

Article

# Uniqueness of the Inflationary Higgs Scalar for Neutron Stars and Failure of Non-Inflationary Approximations

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**Abstract:** Neutron stars are perfect candidates to investigate the effects of a modified gravity theory, since the curvature effects are significant and more importantly, potentially testable. In most cases studied in the literature in the context of massive scalar-tensor theories, inflationary models were examined. The most important of scalar-tensor models is the Higgs model, which, depending on the values of the scalar field, can be approximated by different scalar potentials, one of which is the inflationary. Since it is not certain how large the values of the scalar field will be at the near vicinity and inside a neutron star, in this work we will answer the question, which potential form of the Higgs model is more appropriate in order for it to describe consistently a static neutron star. As we will show numerically, the non-inflationary Higgs potential, which is valid for certain values of the scalar field in the Jordan frame, leads to extremely large maximum neutron star masses; however, the model is not self-consistent, because the scalar field approximation used for the derivation of the potential, is violated both at the center and at the surface of the star. These results show the uniqueness of the inflationary Higgs potential, since it is the only approximation for the Higgs model, that provides self-consistent results.



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## 1. Introduction

The next two decades will possibly bring sensational observational results to the cosmology, theoretical physics and theoretical astrophysics community. All of these observations are related to gravitational wave detections, either stochastic inflationary gravitational waves, such as the LISA [1,2] and DECIGO [3,4], or ordinary astrophysical originating gravitational waves. With regard to astrophysical sources of gravitational waves, neutron stars are in the epicenter of current theoretical and experimental research. This is because neutron stars (NSs) [5–9] are superstars among stars, a wide range of physics research areas must be used to describe these accurately, such as nuclear and high energy physics [10–24], modified gravity can also describe NSs [25–37], and theoretical astrophysics [38–49]. Four decades passed since the first observation of a NS, and to date serious questions remain regarding the inner structure and physics of NSs. The equation of state (EoS) of nuclear matter is still a mystery in addition to the fundamental question whether general relativity (GR) or modified gravity [50–57] controls the physics of the star. A particularly appealing form of modified gravity is scalar-tensor gravity, and many works on NSs in the context of scalar-tensor gravity already exist in the literature [58–71]. Also scalar-tensor gravity is popular in cosmological contexts too [72–84], where viable inflationary models can be realized. The model with the highest importance in scalar-tensor gravity is the Higgs model, since the Higgs boson is the first fundamental (elementary) scalar elementary particle that has ever been observed [85]. The Higgs inflationary potential is capable of producing a viable inflationary era [76] and this occurs for a specific range of values of the scalar field and

the non-minimal coupling constant to the Ricci scalar, usually denoted as  $\xi$ . In a previous work we studied NSs in the context of scalar-tensor theories, using the inflationary Higgs potential [34]. In this work we extend our work to account for different limiting values of the scalar field and the combined non-minimal coupling of the form  $\sim \xi\phi^2$ . We shall be interested in values  $\xi\phi^2 \ll 1$  in Geometrized units. In this approximation, we shall derive the Einstein frame potential and the relevant conformal transformation function  $A(\phi)$ . Accordingly, we shall derive the corresponding Tolman–Oppenheimer–Volkoff (TOV) equations in the Einstein frame, for static NSs, and we shall solve these numerically, assuming piecewise polytropic EoSs [86,87]. We shall find the  $M - R$  relations for static NSs. Our results indicate an important fact, that the only correct description of the Higgs potential for static NSs is the one we developed in Ref. [34]. The results of the current article indicate that the maximum masses of NSs exceed the  $3 M_\odot$  limit, but the approximation  $\xi\phi^2 \ll 1$  fails to hold true at the center and at the surface of the NSs. This result indicates how unique is the inflationary Higgs potential, for providing a self-consistent neutron star phenomenology.

## 2. Non-Inflationary Higgs Scalar-Tensor Gravity in the Einstein Frame and Static NSs Phenomenology

We are interested in extracting the Einstein frame counterpart theory of the Jordan frame Higgs theory, and we shall do so by using a conformal transformation, see [58,79,88–91] for details on conformal transformations. The Jordan frame action of the Higgs model as it appears in cosmological contexts [76], in Geometrized units ( $G = 1$ ) is the following,

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi} \left[ f(\phi)\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) \right] + S_m(\psi_m, g_{\mu\nu}), \quad (1)$$

where  $f(\phi)$  is the non-minimal coupling function and  $U(\phi)$  is the potential, defined as follows,

$$f(\phi) = 1 + \xi\phi^2, \quad U(\phi) = \lambda\phi^4, \quad (2)$$

where  $\phi$  denotes the Jordan frame scalar field. In addition,  $g^{\mu\nu}$ ,  $S_m(\psi_m, g_{\mu\nu})$ ,  $g$  and  $\mathcal{R}$  denote the metric tensor, the action for the matter fluids, the determinant of the metric tensor and the Ricci scalar in the Jordan frame.

Performing the conformal transformation  $\tilde{g}_{\mu\nu} = A^{-2}g_{\mu\nu}$ , where the function  $A(\phi)$  is defined as,

$$A(\phi) = f^{-1/2}(\phi), \quad (3)$$

and the Einstein frame action in terms of the canonical scalar field  $\varphi$  reads,

$$S = \int d^4x \sqrt{-\tilde{g}} \left( \frac{\tilde{\mathcal{R}}}{16\pi} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial^\mu\varphi\partial^\nu\varphi - \frac{V(\varphi)}{16\pi} \right) + S_m(\psi_m, A^2(\varphi)g_{\mu\nu}), \quad (4)$$

where the “tilde” denotes quantities evaluated in the Einstein frame. Specifically,  $\tilde{g}^{\mu\nu}$ ,  $S_m(\psi_m, A^2(\varphi)g_{\mu\nu})$ ,  $\tilde{g}$  and  $\tilde{\mathcal{R}}$  denote the metric tensor, the action for the matter fluids, the determinant of the metric tensor and the Ricci scalar in the Einstein frame.

Recall that  $A(\phi)$  enters in the conformal transformation  $\tilde{g}_{\mu\nu} = A^{-2}g_{\mu\nu}$  and by using Equations (2) and (3) we have,

$$A(\phi) = \left( 1 + \xi\phi^2 \right)^{-1/2}. \quad (5)$$

Also the Einstein frame potential is,

$$V(\varphi) = \frac{U(\phi)}{f^2(\phi)}, \quad (6)$$

and when expressed in terms of  $\phi$  this is written as,

$$V(\phi) = \frac{\lambda\phi^4}{(1 + \xi\phi^2)^2}, \quad (7)$$

Using relation between the Einstein frame canonical scalar field  $\varphi$  and the Jordan frame scalar field  $\phi$ ,

$$\frac{d\varphi}{d\phi} = \frac{1}{\sqrt{4\pi}} \sqrt{\left(\frac{3}{4} \frac{1}{f^2} \left(\frac{df}{d\phi}\right)^2 + \frac{1}{4f}\right)}, \quad (8)$$

and combined with Equation (2), we get,

$$\frac{d\varphi}{d\phi} = \frac{1}{\sqrt{16\pi}} \frac{\sqrt{1 + \xi\phi^2 + 12\xi^2\phi^2}}{1 + \xi\phi^2}. \quad (9)$$

For the Higgs inflationary potential, the field values approximations are the following

$$\xi^2\phi^2 \gg 1, \quad \xi^2\phi^2 \gg \xi\phi^2, \quad (10)$$

however, we shall use another approximation relevant in Higgs potential physics, namely [76],

$$\frac{1}{\sqrt{12\xi}} \ll \phi \ll \frac{1}{\sqrt{\xi}}, \quad (11)$$

which is equivalent to the following two approximations,

$$\xi\phi^2 \ll 1, \quad (12)$$

$$12\xi^2\phi^2 \gg 1. \quad (13)$$

In view of the approximations (12) and (13) we get approximately at leading order,

$$\frac{d\varphi}{d\phi} \simeq \frac{\sqrt{12}}{\sqrt{16\pi}} \xi\phi. \quad (14)$$

Thus, integrating the above we get the final relation between  $\varphi$  and  $\phi$ ,

$$\varphi = \frac{\sqrt{12}}{2\sqrt{16\pi}} \xi\phi^2. \quad (15)$$

At leading order the function  $A(\varphi)$  reads,

$$A(\varphi) = 1 - \frac{2\sqrt{12}}{\sqrt{16\pi}} \varphi, \quad (16)$$

thus, at leading order  $\alpha(\varphi) = \frac{d \ln A}{d\varphi} = -2\sqrt{\frac{16\pi}{12}} \left(1 + 2\sqrt{\frac{16\pi}{12}} \varphi\right)$ . Moreover, the potential as function of  $\varphi$  is,

$$V(\varphi) \simeq \lambda \left(\frac{2\sqrt{16\pi}}{\sqrt{12}}\right)^2 \frac{\varphi^2}{\xi^2}. \quad (17)$$

For phenomenological reasoning [76], we shall choose  $\xi \sim 11.455 \times 10^4$  with  $\lambda = 0.1$ . With regard to the EoS, we shall use a piecewise polytropic EoS, the details of which can be found in [34].

For  $\xi \sim 11.455 \times 10^4$ , the requirement (13) can in principle be satisfied, but the constraint of Equation (12) is not necessarily satisfied. As we will show, this is the case for the non-inflationary Higgs potential, and we shall verify this numerically. For the study we

shall consider static NSs, which are described by a spherically symmetric static spacetime of the form,

$$ds^2 = -e^{\nu(r)} dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (18)$$

where  $m(r)$  denotes the gravitational mass of the stellar object confined inside a radius  $r$ .

For Geometrized units ( $c = G = 1$ ), the TOV equations for the spherically symmetric spacetime are,

$$\frac{dm}{dr} = 4\pi r^2 A^4(\varphi) \epsilon + 2\pi r(r - 2m)\omega^2 + 4\pi r^2 V(\varphi), \quad (19)$$

$$\frac{dv}{dr} = 4\pi r\omega^2 + \frac{2}{r(r - 2m)} \left[ 4\pi A^4(\varphi) r^3 P - 4\pi V(\varphi) r^3 \right] + \frac{2m}{r(r - 2m)}, \quad (20)$$

$$\frac{d\omega}{dr} = \frac{rA^4(\varphi)}{r - 2m} \left( \alpha(\varphi)(\epsilon - 3P) + 4\pi r\omega(\epsilon - P) \right) - \frac{2\omega(r - m)}{r(r - 2m)} + \frac{8\pi\omega r^2 V(\varphi) + r \frac{dV(\varphi)}{d\varphi}}{r - 2m}, \quad (21)$$

$$\frac{dP}{dr} = -(\epsilon + P) \left[ \alpha(\varphi)\omega + 2\pi r\omega^2 + \frac{m - 4\pi r^3(-A^4 P + V)}{r(r - 2m)} \right], \quad (22)$$

$$\frac{d\varphi}{dr} = \omega, \quad (23)$$

The TOV equations must be solved numerically subject to the following initial conditions,

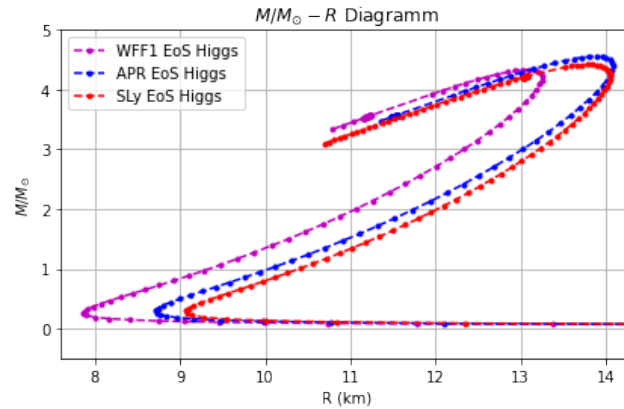
$$P(0) = P_c, \quad m(0) = 0, \quad v(0) = -v_c, \quad \varphi(0) = \varphi_c, \quad \omega(0) = 0, \quad (24)$$

where  $P_c, v_c, \varphi_c$  are the pressure of the NS, the value of the function  $v(r)$  and the value of the scalar field at the center of the NS. The values of  $nu_c$  and  $\varphi_c$  at the center of the star, shall be obtained using a double shooting method, in order for the optimal values of them to be obtained. The requirement for obtaining the optimal values is the scalar field values to vanish at numerical infinity, which proves to be the same numerically as in the inflationary Higgs potential, namely  $r \sim 67.94378528694695$  km in the Einstein frame, see [34]. In addition, for the derivation of the  $M - R$  gravity we need to consider the ADM Jordan frame mass and the Jordan frame radius. Denoting with  $r_E$  the Einstein frame radius at large distances, and  $\frac{d\varphi}{dr} = \frac{d\varphi}{dr} \Big|_{r=r_E}$ , the Jordan frame mass  $M_J \equiv M$  is related to the Einstein frame mass as follows,

$$M_J = A(\varphi(r_E)) \left( M_E - \frac{r_E^2}{2} \alpha(\varphi(r_E)) \frac{d\varphi}{dr} \left( 2 + \alpha(\varphi(r_E)) r_E \frac{d\varphi}{dr} \right) \left( 1 - \frac{2M_E}{r_E} \right) \right), \quad (25)$$

with  $\frac{d\varphi}{dr} = \frac{d\varphi}{dr} \Big|_{r=r_E}$  and  $r_J = Ar_E$ , and  $r_J$  is the Jordan frame radius. The Einstein frame radius  $R_s$  of the star can be obtained by the numerical code by using the condition  $P(R_s) = 0$ , so it is basically determined by the condition that the pressure of the star vanishes at the surface of the star. Accordingly, by finding  $R_s$  we can obtain the Jordan frame radius  $R$  using the relation  $R = A(\varphi(R_s)) R_s$ , where  $\varphi(R_s)$  is the value of the scalar field at the surface of the star. Finally and important note is to verify numerically the validity of the approximation (12) in the Jordan frame. For the numerical analysis, we shall use a freely available PYTHON code pyTOV-STT [92], and we shall derive the solutions for both the interior and the exterior of the NS, using the "LSODA" numerical method. The EoSs we shall use are the WFF1 [93], the SLy [94], and the APR EoS [95]. Let us proceed to the results of our analysis, and we start off with the  $M - R$  graphs for all the EoSs which we present in Figure 1. The purple curve corresponds to the WFF1 EoS, while the red and blue to the SLy and APR EoSs respectively. From the graphs it is apparent that for the non-inflationary Higgs model, the maximum masses are comparably higher with regard to the GR ones. In addition, in Table 1 we present all the maximum masses for all the EoSs corresponding to the alternative Higgs model. As it can be seen in Table 1, the maximum

masses for the alternative Higgs model are quite elevated compared to the GR ones. In addition, the GW170817 constraint which indicates that the radius corresponding to the maximum NS mass must be larger than is satisfied  $R = 9.6^{+0.14}_{-0.03}$  km.

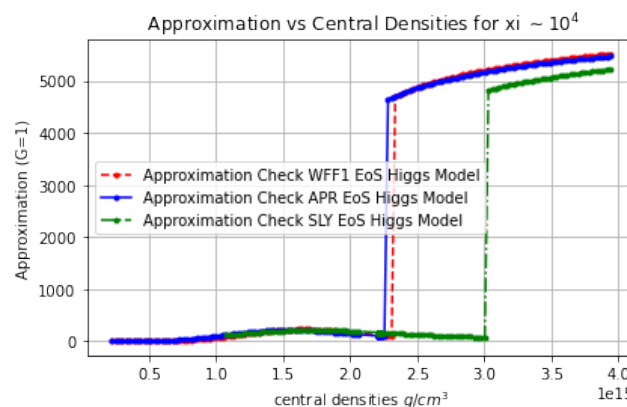


**Figure 1.**  $M - R$  graphs for the alternative non-inflationary Higgs model for the WFF1 EoS (purple curve), the APR EoS (blue curve), and the SLy EoS (red curve). The  $y$ -axis is expressed in  $M/M_{\odot}$  units, with  $M$  denoting the Jordan frame ADM mass, and the  $x$ -axis is the circumferential radius.

**Table 1.** Maximum Masses and the of Static NS for the non-Inflationary Higgs Model and for GR.

Model	APR EoS	SLy EoS	WFF1 EoS
GR	$M_{max} = 2.18739372 M_{\odot}$	$M_{max} = 2.04785291 M_{\odot}$	$M_{max} = 2.12603999 M_{\odot}$
Alternative Higgs $\zeta \sim 10^4$	$M_{max} = 4.55374471 M_{\odot}$	$M_{max} = 4.41766131 M_{\odot}$	$M_{max} = 4.33460622 M_{\odot}$

The results are deemed quite interesting; however, the non-inflationary Higgs model has inherent issues with the approximation (12) as we proved numerically. Particularly it is not satisfied neither at the center nor at the surface of the star. This feature can be clearly seen in Figure 2, where we present the values of  $\zeta\phi^2$  in the Jordan frame for all the EoS for the surface scalar field values. The same applies for the values of the scalar field in the center of the star. Therefore, to our original question whether inflationary scalar potentials or other approximations must be used for static NSs phenomenology, the answer seems to be that only inflationary potentials provide consistent results.



**Figure 2.** The quantity  $\zeta\phi^2$  ( $y$ -axis) in Geometrized units, versus the central densities in CGS units, for  $\zeta \sim 11.455 \times 10^4$ , for the WFF1 (red curve), APR (blue curve) and Sly (green curve) EoSs. As it can be seen the constraint (12) is not satisfied.

### 3. Concluding Remarks

In the field of cosmology, there exist several massive scalar field theories which can potentially play an important role for describing NSs phenomenology. From these theories, the most important is the Higgs inflationary theory in its various forms. Specifically, depending on the scalar field values, the Higgs potential can take various forms, each of which may describe a different era in the cosmological theory. Thus, the question is which approximate Higgs potential can describe in a viable and consistent static NS phenomenology. In this paper we addressed this question for the most fundamental of all the scalar field cosmologies, the Higgs inflationary theory. We considered the theory in the Jordan frame and upon conformally transforming it, we derived the Einstein frame theory. Accordingly, assuming a specific range for the scalar field values, we derived the appropriate quantities which are relevant for studying static NSs in the Einstein frame. For a static spherically symmetric spacetime we derived the TOV equations and we numerically solved them using a double shooting method for optimizing the results. The numerical analysis yielded the Einstein frame masses and radii of the static NS, and also the Einstein frame values of the scalar field, from which we found the corresponding Jordan frame quantities. We constructed the  $M - R$  graphs and we investigated the validity of the approximations holding true for the non-inflationary Higgs model. As we showed, the maximum masses for the alternative Higgs model are quite elevated, compared with the GR case; however, for all the EoSs studies, the approximation we assumed for deriving the theory break down. Thus, although the theory provides interesting result, the inherent structure of it is not correct and consistent. This indicates strongly the suitability of inflationary potentials for studying NSs phenomenology, regardless how well motivated other forms of potentials might be. Moreover, it seems that the approximations for the scalar field values used for deriving the inflationary potentials, are well respected on the surface, center and at numerical infinity of the NS. Hence, in conclusion the Higgs potential that is used for inflationary phenomenology is the only suitable for describing consistently NSs.

We need to note with regard to the EoSs we used, that we used the PAR, and more importantly the WFF1, known as FPS EoS, and the SLy, which both are known to provide a unified description of the crust and core of NSs. However, all these EoSs are to date rather old (nearly 20 years old); thus, it is compelling to incorporate to the analysis more timely and to date EoSs, such as the BSk24 [96,97].

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