A Study of Improving Running Safety of a Railway Wagon with an Independently Rotating Wheel’s Flange

Evgeny Mikhailov 1, Stanislav Semenov 1, Hanna Shvornikova 1, Juraj Gerlici 2*, Maxim Kovtanets 3, Ján Dižo 2*, Miroslav Blatnický 2 and Jozef Harušinec 2

Abstract: The main objective of this work is to study the possibilities of improving the running safety of a railway wagon with independently rotating wheels by changing their design symmetrically mounted on an axle. The article provides a discussion of the advantages and disadvantages of using the independently rotating wheels in a bogie of railway wagons. Their increasing tendency of derailment is described. The influence of a perspective constructive scheme (PKS) of railway wagon wheels in comparison with a traditional constructive scheme (TKS) on running safety due to the climbing of a wheel flange onto a rail is studied. This work introduces a conceptual proposition of a technical solution to railway wheel design as well as containing the results of both analytical calculations as well as the results of multibody simulations. A PKS wheel design for a railway wheel is designed that allows independent rotation of its tread surface and of a guiding surface (i.e., of a flange) to each other, which both are arranged symmetrically on a wheelset axle. It brings features of the distribution of friction forces generating in a flange contact when the wheel with a TKS and with PKS move on a rail. It is possible to conclude with the help of the obtained results that the use of wheels with the PKS is advisable for the reduction of the running resistance as well as for increasing the running safety of railway wagons.

Keywords: perspective constructive scheme; traditional constructive scheme; railway wagon; running safety; running resistance

1. Introduction

A review and analysis of well-known scientific works and the technical literature in the field of rail transport shows that, in recent decades, significant efforts have been made by scientists and engineers to improve the dynamic characteristics of railway wagons. This includes ensuring high speed and smoothness of movement with steady movement in straight sections of track, improving the performance of railway wagons when they enter curves, reducing wear and increasing the life time of wheels and rails. One of the promising ways to achieve these goals could be the use of independently rotating wheels (IRW) in bogies of railway wagons instead of the traditional constructive scheme of a wheel [1–6]. Such bogies have recently been increasingly used on urban low-floor railway wagons (trams) and they are becoming increasingly popular [7,8].

The use of IRW involves a complete or partial separation of the wheels in a wheelset by the coordination of their rotation around a common rotation axis. This allows for the
reduction or complete elimination of longitudinal slippage of the wheels on the rails and, accordingly, a reduction in the running resistance of railway wagons, especially in curved sections of railway tracks. The research results show that the use of IRW in comparison with railway wagons with conventional wheels result in [1,9] a reduction of wagging at high speed, a reduction of wear in the wheel/rail contact including wear of a flange and improving the characteristics of a railway wagon when it enters curves in the railway track due to the practical elimination of the longitudinal components of the creep forces. There is also the study about an application of a differential gear to a rail vehicle bogie similar to road vehicles [10]. This research has showed improved running properties of rail vehicles while running in a curve.

IRW can rotate at different angular speeds. This means that, during their rotational movement, the longitudinal forces of creep do not arise, which forms the control moment and centers a wheelset in a railway track. It leads to an increase in the angle to attack of a wheelset on a rail and the lateral forces and wear of the wheels and rails increases as well. On the other hand, this increases the tendency of derailment of railway wagons with IRW due to the climbing of the railway wheel flange onto the rail head. Obviously, these problems require the additional attention of researchers.

2. A Current State of the Problem

A number of various technical solutions have been proposed to solve the described disadvantages of IRW. Some relate to the setting up some given elastic-dissipative characteristics of a wheels’ torsion joint in a wheelset. The using of sprung wheels mounted on a rigid axle is the simplest solution [5,11,12]. Nowadays, sprung wheels (with rubber elements) are often used in urban railway wagons (trams), which operate at relatively low speeds and with low axle loads. However, an application of such wheel designs for mainline railway wagons is strongly limited based on a series of severe incidents.

More advanced technical solutions include special devices for reducing the torsional rigidity of a wheelset axle. A number of them are focused on ensuring the optimal characteristics of the torsion coupling of the wheels in a wheelset. These devices lock the wheels during running in straight sections of a track and thus completely eliminate their independent rotation. In the vice versa, the wheel coupling devices are unlocked or operate in a limited slip mode while running in track curves. The use of a limited slip differential device [7,13,14] is an example of this approach. It should be noted that the specialists of the German company MBB have been the founders of this solution. At the end of the last century, they carried out large and complex theoretical and experimental studies of a wheelset with the controlled slip. They used a magnetic powder clutch as a slip regulator.

The research of the recent decades proves that the concept of including an IRW is practically feasible by improving the wheel-to-bogie and bogie-to-body interaction characteristics. The application of mechatronic systems for monitoring of a wheelsets position in the horizontal plane for their radial guidance in a curved track section is one of the possible ways to ensure the optimal wheel/rail interaction of railway wagons (including IRW) with minimal lateral forces and low wear [15–20]. In railway wagon bogies with the radial guidance of wheelsets, the angles to attack of the wheelsets to a track are close to zero in the case of their running in curves. This leads to the minimization of the interaction forces in the wheel/rail contact, as well as the running resistance. The known designs of described railway wagon bogies include bogies with self-centering wheelsets, bogies with a semi-forced guidance of wheelsets and bogies with a forced guidance of wheelsets.

Bogies with self-centering wheelsets have shown the low efficiency in curves due to a higher level of rails wear and the higher running resistance when experiments on straight track sections have been performed. Somewhat better indicators for the reduction of flange wear intensity of the wheels in curves has been reached for railway wagons bogies with a semi-forced guidance of wheelsets (a passive mechanism of the radial guidance). However, experiences of the use of the passive mechanisms for the radial guidance of wheelsets
in order to reduce wheel flange wear in curves on European railways has not been very successful [21–25].

Achieving the maximum effect of reducing wheel flange wear intensity and the running resistance in curves is possible by an application of railway wagon bogies with an automatically controlled mechanism for the radial guidance of wheelsets, which works by means of using a mechatronic system [26,27]. The potential benefits that can be achieved in using such a mechatronic system make it an advisable subject for further activities on their development and research [16,17].

The results of a number of studies [28–31] have shown an increased tendency for derailment of railway wagons with IRW due to the climbing of a wheel flange on a rail head. When the IRW moves on a rail, the absence of longitudinal creep forces means that the friction forces in the wheel/rail contact act entirely in the lateral direction. As a result, the risk of climbing of the wheel flange onto the rail head increases at the shortest flange lifting distance [28,31]. Research presented in [29] has shown that the limiting value of the lateral and vertical forces ratio \(Y/Q\) has increased with increasing longitudinal forces in the wheel/rail contacts. Thus, the Nadal criterion characterizing the conditions for climbing of the wheel flange onto the rail head can be softened depending on the level of the longitudinal forces in the wheel/rail contacts. The presence of longitudinal creep forces contributes to the redistribution of the friction force components in the wheel/rail contacts. It reduces the effective friction coefficient when climbing the wheel flange on a rail head and increases the \(Y/Q\) ratio required for climbing the ridge the wheel flange onto the rail [32].

Studies aimed at improving the operational characteristics of railway wagons with IRW in areas described above have shown more or less encouraging results. They certainly should be continued.

In this work, we focus our attention on the use of the perspective constructive scheme (PKS) of a wheel design with independently rotating wheels, which allows independent rotation of a tread wheel surface and a guiding surface, i.e., its flange, including the running safety point of view.

As it is known, when a wheel rolls on a rail with two-point contact, the points of its main and flange contacts perform a rather complex spatial motion along cycloidal trajectories [33–35]. Then, in the flange contact of the traditional constructive scheme (TKS) of a wheel design, the parasitic differential slip occurs associated with the kinematic discrepancy between the geometric parameters of the rolling surfaces of the wheels and the kinematic parameters of the movement [36–38]. The power of the friction forces at the specified slip largely determines the wear size in the contact surfaces and the additional running resistance, especially in the situation, when a railway wagon is passing a curved track section. First of all, this disadvantage of the TKS concerns IRWs, which move with large angles and attack to the rail without their additional direction [2–4]. Attempts to reduce the loss of energy and wear during running on curved track sections such as through lubrication of the contact zones between the wheels and rails and a selection of conformal profiles of wheels and rails and others so far do not present a complete solution this problem.

It is not possible to completely avoid the aforementioned differential parasitic slip of the flanges without changing the traditional monolithic design of the wheel. Therefore, it is expedient to investigate the potential advantages of a principled change of the wheel design. For example, there is a solution which allows an independent rotation of the guiding surface (flange) relative to the tread surface of the wheel around of their common axis [37,38]. This will make it possible to break the closed power contour “lateral edge of the rail head—the wheel flange-rolling surface of the wheel-surface of the rail head” and will significantly reduce the level of differential parasitic slippage of the flanges on the rails. In the research [37,38], the analyses of the possibilities of reducing slippage and, correspondingly, decreasing the friction forces power in the flange contact by using PKS of wheels can be found. However, the issues of the running safety of railway wagons with
such wheel designs have been disclosed. The following text of the article includes the description of the working principle of the proposed technical solution of the PKS including a rotating flange and it also includes analytical calculations and results comparing some parameters of the TKS with the PKS.

Except for the technical solution described above, there are also applications which use a steering system. That system is not new and it dates back at least to the 1990s [39–41]. There is number of research papers that deal with this issue and its application in practice. Among others, there are works of Mei, T.X. and Goodal, R. [42,43] Liang, B., Liu, X.Y. and Iwnicki, S. [44,45], Konowrocki, R. and Kalinowski, D. [46,47], Farhat, N. and Ward, C.P. [48,49], Chudzikiewicz, A. [50–52] and others. The main objective of the steering mechanism is to improve the running safety of a rail vehicle, to better ride comfort for passengers, to lower wear of the contact couple wheel/rail, to extend the life-time of important components of a rail vehicle and a track etc. similar to the concerns described in this study. However, the application of a similar steering system is not being considered at this time for the PKS wheels, the details nor comparison of relevant and comparable results with the steering systems are not included.

3. Materials and Methods

Figure 1 shows schemes of a distribution of the forces for assessment of derailment, when the wheel flange climbs on the rail head. These schemes show external forces and reactions acting in the flange contact of the wheel with the rail at the moment when the wheel begins to egress on this contact. Figure 1 depicts the left wheel and rail, at which we can consider the symmetric principle of the wheelset design. This means that in case of the right wheel and rail, forces would be distributed mirror-image. The modulus and direction of the general reaction $R$ from the rail to the wheel are determined by the magnitude of the vertical $Q$ and horizontal $Y_C$ forces applied in the flange contact and their ratio.

![Figure 1](image-url)

**Figure 1.** Design schemes of the external forces and reactions acting in the contact point of the wheel and the rail at climb of the flange on the rail: (a) Side view; (b) Front view.

Dependencies determining the magnitude of the reactions $R$ acting in the flange contact can be written as follows:

\[ R = \sqrt{P^2 + Y^2}, \]  
\[ N = R \cdot \sin(\pi - \beta - \varphi_0), \]  
\[ \varphi_0 = \arctan\left(\frac{P}{Y}\right). \]
where $\beta$ is the angle of inclination of the wheel flange to the horizontal line, $P$, $Z$ and $N$ are the modules of the corresponding vectors, which directions are clear from Figure 1b and $Y_G$ is the guiding force. The $P$ force is equal to the $Q$ force.

The possibility of independent rotation of the tread wheel surface and of the guiding surface (flange) in the coordinate direction $\varphi$ (around their common axis of rotation) in the PKS wheel adds the additional degree of freedom to the considered mechanical system and it determines the need to verify the safety conditions for climbing the flange onto the rail head in two coordinates ($z$ and $\varphi$) at the same time. We adduct the moments of forces acting on the wheel tread surface and its guiding surface (flange) to the point of the flange contact $B$ and compose the equations of equilibrium of forces and moments for the coordinates $z$ and $\varphi$ (Figure 1a) in the following form:

$$\sum_i F_i^Z = 0, \quad \sum_i M_i^B = 0,$$

(4)

where $\sum_i F_i^Z$ is the sum of the forces acting on the wheel along the axis $O_z$ and $\sum_i M_i^B$ is the sum of the moments of the applied forces about the flange contact point $B$.

Then

$$\sum_i F_i^Z = Y_Z + F_Z - P = 0,$$

(5)

where $Y_Z$ is the vertical component of the reaction of the guiding force $Y$, $F_Z$ is the vertical component of the friction force in the flange contact and $P$ is the total force of the wheel weight and the weight of the part of the wagon structure attributable to it.

The beginning of the wheel derailment process is determined by the beginning separation of the wheel rolling surface, point $A$ (Figure 1), from the rail, the transition of the instantaneous center of rotation of the wheel to point $B$ (Figure 1) of the flange contact and sliding of this point up on the rail head along of the $O_Z$ axis under the influence of the vertical component of the reaction $Y_Z$ from the guiding force upon reaching the following ratio of power, which are attached to this point:

$$Y_Z > P - F_Z,$$

(6)

herein

$$Y_Z = \frac{Y}{\tan \beta}.$$

(7)

The values of the forces $P$ and $Y_Z$ are taken as given and can vary. It is necessary to determine the meaning of the force $F_Z$. Firstly, in order to identify the potential advantages of the wheel with the perspective construction scheme of a wheel design presented in [33,34], the features of the movement of wheels, of both wheel designs on rails with two-point contact, were analyzed. It has been determined that for the TKS of the wheel, the modulus and direction of the friction force vector in the flange contact are uniquely determined by the geometric characteristics of the wheel/rail contact and the angular speed of the wheel rotation. When the wheel with the PKS moves, the direction and modulus of the friction force vector of the flange on the rail depends also on the ratio of the angular velocities of rotation of the wheel tread surface of the wheel and its flange around of their common axis. When the directions and modules of the components of the friction force vector in the flange contact are determined, the following expressions are analytically obtained:

$$F_X^j = \mu \cdot N \cdot \cos \delta^j \cdot \cos \chi^j,$$

(8)

$$F_Y^j = \mu \cdot N \cdot \sin \delta^j,$$

(9)

$$F_Z^j = \mu \cdot N \cdot \cos \delta^j \cdot \sin \chi^j.$$  

(10)
Index “i” in Equations (8)–(10) indicates the belonging value to the particular wheel design (the values of the corresponding quantities for the PKS of the wheel will be denoted by the sign “∗”).

The parameters \( \delta^i \) and \( \chi^i \) used in Equations (8)–(10) are determined from the geometric relations in [33,34] as follows:

\[
\delta^i = \arctan \left( \frac{\sin \chi^i}{\tan \beta} \right), \quad \chi^i = \arctan \left( \frac{r_W + h_F \cdot \tan \psi \cdot \tan \beta}{h_F} \right),
\]

where \( K_W = \frac{\phi_W}{\phi_F} \), \( \psi \) is the angle to attack of the wheel on the rail and \( \beta \) is the angle of inclination of the wheel flange to the horizontal line.

The \( r_W \) and \( h_F \) values are shown in Figure 1.

The value of the vertical component of the friction force in the flange contact:

- For a wheel with the TKS:

\[
F_Z = \mu \cdot N \cdot \cos \delta \cdot \sin \chi = \mu \cdot \sqrt{Q^2 + Y_G^2} \cdot \sin \left[ \beta + \arctan \left( \frac{Q}{Y_G} \right) \right] \cdot \cos \delta \cdot \sin \chi; \quad (13)
\]

- For a wheel with the PKS:

\[
F_Z^∗ = \mu \cdot N \cdot \cos \delta^∗ \cdot \sin \chi^∗ = \mu \cdot \sqrt{Q^2 + Y_G^2} \cdot \sin \left[ \beta + \arctan \left( \frac{Q}{Y_G} \right) \right] \cdot \cos \delta^∗ \cdot \sin \chi^∗. \quad (14)
\]

Then, the condition (6) can be written as:

\[
\frac{Y_G}{\tan \beta} > Q^i - \mu \cdot \sqrt{Q^2 + Y_G^2} \cdot \sin \left[ \beta + \arctan \left( \frac{Q}{Y_G} \right) \right] \cdot \cos \delta^i \cdot \sin \chi^i; \quad (15)
\]

Let’s we characterize the stability condition of the wheel on the rail against climbing the wheel flange by the coefficient \( k_S^i \):

\[
k_S^i = \left\{ \frac{Q}{Y_G} - \mu \cdot \sqrt{Q^2 + Y_G^2} \cdot \sin \left[ \beta + \arctan \left( \frac{Q}{Y_G} \right) \right] \cdot \cos \delta^i \cdot \sin \chi^i \right\} \cdot \tan \beta. \quad (16)
\]

The process of climbing the wheel flange onto the rail can begin at values \( k_S^i < 1 \).

In the next step of the research, a multibody model of a railway wagon bogie was created. The multibody model was set up in Simpack software (Dessault Systèmes, Velizy-Villacoublay, France). This software is a commercial multibody simulation software used for simulating non-linear motion of large and complex multibody systems. Parameters of the bogie correspond to a passenger railway wagon and the main data are listed in Tables 1 and 2.

Table 1. The main parameters of a railway wagon bogie.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight per a wheelset</td>
<td>13.8</td>
<td>t</td>
</tr>
<tr>
<td>Wheelbase</td>
<td>2560</td>
<td>mm</td>
</tr>
<tr>
<td>Track</td>
<td>1435</td>
<td>mm</td>
</tr>
<tr>
<td>Wheel profile</td>
<td>S1002</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 2. Parameters of individual components of the bogie model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheelset Mass</td>
<td>$m_{W}$</td>
<td>1500</td>
<td>kg</td>
</tr>
<tr>
<td></td>
<td>$I_{Wx}$</td>
<td>830</td>
<td>kg·m²</td>
</tr>
<tr>
<td></td>
<td>$I_{Wy}$</td>
<td>830</td>
<td>kg·m²</td>
</tr>
<tr>
<td></td>
<td>$I_{Wz}$</td>
<td>140</td>
<td>kg·m²</td>
</tr>
<tr>
<td></td>
<td>$l_{W}$</td>
<td>2000</td>
<td>mm</td>
</tr>
<tr>
<td>Frame Mass</td>
<td>$m_{B}$</td>
<td>2660</td>
<td>kg</td>
</tr>
<tr>
<td></td>
<td>$I_{Bx}$</td>
<td>1790</td>
<td>kg·m²</td>
</tr>
<tr>
<td></td>
<td>$I_{By}$</td>
<td>1520</td>
<td>kg·m²</td>
</tr>
<tr>
<td></td>
<td>$I_{Bz}$</td>
<td>3250</td>
<td>kg·m²</td>
</tr>
<tr>
<td>Axlebox Mass</td>
<td>$m_{A}$</td>
<td>175</td>
<td>kg</td>
</tr>
<tr>
<td></td>
<td>$I_{Ax}$</td>
<td>3.50</td>
<td>kg·m²</td>
</tr>
<tr>
<td></td>
<td>$I_{Ay}$</td>
<td>8.55</td>
<td>kg·m²</td>
</tr>
<tr>
<td></td>
<td>$I_{Az}$</td>
<td>8.55</td>
<td>kg·m²</td>
</tr>
</tbody>
</table>

For the purposes of the presented research, a single bogie running on a track was used. A multibody model of the bogie is shown in Figure 2. The bogie consists of rigid bodies connected by spring-damper elements. Individual elements of the railway bogie, such as springs, dampers and others are mounted symmetrically regarding to the longitudinal axis. The interface of wheels and rails is described by means of the FASTSIM contact model [53–56]. It is a well-known wheel/rail contact model which is widely used for simulation analyses of railway wagon bogies or entire wagons running on a track. The principle of symmetry is applied also in this contact model. Contact points are calculated symmetrically regarding to the longitudinal axis, it means, their coordinates are arranged mirrored. In comparison with a standard railway wheel (in our case called as a TKS), the presented model includes modification of the model resulting from the need to model a freely rotating flange. It is depicted by black color of the flange. The detail of railway wheels with independently rotating flange (PKS wheel design) is shown in Figure 3. It is possible to recognize the principle of the symmetrical positions of the contact points regarding to the longitudinal axis.

Figure 2. An illustration of the railway wagon bogie with independently rotating wheel flanges created in Simpack software.
The results of calculating values of the coefficient $k_i^S$ for the wheels of both designs (TKS as well as PKS) depending on the value of the guiding force $Y$ in the flange contact are shown in Figure 4. These results are calculated for the chosen parameters as following: $Q = 125$ kN, $\psi = 0.015$ rad and $K_W = 1.021$.

![Figure 3](image3.png)

**Figure 3.** The detail of the wheel/rail interface of the created multibody model: 1—a wheel tread, 2—a wheel flange, 3—a rail.

![Figure 4](image4.png)

**Figure 4.** Dependency graphs, $k_i^S = f(Y_G)$: (a)—A mathematical model, (b)—A Simpack model.

This figure contains a comparison of achieved results for the mathematical model (Figure 4a) and for the multibody model (Figure 4b). We see that the value of the coefficient $k_i^S$, which characterizes the stability condition of the wheels against derailment of rails, depends on the magnitude of the guiding force $Y$. Practically, it is the same for the wheels of both wheel design schemes. Differences of values of this coefficient $k_i^S$ are determined due to an influence of the wheel design on the distribution of individual components of the total friction force in the contact of the flange. Namely, there are values of the vertical component of this force. Comparing of two graphs shown in Figure 4, we can recognize that multibody simulations of the bogie with independently rotating flanges have reached results, which are very close to the described mathematical model depicted in Figure 4a. Results are similar for both TKS as well as PKS schemes.

In this analyzed case, the considered coefficient $k_i^S$ characterizes the possibility of an occurrence of the process of climbing of a wheel flange onto a rail. The determination of the stability of the wheels of both design schemes of the wheel against climbing onto a rail under the given loading conditions would be more informative.

The safety condition of derailment when the wheel flanges climb onto the rail heads is one of the most important characteristics of any railway wagons [28,31,36]. Traditionally, this stability condition for railway wagons is usually estimated by means of the Nadal criterion $K_N$ [57]. According to this criterion, the following condition must be met in order...
for the flange to slide down the rail head (i.e., for the wheel, which does not climb onto the rail head):

\[ K_N = K_{Nad} \cdot \frac{V}{G} \geq [K_N], \]  

where \([K_N]\) is the permissible value of the stability condition and \(K_{Nad}\) is the normalized Nadal factor reflecting the limiting value of the ratio \(\frac{V}{G}\) and it is given by the formulation:

\[ K_{Nad} = \frac{\tan \beta - \mu}{1 + \mu \cdot \tan \beta}, \]  

where \(G\) is the horizontal force of pressure of the attacking wheel on the rail, \(V\) is the vertical force of pressure of the attacking wheel on the rail, \(\beta\) is the angle of inclination of the wheel flange to the horizontal line and \(\mu\) is the coefficient of friction between the flange and the rail head.

When the simplicity of this criterion is taken into account, it is traditionally widely used in practice for various modifications, since it is well suited for comparing the safety performance of various designs of railway wagons.

However, it should be noted that the Nadal criterion is “dotty”, i.e., it is valid at a given time and it takes into account (in addition to the ratio of the vertical and horizontal loads in the wheel/rail contact) only two parameters: the coefficient of sliding friction between the wheel and the rail \(\mu\) and the angle of inclination of the wheel flange to the horizontal line \(\beta\).

4. Results and Discussion

The classical Nadal criterion does not take into account the influence the important factor of the angle to attack \(\psi\) of the wheel on the rail. However, the main number of wheel derailments when they climb the flange onto the rail was recorded precisely at significant values of the angle \(\psi\). In addition, in our case, it is also necessary to take into account the fact that changing the traditional design of the wheel of a railway wagon leads to an additional degree of freedom of the design. It allows the independent rotation of the wheel tread surface and of the wheel flange around their common axis of rotation. In this case, the direction of action and the values of the friction force components in the flange contact change.

Thereof, when we want to apply the traditional stability criterion for our case, it has to be modified. In particular, the difference of the values of the friction force component has to be taken into account, which acts in the \(yoz\) plane (Figure 1b) for different design options of wheels.

We are interested in the magnitude of the friction force component \(F_{YZ}\). It acts in the \(yoz\) plane and it can be determined in accordance with Equations (9) and (10) by the expression:

\[ F_{YZ} = \sqrt{(F_Y)^2 + (F_Z)^2} = \sqrt{(\mu \cdot N \cdot \sin \delta i)^2 + (\mu \cdot N \cdot \cos \delta i \cdot \sin \chi i)^2} = \]  

\[ = \mu \cdot N \cdot \sqrt{(\sin \delta i)^2 + (\cos \delta i \cdot \sin \chi i)^2}, \]  

(19)

Then, the expression for determining the corresponding component \(\mu_{YZ}^i\) of the friction coefficient can be written as follows:

\[ \mu_{YZ}^i = \mu \cdot \sqrt{(\sin \delta i)^2 + (\cos \delta i \cdot \sin \chi i)^2}. \]  

(20)

Taking into account the differences of the angles values \(\delta i\) and \(\chi i\) for different variants of the design of the wheels, the corresponding expressions for the modified Nadal criterion for comparative calculations can be written in the following form:

- For a wheel with the TKS:
\[
K_N = \frac{\tan \beta - \mu \cdot \sqrt{(\sin \delta^*)^2 + (\cos \delta \cdot \sin \chi)^2}}{1 + \mu \cdot \sqrt{(\sin \delta^*)^2 + (\cos \delta \cdot \sin \chi)^2}} \cdot \left( \frac{V}{G} \right) \geq [K_N]. \tag{21}
\]

- For a wheel with the PKS:

\[
K_N = \frac{\tan \beta - \mu \cdot \sqrt{(\sin \delta^*)^2 + (\cos \delta \cdot \sin \chi)^2}}{1 + \mu \cdot \sqrt{(\sin \delta^*)^2 + (\cos \delta \cdot \sin \chi)^2}} \cdot \left( \frac{V}{G} \right) \geq [K_N]. \tag{22}
\]

In order to assess the influence of the wheel design on the derailment stability when the flange is climbing in, we calculate the ratio \( \Delta K_N = \frac{K_{NS}}{K_{NK}} \) for the same values of the ratio \( \frac{V}{G} \). The graph of the dependency of this indicator \( \Delta K_N = f(\psi, K_W) \) is shown in Figure 5.

This graph indicates that, in a certain range of operating parameters, the values of the Nadal criterion for the PKS of the wheel are less than the corresponding values of this criterion for the TKS wheel. This is due to the influence of the design of the wheel on the distribution of the friction force in the contact along the coordinate axes.

The equilibrium condition of the PKS of the wheel on the rail in the direction of the coordinate \( \phi \) is as follows:

\[
\sum_i M_{i}^\beta = M_X^\beta + M_Z^\beta, \tag{23}
\]

where \( M_X^\beta \) and \( M_Z^\beta \) are the total moments of the longitudinal and vertical forces relatively to their centers of the flange contact, respectively.

Figure 5. Graph of dependency: \( \Delta K_N = f(\psi, K_W) \).

The diagram in Figure 1a shows that:

\[
M_X^\beta = P_X \cdot (r_W + h_F), \tag{24}
\]

It follows that \( M_X^\beta \) is the moment of the longitudinal force \( P_X \) applied in the center of rotation \( O \) of the wheel. The force \( P_X \) equals to the force \( F_X (P_X = F_X) \), where \( F_X = \mu_x \cdot N \) and it is the longitudinal component of the friction force in the flange contact.

The magnitude of the moment \( M_Z^\beta \) is determined by the sum of the weight of the wheel itself and the vertical load on the wheel from the bogie \( P_Z \) as well as by the value \( x_f \) of the “advancing” of the flange contact point:

\[
M_Z^\beta = Q \cdot x_f. \tag{25}
\]
Obviously, the value of the moment $M_Z$ resulting from the same weight of the TKS and PKS of wheels does not depend on their designs features.

If the condition $M_Z > M_X$ is met, the rotation of the flange is possible, relative to the wheel tread surface of the PKS of the wheel, in the opposite direction to the wheel rolling direction.

Figures 6 and 7 show the results of the calculated values of the corresponding moments performed for both considered designs of wheels with the following chosen initial conditions: $Q = 125$ kN, $Y_G = 50$ kN, $K_W = 1.021$, $\mu_r = 0.25$, $r_W = 0.475$ m, $l_f = 0.01$ m.

Graphs depicted in Figures 6 and 7 show that the value of the moment $M_Z$ for the PKS of the wheel will exceed the value $M_X$ even at small angles to attack ($\psi > 0.0017$ rad or about $0.1^\circ$). Under these conditions, climbing of the wheel onto the rail head with the separation of the rolling surface of the wheel from the surface of the rail head is practically impossible, since its flange under the action of the applied system of forces in these conditions tends turn in the opposite direction to the direction of wheel rolling.

The graph of dependencies (Figure 6a) is compared with results obtained from multibody simulations (Figure 6b). We see that waveforms of $M_{x1}$, $M_{x2}$ and $M_z$ have similar tendencies to curves obtained from the mathematical model. The greatest differences of these two approaches are obvious for the $M_{x2}$. Decreasing moment $M_{x2}$ does not causes so small angle $\psi$ in the case of the multibody simulation in comparison with the mathematical model.

The dependency graph $M_i = f(\psi)$ calculated for the same initial conditions data is shown in Figure 7.

The graph in Figure 7 shows that the noted feature of the movement of the PKS elements is characteristic for a certain range of values $K_W$ corresponding to the given geometric parameters of the wheel, when the flange is climbing onto the rail head.

The presented research represents a conceptual proposal of a possible solution of a railway wagons wheelsets, which could contribute to reduce negative effects occurring mainly when a railway wagon enters small curves and runs in them.

So far, the study is based on analytical calculations and their results added by multibody simulations with a particular type of a railway wagon bogie. Future research would be needed to verify the correctness of the current direction of the research and these results. This is intended to be performed on one hand by more detailed calculations using multibody simulations (including an entire railway wagon model, various axle loads etc.) and on the other hand by tests on the built test stand [58,59].
5. Conclusions

The analysis of the results of the theoretical studies carried out to assess the derailment safety of a wheel for a situation, when the flange climbs onto the rail, has allowed us to establish that the conditions for the implementation of the beginning of this process are practically identical for both the TKS as well as the PKS of wheels.

For wheels of both design schemes, calculations of the safety factor for derailment were carried out according to the modified Nadal criterion. It takes into account the effect of the angle to attack of the wheel on the rail and the features of the construction design of a wheel. The calculation results show that the values of the modified Nadal criterion for the PKS wheel are slightly smaller than the corresponding values of this criterion for the TKS of a wheel in a certain range of operating parameters of wheel movement. This is due to the influence of the design of a wheel on the distribution of the components of the friction force in the flange contact along the coordinate axes. However, this may not be crucial for the use of PKS wheels. The indicated decrease in the calculated indicator is observed in a rather narrow range of wheel motion parameters, which is practically not realized under operating conditions (small angles of attack, a stable fixed ratio of the angular velocities of rotation of the support and guide surfaces of the PKS wheel). After all, it is widely known that the overwhelming number of derailments occur at high values of the angle to attack of the wheel on the rail. In the range of high angles of the attack, the calculated values of the modified Nadal criterion are obtained almost identical for both design wheel schemes.

Besides that, calculations have shown that rolling a PKS wheel with the considered parameters and loading modes onto a rail with separation of the rolling surface of the wheel from the rolling surface of the rail head at angles of the wheel attacking onto the rail of more than 0.0017 rad is practically impossible. Under the action of a system of forces applied to the wheel under these conditions, the wheel flange PKS tends to turn in the direction opposite to the direction of wheel rolling.

Thereof, the expediency of using wheels of a perspective construction scheme in bogies of railway wagons with independently rotating wheels to reduce the level of running resistance and increase the safety of this movement is theoretically confirmed. For the practical confirmation of this thesis, further research will be focused on calculations and evaluation of the behavior on models and simulations carried out in multibody software. Moreover, this thesis could be also evaluated by tests on a test bench.

Undoubtedly, the use of wheels of a promising design scheme in railway wagons with IRW in no way can limit the possibilities of using various mechatronic control systems for guiding wheels in the horizontal plane in order to improve the dynamics of wagons, reduce the wear of wheels and rails, reduce running resistance and increase the running safety.

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