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A Family of Fifth and Sixth Convergence Order Methods for Nonlinear Models

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Abstract: We study the local convergence of a family of fifth and sixth convergence order derivative free methods for solving Banach space valued nonlinear models. Earlier results used hypotheses up to the seventh derivative to show convergence. However, we only use the first divided difference of order one as well as the first derivative in our analysis. We also provide computable radius of convergence, error estimates, and uniqueness of the solution results not given in earlier studies. Hence, we expand the applicability of these methods. The dynamical analysis of the discussed family is also presented. Numerical experiments complete this article.

Keywords: Banach spaces; local convergence; divided difference; fréchet derivative; complex dynamics

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1. Introduction

One of the major goals of this study is to arrive at an estimated solution x_* of the equation

$$F(x) = 0, \quad (1)$$

where the operator $F : \mathscr{D} \subseteq X \to Y$ is Fréchet derivable with values in a Banach space Y and $\mathscr{D}$ is a convex subset of a Banach space X. A number of challenging problems in applied sciences and engineering can be formulated for the issue of solving equations of the form (1). This is why the task of approximating solutions of these equations has always been of central significance in mathematics. Closed form solutions to these equations are almost impossible to compute. Therefore, scientists and researchers often focus on iterative techniques to estimate the desired solution. Among the iterative procedures for addressing (1), Newton's approach is the most popular scheme, having a convergence rate of two. In the last few years, a host of researchers have suggested and are currently designing advanced iterative procedures of higher order [1–16] for solving the problem (1).

In the research of iterative schemes, estimating the convergence domain is an important issue. In most cases, the domain of convergence is small. Thus, the enlargement of the convergence domain is necessary without applying any additional condition. Additionally, it is important to estimate precise error bounds in the convergence study of iterative processes. The study of local analysis of an iterative scheme offers radii of convergence balls, error distances, and uniqueness result for a solution. Many authors [17–27] deduced the local results for different iterative processes. In these studies, essential outcomes, like measurements on error estimates, calculable convergence radii, and improved utility of highly efficient iterative algorithms have been derived. Recently, Argyros and George [28] studied the local convergence analysis of a seventh order iterative algorithm without inverses of derivatives. This method can be written, as follows.

$$\begin{aligned} y_n &= x_n - B_n^{-1}F(x_n) \\ z_n &= y_n - (3I - 2B_n^{-1}[y_n, x_n; F])B_n^{-1}F(y_n) \\ x_{n+1} &= z_n - \frac{13}{4}I - B_n^{-1}[z_n, y_n; F](\frac{7}{2}I - \frac{5}{4}B_n^{-1}[z_n, y_n; F])B_n^{-1}F(z_n), \end{aligned} \tag{2}$$

where x_0 is a starting point, $B(x) = [x + F(x), x - F(x); F]$, $B_n = B(x_n)$ and $[.,.;F] : \mathscr{D} \times \mathscr{D} \to \mathscr{L}(X, Y)$ satisfies

$$[u_1, u_2; F](u_1 - u_2) = F(u_1) - F(u_2) \textit{ for each } u_1, u_2 \in \mathscr{D}$$

and

$$[u_1, u_1; F] = F'(u_1) \textit{ for each } u_1 \in \mathscr{D},$$

if F is differentiable.

On the other hand, the study of complex dynamical properties of a family of iterative approaches, applied on second degree polynomials with complex coefficients, offers essential information regarding its reliability and stability. Complex dynamical behaviors of Chebyshev–Halley methods, Kim's family of methods, and other classes of iterative schemes have been described by authors, like Amat et al. [29,30], Argyros and Magreñán [18,19], Cordero et al. [31–33], and others [26,34–36]. In these works, important dynamical planes have been found showing periodical behavior and other convergence properties.

Our ultimate purpose in this article is to derive the local result and dynamical properties of a fifth and sixth convergence order family of iterative techniques. Behl and Martínez [3] designed a family of iterative procedures to address the problem (1), whose iterative steps are defined by

$$\begin{aligned} y_n &= x_n - F'(x_n)^{-1}F(x_n) \\ z_n &= x_n - \alpha F'(x_n)^{-1}(F(x_n) + F(y_n)) - (1 - \alpha)[y_n, x_n; F]^{-1}F(x_n) \\ x_{n+1} &= z_n - [y_n, z_n; F]^{-1}F(z_n), \end{aligned} \tag{3}$$

where $\alpha \in \mathbb{R}$, $[.,.;F] : \mathscr{D} \times \mathscr{D} \to \mathscr{L}(X, Y)$ is a divided difference of order one. It is shown to be order six for $\alpha = -1$ by applying hypotheses up to the seventh Fréchet derivative of F. The usage of these algorithms is restricted due to such hypotheses on the higher order derivative. To show this, we introduce $\Omega = [-\frac{1}{2}, \frac{3}{2}]$ and defined a function F on Ω by

$$F(x) = \begin{cases} x^3 \ln(x^2) + x^5 - x^4, & \text{if } \ x \neq 0 \\ 0, & \text{if } \ x = 0 \end{cases}.$$

The third order derivative F''' of the considered function F is unbounded on $\mathscr{D}$. The local convergence result for the family (3) that is described in [3] does not work for this example. Additioanlly, no research regarding the convergence domain or radius of convergence ball was done in the existing paper [3]. Accordingly, the local analysis theorem for the schemes (3) is established in study by considering a set of assumptions only on F'. In specific, we employ the ω-continuity condition only on F' to expand the utility of this family of methods. Furthermore, dynamical study of the parametric family (3) is presented in the context of scalar nonlinear equations. This analysis is helpful in determining appropriate values of α. Besides this, several anomalies, namely convergence to strange fixed points or m-cycles and divergence to ∞ are observed by applying the procedures that are used in [19,36]. By means of parameter planes and various dynamical planes, these anomalies are shown.

The remaining part of this study is presented in the following manner. Local convergence of the discussed class of algorithms (3) is established in Section 2. Section 3 describes the complex dynamical analysis of this class. The last section contains a series of numerical experiments.

2. Convergence

Let us consider some real functions. Assume:

(a) There exists function $\omega_0 : [0,\infty) \to [0,\infty)$ non-decreasing and continuous such that

$$\omega_0(t) - 1 = 0, \tag{4}$$

has a least root $R_0 \in (0,\infty)$.

Consider function $\omega : [0,R_0) \to [0,\infty)$ non-decreasing and continuous. Define function $g_1 : [0,R_0) \to [0,\infty)$ by

$$g_1(t) = \frac{\int_0^1 \omega((1-\theta)t)\, d\theta}{1-\omega_0(t)}.$$

(b) Equation

$$g_1(t) - 1 = 0 \tag{5}$$

has a least root $r_1 \in (0,R_0)$. Consider functions $\omega_1 : [0,R_0) \to [0,\infty)$ and $\omega_2 : [0,R_0) \times [0,R_0) \to [0,\infty)$ continuous and non-decreasing.

(c) Equation

$$\omega_2(g_1(t)t, t) - 1 = 0 \tag{6}$$

has a least root $R \in (0,R_0)$.

Set $R_1 = \min\{R_0, R\}$. Define the function $g_2 : [0,R_1) \to [0,\infty)$ by

$$\begin{aligned} g_2(t) &= g_1(t) + \frac{|1-\alpha|(\omega_0(t) + \omega_2(g_1(t)t,t)) \int_0^1 \omega_1(\theta t)\, d\theta}{(1-\omega_0(t))(1-\omega_2(g_1(t)t,t))} \\ &\quad + \frac{|\alpha| \int_0^1 \omega_1(\theta g_1(t)t)\, d\theta}{1-\omega_0(t)} g_1(t). \end{aligned}$$

(d) Equation

$$g_2(t) - 1 = 0 \tag{7}$$

has a least root $r_2 \in (0,R_1)$.

(e) Equations

$$\omega_0(g_2(t)t) - 1 = 0 \tag{8}$$

and

$$\omega_2(g_1(t)t, g_2(t)t) - 1 = 0 \tag{9}$$

have least roots $R_2, R_3 \in (0,R_0)$, respectively. Set $R_4 = \min\{R_1, R_2, R_3\}$. Define function $g_3 : [0,R_4) \to [0,\infty)$ by

$$\begin{aligned} g_3(t) = \Bigg[& g_1(g_2(t)t) \\ & + \frac{(\omega_0(g_2(t)t) + \omega_2(g_1(t)t, g_2(t)t)) \int_0^1 \omega_1(\theta g_2(t)t)\, d\theta}{(1-\omega_0(g_2(t)t))(1-\omega_2(g_1(t)t, g_2(t)t)}\Bigg] g_2(t). \end{aligned}$$

(f) Equation

$$g_3(t) - 1 = 0 \tag{10}$$

has a least root $r_3 \in (0,R_4)$. Next,

$$r = \min\{r_k\},\ k = 1,2,3 \tag{11}$$

is proved to be a convergence radius for the method (3). By the definition of r,

$$0 \le \omega_0(t) < 1, \tag{12}$$

$$0 \le \omega_2(g_1(t)t, t) < 1, \tag{13}$$

$$0 \le \omega_0(g_2(t)t) < 1, \tag{14}$$

$$0 \le \omega_2(g_1(t)t, g_2(t)t) < 1 \tag{15}$$

and

$$0 \le g_k(t) < 1 \text{ hold for all } t \in [0, r). \tag{16}$$

It is worth noticing that, by the definition of R_0 and r (see (4), and (11), respectively), we have $r < R_0$, so $\omega_0(r) \le \omega_0(R_0) = 1$. However, in particular $\omega_0(r) < \omega_0(R_0) = 1$, since, otherwise, r is the least positive solution of Equation (4), contradicting the definition of R_0. Hence, $\omega_0(t)$ is not reaching one, even if $t = r$. By a similar argument the rest of the functions in (13)–(16) are not reaching one.

Let $S(x_*, \rho)$, $\overline{S}(x_*, \rho)$ be the open ball and its and closure in X with center $x_* \in X$ and radius $\rho > 0$. The following hypotheses (H) shall be used with the ω functions defined previously. Assume:

(H_1) $F : \mathscr{D} \subseteq X \to Y$ is differentiable, $[.,.;F] : \mathscr{D} \times \mathscr{D} \to \mathscr{L}(X, Y)$, and x_* is a simple solution of (1).
(H_2) $||F'(x_*)^{-1}(F'(x) - F'(x_*))|| \le \omega_0(||x - x_*||)$ for all $x, y \in \mathscr{D}$.
Set $S_0 = \mathscr{D} \cap S(x_*, R_0)$.
(H_3) $||F'(x_*)^{-1}(F'(y) - F'(x))|| \le \omega(||y - x||)$, $||F'(x_*)^{-1}F'(x)|| \le \omega_1(||x - x_*||)$ and $||F'(x_*)^{-1}([y, x; F] - F'(x_*))|| \le \omega_2(||y - x_*||, ||x - x_*||)$ for all $x, y \in S_0$.
(H_4) $\overline{S}(x_*, r) \subset \mathscr{D}$, where r is given in (11).
(H_5) There exist $r_4 \ge r$ such that

$$\int_0^1 \omega_0(\theta r_4)\, d\theta < 1. \tag{17}$$

Set $S_1 = \mathscr{D} \cap S(x_*, r_4)$.

Next, hypotheses (H) and the notation introduced shall be used to develop the analysis of (3).

Theorem 1. *Under the hypothesis (H), further choose a starter point $x_0 \in S(x_*, r) \setminus \{x_*\}$. Subsequently, sequence $\{x_n\}$ developed by Formula (3) is well defined in $S(x_*, r)$, remains in $S(x_*, r)$ for each $n = 0, 1, 2, \ldots$, and converges to x_*. Additionally, the following upper error estimations are valid*

$$||y_n - x_*|| \le g_1(||x_n - x_*||)||x_n - x_*|| \le ||x_n - x_*|| < r, \tag{18}$$

$$||z_n - x_*|| \le g_2(||x_n - x_*||)||x_n - x_*|| \le ||x_n - x_*|| \tag{19}$$

and

$$||x_{n+1} - x_*|| \le g_3(||x_n - x_*||)||x_n - x_*|| \le ||x_n - x_*||, \tag{20}$$

where the functions g_k are defined previously and r is given in (11). Furthermore, the only solution of Equation (1) in the set S_1 that is given in (H_5) is x_*.

Proof. Estimates (18)–(20) shall be proved using induction. Let $v \in S(x_*, r) \setminus \{x_*\}$ be arbitrary. Afterwards, by hypotheses (H_1), (H_2), (11), and (12), we get, in turn, that

$$||F'(x_*)^{-1}(F'(v) - F'(x_*))|| \le \omega_0(||v - x_*||) \le \omega_0(r) < 1. \tag{21}$$

Estimate (21), together with a lemma due to Banach [1,13] on operators with inverses, imply $F'(v)^{-1} \in \mathscr{L}(Y, X)$ with

$$||F'(v)^{-1}F'(x_*)|| \leq \frac{1}{1 - \omega_0(||v - x_*||)}. \tag{22}$$

By (3), for $n = 0$, $v = x_0$ and (22) y_0 is well defined. We can also write

$$\begin{aligned} y_0 - x_* &= x_0 - x_* - F'(x_0)^{-1}F(x_0) \\ &= -\Big[F'(x_0)^{-1}F'(x_*)\Big]\Big[\int_0^1 F'(x_*)^{-1}(F'(x_* + \theta(x_0 - x_*)) - F'(x_0))\, d\theta\, (x_0 - x_*)\Big]. \end{aligned} \tag{23}$$

However, then, (11), (16) (for $k = 0$), (H_3), (22) (for $v = x_0$), and (23) imply, in turn, that

$$\begin{aligned} ||y_0 - x_*|| &\leq \frac{\int_0^1 \omega((1-\theta)||x_0 - x_*||)\, d\theta\, ||x_0 - x_*||}{1 - \omega_0(||x_0 - x_*||)} \\ &\leq g_1(||x_0 - x_*||)||x_0 - x_*|| \leq ||x_0 - x_*|| < r \end{aligned} \tag{24}$$

proving (18) for $n = 0$ and $y_0 \in S(x_*, r)$. To show the existence of z_0, we need to establish the inversibility of linear operator $[y_0, x_0; F]$. Indeed, by (11), (13), (H_3), and (24), we have

$$\begin{aligned} ||F'(x_*)^{-1}([y_0, x_0; F] - F'(x_*))|| &\leq \omega_2(||y_0 - x_*||, ||x_0 - x_*||) \\ &\leq \omega_2(g_1(||x_0 - x_*||)||x_0 - x_*||, ||x_0 - x_*||) \\ &\leq \omega_2(g_1(r)r, r) < 1, \end{aligned} \tag{25}$$

leading to

$$||[y_0, x_0; F]^{-1}F'(x_*))|| \leq \frac{1}{1 - \omega_2(g_1(||x_0 - x_*||)||x_0 - x_*||, ||x_0 - x_*||)}. \tag{26}$$

Next, we get

$$\begin{aligned} & z_0 - x_* \\ &= x_0 - x_* - F'(x_0)^{-1}F(x_0) + F'(x_0)^{-1}F(x_0) - \alpha F'(x_0)^{-1}F(x_0) \\ &- \alpha F'(x_0)^{-1}F(y_0) - (1-\alpha)[y_0, x_0; F]^{-1}F(x_0) \\ &= y_0 - x_* + (1-\alpha)(F'(x_0)^{-1} - [y_0, x_0; F]^{-1})F(x_0) - \alpha F'(x_0)^{-1}F(y_0) \\ &= y_0 - x_* + (1-\alpha)F'(x_0)^{-1}\Bigg(([y_0, x_0; F] - F'(x_*)) \\ &+ (F'(x_*) - F'(x_0))\Bigg)[y_0, x_0; F]^{-1}F(x_0) \\ &- \alpha F'(x_0)^{-1}F(y_0). \end{aligned} \tag{27}$$

Hence, by (22)–(27), (11), and (16) for $k = 2$, we can get

$$\begin{aligned}
&||z_0 - x_*|| \\
&\leq ||y_0 - x_*|| \\
&+ \left(\frac{|1-\alpha|(\omega_0(||x_0 - x_*||) + \omega_2(||y_0 - x_*||, ||x_0 - x_*||))}{(1 - \omega_0(||x_0 - x_*||))(1 - \omega_2(||y_0 - x_*||, ||x_0 - x_*||))} \right) \\
&\times \int_0^1 \omega_1(\theta ||x_0 - x_*||)\, d\theta\, ||x_0 - x_*|| \\
&+ \frac{|\alpha| \int_0^1 \omega_1(\theta ||y_0 - x_*||)\, d\theta\, ||y_0 - x_*||}{1 - \omega_0(||x_0 - x_*||)} \\
&\leq g_2(||x_0 - x_*||)||x_0 - x_*|| \leq ||x_0 - x_*||
\end{aligned} \tag{28}$$

proving (19) for $n = 0$ and $z_0 \in S(x_*, r)$. Hence, it follows that x_1 is well defined and the inversibility of $F'(z_0)$ and $[y_0, z_0; F]$. Indeed, as in (26), and using (14) and (15), we have, in turn

$$\begin{aligned}
||F'(x_*)^{-1}(F'(z_0) - F'(x_*))|| &\leq \omega_0(||z_0 - x_*||) \\
&\leq \omega_0(g_2(||x_0 - x_*||)||x_0 - x_*||) < 1
\end{aligned}$$

and

$$\begin{aligned}
&||F'(x_*)^{-1}([y_0, z_0; F] - F'(x_*))|| \\
&\leq \omega_2(||y_0 - x_*||, ||z_0 - x_*||) \\
&\leq \omega_2(g_1(||x_0 - x_*||)||x_0 - x_*||, g_2(||x_0 - x_*||)||x_0 - x_*||) \\
&\leq \omega_2(g_1(r)r, g_2(r)r) < 1,
\end{aligned}$$

so

$$||F'(z_0)^{-1}F'(x_*)|| \leq \frac{1}{1 - \omega_0(||z_0 - x_*||)} \tag{29}$$

and

$$||[y_0, z_0; F]^{-1}F'(x_*)|| \leq \frac{1}{1 - \omega_2(||y_0 - x_*||, ||z_0 - x_*||)}. \tag{30}$$

Hence, x_1 is well defined. We can have

$$x_1 - x_* = z_0 - x_* - F'(z_0)^{-1}F(z_0) + (F'(z_0)^{-1} - [y_0, z_0; F]^{-1})F(z_0). \tag{31}$$

It then follows from (11), (16) for $k = 3$, (29)–(31) that

$$\begin{aligned}
&||x_1 - x_*|| \\
&\leq \Bigg[g_1(||z_0 - x_*||) \\
&+ \frac{(\omega_0(||z_0 - x_*||) + \omega_2(||y_0 - x_*||, ||z_0 - x_*||)) \int_0^1 \omega_1(\theta ||z_0 - x_*||)\, d\theta\, ||x_0 - x_*||}{(1 - \omega_0(||z_0 - x_*||))(1 - \omega_2(||y_0 - x_*||, ||z_0 - x_*||))} \Bigg] \\
&\times ||z_0 - x_*|| \\
&\leq g_3(||x_0 - x_*||)||x_0 - x_*|| \leq ||x_0 - x_*||,
\end{aligned} \tag{32}$$

proving (20) for $n = 0$, and $x_1 \in S(x_*, r)$. In order to complete the induction for (18)–(20), we simply exchange x_0, y_0, z_0, x_1 by x_m, y_m, z_m, x_{m+1} in the preceding estimations. Subsequently, by the estimation

$$||x_{m+1} - x_*|| \leq q||x_m - x_*|| < r, \ q = g_3(||x_0 - x_*||) \in [0, 1), \tag{33}$$

we get $\lim_{m\to\infty} x_m = x_*$ and $x_{m+1} \in S(x_*, r)$. The uniqueness result is proved by setting $A = \int_0^1 F'(x_* + \theta(q - x_*))\, d\theta$ for some $q \in S_1$ with $F(q) = 0$. Using (H_2) and (H_5), we obtain

$$||F'(x_*)^{-1}(A - F'(x_*))|| \leq \int_0^1 \omega_0(\theta||q - x_*||)\, d\theta$$
$$\int_0^1 \omega_0(\theta r_4) < 1,$$

so $q = x_*$ by $F(q) - F(x_*) = A(q - x_*)$. □

3. Dynamical Analysis of the Discussed Class of Algorithms (3)

The complex dynamical properties of the class (3) are analyzed in detail in this segment. In the research field of iterative algorithms, the dynamical study of a class of iterative processes has emerged as a standard research approach for categorizing various iterative procedures according to their convergence rate. Additionally, it enables the evaluation of their numerical performance in relation to the selected initial estimation. This research allows for the visualization of the set of starting values that converge to a solution or other locations. Moreover, it shows the robustness and effectiveness of an iterative formula.

This report examines the dynamical features of the class of solvers (3). The family (3) can be expressed, when $X = Y = \mathbb{C}$ as:

$$\begin{aligned} y_n &= x_n - \frac{F(x_n)}{F'(x_n)} \\ z_n &= x_n - \alpha F'(x_n)^{-1}(F(x_n) + F(y_n)) - (1-\alpha)\frac{F(x_n)(x_n - y_n)}{F(x_n) - F(y_n)} \\ x_{n+1} &= z_n - \frac{F(z_n)(z_n - y_n)}{F(z_n) - F(y_n)}. \end{aligned} \tag{34}$$

We study the dynamical the class (34) applied on a two degree complex polynomial $\mathscr{H}(z) : \mathbb{C} \to \mathbb{C}$ defined by $\mathscr{H}(z) = (z - s_1)(z - s_2)$. We discuss, by employing the graphical software MATHEMATICA [18,19], the fixed points related to the class (34) and their stability. Besides this, different anomalies in the considered family (34) are shown by means of parameter spaces and several dynamical planes.

Let $\hat{\mathbb{C}}$ stand for the Riemann sphere and $\mathscr{R} : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$ is a rational function. Subsequently, we have the following definitions [18,19,30–32].

Definition 1. *The orbit of a point $z_0 \in \hat{\mathbb{C}}$ is defined as the set $\{z_0, \mathscr{R}(z_0), \mathscr{R}^2(z_0), \ldots, \mathscr{R}^n(z_0)\}$.*

Definition 2. *A point $z_0 \in \hat{\mathbb{C}}$ is called a fixed point of $\mathscr{R}(z)$, if it verifies $\mathscr{R}(z_0) = z_0$. The fixed points that are not related to the roots of the polynomial $\mathscr{H}(z)$ are called strange fixed points.*

Definition 3. *$z_0 \in \hat{\mathbb{C}}$ is called a periodic point of period $m > 1$, if it satisfies $\mathscr{R}^m(z_0) = z_0$ with $\mathscr{R}^n(z_0) \neq z_0$, for each $n < m$. Moreover, a point z_0 is called pre-periodic if it is not periodic but there exists a $l > 0$, such that $\mathscr{R}^l(z_0)$ is periodic.*

Depending on the associated multiplier $|\mathscr{R}'(z_0)|$, the fixed points can be categorized, as follows.

Definition 4. *A fixed point z_0 is called:*

(i) *superattractor if $|\mathscr{R}'(z_0)| = 0$,*
(ii) *attractor if $|\mathscr{R}'(z_0)| < 1$,*
(iii) *repulsor if $|\mathscr{R}'(z_0)| > 1$ and*
(iv) *parabolic if $|\mathscr{R}'(z_0)| = 1$.*

Definition 5. *A point $z_0 \in \hat{\mathbb{C}}$ is called a critical point of $\mathscr{R}(z)$, if it satisfies $\mathscr{R}'(z_0) = 0$. Free critical points are those critical points that are not related to the roots of $\mathscr{H}(z)$.*

Definition 6. *The basin of attraction of an attractor β is defined as $\mathscr{A}(\beta) = \{z_0 \in \hat{\mathbb{C}} : \mathscr{R}^n(z_0) \to \beta$ as $n \to \infty\}$.*

Definition 7. *The Fatou set of the rational function $\mathscr{R}$, $\mathscr{F}(\mathscr{R})$, is the set of points $z_0 \in \hat{\mathbb{C}}$ whose orbits tend to an attractor (fixed point, periodic orbit, or infinity). Its complement in $\hat{\mathbb{C}}$ is the Julia set, $\mathscr{J}(\mathscr{R})$.*

We apply the family of algorithms (34) on $\mathscr{H}(z) = (z - s_1)(z - s_2)$, where $s_1 \neq s_2$. Subsequently, we derive the rational operator

$$\mathscr{Y}_{\mathscr{H}}(z, \alpha) = z^5 \frac{z^3 + 3z^2 + \alpha + 3z + 1}{1 + (\alpha + 1)z^3 + 3z^2 + 3z}, \alpha \in \mathbb{C} \tag{35}$$

by considering the Möbius map $\mathscr{M}(z) = \frac{z - s_1}{z - s_2}$, such that $\mathscr{M}(s_1) = 0$, $\mathscr{M}(s_2) = \infty$ and $\mathscr{M}(\infty) = 1$.

3.1. Stability of the Fixed Points

The fixed points of $\mathscr{Y}_{\mathscr{H}}(z, \alpha)$ are determined by solving $\mathscr{Y}_{\mathscr{H}}(z, \alpha) = z$. Solving the earlier equation is equivalent to solving $z(z - 1)(z^6 + 4z^5 + 7z^4 + (\alpha + 8)z^3 + 7z^2 + 4z + 1) = 0$. It is easy to notice that the points $z = 0$ and $z = \infty$ are the fixed points of $\mathscr{Y}_{\mathscr{H}}(z, \alpha)$ and associated with s_1 and s_2, respectively. It follows from the definition that the point $z = 1$ is a member of the set of strange fixed points of the operator $\mathscr{Y}_{\mathscr{H}}(z, \alpha)$. Moreover, the solutions of $z^6 + 4z^5 + 7z^4 + (\alpha + 8)z^3 + 7z^2 + 4z + 1 = 0$ are other six strange fixed points. These points depend on α and they are denoted by $efp_k(\alpha)$, $k = 1, 2, \ldots, 6$. We consider the expressions $efp_k(\alpha) = Root[1 + 4\#1 + 7\#1^2 + (8 + \alpha)\#1^3 + 7\#1^4 + 4\#1^5 + \#1^6 \&, k]$, $k = 1, 2, \ldots, 6$, for these points using MATHEMATICA software. Figure 1 shows the points $efp_k(\alpha)$ when $efp_k \in \mathbb{R}$, $k = 1, 2, \ldots, 6$.

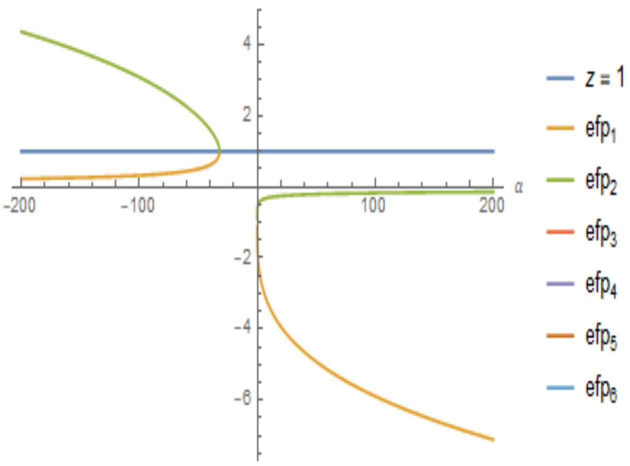


Figure 1. Behavior of $efp_k(\alpha)$, $k = 1, 2, \ldots, 6$.

It is mandatory to obtain $\mathscr{Y}'_{\mathscr{H}}(z, \alpha)$ for describing the stability of $z = 1$ and $efp_k(\alpha)$, $k = 1, 2, \ldots, 6$. We have using (35) that

$$\mathscr{Y}'_{\mathscr{H}}(z, \alpha) = \frac{z^4}{(1 + 3z + 3z^2 + z^3(1 + \alpha))^2} \Big(5(1 + \alpha) + 5z^6(1 + \alpha) + 6z(5 + 2\alpha) + 6z^5(5 + 2\alpha) + z^2(75 + 9\alpha) + z^4(75 + 9\alpha) + 2z^3(50 + 2\alpha + \alpha^2) \Big). \tag{36}$$

It is confirmed from (36) that the points $z = 0$ and $z = \infty$ belong to the set of superattracting fixed points of $\mathscr{Y}_{\mathscr{H}}(z, \alpha)$. Next, we investigate the stability properties of the fixed point $z = 1$. We use the notation $Str_1(\alpha)$ for indicating the stability function $|\mathscr{Y}_H'(1, \alpha)|$ of $z = 1$. The stability functions of efp_k are denoted by $Str_{k+1}(\alpha)$, $k = 1, 2, \ldots, 6$. We discuss the stability of $z = 1$ in the below result.

Theorem 2. *The stability characterization of the strange fixed point $z = 1$ can be described, as follows.*

1. *$z = 1$ is a superattracting strange fixed point for $\alpha = -20$.*
2. *$z = 1$ is an attracting point if $|\alpha + 24| < 8$.*
3. *For $|\alpha + 24| = 8$ the point $z = 1$ is parabolic.*
4. *Lastly, if $|\alpha + 24| > 8$, then $z = 1$ is repulsor.*

Proof. Using Equation (36), we obtain

$$Str_1(\alpha) = \left| \frac{40 + 2\alpha}{\alpha + 8} \right|. \tag{37}$$

It is clear that $Str_1(-20) = 0$. Also, $\left| \frac{40+2\alpha}{\alpha+8} \right| \leq 1$ is equivalent to $|40 + 2\alpha| \leq |\alpha + 8|$. Set $\alpha = a_1 + ia_2$ a complex number. Next,

$$1600 + 4a_1^2 + 160a_1 + 4a_2^2 \leq a_1^2 + 64 + 16a_1 + a_2^2. \tag{38}$$

We obtain

$$512 + a_1^2 + 48a_1 + a_2^2 \leq 0 \tag{39}$$

by simplifying (38). Further, we get form (39), which

$$(a_1 + 24)^2 + a_2^2 \leq 64.$$

Thus, $|Str_1(\alpha)| \leq 1$ if and only if $|\alpha + 24| \leq 8$. $\square$

Thus, the area where $|\alpha + 24| \leq 8$ represents the stability region of $z = 1$. Figure 2 provides a graphical view of this stability area.

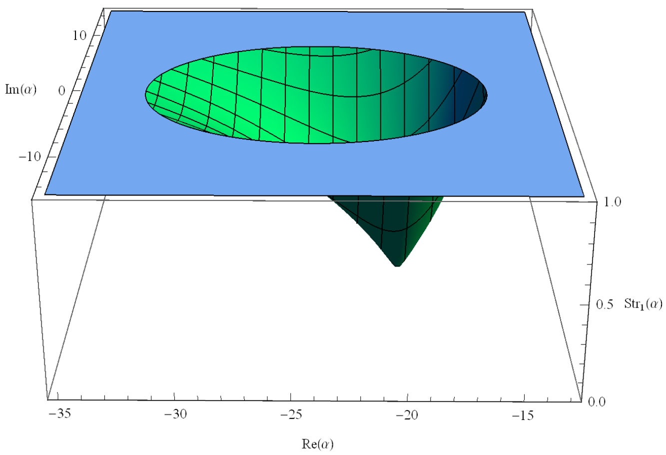
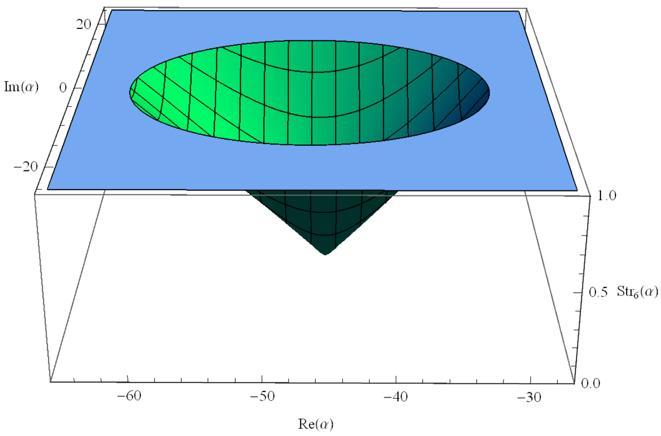
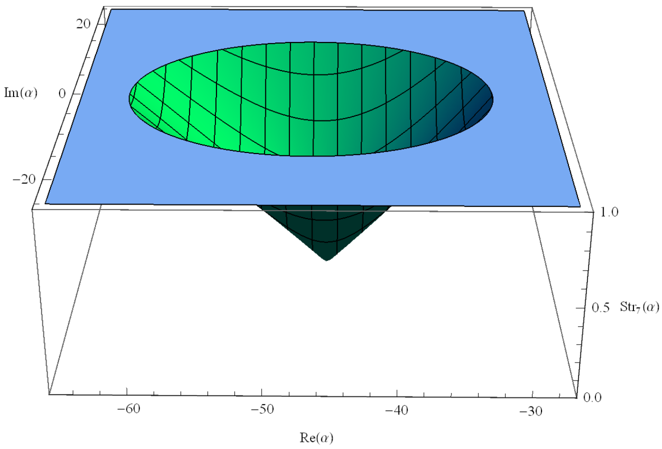


Figure 2. Region of $z = 1$: Stability.

It is extremely difficult to determine the stability of efp_k, $k = 1, 2, \ldots, 6$ in an analytical approach. Nevertheless, the graphical software MATHEMATICA can be employed to visualize the stability areas for the points efp_5 and efp_6. These stability regions are presented in Figure 3a,b, respectively.



(**a**) Stability region of $efp_5(\alpha)$



(**b**) Stability region of $efp_6(\alpha)$

Figure 3. Stability regions.

3.2. Study of Parameter Spaces and Critical Points

We determine the critical points of the operator $\mathscr{Y}_{\mathscr{H}}(z,\alpha)$ by finding the solutions of $\mathscr{Y}'_{\mathscr{H}}(z,\alpha) = 0$. From (36), we have that 0 and ∞ are critical points of $\mathscr{Y}_{\mathscr{H}}(z,\alpha)$. The free critical points of $\mathscr{Y}_{\mathscr{H}}(z,\alpha)$ are the solutions of $5(1+\alpha) + 5z^6(1+\alpha) + 6z(5+2\alpha) + 6z^5(5+2\alpha) + z^2(75+9\alpha) + z^4(75+9\alpha) + 2z^3(50+2\alpha+\alpha^2) = 0$. These points can be represented for $k = 1,\ 2,\ldots,\ 6$ by $fcp_k(\alpha) = Root[5+5\alpha+(30+12\alpha)\#1+(75+9\alpha)\#1^2+(100+4\alpha+2\alpha^2)\#1^3+(75+9\alpha)\#1^4+(30+12\alpha)\#1^5+(5+5\alpha)\#1^6\ \&, k]$ using MATHEMATICA software. Figure 4 provides the behavior of $fcp_k(\alpha)$ (when $fcp_k \in \mathbb{R}$, $k = 1,\ 2,\ldots,\ 6$).

The dynamical analysis of the discussed class of algorithms (34) are described by applying the procedure that was used in [18,19]. We consider the free critical points $fcp_4(\alpha)$ and $fcp_6(\alpha)$ to present the parameter planes that are related to them in Figures 5 and 6, respectively. We use the point $z_0 = fcp_k(\alpha)$ as a starter for the members of the considered family. Various colors are applied on the starting estimation z_0 to show different convergence behavior of the corresponding sequence of iterates $\{z_n\}$ on the complex plane. The convergence of $\{z_n\}$ to 0 or ∞ is denoted by cyan color. Additionally, convergence of $\{z_n\}$ to $z = 1$ is assigned in yellow. We execute maximum 1000 iteration with the tolerance 10^{-6}. Convergence to $efp_k(\alpha)$, $k = 1, 2, \ldots, 6$ is presented in magenta. Other colors like light green, orange, blue, dark orange, dark green, dark red, and white are applied to z_0 if $\{z_n\}$ convergence to m-cycles for $m = 2, 3, 4\ldots, 8$, respectively. Convergence of $\{z_n\}$ to other m-periodic orbits are displayed in black . It is found that there exist non-cyan regions in the parameter spaces. In these areas, the sequence $\{z_n\}$ converges to $efp_k(\alpha)$, $k = 1, 2, \ldots, 6$ or to m-cycles or even to ∞. Therefore, one should avoid these regions while selecting α for practical use. Additionally, there are wide cyan regions in the parameter planes, and this confirms that the family contains some numerically stable iterative elements.

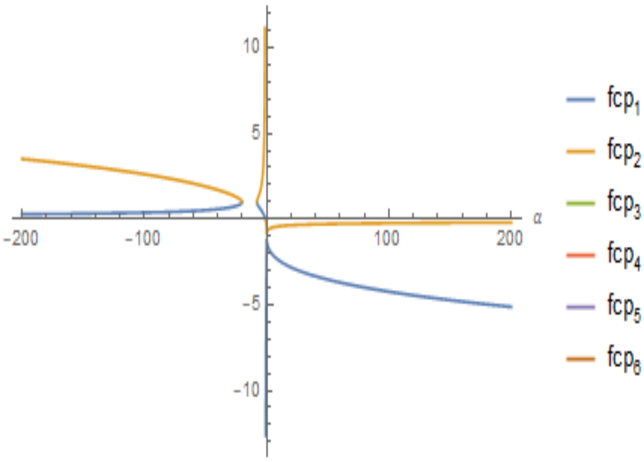


Figure 4. Behavior of free critical points.

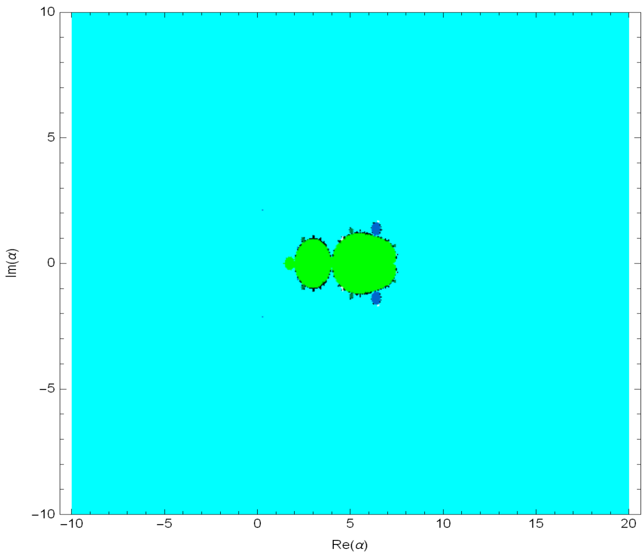


Figure 5. Critical point $fcp_4(\alpha)$ and the parameter plane.

We move forward to discuss several important dynamical planes in order to study some special anomalies. Orange and cyan color are employed to display the convergence to ∞ and 0, respectively. If convergence of the iterative algorithm of the class (34) is not related to either 0 or ∞, then it is indicated in black color. We consider 1000 iterations or the tolerance 10^{-6} as a stopping condition. In Figure 7a,b, yellow regions represent that the iterative elements (for $\alpha = -27$ and $\alpha = -20$) convergence to the point $z = 1$. In Figure 8a,b, appearance of attracting fixed points efp_5 and efp_6 is displayed in black color.

The dynamical plane that is associated with the algorithm extracted from the considered family for $\alpha = -15$ is given in Figure 9a. In this figure, an attracting 2-cycle $\{0.9788 - 0.2047i, 0.9788 + 0.2047i\}$ appeared. In addition to this, the existence of another attracting 2–periodic orbit is displayed in Figure 9b. The existence of a 3-periodic is given in Figure 9c. In these Figs., black color is used to present the convergence of the respective method to various m-periodic orbits, since this convergence is not related to s_1 and s_2.

Finally, we provide dynamical planes for $\alpha = -1$ and $\alpha = -0.5$ in Figure 10a,b, respectively. In Figure 11a,b, dynamical planes for $\alpha = 0$ and $\alpha = 1$ are presented. In these planes, the convergence is related to s_1 or s_2 only; consequently, these algorithms of the discussed class are highly stable. Hence, these iterative techniques are superior to other schemes of the class in terms of practical application.

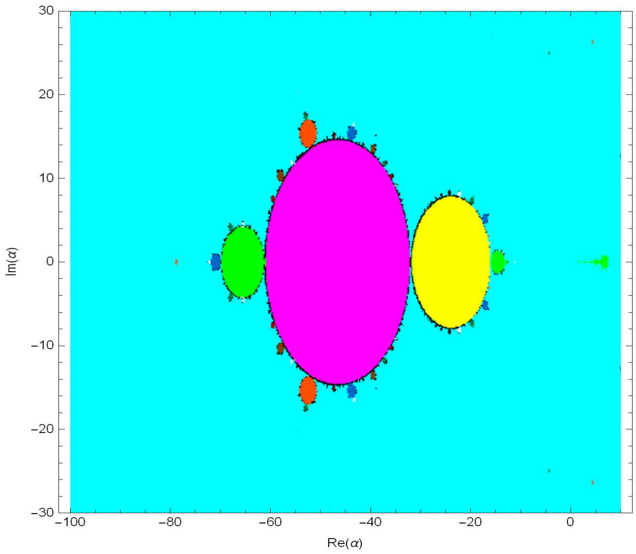
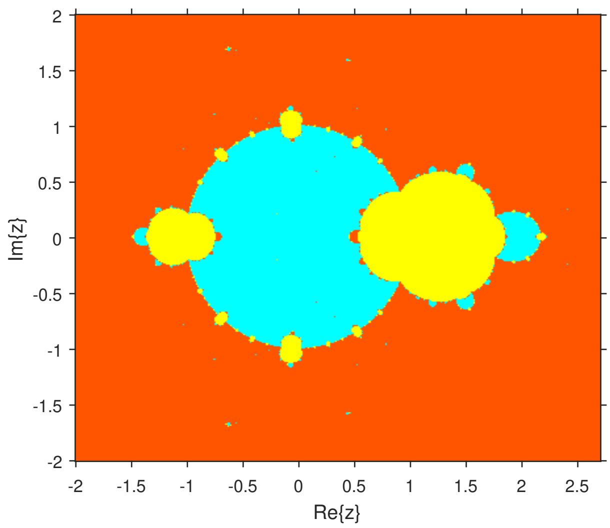
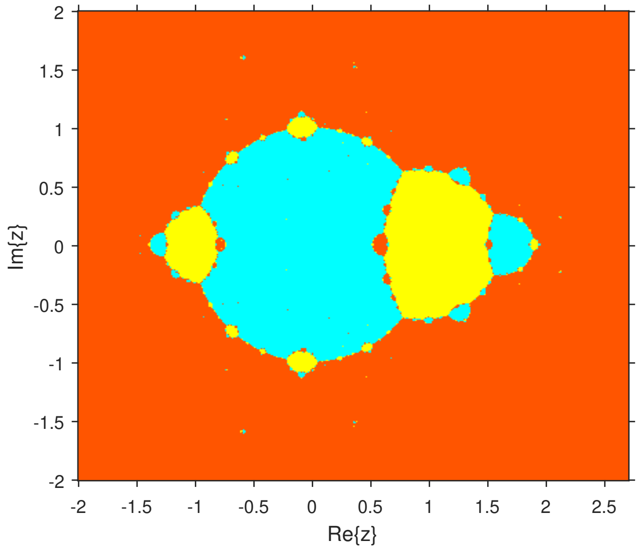


Figure 6. Critical point $fcp_6(\alpha)$ and the parameter plane.



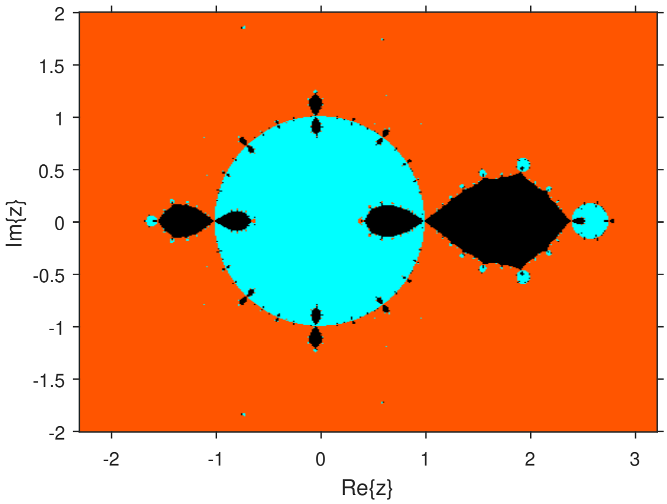
(**a**) $\alpha = -27$

Figure 7. *Cont*.

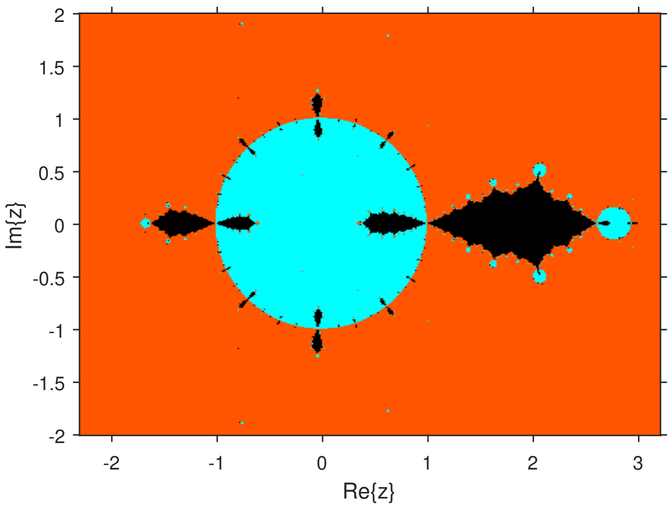


(**b**) $\alpha = -20$

Figure 7. Dynamical Planes.

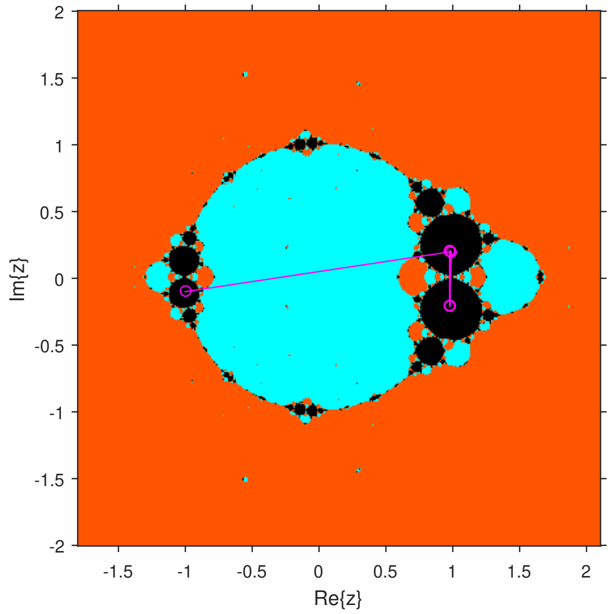


(**a**) $\alpha = -47$

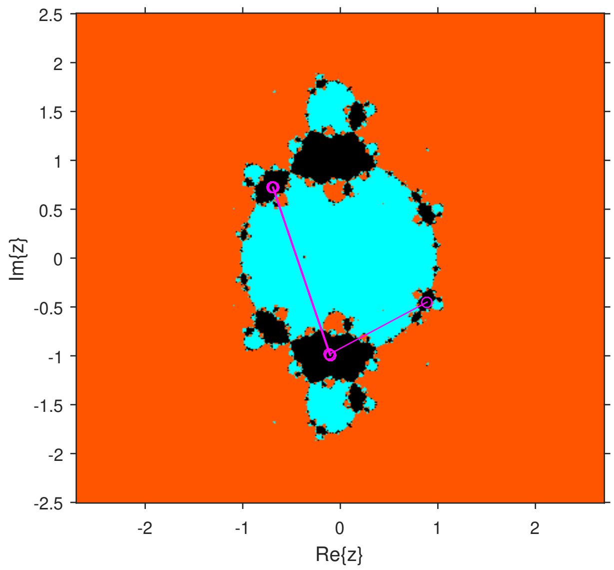


(**b**) $\alpha = -55$

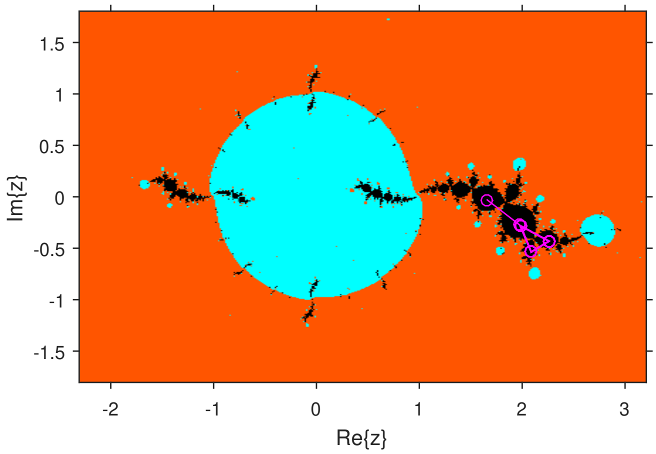
Figure 8. Dynamical Planes.



(**a**) $\alpha = -15$

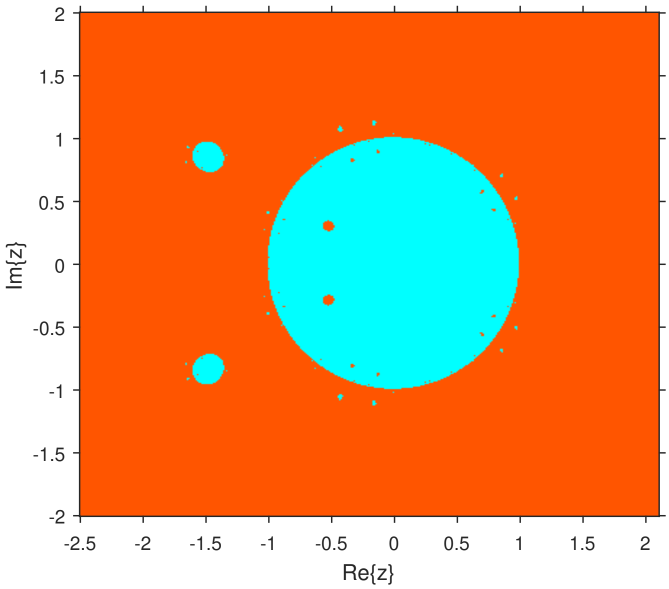


(**b**) $\alpha = 6.5$

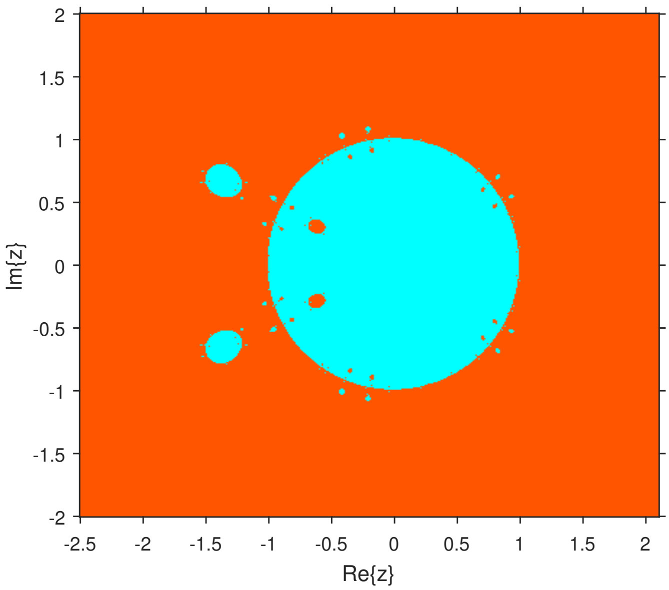


(**c**) $\alpha = -52.5 + 15i$

Figure 9. Dynamical Planes.

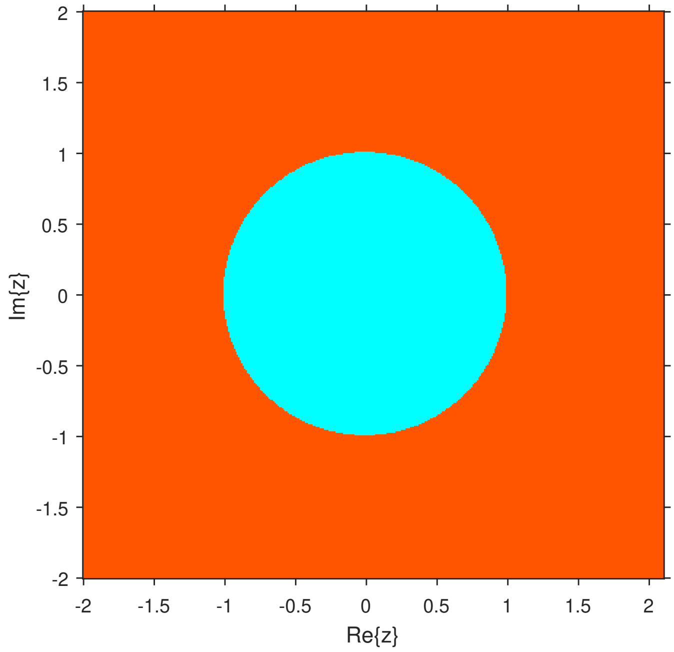


(**a**) $\alpha = -1$

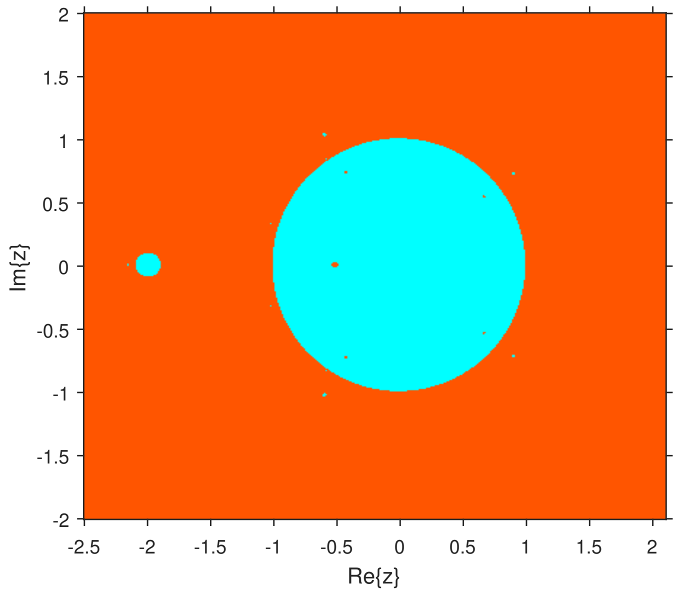


(**b**) $\alpha = -0.5$

Figure 10. Dynamical Planes.



(**a**) $\alpha = 0$



(**b**) $\alpha = 1$

Figure 11. Dynamical Planes.

4. Numerical Examples

We address numerical problems in this segment to explain the validity of our theoretical results. We use the proposed findings to measure the convergence radii for five iterative schemes. These schemes are derived from the discussed family (3) by putting $\alpha = -1$, $\alpha = -0.5$, $\alpha = 0$, $\alpha = 0.75$ and $\alpha = 1$. The divided difference in all of the selected examples is taken as $[x, y; F] = \int_0^1 F'(y + \theta(x - y))\, d\theta$.

Example 1 ([35])**.** *Let $X = Y = \mathbb{R}^3$ and $\mathscr{D} = \overline{S}(0, 1)$. Consider F on $\mathscr{D}$ for $x = (x_1, x_2, x_3)^t$ as*

$$F(x) = (e^{x_1} - 1, \frac{e-1}{2} x_2^2 + x_2, x_3)^t$$

Subsequently, we set for $x_* = (0,0,0)^t$, $\omega_0(t) = (e-1)t$, $\omega(t) = e^{\frac{1}{e-1}}t$, $\omega_1(t) = 2$ *and* $\omega_2(t,s) = \frac{e-1}{2}(t+s)$. *Using Theorem 1, the values of* r *for different values of* α *are calculated and presented in Table 1.*

Table 1. Convergence radii for Example 1.

$\alpha = -1$	$\alpha = -0.5$	$\alpha = 0$	$\alpha = 0.75$	$\alpha = 1$
$r_1 = 0.382692$	$r_1 = 0.382692$	$r_1 = 0.382692$	$r_1 = 0.382692$	$r_1 = 0.382692$
$r_2 = 0.063309$	$r_2 = 0.081572$	$r_2 = 0.115386$	$r_2 = 0.163070$	$r_2 = 0.189799$
$r_3 = 0.076949$	$r_3 = 0.091046$	$r_3 = 0.115386$	$r_3 = 0.148095$	$r_3 = 0.166247$
$r = 0.063309$	$r = 0.081572$	$r = 0.115386$	$r = 0.148095$	$r = 0.166247$

Example 2 ([21])**.** *Let us consider* $X = Y = C[0,1]$ *and* $\mathcal{D} = \overline{S}(0,1)$. *Introduce* F *on* $\mathcal{D}$ *by*

$$F(x)(t) = x(t) - 5\int_0^1 tu\, x(u)^3\, du,$$

where $x(t) \in C[0,1]$. *We have* $x^* = 0$. *Additionally,* $\omega_0(t) = 7.5t$, $\omega(t) = 15t$, $\omega_1(t) = 2$ *and* $\omega_2(t,s) = \frac{7.5}{2}(t+s)$. *The values of* r *for different* α *are obtained by applying Theorem 1 and presented in Table 2.*

Table 2. Convergence radii for Example 2.

$\alpha = -1$	$\alpha = -0.5$	$\alpha = 0$	$\alpha = 0.75$	$\alpha = 1$
$r_1 = 0.066667$	$r_1 = 0.066667$	$r_1 = 0.066667$	$r_1 = 0.066667$	$r_1 = 0.066667$
$r_2 = 0.012408$	$r_2 = 0.016263$	$r_2 = 0.023773$	$r_2 = 0.027609$	$r_2 = 0.029230$
$r_3 = 0.015374$	$r_3 = 0.018380$	$r_3 = 0.023773$	$r_3 = 0.026383$	$r_3 = 0.027459$
$r = 0.012408$	$r = 0.0162627$	$r = 0.023773$	$r = 0.026383$	$r = 0.027459$

Example 3 ([35])**.** *Let us take* $X = Y = C[0,1]$ *and* $\mathcal{D} = \overline{S}(0,1)$. *We introduce the Hammerstein type operator*

$$F(x)(t) = x(t) - \int_0^1 \mathcal{G}(t,u)\, \frac{x(u)^2}{2}\, du,$$

with $x(t) \in C[0,1]$ *and* $\mathcal{G}(t,u) =$ *is defined on* $[0,1] \times [0,1]$ *by*

$$G_1(t,u) = \begin{cases} (1-t)u, & \text{if } u \le t \\ (1-u)t, & \text{if } t \le u \end{cases}.$$

We have $x^* = 0$. *Additionally,* $\omega_0(t) = \omega(t) = 0.125t$, $\omega_1(t) = 2$ *and* $\omega_2(t,s) = \frac{t+s}{16}$. *We apply Theorem 1 to compute the values of* r *for different* α. *Table 3 presents these values.*

Table 3. Convergence radii for Example 3.

$\alpha = -1$	$\alpha = -0.5$	$\alpha = 0$	$\alpha = 0.75$	$\alpha = 1$
$r_1 = 5.333334$	$r_1 = 5.333334$	$r_1 = 5.333334$	$r_1 = 5.333334$	$r_1 = 5.333334$
$r_2 = 0.876774$	$r_2 = 1.128755$	$r_2 = 1.594072$	$r_2 = 2.277393$	$r_2 = 2.666667$
$r_3 = 1.064670$	$r_3 = 1.259079$	$r_3 = 1.594072$	$r_3 = 2.063152$	$r_3 = 2.328353$
$r = 0.876774$	$r = 1.128755$	$r = 1.594072$	$r = 2.063152$	$r = 2.328353$

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and D.S.; visualization, I.K.A., D.S., C.I.A., S.K.P. and S.K.S.; supervision, I.K.A. All authors have read and agreed to the published version of the manuscript.

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