier field. The average number of pairs of scalar particles produced by the barrier \( V(\tau) = -\tau^2 \) per unit time is as in Reference [40]:

\[
W = \frac{1}{4\pi^2 T_0} e^{-\frac{4m^2}{\tau_0}}
\]

Similarly, the probability of producing spinor particles [40] can be found. In Formula (16), instead of the Planck constant \( \hbar \) for massive particles, its diffusion analog \( \hbar = 2mD \) can be used where \( D \) is the diffusion coefficient. For example, for a particle with mass \( m = 10^{-5}\text{g} \) and \( D = 0.002\text{cm}^2/\text{sec} \), the relation \( \frac{T}{\hbar} \approx 10^{20} \). It can enhance the quantum effects of macroscopic objects. Obviously, for massless particles, the Planck constant should be used.

In conclusion, we study Hawking radiation of the scalar field of the compact configuration of the stellar size with \( r_0 \approx 10 \text{ km} \). It is known that thermal energy can turn into radiation. In this regard, we study the thermal radiation (Hawking radiation) of an individual exotic atom and an exotic medium near the scalar surface consisting of a large set of exotic atoms. We solve this problem using the Planck approach. Let there be \( N \) energy cells-oscillators \( \bar{E}_N \), \( N' \) energy cells-oscillators \( \bar{E}_{N'} \), and so on. The task is to find the distribution of energy between individual cells from the group of energy cells \( \bar{E}_N \). Let energy \( E_{N'} \) of this group of cells consist of the exact number \( r \) of quanta \( \bar{E}_0 \), so that \( r = \frac{\bar{E}_0}{\bar{E}_0} \). It is assumed that the energy of quanta of all the cells-oscillators of this group is the same while the cells-oscillators play the role of the simplest models of atoms.

The number of ways of distributing \( r \) quanta over \( N \) cells according to the formula \( N_r^m = \frac{C_{m-1}^{m+n-1}}{m+n-1} \) of combinatorial analysis is \( N_r^N = \frac{(N+r-1)!}{r!(N-1)!} \) from which according to the Sterling formula \( N_r^N \approx \frac{(N+r)^N}{N^N \rho^r} \). Determining the total Boltzmann–Planck entropy of the cells as \( S_N = k_B \ln(N_r^N) \), considering the Sterling formula, we find

\[
S_N = Ns = k_B N \left[ \left(1 + \frac{\bar{E}}{\bar{E}_0}\right) \ln \left( 1 + \frac{\bar{E}}{\bar{E}_0} \right) - \frac{\bar{E}}{\bar{E}_0} \ln \left( \frac{\bar{E}}{\bar{E}_0} \right) \right]
\]

where \( \bar{E} = \frac{\bar{E}}{\bar{E}_0} \) is energy, and \( s = \frac{S_N}{N} \) is single cell entropy, and \( k_B \) is the Boltzmann constant. With the help of thermodynamic relation \( \frac{dS}{dE} = \frac{1}{T} \), we obtain \( \bar{E} = \frac{\bar{E}_0}{\exp \left( \frac{\bar{E}_0}{k_B T} \right)} \). We account for the Rayleigh
formula for the number of natural vibrations in a unit volume per unit frequency \( N_\nu = \frac{dN}{dE_0} = \frac{8\pi^2}{c^3}, \) as the distribution of the intensity over frequencies

\[
I(\nu) = P(E)E_0N_\nu = \frac{8\pi^2E_0}{c^3} \frac{1}{\exp\left(\frac{E_0}{k_BT}\right) - 1}
\]

It is known that thermal energy can turn into radiation as the energy quantum of a self-acting electric field \( E_0 \) can be transformed into photon energy \( E_0 = h\nu. \) In this case, the obtained formula of heat radiation of an exotic atom coincides with the Planck formula describing radiation of the black body. Thermal radiation is similar to Hawking radiation.

Given the formula \( \bar{E} = \frac{E_0}{\exp\left(\frac{E_0}{k_BT}\right) - 1}, \) if \( E_0 = h\nu, \) the cell entropy \( s(x) = k_B\left[\frac{x}{e^{x-1}} - \ln(1 - e^{-x})\right], \)

where \( x = \frac{h\nu}{k_BT}. \) Then the volume density of entropy \( \bar{S} = \frac{s}{E} = k_B \int_0^\infty s(x)N_\nu dv = \frac{k_B}{\pi} \left(k_BT\right)^3 \int_0^\infty s(x)x^2 dx = \frac{4\pi^2k_B}{15}\left(k_BT\right)^3, \) where \( \int_0^\infty \frac{x^3}{e^{x-1}} dx = \frac{x^4}{15}, \int_0^\infty \ln(1 - e^{-x}) dx = \frac{x^4}{15}. \) From relation \( \bar{E} = \frac{E_0}{\exp\left(\frac{E_0}{k_BT}\right) - 1}, \) it follows that the absolute temperature \( T(\bar{E}) = \frac{hv}{k_B \ln(1 + \frac{1}{T})}, \) so that it possesses the following properties \( \lim_{E_0 \to 0} (T) = 0, \frac{dT}{dE} > 0, 0 < T < \infty. \) Entropy of a single cell

\[
s = \int_0^\infty \frac{d\bar{E}}{T(\bar{E})} = \frac{k_B}{h\nu} \int_0^\infty \ln\left(1 + \frac{h\nu}{\bar{E}}\right) d\bar{E} = k_B\left(1 + \frac{\bar{E}}{h\nu}\right) \ln\left(1 + \frac{\bar{E}}{h\nu}\right) - \frac{\bar{E}}{h\nu} \ln\left(\frac{\bar{E}}{h\nu}\right)
\]

where it follows that \( \frac{dS}{dE} = \frac{k_B}{mv} \ln\left(1 + \frac{h\nu}{E}\right) = \frac{1}{T} > 0, \frac{dE}{dS} = -\frac{1}{T^2C_V} < 0, \) and where the heat capacity of a single cell oscillator \( C_V \) coincides with the Einstein quantum heat capacity. \( C_V = k_B\left(\frac{z}{z+2}\right)^2, z = \frac{2}{3}, \)

\[
\lim_{T \to 0} C_V = k_B, \lim_{T \to 0} C_V = 0. \]

From the entropy properties, it follows that in the case of energy constancy, the entropy of the cell is maximum when all the cells have the same temperature. Thus, the entropy of a cell is a monotonous convex function of the energy of a single cell. It reaches its maximum in the state of thermodynamic equilibrium while the state of the system taken as the beginning of the energy reading scale at the same time as the state for the reference points of temperature and entropy.

5. Conclusions

The undertaken research allows us to conclude that for a particular scalar field, the isotropic energy dominant condition can be violated both within the framework of GR and within RTG with Logunov constraints. It eliminates the cosmological singularity since the Penrose theorem on the inevitability of cosmological singularity and singularity of black holes does not work. This means that the problem of cosmological singularity can be solved within the framework of classical rather than quantum geometrodynamics.

In case of collapse, the ordinary matter can turn into a certain scalar field so that inside the massive stars, there can exist spheres-scalar filled with a certain scalar field capable of stopping the collapse. There is no singularity inside the scalar that ensures the solution of a series of important problems and paradoxes connected with singularity and black holes. Scalars can play the role of the so-called “black holes.” Unlike “black holes” the scalars do not contain singularity inside themselves. Relativistic plasma or quintessence near the surface of the scalar can generate a flux of particles of various types which makes them available for observation at present. For the external observer, the “brightness” of the negative scalar decreases; however, nothing unusual happens to it due to the finiteness of its
density. Scalar always possesses a material surface. The time of reaching it for an external observer is infinite. It can be assumed that by observing the so-called “black holes,” we thereby observe the scalars, which from a theoretical point of view are more suitable for this role.

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