First Results on the Revealing of Cognate Ancestors among the Particles of the Primary Cosmic Rays That Gave Rise to Extensive Air Showers Observed by the GELATICA Network

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Abstract: For the data on the observation times and directions of the motion of extensive air showers, which are observed at two stations of the GELATICA network, for the first time we apply the method we have developed previously for identifying pairs of mutually remote extensive air showers, the ancestor particles of which arose, possibly, in a single process. A brief description of the GELATICA network, a review of the properties of used samples of data on shower observations at two stations of the network during the 2019–2021 session, and the result of applying the above method to them are given. Some properties of a single peculiar pair of remote showers are discussed. A side question arose about the cause of the observed temporal asymmetry in the locations of the regions of mutual approach of independent primary cosmic ray particles.

Keywords: cosmic ray ensembles; large scale primary cosmic ray correlations; extensive air showers; pairs of showers; proximity estimations; CREDO collaboration

1. Introduction

The majority of current primary cosmic ray (PCR) research has been focused on the detection and analysis of cosmic particles observed through individual detectors or arrays. Meanwhile, the correlated PCR observations on the global scale remain almost unexplored. Indeed, primary ultra-high-energy cosmic ray (UHECR) particles can initiate cascades already above Earth’s atmosphere, forming the so-called cosmic ray ensembles (CRE)—the phenomena composed of at least two PCR particles, including photons, with a common primary interaction vertex. So, the CRE can be observed as a set of remote extensive air showers (EAS or simply “showers”) generated by ancestor particles with correlated arrival directions and arrival times. The properties of the CREs, particles of which may spread over a significant part of Earth’s area, are indeterminate. Such ensembles could be generated both within well-known processes (e.g., produced in photon-photon or proton-nucleus interactions) and exotic scenarios (e.g., as a result of decay of super-heavy dark matter (DM) particles and subsequent interactions).

Some fundamental problems can be investigated by studying the CRE properties, such as the nature of DM and the explanation of the existence of PCR particles with energies...
exceeding $10^{20}$ eV. The first approach in explanation of their existence can be reduced to usual scenarios describing acceleration processes, in which the PCR particles are accelerated by some particular astrophysical object, e.g., second-order Fermi acceleration, shock waves in interstellar relativistic plasma, unipolar induction mechanisms, etc. However, the CRE investigation can be useful also in the study of unconventional physical scenarios, e.g., interactions of exotic super-heavy matter, extra dimensions, Lorentz invariance violation, cosmic strings, the existence of new particles, influence of parallel worlds, etc. [1–12].

For effective investigation of these fundamental problems in terms of CRE properties, sufficient statistical data on observations of such ensembles are obviously required. These data enable consideration of the phenomenology of the CRE effect, at least. Though, at the current research stage, only a few statements on observations of time-contiguous showers are available [13–18]. Only after a sufficient number of such ensembles of separated showers have been obtained, will it be possible to directly investigate the CRE phenomenology by studying the statistical properties of the resulting set of selected PCR ensembles. At the current preliminary stage of research, it is premature to speculate about the methods that will be useful to ascertain the required phenomenology of CRE, since this will significantly depend both on the results of the data acquisition on the CRE properties and on the future understanding of the problem on a global scale.

The CRE components might be extended over very large areas, as illustrated in Figure 1, and this specific property might make them all unachievable for detection by existing detector systems, observing the EAS, and operating in isolation. Meanwhile, if the detectors are operating as part of a global network, these CREs are naturally more likely to be observed.

**Figure 1.** The cosmic ray ensemble is composed of the cognate primary cosmic ray particles. They are ancestors of the extensive air showers generated in Earth’s atmosphere. Some of them can be observed by ground-based cosmic ray stations, which can measure their arrival times and movement directions.

Both the large PCR arrays (such as the Pierre Auger Observatory [19], Telescope Array [20], and other astrophysical observatories) and smaller arrays of detectors sparsely located around the world can be important for CRE-oriented studies. There are some networks of small separate installations overlapping large areas and applicable for CRE investigations: HiSPARC [21] in The Netherlands, QuarkNet [22] and CHICOS [23] in the USA, CZELTA [24] in the Czech Republic, ALTA [15] in Canada, EEE [25] in Italy, LAAS [26] in Japan, and TAIGA-HiSCORE [27] in RF.

Additionally, our network called GELATICA (GEorgian Large-area Angle and Time Coincidence Array) [28] operates in Georgia.
Therewith, a new international association investigating the problem, i.e., the Cosmic-Ray Extremely Distributed Observatory (CREDO) \cite{29,30}, has been organized some years ago. The GELATICA network has been part of the CREDO Memorandum of Understanding \cite{31} since 2019.

2. Materials and Methods

2.1. The Method of Ascertainment of the Possible Historical Proximity of Two Remote Showers

Every station of the GELATICA network can estimate both arrival time and front movement direction of any observable EAS generated above the installation by the PCR particle with sufficient energy. (From this time onward the EAS-observing installations with such possibilities will be referred as “EAS goniometers” or simply “goniometers”) During the simultaneous observations of showers by the network’s installations, the data samples of the mentioned values are collected about the showers traversing every goniometer. The possible cosmic ray ensembles can be revealed among the PCR particles which have generated the remote showers observed by the network’s goniometers by determining the historical proximity (that is of possible “affinity”) of the ancestor particles of both showers in every pair of showers associated with the possible CRE. The method of ascertainment of the possible historical proximity of two remote showers has been proposed earlier in our papers \cite{32,33}.

The required measure of the desired historical proximity of two showers is determined as the minimal distance between the two moving ancestor PCR particles. These movements are considered with respect to some inertial frame of reference. So, the distance between the simultaneous positions of two ancestor particles is a space-like Lorentz invariant. The data required for estimation of this minimal distance are available from the showers’ observation results. These data are:

1. The observation times \(t\) and the dispersions \(\sigma_\tau^2\) of their estimations;
2. The coordinates \(r\) of positions of the showers’ cores at the observation moments and the covariance matrices \(M\) of their estimations;
3. The unit directional vectors \(n\) of the shower fronts’ movement directions and the covariance matrices \(D\) of their estimations.

These data allow estimation of two useful values: the time moment \(\tau\) of the closest approximation of two ancestor particles (the timing begins at the average moment of both EAS observations):

\[
\tau = -\frac{\delta r^T \delta n}{c \sqrt{\delta n^T \delta n}}, \text{ c-speed of light} \quad (1)
\]

and the vector \(\Delta\) connecting the ancestor particles at this moment:

\[
\Delta = \delta r - <n> (c \delta t) - \delta n (\delta r^T \delta n)/c \cdot (\delta n^T \delta n) \quad (2)
\]

Here, the difference values are used:

\[
\delta t = t_2 - t_1, \quad \delta r = r_2 - r_1, \quad \delta n = n_2 - n_1
\]

and the vector \(<n> = (n_2 + n_1)/2\) defines the average movement direction for both ancestors. The indexes numerate the two showers observed by two remote goniometers. Right, the length of the vector \((2) \Delta = |\Delta|\) is used as the required measure of the two-shower historical proximity. The method of estimation of the respective dispersion \(\sigma^2\) is explained in previous papers \cite{32,33}.

Since the cognate ancestor particles of two observed showers have their closest approach moment before the earliest shower arrival moment detected by two goniometers, the Lorentz invariant dimensionless kinematic parameter of temporal sequencing:

\[
S = \text{arsinh} \left( c \left( \frac{\tau + |\delta t|/2}{|\delta r|} \right) \right) \quad (3)
\]
applicable for this pair of the observed showers, has to be negative. Then, the minimal distance \( \Delta \) between the ancestor particles must be sufficiently small in comparison with the estimation of standard deviation \( \sigma_\Delta \) of this distance value. That is why the Lorentz invariant dimensionless kinematic parameter of spatial proximity:

\[
p = \ln(k_\sigma \cdot \sigma_\Delta / \Delta)
\]  

has to be positive for really or possibly cognate pair of ancestor particles. Here, the optional coefficient \( k_\sigma \) adjusts the strictness of the proximity definition. The value of \( k_\sigma \) coefficient has to be assigned under the agreement.

At last, the affinity criterion is defined with the use of the two latest aggregating kinematic parameters as a value proportional to the angle between the proximity axis and the direction to the \((P,S)\) point from \((0,0)\) point:

\[
K = \left( \frac{2}{\pi} \right) \cdot \text{angle} \ (P,S)
\]  

Obviously, it varies in the range \( 0 \leq K \leq 4 \) and its value can belong to three zones with meanings specified below:

- **Zone 1**: \( 0 < K \leq 2 \)—Unrealizable ancestors approach in the future (they are absorbed already),
- **Zone 2**: \( 2 < K \leq 3 \)—Unreliable ancestors proximity in the past (the value of \( \Delta \) is too big in comparison with \( \sigma_\Delta \)),
- **Zone 3**: \( 3 < K \leq 4 \)—Possibly historically related ancestors of the two showers,

The application of this method of cognate showers revelation to the data on extensive air showers observation was obtained by GELATICA network in 2019–2021 in sessions by two remote goniometers, as shown below.

### 2.2. Short Description of the GELATICA Network

The goniometers of the GELATICA network are located in Georgia in the roof spaces of several universities in Tbilisi and Telavi. Their locations are shown in Figure 2 and specified in Table 1, while the mutual distances are listed in Table 2.

![Figure 2. GELATICA net of EAS observing stations in Georgia. The locations of goniometers AIP, GTU, HEP, and TEL are shown. Distances between these goniometers are listed in Table 2.](image)
Table 1. Location of goniometers of the GELATICA network.

<table>
<thead>
<tr>
<th>City</th>
<th>Goniometer’s Designation</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tbilisi</td>
<td>AIP</td>
<td>E. Andronikashvili Institute of Physics (Tbilisi State University), building II</td>
</tr>
<tr>
<td></td>
<td>GTU</td>
<td>Georgian Technical University, building IV</td>
</tr>
<tr>
<td></td>
<td>HEP</td>
<td>High Energy Physics Institute (Tbilisi State University)</td>
</tr>
<tr>
<td>Telavi</td>
<td>TEL</td>
<td>J. Gogebashvili Telavi State University, building I</td>
</tr>
</tbody>
</table>

Table 2. Distances between goniometers of the GELATICA network.

<table>
<thead>
<tr>
<th></th>
<th>GTU</th>
<th>HEP</th>
<th>TEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIP</td>
<td>(2792 ± 9) m</td>
<td>(2727 ± 9) m</td>
<td>(63,613 ± 9) m</td>
</tr>
<tr>
<td>GTU</td>
<td>(5519 ± 8) m</td>
<td>(60,945 ± 8) m</td>
<td></td>
</tr>
<tr>
<td>HEP</td>
<td>(5519 ± 8) m</td>
<td>(60,945 ± 8) m</td>
<td>(66,346 ± 8) m</td>
</tr>
</tbody>
</table>

The measurements at a typical installation are performed using a set of four 0.25 m$^2$ scintillation detectors located in a common horizontal plane at the corners of approximately 10 m × 10 m (up to 13 m × 13 m) side square, with the use of a standard QuarkNet Data Acquisition Card [34], i.e., by means of the so-called planar EAS goniometer. This is a version of the well-known and extensively used method suggested in 1953 by B. Rossi et al. [35]. This type of EAS station has no estimation possibility either of the shower’s energy or of the shower core’s position.

The Global Positioning System (GPS) component of the equipment used allows the EAS front arrival moment via Universal Coordinated Time (UTC) estimation with $\sigma_t = 1 \mu s$ accuracy. The scintillation detector system measures the relative time delays of EAS charged particles’ passages through the detectors in increments of 1.25 ns. This information allows estimation of the directing unit vector of the tangential plane of the EAS front (which coincides with unit vector $\mathbf{n}$ of the shower front’s movement direction within the estimation error) with the corresponding covariance matrix $D$ of this vector’s components estimations. The method of estimation used for calculations of the directing vector $\mathbf{n}$ and covariance matrix $D$ for any observed shower is described in [36].

It should be emphasized that these values are defined with respect to the local Cartesian horizontal coordinate system (HCS), specific for each goniometer, with the axes directed (locally) to the east (x-axis), north (y-axis), and to the Zenith (z-axis).

The typical error of EAS movement direction estimation usually achieved by the GELATICA goniometers [37–39] (integral of statistical and systematic error estimations in respect of the equivalent spherical angles) is approximately $3^\circ \times \sec(\theta)$ for the zenith angle estimation and $3^\circ \times \csc(\theta)$ for the azimuth angle estimation, where $\theta$ is a zenith angle. (Certainly, every shower event has its own individual estimation of the covariance matrix $D$ of the components of the unit directional vector’s estimations.) These average errors are slightly different for individual installations. All details on the individual features of existing and formerly operating goniometers are available on the GELATICA site [28], together with the method of their estimation. The main calculated characteristics of the goniometers AIP and TEL. Observations data used in this paper are shown in Table 3.
Table 3. Main properties of the GELATICA goniometers AIP and TEL.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>AIP</th>
<th>TEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geographic latitude $\psi_o$</td>
<td>41.720329° N</td>
<td>41.910301° N</td>
</tr>
<tr>
<td>Geographic longitude $\lambda_o$</td>
<td>44.744092° E</td>
<td>45.468106° E</td>
</tr>
<tr>
<td>Local altitude $h_o$</td>
<td>495 m a.s.l</td>
<td>845 m a.s.l</td>
</tr>
<tr>
<td>Approximate dimensions</td>
<td>13.0 m × 13.0 m</td>
<td>10.2 m × 11.2 m</td>
</tr>
<tr>
<td>Upright atmospheric mass depth</td>
<td>972.8 g/cm²</td>
<td>936.4 g/cm²</td>
</tr>
<tr>
<td>Air density at actual location</td>
<td>1167 g/m³</td>
<td>1129 g/m³</td>
</tr>
<tr>
<td>Total number of the observed showers</td>
<td>27,147</td>
<td>230,178</td>
</tr>
<tr>
<td>Estimated observation rate of the showers</td>
<td>$(2.66 \pm 0.1)$ h⁻¹</td>
<td>$(24.5 \pm 0.8)$ h⁻¹</td>
</tr>
</tbody>
</table>

In spite of the very similar total dimensions of both goniometers, the difference in total number of the observed EAS is striking. The major cause of this discrepancy is the constructional dissimilarity between the types of the scintillation detectors used in both installations, inducing the difference in the threshold of average sensitivity of all detectors. So, the calculated lowest energies of the observable showers differ significantly. That is why the ratio of the estimated observation rates of the observed showers is close to 9, generating the mentioned difference in the total number of the observed showers.

2.3. Choice of the Single Common Inertial Coordinate System and Complete Estimated Data Representation in this Special Reference Frame

As it is mentioned previously, the method of ascertainment of the possible historical proximity of two remote showers uses all necessary data defined in a single inertial system. Since all exploiting goniometers are located on the Earth’s surface in different points, the required choice of the common reference frame must take into account both this difference in location and the total motion of the Earth. The influence of the gravitational field onto the relativistic motions is negligible within the solar system. As well the variation of Earth’s velocity in the motion around the Sun within short time intervals, considerable for the current research, is negligible. All the more, the motion of the Solar System in the galaxy is negligible. So, only the proper rotation of the Earth with respect to the “fixed stars” is essential. That is, the Cartesian analog of the usual astronomic Equatorial Coordinate System (ECS) should be used. Moreover, since the connection of EAS events with the directions in the space is not significant in the present research, the initial orientation of the currently constructing reference frame is optional. So, the initial moment of 1 November 2019 is taken as the zero moment of this investigation and Earth’s orientation at this moment is taken as the initial one.

So, the origin $O$ of ECS is located in Earth’s center. The first axis $OX$ is directed to the geographic point $(\psi_x = 0°, \lambda_x = 0°)$, the second axis $OY$ is directed to the geographic point $(\psi_y = 0°, \lambda_y = 90°)$, and the third axis $OZ$—to the North Pole $(\psi_z = 90°, \lambda_z = 0°)$. At the initial coordinate time moment $t_0 = 0$, every local position on Earth’s surface coincides with its geographic coordinates $(\psi_o, \lambda_o)$. Any time value is a UTC moment measured by the GPS equipment. At any moment $t$, the coordinates of the same point are

$$\psi(t) = \psi_o; \lambda(t) = \lambda_o + \Omega_E \cdot t$$
Here, the angular velocity $\Omega_E$ of the Earth’s rotation with respect to the ECS frame is used. In accordance with the International Earth Rotation Service (IERS) [40], it has a nominal magnitude:

$$\Omega_E = 72.92115146064 \times 10^{-6} \text{ rad/s}$$

Here, neither precession of the Earth nor accidental variations of Earth’s rotation, etc., are taken into account as the significant time intervals are sufficiently small. Neither the estimation errors of this value should be taken into account as the estimation errors of values measured by the used installations are much bigger.

At the moment, $t$ the Cartesian coordinates of a point with fixed geographic coordinates $(\psi_o, \lambda_o)$ located at the altitude $h_o$ are:

$$\mathbf{r}(\psi_o, \lambda_o, h_o, t) = \begin{pmatrix} R_o(\psi_o, h_o) \cdot \cos \psi_o \cdot \cos \lambda(t) \\ R_o(\psi_o, h_o) \cdot \cos \psi_o \cdot \sin \lambda(t) \\ R_o(\psi_o, h_o) \cdot \sin \psi_o \end{pmatrix}$$

Here, $R_o(\psi_o, h_o) = \sqrt{R^2_{\text{equ}} \cos^2 \psi_o + R^2_{\text{pol}} \sin^2 \psi_o + h_o}$ is the distance of the mentioned point from Earth’s center, and the equatorial and polar radii of Earth’s ellipsoid parameters by the IERS [40] are:

$$R_{\text{equ}} = 6,378,137 \text{ m}; \quad f_c = 1/298.257223563; \quad R_{\text{pol}} = R_{\text{equ}}(1 - f_c) \approx 6,356,752 \text{ m};$$

Every unit directing vector $\mathbf{n}$ of the shower’s front motion and the covariance matrix $\mathbf{D}$ of its components are estimated with respect to the local HCS, while the EAS occurrence properties must be represented in the common global Equatorial Coordinate System (ECS).

The UTC time estimation can be used as a common coordinate time in this global ECS as the relativistic correction is negligible due to sufficiently low velocity of Earth’s rotating surface. For calculation of the ECS-components of the vector $\mathbf{n}$ and covariance matrix $\mathbf{D}$ the time-dependent rotation matrix $\mathbf{W}$, specific for every observation point $(\psi_o, \lambda_o)$, should be used:

$$\mathbf{W}(\lambda_o, \psi_o, t) = \begin{pmatrix} -\sin(\lambda(t)) & -\sin(\psi_o) \cos(\lambda(t)) & \cos(\psi_o) \cos(\lambda(t)) \\ \cos(\lambda(t)) & -\sin(\psi_o) \sin(\lambda(t)) & \cos(\psi_o) \sin(\lambda(t)) \\ 0 & \cos(\psi_o) & \sin(\psi_o) \end{pmatrix}$$

Therefore, the required ECS components of measured quantities are:

$$\mathbf{n}(\mathbf{n}; \lambda_o, \psi_o, t) = \mathbf{W}(\lambda_o, \psi_o, t) \cdot \mathbf{n}$$

$$\mathbf{D}(\mathbf{D}; \lambda_o, \psi_o, t) = \mathbf{W}(\lambda_o, \psi_o, t) \cdot \mathbf{D} \cdot \mathbf{W}^{-1}(\lambda_o, \psi_o, t)$$

For the desired estimations of sequencing (3) and proximity (4) parameters, the ECS coordinates $\mathbf{r}$ of both showers’ core positions at the observation moments and their components’ covariance matrixes $\mathbf{M}$ are required. Meanwhile, the used planar goniometers have no possibility of estimating of the shower’s core position. Since the distance between the AIP and TEL goniometer ([63,613 ± 9] m, see Table 2) is much greater than the calculated values of the root mean square distances to the shower cores $\rho_c$ for both goniometers (150 m and 165 m, Table 3), the own (time-dependent) positions of the goniometers may be used to approximate the shower’s core ECS positions $\mathbf{r}$. The local HCS covariance matrix of components of this vector can be approximated as

$$\mathbf{M} = \begin{pmatrix} \rho_c^2 / 2 & 0 & 0 \\ 0 & \rho_c^2 / 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
and the ECS components of this value are calculated as:

\[ M\left(M;\lambda_o,\psi_o,t\right) = W\left(\lambda_o,\psi_o,t\right) \cdot M \cdot W^{-1}\left(\lambda_o,\psi_o,t\right) \]

All local HCS data on the EAS observations obtained during the 2019–2021 session by AIP and TEL goniometers have been recalculated into the common global ECS representation. The main features of these observations are shown in Table 4, while the short display of the data in this last form is shown further.

**Table 4.** Main properties of EAS observations’ set obtained by GELATICA goniometers AIP and TEL during the 2019–2021 session.

<table>
<thead>
<tr>
<th>Obtained Quantity</th>
<th>AIP</th>
<th>TEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total numbers of the observed showers</td>
<td>27,147</td>
<td>23,017</td>
</tr>
<tr>
<td>The overall time of simultaneous observations</td>
<td>9329 h</td>
<td>388.7 days</td>
</tr>
<tr>
<td>The numbers of the observed showers during the simultaneous observations’ period</td>
<td>24,325</td>
<td>213,997</td>
</tr>
<tr>
<td>Estimated observation rates of the showers upon the simultaneous observations’ period</td>
<td>(2.61 ± 0.02) h(^{-1})</td>
<td>(22.9 ± 0.5) h(^{-1})</td>
</tr>
</tbody>
</table>

Spreading of the EAS arrival directions, observed by AIP and TEL goniometers, are shown in Figure 3 on the celestial sphere in the mentioned special equatorial coordinate system. The shower arrival direction’s declination and hour angle correspond to unit vector (−n), opposite to the shower’s movement direction with respect to the ECS.

![Figure 3. Spreading of the EAS arrival directions shown on the celestial sphere in the equatorial coordinate system.](image-url)

Next, Figures 4 and 5 display the histograms of the ECS declinations’ and hour angles’ distributions of the arrival directions of all showers observed during the 2019–2021 session by AIP and TEL goniometers. The arrival directions are distributed almost evenly in hour angles due to the uniform rotation of the Earth, while the declination distributions are determined by the shower’s arrival directions’ distributions in zenith angles [37,38] defined in both local HCSs. They are concentrated closely around both goniometer locations’ geographic latitudes due to EASs’ arrival concentration near Zenith in the local reference frame. The declination distributions are limited in the south direction on account of the EASs’ restriction by the local horizons.
3. Results

Search for Possibly Cognate Ancestors of Extensive Air Showers among the Observed Pairs of Remote Showers

Only the showers observed in sufficiently short periods can have cognate ancestors. That is why the remote showers observed by turns with AIP and TEL goniometers within the time interval of 60 s are selected for detection of shower pairs with possibly cognate ancestor particles. A total of 18,523 pairs appeared in this selected sample. (The whole number of pairs of remote showers during the period of simultaneous observations is...
The values of temporal sequencing $S(3)$ and spatial proximity $P(4)$ are calculated for each pair in this sample in concordance with [32,33] (the magnitude $k_\sigma = 3$ is taken for the optional coefficient $k_\sigma$ in the definition (4) of the proximity $P$ parameter). Their positions on the $(P, S)$ plane are shown in Figure 6, together with histograms of the affinity criterion $K(5)$.

Figure 6. (a) Positions of all selected pairs of showers on the plane of Lorentz invariant parameters $(P, S)$ describe the possible historical proximity of their ancestors. (b) The respective histogram of the showers’ affinity criterion $K$. Every bin in this histogram is enlarged by unit for improvement of visualization. The set of $(P, S)$ points is limited at big magnitudes of parameter $S$ because of the 60 s restriction on the showers’ observation time difference used for the selection of shower pairs in this research. The position of the only pair of showers whose ancestors have approached each other before both showers’ observations is shown in blue. Zone 1 (brown): unrealizable ancestors’ approach in the future (they are absorbed already); Zone 2 (blue): unreliable ancestors’ proximity in the past; Zone 3 (green): possibly historically proximal ancestor particles of the two showers.

Not a single pair of showers with possibly related ancestral particles is observed. Moreover, only one pair in this sample of pairs has ancestors approached before the showers’ observations, though the approaching distance $\Delta$ is too big in comparison with the respective standard deviation. The properties of this single peculiar EAS pair are discussed below. In regard to other pairs in the sample, only bunches of neutrinos generated during the EASs’ developments have the opportunity to approach each other after the showers’ observations. Indeed, some pairs of neutrino bunches even closely approach in the future, at $(p > 0, S > 0)$, since their proximity parameter $P$ in Figure 6 reaches the magnitude of 3.2, and this is quite a reliable proximity.

4. Discussion

Despite the fact that in the obtained sample of shower pairs there is not a single pair that could be considered as a possible manifestation of the CRE phenomenon, the only stand-out pair of showers (Figure 6), in which the ancestor particles approached before the observation of the showers themselves, deserves separate consideration. Estimates of the properties of the showers of this pair are listed in Table 5. It is noteworthy that they are observed at the same instant of time, with an accuracy of 1 $\mu$s. Using the known average rates of shower observations by both goniometers (Table 4), it is easy to estimate that the expected probability of observing a pair of independent showers during a small time interval of 1 $\mu$s is only $5 \times 10^{-18}$. Really actualized pair of showers with such a rare possibility can indeed be considered peculiar.
Table 5. Estimated main features of two showers forming the single peculiar EAS pair observed by GELATICA goniometers AIP and TEL during the 2019–2021 session.

<table>
<thead>
<tr>
<th>Estimated Quantity</th>
<th>AIP</th>
<th>TEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation day</td>
<td>23 February 2021</td>
<td>23 February 2021</td>
</tr>
<tr>
<td>Observation UTC time</td>
<td>$t_1 = 29,064.3925780$ s</td>
<td>$t_2 = 29,064.3925780$ s</td>
</tr>
<tr>
<td>Estimated directional unit vectors of the EASs’ front movement</td>
<td>$n_1 = \begin{pmatrix} -0.3536 \ 0.6956 \ -0.6254 \end{pmatrix}$</td>
<td>$n_2 = \begin{pmatrix} -0.0432 \ 0.5234 \ -0.8510 \end{pmatrix}$</td>
</tr>
<tr>
<td>Covariance matrixes of estimations of the components of the directional unit vectors</td>
<td>$D_1 = \begin{pmatrix} 125 &amp; 35 &amp; -32 \ 35 &amp; 72 &amp; 60 \ -32 &amp; 60 &amp; 85 \end{pmatrix} \cdot 10^{-6}$</td>
<td>$D_2 = \begin{pmatrix} 12574 &amp; 1144 &amp; 66 \ 1144 &amp; 9569 &amp; 5827 \ 66 &amp; 5827 &amp; 3581 \end{pmatrix} \cdot 10^{-6}$</td>
</tr>
<tr>
<td>Coordinates of the observation points (they are opposite vectors definitionally)</td>
<td>$r_{AIP} = \begin{pmatrix} -30.002 \ -10.560 \ -0.141 \end{pmatrix}$ km</td>
<td>$r_{TEL} = \begin{pmatrix} 30.002 \ 10.560 \ 0.141 \end{pmatrix}$ km</td>
</tr>
<tr>
<td>Local directional unit vectors of the EASs’ front movement</td>
<td>$n_{AIP} = \begin{pmatrix} -0.2322 \ 0.0306 \ -0.9722 \end{pmatrix}$</td>
<td>$n_{TEL} = \begin{pmatrix} -0.0450 \ -0.2854 \ -0.9574 \end{pmatrix}$</td>
</tr>
<tr>
<td>Equivalent average angular error of the direction estimations</td>
<td>0.7°</td>
<td>6.5°</td>
</tr>
</tbody>
</table>

However, the reconstruction of the trajectories of the ancestor particles of this pair, shown in Figure 7, illustrates that in reality, these EASs’ ancestors are clearly independent. The kinematic parameters of the considered pair of showers are presented in Table 6. The ancestors of these showers approached to a distance of $(59.1 \pm 16.5)$ km in the stratopause of Earth’s atmosphere at altitudes of about 54 km, moving at an angle of about 24.3° to each other. The reconstructed time dependence of the distance between EAS ancestors in this pair is shown in Figure 8.

Figure 7. Spatial arrangement of the ancestor particle trajectories of the single peculiar pair of showers in the sample under study. Approximate cones of 90% reliability in estimation of the arrival directions of the showers’ ancestors are shown. The images of emerging EASs are displayed conventionally distorted for better visibility. The red color line connects the positions of the ancestor particles at the moment of closest approach. (yellow - the error coordinates of the observation points, green - only for visual contrast).
Table 6. Aggregating kinematic parameters of the single peculiar EAS pair observed by GELATICA goniometers AIP and TEL during the 2019–2021 session.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations’ day</td>
<td>23 February 2021</td>
</tr>
<tr>
<td>Observations’ common UTC time</td>
<td>( t = 29,064.392578 \text{ s} )</td>
</tr>
<tr>
<td>The interval of time from the ancestors’ closest approach till the common observations’ moment</td>
<td>( \tau = (188 \pm 24) \text{ (\mu\text{s}) } )</td>
</tr>
<tr>
<td>The Lorentz invariant spacelike interval of the ancestors’ closest approach</td>
<td>( \Delta = (59.1 \pm 16.5) \text{ km} )</td>
</tr>
<tr>
<td>The value of the temporal sequencing parameter</td>
<td>( S = -0.798 )</td>
</tr>
<tr>
<td>The value of the spatial proximity parameter (with use of optional coefficient ( k_\alpha = 3 ))</td>
<td>( P = -0.174 )</td>
</tr>
<tr>
<td>The value of the affinity criterion</td>
<td>( K = 2.863 )</td>
</tr>
</tbody>
</table>

Figure 8. Time dependence of the distance between the ancestor particles of both showers in the observed single peculiar pair of showers. The inset shows on an enlarged scale the vicinity of the time of observation of both showers.

The approach of the ancestor particles occurred at the time moment \( \tau = -(188 \pm 24) \text{ \(\mu\text{s}\) } \) before the observation of both showers. During this time, each ancestor walked \((56.3 \pm 7.3) \text{ km}\). In Figure 8, one can also notice that the error in estimating the distance \( \Delta \) of the closest approach of the ancestors is greater than the error in the variable distance between moving particles at the same moment. This is due to the fact that time, the argument of the last function, is defined exactly, while the error in estimating the time interval \( \tau \) is also implicitly involved in the estimation of the error in the quantity \( \Delta \).

All of the above merely demonstrates the obvious possibility of observing a very close temporal approach of observations of completely independent showers. However, the kinematic properties of the remaining pairs of showers in the sample under consideration, characterized by locations of the points depicting them on the plane \((P, S)\) (Figure 6), reveal a strange feature of the sample under investigation. Indeed, for all observed pairs of showers (with the exception of the only pair just considered), the approach occurred after the observations of showers. (Of course, only the neutrino bunches generated during the developments of both EASs in the Earth’s atmosphere are already approaching). Such a strange asymmetry in the temporal location of the regions of approaches is perplexing. Indeed, if there had been no Earth in the path of all these ancestor particles, then the real PCR particles would have approached. If the density of cosmic rays is spatially homogeneous and moving directions of PCR particles are really isotropic, it is difficult to...
explain why they approach each other only after a conditional, failed observation. Perhaps this is an implicit effect of the conditions of EAS observations or of the selection of pairs of showers, or even some kind of geometric selection.

5. Conclusions

The study of the obtained sample of EAS pairs observed independently by the mutually remote goniometers AIP and TEL (they are the members of the GELATICA network) confirmed that the desired pairs of showers, which could form parts of cosmic ray ensembles, are extremely rare. A long continuous process of showers’ observations at as many remote cosmic ray stations as possible is needed to estimate both the times of EASs observations and the directions of their motion. The method of ascertainment of the desired historically proximal showers proposed in [32,33] proved to be effective in solving the problem of searching for manifestations of the CRE effect. However, a side question arose about the cause of the observed temporal asymmetry in the locations of the regions of the mutual approach of independent PCR particles. It can be expected that this is a manifestation of some implicit conditions for EAS observations or of the selection of shower pairs for investigation.

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References


