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Emission of Photons by Quasiparticles in Weyl Semimetals

Alexander Andrianov [1,2], Roberto Soldati [3] and Dmitri Vassilevich [4,5,*]

[1] Department of Theoretical Physics, Saint-Petersburg State University, St. Petersburg 198904, Russia; a.andrianov@spbu.ru
[2] Departament d'Estructura i Constituents de la Matèria and Institut de Ciències del Cosmos (ICCUB), Universitat de Barcelona, 08028 Barcelona, Spain
[3] Dipartimento di Fisica e Astronomia, Universitá di Bologna, Istituto Nazionale di Fisica Nucleare-Sezione di Bologna, 40127 Bologna, Italy; roberto.soldati@unibo.it
[4] Center of Mathematics, Computation and Cognition, Universidade Federal do ABC, Santo André 09210-580, SP, Brazil
[5] Physics Department, Tomsk State University, Tomsk 634050, Russia
* Correspondence: dmitry.vasilevich@ufabc.edu.br

Received: 5 April 2020; Accepted: 12 May 2020; Published: 25 May 2020

Abstract: We show that quasiparticles in Weyl semimetals may decay with emission of a single photon. We study the spectrum of emitted photons and estimate the decay rates.

Keywords: Weyl semimetals; Lorentz violating theories; decays of quasiparticles

1. Introduction

Many modern materials provide condensed matter realizations of the Dirac equation, thus hinting to the possibility of a quantum field theory description. Among these materials, a special place belongs to Weyl semimetals (see [1] for a recent review). Most of the Weyl semimetals have a constant axial vector field in the bulk, which leads to various exciting phenomena: the presence of Fermi arc states on the boundary and the chiral magnetic effect [2] as a manifestation of the chiral anomaly.

Stretching too wide the analogies to relativistic field theory may, however, be misleading. The Lorentz invariance in Weyl semimetals is violated by the presence of the axial vector and by the difference between characteristic propagation speeds for photons and quasiparticles. Thus, processes that are strictly forbidden in a relativistic physics may become possible in Weyl semimetals. We study one such process: emission of a single photon by a quasiparticle.

The purpose of this short note is to show that the effect exists and to estimates its magnitude. To achieve this purpose, we use a lot of simplifying assumptions, which include a small mass approximation and a particular initial state. As we show below, the effect is not negligible.

This paper is organized as follows. The solutions of Dirac equation are analyzed in the next section. The kinematic regions for the decay are found in Section 3, while the decay probability is calculated in Section 4.

2. Spectrum of Quasiparticles

The Dirac Lagrangian that governs free propagation of quasiparticles in Weyl semimetals can be written as [1]

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu \partial'_\mu - m - b_\mu \gamma^\mu \gamma^5\right)\psi\,. \tag{1}$$

Here and in what follows, the prime near any vector V means the rescaling of all spatial components with the Fermi velocity v_F,

$$V'_0 = V_0, \qquad V'_a = v_F V_a, \qquad a = 1,2,3.$$

The axial vector b^μ is assumed to be space-like. By a suitable choice of the coordinate system, it can be directed along the positive x^3 axis

$$b^\mu \equiv \delta^{\mu 3} b, \qquad b > 0. \tag{2}$$

The γ-matrices satisfy $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ with $g = \mathrm{diag}\,(+1,-1,-1,-1)$.

By passing to the Fourier modes $\psi \sim e^{-ip_\mu x^\mu}$, the Dirac equation is transformed into the following form

$$\left(\gamma^\mu p'_\mu - m - b\gamma^3\gamma^5\right)\psi = 0 \tag{3}$$

To solve this equation, we introduce the projectors $\mathcal{P}_\pm = \frac{1}{2}(1 \pm \gamma^0\gamma^3\gamma^5)$ and corresponding spinors $u_\pm = \mathcal{P}_\pm\psi$. The Dirac equation then reads

$$(p_0 + p'_3\gamma_5 - m\gamma_0 + b)u_+ + p'_j\gamma_0\gamma^j u_- = 0 \tag{4}$$

$$(p_0 - p'_3\gamma_5 - m\gamma_0 - b)u_- + p'_j\gamma_0\gamma^j u_+ = 0 \tag{5}$$

where $j = 1,2$. This yields

$$\left(p_0^2 - (\vec{p}')^2 - m^2 - b^2 - 2b(p'_3\gamma_5 \mp m\gamma_0)\right)u_\pm = 0\,. \tag{6}$$

Further splitting is done with the help of the following projectors

$$\mathcal{Q}^+_+ = \frac{1}{2}\left(1 + \frac{p'_3\gamma_5 - \gamma_0 m}{\sqrt{(p'_3)^2 + m^2}}\right) \tag{7}$$

$$\mathcal{Q}^-_+ = \frac{1}{2}\left(1 - \frac{p'_3\gamma_5 - \gamma_0 m}{\sqrt{(p'_3)^2 + m^2}}\right) \tag{8}$$

$$\mathcal{Q}^+_- = \frac{1}{2}\left(1 + \frac{p'_3\gamma_5 + \gamma_0 m}{\sqrt{(p'_3)^2 + m^2}}\right) \tag{9}$$

$$\mathcal{Q}^-_- = \frac{1}{2}\left(1 - \frac{p'_3\gamma_5 + \gamma_0 m}{\sqrt{(p'_3)^2 + m^2}}\right) \tag{10}$$

The square roots in the formulas above are all positive.

Let us define

$$u^+_+ = \mathcal{Q}^+_+ u_+, \quad u^-_+ = \mathcal{Q}^-_+ u_+, \quad u^+_- = \mathcal{Q}^+_- u_-, \quad u^-_- = \mathcal{Q}^-_- u_- \tag{11}$$

Then, for u^+_+ and u^+_-, the dispersion relation reads

$$p_0^2 - (\vec{p}')^2 - m^2 - b^2 - 2b\sqrt{(p'_3)^2 + m^2} = 0\,. \tag{12}$$

For u^-_+ and u^-_-, we have

$$p_0^2 - (\vec{p}')^2 - m^2 - b^2 + 2b\sqrt{(p'_3)^2 + m^2} = 0\,. \tag{13}$$

(See [3] for a comprehensive analysis of dispersion relations in Weyl semimetals.)

One can easily see that $u^{\pm}_{\pm}$ are linearly independent and thus form a basis.

In this paper, we analyze the decays of quasiparticles with the emission of a photon. Let us assume that the initial and final quasiparticles obey the same dispersion law. Let us take Equation (12) to be more specific. Let us denote the momentum of initial quasiparticle by p and of the final by q. The momentum of emitted photon is then $p - q$. We have for the 0th components of momenta

$$\sqrt{p'^2_\perp + (b + \sqrt{p'^2_3 + m^2})^2} - \sqrt{q'^2_\perp + (b + \sqrt{q'^2_3 + m^2})^2} = |\vec{p} - \vec{q}| \tag{14}$$

Let us use the inequality

$$|A - B| \geq ||A| - |B|| \tag{15}$$

valid for any vectors A and B for $A = (p'_\perp, b + \sqrt{p'^2_3 + m^2})$ and $B = (q'_\perp, b + \sqrt{q'^2_3 + m^2})$.

$$\begin{aligned}
&\sqrt{p'^2_\perp + (b + \sqrt{p'^2_3 + m^2})^2} - \sqrt{q'^2_\perp + (b + \sqrt{q'^2_3 + m^2})^2} \\
&\leq \sqrt{(p'_\perp - q'_\perp)^2 + (\sqrt{p'^2_3 + m^2} - \sqrt{q'^2_3 + m^2})^2} \\
&\leq \sqrt{(p'_\perp - q'_\perp)^2 + (p'_3 - q'_3)^2} \\
&< |\vec{p} - \vec{q}|.
\end{aligned}$$

To pass from the second line to the third, we use the same inequality applied to 2-vectors $A = (p'_3, m)$ and $B = (q'_3, m)$. The last line follows from $v_F < 1$. Thus, Equation (14) cannot be satisfied. Consequently, initial and final quasiparticles have to satisfy different dispersion relations.

Let us suppose that the mass gap parameter m is much smaller than the third components, p'_3 and q'_3, of rescaled momenta of the fermions involved in the decay process (starting with the next section, we assume that m is much smaller than other dimensional parameters as well, while no relations between the components of momenta and b are imposed). In this approximation, we write

$$\mathcal{Q}^+_+ = \frac{1}{2}\left(1 + \gamma_5 - \gamma_0 \frac{m}{p'_3}\right) \tag{16}$$

$$\mathcal{Q}^-_+ = \frac{1}{2}\left(1 - \gamma_5 + \gamma_0 \frac{m}{p'_3}\right) \tag{17}$$

$$\mathcal{Q}^+_- = \frac{1}{2}\left(1 + \gamma_5 + \gamma_0 \frac{m}{p'_3}\right) \tag{18}$$

$$\mathcal{Q}^-_- = \frac{1}{2}\left(1 - \gamma_5 - \gamma_0 \frac{m}{p'_3}\right) \tag{19}$$

Let us take a particular representation of the γ-matrices:

$$\gamma^0 = \tau_1 \otimes 1_2, \qquad \gamma^1 = i\tau_2 \otimes \sigma_2, \qquad \gamma^2 = -i\tau_2 \otimes \sigma_1, \qquad \gamma^3 = -i\tau_3 \otimes 1_2,$$

where $\{\tau\}$ and $\{\sigma\}$ are two sets of Pauli matrices. Then,

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -\tau_2 \otimes \sigma_3.$$

Up to normalization factors,

$$u^+_+(p) = \left[\begin{pmatrix} 1 \\ -i \end{pmatrix} - \frac{m}{2p'_3}\begin{pmatrix} -i \\ 1 \end{pmatrix}\right] \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{20}$$

$$u^-_+(p) = \left[\begin{pmatrix} 1 \\ i \end{pmatrix} + \frac{m}{2p'_3}\begin{pmatrix} i \\ 1 \end{pmatrix}\right] \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{21}$$

$$u^+_-(p) = \left[\begin{pmatrix} 1 \\ i \end{pmatrix} + \frac{m}{2p'_3}\begin{pmatrix} i \\ 1 \end{pmatrix}\right] \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{22}$$

$$u^-_-(p) = \left[\begin{pmatrix} 1 \\ -i \end{pmatrix} - \frac{m}{2p'_3}\begin{pmatrix} -i \\ 1 \end{pmatrix}\right] \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{23}$$

The coupling to electromagnetic field is done by replacing $\partial_\mu \to \partial_\mu - ieA_\mu$. Thus, to compute the decay amplitudes, we have to evaluate the matrix elements $\bar{u}\gamma^\mu u'$ where u is u^+_+ or u^+_- and u' is u^-_+ or u^-_-. Non-zero matrix elements read

$$\left(u^+_+\right)^\dagger(p)u^-_+(q) = \tfrac{im}{q'_3} - \tfrac{im}{p'_3}, \qquad \left(u^+_-\right)^\dagger(p)u^-_-(q) = \tfrac{im}{q'_3} - \tfrac{im}{p'_3} \tag{24}$$

$$\left(u^+_+\right)^\dagger(p)\alpha^1 u^-_-(q) = -\tfrac{m}{q'_3} + \tfrac{m}{p'_3}, \qquad \left(u^+_-\right)^\dagger(p)\alpha^1 u^-_+(q) = \tfrac{m}{q'_3} - \tfrac{m}{p'_3} \tag{25}$$

$$\left(u^+_+\right)^\dagger(p)\alpha^2 u^-_-(q) = \tfrac{im}{q'_3} - \tfrac{im}{p'_3}, \qquad \left(u^+_-\right)^\dagger(p)\alpha^2 u^-_+(q) = \tfrac{im}{q'_3} - \tfrac{im}{p'_3} \tag{26}$$

$$\left(u^+_+\right)^\dagger(p)\alpha^3 u^-_+(q) = \tfrac{im}{q'_3} + \tfrac{im}{p'_3}, \qquad \left(u^+_-\right)^\dagger(p)\alpha^3 u^-_-(q) = -\tfrac{im}{q'_3} - \tfrac{im}{p'_3}, \tag{27}$$

where $\vec{\alpha} \equiv \gamma^0\vec{\gamma}$. It is important to note that all matrix elements in the equations above are linear in m. Thus, if one is interested in the leading order of the small mass expansion only, one can compute all other quantities at $m = 0$.

The reason behind vanishing of the matrix elements in Equations (24)–(27) at $m = 0$ is the chiral symmetry of massless theory. At $m = 0$, the projectors in Equations (7)–(10) become the chirality projectors. Consequently, the matrix elements in Equations (24)–(27) become the matrix elements of electromagnetic field between the states of different chiralities. They have to vanish. Another way to arrive at the same conclusion is observe that when $m = 0$ the axial vector field b can be removed by a chiral gauge transformation. For $b = 0$, the decays of spinors with emission of a single photon are forbidden. Note that, since we do not consider any loop diagrams, the axial anomaly does not destroy the symmetry.

The modes corresponding to $u^\pm_\pm$ are not independent but rather related through the Dirac equation that has two independent solutions

$$v^+(p) = (p'_2 + ip'_1)u^+_+ - (p_0 + p'_3 + b)u^+_- \tag{28}$$

$$v^-(p) = (p'_2 + ip'_1)u^-_+ - (p_0 - p'_3 + b)u^-_- \tag{29}$$

for $m = 0$.

3. Kinematic Regions for the Decays

Let us remind that the states with dispersion relation in Equation (12) can decay into the states with the dispersion relation in Equation (13), and vice versa. Final and initial states cannot have the same dispersion relation. It is clear that with the sign convention in Equation (2) the states in Equation (12) allow for higher values of p_0 than the states in Equation (13) for the same values of spatial momenta. This energy surplus is used to create a photon. Based on these qualitative arguments (which can be confirmed by direct calculations), we conclude that the decays we are looking for is of the initial

states of the type in Equation (12) with the spinors in Equation (28) to the states in Equation (13) with the spinors in Equation (29).

Let us make a simplifying assumption that, in the initial state,

$$p_\perp = 0. \tag{30}$$

To further simplify the notations, we fix $p_3 > 0$. This does not affect the kinematic analysis since Equations (12) and (13) are not sensitive to the sign of p_3. We do not impose any restrictions on the momenta q of the final quasiparticle. As explained in the previous section, in our approximation, we may take $m = 0$ in analyzing the kinematics. The energy conservation condition yields

$$p_3' + b = \sqrt{{q_\perp'}^2 + \left(b - |q_3'|\right)^2} + \sqrt{(p_3 - q_3)^2 + q_\perp^2}. \tag{31}$$

The momentum $q_\perp$ appears under both square roots on the right hand side of the equation above. Under the first square root, $q_\perp^2$ is multiplied by v_F^2 (which is a very small quantity) and thus may be neglected as compared to $q_\perp^2$ under the second square root (by using twice the obvious inequality $\sqrt{a} \le \sqrt{a+b} \le \sqrt{a} + \sqrt{b}$ valid for any nonnegative a and b, one can easily shows that the relative error induced by neglecting $q_\perp'$ is less than v_F in the whole range of parameters). Thus,

$$p_3' + b = |b - |q_3'|| + \sqrt{(p_3 - q_3)^2 + q_\perp^2}. \tag{32}$$

This equation can be solved for $q_\perp$ if an only if

$$p_3' + b - |b - |q_3'|| \ge |p_3 - q_3|. \tag{33}$$

This inequality is easy to solve. There are no solutions for $q_3 < 0$. For $q_3 > 0$, one has to distinguish two cases:

$$b > q_3': \quad |p_3' - q_3'| < 2p_3' v_F, \tag{34}$$

$$b < q_3': \quad |p_3' - q_3'| < 2b v_F\,. \tag{35}$$

We neglect v_F^2 corrections on the right hand sides of both inequalities. Both regions are quite narrow, and the frequencies ω of emitted photons are also peaked. In the region in Equation (34), $\omega \simeq p_3' + q_3' \simeq 2p_3'.$, while in Equation (35) $\omega \simeq p_3' - q_3' + 2b \simeq 2b$.

Let us estimate the effect of a non-unit refraction index n. The second square root in Equation (31), which represents the energy of emitted photon, has to be divided by n. The initial form of this equation is recovered with the replacements $v_F \to n v_F$ and $b \to nb$. The analysis proceeds exactly as before. After returning to the original parameters, the inequalities in Equations (34) and (35) receive the factors of n on the right hand sides. Thus, the kinematic regions become n times wider.

4. Decay Rates in the Small Mass Approximation

We are interested in the decays where the initial fermion is in the state described by $v^+(p)$, while in the final state we have $v^-(q)$. Since assume that $p_\perp = 0$, we can also take $u_-^+(p)$ to describe the initial state (see Equation (28)). Relevant matrix elements of the electromagnetic field are easily computed:

$$\left(u_-^+(p)\right)^\dagger v^-(q) = -(q_0 - q_3' + b)\left(\tfrac{im}{q_3'} - \tfrac{im}{p_3'}\right) \tag{36}$$

$$\left(u_-^+(p)\right)^\dagger \alpha^1 v^-(q) = (q_2' + i q_1')\left(\tfrac{m}{q_3'} - \tfrac{m}{p_3'}\right) \tag{37}$$

$$\left(u_-^+(p)\right)^\dagger \alpha^2 v^-(q) = (q_2' + i q_1')\left(\tfrac{im}{q_3'} - \tfrac{im}{p_3'}\right) \tag{38}$$

$$\left(u_-^+(p)\right)^\dagger \alpha^3 v^-(q) = (q_0 - q_3' + b)\left(\tfrac{im}{q_3'} + \tfrac{im}{p_3'}\right) \tag{39}$$

One can check that these matrix elements satisfy the transversality condition

$$\left(u_-^+(p)\right)^\dagger \alpha^\mu v^-(q)(q'-p')_\mu = 0 \tag{40}$$

to the linear order in m.

Let us estimate the decay probabilities. All relevant formulas for normalizations, integration measures, etc. are taken from [4]. The normalized initial and final fermion states read

$$\psi_{\rm i}(p) = N_{\rm i} u_-^+(p), \qquad N_{\rm i} = 2^{-1/2} \tag{41}$$

$$\psi_{\rm f}(q) = N_{\rm f} v^-(q), \qquad N_{\rm f} = (4q_0(q_0+b-q_3'))^{-1/2} \tag{42}$$

respectively. The emitted photon may be in two polarization states given by the formulas

$$A_{(1)}(k) = N_{\rm p,1}(0, k_2, -k_1, 0), \qquad N_{\rm p,1} = (2k_\perp^2 k_0)^{-1/2} \tag{43}$$

$$A_{(2)}(k) = N_{\rm p,2}(0, k_1 k_3^2, k_2 k_3^2, -k_3(k_1^2+k_2^2)), \qquad N_{\rm p,2} = (2k_0^3 k_\perp^2 k_3^2)^{-1/2}. \tag{44}$$

Here, k denotes the 3-momentum of photon. These two states correspond to the TE and TM modes with respect to the x^3 direction.

The differential decay probability

$$dw = \frac{1}{(2\pi)^4}|\mathcal{A}|^2\delta^4(p-q-k)\frac{d^3q}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\,. \tag{45}$$

is expressed though the interaction vertex computed with normalized states

$$\mathcal{A} \equiv eA_i'(k)\psi_{\rm i}(p)^\dagger \alpha^i \psi_{\rm f}(q)\,. \tag{46}$$

To get the full decay probability, we have to integrate Equation (45) over the spatial components of k and q. The integration over k removes three of the four delta functions and enforces the spatial momentum conservation. To compute the integral over q, we write $d^3q = d^2q_\perp dq_3 = \pi dq_\perp^2 dq_3$ (where we use the rotational symmetry of integrand to integrate over the angular variable on $q_\perp$ plane). To integrate over $dq_\perp^2$, we use the remaining delta function, so that $q_\perp^2$ has to be expressed through other momenta with the help of equation

$$\sqrt{(p_3-q_3)^2+q_\perp^2} + |b-|q_3'|| = p_3'+b. \tag{47}$$

In this equation, we neglect $q_\perp'^2$ as compared to $q_\perp^2$ on the left hand side. This integration also produces a Jacobian factor $2k_0$ and enforces the integration limits for q_3 as prescribed by Equations (34) and (35).

The vertices in Equation (46) for the photons described by Equations (43) and (44) read

$$\mathcal{A}_{(1)} = eN_{\rm i}N_{\rm f}N_{\rm p,1}\frac{m(q_1'^2+q_2'^2)(q_3'-p_3')}{q_3'p_3'} \tag{48}$$

$$\mathcal{A}_{(2)} = eN_{\rm i}N_{\rm f}N_{\rm p,2}\frac{im}{v_F^2 p_3' q_3'}q_\perp'^2(q_3'-p_3')\left[(q_3'-p_3')^2+(q_3'+p_3')(q_0-q_3'+b)\right], \tag{49}$$

respectively.

Since the kinematic regions in Equations (34) and (35) are very narrow, without losing too much we may suppose that q_3' and p_3' are both larger or both smaller than b. In the region in Equation (35), this means $p_3' > b$. For simplicity, we also assume that $p_3' - b \gg v_F b$. This is a technical assumption

which ensures that the q_3' is in the kinematic region in Equation (35) and allows performing all integrations analytically. Here, we can use the following approximate relations

$$q_\perp'^2 \simeq 4b^2 v_F^2 - (q_3' - p_3')^2, \tag{50}$$

$$q_0 \simeq p_3' - b, \tag{51}$$

$$q_0 + b - q_3' \simeq \frac{q_\perp'^2}{2(p_3' - b)}. \tag{52}$$

The corrections to these formulas are of higher order in v_F. With these approximate formulas, one can derive simple analytic formulas for the total decay probabilities. For final photons described by Equation (43), we have

$$\mathcal{W}^{(1)}_{p_3'>b} = \int dw \simeq \frac{e^2 m^2 v_F^2}{8(2\pi)^9 p_3'^4} \int_{p_3-2b}^{p_3+2b} dq_3 (q_3' - p_3')^2 = \frac{2e^2 m^2 v_F^4 b^3}{3(2\pi)^9 p_3'^4}. \tag{53}$$

Similarly, for the second photon polarization in Equation (44), one obtains

$$\mathcal{W}^{(2)}_{p_3'>b} \simeq \frac{4e^2 m^2 v_F^4 b^3 (3b^2 - 10bp_3' + 15p_3'^2)}{15(2\pi)^9 p_3'^4 (p_3' - b)^2} \tag{54}$$

In the other region in Equation (34), when $p_3', q_3' < b$ and $b - p_3' \gg v_F b$, we can write

$$q_\perp'^2 \simeq 4p_3' v_F^2 - (q_3' - p_3')^2, \tag{55}$$

$$q_0 \simeq b - p_3', \tag{56}$$

$$q_0 + b - q_3' \simeq 2(b - p_3'). \tag{57}$$

The total decay probabilities become

$$\mathcal{W}^{(1)}_{p_3'<b} \simeq \frac{4e^2 m^2 v_F^6 p_3'}{15(2\pi)^9 (b - p_3')^2}. \tag{58}$$

for the polarization in Equation (43) and

$$\mathcal{W}^{(2)}_{p_3'<b} \simeq \frac{16e^2 m^2 v_F^2}{3(2\pi)^9 p_3'} \tag{59}$$

for the polarization in Equation (44), respectively.

Note that these formulas have been derived assuming that $|b - p_3'|$ is finite. The apparent singularity in Equations (58) and (54) at $b = p_3'$ signals of a crossover behavior to a regime with a different dependence on v_F.

To estimate the order of this effect, let us take $v_F = (500)^{-1}$, $m = 0.1\,\mathrm{eV}$ and $p_3' = 0.3\,\mathrm{eV}$. Then, $\mathcal{W}^{(2)}_{p_3'<b} \simeq 800\,\mathrm{s}^{-1}$. This is a small number. However, there are ways to improve this result. As explained at the end of Section 3, a non-unit refraction index of the material widens the allowed kinematic regions. This can lead to a significant effect after the integration. In addition, getting rid of the small mass approximation is going to increase the decay probability (since this means going away from the point $m = 0$ where the decay amplitudes vanish). Without the small mass approximation, analytical results are hardly possible. One would have to rely on numerical methods. We may hope to get in this way the lifetime of the order of about 10 ms. To compare, we note that this is already of the same order as the characteristic time scale of electronic cooling through interaction with phonons in Weyl and Dirac semimetals [5]. This makes the effect phenomenologically significant, especially taking into account a very specific spectrum of emitted photons that have their frequencies sharply peaked at $2p_3'$.

The other decay probabilities in Equations (53), (58), and (54) are damped by higher powers of v_F^2 and thus are less important. Unfortunately, we cannot suggest any physical explanation for the distribution of powers of v_F.

We have to stress that we have studied just a single possible relative orientation (parallel) of the initial state momentum and the axial vector b. By repeating the computations of Section 3, one can easily show that in the opposite case: when $\vec{p}$ is normal to $\vec{b}$, the kinematic region for $m = 0$ is empty. Thus, there are no decays at least in the m^2 order of the small mass expansion. Therefore, we may assume that the case of the initial momentum $\vec{p}$ parallel to $\vec{b}$ indeed represents the main effect.

The temporal component of b plays the role of a chiral chemical potential. The physics in this case is quite different to what we have considered here. One cannot however exclude interesting decays due to the b_0 component.

5. Conclusions

The main message of this work is that, in contrast to the intuition obtained through working in Lorentz invariant field theories, the quasiparticles in Weyl semimetals may decay emitting a single photon. We study this effect in a small mass approximations and demonstrated that it is small but not too small. We argue that giving up the small mass approximation and taking into account the refraction index of the bulk of Weyl semimetals may lead to a considerable enhancement of the decay probability. Besides, it is interesting to study the effects of chemical potential and of the temperature.

Author Contributions: All authors contributed equally to this work. Investigation: A.A., R.S., D.V.; Analysis: A.A., R.S., D.V.; Manuscript development and revision: A.A., R.S., D.V. All authors have read and agreed to the published version of the manuscript.

Funding: This work was started during the visit of A.A. to UFABC supported by the Sao Paulo Research Foundation (FAPESP), project 2014/23772-1. The work of A.A. was supported by the project RFBR 18-02-00264. The work of D.V. was supported in parts by FAPESP, project 2016/03319-6; by CNPq, projects 428951/2018-0 and 305594/2019-2; by the RFBR project 18-02-00149-a; and by the Tomsk State University Competitiveness Improvement Program.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Armitage, N.P.; Mele, E.J.; Vishwanath, A. Weyl and Dirac Semimetals in Three Dimensional Solids. *Rev. Mod. Phys.* **2018**, *90*, 015001.
2. Burkov, A.A. Chiral anomaly and transport in Weyl metals. *J. Phys. Condens. Matter* **2015**, *27*, 113201. [CrossRef] [PubMed]
3. Tabert, C.J.; Carbotte, J.P. Optical conductivity of Weyl semimetals and signatures of the gapped semimetal phase transition. *Phys. Rev. B* **2016**, *93*, 085442. [CrossRef]
4. Akhiezer, A.I.; Berestetskii, V.B. *Quantum Electrodynamics*; Nauka: Moscow, Russia, 1981. (In Russian)
5. Lundgren, R.; Fiete, G.A. Electronic cooling in Weyl and Dirac semimetals. *Phys. Rev. B* **2015**, *92*, 125139. [CrossRef]