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Symmetry Breaking of Universal Type and Particular Types

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Abstract: The concepts of symmetry and its breakdown are investigated in two different terms according to whether the resulting asymmetry is universal or only obtained for a special configuration: we illustrate this by considering, in the first case, an example from the standard model of particles with some consequences for cosmological scenarios; and in the second case, we consider an example from specific solutions for the particle dynamics, and an example for a toy model of entangled spins.

Keywords: symmetry breaking; standard model; spinor fields; entangled spins

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1. Introduction

In physics, one of the most important concepts is that of symmetry: from general coordinate covariance, through local Lorentz covariance, to gauge covariance, symmetry is the basis upon which all kinematical quantities are defined. Furthermore, when such kinematic quantities are coupled together into dynamic field equations, the requirement of covariance is also capable of restricting the possible ways in which this coupling can be achieved, to the point that, when renormalizability is further assumed, the possible terms within the Lagrangian are reduced to just a few.

For this reason, symmetry principles play a fundamental role. Nevertheless, starting from a theory that is symmetric, we are eventually forced to address the fact that nature is obviously not symmetric. Symmetry must be followed by a breakdown, giving asymmetry.

Because every symmetry is represented as some invariance under transformations, symmetry breaking must be described by fixing some parameter in those transformations, leading to the fact that a specific physical situation described by a Lagrangian has less symmetry than the general Lagrangian. However, as parameters can be either universal or proper to specific situations, it follows that there are two types of asymmetries according to whether they are obtained for the universe as a whole or for specific subsystems.

So, in the first case, symmetry breaking is of the type we have, for example, in the standard model, where the Higgs vacuum $\phi^2 = v^2$ is given in terms of a universal parameter [1–5]. The concept of symmetry breaking in the standard model is perhaps the prototypical example of symmetry breaking upon which subsequent types of breakdown have been achieved, from different manners to achieve it spontaneously or dynamically [6–8] (other particular types are also the nonlinear realizations, as discussed in [9,10]).

In the second case, symmetry breaking is of the type we have, for instance, when looking for solutions of field equations, where a choice of boundary conditions is different for different solutions. This type of symmetry breaking is less known as a way to have a breakdown, but it is not less common, with examples that are found in basic physical situations—from the fact that, even in presence of a central Newtonian gravitational attraction, the planetary orbits can also be elliptic, to the fact that, even in presence of a central Coulomb electrostatic force, the electronic orbitals can also display angular harmonics.

In the following, we will review some of the symmetries and their breaking both for the case of universal type and for the case of particular type. In the last case, we will deepen the discussion by presenting two distinct notable cases.

2. Symmetry Breaking of the Universal Type: The Standard Model

We will begin the treatment by recalling the standard model in a manner that is slightly different from the way it is usually presented, so to highlight specific features of interest. Because the SM is the theory that is symmetric under the group $\mathrm{SU}(2)\times\mathrm{U}(1)$, we start by giving generalities about these transformations and their properties.

In the most general form, $\mathrm{SU}(2)$ transformations are defined by means of the Pauli matrices $\vec{\boldsymbol{\sigma}}$, which are such that $\boldsymbol{\sigma}^{A}\boldsymbol{\sigma}^{B}=\delta^{AB}\mathbb{I}+i\varepsilon^{ABC}\boldsymbol{\sigma}^{C}$, where δ^{AB} and ε^{ABC} are the Kronecker delta and the Levi-Civita symbol in 3-dimensional spaces. Then, in terms of the Pauli matrices and the parameters $\vec{\theta}$, we have that

$$\boldsymbol{U}=e^{-\frac{i}{2}\vec{\boldsymbol{\sigma}}\cdot\vec{\theta}} \tag{1}$$

is the most general form of an $\mathrm{SU}(2)$ transformation. It is possible to make this expression explicit by defining

$$y^{2}=\vec{\theta}/2\cdot\vec{\theta}/2 \tag{2}$$

and hence

$$X=\cos y \tag{3}$$

$$\vec{Z}=\frac{1}{2}\frac{\sin y}{y}\vec{\theta} \tag{4}$$

so that we can now write

$$\boldsymbol{U}=X\mathbb{I}-i\vec{Z}\cdot\vec{\boldsymbol{\sigma}} \tag{5}$$

in explicit form. The most general form of $\mathrm{SU}(2)\times\mathrm{U}(1)$ transformations is then given by the product of the above times a unitary phase α and, since they commute, we have that they can be written according to

$$\boldsymbol{S}=\boldsymbol{U}e^{\frac{i}{2}\alpha}=(X\mathbb{I}-i\vec{Z}\cdot\vec{\boldsymbol{\sigma}})e^{\frac{i}{2}\alpha} \tag{6}$$

as it is obvious. The doublet of complex scalar fields that transforms according to the transformation

$$\Phi\rightarrow\boldsymbol{S}\Phi \tag{7}$$

which is known as Higgs field. From it, we can build

$$\Phi^{\dagger}\vec{\boldsymbol{\sigma}}\Phi=\vec{S} \tag{8}$$

$$\Phi^{\dagger}\Phi=P \tag{9}$$

which are both real quantities, such that

$$\Phi\Phi^{\dagger}=\tfrac{1}{2}P\mathbb{I}+\tfrac{1}{2}\vec{S}\cdot\vec{\boldsymbol{\sigma}} \tag{10}$$

as well as

$$\vec{S}\cdot\vec{S}=P^{2} \tag{11}$$

which are valid as general geometric identities, known as Fierz identities.

Because the Higgs field transforms in this way, we can prove that one can always find a gauge, in which

$$\Phi=\phi\boldsymbol{R}^{-1}\left(\begin{array}{c}0\\1\end{array}\right) \tag{12}$$

for some $\boldsymbol{R}$, and in terms of ϕ being a generic real scalar field, and the only degree of freedom, called a module. Then,

$$\vec{S} = \phi^2 \vec{s} \tag{13}$$

and

$$P = \phi^2 \tag{14}$$

such that

$$\Phi\Phi^\dagger = \tfrac{1}{2}\phi^2(\mathbb{I} + \vec{s}\cdot\vec{\sigma}) \tag{15}$$

as well as

$$\vec{s}\cdot\vec{s} = 1 \tag{16}$$

is the normalized vector of isospin. Written in polar form, its 4 real components are rearranged into that very special configuration, where the real scalar degree of freedom is isolated from the 3 real components, that are passed on into the gauge through the 3 parameters of the $\boldsymbol{R}$ matrix, known with the name of Goldstone bosons [11,12].

It can be seen from the expression (6) that, by introducing

$$(\partial_\mu X\vec{Z} - X\partial_\mu\vec{Z}) + \vec{Z}\times\partial_\mu\vec{Z} = -\frac{1}{2}\partial_\mu\vec{\zeta} \tag{17}$$

we can always write

$$\boldsymbol{S}^{-1}\partial_\mu\boldsymbol{S} = -\frac{i}{2}\partial_\mu\vec{\zeta}\cdot\vec{\sigma} + \frac{i}{2}\partial_\mu\alpha\mathbb{I} \tag{18}$$

as a general identity. Upon the introduction of two gauge fields, $\vec{A}_\mu$ and B_μ, which transforms according to

$$g\vec{A}_\mu\cdot\vec{\sigma} \to \boldsymbol{U}\left[(g\vec{A}_\mu - \partial_\mu\vec{\zeta})\cdot\vec{\sigma}\right]\boldsymbol{U}^{-1} \tag{19}$$

$$g'B_\mu \to g'B_\mu - \partial_\mu\alpha \tag{20}$$

with coupling constants g and g', we can see that

$$D_\mu\Phi = \nabla_\mu\Phi - \tfrac{i}{2}\left(g\vec{A}_\mu\cdot\vec{\sigma} - g'B_\mu\mathbb{I}\right)\Phi \tag{21}$$

is the gauge covariant derivative of the Higgs field [13].

Since from the polar form of the Higgs, we can see that

$$\boldsymbol{R}^{-1}\partial_\mu\boldsymbol{R} = -\frac{i}{2}\partial_\mu\vec{\xi}\cdot\vec{\sigma} + \frac{i}{2}\partial_\mu\xi\mathbb{I} \tag{22}$$

where ξ and $\vec{\xi}$ are the Goldstone modes, we can define

$$g\vec{M}_\mu = g\vec{A}_\mu - \partial_\mu\vec{\xi} \tag{23}$$

$$g'N_\mu = g'B_\mu - \partial_\mu\xi \tag{24}$$

which are proven to be true vector fields, from which

$$D_\mu\Phi = \left[\nabla_\mu \ln\phi - \tfrac{i}{2}\left(g\vec{M}_\mu\cdot\vec{\sigma} - g'N_\mu\mathbb{I}\right)\right]\Phi \tag{25}$$

is the gauge covariant derivative of the Higgs field, and

$$\nabla_\mu \vec{s} = g\vec{M}_\mu \times \vec{s} \tag{26}$$

are general identities. The Goldstone states are absorbed by the gauge fields as their longitudinal components [14].

As for the dynamics, we will consider the following Lagrangian:

$$\mathscr{L} = D_\mu \Phi^\dagger D^\mu \Phi - \tfrac{1}{2}\lambda^2\left(v^2 - \Phi^\dagger \Phi\right)^2 \tag{27}$$

with the v and λ constants and $\mathrm{SU}(2)\times\mathrm{U}(1)$ invariant.

Plugging the polar form of the gauge covariant derivative, we obtain the following polar form of the Lagrangian:

$$\mathscr{L} = \nabla_\mu \phi \nabla^\mu \phi + \tfrac{1}{4}\phi^2 (g^2 \vec{M}_\mu \cdot \vec{M}^\mu + g'^2 N^\mu N_\mu - 2gg' N^\mu \vec{M}_\mu \cdot \vec{s}) - \tfrac{1}{2}\lambda^2\left(v^2 - \phi^2\right)^2 \tag{28}$$

where we have no information about the direction of the isospin. If it is along the third axis of the internal space,

$$-g\vec{M}_\mu \cdot \vec{s} = gM^3_\mu \tag{29}$$

and then the Lagrangian becomes

$$\begin{aligned}\mathscr{L} = \nabla_\mu \phi \nabla^\mu \phi + \tfrac{1}{4}\phi^2 [g^2 (M^1_\mu M^\mu_1 + M^2_\mu M^\mu_2) + \\ + (gM^3_\mu + g' N_\mu)(gM^\mu_3 + g' N^\mu)] - \tfrac{1}{2}\lambda^2\left(v^2 - \phi^2\right)^2\end{aligned} \tag{30}$$

so that, after diagonalizing as

$$\tfrac{1}{\sqrt{2}}\left(M^1_\mu \pm iM^2_\mu\right) = W^\pm_\mu \tag{31}$$

$$gM^\mu_3 + g' N^\mu = \sqrt{g^2 + g'^2} Z^\mu \tag{32}$$

$$-g' M^\mu_3 + gN^\mu = \sqrt{g^2 + g'^2} A^\mu \tag{33}$$

we eventually obtain

$$\mathscr{L} = \nabla_\mu \phi \nabla^\mu \phi + \tfrac{1}{4}\phi^2 [2g^2 W^+ W^- + (g^2 + g'^2) Z^2] - \tfrac{1}{2}\lambda^2\left(v^2 - \phi^2\right)^2 \tag{34}$$

as it is easy to see. The potential is defined so to be zero at its minimum. Minimizing the potential with respect to the field, we obtain $\left(v^2 - \phi^2\right)\phi = 0$, showing that there are two equilibria, one for $\phi = 0$ and one for $\phi^2 = v^2$—exactly as in the usual version of the standard model. The first gives to the potential the value $\frac{1}{2}\lambda^2 v^4$, while the second gives to the potential the value 0, so that the former is the unstable configuration and the latter is the stable configuration. The former is also the configuration that preserves the symmetry of the vacuum, while the latter does not preserve such a symmetry, as is clear from the fact that, in this case, the following Higgs vacuum:

$$\Phi = v\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{35}$$

is not annihilated by the generator

$$\frac{1}{2}(\mathbb{I} - \sigma^3) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{36}$$

as it is easy to check. The above coefficients show that

$$Q = \frac{1}{2}(Y - T^3) \tag{37}$$

is the relation tying the hypercharge and the third component of the isospin to the electric charge. The symmetric configuration is the one that in the standard model is assumed to be the configuration of the initial state of the universe, due to arguments of symmetry that are encoded by the cosmological principle. However, it is unstable. The stable configuration is the one that is assumed to be the configuration toward which the system would tend, since it is generally assumed that any system would tend toward configurations of least possible energy. Nevertheless, it is asymmetric. This gives us the general evolution of the Higgs field configuration in the standard model. The universe starts with an initial configuration that is assumed to be symmetric and thus given by the $\phi = 0$ state. In this case, we obtain a positive cosmological constant term. To see this, it is necessary to write the Lagrangian in the presence of gravity. Given the metric $g_{\mu\nu}$, and the following metric-compatible symmetric connection:

$$\Lambda^{\sigma}_{\alpha\nu} = \frac{1}{2} g^{\sigma\rho}(\partial_\alpha g_{\rho\nu} + \partial_\nu g_{\alpha\rho} - \partial_\rho g_{\alpha\nu}) \tag{38}$$

it is possible to set the following conventional definition:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\sigma\nu} - \partial_\nu \Gamma^{\rho}{}_{\sigma\mu} + \Gamma^{\rho}{}_{k\mu}\Gamma^{k}{}_{\sigma\nu} - \Gamma^{\rho}{}_{k\nu}\Gamma^{k}{}_{\sigma\mu} \tag{39}$$

for the Riemann curvature tensor. Another convention sets

$$R^{\rho}{}_{\sigma\rho\nu} = R_{\sigma\nu} \tag{40}$$

for the Ricci curvature tensor, and then

$$R_{\sigma\nu} g^{\sigma\nu} = R \tag{41}$$

is the Ricci curvature scalar. This is used to assign the gravitational Lagrangian in addition to (27), so that we have the full

$$\mathscr{L} = -R + D_\mu \Phi^\dagger D^\mu \Phi - \frac{1}{2}\lambda^2 \left(v^2 - \Phi^\dagger \Phi\right)^2 \tag{42}$$

having normalized the Newton constant. Its variation gives the Einstein gravitational field equations as follows:

$$R_{\alpha\nu} - \frac{1}{2} R g_{\alpha\nu} = D_\alpha \Phi^\dagger D_\nu \Phi - \frac{1}{2} D_\mu \Phi^\dagger D^\mu \Phi g_{\alpha\nu} + \frac{1}{4}\lambda^2 \left(v^2 - \Phi^\dagger \Phi\right)^2 g_{\alpha\nu} \tag{43}$$

with the convention that the sign of the time–time component be always positive, as it is needed due to the fact that this is the energy density. This equation can be contracted as

$$-R = -D_\mu \Phi^\dagger D^\mu \Phi + \lambda^2 \left(v^2 - \Phi^\dagger \Phi\right)^2 \tag{44}$$

which can be substituted back into the initial one, to give

$$R_{\alpha\nu} = D_\alpha \Phi^\dagger D_\nu \Phi - \frac{1}{4}\lambda^2 \left(v^2 - \Phi^\dagger \Phi\right)^2 g_{\alpha\nu} \tag{45}$$

as the gravitational equation. In the configuration we have at the beginning of the universe

$$R_{\alpha\nu} = -\frac{1}{4}\lambda^2 v^4 g_{\alpha\nu} \tag{46}$$

giving an effective cosmological constant $\Lambda_{\text{effective}} = \left|\lambda v^2/2\right|^2$ for the convention we have chosen here above. This means that the cosmological constant is positive in our convention, or equivalently that it gives rise to an expansion. In fact, as is well known, when the Ricci tensor is proportional to the metric tensor, with negative proportionality constant, that is when the effective cosmological constant has positive value, the metric solution is that of a Friedmann universe with a scale factor given by an exponential function of the cosmological time [15]. As such, an inflationary epoch takes place [16]. There is no mass generation up to this point. Nevertheless, such a condition is unstable. So, it will spontaneously move toward a stable state. This is given by the $\phi^2 = v^2$ state. In this case, the new vacuum has a value for which the cosmological constant term is canceled, quenching inflation. However, with respect to the new vacuum, the symmetry has now been broken. Mass generation is carried out by noticing that, for the vacuum $\phi^2 = v^2$, we can choose the re-parametrization

$$\phi = v + H \tag{47}$$

since, in this case, $H = 0$ corresponds to the $\phi^2 = v^2$ constraint—that is, to the vacuum. Then,

$$\begin{aligned} \mathscr{L} = \nabla_\mu H \nabla^\mu H - \tfrac{1}{4} m_H^2 H^4/v^2 - m_H^2 H^3/v - m_H^2 H^2 + \\ + (H^2/v^2 + 2H/v)(m_W^2 W^+ W^- + \tfrac{1}{2} m_Z^2 Z^2) + (m_W^2 W^+ W^- + \tfrac{1}{2} m_Z^2 Z^2) \end{aligned} \tag{48}$$

with $2\lambda^2 v^2 = m_H^2$ as the Higgs mass together with $g^2 v^2 = 2m_W^2$ and $v^2(g^2 + g'^2) = 2m_Z^2$ as the two vector boson masses like it is carried out in the usual standard model. Notice that because the effective Lagrangian has to give the known 4-fermion interaction, the value of the vacuum can be fixed by the Fermi constant at $v \approx 200$ GeV, as the scale of the symmetry breaking.

It is important to highlight that after the symmetry group $\mathrm{SU}(2) \times \mathrm{U}(1)$ is broken, there still remains a residual symmetry for the full Lagrangian, given by a $\mathrm{U}(1)$ group, associated with the charge $\frac{1}{2}(Y - T^3) = Q$, which gives rise to an Abelian gauge field recognized as electrodynamics. It is also important to underline that there is yet another residual symmetry for the Higgs potential, given by the custodial $\mathrm{SU}(2)$ group. It is not within the scope of the paper to account for either of these residual symmetries in the following.

The known phenomenology, apart from the dynamical structure of the Lagrangian, is obtained by the conditions $s_a = -(0, 0, 1)$ and $\phi = v + H$, which together form the full symmetry-breaking conditions. Both are universal in the sense that they are a choice of fields and parameters that is the same throughout the whole cosmological setting.

Next, we will observe that this is not always the case.

3. Symmetry Breaking of Particular Type: Spinorial Fields

We continue the presentation introducing the concept of tetrads, or frames, in the general theory of relativity. It is important to specify from the start that, here, with general relativity, we do not mean the dynamical theory of Einsteinian gravity, obtained by assigning the field equations that link the space–time curvature to the energy tensor and interpreting the space–time curvature as gravitation. Here, with general relativity, we mean the kinematical theory that implements the principle of general covariance under curvilinear coordinate transformations by employing tensor quantities. As such, we are simply meaning to retain the possibility to study fields in whatever system of reference, whether or not the space–time has a curvature. All we say is valid, regardless the space–time dynamical character, and regardless of the fact that such dynamical character be determined by Einstein gravity or any of its extensions.

The initial point is that of assuming the existence of a metric tensor $g_{\mu\nu} = g_{\nu\mu}$ with inverse $g^{\mu\nu} = g^{\nu\mu}$, such that we have $g_{\mu\rho} g^{\rho\alpha} = \delta^\alpha_\mu$, where δ^α_μ is the Kronecker delta. We specify that this metric need not be in Minkowskian form, because we retain the right to employ whatever system of coordinates, even in flat space–time. Nonetheless, we can always introduce bases of vectors e^μ_a and e^a_μ, such that

$$e^\mu_a e^b_\mu = \delta^b_a \qquad e^\mu_a e^a_\nu = \delta^\mu_\nu \tag{49}$$

are called tetrads, and for which

$$e^a_\mu e^b_\nu g^{\mu\nu} = \eta^{ab} \quad e^\mu_a e^\nu_b g_{\mu\nu} = \eta_{ab} \tag{50}$$

where η is the Minkowskian matrix. The fact that we can always choose to do this comes from the fact that we can always make the ortho-normalization procedure on the tetradic fields. The tetradic fields have two indices, the Latin one indicates what vector of the basis we choose, and the Greek one denotes what component of that vector we pick. As with for the metric, the Greek index is associated with a general coordinate transformation. The Latin index is associated with a new type of transformation shuffling vectors within the basis, and, consequently, this transformation must be a Lorentz transformation, since we wish the Minkowskian matrix to be preserved. Indicating such a real Lorentz transformation as Λ, we have

$$e^a_\nu \to (\Lambda)^a_b e^b_\nu \qquad\qquad e^\nu_a \to (\Lambda^{-1})^b_a e^\nu_b \tag{51}$$

as the transformation on tetrads. We also introduce a set of Clifford matrices γ^a, verifying the relations

$$\{\gamma^a, \gamma^b\} = 2\mathbb{I}\eta^{ab} \tag{52}$$

where $\mathbb{I}$ is the identity matrix. It is then possible to define

$$\tfrac{1}{4}\left[\gamma^a, \gamma^b\right] = \boldsymbol{\sigma}^{ab} \tag{53}$$

and one can easily verify that the $\boldsymbol{\sigma}^{ab}$, thus defined, satisfies the commutation relations defining the complex Lorentz algebra; therefore, they are the generators of the complex Lorentz group. We also have that

$$2i\boldsymbol{\sigma}_{ab} = \varepsilon_{abcd}\boldsymbol{\pi}\boldsymbol{\sigma}^{cd} \tag{54}$$

implicitly defining the $\boldsymbol{\pi}$ matrix (this matrix is usually denoted as a gamma matrix with an index five, but as this index has no sense in four-dimensional space–times, here we will adopt a notation without any index at all), whose existence proves that the complex Lorentz group is reducible. From them,

$$\gamma_i\gamma_j\gamma_k = \gamma_i\eta_{jk} - \gamma_j\eta_{ik} + \gamma_k\eta_{ij} + i\varepsilon_{ijkq}\boldsymbol{\pi}\gamma^q \tag{55}$$

which are valid as geometric identities.

In the most general form the complex Lorentz transformation is by means of the Clifford matrices γ_i defined above. In terms of these, and specifically the sigma matrices $\boldsymbol{\sigma}^{ab}$ belonging to the Lorentz algebra, and the parameters $\theta_{ab} = -\theta_{ba}$, we have that

$$\boldsymbol{\Lambda} = e^{\frac{1}{2}\theta_{ab}\boldsymbol{\sigma}^{ab}} \tag{56}$$

in the most general case. A more compact form can be written by defining

$$a = -\tfrac{1}{8}\theta_{ij}\theta^{ij} \tag{57}$$

$$b = \tfrac{1}{16}\theta_{ij}\theta_{ab}\varepsilon^{ijab} \tag{58}$$

and then

$$2x^2 = a + \sqrt{a^2 + b^2} \tag{59}$$

$$2y^2 = -a + \sqrt{a^2 + b^2} \tag{60}$$

so that we can introduce

$$\cos y \cosh x = X \tag{61}$$
$$\sin y \sinh x = Y \tag{62}$$
$$\left(\frac{x \sinh x \cos y + y \sin y \cosh x}{x^2+y^2}\right)\theta^{ab} + \left(\frac{x \cosh x \sin y - y \cos y \sinh x}{x^2+y^2}\right)\frac{1}{2}\theta_{ij}\varepsilon^{ijab} = Z^{ab} \tag{63}$$

allowing us to write

$$\boldsymbol{\Lambda} = X\mathbb{I} + Yi\boldsymbol{\pi} + \frac{1}{2}Z^{ab}\boldsymbol{\sigma}_{ab} \tag{64}$$

in the most compact and explicit way. The most complete complex Lorentz and phase transformation is therefore

$$\boldsymbol{S} = \boldsymbol{\Lambda}e^{iq\alpha} = (X\mathbb{I} + Yi\boldsymbol{\pi} + \frac{1}{2}Z^{ab}\boldsymbol{\sigma}_{ab})e^{iq\alpha} \tag{65}$$

called spinorial transformation. Any column of 4 complex functions that—under general coordinate transformations—are scalars, but which, for a spinorial transformation, are converted according to the following law:

$$\psi \rightarrow \boldsymbol{S}\psi \tag{66}$$

is called spinorial field. It is possible to prove that with the adjoint $\overline{\psi} = \psi^\dagger\boldsymbol{\gamma}^0$, we can construct the quantities

$$\Sigma^{ab} = 2\overline{\psi}\boldsymbol{\sigma}^{ab}\boldsymbol{\pi}\psi \tag{67}$$
$$M^{ab} = 2i\overline{\psi}\boldsymbol{\sigma}^{ab}\psi \tag{68}$$
$$S^a = \overline{\psi}\boldsymbol{\gamma}^a\boldsymbol{\pi}\psi \tag{69}$$
$$U^a = \overline{\psi}\boldsymbol{\gamma}^a\psi \tag{70}$$
$$\Theta = i\overline{\psi}\boldsymbol{\pi}\psi \tag{71}$$
$$\Phi = \overline{\psi}\psi \tag{72}$$

which are all real tensors, and such that

$$\psi\overline{\psi} \equiv \frac{1}{4}\Phi\mathbb{I} + \frac{1}{4}U_a\boldsymbol{\gamma}^a + \frac{i}{8}M_{ab}\boldsymbol{\sigma}^{ab} - \frac{1}{8}\Sigma_{ab}\boldsymbol{\sigma}^{ab}\boldsymbol{\pi} - \frac{1}{4}S_a\boldsymbol{\gamma}^a\boldsymbol{\pi} - \frac{i}{4}\Theta\boldsymbol{\pi} \tag{73}$$

as well as

$$\Sigma^{ab} = -\frac{1}{2}\varepsilon^{abij}M_{ij} \tag{74}$$

with

$$M_{ab}\Theta + \Sigma_{ab}\Phi = U_{[a}S_{b]} \tag{75}$$

alongside to

$$U_aS^a = 0 \tag{76}$$
$$U_aU^a = -S_aS^a = \Theta^2 + \Phi^2 \tag{77}$$

as is straightforward to prove, called Fierz identities.

Let us settle on the general case, where $\Theta^2 + \Phi^2 \neq 0$ (a parallel analysis could also be carried out in the case in which $\Theta = \Phi \equiv 0$; although, in this case, we talk about a very specific type of spinors that we are not going to treat in the present work). Then, we have that it is always possible to write the spinor as

$$\psi = \phi e^{-\frac{i}{2}\beta\pi} \boldsymbol{L}^{-1} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \tag{78}$$

in chiral representation, with $\boldsymbol{L}$ a Lorentz transformation and with ϕ and β that are real scalar and pseudo-scalar fields,and the only degrees of freedom,called module and Yvon–Takabayashi angle. We then can compute

$$S^a = 2\phi^2 s^a \tag{79}$$
$$U^a = 2\phi^2 u^a \tag{80}$$

as well as

$$\Theta = 2\phi^2 \sin\beta \tag{81}$$
$$\Phi = 2\phi^2 \cos\beta \tag{82}$$

from which

$$\psi\overline{\psi} \equiv \frac{1}{2}\phi^2 e^{-i\beta\pi}(e^{i\beta\pi} + u_a\gamma^a)(e^{-i\beta\pi} - s_a\gamma^a\boldsymbol{\pi}) \tag{83}$$

and

$$u_a s^a = 0 \tag{84}$$
$$u_a u^a = -s_a s^a = 1 \tag{85}$$

are the normalized velocity vector and spin axial vector, as well known. Written in polar form, the 8 real components of the spinor can be rearranged in such a way that the 2 real scalar degrees of freedom are isolated from the 6 real components that can always be transferred into the frame through the 6 parameters of the Lorentz transformation $\boldsymbol{L}$, which can be identified as Goldstone bosons.

As above, it can be seen from (65) that defining

$$(\partial_\mu X Z^{ab} - X\partial_\mu Z^{ab}) + \frac{1}{2}(\partial_\mu Y Z_{ij} - Y\partial_\mu Z_{ij})\varepsilon^{ijab} + \partial_\mu Z^{ak} Z^b{}_k = -\partial_\mu \zeta^{ab} \tag{86}$$

allows us to write

$$\boldsymbol{S}^{-1}\partial_\mu \boldsymbol{S} = \frac{1}{2}\partial_\mu \zeta_{ab}\boldsymbol{\sigma}^{ab} + iq\partial_\mu\alpha\mathbb{I} \tag{87}$$

as a general identity. With the spin connection $\Omega_{ij\mu}$ and the gauge field A_μ given in terms of their transformations

$$\tfrac{1}{2}\Omega_{ij\mu}\boldsymbol{\sigma}^{ij} \to \boldsymbol{\Lambda}\left[\tfrac{1}{2}(\Omega_{ij\mu} - \partial_\mu\zeta_{ij})\boldsymbol{\sigma}^{ij}\right]\boldsymbol{\Lambda}^{-1} \tag{88}$$
$$A_\mu \to A_\mu - \partial_\mu\alpha \tag{89}$$

we have that

$$\boldsymbol{\nabla}_\mu\psi = \partial_\mu\psi + \frac{1}{2}\Omega_{ij\mu}\boldsymbol{\sigma}^{ij}\psi + iqA_\mu\psi \tag{90}$$

is the spinorial covariant derivative of the spinor field [17].

From the polar form of the spinor we can also see that

$$\boldsymbol{L}^{-1}\partial_\mu \boldsymbol{L} = iq\partial_\mu\xi\mathbb{I} + \frac{1}{2}\partial_\mu\xi^{ab}\boldsymbol{\sigma}_{ab} \tag{91}$$

where ξ and ξ^{ab} are the Goldstone states, so defining

$$q(\partial_\mu\xi - A_\mu) \equiv P_\mu \tag{92}$$
$$\partial_\mu\xi_{ij} - \Omega_{ij\mu} \equiv R_{ij\mu} \tag{93}$$

which are true tensor fields, we have that

$$\boldsymbol{\nabla}_\mu\psi = (-\frac{i}{2}\nabla_\mu\beta\boldsymbol{\pi} + \nabla_\mu \ln\phi\mathbb{I} - iP_\mu\mathbb{I} - \frac{1}{2}R_{ij\mu}\boldsymbol{\sigma}^{ij})\psi \tag{94}$$

as spinorial covariant derivatives, such that

$$\nabla_\mu s_i = R_{ji\mu}s^j \tag{95}$$
$$\nabla_\mu u_i = R_{ji\mu}u^j \tag{96}$$

are general identities. The Goldstone states are absorbed by the gauge field and spin connection thus becoming the longitudinal components of the P_μ and $R_{ji\mu}$ tensors [14].

As for the dynamics, we consider the Dirac equations

$$i\boldsymbol{\gamma}^\mu\boldsymbol{\nabla}_\mu\psi - XW_\mu\boldsymbol{\gamma}^\mu\boldsymbol{\pi}\psi - m\psi = 0 \tag{97}$$

with W_μ axial vector torsion and X torsion–spin coupling constant, added to be in the most general case [17].

In polar form, these equations decompose according to

$$B_\mu - 2P^\iota u_{[\iota}s_{\mu]} + (\nabla\beta - 2XW)_\mu + 2s_\mu m\cos\beta = 0 \tag{98}$$
$$R_\mu - 2P^\rho u^\nu s^\alpha\varepsilon_{\mu\rho\nu\alpha} + 2s_\mu m\sin\beta + \nabla_\mu \ln\phi^2 = 0 \tag{99}$$

with $R_{\mu a}{}^{a} = R_\mu$ and $\frac{1}{2}\varepsilon_{\mu\alpha\nu\iota}R^{\alpha\nu\iota} = B_\mu$ and which can be proven equivalent to the Dirac equations. In fact they are two special Gordon decompositions, which, in polar form, possess the same information of the Dirac Equations [18].

We have no information about the direction of velocity and spin, although we can always boost in the rest frame and there align the spin along the third axis. In doing so, we obtain the possibility to choose P_ν and $R_{ij\nu}$ in ways that might allow us to find spinorial field solutions that could be written in the radial and angular coordinates, without variable separability [19,20]. As solutions of this kind depend on the elevation angle, the spinor symmetry can never be more than an axial symmetry even if the background had spherical symmetry. The fact that the solution of a given equation has less symmetry than that very equation tells that symmetry breaking occurred. It is of particular type as it occurs only for that specific solution. Furthermore, it occurs only for that solution due to the boundary conditions.

Similarly to the previous case, we have that situations of a given symmetry allow the system to be reconfigured into a form in which some degrees of freedom, recognized as Goldstone states, are transferred into gauge fields, and symmetry breaking can occur. Differently from the previous case, where the symmetry breaking meant selecting a configuration of different fields, now symmetry breaking means selecting a configuration of different components within a single field. This difference is notable.

Nevertheless, for this last case, symmetry breaking of particular type can occur in more than one way. In the case described in this section, symmetry breaking of particular type occurs because some solution is defined in terms of boundary conditions that differ for different solutions but which for a single solution they are defined once and for all.

However, we could also have cases in which additional properties of a given solution are determined by boundary conditions that differ even for a single solution. To be more mathematically detailed, boundary conditions can be fixed for ϕ and β, since these fields are determined by the field equations. On the other hand, there can be no fixing $\boldsymbol{L}$, since there is no way to determine it from any field equation. So, for example, a given spinor solution representing an electronic orbital in a hydrogen-like atom has a fixed form of the module. Yet, there is no determined information about the direction of its spin.

For this last instance, one can ask if there can be any relation between the Goldstone degrees of freedom contained within $\boldsymbol{L}$, and some properties of observable quantities, such as the spin orientation. This is what we intend to do in the next section—that is, exploiting Goldstone states of spinor fields to discuss a possible mechanism for entangled spins that is compatible with relativistic principles. Therefore, a comparison with the relativistic version of the de Broglie–Bohm theory with spin will naturally arise.

4. Symmetry Breaking of Particular Type: Spin Entanglement

The polar decomposition applied to the case of spinors has considerable advantage when it comes to seeing what are the physical degrees of freedom, or finding solutions to the Dirac equations. However, it has also another utility in providing the equivalent of the Madelung decomposition in the relativistic case [21]. So, relativistic spinors can also be interpreted from the hydrodynamic perspective [22].

To better see this, consider the polar form of Dirac Equations (98) and (99), written as follows:

$$Y_\mu - P^\iota u_{[\iota} s_{\mu]} + m s_\mu \cos\beta = 0 \tag{100}$$

$$Z_\mu + P^\rho u^\nu s^\alpha \varepsilon_{\mu\rho\nu\alpha} - m s_\mu \sin\beta = 0 \tag{101}$$

where $(\nabla\beta - 2XW + B)_k = 2Y_k$ and $(\nabla \ln\phi^2 + R)_k = -2Z_k$ are potentials. So, we can invert the momentum [23] as

$$P^\rho = m\cos\beta u^\rho + Y_\nu u^{[\nu} s^{\rho]} + Z_\mu s_\alpha u_\nu \varepsilon^{\mu\alpha\nu\rho} \tag{102}$$

after straightforward manipulation. This form shows that the momentum P_ν is not just the kinematic momentum mu_ν, but there are a number of corrections. One is in the correction due to the Yvon–Takabayashi angle $\cos\beta$, which expresses the effects of internal dynamics [18]. The others are proportional to the spin axial vector and due to the Y_ν and Z_μ potentials. These are given by some external contributions of W_α and $R_{ij\alpha}$ plus the derivatives of the β and $\ln\phi^2$, and as such they can be seen as the quantum potentials in relativistic version with spin. The fact that they are first-order differentials is the consequence of their relativistic essence, and the existence of a second quantum potential is the consequence of the internal structure that comes from the presence of spin. Both potentials are in terms multiplying the spin axial vector and, consequently, they disappear in the macroscopic approximation. That the macroscopic approximation be encoded by the condition $s_a \to 0$ is clear from the fact that, if we were not to normalize $\hbar = 1$, then the spin axial vector would be multiplied by the Planck constant, and $\hbar \to 0$ is the definition of the non-quantum limit.

To better see this fact, let us write the expression of the energy of the spinorial field in its polar form. We have

$$E^{\rho\sigma} = \frac{i}{4}(\overline{\psi}\gamma^\rho \boldsymbol{\nabla}^\sigma\psi - \boldsymbol{\nabla}^\sigma\overline{\psi}\gamma^\rho\psi + \overline{\psi}\gamma^\sigma\boldsymbol{\nabla}^\rho\psi - \boldsymbol{\nabla}^\rho\overline{\psi}\gamma^\sigma\psi) - \frac{1}{2}X(W^\sigma\overline{\psi}\gamma^\rho\boldsymbol{\pi}\psi + W^\rho\overline{\psi}\gamma^\sigma\boldsymbol{\pi}\psi) \tag{103}$$

which in polar form becomes

$$E^{\rho\sigma} = \phi^2\left[(\nabla\beta/2 - XW)^\sigma s^\rho + (\nabla\beta/2 - XW)^\rho s^\sigma + P^\sigma u^\rho + P^\rho u^\sigma - \frac{1}{4}(R_{ij}{}^\sigma \varepsilon^{\rho ijk} + R_{ij}{}^\rho \varepsilon^{\sigma ijk}) s_k\right] \tag{104}$$

as it is straightforward to see. Employing (102) gives

$$E^{\rho\sigma}=2\phi^2 m\cos\beta u^\sigma u^\rho+\tfrac{1}{2}\phi^2\Big[2Y^\sigma s^\rho+2Y^\rho s^\sigma-2Y_k(s^{[k}u^{\sigma]}u^\rho+s^{[k}u^{\rho]}u^\sigma)+$$
$$+2Z_k s_j u_i(u^\rho\varepsilon^{kji\sigma}+u^\sigma\varepsilon^{kji\rho})-B^\sigma s^\rho-B^\rho s^\sigma-\tfrac{1}{2}(R_{ij}{}^{\sigma}\varepsilon^{\rho ijk}+R_{ij}{}^{\rho}\varepsilon^{\sigma ijk})s_k\Big] \tag{105}$$

which is general. In macroscopic limit $s_j\to 0$ we obtain

$$P^\mu\approx m u^\mu\cos\beta \tag{106}$$

as well as

$$E^{\rho\sigma}\approx 2\phi^2 m\cos\beta u^\sigma u^\rho \tag{107}$$

with torsion decoupling from the spinor, so that we can neglect it. The full energy with electrodynamics is

$$T^{\rho\sigma}\approx 2\phi^2 m\cos\beta u^\sigma u^\rho+\tfrac{1}{4}F^2 g^{\rho\sigma}-F^{\rho}{}_{\alpha}F^{\sigma\alpha} \tag{108}$$

and because $\nabla_\rho T^{\rho\sigma}=0$, then

$$q2\phi^2 u_\alpha F^{\sigma\alpha}=m2\phi^2 u^\eta\nabla_\eta(u^\sigma\cos\beta) \tag{109}$$

having used Maxwell equations and the conservation of the electrodynamic current. Simplifying the module and employing again (102), we eventually obtain

$$u^\eta\nabla_\eta P^\sigma=qF^{\sigma\alpha}u_\alpha \tag{110}$$

which is the Lorentz force in the Newton law. Notice that we have never used any assumption on the module being localized in order to obtain the macroscopic approximation, which means that, in the present derivation, all points and not only the peak of the matter distribution do follow the classical trajectory. In absence of electrodynamics,

$$u^\eta\nabla_\eta P^\sigma=0 \tag{111}$$

is the Newton law. Therefore the mass can be simplified, obtaining the equivalence principle, and if $\beta\to 0$, then we have $u^\eta\nabla_\eta u^\sigma=0$ identically, which is merely the geodesic equation. Furthermore, the entire derivation could have been obtained also without the macroscopic approximation, in which case we would obtain the particle trajectories with corrections due to the quantum potentials. The full form of the final expressions is too complicated to be insightful, but even without them one can already understand that they constitute the guiding equation. In fact, after having solved for P^σ, we can use

$$P^\sigma=m\frac{d}{ds}x^\sigma \tag{112}$$

and solve for $x^\sigma=x^\sigma(s)$, giving the position of the particle in terms of the length parameter s and which is therefore the trajectory of the particle, as in the de Broglie–Bohm theory [24].

As we had already mentioned, the above derivation was based on no assumption regarding the localization of the matter distribution, which made it more general than the Ehrenfest theorem on the classical limit. However, it also allows a novel definition of particles that does not require them to be the manifestation of a localized module. Furthermore, in fact in the de Broglie–Bohm theory, particles are not the peak of the module, but yet another entity that has to be postulated independently and which rides on the module according to the guiding equation. This interpretation, however, is a weak point of the de Broglie–Bohm theory. In fact, in this case, the motion of a particle would be determined by the module, that is the wave function, which in principle depends on the configuration of all other particles; whereas, this link among all particles of the universe is non-local enough to ensure entanglement of all particles, this entanglement is mediated by the wave function, which is physical. Hence, any non-local behavior is also physical. The possibility that acausal propagation may be observable creates some compatibility issue with relativity. Some attempts to have these problems circumvented by writing the de Broglie–Bohm theory in relativistic form were made immediately by Bohm himself [25] and later by them and co-workers [26]; although, full covariance was not granted for many-particle systems (further attempts to solve these issues are more recent [27,28], but there does not seem to be a general consensus yet). Consequently, here, we would like to take a different route, one that does not involve considering the supplementary idea of particles, however many they may be. We will instead interpret particles as the manifestation of a localized module. The task is then to find different ways to explain correlation between states. So, in the following, there are then two things we will have to do. One is to justify somehow how the module can be localized. The other is explaining how two states can be linked non-locally, but in full compatibility with relativity.

The first of these two problems may be treated by considering that in full, the Dirac equations also contain the torsion of the space–time. Consider then the Dirac equations given in the following alternative form:

$$\nabla_\mu \ln\phi^2 - G_\mu + 2ms_\mu \sin\beta = 0 \tag{113}$$

$$\nabla_\mu\beta - 2XW_\mu - K_\mu + 2ms_\mu \cos\beta = 0 \tag{114}$$

with $G_\mu = -R_\mu + 2P^\rho u^\nu s^\alpha \varepsilon_{\mu\rho\nu\alpha}$ and $K_\mu = -B_\mu + 2P^\iota u_{[\iota} s_{\mu]}$ as yet another type of potentials. Via the straightforward manipulation of these equations, we can obtain

$$\nabla^\mu\left(\phi^2\nabla_\mu\beta\right) - (8X^2M^{-2}\phi^2 m\sin\beta + 2XW\cdot G + \nabla_\mu K^\mu + K_\mu G^\mu)\phi^2 = 0 \tag{115}$$

$$|\nabla\beta/2|^2 - m^2 - \phi^{-1}\nabla^2\phi + \tfrac{1}{2}(\nabla_\mu G^\mu + \tfrac{1}{2}G^2 - \tfrac{1}{2}K^2 - 2XW\cdot K - 2X^2W^2) = 0 \tag{116}$$

the first being a continuity equation and the second being a Hamilton–Jacobi equation. Particularly interesting for us is the HJ equation for $\beta \to 0$, because in this case

$$\nabla^2\phi + X^2W^2\phi + XW\cdot K\phi - \tfrac{1}{2}(\nabla_\mu G^\mu + \tfrac{1}{2}G^2 - \tfrac{1}{2}K^2)\phi + m^2\phi = 0 \tag{117}$$

with W_μ left explicitly. As for the field equations for the propagating torsion field [29], they can be taken in their effective approximation $M^2W^\mu = 2X\phi^2 s^\mu$ which, upon a direct substitution, furnish the effective HJ equations

$$\nabla^2\phi - 4X^4M^{-4}\phi^5 + 2X^2M^{-2}K\cdot s\phi^3 - \tfrac{1}{2}(\nabla_\mu G^\mu + \tfrac{1}{2}G^2 - \tfrac{1}{2}K^2 - 2m^2)\phi = 0 \tag{118}$$

which are now written in the form of Klein–Gordon equations for the module. They are nonlinear with negative sign for the highest-order potential, which make them candidate equations for soliton solutions. So, localized modules are dynamically justified by torsion.

However, in practice, it is very difficult to actually find solutions of such nonlinear field equations, though some approximated solutions can be found, such as those of [18], or those of [19,20]. Either way, the solution are localized and regular matter distributions.

To face the issue of entanglement, we begin by recalling some general features of the theory presented so far which may be of help. First of all, as it is well known, the Dirac equations contain the spinor field and its dynamical properties but also the tetradic fields. These tetrad fields are important for two reasons. A first is that, without them, we cannot write the spinor equation, highlighting how much the spinor fields are sensitive to the underlying structure of the background. Another is that tetrads contain more information as compared with the metric within the same background. In a given background of an assigned metric, a basis of tetrads have a richer structure which can be felt by spinor fields. Secondly, both information about frame of reference and gravitational effects are generally found within tetrad fields; although, only gravity can be found in the curvature tensor and henceforth determined by field equations with a source. So, the information about pure geometry that can be found inside non-trivial tetrads in flat space–time remains undetermined. Genuine geometric effects in tetrads have no propagation, and no acausal behavior can be imputed to them. Non-local actions are therefore not forbidden.

In encoding what we can know about physics, tetrads complement the information contained in the spinor field and without being predetermined. Flat space–time does not imply that pure geometric effects cannot be present, and in fact the tetrads can still be non-trivial, entering in the Dirac equations in a way that can have an impact on the spinor field. So, in the following, we will work out some consequences of a toy model based on an exact solution of the Dirac equations in a perfectly flat space–time.

Consider then the Minkowski metric, thus zero connection and flat space–time. We can write tetrads and spin connection as those found in [18–20]. Whatever its form, a solution is in general constituted by an assigned module and Yvon–Takabayashi angle that have to solve (98) and (99) in a specific background that is given. Equivalently, we can also write the spinor field according to the form

$$\psi = \phi e^{-\frac{i}{2}\beta\pi} e^{-iq\alpha} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \tag{119}$$

solving (97) for a specific set of tetradic fields that is also given as background. These are general results [18,19]. For this solution, however, it is also possible to assign a very special alternative form that is given for the same module and Yvon–Takabayashi angle. Quite simply it is

$$\psi = \phi e^{-\frac{i}{2}\beta\pi} e^{-iq\alpha} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \tag{120}$$

corresponding to the very same material distribution but with an opposite spin. This is not a surprise because it is well known that spinors have two basic spin orientations, as wanted by the Pauli principle. The two solutions above differ from (78) for the fact that they have been taken in their rest frame and spin aligned along the third axis, as also customary. However, nonetheless, one might wonder what additional information could be encoded within the $\boldsymbol{L}^{-1}$ matrix. To keep things simple, we will still remain in the rest frame. However, there we consider a rotation of the form

$$\boldsymbol{L}^{-1} = \begin{pmatrix} \cos\zeta/2 & \sin\zeta/2 & 0 & 0 \\ -\sin\zeta/2 & \cos\zeta/2 & 0 & 0 \\ 0 & 0 & \cos\zeta/2 & \sin\zeta/2 \\ 0 & 0 & -\sin\zeta/2 & \cos\zeta/2 \end{pmatrix} \tag{121}$$

with $\zeta = \omega t$ and ω constant. The appearance of such new term determines the appearance of an additional

$$\boldsymbol{L}^{-1}\partial_t \boldsymbol{L} = -\omega\boldsymbol{\sigma}_{13} \tag{122}$$

so that (91) yields

$$\partial_t \xi_{13} = -\omega \tag{123}$$

as Goldstone mode of this state. Because of (93), we have that $\Omega_{13t} = \partial_t \xi_{13}$ and, consequently, we obtain

$$\Omega_{13t} = -\omega \tag{124}$$

as additional component of the spin connection. As it is clear, there is no contribution to the curvature, for which we still have flatness. The general form of the spinor (78) in both the above cases (119) and (120) is therefore

$$\psi = \phi e^{-\frac{i}{2}\beta\pi} e^{-iq\alpha} \left(\begin{array}{c} \cos\zeta/2 \\ -\sin\zeta/2 \\ \cos\zeta/2 \\ -\sin\zeta/2 \end{array}\right) \tag{125}$$

having $s^3 = \cos\zeta$ alongside to

$$\psi = \phi e^{-\frac{i}{2}\beta\pi} e^{-iq\alpha} \left(\begin{array}{c} \sin\zeta/2 \\ \cos\zeta/2 \\ \sin\zeta/2 \\ \cos\zeta/2 \end{array}\right) \tag{126}$$

with $s^3 = -\cos\zeta$ therefore showing the opposition of the two spin orientations. Then, while maintaining opposite orientation, both spins display a flipping that depends on ζ over time. The Goldstone state has no dependence on spatial coordinates and it will remain the same even when the two solutions have space-like distance. Now, suppose that a measurement be performed on the first solution so to force it to collapse onto the stateof definite spin. If we perform a measurement fixing the first spinor to its form

$$\psi = \phi e^{-\frac{i}{2}\beta\pi} e^{-iq\alpha} \left(\begin{array}{c} 1 \\ 0 \\ 1 \\ 0 \end{array}\right) \tag{127}$$

then $s^3 = 1$, hence showing that the spin is in the up-configuration. Since this state would still have to be solution of the Dirac equation, the spin connection collapses onto the case in which we have that $\omega = 0$, and because this is a constant, it will remain in this state, so that

$$\Omega_{13t} = 0 \tag{128}$$

and since the spin connection is uniquely defined as background, this must also be the value of the spin connection of the second state. As we want this state to still be a solution of the Dirac equation, then

$$\psi = \phi e^{-\frac{i}{2}\beta\pi} e^{-iq\alpha} \left(\begin{array}{c} 0 \\ 1 \\ 0 \\ 1 \end{array}\right) \tag{129}$$

with $s^3 = -1$ and so that the spin is now in the down-configuration. Summarizing, forcing the first solution into a spin-up state implies, through the $\omega = 0$ condition, that a spin-down state be fixed for the second solution, and the full process can take place no matter how distant the two solutions are. Notice that such process would have been exactly the same if we had the first solution collapse onto the spin-down state and the second solution collapse onto the spin-up state. This uniform spin flip guarantees lack of predetermination in spin orientation, and thus results are statistically distributed as it is necessary in quantum mechanics. Yet, a measurement fixing one spin also fixes the other spin, and it does so immediately. This process is mediated by the spin connection, and in particular by the component that arises as Goldstone state $\boldsymbol{L}^{-1}\partial_\nu \boldsymbol{L}$ in the structure of the spinor field. This degree of freedom does not encode physical interactions since it gives rise to no contribution in the curvature, and therefore it can not be determined by any field equation. So, the information that is transferred between the two spinors through their common Goldstone state is not restricted to be causal as it does not have any propagation in the first place. Hence, compatibility with the principles of relativity is clear.

As an example, let us next try to apply such a concept for a specific solution, that is that of [18], of which we will consider only the exterior branch. We will have two wave functions with an opposite spin orientation. Furthermore, we will work in spherical coordinates for compactness. Of these two opposite-spin wave functions, the first that we shall consider is given according to the following expression

$$\psi = \frac{K}{r\sqrt{\sin\theta}} e^{-r\sqrt{\varepsilon(2m-\varepsilon)}} e^{-it(m-\varepsilon)} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \tag{130}$$

with tetrads

$$e_0^t = \cosh\alpha \quad e_2^t = -\sinh\alpha \tag{131}$$
$$e_1^r = -1 \tag{132}$$
$$e_3^\theta = \frac{1}{r} \tag{133}$$
$$e_0^\varphi = -\frac{1}{r\sin\theta}\sinh\alpha \quad e_2^\varphi = \frac{1}{r\sin\theta}\cosh\alpha \tag{134}$$

giving spin connection

$$\Omega_{13\theta} = -1 \tag{135}$$
$$\Omega_{01\varphi} = -\sin\theta\sinh\alpha \tag{136}$$
$$\Omega_{03\varphi} = \cos\theta\sinh\alpha \tag{137}$$
$$\Omega_{12\varphi} = -\sin\theta\cosh\alpha \tag{138}$$
$$\Omega_{23\varphi} = -\cos\theta\cosh\alpha \tag{139}$$

where $\sinh\alpha = \sqrt{\varepsilon(2m-\varepsilon)}/(m-\varepsilon)$ with $m > \varepsilon > 0$ and K a generic constant. This corresponds to the spin-up case and it is a solution of the Dirac equations. Similarly, it is possible to consider the alternative wave function

$$\psi = \frac{K}{r\sqrt{\sin\theta}} e^{-r\sqrt{\varepsilon(2m-\varepsilon)}} e^{-it(m-\varepsilon)} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \tag{140}$$

with tetrads

$$e_0^t = \cosh\alpha \quad e_2^t = -\sinh\alpha \tag{141}$$
$$e_1^r = 1 \tag{142}$$
$$e_3^\theta = -\frac{1}{r} \tag{143}$$
$$e_0^\varphi = -\frac{1}{r\sin\theta}\sinh\alpha \quad e_2^\varphi = \frac{1}{r\sin\theta}\cosh\alpha \tag{144}$$

giving spin connection

$$\Omega_{13\theta} = -1 \tag{145}$$
$$\Omega_{01\varphi} = \sin\theta \sinh\alpha \tag{146}$$
$$\Omega_{03\varphi} = -\cos\theta \sinh\alpha \tag{147}$$
$$\Omega_{12\varphi} = \sin\theta \cosh\alpha \tag{148}$$
$$\Omega_{23\varphi} = \cos\theta \cosh\alpha \tag{149}$$

where $\sinh\alpha = \sqrt{\varepsilon(2m-\varepsilon)}/(m-\varepsilon)$ with $m > \varepsilon > 0$ and K generic constant. This corresponds to the spin-down case and it is a solution of the Dirac equations. As easy to see these solutions are square-integrable (albeit their energy has a logarithmic divergence near the origin of the radial coordinate). By applying now the rotation (121), we will obtain that the first spinor becomes of the form (125) as

$$\psi = \frac{K}{r\sqrt{\sin\theta}} e^{-r\sqrt{\varepsilon(2m-\varepsilon)}} e^{-it(m-\varepsilon)} \begin{pmatrix} \cos\zeta/2 \\ -\sin\zeta/2 \\ \cos\zeta/2 \\ -\sin\zeta/2 \end{pmatrix} \tag{150}$$

with the real representation of (121) inducing the corresponding rotation on the tetrads

$$e_0^t = \cosh\alpha \quad e_2^t = -\sinh\alpha \tag{151}$$
$$e_1^r = -\cos\zeta \quad e_3^r = -\sin\zeta \tag{152}$$
$$e_1^\theta = -\frac{1}{r}\sin\zeta \quad e_3^\theta = \frac{1}{r}\cos\zeta \tag{153}$$
$$e_0^\varphi = -\frac{1}{r\sin\theta}\sinh\alpha \quad e_2^\varphi = \frac{1}{r\sin\theta}\cosh\alpha \tag{154}$$

and hence on the spin connection

$$\Omega_{13t} = -\omega \tag{155}$$
$$\Omega_{13\theta} = -1 \tag{156}$$
$$\Omega_{01\varphi} = -\sin(\theta+\zeta)\sinh\alpha \tag{157}$$
$$\Omega_{03\varphi} = \cos(\theta+\zeta)\sinh\alpha \tag{158}$$
$$\Omega_{12\varphi} = -\sin(\theta+\zeta)\cosh\alpha \tag{159}$$
$$\Omega_{23\varphi} = -\cos(\theta+\zeta)\cosh\alpha \tag{160}$$

while the second spinor becomes of the form (126) as

$$\psi = \frac{K}{r\sqrt{\sin\theta}} e^{-r\sqrt{\varepsilon(2m-\varepsilon)}} e^{-it(m-\varepsilon)} \begin{pmatrix} \sin\zeta/2 \\ \cos\zeta/2 \\ \sin\zeta/2 \\ \cos\zeta/2 \end{pmatrix} \tag{161}$$

with the real representation inducing the corresponding rotation on the tetrads

$$e_0^t = \cosh\alpha \quad e_2^t = -\sinh\alpha \tag{162}$$
$$e_1^r = \cos\zeta \quad e_3^r = \sin\zeta \tag{163}$$
$$e_1^\theta = \frac{1}{r}\sin\zeta \quad e_3^\theta = -\frac{1}{r}\cos\zeta \tag{164}$$
$$e_0^\varphi = -\frac{1}{r\sin\theta}\sinh\alpha \quad e_2^\varphi = \frac{1}{r\sin\theta}\cosh\alpha \tag{165}$$

and hence on the spin connection

$$\Omega_{13t} = -\omega \tag{166}$$
$$\Omega_{13\theta} = -1 \tag{167}$$
$$\Omega_{01\varphi} = \sin(\theta+\zeta)\sinh\alpha \tag{168}$$
$$\Omega_{03\varphi} = -\cos(\theta+\zeta)\sinh\alpha \tag{169}$$
$$\Omega_{12\varphi} = \sin(\theta+\zeta)\cosh\alpha \tag{170}$$
$$\Omega_{23\varphi} = \cos(\theta+\zeta)\cosh\alpha \tag{171}$$

and they are both solutions of the Dirac equations. Hence, we see thatboth wave functions display the above uniform rotation, with the spin connection that has generated the additional component $\Omega_{13t} = -\omega$ exactly as we discussed above. Notice that such a component does not depend on the variables of the system, but note also that it is not an absolute constant. The independence on the position of the particle means that the dynamics will remain the same even if the two particles were separated. However, any observation breaking the rotation by fixing $\omega = 0$ will have the effect of producing the collapse of both spinors simultaneously. In fact, suppose that an observation were performed at a time for which more or less $\omega t = 2n\pi$, then a solution (150)–(160) would be (130)–(139) plus the $\Omega_{13t} = -\omega$ condition and (161)–(171) as (140)–(149), plus the $\Omega_{13t} = -\omega$ condition. Now, if the system were disturbed so that $\omega = 0$, then the rotation would stop, simultaneously locking the first solution to the spin-up state and the second solution to the spin-down state. If we had about $\omega t = 2n\pi + \pi$, then solution (150)–(160) would be (140)–(149) plus the $\Omega_{13t} = -\omega$ condition and (161)–(171) as (130)–(139) plus the $\Omega_{13t} = -\omega$ condition. Now, if the system were disturbed so that $\omega = 0$, then the rotation would stop, simultaneously locking the first solution to spin-down states and the second solution to spin-up states. This is what we had discussed.

Contrary to the de Broglie–Bohm interpretation, where, as already mentioned, entanglement is due to observable degrees of freedom, and thus non-local effects are real, here the correlation of two observables occurs through the Goldstone degrees of freedom, which have no local restriction given that their propagation is not restricted by anything. With the original terminology [30–32], we may say that, in the de Broglie–Bohm interpretation, non-local hidden variables are the positions of the particles, while here the non-local hidden variables are the Goldstone state of the spinorial field contained within the $\boldsymbol{L}$ matrix.

The single measurement is also completely determined through the knowledge of the parameter $\zeta = \omega t$ and ultimately on t, but this requires the knowledge of the initial time t_0 as boundary condition. Knowledge of this boundary condition is therefore the condition in terms of which of all possible states only a special state is selected hence entailing a form of symmetry breaking. It is of particular type as it occurs for a special state. Furthermore, again it occurs only for that special state due to specific types of boundary conditions.

Analogies with the previous case are found in the fact that both types of symmetry breaking are specific to one given wave function. Differently from the previous case, where symmetry breaking meant choosing one solution among many, here, symmetry breaking means choosing a specific observable property for a given solution.

5. Conclusions

In this paper, we have considered symmetry breaking occurring in two situations, universally and particularly, and we have discussed these ideas in terms of three possible examples. The first was about cosmological effects of the standard model of particle physics. We recalled the way in which symmetry gets broken by the Higgs vacuum and by the choice of the specific gauge field configuration selected by the Higgs isospin. Since these selections occur throughout the universe, symmetry breaking is universal.

The second case was a situation that was formally analogous to the one of the Higgs field. We have shown how, similarly to the Higgs field, general spinor fields are written in polar form. For them, we identified the Goldstone states and showed that they are absorbed as longitudinal components in the P_α and $R_{ij\alpha}$ tensors. So, we have seen that there arises a specific spin axial vector and we have recalled how its direction selects a specific spinor field as solution to the Dirac equations. Because the selection of this solution is due to boundary conditions that are valid for such a case solely, symmetry breaking is particular.

The third case was an entirely different situation, that was regarding the process of measurement within general quantum mechanical systems. After having presented the version of the de Broglie–Bohm formalism that was written in a wholly relativistic form and in presence of spin, we had shown a toy model in which a pair of spin-up and spin-down states were found to possess a uniform rotation that could have been maintained over large distances but which could also be made to collapse for one spin state instantly forcing a collapse of the other spin state. This toy model for pairs of entangled spins does not have compatibility problems with relativity because, in it, the correlation of two states is ensured by the component of the spin connection arising from the Goldstone degrees of freedom of the spinor field, which have a peculiar property. They are given by the term $\boldsymbol{L}^{-1}\partial_\nu \boldsymbol{L}$, and, as such, they do not contribute to the curvature tensor, showing that they cannot carry any gravitational information but only frame-related types of information; whereas, the gravitational information would go into the curvature and, as such, it would have to verify Einstein equations ensuring the causal propagation of all gravitational degrees of freedom, where information about frames does not go in the curvature. So, for it, there is no field equation restricting its propagation, while an interaction mediated by some physical field would have to respect physical locality, entanglement—as described by Goldstone states—does not have to obey constraints. In other words, even if locality must be ensured for all fields that are the solutions of field equations, not all fields are solutions of field equations. There might be non-local objects even in a full relativistic environment. Furthermore, employing them to have a description of a non-local action is compatible with relativity. In our toy model, they are the Goldstone states of spinor fields, and they are recognized as non-local hidden variables. These are fixed by boundary conditions, such as the time t_0 that makes the wave function collapse onto a single state. As the boundary conditions pick only one state, the resulting symmetry breaking is of course of particular type.

To conclude this discussion about symmetry breaking types, we would like to stress that the example of universal type and the first example of particular type seem to be more alike than the two examples of particular type. In fact, the example of universal type and the first example of particular type are different for the fact that in the former the symmetry group is among different fields, while, in the latter, the symmetry group is among different components of the same field—apart from this they are analogous in all respects.

The two examples of particular types are different in the fact that, while in the former, the choice of the boundary conditions is made for β and ϕ^2, in the latter, the choice of the boundary conditions is made for the Goldstone states of the spinor fields; although the Dirac equations specify the physical properties of the wave function, no equation can determine the propagation of its Goldstone states. Hence, one type of boundary conditions is the usual type needed to specify the solution of a given field equation and the other type of boundary conditions is a new type that makes the wave function collapse onto a special state.

In our toy model, the property in exam was the orientation of the spin axial vector, which cannot be determined by field equations and yet it is necessary to establish the results of spin observations. This analysis points toward the importance of Goldstone states of a spinor field in encoding information that is not fully determined and yet observable.

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