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Some Single-Valued Neutrosophic Uncertain Linguistic Maclaurin Symmetric Mean Operators and Their Application to Multiple-Attribute Decision Making

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Abstract: The Maclaurin symmetric mean (MSM) operator has a good aggregation effect. It can capture the relationships between multiple input parameters, and the neutrosophic uncertain linguistic numbers can well represent some indeterminate and incomplete information. In this paper, we combine the MSM operator with the singled-valued neutrosophic uncertain linguistic set and propose some MSM operators based on single-valued neutrosophic uncertain linguistic environment, such as single-valued neutrosophic uncertain linguistic Maclaurin symmetric mean(SVNULMSM) operator and single-valued neutrosophic uncertain linguistic generalized Maclaurin symmetric mean(SVNULGMSM) operator. First of all, according to the neutrosophic set and uncertain linguistic numbers, we propose the single-valued neutrosophic uncertain linguistic numbers and give some operating rules. Furthermore, considering the influence of attribute weight on the results, we introduce the weighted SVNULMSM operator and weighted SVNULGMSM operator. Then, we propose a method to deal with MSDM problems and give the specific steps to solve the problem. Finally, an investment example is used to verify the effectiveness of our method, and the superiority of the method is proved by comparing with other methods.

Keywords: multiple-attribute decision making; Maclaurin symmetric mean operator; uncertain linguistic number; neutrosophic set

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1. Introduction

Multi-attribute decision making problems are very common in many research fields. In real life, many uncertain and inconsistent pieces of information cannot be described by specific values. In order to solve this problem, Zadeh [1] firstly proposed the concept of fuzzy set theory. However, a fuzzy set cannot express the non-membership degree. Then, Atanassov [2] presented the concept of intuitionistic fuzzy sets (IFSs), which involve degrees of membership and degrees of non-membership. The IFS includes the membership $T(x)$ and non-membership $F(x)$, and $T(x), F(x) \in [0,1]$, $0 \leq T(x) + F(x) \leq 1$. Zwick et al. [3] proposed triangular IFS. Zeng and Li [4] defined trapezoidal IFS. Yager [5] discuss the need for obtaining aggregation operations on ordinal-based intuitionistic fuzzy subsets (OBIFS) and applied fuzzy multi-criteria technology to mobile "apps." Verma [6] defined some new operational-laws for LIFNs based on the linguistic scale function (LSF) and proposed a generalized linguistic intuitionistic fuzzy weighted average (GLIFWA) operator for aggregating LIFNs. However, IFSs cannot process indeterminate information and inconsistent information. Therefore, Smarandache [7] introduced the neutrosophic set (NS), which is composed of truth membership $T(x)$, falsity membership $F(x)$, and indeterminacy membership $I(x)$, and they are independent of each other. NS is an extension of IFS. Wang [8] presented interval NS(INS), which can represent the function of truth membership, falsity membership, and indeterminacy membership by the interval value.

Wang et al. [9] extended some new sets of NSs and proposed a single-valued neutrosophic set (SVNS). Fan [10] defined a new form of SVNULS and expressed the weight of each expert in the uncertain linguistic part. Ji [11] established an MABAC–ELECTRE method under SVNUL environments and used it to solve the problem of outsourcing provider selection. Kamac [12] proposed a Hamming distance-based formula to measure the distance between two SVNULNs and presented a game theory model based on the framework of LSVNS technique.

In daily life, in addition to multi-attribute decision making problems that can be processed by qualitative information, some uncertain information is difficult to be described by precise numbers. At this time, decision makers usually use some language terms (LTs), such as "good", "very good", "bad", and "very bad". Therefore, Herrera and Herrera-Viedma [13] proposed the LTs to deal with this kind of non-qualitative information. However, linguistic variables can only express membership, and cannot express non-membership. On this basis, Wang [14] presented the concept of the intuitionistic linguistic set, which combines the concepts of linguistic variables and intuitionistic fuzzy sets. After that, Ye [15] proposed the SVNLNs and used LTs to describe the truth membership, indeterminacy membership, and falsity membership.

As an effective tool, aggregation operators are widely used in MADM problems [16]. Yager [17] proposed the ordered weighted average (OWA) operator [18] for multi-attribute decision making. Bonferroni presented the Bonferroni mean (BM) [19–22] operator, which can capture the interrelationships among multiple input arguments. Furthermore, Beliakov [23] proposed the Heronian mean (HM) [24–26] operators. Like the BM operator, it can also capture the interrelationships among multiple input arguments. However, since they can only reflect the interrelationships between two arguments, they cannot effectively deal with multi-attribute value decision making problems. Therefore, in order to solve this problem, Maclaurin introduced the MSM operator, which can capture the interrelationships between multiple input arguments [27]. Qin and Liu [28] extended the intuitionistic fuzzy numbers to MSM operator. Ju et al. [29] introduced some novel weighted intuitionistic linguistic MSM operators. Liu and Qin [30] combined the MSM operator with linguistic intuitionistic fuzzy numbers to deal with the MADM problem. Zhong Y [31] proposed hesitant fuzzy power Maclaurin symmetric mean operators. Furthermore, Qin [32] proposed a new method with which to solve the MADM problem by combining MSM operators with uncertain linguistic variables. In addition, it is also an effective method for applying an MSM operator to hesitant fuzzy sets [33,34].

As we know, an MSM operator can capture the interrelationships between multiple input arguments, and language terms can describe non-qualitative information well. Meanwhile, a neutrosophic set has advantages in expressing incomplete, indeterminate, and inconsistent information. Therefore, the purpose of this paper is to apply the MSM operator to the neutrosophic uncertain linguictic numbers environment, and propose some Maclaurin symmetric mean operators based on single-valued neutrosophic uncertain linguistic numbers, including the weighted SVNULMSM operator and the weighted SVNULGMSM operator. Then, we give some definitions and properties and prove them. Finally, an investment case is used to verify the effectiveness of the proposed method, and compared it with other methods to prove its advantages.

The complete structure of the paper is as follows: In Section 2, we give the definitions and properties of uncertain linguistic numbers, the single-valued neutrosophic set, the Maclaurin symmetric mean (MSM) operator, and the generalized Maclaurin symmetric mean (GMSM) operator. In Section 3, we introduce the single-valued neutrosophic uncertain linguistic set composed of the uncertain linguistic numbers and the single-valued neutrosophic set, and give some operational rules. In Section 4, we extend MSM operators to single-valued neutrosophic uncertain linguistic numbers, and propose the SVNULMSM operator, SVNULGMSM operator, WSVNULMSM operator, and WSVNULGMSM operator. In Section 5, we introduce the decision making methods based on the WSVNULMSM and WSVNULGMSM operators and give the specific operation steps. In Section 6, we demonstrate the effectiveness of the method with an investment example. In Section 7, we give the conclusions.

2. Preliminaries

2.1. The Uncertain Linguistic Numbers

Let $S = \{s_0, s_1, \ldots, s_{l-1}\}$ be a linguistic set, and $s_i(i = 1, 2, \ldots, l-1)$ is a linguistic number; l is an odd value. For example, when $l = 7$, $S = (s_0, s_1, s_2, s_3, s_4, s_5, s_6) =$ (extremely poor, very poor, poor, medium, good, very good, extremely good).

Let s_i and s_j be any two linguistic numbers, and the characteristics are as follows: [35,36]

1. If $i > j$, then $s_i > s_j$.
2. There exists a negative operator: neg(s_i) = s_j, where $j = l - 1 - i$.
3. If $s_i >= s_j$, $max(s_i, s_j) = s_i$.
4. If $s_i <= s_j$, $min(s_i, s_j) = s_i$.

Let $\bar{S} = \{s_\alpha | \alpha \in R^+\}$ be a continuous linguistic set, and $\bar{S}$ meets the strictly monotonically increasing condition. Some operational rules are defined as follows [37]:

$$\beta s_i = s_{\beta \times i} \tag{1}$$

$$s_i + s_j = s_{i+j} \tag{2}$$

$$s_i \times s_j = s_{i \times j} \tag{3}$$

$${s_i}^n = s_{i^n} \tag{4}$$

Definition 1 ([8])**.** *Let $\tilde{s} = [s_a, s_b]$,$s_a, s_b \in \bar{S}$, $a \le b$ be an uncertain linguistic variable, where s_a is the lower limit of $\tilde{s}$ and s_b is the upper limit of $\tilde{s}$. $\tilde{s}$*

Suppose $\tilde{s_1} = [s_{a1}, s_{b1}]$ and $\tilde{s_2} = [s_{a2}, s_{b2}]$ are two uncertain linguistic variables. The operation laws are defined as follows [35,36,38]:

$$\tilde{s_1} + \tilde{s_2} = [s_{a1}, s_{b1}] + [s_{a2}, s_{b2}] = [s_{a1+a2-\frac{a1\times b1}{l}}, s_{b1+b2-\frac{b1\times b2}{l}}] \tag{5}$$

$$\tilde{s_1} \times \tilde{s_2} = [s_{a1}, s_{b1}] \times [s_{a2}, s_{b2}] = [s_{\frac{a1\times a2}{l}}, s_{\frac{b1\times b2}{l}}] \tag{6}$$

$$\lambda\tilde{s_1} = \lambda[s_{a1}, s_{b1}] = [s_{l-l(1-\frac{a1}{l})^\lambda}, s_{l-l(1-\frac{b1}{l})^\lambda}], \lambda \ge 0 \tag{7}$$

$$\tilde{s_1}^\lambda = [s_{a1}, s_{b1}]^\lambda = [s_{l(\frac{a1}{l})^\lambda}, s_{l(\frac{b1}{l})^\lambda}], \lambda \ge 0 \tag{8}$$

2.2. The Single-Valued Neutrosophic Set (SVNS)

Definition 2 ([7])**.** *Let X be a universe of discourse, with a generic element in X denoted by x. A SVNS A in X is*

$$A = \{x, \langle T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\} \tag{9}$$

where $T_A(x)$, $I_A(x)$, and $F_A(x)$ are the truth membership, indeterminacy membership, and falsity membership functions. For each points x in X, $T_A(x), I_A(x), F_A(x) \in [0, 1]$, and $0 \le T_A(x)+I_A(x)+F_A(x) \le 1$.

Definition 3 ([7])**.** *Suppose $A = \{x, \langle T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$, $B = \{x, \langle T_B(x), I_B(x), F_B(x)\rangle \mid x \in X\}$ be two SVNSs, and if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$ for all x in X, then $A \le B$.*

2.3. Maclaurin Symmetric Mean Operator

Definition 4 ([27–30])**.** *Let $x_i(i = 1, 2, \ldots, n)$ be a collection of non-negative real numbers. A MSM operator of dimensions n is a mapping $MSM^{(m)} : (R^+)^n \to R^+$, and it can be defined as follows:*

$$MSM^{(m)}(x_1, x_2, \ldots, x_n) = \left(\frac{\sum_{1\le i_1<i_2\cdots<i_m\le n} \prod_{j=1}^{m} x_{i_j}}{C_n^m}\right)^{\frac{1}{m}} \tag{10}$$

where $(i_1, \ldots, i_m)$ traverses all the m-tuple combinations of $(1, \ldots, n)$, and $C_n^m = \frac{n!}{m!(n-m)!}$ is the binomial coefficient. Furthermore, the x_{i_j} mean refers to the i_jth element in a particular arrangement.

Property 1. *The $MSM^{(m)}$ operator has the following properties:*

1. *Idempotency. If $x_i = x$ for each i, and then $MSM^{(m)}(x, , \ldots, x) = x$.*
2. *Monotonicity. If $x_i \le y_i$ for all i, $MSM^{(m)}(x_1, , \ldots, x_n) \le MSM^{(m)}(y_1, , \ldots, y_n)$.*
3. *Boundedness. $min\{x_1, \ldots, x_n\} \le MSM^{(m)}(x_1, \ldots, x_n) \le max\{x_1, \ldots, x_n\}$.*

Furthermore, when m takes different values, the $MSM^{(m)}$ operator can be converted to some special forms, as follows:

1. *When m = 1, the $MSM^{(m)}$ operator reduces to the average operator.*

$$MSM^{(1)}(x_1, \ldots, x_n) = \left(\frac{\sum_{1\le i_1\le n} x_{i_1}}{C_n^1}\right) = \frac{\sum_{i=1}^{n} x_i}{n} \tag{11}$$

2. *When m = 2, the $MSM^{(m)}$ operator reduces to the Bonferroni mean(BM) operator (p = q = 1).*

$$\begin{aligned} MSM^{(2)}(x_1, \ldots, x_n) &= \left(\frac{\sum_{1\le i_1<i_2\le n} \prod_{j=1}^{2} x_{i_j}}{C_n^2}\right)^{\frac{1}{2}} \\ &= \left(\frac{2\sum_{1\le i_1<i_2\le n} x_{i_1} x_{i_2}}{n(n-1)}\right)^{\frac{1}{2}} \\ &= \left(\frac{\sum_{i,j=1, i\ne j}^{n} x_i x_j}{n(n-1)}\right)^{\frac{1}{2}} \\ &= BM^{1,1}(x_1, \ldots, x_n) \end{aligned} \tag{12}$$

3. *When m = 3, the $MSM^{(m)}$ operator reduces to the generalized Bonferroni mean (GBM) operator (p = q = r = 1).*

$$\begin{aligned} MSM^{(3)}(x_1, \ldots, x_n) &= \left(\frac{\sum_{1\le i_1<i_2<i_3\le n} \prod_{j=1}^{3} x_{i_j}}{C_n^3}\right)^{\frac{1}{3}} \\ &= \left(\frac{6\sum_{1\le i_1<i_2<i_3\le n} x_{i_1} x_{i_2} x_{i_3}}{n(n-1)(n-2)}\right)^{\frac{1}{3}} \\ &= \left(\frac{\sum_{i,j,k=1, i\ne j\ne k}^{n} x_i^1 x_j^1 x_k^1}{n(n-1)(n-2)}\right)^{\frac{1}{3}} \\ &= GBM^{1,1,1}(x_1, \ldots, x_n) \end{aligned} \tag{13}$$

4. *When m = n, the $MSM^{(m)}$ operator reduces to the geometric mean operator.*

$$MSM^{(n)}(x_1, \ldots, x_n) = \left(\frac{\sum_{1\le i_1<\cdots<i_n\le n} \prod_{j=1}^{n} x_{i_j}}{C_n^n}\right)^{\frac{1}{n}} = \left(\prod_{j=1}^{n} x_j\right)^{\frac{1}{n}} \tag{14}$$

Definition 5 ([27–31])**.** *Let x_i be the set of non-negative real numbers and $p_1, \ldots, p_m \ge 0$. A generalized MSM operator of dimension n is a mapping $GMSM^{(m,p_1,\ldots,p_m)} : (R^+)^n \to R^+$, and it is defined below:*

$$GMSM^{(m,p_1,\ldots,p_m)}(x_1, \ldots, x_n) = \left(\frac{\sum_{1\le i_1<\cdots<i_m\le n} \prod_{j=1}^{m} x_{i_j}^{p_j}}{C_n^m}\right)^{\frac{1}{p_1+p_2+\cdots+p_m}} \tag{15}$$

where $(i_1, i_2, \ldots, i_m)$ traverses all the m-tuple combination of $(1, 2, \ldots, n)$, and $C_n^m = \frac{n!}{m!(n-m)!}$ is the binomial coefficient.

Property 2. *The $GMSM^{(m,p_1,\ldots,p_m)}$ operator has the following properties:*

1. *Idempotency. If $x_i = x$ for each i, and then $GMSM^{(m,p_1,\ldots,p_m)}(x, \ldots, x) = x$*
2. *Monotonicity. if $x_i \leq y_i$ for each i, and then $GMSM^{(m,p_1,\ldots,p_m)}(x_1, x_2, \ldots, x_n) \leq GMSM^{(m,p_1,\ldots,p_m)}(y_1, y_2, \ldots, y_n)$*
3. *Boundedness. $min\{x_1, \ldots, x_n\} \leq GMSM^{(m,p_1,\ldots,p_m)}(x_1, x_2, \ldots, x_n) \leq max\{x_1, \ldots, x_n\}$*

Furthermore, when m takes different values, the $GMSM^{(m,p_1,\ldots,p_m)}$ operator can be converted to some special forms, as follows:

1. *When m = 1, the $GMSM^{(m,p_1,\ldots,p_m)}$ operator is as follows:*

$$GMSM^{(1,p_1)}(x_1, x_2, \ldots, x_n) = \left(\frac{\sum_{1\leq i_1\leq n} x_{i_1}^{p_1}}{C_n^1}\right)^{\frac{1}{p_1}} = \left(\frac{\sum_{i=1}^{n} x_i^{p_1}}{n}\right)^{p_1} \tag{16}$$

2. *When m = 2, the $GMSM^{(m,p_1,\ldots,p_m)}$ operator is as follows:*

$$\begin{aligned} GMSM^{(2,p_1,p_2)}(x_1, x_2, \ldots, x_n) &= \left(\frac{\sum_{1\leq i_1<i_2\leq n} x_{i_1}^{p_1} x_{i_2}^{p_2}}{C_n^2}\right)^{\frac{1}{p_1+p_2}} \\ &= \left(\frac{2\sum_{1\leq i<j\leq n} x_i^{p_1} x_j^{p_2}}{n(n-1)}\right)^{\frac{1}{p_1+p_2}} \\ &= \left(\frac{\sum_{i,j=1,i\neq j}^{n} x_i^{p_1} x_j^{p_2}}{n(n-1)}\right)^{\frac{1}{p_1+p_2}} \\ &= BM^{p_1,p_2} \end{aligned} \tag{17}$$

3. *When m = 3, the $GMSM^{(m,p_1,\ldots,p_m)}$ operator is as follows:*

$$\begin{aligned} GMSM^{(2,p_1,p_2,p_3)}(x_1, x_2, \ldots, x_n) &= \left(\frac{\sum_{1\leq i_1<i_2<i_3\leq n} x_{i_1}^{p_1} x_{i_2}^{p_2} x_{i_3}^{p_3}}{C_n^3}\right)^{\frac{1}{p_1+p_2+p_3}} \\ &= \left(\frac{6\sum_{1\leq i<j<k\leq n} x_i^{p_1} x_j^{p_2} x_k^{p_3}}{n(n-1)(n-2)}\right)^{\frac{1}{p_1+p_2+p_3}} \\ &= \left(\frac{\sum_{i,j,k=1,i\neq j\neq k}^{n} x_i^{p_1} x_j^{p_2} x_k^{p_3}}{n(n-1)(n-2)}\right)^{\frac{1}{p_1+p_2+p_3}} \\ &= GBM^{p_1,p_2,p_3} \end{aligned} \tag{18}$$

4. *When m = n, the $GMSM^{(m,p_1,\ldots,p_m)}$ operator is as follows:*

$$\begin{aligned} GMSM^{(n,p_1,\ldots,p_n)}(x_1, \ldots, x_n) &= \left(\frac{\sum_{1\leq i_1<\cdots<i_n\leq n} \prod_{j=1}^{n} x_{i_j}^{p_j}}{C_n^n}\right)^{\frac{1}{p_1+p_2+\cdots+p_n}} \\ &= \left(\prod_{j=1}^{n} x_j^{p_j}\right)^{\frac{1}{p_1+p_2+\cdots+p_n}} \end{aligned} \tag{19}$$

5. *When $p_1 = p_2 = \cdots = p_m = 1$, the $GMSM^{(m,p_1,\ldots,p_m)}$ operator is as follows:*

$$\begin{aligned} GMSM^{(m,1,\ldots,1)}(x_1, \ldots, x_n) &= \left(\frac{\sum_{1\leq i_1<\cdots<i_n\leq n} \prod_{j=1}^{m} x_{i_j}^{1}}{C_n^m}\right)^{\frac{1}{m}} \\ &= \left(\frac{\sum_{1\leq i_1<\cdots<i_m\leq n} \prod_{j=1}^{m}}{C_n^m}\right)^{\frac{1}{m}} \\ &= MSM^{(m)}(x_1, \ldots, x_n) \end{aligned} \tag{20}$$

3. The Single-Valued Neutrosophic Uncertain Linguistic Set

Definition 6 ([10]). *Let X be a universe of discourse, with a generic element in X denoted by x, and $[S_{\mu(x)}, S_{\nu(x)}] \in \tilde{S}$, then*

$$A = \{\langle x, [S_{\mu(x)}, S_{\nu(x)}], (T(x), I(x), F(x))\rangle | x \in X\} \tag{21}$$

where $S_{\mu(x)}, S_{\nu(x)} \in \bar{S}$, and $T(x)$, $I(x)$, $F(x) \in [0, 1]$.

Definition 7 ([24]). *Let $A = \{\langle x, [S_\mu, S_\nu], (T, I, F)\rangle | x \in X\}$ be an SVNULS, and $a = \{\langle [S_\mu, S_\nu], (T, I, F)\rangle\}$ is called an SVNULN.*

Suppose $a_1 = \langle [S_{\mu_1}, S_{\nu_1}], (T_1, I_1, F_1)\rangle$ and $a_2 = \langle [S_{\mu_2}, S_{\nu_2}], (T_2, I_2, F_2)\rangle$ are any two SVNULNs, and the operational laws are defined as follows:

$$a_1 + a_2 = \langle [S_{\mu_1+\mu_2-\frac{\mu_1\mu_2}{l}}, S_{\nu_1+\nu_2-\frac{\nu_1\nu_2}{l}}], (T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2)\rangle \tag{22}$$

$$a_1 \times a_2 = \langle [S_{\frac{\mu_1\mu_2}{l}}, S_{\frac{\nu_1\nu_2}{l}}], (T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2)\rangle \tag{23}$$

$$\lambda a_1 = \langle [S_{l-l(1-\frac{\mu_1}{l})^\lambda}, S_{l-l(1-\frac{\nu_1}{l})^\lambda}], (1 - (1 - T_1)^\lambda, I_1^\lambda, F_1^\lambda)\rangle \tag{24}$$

$$a_1^\lambda = \langle [S_{l(\frac{\mu_1}{l})^\lambda}, S_{l(\frac{\nu_1}{l})^\lambda}], T_1^\lambda, 1 - (1 - I_1)^\lambda, 1 - (1 - F_1)^\lambda)\rangle \tag{25}$$

These operational results are still SVNULNs.

Definition 8 ([24]). *Suppose $a_1 = \langle [S_{\mu_1}, S_{\nu_1}], (T_1, I_1, F_1)\rangle$ is an SVNULN, and the expectation value $E(a_1)$ of a_1 can be defined as follows:*

$$E(a_1) = \frac{1}{3}(2 + T_1 - I_1 - F_1) \times S_{\frac{\mu_1+\nu_1}{2}} = S_{l-l(1-\frac{\mu_1+\nu_1}{2l})^{\frac{1}{3}(2+T_1-I_1-F_1)}} \tag{26}$$

Definition 9 ([24]). *Suppose $a_1 = \langle [S_{\mu_1}, S_{\nu_1}], (T_1, I_1, F_1)\rangle$ is an SVNULN, and the accuracy function $H(a_1)$ of a_1 can be defined as follows:*

$$H(a_1) = (T_1 + I_1 + F_1) \times S_{\frac{\mu_1+\nu_1}{2}} = S_{l-l(1-\frac{\mu_1+\nu_1}{2l})^{T_1+I_1+F_1}} \tag{27}$$

Definition 10 ([24]). *Let $a_1 = \langle [S_{\mu_1}, S_{\nu_1}], (T_1, I_1, F_1)\rangle$ and $a_2 = \langle [S_{\mu_2}, S_{\nu_2}], (T_2, I_2, F_2)\rangle$ be any two SVNULNs; then*

1. *If $E(a_1) > E(a_2)$, then $a_1 > a_2$*
2. *If $E(a_1) = E(a_2)$, then*

   If $H(a_1) > H(a_2)$, then $a_1 > a_2$

   If $H(a_1) = H(a_2)$, then $a_1 = a_2$

4. Some Single-Valued Neutrosophic Uncertain Linguistic Maclaurin Symmetric Mean Operators

In this section, we propose SVNULMSM operators, SVNULGMSM operators, WSVNULMSM operators, and WSVNULGMSM operators.

4.1. The SVNULMSM Operator

Definition 11. *Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle (i = 1, \ldots, n)$ be a collection of SVNULNs. The SVNULMSM operator $\Omega^n \to \Omega$ is*

$$SVNULMSM^{(m)}(a_1, \ldots, a_n) = (\frac{\sum_{1\le i_1<\cdots<i_m\le n} \prod_{j=1}^m a_{i_j}}{C_n^m})^{\frac{1}{m}} \tag{28}$$

Ω is a set of SVNULNs and $m = 1, \ldots, n$.

Theorem 1. *Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle (i = 1, \dots, n)$ be a collection of SVNULNs, and $m = 1, \dots, n$. Then,*

$$SVNULMSM^{(m)}(a_1, \dots, a_n) = \left(\frac{\sum_{1\le i_1<\cdots<i_m\le n}\prod_{j=1}^{m} a_{i_j}}{C_n^m}\right)^{\frac{1}{m}} = \langle [S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\mu_{i_j}(k)}{l}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}}, S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\nu_{i_j}(k)}{l}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}}],$$
$$((1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m} T_{i_j(k)})^{\frac{1}{C_n^m}})^{\frac{1}{m}}, 1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-I_{i_j(k)}))^{\frac{1}{C_n^m}})^{\frac{1}{m}},$$
$$1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-F_{i_j(k)}))^{\frac{1}{C_n^m}})^{\frac{1}{m}})\rangle \tag{29}$$

where $k = 1, 2, \dots, C_n^m$, $a_{i_j(k)}$ represents the i_jth element in the kth permutation.

Proof.

$$a_{i_j(k)} = \langle [S_{\mu_{i_j(k)}}, S_{\nu_{i_j(k)}}], (T_{i_j(k)}, I_{i_j(k)}, F_{i_j(k)})\rangle (j = 1, \dots, m)$$
$$\Rightarrow \prod_{j=1}^{m} a_{i_j(k)} = \langle [S_{l\prod_{j=1}^{m}(\frac{\mu_{i_j(k)}}{l})}, S_{l\prod_{j=1}^{m}(\frac{\nu_{i_j(k)}}{l})}]$$
$$, (\prod_{j=1}^{m} T_{i_j(k)}, 1-\prod_{j=1}^{m}(1-I_{i_j(k)}), 1-\prod_{j=1}^{m}(1-F_{i_j(k)}))\rangle$$
$$\Rightarrow \sum_{1\le i_1<\cdots<i_m\le n}\prod_{j=1}^{m} a_{i_j(k)} = \langle [S_{l-l\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\mu_{i_j(k)}}{l}))}, S_{l-l\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\nu_{i_j(k)}}{l}))}]$$
$$, (1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(T_{i_j(k)})), \prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-I_{i_j(k)})), \prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-F_{i_j(k)})))\rangle$$
$$\Rightarrow \left(\frac{\sum_{1\le i_1<\cdots<i_m\le n}\prod_{j=1}^{m} a_{i_j}}{C_n^m}\right)^{\frac{1}{m}}$$
$$= \langle [S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\mu_{i_j}(k)}{l}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}}, S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\nu_{i_j}(k)}{l}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}}],$$
$$((1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m} T_{i_j(k)})^{\frac{1}{C_n^m}})^{\frac{1}{m}}, 1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-I_{i_j(k)}))^{\frac{1}{C_n^m}})^{\frac{1}{m}},$$
$$1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-F_{i_j(k)}))^{\frac{1}{C_n^m}})^{\frac{1}{m}})\rangle$$

□

Property 3. *Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle (i = 1, \dots, n)$ be a collection of SVNULNs, and $m = 1, 2, \dots, n$. There are some properties of $SVNULMSM^{(m)}$ operator shown below.*

1. *Idempotency. If the SVNULN $a_i = a = \langle [S_{\mu_a}, S_{\nu_a}], (T_a, I_a, F_a)\rangle$ for each $i (i = 1, 2, \dots, n)$, then $SVNULMSM^{(m)}(a, \dots, a) = a = \langle [S_{\mu_a}, S_{\nu_a}, (T_a, I_a, F_a)]\rangle$.*
2. *Commutativity. If $(a_1', \dots, a_n')$ is a permutation of $(a_1, \dots, a_n)$. Then $SVNULMSM^{(m)}(a_1', \dots, a_n') = SVNULMSM^{(m)}(a_1, \dots, a_n)$.*
3. *Monotonicity. Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle$ and $a_i' = \langle [S_{\mu_i'}, S_{\nu_i'}], (T_i', I_i', F_i')\rangle (i = 1, 2, \dots, n)$ be two collections of neutrosophic uncertain linguistic numbers, and if $a_i \le a_i'$—i.e., $S_{\mu_i} \le S_{\mu_i'}$, $S_{\nu_i} \le S_{\nu_i'}$, $T_i \le T_i'$, $I_i \ge I_i'$, and $F_i \ge F_i'$, for all i—then, $SVNULMSM^{(m)}(a_1, \dots, a_n) \le SVNULMSM^{(m)}(a_1', \dots, a_n')$.*
4. *Boundedness. $min\{a_1, \dots, a_n\} \le SVNULMSM^{(m)}(a_1, \dots, a_n) \le max\{a_1, \dots, a_n\}$.*

Proof. 1. If each $a_i = a$, then we get the equation below:
$SVNULMSM^{(m)}(a, \ldots, a)$
$= \langle [S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\mu_a}{l}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}}, S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\nu_a}{l}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}}],$
$((1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}T_a)^{\frac{1}{C_n^m}})^{\frac{1}{m}}, 1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-I_a))^{\frac{1}{C_n^m}})^{\frac{1}{m}},$
$1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-F_a))^{\frac{1}{C_n^m}})^{\frac{1}{m}})\rangle = \langle [S_{\mu_a}, S_{\nu_a}], (T_a, I_a, F_a)$

2. This property is clear and the proof is omitted.
3. if $S_{\mu_i} \leq S_{\mu_i'}$, $S_{\nu_i} \leq S_{\nu_i'}$, $T_i \leq T_i'$, $I_i \geq I_i'$, and $F_i \geq F_i'$ for all i, we can know $\prod_{j=1}^{m}\mu_i \leq \prod_{j=1}^{m}\mu_i'$, $\prod_{j=1}^{m}\nu_i \leq \prod_{j=1}^{m}\nu_i'$, $\prod_{j=1}^{m}T_i \leq \prod_{j=1}^{m}T_i'$, $\prod_{j=1}^{m}I_i \leq \prod_{j=1}^{m}I_i'$, $\prod_{j=1}^{m}F_i \leq \prod_{j=1}^{m}F_i'$, then $l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\mu_i}{l}))^{\frac{1}{C_n^m}})^{\frac{1}{m}} \leq l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\mu_i'}{l}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}$, c$l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\nu_i}{l}))^{\frac{1}{C_n^m}})^{\frac{1}{m}} \leq l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\nu_i'}{l}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}$, $(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}T_i)^{\frac{1}{C_n^m}})^{\frac{1}{m}} \leq (1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}T_i')^{\frac{1}{C_n^m}})^{\frac{1}{m}}$, $1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-I_i))^{\frac{1}{C_n^m}})^{\frac{1}{m}} \geq 1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-I_i'))^{\frac{1}{C_n^m}})^{\frac{1}{m}}$,
$1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-F_i))^{\frac{1}{C_n^m}})^{\frac{1}{m}} \geq 1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-F_i'))^{\frac{1}{C_n^m}})^{\frac{1}{m}}$.
Thus, we can get
$SVNULMSM^{(m)}(a_1, \ldots, a_n) \leq SVNULMSM^{(m)}(a_1', \ldots, a_n')$
4. Let $min(a_1, \ldots, a_n) = a_l = SVNULMSM^{(m)}(a_l, \ldots, a_l)$, $max(a_1, \ldots, a_n) = a_h = SVNULMSM^{(m)}(a_h, \ldots, a_h)$. According to the monotonicity, if $a_l \leq a_i$ and $a_i \leq a_h$ for all i, we have $a_l = SVNULMSM^{(m)}(a_l, \ldots, a_l) \leq SVNULMSM^{(m)}(a_1, \ldots, a_n)$ and $SVNULMSM^{(m)}(a_1, \ldots, a_n) \leq a_h = SVNULMSM^{(m)}(a_h, \ldots, a_h)$.
□

Furthermore, when m takes different values, the $SVNULMSM^{(m)}$ operator can be converted to some special forms, which are as follows:

1. *When m = 1, we have the formula below.*

$$\begin{aligned} SVNULMSM^{(1)}(a_1, \ldots, a_n) &= \frac{\sum_{1 \leq i_1 \leq n} a_{i_1}}{C_n^1} = \frac{\sum_{i=1}^{n} a_i}{C_n^1} \\ &= \langle [S_{l(1-\prod_{k=1}^{n}(1-\frac{\mu_k}{l})^{\frac{1}{n}})}, S_{l(1-\prod_{k=1}^{n}(1-\frac{\nu_k}{l})^{\frac{1}{n}})}], \\ &(1-\prod_{k=1}^{n}(1-T_k)^{\frac{1}{n}}, \prod_{k=1}^{n} I_k)^{\frac{1}{n}}, \prod_{k=1}^{n} F_k)^{\frac{1}{n}} \end{aligned} \tag{30}$$

2. *When m = 2, we have the formula below.*

$$\begin{aligned} SVNULMSM^{(2)}(a_1, \ldots, a_n) &= (\frac{\sum_{1 \leq i_1 < i_2 \leq n} a_{i_1} a_{i_2}}{C_n^2})^{\frac{1}{2}} \\ = \langle [S_{l(1-\prod_{k=1}^{C_n^2}(1-\frac{\mu_{i_1(k)}}{l}\frac{\mu_{i_2(k)}}{l})^{\frac{1}{C_n^2}})^{\frac{1}{2}}}, &S_{l(1-\prod_{k=1}^{C_n^2}(1-\frac{\nu_{i_1(k)}}{l}\frac{\nu_{i_2(k)}}{l})^{\frac{1}{C_n^2}})^{\frac{1}{2}}}], \\ &(1-(\prod_{k=1}^{C_n^2}(1-T_{i_1(k)}T_{i_2(k)})^{\frac{1}{C_n^2}})^{\frac{1}{2}}, \\ 1-(1-\prod_{k=1}^{C_n^2}&(1-(1-I_{i_1(k)}(1-I_{i_2(k)}))^{\frac{1}{C_n^2}})^{\frac{1}{2}}), \\ 1-(1-\prod_{k=1}^{C_n^2}&(1-(1-I_{i_1(k)}(1-I_{i_2(k)}))^{\frac{1}{C_n^2}})^{\frac{1}{2}}))\rangle \end{aligned} \tag{31}$$

3. *When m = n, we have the formula below.*

$$
\begin{aligned}
SVNULMSM^{(n)}(a_1,\ldots,a_n) &= \left(\frac{\sum_{1\le i_1<\cdots<i_n\le n}\prod_{j=1}^{n} a_{i_j}}{C_n^n}\right)^{\frac{1}{n}} \\
&= \langle [S_{l(\prod_{j=1}^{n}(\frac{\mu_j}{l}))^{\frac{1}{n}}}, S_{l(\prod_{j=1}^{n}(\frac{\nu_j}{l}))^{\frac{1}{n}}}], \\
&((\prod_{j=1}^{n} T_j)^{\frac{1}{n}}, 1-(\prod_{j=1}^{n}(1-I_j))^{\frac{1}{n}}, 1-(\prod_{j=1}^{n}(1-F_j))^{\frac{1}{n}})\rangle
\end{aligned}
\tag{32}
$$

4.2. The SVNULGMSM Operator

Definition 12. *Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle (i = 1,\ldots,n)$ be a collection of SVNULNs. The SVNULGMSM operator $\Omega^n \to \Omega$ is*

$$
SVNULGMSM^{(m,p_1,\ldots,p_m)}(a_1,\ldots,a_n) = \left(\frac{\sum_{1\le i_1<\cdots<i_m\le n}\prod_{j=1}^{m} a_{i_j}^{p_j}}{C_n^m}\right)^{\frac{1}{p_1+\cdots+p_m}} \tag{33}
$$

where Ω a set of SVNULNs and $m = 1,\ldots,n$.

Theorem 2. *Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle (i = 1,\ldots,n)$ be a collection of SVNULNs, and $m = 1,\ldots,n$. Then,*

$$
\begin{aligned}
SVNULGMSM^{(m,p_1,\ldots,p_m)}(a_1,\ldots,a_n) &= \left(\frac{\sum_{1\le i_1<\cdots<i_m\le n}\prod_{j=1}^{m} a_{i_j}^{p_j}}{C_n^m}\right)^{\frac{1}{p_1+\cdots+p_m}} \\
&= \langle [S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\mu_{i_j(k)}}{l})^{p_j})^{\frac{1}{C_n^m}})^{\frac{1}{p_1+\cdots+p_m}}}, \\
&S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\nu_{i_j(k)}}{l})^{p_j})^{\frac{1}{C_n^m}})^{\frac{1}{p_1+\cdots+p_m}}}], \\
&((1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(T_{i_j(k)})^{p_j})^{\frac{1}{C_n^m}})^{\frac{1}{p_1+\cdots+p_m}}, \\
&1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-I_{i_j(k)})^{p_j})^{\frac{1}{C_n^m}})^{\frac{1}{p_1+\cdots+p_m}}, \\
&1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-F_{i_j(k)})^{p_j})^{\frac{1}{C_n^m}})^{\frac{1}{p_1+\cdots+p_m}})\rangle
\end{aligned}
\tag{34}
$$

where $k = 1,\ldots,C_n^m$, $a_{i_j(k)}$ represents the i_jth element in the kth permutation. The proof of Theorem 2 is similar to that of Theorem 1, so it is omitted here.

Property 4. *Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle (i = 1,\ldots,n)$ be a collection of SVNULNs, and $m = 1,\ldots,n$. There are some properties of $SVNULGMSM^{(m)}$ operator shown below.*

1. *Idempotency. If the SVNULNs $a_i = a = \langle [S_{\mu_a}, S_{\nu_a}], (T_a, I_a, F_a)\rangle$ for each $i (i = 1,2,\ldots,n)$, then $SVNULGMSM^{(m,p_1,\ldots,p_n)}(a,\ldots,a) = \langle [S_{\mu_a}, S_{\nu_a}], (T_a, I_a, F_a)\rangle$.*
2. *Commutativity. Let $(a_1^{'},\ldots,a_n^{'})$ be a permutation of $(a_1,\ldots,a_n)$. Then $SVNULGMSM^{(m,p_1,\ldots,p_n)}(a_1^{'},\ldots,a_n^{'}) = SVNULGMSM^{(m,p_1,\ldots,p_n)}(a_1,\ldots,a_n)$.*
3. *Monotonicity. Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle$ and $a_i^{'} = \langle [S_{\mu_i^{'}}, S_{\nu_i^{'}}], (T_i^{'}, I_i^{'}, F_i^{'})\rangle (i = 1,2,\ldots,n)$ be two collections of neutrosophic uncertain linguistic numbers, and if $a_i \le a_i^{'}$—i.e., $S_{\mu_i} \le S_{\mu_i^{'}}$, $S_{\nu_i} \le S_{\nu_i^{'}}$, $T_i \le T_i^{'}$, $I_i \ge I_i^{'}$, and $F_i \ge F_i^{'}$, for all i—then $SVNULGMSM^{(m,p_1,\ldots,p_n)}(a_1,\ldots,a_n) \le SVNULGMSM^{(m,p_1,\ldots,p_n)}(a_1^{'},\ldots,a_n^{'})$.*
4. *Boundedness. $min\{a_1,\ldots,a_n\} \le SVNULGMSM^{(m,p_1,\ldots,p_n)}(a_1,\ldots,a_n) \le max\{a_1,\ldots,a_n\}$.*

The proof of Property 4 is similar to that of Property 3; it is omitted here.

When m takes different values, the $SVNULGMSM^{(m,p_1,\ldots,p_n)}$ operator can be converted to some special forms, which are as follows:

1. *When m = 1, we have the formula below.*

$$\begin{aligned}
&SVNULGMSM^{(m,p_1)}(a_1,\ldots,a_n) = \left(\frac{\sum_{1\le i_1\le n} a_{i_1}^{p_1}}{C_n^1}\right)^{\frac{1}{p_1}} \\
&= \langle [S_{l(1-\prod_{k=1}^{n}(1-(\frac{\mu_k}{l})^{p_1})^{\frac{1}{n}})^{\frac{1}{p_1}}}, S_{l(1-\prod_{k=1}^{n}(1-(\frac{\nu_k}{l})^{p_1})^{\frac{1}{n}})^{\frac{1}{p_1}}}], \\
&((1-\prod_{k=1}^{n}(1-(T_k)^{p_1})^{\frac{1}{n}})^{\frac{1}{p_1}}, \\
&1-(1-\prod_{k=1}^{n}(1-(1-I_k)^{p_1})^{\frac{1}{n}})^{\frac{1}{p_1}}, \\
&1-(1-\prod_{k=1}^{n}(1-(1-F_k)^{p_1})^{\frac{1}{n}})^{\frac{1}{p_1}})\rangle
\end{aligned} \tag{35}$$

2. *When m = 2, we have the formula below.*

$$\begin{aligned}
&SVNULGMSM^{(m,p_1,p_2)}(a_1,\ldots,a_n) = \left(\frac{\sum_{1\le i_1< i_2\le n} a_{i_1}^{p_1} a_{i_2}^{p_2}}{C_n^2}\right)^{\frac{1}{p_1+p_2}} \\
&= \langle [S_{l(1-\prod_{k=1}^{C_n^2}(1-(\frac{\mu_{i_1(k)}}{l})^{p_1}(\frac{\mu_{i_2(k)}}{l})^{p_2})^{\frac{1}{C_n^2}})^{\frac{1}{p_1+p_2}}}, \\
&S_{l(1-\prod_{k=1}^{C_n^2}(1-(\frac{\nu_{i_1(k)}}{l})^{p_1}(\frac{\nu_{i_2(k)}}{l})^{p_2})^{\frac{1}{C_n^2}})^{\frac{1}{p_1+p_2}}}], \\
&((1-\prod_{k=1}^{C_n^2}(1-(T_{i_1(k)})^{p_1}(T_{i_2(k)})^{p_2})^{\frac{1}{C_n^2}})^{\frac{1}{p_1+p_2}}, \\
&1-(1-\prod_{k=1}^{C_n^2}(1-(1-I_{i_1(k)})^{p_1}(1-I_{i_2(k)})^{p_2})^{\frac{1}{C_n^2}})^{\frac{1}{p_1+p_2}}, \\
&1-(1-\prod_{k=1}^{C_n^2}(1-(1-F_{i_1(k)})^{p_1}(1-F_{i_2(k)})^{p_2})^{\frac{1}{C_n^2}})^{\frac{1}{p_1+p_2}})\rangle
\end{aligned} \tag{36}$$

3. *When m = n, we have the formula below.*

$$\begin{aligned}
&SVNULGMSM^{(n,p_1\ldots,p_n)}(a_1,\ldots,a_n) = \left(\frac{\sum_{1\le i_1<\cdots<i_n\le n}\prod_{j=1}^{n} a_{i_j}^{p_j}}{C_n^n}\right)^{\frac{1}{p_1+\cdots+p_n}} \\
&= \langle [S_{l(\prod_{j=1}^{n}(\frac{\mu_j}{l})^{p_j})^{\frac{1}{p_1+\cdots+p_n}}}, S_{l(\prod_{j=1}^{n}(\frac{\nu_j}{l})^{p_j})^{\frac{1}{p_1+\cdots+p_n}}}], \\
&((\prod_{j=1}^{n}(T_j)^{p_j})^{\frac{1}{p_1+\cdots+p_n}}, 1-(\prod_{j=1}^{n}(1-I_j)^{p_j})^{\frac{1}{p_1+\cdots+p_n}}, \\
&1-(\prod_{j=1}^{n}(1-F_j)^{p_j})^{\frac{1}{p_1+\cdots+p_n}})\rangle
\end{aligned} \tag{37}$$

4.3. The Weighted SVNULMSM Operator and Weighted SVNULGMSM Operator

In this subsection, we will introduce the weighted SVNULMSM operator and weighted SVNULGMSM operator.

Definition 13. *Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle (i = 1, \ldots, n)$ be a collection of SVNULNs. Let $\omega = (\omega_1, \ldots, \omega_2)^T$ be the weight vector, which satisfies $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i > 0$ $(i = 1, \ldots, n)$. Each ω_i represents the importance degree of a_i. The $WSVNULMSM$ operator $\Omega^n \to \Omega$ is*

$$WSVNULMSM^{(m)}(a_1, \ldots, a_n) = (\frac{\sum_{1\le i_1<\cdots<i_m\le n} \prod_{j=1}^{m} (n\omega_{i_j} a_{i_j})}{C_n^m})^{\frac{1}{m}} \tag{38}$$

where Ω is a set of SVNULNs and $m = 1, \ldots, n$.

Based on the calculation laws for SVNULNs described earlier, the $WSVNULMSM$ operator can be expressed as follows:

Theorem 3. *Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle (i = 1, \ldots, n)$ be a collection of SVNULNs, and $m = 1, \ldots, n$, then*

$$\begin{aligned} WSVNULMSM^{(m)}(a_1, \ldots, a_n) &= (\frac{\sum_{1\le i_1<\cdots<i_m\le n} \prod_{j=1}^{m} (n\omega_{i_j} a_{i_j})}{C_n^m})^{\frac{1}{m}} \\ &= \langle [S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(1-\frac{\mu_{i_j(k)}}{l})^{n\omega_{i_j}}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}}, \\ &\quad S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(1-\frac{\nu_{i_j(k)}}{l})^{n\omega_{i_j}}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}}], \\ &\quad ((1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(1-T_{i_j(k)})^{n\omega_{i_j}}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}, \\ &\quad 1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(I_{i_j(k)})^{n\omega_{i_j}}))^{\frac{1}{c_n^m}})^{\frac{1}{m}}, \\ &\quad 1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(F_{i_j(k)})^{n\omega_{i_j}}))^{\frac{1}{c_n^m}})^{\frac{1}{m}})\rangle \end{aligned} \tag{39}$$

where $k = 1, \ldots, C_n^m$,$a_{i_j(k)}$ represents the i_jth element in the kth permutation.

The proof of Theorem 3 is similar to that of Theorem 1; it is omitted here.

Property 5. *Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle (i = 1, \ldots, n)$ be a collection of SVNULNs, and $m = 1, \ldots, n$. The properties of the $WSVNULMSM^{(m)}$ operator are shown below.*

1. *Reducibility. When $\omega = (\frac{1}{n}, \ldots, \frac{1}{n})^T$. Then $WSVNULMSM^{(m)}(a_1, \ldots, a_n) = SVNULMSM^{(m)}(a_1, \ldots, a_n)$*
2. *Monotonicity. Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle$ and $a_i' = \langle [S_{\mu_i'}, S_{\nu_i'}], (T_i', I_i', F_i')\rangle (i = 1, 2, \ldots, n)$ be two collections of SVNULNs, and if $a_i \le a_i'$—i.e., $S_{\mu_i} \le S_{\mu_i'}$, $S_{\nu_i} \le S_{\nu_i'}$, $T_i \le T_i'$, $I_i \ge I_i'$, and $F_i \ge F_i'$, for all i—then $WSVNULMSM^{(m)}(a_1, \ldots, a_n) \le WSVNULMSM^{(m)}(a_1', \ldots, a_n')$.*
3. *Boundedness. $min\{a_1, \ldots, a_n\} \le WSVNULMSM^{(m)}(a_1, \ldots, a_n) \le max\{a_1, \ldots, a_n\}$.*

Proof. 1. If $\omega = (\frac{1}{n}, \ldots, \frac{1}{n})^T$, then

$$WSVNULMSM^{(m)}(a_1, \ldots, a_n) = (\frac{\sum_{1 \le i_1 < \cdots < i_m \le n} \prod_{j=1}^{m}(n\omega_{i_j} a_{i_j})}{C_n^m})^{\frac{1}{m}}$$
$$= \langle [S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(1-\frac{\mu_{i_j(k)}}{l})^{n\frac{1}{n}}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}},$$
$$S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(1-\frac{\nu_{i_j(k)}}{l})^{n\frac{1}{n}}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}}],$$
$$((1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(1-T_{i_j(k)})^{n\frac{1}{n}}))^{\frac{1}{C_n^m}})^{\frac{1}{m}},$$
$$1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(I_{i_j(k)})^{n\frac{1}{n}}))^{\frac{1}{c_n^m}})^{\frac{1}{m}},$$
$$1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(F_{i_j(k)})^{n\frac{1}{n}}))^{\frac{1}{c_n^m}})^{\frac{1}{m}})\rangle$$
$$= \langle [S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(\frac{\mu_{i_j(k)}}{l}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}},$$
$$S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1\frac{\nu_{i_j(k)}}{l}))^{\frac{1}{C_n^m}})^{\frac{1}{m}}}],$$
$$((1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(T_{i_j(k)}))^{\frac{1}{C_n^m}})^{\frac{1}{m}},$$
$$1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-I_{i_j(k)})^{\frac{1}{c_n^m}})^{\frac{1}{m}},$$
$$1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-F_{i_j(k)}))^{\frac{1}{c_n^m}})^{\frac{1}{m}})\rangle$$
$$= SVNULMSM^{(m)}(a_1, \ldots, a_n)$$

2. The proofs of monotonicity and boundedness are similar to those for Property 3, so are omitted here.

□

When m takes different values, the $WSVNULMSM^{(m)}$ operator can be converted to some special forms, which are shown in the following:

1. *When m = 1, we have the formula below.*

$$WSVNULMSM^{(1)}(a_1, \ldots, a_n) = \frac{\sum_{1 \le i_1 \le n}(n\omega_{i_1} a_{i_1})}{C_n^1}$$
$$= \langle [S_{l(1-\prod_{k=1}^{n}(1-\frac{\mu_k}{l})^{\omega_k})}, S_{l(1-\prod_{k=1}^{n}(1-\frac{\nu_k}{l})^{\omega_k})}],$$
$$((1-\prod_{k=1}^{n}(1-T_k)^{\omega_k}), \prod_{k=1}^{n}(I_k)^{\omega_k}, \prod_{k=1}^{n}(F_k)^{\omega_k})\rangle \quad (40)$$

2. *When m = 2, we have the formula below.*

$$
\begin{aligned}
&WSVNULMSM^{(2)}(a_1,\ldots,a_n) = (\frac{\sum_{1\le i_1<i_2\le n}\prod_{j=1}^{2}(n\omega_{i_j}a_{i_j})}{C_n^2})^{\frac{1}{2}} \\
&= \langle [S_{l(1-\prod_{k=1}^{C_n^2}(1-(1-(1-\frac{\mu_{i_1(k)}}{l})^{n\omega_{i_1}})(1-(1-\frac{\mu_{i_2(k)}}{l})^{n\omega_{i_2}}))^{\frac{1}{C_n^2}})^{\frac{1}{2}}}, \\
&S_{l(1-\prod_{k=1}^{C_n^2}(1-(1-(1-\frac{\nu_{i_1(k)}}{l})^{n\omega_{i_1}})(1-(1-\frac{\nu_{i_2(k)}}{l})^{n\omega_{i_2}}))^{\frac{1}{C_n^2}})^{\frac{1}{2}}}], \\
&((1-\prod_{k=1}^{C_n^2}(1-(1-(1-T_{i_1(k)})^{n\omega_{i_1}})(1-(1-T_{i_2(k)})^{n\omega_{i_2}}))^{\frac{1}{C_n^2}})^{\frac{1}{2}}, \\
&1-(1-\prod_{k=1}^{C_n^2}(1-(1-(I_{i_1(k)})^{n\omega_{i_1}})(1-(I_{i_2(k)})^{n\omega_{i_2}}))^{\frac{1}{C_n^2}})^{\frac{1}{2}}, \\
&1-(1-\prod_{k=1}^{C_n^2}(1-(1-(F_{i_1(k)})^{n\omega_{i_1}})(1-(F_{i_2(k)})^{n\omega_{i_2}}))^{\frac{1}{C_n^2}})^{\frac{1}{2}})\rangle
\end{aligned}
\tag{41}
$$

3. *When m = n, we have the formula below.*

$$
\begin{aligned}
&WSVNULMSM^{(n)}(a_1,\ldots,a_n) = (\frac{\sum_{1\le i_1<\cdots<i_m\le n}\prod_{j=1}^{n}(n\omega_{i_j}a_{i_j})}{C_n^n})^{\frac{1}{n}} \\
&= \langle [S_{l(1-\prod_{j=1}^{n}(1-(1-\frac{\mu_j}{l})^{n\omega_j}))^{\frac{1}{n}}}, S_{l(1-\prod_{j=1}^{n}(1-(1-\frac{\nu_j}{l})^{n\omega_j}))^{\frac{1}{n}}}], \\
&((\prod_{j=1}^{n}(1-(1-T_j)^{n\omega_j}))^{\frac{1}{n}}, \\
&1-(\prod_{j=1}^{n}(1-(I_j)^{n\omega_j}))^{\frac{1}{n}}, \\
&1-(\prod_{j=1}^{n}(1-(F_j)^{n\omega_j}))^{\frac{1}{n}})\rangle
\end{aligned}
\tag{42}
$$

Definition 14. *Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i)\rangle (i = 1,\ldots,n)$ be a collection of SVNULNs. Let $\omega = (\omega_1,\ldots,\omega_2)^T$ be the weight vector, which satisfies $\sum_{i=1}^{n}\omega_i = 1$ and $\omega_i > 0$ $(i = 1,\ldots,n)$. Each ω_i represents the importance degree of a_i. The $WSVNULGMSM$ operator $\Omega^n \to \Omega$ is*

$$
WSVNULGMSM^{(m,p_1,\ldots,p_m)}(a_1,\ldots,a_n) = (\frac{\sum_{1\le i_1<\cdots<i_m\le i_n}\prod_{j=1}^{n}(b\omega_{i_j}a_{i_j})^{p_j}}{C_n^m})^{\frac{1}{p_1+\cdots+p_m}}
\tag{43}
$$

where Ω is a set of SVNULNs and $m = 1,\ldots,n$.

Theorem 4. *Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i) \rangle (i = 1, \ldots, n)$ be a collection of SVNULNs, and $m = 1, \ldots, n$, then*

$$\begin{aligned} & WSVNULGMSM^{(m,p_1,\ldots,p_m)}(a_1,\ldots,a_n) \\ & = \left(\frac{\sum_{1\le i_1<\cdots<i_m\le i_n}\prod_{j=1}^{n}(b\omega_{i_j}a_{i_j})^{p_j}}{C_n^m}\right)^{\frac{1}{p_1+\cdots+p_m}} \\ & = \langle [S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(1-\frac{\mu_{i_j(k)}}{l})^{n\omega_{i_j}})^{p_j})^{\frac{1}{C_n^m}})^{\frac{1}{p_1+\cdots+p_m}}}, \\ & S_{l(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(1-\frac{\nu_{i_j(k)}}{l})^{n\omega_{i_j}})^{p_j})^{\frac{1}{C_n^m}})^{\frac{1}{p_1+\cdots+p_m}}}], \\ & ((1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(1-T_{i_j(k)})^{n\omega_{i_j}})^{p_j})^{\frac{1}{C_n^m}})^{\frac{1}{p_1+\cdots+p_m}}, \\ & 1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(I_{i_j(k)})^{n\omega_{i_j}})^{p_j})^{\frac{1}{C_n^m}})^{\frac{1}{p_1+\cdots+p_m}}, \\ & 1-(1-\prod_{k=1}^{C_n^m}(1-\prod_{j=1}^{m}(1-(I_{i_j(k)})^{n\omega_{i_j}})^{p_j})^{\frac{1}{C_n^m}})^{\frac{1}{p_1+\cdots+p_m}})\rangle \end{aligned} \tag{44}$$

where $k = 1, \ldots, C_n^m$,$a_{i_j(k)}$ represents the i_jth element in kth permutation.

The proof of Theorem 4 is similar to that of Theorem 1; it is omitted here.

Property 6. *Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i) \rangle (i = 1, \ldots, n)$ be a collection of SVNULNs, and $m = 1, \ldots, n$. The properties of the $WSVNULGMSM^{(m,p_1,\ldots,p_m)}$ operator are shown below.*

1. *Reducibility. When $\omega = (\frac{1}{n}, \ldots, \frac{1}{n})^T$, then $WSVNULGMSM^{(m,p_1,\ldots,p_m)}(a_1, \ldots, a_n) = SVNULGMSM^{(m,p_1,\ldots,p_m)}(a_1, \ldots, a_n)$*
2. *Monotonicity. Let $a_i = \langle [S_{\mu_i}, S_{\nu_i}], (T_i, I_i, F_i) \rangle$ and $a_i' = \langle [S_{\mu_i'}, S_{\nu_i'}], (T_i', I_i', F_i') \rangle (i = 1, 2, \ldots, n)$ be two collections of SVNULNs, and if $a_i \le a_i'$—i.e., $S_{\mu_i} \le S_{\mu_i'}$,$S_{\nu_i} \le S_{\nu_i'}$, $T_i \le T_i'$,$I_i \ge I_i'$, and $F_i \ge F_i'$, for all i—then $WSVNULGMSM^{(m,p_1,\ldots,p_m)}(a_1, \ldots, a_n) \le WSVNULGMSM^{(m,p_1,\ldots,p_m)}(a_1', \ldots, a_n')$.*
3. *Boundedness. $min\{a_1, \ldots, a_n\} \le WSVNULGMSM^{(m,p_1,\ldots,p_m)}(a_1, \ldots, a_n) \le max\{a_1, \ldots, a_n\}$*

The proof is similar to that of Property 3 and is omitted.

When m takes different values, the $WSVNULGMSM^{(m,p_1,\ldots,p_m)}$ operator can be converted to some special forms, as follows:

1. *When m = 1, we have the formula below.*

$$\begin{aligned} & WSVNULGMSM^{(1,p_1)}(a_1,\ldots,a_n) = \left(\frac{\sum_{1\le i_1\le n}(n\omega_{i_1}a_{i_1})^{p_1}}{C_n^1}\right)^{\frac{1}{p_1}} \\ & = \langle [S_{l(1-\prod_{k=1}^{n}(1-(1-(1-\frac{\mu_{i_1(k)}}{l})^{n\omega_{i_1}})^{p_1})^{\frac{1}{n}})^{\frac{1}{p_1}}}, \\ & S_{l(1-\prod_{k=1}^{n}(1-(1-(1-\frac{\nu_{i_1(k)}}{l})^{n\omega_{i_1}})^{p_1})^{\frac{1}{n}})^{\frac{1}{p_1}}}], \\ & ((1-\prod_{k=1}^{n}(1-(1-(1-T_{i_1(k)})^{n\omega_{i_1}})^{p_1})^{\frac{1}{n}})^{\frac{1}{p_1}}, \\ & 1-(1-\prod_{k=1}^{n}(1-(1-(I_{i_1(k)})^{n\omega_{i_1}})^{p_1})^{\frac{1}{n}})^{\frac{1}{p_1}}, \\ & 1-(1-\prod_{k=1}^{n}(1-(1-(F_{i_1(k)})^{n\omega_{i_1}})^{p_1})^{\frac{1}{n}})^{\frac{1}{p_1}})\rangle \end{aligned} \tag{45}$$

2. *When m = 2, we have the formula below.*

$$WSVNULGMSM^{(2,p_1,p_2)}(a_1,\ldots,a_n) = \left(\frac{\sum_{1\le i_1<i_2\le n}\prod_{j=1}^{2}(n\omega_{i_j}a_{i_j})^{p_j}}{C_n^2}\right)^{\frac{1}{p_1+p_2}}$$
$$= \langle [S_{l(1-\prod_{k=1}^{C_n^2}(1-(1-(1-\frac{\mu_{i_1(k)}}{l})^{n\omega_{i_1}})^{p_1}(1-(1-\frac{\mu_{i_2(k)}}{l})^{n\omega_{i_2}})^{p_2})^{\frac{1}{C_n^2}})^{\frac{1}{p_1+p_2}}},$$
$$S_{l(1-\prod_{k=1}^{C_n^2}(1-(1-(1-\frac{\nu_{i_1(k)}}{l})^{n\omega_1})^{p_1}(1-(1-\frac{\nu_{i_2(k)}}{l})^{n\omega_2})^{p_2})^{\frac{1}{C_n^2}})^{\frac{1}{p_1+p_2}}}],$$
$$((1-\prod_{k=1}^{C_n^2}(1-(1-(1-T_{i-1(k)})^{n\omega_{i_1}})^{p_1}(1-(1-T_{i_2(k)})^{n\omega_{i_2}})^{p_2})^{\frac{1}{C_n^2}})^{\frac{1}{p_1+p_2}}, \tag{46}$$
$$1-(1-\prod_{k=1}^{C_n^2}(1-(1-(I_{i_1(k)})^{n\omega_{i-1}})^{p_1}(1-(I_{i_2(k)})^{n\omega_{i_2}})^{p_2})^{\frac{1}{C_n^2}})^{\frac{1}{p_1+p_2}},$$
$$1-(1-\prod_{k=1}^{C_n^2}(1-(1-(F_{i_1(k)})^{n\omega_{i-1}})^{p_1}(1-(F_{i_2(k)})^{n\omega_{i_2}})^{p_2})^{\frac{1}{C_n^2}})^{\frac{1}{p_1+p_2}})\rangle$$

3. *When m = n, we have the formula below.*

$$WSVNULGMSM^{(n,p_1,\ldots,p_n)}(a_1,\ldots,a_n)$$
$$= \left(\frac{\sum_{1\le i_1<\cdots<i_n\le n}\prod_{j=1}^{n}(n\omega_{i_j}a_{i_j})^{p_j}}{C_n^n}\right)^{\frac{1}{p_1+\cdots+p_n}}$$
$$= \langle [S_{l(\prod_{j=1}^{n}(1-(1-\frac{\mu_j}{l})^{n\omega_j})^{p_j})^{\frac{1}{p_1+\cdots+p_n}}}, S_{l(\prod_{j=1}^{n}(1-(1-\frac{\nu_j}{l})^{n\omega_j})^{p_j})^{\frac{1}{p_1+\cdots+p_n}}}],$$
$$((\prod_{j=1}^{n}(1-(1-T_j)^{n\omega_j})^{p_j})^{\frac{1}{p_1+\cdots+p_n}}, \tag{47}$$
$$1-(\prod_{j=1}^{n}(1-(I_j)^{n\omega_j})^{p_j})^{\frac{1}{p_1+\cdots+p_n}},$$
$$1-(\prod_{j=1}^{n}(1-(F_j)^{n\omega_j})^{p_j})^{\frac{1}{p_1+\cdots+p_n}})\rangle$$

5. MCDM Approach Based on WSVNULMSM Operator and WSVNULGMSM Operator

In this section, we apply the proposed SVNULMSM operator and SVNULGMSM operator to cope with a MADM issue. Suppose $A = \{A_1,\ldots,A_m\}$ is a set of alternatives, and let $C = \{C_1,\ldots,C_n\}$ be a collection of attributes. The weight vector of the attribute is $\omega = (\omega_1,\ldots,\omega_n)^T$ while satisfying $\sum_{i=1}^{n}\omega_i = 1(\omega_i \ge 0, i = 1,\ldots,n)$, and each ω_i represents the importance of C_j. Let $D = \{D_1,\ldots,D_t\}$ be the set of decision makers, and $\lambda = (\lambda_1,\ldots,\lambda_t)^T$ be the weight vector of decision makers $D_s(s = 1,\ldots,t)$, and $\sum_{s=1}^{t}\lambda_s = 1(\lambda_s \ge 0, s = 1,\ldots,t)$. $R^{(s)} = [R_{ij}^{(s)}]_{m\times n}$ is the decision matrix, where $R_{ij}^{(s)} = \langle [S_{\mu_{ij}}^{(s)}, S_{\nu_{ij}}^{(s)}], (T_{ij}^{(s)}, I_{ij}^{(s)}, F_{ij}^{(s)})\rangle$ is attribute value given by the decision maker D_s for alternative A_i with respect to attribute C_j. The main steps as follows:

Step 1 Utilize the WSVNULMSM operator:

$$R_i^{(s)} = SVNULMSM(R_{i_1}^{(s)}, R_{i_2}^{(s)},\ldots,R_{i_n}^{(s)}) \tag{48}$$

Utilize the WSVNULGMSM operator:

$$R_i^{(s)} = SVNULGMSM(R_{i_1}^{(s)}, R_{i_2}^{(s)},\ldots,R_{i_n}^{(s)}) \tag{49}$$

We use Definition 12 and Definition 13 to aggregate the attribute values of each alternative for decision maker D_s, and obtain the overall preference value $R_i^{(s)}$ corresponding to alternative A_i.

Step 2 Utilize the WSVNULMSM operator:

$$R_i = SVNULMSM(R_i^{(s)}, R_i^{(s)}, \ldots, R_i^{(s)}) \tag{50}$$

Utilize the WSVNULGMSM operator:

$$R_i = SVNULGMSM(R_i^{(s)}, R_i^{(s)}, \ldots, R_i^{(s)}) \tag{51}$$

According to the aggregation value $R_i^{(s)}$ from the above, we also use Definition 12 and Definition 13 to aggregate the evaluation values of the decision maker.

Step 3 Calculate the value $E(R_i)$ of R_i according to Definition 7.

Step 4 According to Definition 9, rank the alternatives.

6. Illustrative Example

In this section, we provide an investment example (adapted from [24]) to illustrate the application of WSVNULMSM and WSVNUGMSM operators.

There are four alternatives, including a car company (A1), a food company (A2), a computer company (A3), and an arms company (A4); and these companies evaluated by three decision makers, D_1, D_2, and D_3. The weight vector of the decision makers is $\lambda = (0.314, 0.355, 0.331)^T$. We use the following attributes: C1 (the risk index), C2 (the growth index), and C3 (the social-political impact index). Suppose the attribute weight vector is $\omega = (0.4, 0.2, 0.4)^T$. Decision makers use linguistic term set $S = S_0, S_1, S_2, S_3, S_4, S_5, S_6$ to express their evaluation results. The decision matrices $R^{(s)} = [R_{ij}^{(s)}]_{4\times4}(s = 1, 2, 3)$ as listed in Tables 1–3.

Table 1. Decision matrix $R^{(1)}$.

	C1	C2	C3
A1	$\langle[s_5, s_5], (0.265, 0.350, 0.385)\rangle$	$\langle[s_2, s_3], (0.330, 0.390, 0.280)\rangle$	$\langle[s_5, s_6], (0.245, 0.275, 0.480)\rangle$
A2	$\langle[s_4, s_5], (0.345, 0.245, 0.410)\rangle$	$\langle[s_5, s_5], (0.430, 0.290, 0.280)\rangle$	$\langle[s_3, s_4], (0.245, 0.375, 0.380)\rangle$
A3	$\langle[s_3, s_4], (0.365, 0.300, 0.335)\rangle$	$\langle[s_4, s_4], (0.480, 0.315, 0.205)\rangle$	$\langle[s_4, s_5], (0.340, 0.370, 0.290)\rangle$
A4	$\langle[s_6, s_6], (0.430, 0.300, 0.270)\rangle$	$\langle[s_2, s_3], (0.460, 0.245, 0.295)\rangle$	$\langle[s_3, s_4], (0.310, 0.520, 0.170)\rangle$

Table 2. Decision matrix $R^{(2)}$.

	C1	C2	C3
A1	$\langle[s_3, s_4], (0.125, 0.470, 0.405)\rangle$	$\langle[s_3, s_4], (0.220, 0.420, 0.360)\rangle$	$\langle[s_3, s_4], (0.345, 0.490, 0.165)\rangle$
A2	$\langle[s_5, s_6], (0.355, 0.315, 0.330)\rangle$	$\langle[s_3, s_4], (0.300, 0.370, 0.330)\rangle$	$\langle[s_4, s_5], (0.205, 0.630, 0.165)\rangle$
A3	$\langle[s_4, s_5], (0.315, 0.380, 0.305)\rangle$	$\langle[s_4, s_4], (0.330, 0.565, 0.105)\rangle$	$\langle[s_2, s_3], (0.280, 0.520, 0.200)\rangle$
A4	$\langle[s_5, s_5], (0.365, 0.365, 0.270)\rangle$	$\langle[s_4, s_5], (0.355, 0.320, 0.325)\rangle$	$\langle[s_2, s_3], (0.425, 0.485, 0.090)\rangle$

Table 3. Decision matrix $R^{(3)}$.

	C1	C2	C3
A1	$\langle[s_5, s_5], (0.260, 0.425, 0.315)\rangle$	$\langle[s_3, s_4], (0.220, 0.450, 0.330)\rangle$	$\langle[s_4, s_5], (0.255, 0.500, 0.245)\rangle$
A2	$\langle[s_4, s_5], (0.270, 0.370, 0.360)\rangle$	$\langle[s_5, s_5], (0.320, 0.215, 0.465)\rangle$	$\langle[s_2, s_3], (0.135, 0.575, 0.290)\rangle$
A3	$\langle[s_4, s_4], (0.245, 0.465, 0.290)\rangle$	$\langle[s_5, s_5], (0.250, 0.570, 0.180)\rangle$	$\langle[s_1, s_3], (0.175, 0.660, 0.165)\rangle$
A4	$\langle[s_3, s_4], (0.390, 0.340, 0.270)\rangle$	$\langle[s_3, s_4], (0.305, 0.475, 0.220)\rangle$	$\langle[s_4, s_5], (0.465, 0.485, 0.050)\rangle$

6.1. The Decision Making Method Based on the WSVNULMSM Operator

We can give $m = \frac{n}{2}$, so m = 1 or m = 2.

(1) When $m = 1$, the steps are shown below.

Step 1 Get the aggregate values of each alternative for decision maker D_s with the WSVNULMSM operator.

$$\begin{aligned}
R_1^{(1)} &= \langle [s_{4.6805}, s_{6.0000}], (0.2707, 0.3248, 0.3946) \rangle \\
R_2^{(1)} &= \langle [s_{3.9523}, s_{4.6805}], (0.3257, 0.3004, 0.3685) \rangle \\
R_3^{(1)} &= \langle [s_{3.6478}, s_{4.4843}], (0.3804, 0.3295, 0.2866) \rangle \\
R_4^{(1)} &= \langle [s_{6.0000}, s_{6.0000}], (0.3913, 0.3590, 0.2284) \rangle \\
R_1^{(2)} &= \langle [s_{3.0000}, s_{4.0000}], (0.2384, 0.4673, 0.2762) \rangle \\
R_2^{(2)} &= \langle [s_{4.3562}, s_{6.0000}], (0.2872, 0.4292, 0.2501) \rangle \\
R_3^{(2)} &= \langle [s_{3.3610}, s_{4.2174}], (0.3043, 0.4664, 0.2081) \rangle \\
R_4^{(2)} &= \langle [s_{4.0000}, s_{4.4482}], (0.3878, 0.3983, 0.1806) \rangle \\
R_1^{(3)} &= \langle [s_{4.3562}, s_{4.8513}], (0.2502, 0.4588, 0.2875) \rangle \\
R_2^{(3)} &= \langle [s_{3.7026}, s_{4.4482}], (0.2297, 0.3959, 0.3475) \rangle \\
R_3^{(3)} &= \langle [s_{3.4881}, s_{3.9523}], (0.2188, 0.5572, 0.2104) \rangle \\
R_4^{(3)} &= \langle [s_{3.4492}, s_{4.4843}], (0.4059, 0.4190, 0.1320) \rangle
\end{aligned}$$

Step 2 Get the aggregate values for each alternative with the WSVNULMSM operator.

$$\begin{aligned}
\tilde{R_1} &= \langle [s_{4.1005}, s_{4.8513}], (0.2526, 0.4143, 0.3131) \rangle \\
\tilde{R_2} &= \langle [s_{4.0324}, s_{6.0000}], (0.2813, 0.3736, 0.3150) \rangle \\
\tilde{R_3} &= \langle [s_{3.4959}, s_{4.2264}], (0.3029, 0.4436, 0.2309) \rangle \\
\tilde{R_4} &= \langle [s_{6.0000}, s_{6.0000}], (0.3949, 0.3920, 0.1753) \rangle
\end{aligned}$$

Step 3 Calculate the value $E(R_i)$ of R_i.

$E(R_1) = s_{3.0106}$, $E(R_2) = s_{3.7024}$, $E(R_3) = s_{2.5724}$, $E(R_4) = s_6$

Step 4 According to Definition 7, rank the alternatives:

$A_4 > A_2 > A_1 > A_3$.

(2) When $m = 2$, the steps are shown below.

Step 1 Get the aggregate values of each alternative for decision maker D_s with the WSVNULMSM operator.

$$\begin{aligned}
R_1^{(1)} &= \langle [s_{4.1408}, s_{4.7344}], (0.2682, 0.3547, 0.4002) \rangle \\
R_2^{(1)} &= \langle [s_{3.9093}, s_{4.5528}], (0.3216, 0.3232, 0.3745) \rangle \\
R_3^{(1)} &= \langle [s_{3.5452}, s_{4.2100}], (0.3782, 0.3467, 0.2938) \rangle \\
R_4^{(1)} &= \langle [s_{3.4510}, s_{4.2006}], (0.3831, 0.3760, 0.2643) \rangle \\
R_1^{(2)} &= \langle [s_{2.9199}, s_{3.8948}], (0.2122, 0.4762, 0.3334) \rangle \\
R_2^{(2)} &= \langle [s_{3.9340}, s_{4.9269}], (0.2735, 0.4659, 0.2956) \rangle \\
R_3^{(2)} &= \langle [s_{3.1586}, s_{3.8256}], (0.2984, 0.5004, 0.2144) \rangle \\
R_4^{(2)} &= \langle [s_{3.4190}, s_{4.2062}], (0.3730, 0.4091, 0.2487) \rangle \\
R_1^{(3)} &= \langle [s_{3.9340}, s_{4.5840}], (0.2410, 0.4739, 0.3147) \rangle \\
R_2^{(3)} &= \langle [s_{3.5393}, s_{4.2062}], (0.2225, 0.4065, 0.3846) \rangle \\
R_3^{(3)} &= \langle [s_{3.1297}, s_{3.9093}], (0.2139, 0.5816, 0.2312) \rangle \\
R_4^{(3)} &= \langle [s_{3.2394}, s_{4.2100}], (0.3810, 0.4492, 0.2065) \rangle
\end{aligned}$$

Step 2 Get the aggregate values for each alternative with the WSVNULMSM operator.

$$\tilde{R_1} = \langle [s_{3.6561}, s_{4.4072}], (0.2394, 0.4361, 0.3501) \rangle$$
$$\tilde{R_2} = \langle [s_{3.7901}, s_{4.5549}], (0.2708, 0.3995, 0.3527) \rangle$$
$$\tilde{R_3} = \langle [s_{3.2736}, s_{3.9793}], (0.2930, 0.4792, 0.2473) \rangle$$
$$\tilde{R_4} = \langle [s_{3.3666}, s_{4.2022}], (0.3787, 0.4120, 0.2403) \rangle$$

Step 3 Calculate the value $E(R_i)$ of R_i.
$E(R_1) = s_{2.5031}$, $E(R_2) = s_{2.7130}$, $E(R_3) = s_{2.3030}$, $E(R_4) = s_{2.6180}$
Step 4 According to Definition 7, rank the alternatives:
$A_2 > A_4 > A_1 > A_3$.

6.2. The Method Based on the WSVNULGMSM Operator

When $m = 1$, $p = 1$, the $WSVNULGMSM^{(1)}$ operator is the same as the $WSVNU$ $LMSM^{(1)}$ operator. The steps are omitted here. When $m = 2$, the steps are below.
Step 1 Get the aggregate values of each alternative for decision maker D_s with the WSVNULGMSM operator.

$$R_1^{(1)} = \langle [s_{4.3162}, s_{5.0320}], (0.2668, 0.3354, 0.4112) \rangle$$
$$R_2^{(1)} = \langle [s_{3.8062}, s_{4.4423}], (0.3093, 0.3364, 0.3688) \rangle$$
$$R_3^{(1)} = \langle [s_{3.7000}, s_{4.4027}], (0.3758, 0.3518, 0.2864) \rangle$$
$$R_4^{(1)} = \langle [s_{3.2189}, s_{4.0407}], (0.3686, 0.4010, 0.2427) \rangle$$
$$R_1^{(2)} = \langle [s_{2.9535}, s_{3.9257}], (0.2527, 0.4757, 0.2906) \rangle$$
$$R_2^{(2)} = \langle [s_{3.8869}, s_{4.8509}], (0.2556, 0.5022, 0.2638) \rangle$$
$$R_3^{(2)} = \langle [s_{2.9426}, s_{3.5980}], (0.2965, 0.5097, 0.2108) \rangle$$
$$R_4^{(2)} = \langle [s_{3.1004}, s_{3.9959}], (0.3896, 0.4227, 0.2123) \rangle$$
$$R_1^{(3)} = \langle [s_{3.8869}, s_{4.6467}], (0.2467, 0.4795, 0.2979) \rangle$$
$$R_2^{(3)} = \langle [s_{3.3964}, s_{4.9959}], (0.2073, 0.4275, 0.3625) \rangle$$
$$R_3^{(3)} = \langle [s_{3.0819}, s_{3.8062}], (0.2057, 0.6036, 0.2121) \rangle$$
$$R_4^{(3)} = \langle [s_{3.4458}, s_{4.4027}], (0.4053, 0.4617, 0.1699) \rangle$$

Step 2 Get the aggregate values for each alternative with the WSVNULGMSM operator.

$$\tilde{R_1} = \langle [s_{3.6816}, s_{4.5119}], (0.2543, 0.4453, 0.3203) \rangle$$
$$\tilde{R_2} = \langle [s_{3.6683}, s_{4.4066}], (0.2473, 0.4315, 0.3293) \rangle$$
$$\tilde{R_3} = \langle [s_{3.1774}, s_{3.8778}], (0.2671, 0.5144, 0.2247) \rangle$$
$$\tilde{R_4} = \langle [s_{3.2901}, s_{4.1980}], (0.3932, 0.4331, 0.1988) \rangle$$

Step 3 Calculate the value $E(R_i)$ of R_i.
$E(R_1) = s_{2.6061}$, $E(R_2) = s_{2.5512}$, $E(R_3) = s_{2.1903}$, $E(R_4) = s_{2.6214}$
Step 4 According to Definition 7, rank the alternatives:
$A_4 > A_1 > A_2 > A_3$.

6.3. Comparative Analysis and Discussion

Based on the experiment in Sections 6.1 and 6.2, we acquired the ranking results shown in Table 4. According to Table 4, we can achieve the same ranking results when $m = 1$. This is because the relationship between the attributes is not considered. When $m = 2$, we can see that the ranking results are slightly different. It can be stated that we should consider the relationship between the attributes.

From Table 5, we see the comparisons for when p_1 and p_2 have different values. When $p_1 = 0$, we do not need to consider the relationship between multiple attributes, so the ranking results are the same: $A_4 > A_1 > A_2 > A_3$. When $p_2 = 0$, the relationship between the attributes also does not need to be considered, and the ranking results are $A_4 > A_2 > A_1 > A_3$. When p_1 and p_2 are not equal to zero, we should consider the interrelationship between the attributes, and we find the best alternative to be the arms company (A_4). According to the sorting results, we can know that the number of parameters has a great influence on the results. Therefore, sometimes the best alternative was the food company (A_2). Meanwhile, the worst alternative was the computer company (A_3).

Table 4. A comparison of different operators.

Operator	m	P_1	P_2	Ranking
$WSVNULMSM^{(m)}$	1	-	-	$A_4 > A_2 > A_1 > A_3$
	2	-	-	$A_2 > A_4 > A_1 > A_3$
$WSVNULGMSM^{(m)}$	1	1	-	$A_4 > A_2 > A_1 > A_3$
	2	1	2	$A_4 > A_1 > A_2 > A_3$

Table 5. A comparison of different values of P_1 and p_2 when $m = 2$.

Operator	P_1	P_2	$E_{r_i}(i = 1, 2, 3, 4)$	Ranking
$WSVNULGMSM^{(m)}$	0	1	$E_{r_1} = S_{2.7367}$ $E_{r_2} = S_{2.2580}$ $E_{r_3} = S_{1.7169}$ $E_{r_4} = S_{3.0104}$	$A_4 > A_1 > A_2 > A_3$
	0	2	$E_{r_1} = S_{2.8339}$ $E_{r_2} = S_{2.3480}$ $E_{r_3} = S_{1.8018}$ $E_{r_4} = S_{3.1106}$	$A_4 > A_1 > A_2 > A_3$
	0	3	$E_{r_1} = S_{2.9203}$ $E_{r_2} = S_{2.4411}$ $E_{r_3} = S_{1.8866}$ $E_{r_4} = S_{3.2059}$	$A_4 > A_1 > A_2 > A_3$
	1	0	$E_{r_1} = S_{2.7756}$ $E_{r_2} = S_{4.0998}$ $E_{r_3} = S_{2.7348}$ $E_{r_4} = S_{6.0000}$	$A_4 > A_2 > A_1 > A_3$
	1	1	$E_{r_1} = S_{2.5031}$ $E_{r_2} = S_{2.7130}$ $E_{r_3} = S_{2.3030}$ $E_{r_4} = S_{2.6180}$	$A_2 > A_4 > A_1 > A_3$
	1	2	$E_{r_1} = S_{2.6061}$ $E_{r_2} = S_{2.5512}$ $E_{r_3} = S_{2.1903}$ $E_{r_4} = S_{2.6214}$	$A_4 > A_1 > A_2 > A_3$
	1	3	$E_{r_1} = S_{2.7066}$ $E_{r_2} = S_{2.5273}$ $E_{r_3} = S_{2.1670}$ $E_{r_4} = S_{2.7212}$	$A_4 > A_1 > A_2 > A_3$
	2	0	$E_{r_1} = S_{2.9080}$ $E_{r_2} = S_{4.1561}$ $E_{r_3} = S_{2.7983}$ $E_{r_4} = S_{6.0000}$	$A_4 > A_2 > A_1 > A_3$
	2	1	$E_{r_1} = S_{2.5839}$ $E_{r_2} = S_{3.0059}$ $E_{r_3} = S_{2.4821}$ $E_{r_4} = S_{2.9243}$	$A_2 > A_4 > A_1 > A_3$
	2	2	$E_{r_1} = S_{2.6329}$ $E_{r_2} = S_{2.7833}$ $E_{r_3} = S_{2.3747}$ $E_{r_4} = S_{2.7091}$	$A_2 > A_4 > A_1 > A_3$

Table 5. *Cont.*

Operator	P_1	P_2	$E_{r_i}(i=1,2,3,4)$	Ranking
$WSVNULGMSM^{(m)}$	2	3	$E_{r_1}=S_{2.7067}$ $E_{r_2}=S_{2.7001}$ $E_{r_3}=S_{2.3338}$ $E_{r_4}=S_{2.7154}$	$A_4>A_1>A_2>A_3$
	3	0	$E_{r_1}=S_{3.0236}$ $E_{r_2}=S_{4.2111}$ $E_{r_3}=S_{2.8640}$ $E_{r_4}=S_{6.0000}$	$A_4>A_2>A_1>A_3$
	3	1	$E_{r_1}=S_{2.6940}$ $E_{r_2}=S_{3.2013}$ $E_{r_3}=S_{2.5887}$ $E_{r_4}=S_{3.2176}$	$A_4>A_2>A_1>A_3$
	3	2	$E_{r_1}=S_{2.7075}$ $E_{r_2}=S_{2.9661}$ $E_{r_3}=S_{2.4879}$ $E_{r_4}=S_{2.9075}$	$A_2>A_4>A_1>A_3$
	3	3	$E_{r_1}=S_{2.7610}$ $E_{r_2}=S_{2.8566}$ $E_{r_3}=S_{2.4449}$ $E_{r_4}=S_{2.8235}$	$A_2>A_4>A_1>A_3$

In order to prove the effectiveness of the method proposed in this paper, we have performed a comparison with Liu's [24] method, and the results are shown in Table 6.

As shown in Table 6, the best choice found by the other method was A_4, which is the same as in our former results. Obtaining the same ranking results shows that our method is effective and reasonable. When $m=1$, our method had the same values as Liu's method [24]. This is because neither method considers the relationship between attributes. When $m=2$, the ranking results are different from those of Liu's method [24]. This is because the method proposed by Liu [24] only considers the relationship between two input parameters, whereas our method takes the relationship between multiple parameters into account, which effectively solves the problem of multiple input parameters. Therefore, our method has a wider range of applications.

Table 6. Comparison of different methods.

Methods	Operator	Ranking
Methods in this paper	$WSVNULMSM^{(m)} m=1$	$A_4>A_2>A_1>A_3$
	$WSVNULMSM^{(m)} m=2$	$A_2>A_4>A_1>A_3$
	$WSVNULGMSM^{(m)} m=1$	$A_4>A_2>A_1>A_3$
	$WSVNULGMSM^{(m)} m=2$	$A_4>A_1>A_2>A_3$
other methods	$NULNIGWHM(p=q=1)$	$A_4>A_2>A_3>A_1$
	$NULNIGGWHM(p=q=1)$	$A_4>A_2>A_1>A_3$

7. Conclusions

SVNULNs can well represent incomplete, indeterminate, and inconsistent information. Meanwhile, as an effective aggregation tool, the MSM operator can consider the relationship between multiple input parameters. In this paper, we combined the neutrosophic uncertain linguistic numbers with MSM operators, and proposed some MSM operators based on a neutrosophic uncertain linguistic environment, including the SVNULMSM operator, SVNULGMSM operator, weighted SVNULMSM operator, and weighted SVNUL-

GMSM operator. Then, we proposed a method for solving the MADM problem by using a WSVNULMSM operator and a WSVNULGMSM operator.

Finally, the effectiveness of the proposed method was proved by comparing it with other methods. According to the same ranking results, the rationality of the method was proved. The research on the determination of parameter values is not thorough, so we will further study and explore the area. In the future, we will further extend the aggregation operator and apply it to a wider range of fields, such as fault diagnosis, financial analysis, and algorithm selection. In future research, we will extend the MSM operator to other fuzzy environments, such as an intuitionistic fuzzy set, and further validate the effectiveness of MSM operator.

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